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Recent advances on notch effects in metal fatigue: A review

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Abstract

Notch features including holes, fillets, shoulders, and grooves commonly exist in engineering components. When subjected to external loads, these geometrical discontinuities generally act as stress raisers and thus present significant influences on the component strength and life, which are more remarkable under complex loading paths. Accordingly, numerous theories and approaches have been developed to address notch effects in metal fatigue as well as damage modelling and life predictions, which aim to provide theoretical support for structural optimal design and integrity evaluation. However, most of them are self-styled or focus on specific objects, which limits their engineering applicability. This review recalls recent developments and achievements in notch fatigue modelling and analysis of metals. In particular, four commonly used methods for fatigue evaluation of metallic notched components/structures are summarized and elaborated, namely, nominal stress approaches, local stress-strain approaches, and critical distance theories and weighting control parameters-based approaches, which intend to provide a reference for further research on notch fatigue analysis and promote the integration and/or development among different approaches for practice.

KEYWORDS

fatigue, local stress-strain approach, nominal stress approach, notch, theory of critical distance, weighting control parameter

1 | INTRODUCTION

With technical advances, higher requirements are raised during the design of critical components within major equipment, like engine turbines. Nowadays, design guidelines include such requirements, but they are not limited to ensuring the operational safety of these components during service. Specifically, the physical tear and wear of components with the environmental influence of structures or machines is also considered for the optimal design.[1,2] According to different functional requirements of engineering components, environmental concerns demand eco-friendly material usage but at same time achieving costs reduction and requiring better balanced designs of structures and machines by introducing empirical or semi-empirical safety factors.[3-5] In order to meet the above-mentioned design requirements, it is critical to achieve the accurate calculation of fatigue strength or lifetime, rather than using empirical methods or life divergence coefficients, which might greatly reduce the optional design space for engineering components.[6-9] This is particularly important when considering the notch effect on the fatigue resistance of components, which varies considerably with notch geometric features.[10-13] Under external loads, geometric discontinuities unavoidably raise stress concentrations as well as complex stress-strain responses within the structure, in which the structural geometry suddenly changes (see Figure 1) and thus result in complex fatigue problems and induce surface fatigue cracks, even under conditions of inconspicuous plastic deformation.[14-16] According to this, the complete understanding of notch effect is pivotal to ensure safety and durability of engineering components.

To provide a quantitative description for stress concentration severity of notched components, the concept of theoretical stress concentration factor K_t is defined by:

$$K_t = \sigma_{\max}/S, \tag{1}$$

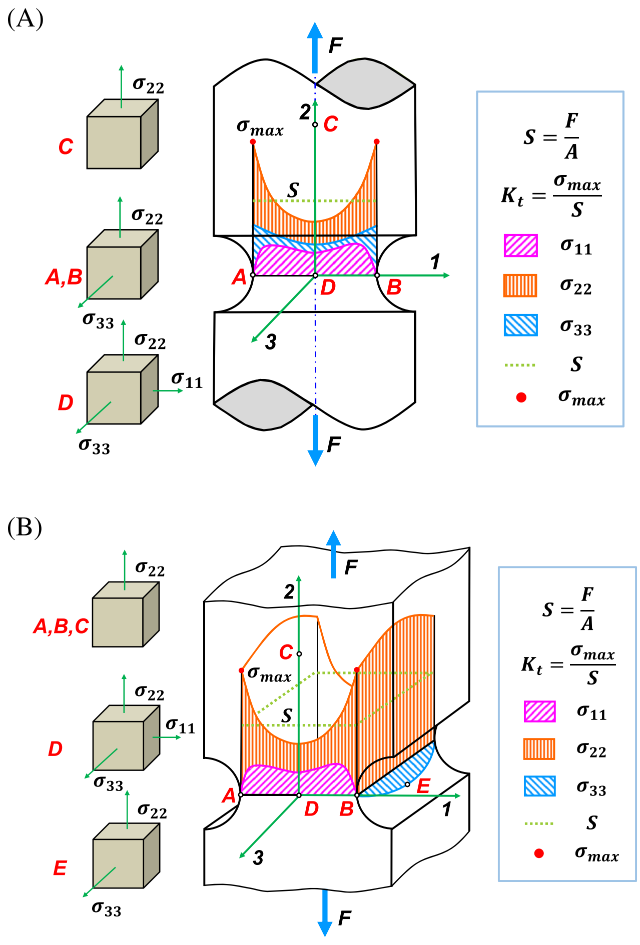

FIGURE 1 Geometrical interpretation of stress distributions under uniaxial loadings: (a) axisymmetric notched body and (b) prismatic notched body [Colour figure can be viewed at wileyonlinelibrary.com]

where $\sigma_{\max}$ and S are the maximum stress at notch tip and the nominal normal stress under pure elastic state, respectively. However, it is widely accepted that the use of K_t yields nonconservative predictions during fatigue lifetime evaluation of ductile materials or structures with sharp notches.[17]

As is shown in Figure 1, because of the inhomogeneous stress distribution, when regions near the surface fleetly exceed the material yield strength and plastic deformation occurs, neighbouring materials further inside the body still support the highly stressed area since they are less stressed, often in elastic regime, and thus slows the progression of crack initiation, propagation, and eventual fracture. The above-mentioned phenomenon explains why actual fatigue lives of notched parts under cyclic loadings are generally longer than those predicted by utilizing the peak stress-strain data. Moreover, components with a stepped decreasing stress at the notch root can withstand more fatigue damage than those with a smaller stress gradient.[18] Figure 2 presents the *S-N* curves of smooth and notched TC4 specimens. Note that K_t cannot effectively characterize the relationship between fatigue strengths of smooth and notched specimens (notch effect), since it generally yields conservative predictions on the endurance of engineering components under cyclic loadings.

In view of these cases, the fatigue strength reduction factor K_f is introduced,[17] which depicts the influence of notch effect on fatigue strength of interest:

$$K_f = S_e/S_n, \tag{2}$$

where S_e and S_n are the fatigue strengths of smooth and notched specimen, respectively. Note from Figure 2 that K_f varies with fatigue lifetime. Moreover, values of K_f are different under diverse stress ratios.[20] Generally, K_f is calculated from empirical equations based on statistics, crack growth, or reversed yielding.[17] Herein, three commonly used formulae for K_f are given below, which can be utilized for life assessment of notched components with simple geometries under uniaxial loadings[20]:

$$\text{Seibel}: \quad K_f = \frac{K_t}{1+\sqrt{1+a_1\chi}} \tag{3}$$

$$\text{Perterson}: \quad K_f = 1+\frac{K_t-1}{1+a_2/\rho}, \tag{4}$$

$$\text{Neuber}: \quad K_f = 1+\frac{K_t=1}{1+\sqrt{a_3/\rho}} \tag{5}$$

where χ is the relative stress gradient[8] and a_1, a_2, and a_3 are material constants for Seibel, Peterson, and Neuber

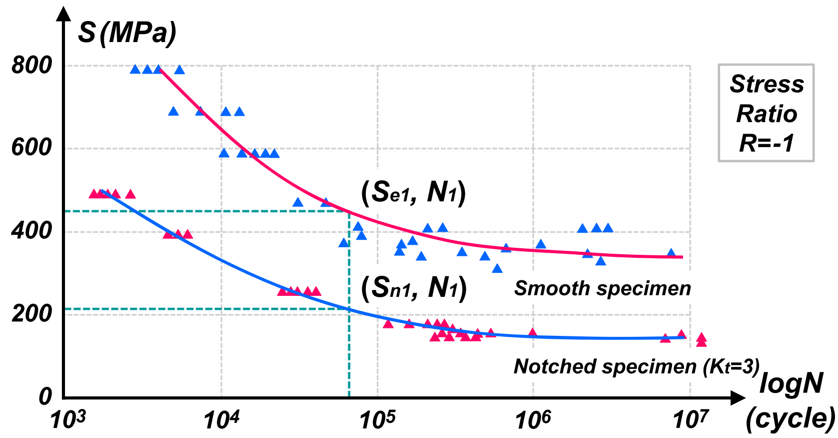

FIGURE 2 *S-N* curves of smooth and notched TC4 specimens[19] [Colour figure can be viewed at wileyonlinelibrary.com]

models, respectively; ρ is the radius of notch root. In addition, Qylafku et al[1] and Yao[20] summarized the earlier formulae of K_f for fatigue life assessment.

Recently, Mäde et al[21,22] coupled the Coffin-Manson-Basquin (CMB) equation and Seibel's empirical formula into probabilistic low cycle fatigue (LCF) risk evaluation of turbine vanes. Specifically, the notch support factor n_χ is utilized directly in the form of $1+A_\chi\times\chi^{k_\chi}$, where A_χ and k_χ are material-dependent notch support parameters. Considering that the relative stress gradient obtained in the boundary element cannot effectively describe the impact of stress gradient on fatigue strength, Liao et al[8] defined χ_{elem} to consider the inhomogeneous stress distribution in the boundary elements. In addition, they developed a notch support factor n'_χ based on stress gradient to reflect and describe the notch support effect under multiaxial stress-strain states, which can be coupled with other multiaxial fatigue criteria and has been utilized for fatigue strength assessment of a compressor blade-disc attachment subjected to given load spectra.

However, although the empirical formulae of K_f enable quick assessment of fatigue strength of notched components, they are after all empirical, thus their universality among diverse structures and materials requires further verification. Moreover, they are generally applicable only in uniaxial cases. Except for the traditional empirical approaches mentioned above, recently, great developments have been made on accounting for fatigue notch effects. However, most of them constitute classes by themselves or aim at specific objects and thus hinder communication between approaches; improving their robustness and then establishing a general analytical framework are still lacking and desired. To help academics and engineers understand and grasp developing status and trends in notch fatigue assessment, this review summarizes recent progress in this field. In particular, methods for notch fatigue analysis are introduced from the nominal stress approaches (NSAs) (see Section 2) to local stress-strain approaches (LSSAs) (see Section 3), critical distance theory and its variants (see Section 4), and then to weighting control parameter-relevant approaches (see Section 5). Finally, further discussion and conclusions are given in Sections 6 and 7, respectively.

