Microeconomia II

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Lista de Exercícios

Ch 8 : Simultaneous-Move Games

Problema 1 Apresente as seguintes definições:

(a) Estratégia Estritamente Dominante.

(b) Estratégia Estritamente Dominada.

(c) Estratégias Racionalizáveis

(d) Equilíbrio de Nash

(e) Equilíbrio de Nash em Estratégia Mista

(f) Equilíbrio Nash Bayesiano

Problema 2 There are I firms in an industry. Each can try to convince Congress to give the industry a subsidy. Let h_i denote the number of hours of effort put in by firm i, and let $c_i(h_i) = w_i(h_i)^2$, where w_i is a positive constant, be the cost of this effort to firm i. When the effort levels of the firms are $(h_1, ..., h_I)$, the value of the subsidy that gets approved is $\alpha \sum_i h_i + \beta(\prod_i h_i)$,where α and β are constants. Consider a game in which the firms decide simultaneously and independently how many hours they will each devote to this effort. Show that each firm has a strictly dominant strategy if and only if $\beta = 0$. What is firm i's strictly dominant strategy when this is so? (MWG 8.B.1)

Problema 3 Consider the Cournot duopoly model (discussed extensively in Chapter 12) in which two firms, 1 and 2, simultaneously choose the quantities they will sell on the market, q_1 and q_2. The price each receives for each unit given these quantities is $P(q_1, q_2) = a - b(q_1 + q_2)$. Their costs are c per unit sold. (MGW 8.B.5)

(a) Argue that successive elimination of strictly dominated strategies yield a unique prediction in this game.

(b) Would this be true if there were three firms instead of two?

Problema 4 Consider a game Γ_N with players 1, 2, and 3 in which $S_1 = \{L, M, R\}$, $S_2 = \{U, D\}$, and $S_3 = \{l, r\}$. Player 1's payoffs from each of these three strategies conditional on the strategies choices of players 2 and

		Player 3's Strategy	
		ℓ	r
Player 2's Strategy	U	$\pi + 4\varepsilon, \pi - \eta, \pi - 4\varepsilon$	$\pi - 4\varepsilon, \pi + \frac{\eta}{2}, \pi + 4\varepsilon$
	D	$\eta + 4\varepsilon, \pi + \frac{\eta}{2}, \pi - 4\varepsilon$	$\pi - 4\varepsilon, \pi - \eta, \pi + 4\varepsilon$

3 are depicted as (u_L, u_M, u_R) in each of the four boxes shown below, where $(\phi, \epsilon, \eta) >> 0$. Assume that $\eta < 4\epsilon$. (MGW 8.C.4)

(a) Argue that (pure) strategy M is never a best response for player 1 to any independent randomizations by players 2 and 3.

(b) Show that (pure) strategy M is not strictly dominated.

(c) Show that (pure) strategy M can be a best response if player 2's and player 3's randomizations are allowed to be correlated.

Problema 5 Consider the following reduced form game, where rows are the strategies for player one and columns are the strategies for player two. The payoffs, as usual, are listed such that the first number represents the payoff of the player one and the second represents the payoff of player two.

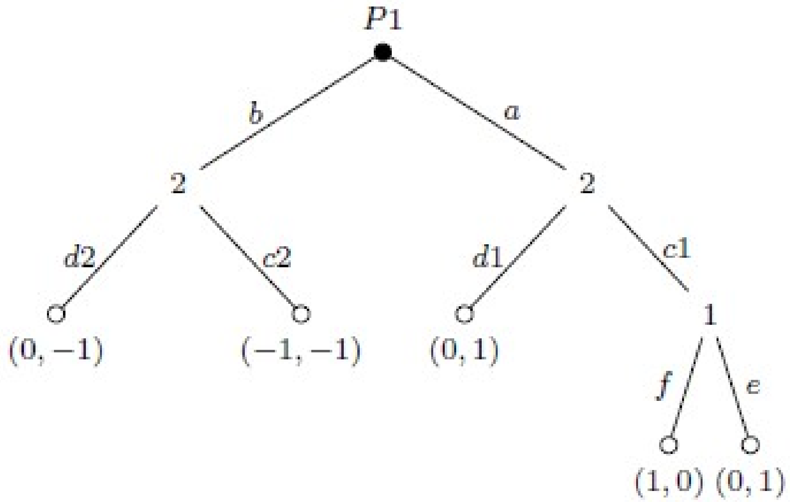
	0	1	2	3	4	5
0	4, 5	4, 14	4, 13	4, 12	4, 11	4, 10
1	13, 5	3, 4	3, 13	3, 12	3, 11	3, 10
2	12, 5	12, 4	2, 3	2, 12	2, 11	2, 10
3	11, 5	11, 4	11, 3	1, 2	1, 11	1, 10
4	10, 5	10, 4	10, 3	10, 2	0, 1	0, 10

(a) Apply the iterated elimination of strictly dominated pure strategies to find the equilibrium of the game. Say exactly in what order you eliminated rows and columns and the reasons for elimination.

(b) Apply the iterated elimination of weakly dominated pure strategies to find the equilibrium of the game. Say exactly in what order you eliminated rows and columns and the reasons for elimination.

(c) Determine the set of rationalizable pure strategies for this game. Say exactly why the set of pure strategies is rationalizable.

