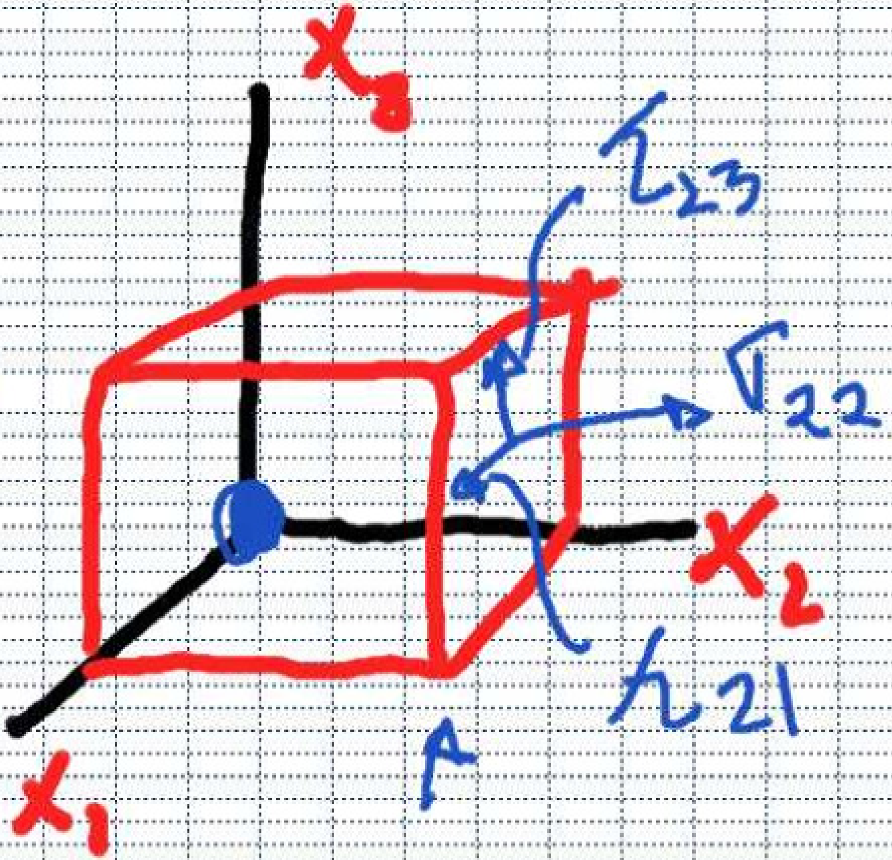
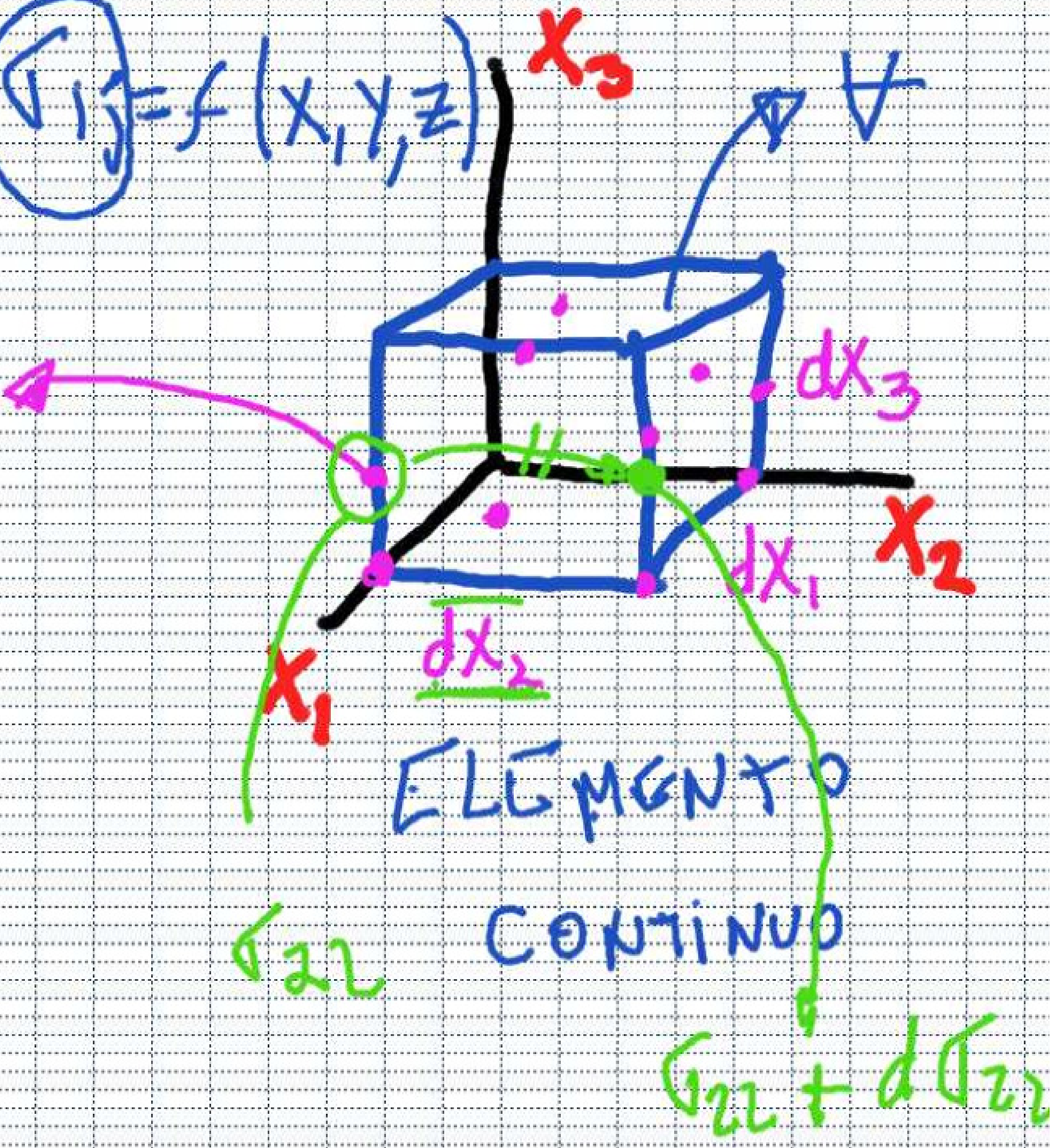
$\sigma_{ij} = f(x, y, z)$