2 | NOMINAL STRESS APPROACHES

NSAs are established under the assumption that two parts manufactured from the same material own identical fatigue life if they possess the same theoretical stress concentration factor K_t and nominal stress S,[20,23] as is illustrated in Figure 3.

When employing the NSA for fatigue life evaluation, stress-life (*S-N*) curves (see Figure 2) are normally needed. Specifically, there are two strategies[20]: (a) evaluating the fatigue lives of the notched components by introducing the *S-N* curves of notched specimens with the same K_t and (b) converting a smooth *S-N* curve into the notched specimens' *S-N* curves possessing the same K_t.

The first strategy can achieve precise predictions while its weak point is that massive experiments or a big database is required because of the complex geometric forms of notched components and diversified boundary conditions, which take time and increase costs. By employing the first strategy, Tanaka and Akiniwa[24] deduced the crack propagation law from the *S-N* curve in the very high cycle fatigue (VHCF) and further applied the crack propagation law to evaluate the influence of geometry, loading mode, and residual stresses on the *S-N* curves.

The second strategy, by contrast, benefits from its economic advantage and is widely used in engineering practice. Collins[25] corrected the fatigue endurance limit in the *S-N* curve by introducing the fatigue notch reduction factor K_f, which ensures the applicability of the modified

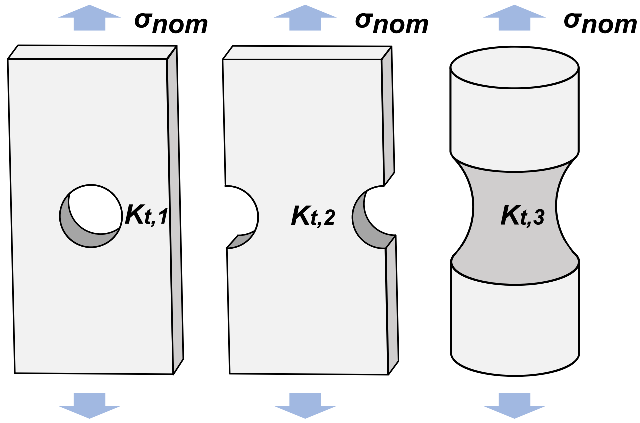

FIGURE 3 Schematic diagram of the nominal stress approach: specimens with same K_t ($K_{t,1} = K_{t,2} = K_{t,3}$) should present the same fatigue life [Colour figure can be viewed at wileyonlinelibrary.com]

S-N curve for notched components in NSA-based fatigue life assessment. Qiao and Xu[26] developed a two-parameter–based NSA by introducing the S_a (the amplitude of the nominal stress)-S_m (the mean value of the nominal stress)-N spatial curved surface equation for model optimization, the prediction ability of the new model considering the influence of mean stress and stress amplitude being improved. For coupling with the NSA, Liu et al[27] studied the effect of hole quality on the fatigue life of Al 2A12-T4 notched specimens with open holes. By using the *S-N* curves under symmetric loads with various K_t, the *S-N* curve with a certain K_t can be attained by polynomial interpolation, and finally the predicted life can be figured out by consulting the *S-N* curve with the stress amplitude S_a. However, the above-mentioned strategies of the NSA only serve uniaxial loadings, which ignore the influence of multiaxial loadings on fatigue life, especially under the nonproportional loading cases. Therefore, inaccurate results are obtained when applying the traditional NSA to multiaxial fatigue analysis. To widen the scope of the NSA for multiaxial loads (both proportional and nonproportional cases), employing S and K_t to form a proper equivalent nominal stress σ_{eq} is a commonly used strategy.[23]

On the basis of the von Mises yield criterion, Gough and Pollard[28] developed a new definition of the equivalent nominal stress under multiaxial loadings, known as the Gough-Pollard criterion:

$$\sigma_{eq} = \sqrt{(\sigma_{be} \cdot K_{tb})^2 + 3(\tau_n \cdot K_{tt})^2}, \tag{6}$$

where σ_{be} and τ_n are respectively the bending stress and the torsional stress in the net area; K_{tb} is the theoretical bending stress concentration factor, and K_{tt} is the theoretical torsional stress concentration factor. The Gough-Pollard criterion works well under uniaxial and multiaxial proportional loadings, while it fails to predict fatigue damage under nonproportional loading cases.

To further consider the impact of nonproportional hardening, Grubisic and Simbürger[29] introduced the phase difference θ for model correction, namely, the Grubisic-Simburger criterion:

$$\sigma_{eq} = \sqrt{1 + \frac{3}{4}K^2 + \sqrt{1 + \frac{3}{2}K^2\cos 2\theta + \frac{9}{16}K^4}}, \tag{7}$$

where $K = 2\tau_a/\sigma_a$, σ_a, and τ_a are the stress amplitudes resulting from bending and torsion in the notch root, respectively, which can be extracted by linear elastic finite element analysis (FEA). In particular, the Grubisic-Simburger criterion degenerates into the Gough-Pollard criterion under proportional loadings when $\theta = 0$.

Moreover, in research of the fatigue strength of weld joints made of fine-grained steels StE 290 and 460 under multiaxial loadings, Sonsino[30] considered the out-phase loading effect by incorporating the interplay of local shear stresses acting in diverse material surface planes. Besides, a size effect factor f_Q was added to characterize the size effects attributed to weld geometry and loading mode. Specifically, σ_{eq} is constructed as follows (Sonsino model):

$$\sigma_{eq}(\delta) = \sigma_{eq}(\delta = 0^\circ) \cdot \frac{\tau_{arith}(\delta)}{\tau_{arith}(\delta = 0^\circ)} \cdot \sqrt{G \cdot \exp\left[1 - \left(\frac{\theta - 90^\circ}{90^\circ}\right)^2\right]}, \tag{8}$$

where $\sigma_{eq}(\delta = 0^\circ) = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + f_Q^2 3\tau_{xy}^2}$; the effective shear stress $\tau_{arith} = \frac{1}{\pi}\int_0^\pi \tau_n(\varphi)d\varphi$; the size effect factor $f_Q = \frac{\sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y}}{\sqrt{3}\tau_{xy}}$; and the sliding modulus (the ratio of normalized stress gradients or stress concentration factors) $Q = \frac{1 + K_{ta}}{1 + K_{tt}}$ or $\frac{1 + K_{ta}}{1 + K_{tt}}$. A schematic diagram of the interference plane and stresses is presented in Figure 4.

Recently, by performing fatigue tests on thin-walled tubular specimens of LY12CZ aluminium alloys under diversiform multiaxial load paths, Wu and Wang[31] developed a new definition of σ_{eq}, as follows (Wu-Wang model):

$$\sigma_{eq} = \sigma(t) + \lambda \sin\theta\tau(t), \tag{9}$$

where λ is 0.5 under proportional loadings and 1 under nonproportional loadings. In detail, the Wu-Wang model

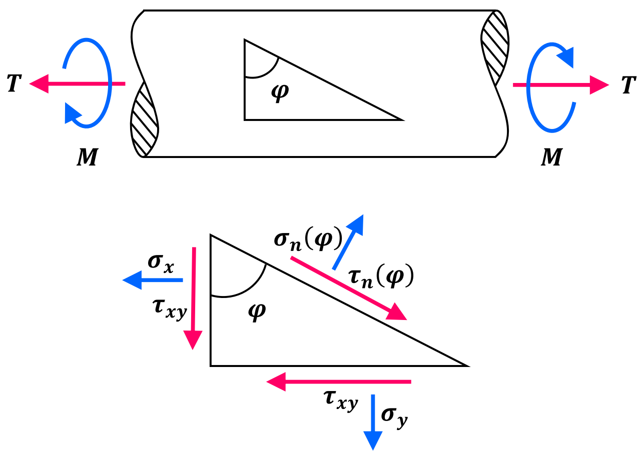

FIGURE 4 Interference plane and stresses [Colour figure can be viewed at wileyonlinelibrary.com]

can characterize the stress situations of notched specimens under various multiaxial load levels and accounts for the influence of the nonproportionality between the tensile and torsional loads on fatigue lives.

3 | LOCAL STRESS-STRAIN APPROACHES

Differing from the NSA, LSSAs consider that fatigue strength and lifetime of notched components are governed by the maximum local stresses and strains at the notch tip. The LSSAs have been developed[32] assuming that fatigue damage of components starts from the regions with the maximum local strain, and generally, plastic deformations appear before crack initiation. In other words, the local plastic deformation is a prerequisite for crack initiation and propagation. Therefore, the LSSA regards the maximum local stress-strain evaluation at the notch root as the determining factor for the fatigue strength of components. Such approaches assume that the number of loading cycles required for crack initiation of notched components and smooth specimens made of the same material are equal if the local stress-strain response at the notch tip of notched components is the same as that of smooth specimens.

The general process of LSSA-based fatigue lifetime assessment can be summarized as follows: (a) firstly, establish a relationship between the stress-strain response at the notch tip of the component and the load history; (b) then, apply the rain-flow cycle counting method to analyse the stress-strain history; (c) finally, the fatigue lifetime of the component is estimated by correlating an appropriate *S-N* curve.[33] It is worth noting that the accurate extraction of the local stress-strain response at the notch root is the vital step of this method.[20]

Until now, three strategies are commonly used to obtain the stress-strain histories at the notch root[34]: the experimental approaches, elasto-plastic FEA, and approximate calculation approaches. Among them, the experimental approaches can present objective and precise stress-strain relationships. However, for notched components, it is sometimes difficult to measure their strain using traditional apparatus like extensometers or strain gauges. Recently, noncontact full-field strain measurement systems have been developed (eg, video image correlation [VIC]-3D), which adopt the optimized 3D digital image correlation algorithm and support the measurement of the shape, displacement, and strain data in experiments.[35] But these systems only allow surface data to be extracted and processed.[36] Moreover, for complicated structures (turbine blades for instance), the installation of measuring equipment will be difficult.