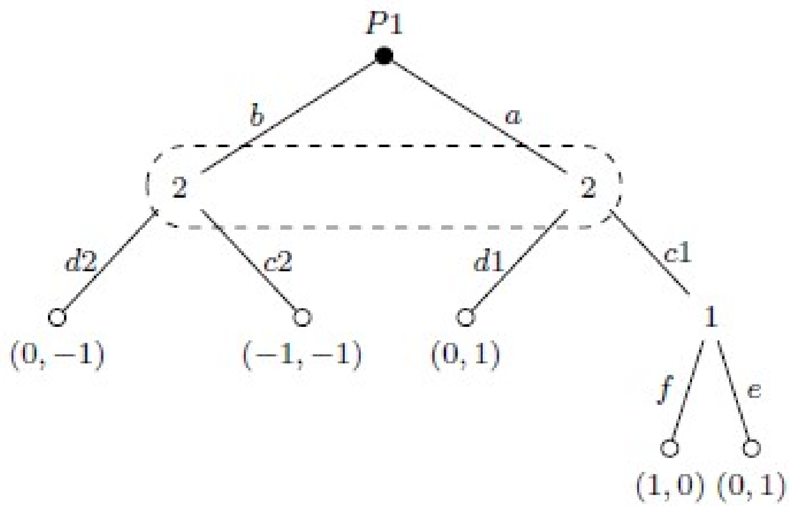
Problema 6 Considere o seguinte Jogo:



(a) Escreva o jogo na forma normal, enuncie as estratégias de cada um dos jogadores.

(b) Encontre os equilíbrios de Nash em Estratégias Puras

Considere agora a seguinte variação do jogo:



(c) Escreva o jogo na forma normal, enuncie as estratéegias de cada um dos jogadores.

(d) Encontre os equilíbrios de Nash em Estratéegias Puras

Problema 7 Considere a seguinte matriz de payoffs:

		Jogador 2	
		L	H
Jogador 1	L	$2, 4$	$1, 0$
	H	$0, 1$	$4, 5$

(a) Encontre os equilíbrios de Nash em estratégias puras.

(b) Encontre os equilíbrios de Nash em estratégias mistas.

Problema 8 Considere a seguinte matriz de payoffs:

		Jogador 2	
		D	H
Jogador 1	D	$1/2, 1/2$	$0, 1$
	H	$1, 0$	$1/2(1-c), 1/2(1-c)$

(a) Encontre os equilíbrios de Nash em estratégias puras e mistas quando $c < 1$.

(b) Encontre os equilíbrios de Nash em estratéegias puras e mistas quando $c > 1$.

(c) Encontre os equilíbrios de Nash em estratégias puras e mistas quando $c = 1$.

Problema 9 Consider a first-price sealed-bid auction of an object with two bidders. Each bidder i's valuation of the object is v_i, which is known to both bidders. The auction rules are that each player submits a bid in a sealed envelope. The envelopes are then opened, and the bidder who has submitted the highest bid gets the object and pays the auctioneer the amount of his bid. If the bidders submit the same bid, each gets the object with probability $\frac{1}{2}$. Bids must be in dollar multiples (assume that valuations are also). (MGW 8.D.3)

(a) Are any strategies strictly dominated?

(b) Are any strategies weakly dominated?

(c) Is there a Nash equilibrium? What is it? Is it unique?

Problema 10 Consider any two-player of the following form (where letters indicate arbitrary payoffs):

		Player 2	
		b_1	b_2
Player 1	a_1	u, v	l, m
	a_2	w, x	y, z

Show that a mixed strategy Nash equilibrium always exists in this game. [Hint: Define player I's strategy to be his probability of choosing action a_1 and player 2's to be his probability of choosing b_1; ten examine the best-response correspondences of the two players.] (MGW 8.D.6)

Problema 11 Consider the following strategic situation. Two opposed armies are poised to seize an island. Each army's general can choose either "attack"or "not attack". In addition, each any army is either "strong"or "weak"with equal probability (the draws for each army are independent), and an army's type is known only to its general. Payoffs are as follows: The island is worth M if captured. An army can capture the island either by attacking when its opponent does not or by attacking when its rival does if it is strong and its rival is weak. If two armies of equal strength both attack, neither captures the island. An army also has a "cost"of fighting, which is s if it is strong and w if it is weak, where $s < W$. There is no cost of attacking if its rival does not. Identify all pure strategy Bayesian Nash equilibria of this game. (MGW 8.E.1)

Problema 12 Consider the first-price sealed bid auction of Exercise 8.D.3, but now suppose that each bidder i observes only his own valuation v_i. This valuation is distributed uniformly and independently on $[0, \overline{v}]$ for each bidder. (MGW 8.E.2)

(a) Derive a symmetric (pure strategy) Bayesian Nash equilibrium of this auction. (You should now suppose that bids can be any real number.) [Hint: Look for an equilibrium in which bidder i's bid is a linear function of his valuation.]

(b) What if there are I bidders? What happens to each bidder's equilibrium bid function $s(v_i)$ as I increases?

Problema 13 Consider the linear Cournot model described in Exercise 8.B.5. Now, however, suppose that each firm has probability μ of having unit costs of c_L and $(1 - \mu)$ of having unit costs of c_H, where $c_H > c_L$. Solve for the Bayesian Nash equilibrium. (MGW 8.E.3)