REPRESENTAÇÃO
PONTO CONTÍNUO



ELEMENTO
CONTINUO

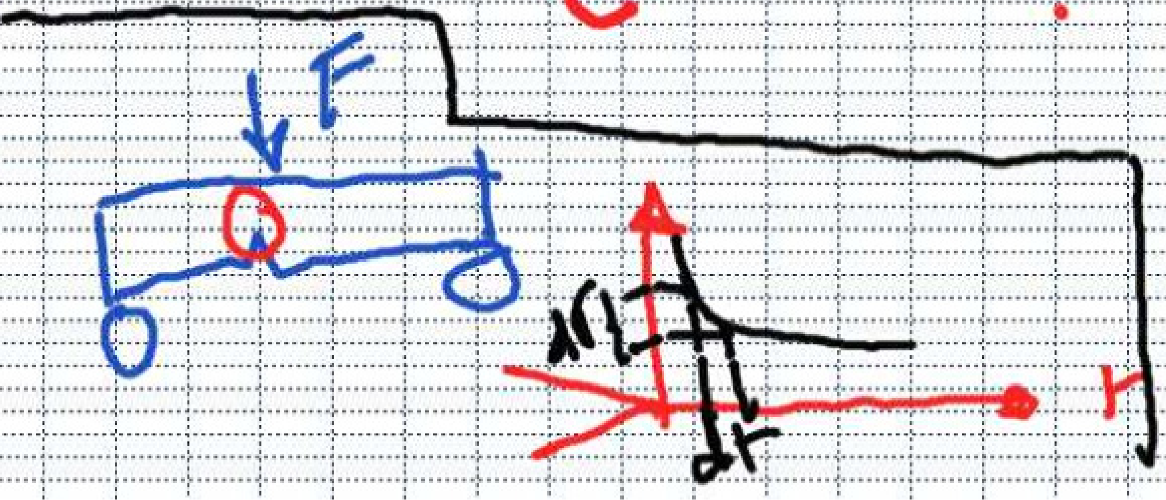
EXEMPLO

$$\sigma_{ij} = \begin{bmatrix} 21 & -63 & 42 \\ -63 & 0 & 84 \\ 42 & 84 & -21 \end{bmatrix} \quad [MPa]$$