Alternative experimental techniques to the VIC exist. Among them, Liew et al[37] measured notch root strains of WE43-T6 magnesium alloy specimens using electronic speckle pattern interferometry (ESPI); however, it also cannot avoid the limitations in the usage of VIC-3D systems. Specifically, acquisition of stress-strain states inside the body needs the support of FEA technique. In addition, both strains and stresses are required, but experimental techniques are devoted to displacement and strain measurements. Compared with the experimental approach, FEA provides precise calculation of stress-strain histories both on surface and inside the body. With the improvement of finite element technology and constitutive models, the computational accuracy improves, but more computing resources are required. This means longer analysis periods for complicated structures especially under complex loading conditions.

As an alternative to elasto-plastic FEA approach, approximate calculation approaches have found supports from engineers because of their simplicity in form and high efficiency.[38,39] Specifically, the main idea is to transform the nominal stress or load spectrum acting in the structure into the local stress-strain at critical regions via analytical elasto-plastic analysis or other strategies combined with a cyclic hysteresis loop.[40] Among them, the Neuber approximation formula is the most well-known in practice.[41] On the basis of the pure shear fatigue testing of prismatic notched parts, Neuber established a relationship between the theoretical stress concentration factor K_t and the geometrical mean of the elasto-plastic stress and strain concentration factors:

$$K_t = \sqrt{K_\sigma \cdot K_\varepsilon}, \tag{10}$$

where K_σ is the ratio of the local elasto-plastic stress at the notch root to the applied nominal stress, and K_ε is the ratio of the local elasto-plastic strain at the notch root to the applied nominal strain. Specifically, under pure elastic case, $K_\sigma = K_\varepsilon = K_t$.

Alternatively, as is presented in Figure 5, the Neuber rule can be rewritten in the form

$$\sigma^e \cdot \varepsilon^e = \sigma^a \cdot \varepsilon^a, \tag{11}$$

where σ^e and ε^e are the fictitious stress and strain under pure elastic states, respectively; σ^a and ε^a denote the real elasto-plastic stress and strain, respectively.

To reconstruct Equation (11), the Neuber rule can be transformed into the following form:

$$\sigma^a \cdot \varepsilon^a = \sigma^e \cdot \varepsilon^e = (K_t \cdot S) \cdot \left(K_t \cdot \frac{S}{E}\right) = \frac{K_t^2 S^2}{E} = C, \tag{12}$$

where E is the elastic modulus and $C = K_t^2 S^2 / E$ is the Neuber constant.

Generally, Equation (12) is expressed in terms of stress and strain ranges for stress/strain calculation in fatigue analysis[42]:

$$\Delta\sigma\Delta\varepsilon = \frac{K_t^2 \Delta S^2}{E} = C. \tag{13}$$

Topper et al[43] applied the Neuber rule to fatigue evaluation of notched parts subjected to uniaxial cyclic loadings, which led to a sound accordance to the fatigue tests.

Moftakhar et al[44] extended the Neuber rule from uniaxial to the multiaxial case and deduced the tensorial form of the Neuber rule as follows:

$$\sigma_{ij}^e \varepsilon_{ij}^e = \sigma_{ij}^a \varepsilon_{ij}^a, \tag{14}$$

where the symbols with superscript e are the stress/strain data under fictitious elastic states and the symbols with superscript a are the stress/strain data under elasto-plastic conditions, respectively. In particular, the constitutive equations of the material are as follows[45]:

$$\begin{aligned}
\varepsilon_1^a &= \frac{-\upsilon}{E}\left(\sigma_2^a + \sigma_3^a\right) - \frac{f(\bar{\sigma}^a)}{2\bar{\sigma}^a}\left(\sigma_2^a + \sigma_3^a\right),\\
\varepsilon_2^a &= \frac{1}{E}\left(\sigma_2^a \upsilon \sigma_3^a\right) + \frac{f(\bar{\sigma}^a)}{2\bar{\sigma}^a}\left(2\sigma_2^a - \sigma_3^a\right),\\
\varepsilon_3^a &= \frac{1}{E}\left(\sigma_3^a - \upsilon \sigma_2^a\right) + \frac{f(\bar{\sigma}^a)}{2\bar{\sigma}^a}\left(2\bar{\sigma}_3^a - \sigma_2^a\right),
\end{aligned} \tag{15}$$

where $\bar{\sigma}^a = \sqrt{(\sigma_2^a)^2 + \sigma_2^a \sigma_3^a + (\sigma_3^a)^2}$ and the symbols marked with "–" represent von Mises equivalent stresses/strains, respectively.

However, experiments show that the Neuber rule generally overestimates the values of local stress and strain.[13] To improve precision in notch fatigue analysis, Topper et al[43] and Yao[20] modified the Neuber rule by substituting K_t with K_f, which closely relates to stress gradients and applied stress levels:

$$\sigma^a \cdot \varepsilon^a = \frac{K_f^2 S^2}{E} = C. \tag{16}$$

Similarly, for cyclic loading cases, Equation (13) can be rearranged as

$$\Delta\sigma^a \cdot \Delta\varepsilon^a = \frac{K_f^2 \Delta S^2}{E} = C. \tag{17}$$

Moreover, as the LSSA extends to multiaxial cases combining with multiaxial fatigue damage parameters, multiaxial strain/stress responses at the notch root can be obtained. To extend the Neuber rule deduced form hyperbolic notch profile under monotonic shear loadings to multiaxial stress cases, Hoffmann and Seeger[46,47] developed approximation formulas for equivalent stresses and strains (namely, Hoffmann-Seeger rule) by assuming that during cyclic loadings, the ratio of minimum principal strain components at the notch root remains unchanged:

$$\bar{\sigma}^a \cdot \bar{\varepsilon}^a = \bar{\sigma}^e \cdot \bar{\varepsilon}^e, \tag{18}$$

where $\bar{\sigma}^a$ and $\bar{\varepsilon}^a$ are respectively the elasto-plastic equivalent stress and strain at the notch root and $\bar{\sigma}^e$ and $\bar{\varepsilon}^e$ are those values obtained assuming that the material remains pure elastic.

Another commonly used approach for local stress/strain calculation is the equivalent strain energy density (ESED) rule. Inspired by the Neuber rule, assuming that the change of strain energy density at the notch

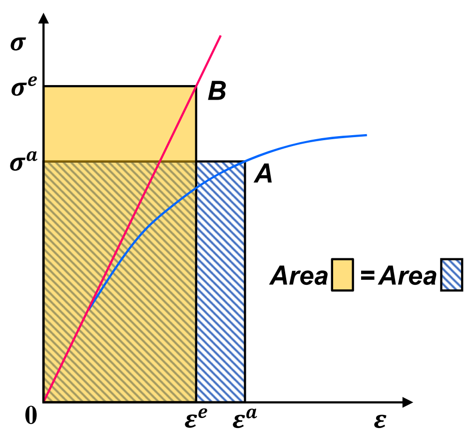

FIGURE 5 Graphical representation of the Neuber rule [Colour figure can be viewed at wileyonlinelibrary.com]

tip is statistically insignificant if the localized plasticity is encompassed by elasticity predominant material, Molski and Glinka developed the ESED rule[48]:

$$\int \sigma^{\mathrm{a}} d\varepsilon^{\mathrm{a}} = \int \sigma^{\mathrm{e}} d\varepsilon^{\mathrm{e}}. \tag{19}$$

In another words, the strain energy density at the notch tip calculated under elasto-plastic state will be identical to the hypothetical elastic strain energy, as is depicted in Figure 6.

For monotonic loading cases, the ESED rule results as

$$\frac{K_{\mathrm{t}}^2 S^2}{E} = \frac{\sigma^2}{E} + \frac{2\sigma}{n+1}\left(\frac{\sigma}{K}\right)^{1/n}, \tag{20}$$

where K and n are the strain hardening coefficient and the strain hardening exponent, respectively.

Similarly, for cyclic loading cases, it may be written as

$$\frac{K_{\mathrm{t}}^2 \Delta S^2}{4E} = \frac{\Delta\sigma^2}{4E} + \frac{\Delta\sigma}{n'+1}\left(\frac{\Delta\sigma}{2K'}\right)^{1/n'}, \tag{21}$$

where K' and n' are the cyclic strength coefficient and the cyclic strain hardening exponent, respectively.

According to the research performed by Glinka et al,[49] the ESED model is better than the Neuber rule in elasto-plastic notch stress/strain calculation for notched specimens under monotonic loadings. Later, Glinka[50] illustrated the intrinsic relationship between the Neuber rule and the ESED rule from the perspective of energy and graphically compared their relationships, which shows that the Neuber rule overestimates the stress/strain at the notch root than actual values, while

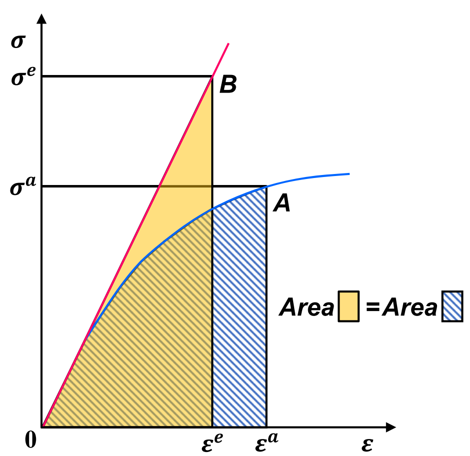

FIGURE 6 Graphical representation of the equivalent strain energy density (ESED) rule [Colour figure can be viewed at wileyonlinelibrary.com]

the ESED rule produces conservative predictions. Moreover, by analysing the energy of the elasto-plastic body under monotonic/cyclic loadings, Ye et al[51] pointed out that the Neuber rule is essentially a special case of the ESED rule when the plastic dissipation is ignored. Considering the actual physical behaviour occurring at the notch tip during cyclic plastic deformation, an improved version of ESED method was developed, which only considers the dissipation of heat energy and regards the stored energy as a contribution to local stress/strain ranges.[51]

For monotonic loading cases, the improved ESED rule by Ye et al[51] results as:

$$\frac{K_{\mathrm{t}}^2 S^2}{E} = \frac{\sigma^2}{E} + \frac{(2-n)\sigma}{n+1}\left(\frac{\sigma}{K}\right)^{1/n}. \tag{22}$$

For cyclic loading cases, it results as:

$$\frac{K_{\mathrm{t}}^2 \Delta S^2}{4E} = \frac{\Delta\sigma^2}{4E} + \frac{(2-n')\Delta\sigma}{2(n'+1)}\left(\frac{\Delta\sigma}{2K'}\right)^{1/n'}. \tag{23}$$

Results show that the prediction accuracy of the modified ESED method has been further improved compared with the original ESED method in local nonlinear stress-strain behaviour simulation.[51]

On the basis of the research performed by Moftakhar et al[44] and Glinka et al,[49] Singh et al[52] and Lim et al[53] developed the incremental forms of the Neuber and the ESED rules, respectively (see Figures 7 and 8), which can serve for stress/strain calculation under nonproportional loading conditions.

