Lista de exercícios 2 (do livro de Boyce e Di Prima)

Problems

In each of Problems 1 through 11:

a. Seek power series solutions of the given differential equation about the given point x_0; find the recurrence relation that the coefficients must satisfy.

b. Find the first four nonzero terms in each of two solutions y_1 and y_2 (unless the series terminates sooner).

c. By evaluating the Wronskian $W[y_1, y_2](x_0)$, show that y_1 and y_2 form a fundamental set of solutions.

d. If possible, find the general term in each solution.

1. $y'' - y = 0, \quad x_0 = 0$

2. $y'' + 3y' = 0, \quad x_0 = 0$

3. $y'' - xy' - y = 0, \quad x_0 = 0$

4. $y'' - xy' - y = 0, \quad x_0 = 1$

5. $y'' + k^2x^2y = 0, \quad x_0 = 0, \;\; k$ a constant

6. $(1 - x)y'' + y = 0, \quad x_0 = 0$

7. $y'' + xy' + 2y = 0, \quad x_0 = 0$

8. $xy'' + y' + xy = 0, \quad x_0 = 1$

9. $(3 - x^2)y'' - 3xy' - y = 0, \quad x_0 = 0$

10. $2y'' + xy' + 3y = 0, \quad x_0 = 0$

11. $2y'' + (x + 1)y' + 3y = 0, \quad x_0 = 2$

In each of Problems 12 through 14:

a. Find the first five nonzero terms in the solution of the given initial-value problem.

G **b.** Plot the four-term and the five-term approximations to the solution on the same axes.

c. From the plot in part b, estimate the interval in which the four-term approximation is reasonably accurate.

12. $y'' - xy' - y = 0, \quad y(0) = 2, \quad y'(0) = 1;$ see Problem 3

13. $y'' + xy' + 2y = 0, \quad y(0) = 4, \quad y'(0) = -1;$ see Problem 7

14. $(1 - x)y'' + xy' - y = 0, \quad y(0) = -3, \quad y'(0) = 2$

15. **a.** By making the change of variable $x - 1 = t$ and assuming that y has a Taylor series in powers of t, find two series solutions of

$$y'' + (x - 1)^2y' + (x^2 - 1)y = 0$$

in powers of $x - 1$.

b. Show that you obtain the same result by assuming that y has a Taylor series in powers of $x - 1$ and also expressing the coefficient $x^2 - 1$ in powers of $x - 1$.

16. Prove equation (10).

17. Show directly, using the ratio test, that the two series solutions of Airy's equation about $x = 0$ converge for all x; see equation (20) of the text.

18. **The Hermite Equation.** The equation

$$y'' - 2xy' + \lambda y = 0, \quad -\infty < x < \infty,$$

where λ is a constant, is known as the Hermite[5] equation. It is an important equation in mathematical physics.

a. Find the first four nonzero terms in each of two solutions about $x = 0$ and show that they form a fundamental set of solutions.

b. Observe that if λ is a nonnegative even integer, then one or the other of the series solutions terminates and becomes a polynomial. Find the polynomial solutions for $\lambda = 0, 2, 4, 6, 8$, and 10. Note that each polynomial is determined only up to a multiplicative constant.

c. The Hermite polynomial $H_n(x)$ is defined as the polynomial solution of the Hermite equation with $\lambda = 2n$ for which the coefficient of x^n is 2^n. Find $H_0(x), H_1(x), \ldots, H_5(x)$.

19. Consider the initial-value problem $y' = \sqrt{1 - y^2}$, $y(0) = 0$.

a. Show that $y = \sin x$ is the solution of this initial-value problem.

b. Look for a solution of the initial-value problem in the form of a power series about $x = 0$. Find the coefficients up to the term in x^3 in this series.

In each of Problems 20 through 23, plot several partial sums in a series solution of the given initial-value problem about $x = 0$, thereby obtaining graphs analogous to those in Figures 5.2.1 through 5.2.4 (except that we do not know an explicit formula for the actual solution).

G **20.** $y'' + xy' + 2y = 0, \;\; y(0) = 0, \;\; y'(0) = 1;$ see Problem 7

G **21.** $(4 - x^2)y'' + 2y = 0, \quad y(0) = 0, \quad y'(0) = 1$

G **22.** $y'' + x^2y = 0, \quad y(0) = 1, \quad y'(0) = 0;$ see Problem 5

G **23.** $(1 - x)y'' + xy' - 2y = 0, \quad y(0) = 0, \quad y'(0) = 1$

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[5] Charles Hermite (1822–1901) was an influential French analyst and algebraist. An inspiring teacher, he was professor at the École Polytechnique and the Sorbonne. He introduced the Hermite functions in 1864 and showed in 1873 that e is a transcendental number (that is, e is not a root of any polynomial equation with rational coefficients). His name is also associated with Hermitian matrices (see Section 7.3), some of whose properties he discovered.