Moreover, as is illustrated in Figure 9, assuming that the total strain energy increment of the hypothetical elastic notch tip input stresses is equal to the total strain energy density of the real elasto-plastic material response at the notch root, Buczynski and Glinka[54] developed an incremental deviatoric Neuber rule.

By defining a structural yield surface in the nominal stress space with anisotropic plasticity theory, which embodies both the anisotropic geometry factors and the isotropic material properties, Barkey et al[55] established an analytical approach to simulate elasto-plastic notch strains of notched specimens under proportional/nonproportional multiaxial loadings. In addition, Köttgen et al[56] extended the Barkey method by integrating the influence of notch effect into the constitutive equation. Seshadri and Kizhatil[57] developed a generalized local stress-strain (GLOSS) analysis procedure for notch root inelastic strain estimation based on two linear elastic FEA per point on the load versus notch strain curve. In particular, the two linear elastic

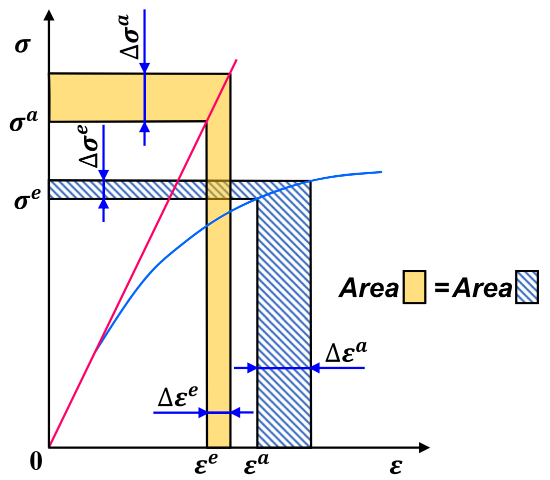

FIGURE 7 Graphical representation of the incremental form of the Neuber rule [Colour figure can be viewed at wileyonlinelibrary.com]

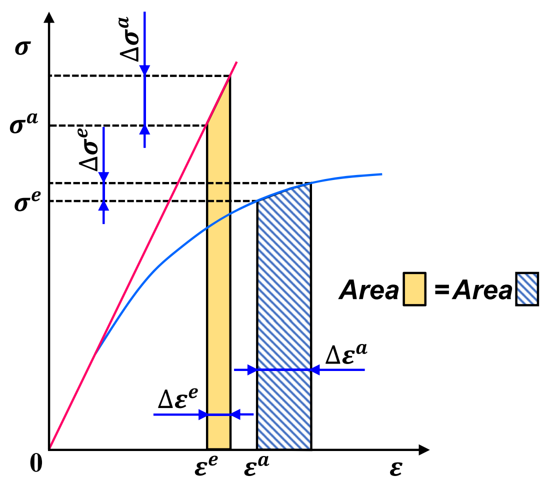

FIGURE 8 Graphical representation of the incremental form of the equivalent strain energy density (ESED) rule [Colour figure can be viewed at wileyonlinelibrary.com]

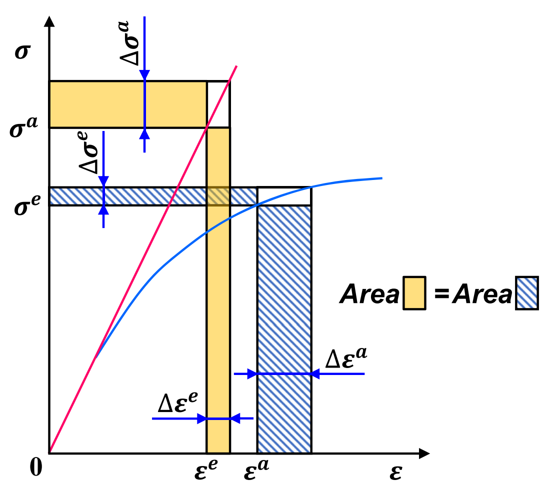

FIGURE 9 Graphical representation of the incremental deviatoric Neuber rule [Colour figure can be viewed at wileyonlinelibrary.com]

FEA consists of two steps: One is carried out for a given component configuration subjected to various mechanical/thermal loadings by assuming that the entire material is linear elastic; another is then performed after reducing the elastic moduli of all elements that exceed the yield stress, assuming an elastic-perfectly plastic constitutive relationship.

Recently, researchers found that notch correction approaches can be applied, combining them with cyclic plasticity models to deduce local stress-strain histories from the pseudo-elastic stress-strain at the notch region.[58,59] On the basis of a simplified thermodynamics analysis of cyclic plastic deformation, Ye et al[59] established an energy transition relationship to describe the elasto-plastic stress-strain behaviour at the notch tip under multiaxial loadings. Coupling with the Garud cyclic plasticity model[60] and the incremental deviatoric Neuber rule[54] and in order to provide the required seven independent expressions (namely, the seven fictitious linear elastic stress/strain components $\left(\sigma_{ij}^{e}, \varepsilon_{ij}^{e}\right)$ for calculating the actual elastic-plastic stress and strain components $\left(\sigma_{ij}^{a}, \varepsilon_{ij}^{a}\right)$ at the notch tip) to obtain all unknown stress/strain components, Ince and Glinka[61] proposed a simple analytical multiaxial notch analysis algorithm to simulate the elastic-plastic notch-root material behaviour of the notch components under multiaxial nonproportional cyclic loadings, see Figure 10.

Furthermore, coupling with a newly defined multiaxial fatigue damage parameter, it has been applied for fatigue analysis of SAE 1045 and SAE 1070 steels notched specimens subjected to proportional/nonproportional loadings.[38,62] Gates and Fatemi[63] studied the fatigue behaviour of Al 2024-T3 alloy using thin-walled tubular samples with a circular transverse hole; results showed

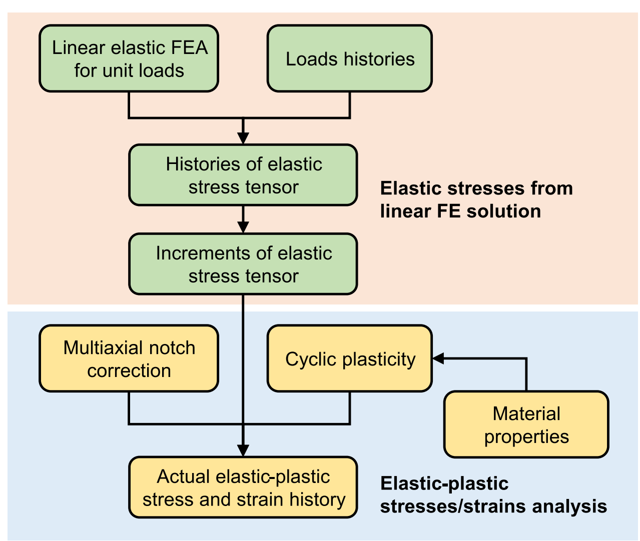

FIGURE 10 Algorithm flow for notch stress/strain analysis [Colour figure can be viewed at wileyonlinelibrary.com]

that the LSSA based on the Neuber rule with K_f correlate well with both axial and torsional data in the LCF regime, but that is no longer applicable in the mid-high cycle fatigue regimes, while utilizing K_t in the Neuber rule gives overly conservative life estimations. In addition, the Fatemi-Socie (FS) criterion[64] raised on the basis of the shear failure mechanisms is well correlated with the experimental fatigue lifetime of various materials under different loading conditions. Moreover, the theory of critical distances (TCD) has also been utilized for coupling analysis with pseudo stress-based plasticity model, which provides better multiaxial fatigue database correlation compared with K_f-based approaches, while small differences between the different TCD methods (including point approach and line approach) were observed.[65]

In virtue of the underestimated notch-tip stresses/strains employing the ESED rule,[50] Li et al[66] introduced a factor $((1+\upsilon_e)/(1+\upsilon_{eff}))$ to modify the heat energy dissipation, where υ_e and υ_{eff} are respectively the elastic and effective Poisson ratios. Coupling with the plasticity model developed by Jiang and Sehitoglu[67,68] for multiaxial elasto-plastic stress-strain responses analysis, the predicted results are in fairly good agreement with experimental results. Besides, Meggiolaro et al[69] developed a unified notch rule (UNR) and widened its application scopes into multiaxial proportional loading histories simulation, which could reproduce the Neuber or ESED rules and interpolate their notch-root stress-strain responses through an accommodation parameter α_U. Tao et al[39] developed a new pseudo stress correction approach for local strains estimation of notched structures under multiaxial loadings. Coupling with the model raised by Kanazawa et al,[70] Tao et al[39] considered the additional hardening attributed to the nonproportional external loadings by substituting pseudo notch strains for measured strains. Additionally, by employing the Wang-Brown reversal counting method, the estimation approach of nonproportionality factor (F) has been extended from multiaxial constant amplitude loading cases to variable amplitude cases. Mourão et al[71] estimated the fatigue damage of offshore structures on the basis of hot-spot and notch strain approaches.

On the basis of massive experimental evidences for model validation, note that the LSSAs generally underestimate fatigue lives. Moreover, fatigue crack initiation is associated with cyclic slip, thus not only the maximum stress at the notch tip is crucial, but also the stress acting on the lower material particles.[17] Therefore, researchers developed critical distance theory as well as weighting control parameter relevant approaches for a better description of notch effects in metal fatigue.

4 | CRITICAL DISTANCE THEORY AND ITS APPLICATIONS TO FATIGUE

The TCD was firstly proposed by Neuber at the beginning of the last century. In research of the influence of stress concentration on fatigue resistance of notched structures (namely notch effect), Neuber used the averaging stress over a certain distance starting at the "hot spot" of the notch as the effective stress for fatigue strength analysis, namely, the line method (LM); see Figure 11. Later, Peterson[72] simplified the LM by substituting the averaging stress over a deterministic distance with the stress at a certain distance from the notch root as the effective stress for fatigue evaluation, which yields excellent predictions, abbreviated hereinafter as the point method (PM). Moreover, there are also the area method (AM)[73] and the volume method (VM),[74] which averages the stresses over the area in the vicinity of the notch within a radius of $1.32L$ and the hemispherical volume within a radius of $1.54L$ as the effective stress, respectively.

On the basis of concepts in Figure 11, Taylor[75] unified the aforementioned four fatigue strength analysis methods, introducing formally the concept of critical distance into the TCD. In particular, his expressions can be summarized as follows:

$$\text{PM}: \qquad \sigma(L/2) = \sigma_o, \tag{24}$$

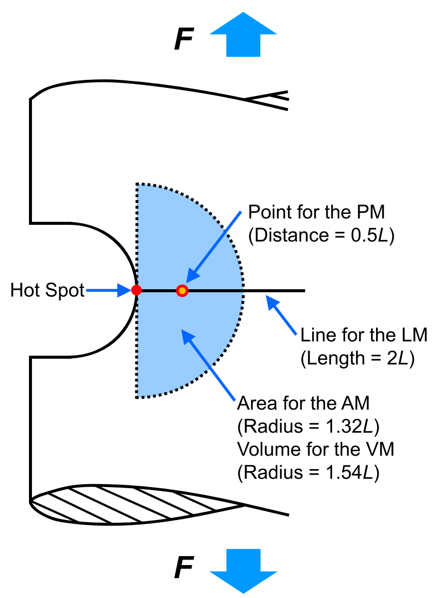

FIGURE 11 Different forms of theory of critical distance (point method [PM], line method [LM], area method [AM], and volume method [VM]) [Colour figure can be viewed at wileyonlinelibrary.com]

$$\text{LM}: \quad \frac{1}{2L}\int_0^{2L}\sigma(r)dr=\sigma_o, \tag{25}$$

$$\text{AM}: \quad \frac{2}{\pi(1.32L)^2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_0^{1.32L}\sigma(r,\theta)rdrd\theta=\sigma_o, \tag{26}$$

$$\text{VM}: \quad \frac{3}{2\pi(1.54L)^3}\int_0^{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_0^{1.54L}\sigma(r,\theta,\varphi)r^2\sin\theta drd\theta d\varphi=\sigma_o. \tag{27}$$

In general, the value of the critical distance for fatigue limit predictions can be obtained by:

$$L=\frac{1}{\pi}\left(\frac{\Delta K_{th}}{\Delta\sigma_o}\right)^2, \tag{28}$$

where ΔK_{th} denotes the range of the threshold stress intensity factor and $\Delta\sigma_o$ represents the range of the plain fatigue limit.

Susmel[76] proposed a unified approach for strength estimation of notched structures in the high cycle fatigue (HCF) regime, in which the linear elastic stress state of the structural volume provides a practical solution for extraction of the fatigue damage related to the overall damaging fatigue process zone. Specifically, the structural volume (can be defined with AM or VM) was found to be constant and independent of both loading type and geometrical features but varies on the basis of the material type. However, its effectiveness under out-of-phase multiaxial fatigue loadings remains to be further verified. Moreover, Susmel and Taylor[77] investigated and reinterpreted the fatigue analysis of conventional high cycle multiaxial fatigue rules with TCD. In particular, in their work, the criteria proposed by Crossland [78], Ballard et al. [79], Papadopoulos [80], Matake [81], and McDiarmid [82], respectively, as well as the modified Wöhler curve method were chosen for feasibility verification. Results show that the critical plane approaches (CPAs) are the exclusive multiaxial fatigue criterion that can coherently be reinterpreted in terms of the TCD. Castro et al[83] also conducted similar work and developed a methodology that considers the impact of load ratio, R, on the threshold stress intensity range. In addition, they pointed out that the coupling analysis of multiaxial fatigue criteria with the TCD should be consistent with the criterion itself.

As is aforementioned, the critical distance in HCF analysis is considered as a material characteristic length constant closely related to the grain size as well as its ability to retard crack growth of the material. However, for fatigue analysis in the medium cycle fatigue (MCF) regime, Susmel and Taylor[84] treated the value of the critical distance as a variance changing with the number of cycles to failure N_f:

$$L=AN_f^B, \tag{29}$$

where A and B are model-relevant parameters. In detail, only linear elastic finite element simulation is required in the analysis procedure. Furthermore, by employing the MWCM[85] to quantify the fatigue damage, combining the classical three-point rain-flow counting method with the Miner rule for calculating the accumulative damage, Susmel and Taylor extended the model to variable amplitude uniaxial/multiaxial fatigue loading situations.[86] However, a question perplexing the authors was finding a physical explanation for the effectiveness of the TCD when dealing with notch fatigue issues. Thus, validity verification research works have been subsequently performed. Specifically, in their following research,[87] they pointed out that the critical distance can also be regarded as a constant in the LCF/MCF regime. By coupling the Manson-Coffin equation with PM/LM for notch fatigue analysis under symmetrical loadings, the model in Susmel and Taylor[87] leads to a sound accordance to the fatigue tests. Moreover, for the cases with $R > -1$, the Smith-Watson-Topper (SWT) model[88] was employed to consider the detrimental influence of nonzero mean stresses on fatigue strength of components. Referring to the idea of LSSA, Wu et al[65,66,89,90] initially analysed the fatigue strength of notch components on the basis of a newly defined critical plane-based multiaxial fatigue model. However, they only considered stress-strain data collected from notch tip and overlooked the impact of notch effect, thus the predicted results overall tend to be conservative. To characterize the effect of inhomogeneous damage parameter distribution on fatigue strength, they incorporated the TCD correction. In particular, the values of critical distance were treated as a material constant, but they varied for different types of notched specimens. Lately, Wang et al[91] repeated the above-mentioned work in fatigue lifetime prediction of plate TA19 alloy specimens with a central circular hole. In their research, the decision that regards the critical distance as a fatigue life related function yields better correlation with experimental data than others.

Lanning et al[92] modified the PM/LM of the TCD by substituting the stress with the quantities of mean stress, stress range, and elastic strain energy, which may contribute to the fatigue process. Besides, a weighting function was included for model calibration. The stress range-modified TCD criterion presented more accurate results than that modified by mean stress and elastic strain energy.

Differing from Lanning's ideas of substituting the damage relevant parameter, in LCF analysis of notched directionally solidified (DS) Ni-based superalloy, Yang et al[93] and Wang and Yang[92] corrected Equation (29) by incorporating the linear elastic stress concentration factor K_t:

$$K_t \times L = AN_f^B. \quad (30)$$

On the basis of experimental evidence, Huang et al[94] further modified the expression as:

$$K_t^m \times L = AN_f^B. \quad (31)$$

Results show that the modified equations provide better correlations with the experimental data than the previous one. Subsequently, Hu et al[95] studied the effect of notch geometry on the fatigue strength as well as the value of critical distance for TC4 titanium alloy. On the basis of vast test data, they concluded that K_t related model proposed by Wang and Yang[96] cannot perfectly account for the variation of the critical distance with the geometry sizes.

In LCF analysis of notched components manufactured by single-crystal superalloy, Leidermark et al[97] also discussed the relationship between the critical distance values and the fatigue lives, in which a combined critical plane-critical distance analysis has been performed. Specifically, the plane of the maximum total shear strain range was selected as the critical slip system. By comparing the model prediction errors, the strategy treating the critical distance as a function of fatigue lifetime indicated the acceptable correlation between the predicted and experimental data. Xin et al[98] performed life assessment for TC4 alloy notched samples in LCF regime based on the TCD, and the relationship between the critical distance and fatigue lifetime can also be expressed by a power function as Equation (29). In addition, it is suggested that the correlation between critical distance and fatigue lifetime, load ratio, and stress concentration factor be considered for more accurate fatigue life prediction. Recently, taking the case of FS criterion, Liao et al[99] studied different coupling sequences of the CPA with the TCD for multiaxial fatigue assessment of notched components. Moreover, the impact of employing the PM and the LM of the TCD on predicting performance as well as the rationality of treating the critical distance as a characteristic material constant or as a fatigue lifetime-related function are also discussed. By using experimental results of Al 7050-T7451 and GH4169 alloys for model verification and comparison, result shows that the procedures that apply the CPA before the TCD and treat the critical distance as a function related to fatigue lifetime show higher accuracy in fatigue life assessment than others.

Bourbita and Rémy[100] coupled critical distance and strain energy density model into fatigue lifetime prediction of notched structures, which seems to be a promising method to assess the lifetime to initiation in single crystal superalloy parts at elevated temperatures. On the basis of a newly defined strain energy gradient concept, Zhu et al[101] also performed LCF life evaluation of turbine discs using the TCD. In particular, the critical distance can be formulated as a load ratio and peak stress-related function independent of fatigue lifetime:

$$L = \alpha_1 \times e^{b \times R} \sigma_{\max}^{\beta_1}. \quad (32)$$

Moreover, by introducing a weight function to account for the position effect within the effective damage zone, a general workflow for strain energy gradient–based LCF life assessment is established and elaborated through a case study on fatigue life estimation of a high pressure turbine disc.

Considering the difficulty of measuring the threshold range by experiments for calculating the critical distance according to Equation (28), Santus et al[102,103] intended to obtain the length of critical distance by using notched specimens. Taking rounded V-notched specimens for instance, a substitute to the crack threshold stress intensity factor for determining the value of critical distance and a computation framework to avoid the specific FEA on specimens were presented. In particular, notched samples with small radius were suggested for solving the critical distance and thus ensure an accurate strength evaluation of blunter structures. Moreover, by combining the TCD and multiaxial fatigue models (like FS[104,105] and Carpinteri criteria[106-108]), they further developed a model that can incorporate the impact of mean stress into the fatigue strength calculation and noted that LM is overall more accurate than PM, especially in those cases where high stress gradients occur.[109]

Recently, Luo et al[110] combined the CPA and the TCD for fatigue life prediction of thin-walled notched specimens with a circular hole. According to the physical mechanism that fatigue crack initiation is dominated by shear stress (slips bands formation), the critical plane is settled as the plane that passes through the fatigue critical point and is defined by the maximal shear stress amplitude. By introducing test results of GH4169, LY12CZ, and TC4 alloys for model validation, it provides accurate predictions for both the crack initiation positions/directions and fatigue lifetimes. However, the study was based on the 2D level only, and the transformation from the 2D shell structures to the multiaxial fatigue evaluation of 3D notched structures remains unexplored.

As Taylor[111] mentioned, the TCD provides a bridge between different parts of subjects thus facilitates its coupling analysis with other methods developed from different perspectives and its application in engineering practice. For instance, Jiang and Sehitoglu[112] and Akama et al[113] coupled the CPA and the TCD for contact fatigue analysis between wheel and rail. Sun et al[114] modified the TCD on the basis of the relative stress gradient and applied it to HCF analysis of crankshaft components. On the basis of the TCD, Adriano et al[115] investigated the influence of the size of the fatigue process zone on fatigue lifetime assessment performed on aluminium wires with geometric discontinuities. Specifically, a new method to assess the equivalent stress inside the spherical material control volume defined by VM is developed, and the strategy using the averaging stress tensor associated to the mesh nodes contained in the sphere for model calibration provides more precise predictions for several types of test samples. Ahmed and Susmel[116] formulated a novel approach for static assessment of plain/notched polylactide 3D-printed with different infill levels. By combining the equivalent homogenized material concept with the TCD, their methodology provided sound correlations with experimental fatigue data, returning estimations falling within ±20% error interval.

To conclude, the exclusive talent of the TCD in coupling analysis with other methods has been validated by a wealth of practices in notch fatigue analysis.[99] Therefore, further attempts can be made through its combination with other approaches.

5 | WEIGHTING CONTROL PARAMETERS–BASED APPROACHES

This section addresses recent models evaluating the fatigue damage by using either averaged or weighted control parameters. In particular, the control parameter can be defined by stress or strain or both, like strain energy density.

5.1 | Stress/strain-based control parameters

Until now, except for the TCD, the first approach for fatigue analysis using weighting stress control parameter is the stress field intensity (SFI) approach developed by Yao,[117] which assumes that fatigue damage is related with the stress response within the local damage zone; see Figure 12.

The SFI function σ_{FI} is given as:

$$\sigma_{FI} = \frac{1}{V}\int_{\Omega} f(\sigma_{ij})\varphi_\sigma(\vec{r})\,dv. \tag{33}$$

In detail, σ_{FI} consists of three components: (a) local fatigue failure region Ω, where V is its volume; (b) function of the equivalent stress, $f(\sigma_{ij})$; (c) weight function, $\varphi_\sigma(\vec{r})$, defined as:

$$\varphi_\sigma(\vec{r}) = 1 - \chi r(1+\sin\theta), \tag{34}$$

where r represents the length from arbitrary point in Ω to the critical one and θ denotes the direction angle as is shown in Figure 12.

Assuming that notched components whose SFI history at the notch tip are identical to that of the smooth example, after giving the σ_{FI}, fatigue lifetime of the critical region can be determined by consulting the material S-N curve. Despite the SFI approach reasonably characterizing the influence of notch effect on fatigue strength/life, in engineering reality, obtaining the radii of the fatigue failure region Ω requires pretests, and calculation of the σ_{FI} is a complex and time-consuming process, which limits its application in engineering

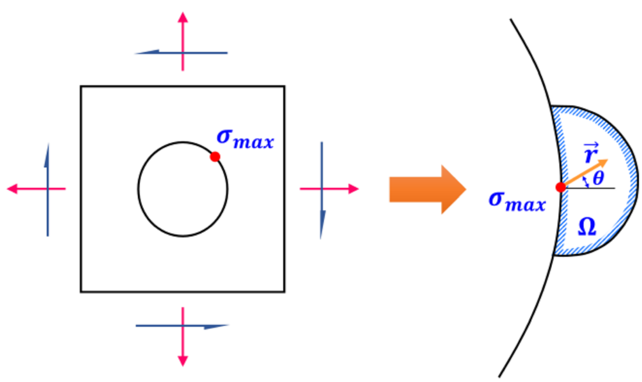

FIGURE 12 Schematic diagram of the stress field intensity (SFI) approach [Colour figure can be viewed at wileyonlinelibrary.com]

practice.[118] Although the concept of field intensity indeed provides new inspiration on the research of notch effects, Qylafku et al[119] summarized the drawbacks of the SFI approach: (a) whether the stresses employed are elastic or elasto-plastic remains unclear and (b) the behaviour of the distribution of stress as a function related to the distance from the notch root is still unknown.

Inspired by the SFI approach, Shang et al[120] developed a local stress-strain field intensity approach for fatigue lifetime evaluation of notched members, in which the strain field intensity is defined as below:

$$\varepsilon_{\mathrm{FI}} = \frac{1}{V}\int_{\Omega} f\left(\varepsilon_{\mathrm{ij}}\right)\varphi_{\varepsilon}\left(\vec{r}\right)dv. \tag{35}$$

In particular, the stress weight function is given by $\varphi_{\sigma}\left(\vec{r}\right) = 1-\left(1-\frac{\sigma_{\mathrm{eq,i}}}{\sigma_{\mathrm{peak}}}\right)r(1+\sin\theta)$, and the strain weight function is provided by $\varphi_{\varepsilon}\left(\vec{r}\right) = 1-\left(1-\frac{\varepsilon_{\mathrm{eq,i}}}{\varepsilon_{\mathrm{peak}}}\right)r(1+\sin\theta)$, where $\left(1-\frac{\sigma_{\mathrm{eq,i}}}{\sigma_{\mathrm{peak}}}\right)$ and $\left(1-\frac{\varepsilon_{\mathrm{eq,i}}}{\varepsilon_{\mathrm{peak}}}\right)$ characterize the influence of stress and strain gradient, respectively; σ_{peak} and $\varepsilon_{\mathrm{peak}}$ are the peak value of stress/strain in the notch root, respectively. Then, fatigue lifetimes of notched specimens can be calculated by applying the proposed local stress-strain field method coupled with the Manson-Coffin and the SWT damage criteria. Moreover, combining the local stress-strain field model with the Neuber rule, rain-flow counting method, and damage accumulation rule, they further extended the local stress-strain field approach into fatigue lifetime prediction under random loading paths.[120]

Later, considering the fatigue failure requires a certain physical volume, namely, the fatigue failure process volume, Kadi et al,[121] Pluvinage,[122] and Krzyzak et al[123] developed a volumetric approach. It is assumed that the corresponding fatigue initiation criterion is dependent on the effective stress as well as the effective distance. Specifically, the effective stress is the averaging stress acting in the cylindrical process volume, and the effective distance defines its boundary.

On the basis of the SFI and the volumetric approach, through considering the physical mechanism of fatigue initiation and, particularly, on the necessity for a physical volume in which the fatigue process can proceed, authors in previous studies [119,124-126] proposed a modified SFI approach, hereafter called as effective stress approach (ESA). In particular, to consider the effect of the real elastic-plastic stress distribution on fatigue strength, Qylafku et al[1] extended the expression of the relative stress gradient as:

$$\chi^{*}(r) = \frac{1}{\sigma(r,\theta=0)}\frac{\partial\sigma_{yy}(r,\theta=0)}{\partial r}. \tag{36}$$

Similar to the SFI approach, the relative stress gradient can reflect the contribution of stress at different locations within the damage zone to the overall fatigue by introducing a weight function. In addition, considering the process of determining the radius of the fatigue failure region of the SFI approach is too complex. Pluvinage redefined the boundary of the fatigue damage zone by introducing an effective distance concept. As is shown in Figure 13, according to the distribution of stress and relative stress gradient at the notch tip, the distribution of elasto-plastic stresses near the notch tip can be conveniently divided into three regions in the double logarithmic coordinates. Region I is the "high stress" region, which contains the maximum stress; region II is the transition region between region I and region III, which exhibits a linear relationship in the double logarithmic coordinates; and the stress distribution in region III can be formulated by a power function:

$$\sigma_{yy} = \frac{C^{*}}{x^{\alpha}}, \tag{37}$$

where α and C^{*} are material constants related to load paths and geometries. Since region III is far from the fatigue damage zone, the impact of the stress distribution on the fatigue failure is basically negligible.

Region I is the effective damage zone of the ESA, and its boundary D_{eff} is defined as the distance from the surface of the notch to the inside of the notch until its stress distribution reaches its first inflexion point (ie, the first minimum value of the relative stress gradient). After defining the boundaries of the effective damage zone, Qylafku et al defined the effective stress as:

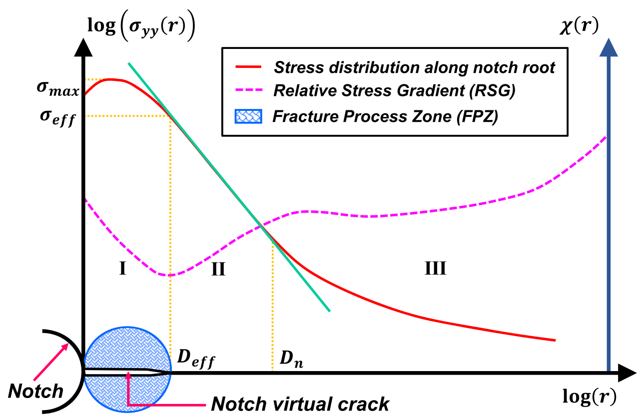

FIGURE 13 Typical elastic-plastic stress and relative stress gradient distributions in the vicinity of an external notch [Colour figure can be viewed at wileyonlinelibrary.com]

$$\sigma_{\text{eff}} = \frac{1}{D_{\text{eff}}} \int_0^{D_{\text{eff}}} \sigma(r,\theta=0) \times \varphi\left(r,\chi^*(r)\right) dr, \tag{38}$$

where $\varphi(r, \chi^*(r))$ is the weight function, and its expression is as follows:

$$\varphi\left(r,\chi^*(r)\right) = 1 - \left|\chi^*(r)\right| \cdot r. \tag{39}$$

In the same way as the SFI approach, after calculating σ_{eff}, the fatigue lifetime of the notched part can also be obtained by consulting the *S-N* curve of smooth samples. Compared with the SFI method, the ESA is more convenient in determining the boundary of the effective damage zone, which greatly simplifies the analysis process and provides a new solution for notch fatigue analysis.

Liu et al[127] also introduced a method to consider the impact of the stress field on fatigue life by coupling the TCD with FEA. To begin with, a critical plane is set as the plane with the maximal shear stress range, and the damage parameters are the maximal effective shear stress amplitude as well as the maximum effective normal stress, which can be calculated by averaging the stress in the hemisphere volume around the peak stress point (using the VM of the TCD with critical radius 1.54L to define the fatigue process zone):

$$\tau_R = \frac{1}{V} \int f(\tau) dv, \tag{40}$$

$$\sigma_{n,\text{FI}} = \frac{1}{V} \int f(\sigma_n) dv. \tag{41}$$

Finally, the fatigue lifetime can be evaluated with the following expression:

$$\sqrt{3(\Delta\tau_{\max,\text{FI}}/2)^2 + k\sigma_{n,\max,\text{FI}}^2} = \sigma_{\text{f}}'(2N_{\text{f}})^b. \tag{42}$$

Actually, there is not a specified or identified frontier between the TCD as well as its variants and weighting stress-based control parameter-relevant approaches. For instance, on one hand, LM, AM, and VM of the TCD can be viewed as a special case of stress averaging; on the other hand, if weight functions of the SFI and the ESA are defined as 1, then they degenerate into VM of the TCD (only small difference in the definition of the boundary of the effective damage area). However, considering wide coupling application of TCD with other methods, the TCD and its nuances are divided into a single section of this review.

Moreover, there is another group of methods proposed to perform notch fatigue analysis using the volume experiencing high stress levels, which are hereafter referred collectively as the "highly stressed volume (HSV)" approach. However, they applied neither averaged nor weighted stress in the HSV as damage parameters but deduce the fatigue strength through the value of HSV. Hooreweder et al[17] analysed the fatigue strength of notched aluminium samples using the HSV method. By defining the most HSV of an arbitrary specimen as the reference volume, V_0, the following expression can be deduced:

$$\sigma_{\text{a,max}} = \sigma_{A0} \left(\frac{V_0}{V_{90}}\right)^{v*}, \tag{43}$$

where σ_{A0} refers to the fatigue limit of a smooth specimen with volume V_0, v^* is a model parameter fitted from test data of specimens with different geometries and sizes, and V_{90} is the maximal stressed and/or strained material volume where the parts subjected to more than 90% of the maximal stresses/strains are included. Then, the amplitude of effective fatigue stress can be given as follows[128]:

$$\bar{\sigma}_{\text{a}} = \sigma_{\text{a,max}} \left(\frac{V_{90}}{V_0}\right)^{v*}. \tag{44}$$

Thus, fatigue lifetime of arbitrary notched components can be assessed by consulting the *S-N* curve of the V_0 specimen (also named as the reference specimen). By employing the fatigue testing data of notched Al 7075-T7531 specimens for model validation, results show that the HSV can be chosen as an accurate and effective design parameter to consider the influence of notch effect on aluminium components. Muñiz-Calvente et al[129] developed a probabilistic scale effect concept that is applicable to account for both volume and stress gradient effects on fatigue behaviour and allows a suitable and reliable consideration of the statistical size effect. Recently, Ai et al[130] presented a method to consider both notch and size effects to describe the fatigue lifetime distribution of test samples with diverse geometries by using the HSV approach. In particular, a dynamic model coefficient considering the impact of different maximum local stresses has been proposed to model the size effect of HSV combining with the Weibull distribution, which presents good correlations between model predictions and experimental results.

5.2 | Strain energy density-based control parameters

Since the first work performed by Beltrami, the concept of strain energy density has been widely utilized in both static and fatigue behaviour assessment of smooth and notched structures.[131-134] Considering the methods developed by averaging or weighting the stress/strain-based control parameters, one may not quantify well the real fatigue damage and link the corresponding failure mechanism. Various researchers intended to analyse the notch effect from the perspective of energy dissipation, which shows superiority in correlating microscopic and macroscopic experimental evidences.[135]

Lazzarin and Zambardi[136] developed a finite volume energy-based method for static and fatigue behaviour analysis of structures with sharp V-shaped notches. On the basis of an averaging energy term in a small but finite volume of material surrounding V-shaped notch roots (which degenerates into an area in two-dimensional cases), the method can accurately predict both static and fatigue behaviour of severely notched structures. Then, they discussed the possibility of applying the above-mentioned method into fatigue analysis of blunt V-notches.[137] Specifically, some expressions for the strain energy averaging in a finite volume surrounding the V-notch roots have been presented to simplify local strain energy calculations. Besides, it is worth mentioning that the method is very sensitive to R_c, which defines the boundaries of the control area. For an ideal brittle material, R_c depends on the ultimate tensile strength and the fracture toughness.

Berto et al[138] also analysed the multiaxial fatigue strength of 39NiCrMo3 notched samples by using the averaging local strain energy density in a certain control volume enclosing the V-notch tip; see Figure 14.

In the presence of an axis-symmetric part weakened by a circumferential V-shaped notch, under linear elastic hypothesis, the averaging strain energy density over a control volume can be expressed according to the following equation:

$$\Delta\bar{W} = \frac{1}{E}\left[e_1 \times \frac{\Delta K_1^2}{R_1^{2(1-\lambda_1)}} + e_3 \times \frac{\Delta K_3^2}{R_3^{2(1-\lambda_3)}}\right], \tag{45}$$

where ΔK_1 and ΔK_3 denote the notch stress intensity factors under mode I and mode III, respectively; R_1 and R_3

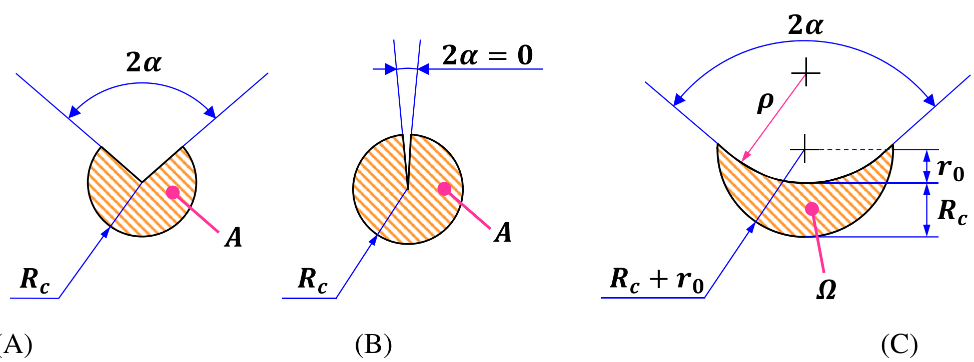

FIGURE 14 Control volume (area) for (a) sharp V-notch, (b) crack, and (c) blunt V-notch [Colour figure can be viewed at wileyonlinelibrary.com]

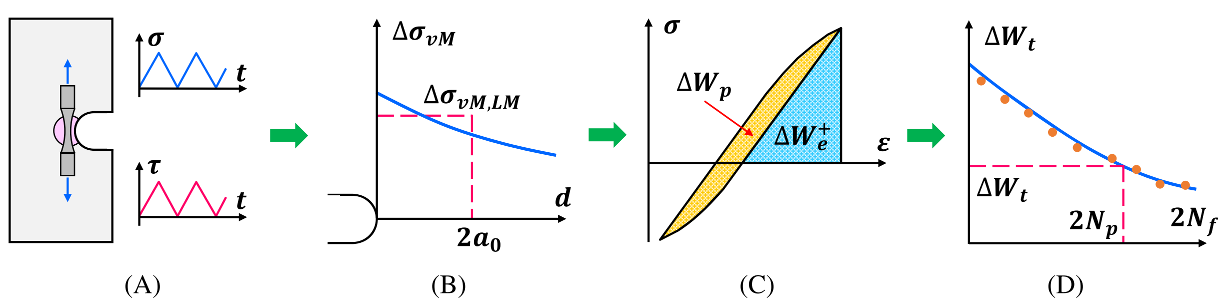

FIGURE 15 Fatigue life prediction approach based on the total strain energy density evaluated at the initiation regions from hysteresis loops obtained through the equivalent strain energy density (ESED) concept and an average stress given by the line method (LM) of the theory of critical distances (TCD): (a) reduction of the multiaxial stress-strain state to equivalent uniaxial one; (b) computation of the effective stress in the fatigue process zone (von Mises equivalent stress range); (c) calculation of the total strain energy density; and (d) life evaluation utilizing the fatigue master curve [Colour figure can be viewed at wileyonlinelibrary.com]

represent the radii of the control volume under loading modes I and III, respectively; e_1 and e_3 are two parameters for describing the effect of all stresses and strains over the control volume. In particular, the control volume is found to be closely related to the loading mode. Moreover, this method has also been successfully applied for multiaxial fatigue strength assessment of severely notched cast iron (EN-GJS400),[139] 45 steel (QT),[140] and 40CrMoV13.9 notched specimens.[141]

Recently, Branco et al[142] summarized a general framework for multiaxial fatigue analysis of notched parts on the basis of the concept of total strain energy density, as is illustrated in Figure 15. Specifically, it is assumed that both the smooth and notched specimens accumulate equal damage and show identical number of cycles to failure if their stress-strain responses at the initiation sites are the same; moreover, fatigue failure happens when the total strain energy density (sum of the plastic and the positive elastic components) at the initiation regions reaches a threshold value. In general, it presents excellent accuracy in fatigue lifetime assessment of plain and notched components under multiaxial loadings.[143] Moreover, this approach can also be extended to variable amplitude fatigue loading cases.[144]

Inspired by the SFI and total strain energy density-based models, Liao and Zhu[128] developed an energy field intensity (EFI) approach for multiaxial notch fatigue analysis, as is schematized in Figure 16.

Firstly, to facilitate the calculation of energy dissipation under complex multiaxial loading history, a modified Ellyin model has been presented, which meanwhile simplifies the master W_f - N_f curve. Then, taking the SFI approach as a reference, by defining a proper weight function and the effective damage zone, the concept of EFI is elaborated, which inherits the merits of energy criteria and considers the interactive influence of critical domains. Better correlation of model predictions with test data of GH4169 and Al 7050-T7451 alloys is provided by applying the EFI model, which shows a promising direction to account for notch effect of arbitrary geometry features.

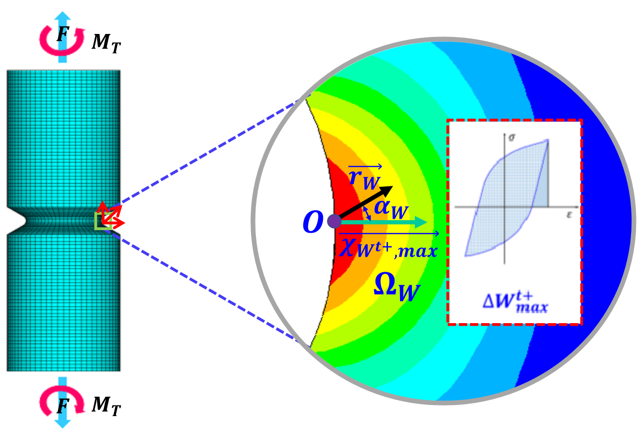

FIGURE 16 Schematic diagram of the energy field intensity (EFI) approach [Colour figure can be viewed at wileyonlinelibrary.com]

To sum up, among weighting control parameters–based methods, both the SFI[117] and the EFI[128] approaches seem to provide reasonable explanations on notch effect, as they highlight the significance of effective damage zone in notch fatigue analysis and meanwhile distinguish different contributions of material blocks inside it to the overall fatigue by introducing corresponding weight functions. In particular, comparing with the von Mises equivalent stress damage parameter, the total strain energy-based damage parameter can present a better correlation between the damage and the tested life under multiaxial stress states and unify micro phenomena and macro experimental evidences of interest, which provides an efficient solution for notch fatigue analysis under multiaxial loading cases.

6 | DISCUSSION

Subjected to fluctuating loadings, engineering components normally fail under low stress levels attributed to irreversible microplastic deformation followed by crack propagation and eventual failure. Generally, cracks initiate from locations with geometric discontinuities (notches), which work as stress raisers and have shown a tremendous influence on fatigue lifetime. Unfortunately, macroscopic notches are inevitable in engineering practice because of various functional requirements. Therefore, it is of great necessity to effectively characterize the notch effect in fatigue strength evaluation and structural integrity analysis. This review intends to present a systematic summary on recent progress in notch fatigue analysis, in order to help researchers and engineers in this field to enhance their understanding on recent progress, and prospective aspects, discussions, and conclusions are given as follows:

(1) At the start of the study on stress concentration, the concept of theoretical stress concentration factor K_t was developed; however, it is insufficient to effectively address its influence on fatigue strength. Later, the fatigue strength reduction factor, K_f, was proposed, which can be calculated from various empirical equations and bring convenience for fatigue life assessment. But its drawbacks include the exclusive applicability under uniaxial loadings and the unknown universal applicability among diverse structures and materials.

(2) For NSAs, they generally lack a link to real fatigue failure mechanisms and do not consider nonproportional

additional hardening as well as local plasticity. Besides, they cannot locate critical regions where cracks might initiate.

(3) Local stress-strain analysis based on analytical method is a promising direction, which can achieve rapid analysis of stress/strain response under complex loading histories and is more efficient than the finite element method. In general, LSSA is a relatively mature method for fatigue analysis in LCF regime, which shows higher accuracy than NSA. However, it only takes the strain-strain response of the critical point into account and thus normally underestimates real fatigue strength for its inconsistency with fatigue damage mechanism.

(4) Methods incorporating the concept of fatigue process zone (line, area, or volume) are drawing major concerns, though their definitions are diverse. The idea that fatigue failure requires a specific physical zone is becoming a consensus, and the core issue is to integrate an appropriate damage parameter and establish its linkage with fatigue lifetime. In the present paper, they are divided into two groups: critical distance theory and its variants and weighting control parameter–relevant approaches. But strictly speaking, this is not a sharp division between them since they share the same core concept.

(5) The TCD agrees well with the fatigue failure mechanism and can accurately locate the exact failure position. In particular, it can be utilized for coupling analysis with other methods and is commonly used in 2D cases for its convenient calculation. However, there are also several points to be explored and strengthened, including quick and easy access to the critical distance L, calculation of equivalent stress subjected to multiaxial loading, and the applied direction of TCD under complex geometry and stress states.

(6) Among weighting control parameter-relevant approaches, some of them average the parameters within control volumes directly, while others introduce weight functions to quantify contributions of different positions to the whole fatigue of the structure and then generate a single damage parameter to characterize the severity of fatigue damage. Some of them utilize the stress/strain while others use strain energy density to construct damage parameter and link it to fatigue lifetime. Generally, methods incorporating weight functions are more reasonable than those by averaging the parameters within control volumes directly, since damage parameters considering the role of each point in overall damage accumulation are diverse and depend both on the relative distance between this point and the notch tip and on the stress/strain energy density gradient. Moreover, compared with strategies using the stress/strain response as control parameters, strain energy density-based approaches seem more effective for their superiority in unifying micro phenomena and macro experimental evidences of interest, while it is relatively complex to define an equivalent stress/strain–based damage parameter under multiaxial stress states. Recently, the EFI approach developed by Liao and Zhu[128] integrates the advantages of both and provides an efficient solution to deal with notch effect of diversiform geometry features under multiaxial loadings.

(7) On the basis of the comprehensive review on notch effects in metal fatigue, some prospective aspects deserve further investigations summarized as below: (a) study the mechanism of notch fatigue failure from the macroscopic and microcosmic perspectives in combination with experiments, realize the modelling and evaluation of the overall damage of the notch, and characterize the contribution of different material blocks inside the effective damage region to the overall fatigue failure; (b) quick analytical calculation of local stress and strain in the notched region and its combination with methods for notch fatigue analysis to achieve convenient and efficient fatigue strength evaluation, as recent analyses based on FEA are generally inefficient; (c) establish a general analytical framework suitable for diversiform notch geometries and load types, further incorporating the influences of multiple factors such as multiaxial fatigue,[99] creep,[145] and size effect[91,130,146,147] on fatigue strength of the interest; (d) build a database of notched fatigue test results, summing up the applicability of each method under different loading conditions or geometry cases, which facilitates communication and innovation among different methods. In particular, the coupling analysis of notch and size effects is the premise of achieving the extrapolation of fatigue strength of large-scale components/structures using experimental data of small-scale specimens collected in laboratory.

7 | CONCLUSIONS

The influence of notch effect on the reduction of fatigue lifetime is a hurdle that cannot be bypassed in structural design of engineering components. Only by accurately grasping the influence of notch features on the fatigue strength can theoretical methods compatible with fatigue failure mechanism be established, thus avoiding conservative design and further achieving optimal design of structures. In order to help researchers and engineers acquire the research status of notch fatigue analysis, this work systematically reviews traditional notch fatigue analysis methods and recent advances in the past decades and classifies and summarizes the mainstream methods as follows.

(1) Traditional notched fatigue analysis methods are generally based on fatigue test data of typical notched examples. These empirical equations often present significant errors when applied to fatigue life prediction and are not suitable for the structural integrity assessment of modern critical components. NSAs are simple in form and application, but their high error index also gradually eliminates them in engineering practice.

(2) Nowadays, analytical calculation of the local stress/strain at the notch root has attracted the attention of the field experts. Simple and efficient analytic algorithms for stress and strain calculation are becoming a new research hotspot; however, only paying attention to the damage of the critical point cannot meet the demand of notch fatigue analysis, but reasonably combining the stress and strain history in the local effective damage zone to establish its relationship with fatigue life still deserves further investigation and validation.

(3) From the critical distance theory and its variants as well as weighting control parameters-based approaches, it can be concluded that more attention is being paid to the concept of effective damage zone. By delineating an effective damage zone closely related to notch fatigue and weighting and integrating the selected control parameters in the effective damage zone to obtain a weighted control parameter, its relationship with fatigue life can be built, which has become a promising solution for structural strength and integrity analysis.

(4) To conclude, in future research, experts on notch effects in metal fatigue field should focus on the following aspects: (a) investigating the mechanism of notch fatigue failure from both macroscopic and microcosmic perspectives and finding or developing an effective parameter to characterize the failure process, (b) improving the analysis efficiency by coupling with analytical calculation, and (c) developing a general analytical framework for notch fatigue analysis and further considering the influences of multiple factors on fatigue strength, especially the joint analysis of both notch and size effects.

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NOMENCLATURE

C	Neuber constant
E	elastic modulus
K	strain hardening coefficient
K'	cyclic strain hardening coefficient
K_f	fatigue strength reduction factor
K_t	theoretical stress concentration factor
L	critical distance
N_f	number of cycles to failure
n	strain hardening exponent
n'	cyclic strain hardening exponent
R	stress R-ratio
υ_e	elastic Poisson ratio
υ_{eff}	effective Poisson ratio
AM	area method of the TCD
CMB	Coffin-Manson-Basquin
CPA	critical plane approach
EFI	energy field intensity
ESA	effective stress approach
ESED	equivalent strain energy density
ESPI	electronic speckle pattern interferometry
FEA	finite element analysis
FS	Fatemi-Socie
GLOSS	generalized local stress-strain
HCF	high cycle fatigue
HSV	highly stressed volume
LCF	low cycle fatigue
LM	line method of the TCD
LSSA	local stress-strain approach
MCF	medium cycle fatigue
MWCM	modified Wöhler curve method
NSAs	nominal stress approaches
PM	point method of the TCD
SFI	stress field intensity
SWT	Smith-Watson-Topper
TCD	theory of critical distance
UNR	unified notch rule
VHCF	very high cycle fatigue
VM	volume method of the TCD

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