$\rightarrow$ TENSOR CAUCHY

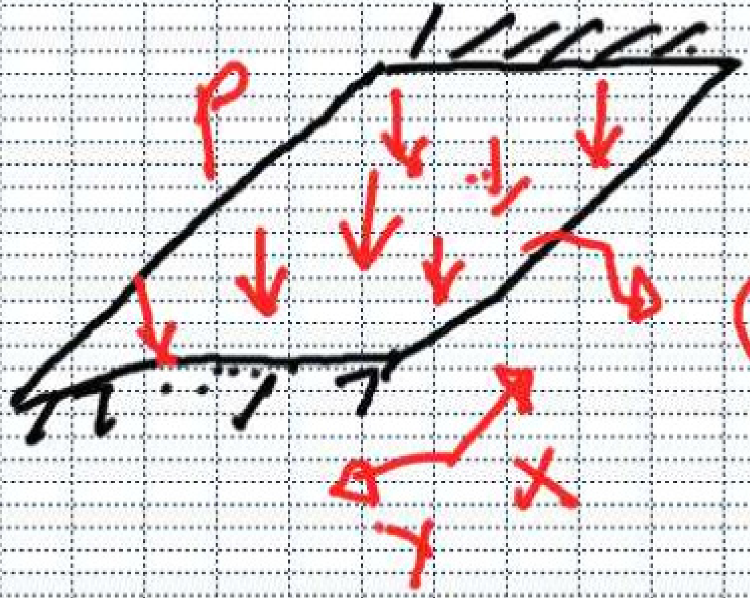
$$\hat{n} = 3\hat{e}_1 + 6\hat{e}_2 - 2\hat{e}_3$$

$$\vec{t}^{(\hat{n})} = ? \rightarrow \vec{t} = \begin{bmatrix} t_1 \\ t_2 \\ t_3 \end{bmatrix}$$



$$\sigma_{ij} = \begin{bmatrix} \dfrac{W\cdot(L-x)}{S} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$S = \frac{I_z}{Y}$$



$t << a, b$

$$\sigma_{ij} = \begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{bmatrix}$$

$$|\hat{n}| = 1$$

$$\hat{n} = (3, 6, -2)$$

$$\frac{\hat{n}}{|n|} = \frac{1}{\sqrt{9+36+4}} (3, 6, -2)$$

$$\hat{n} = (3/7, 6/7, -2/7)$$

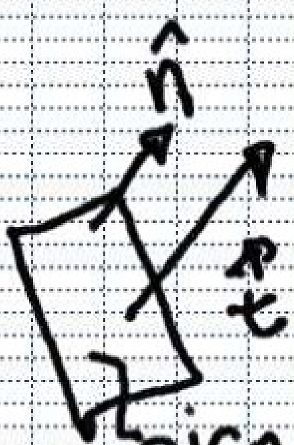
RELAÇÃO CAUCHY

$$\vec{t}^{(\hat{n})} = \sigma_{ij} \cdot \hat{n}$$

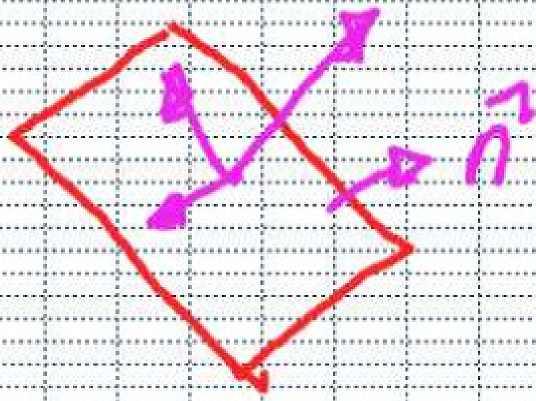
$$\begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} = \begin{bmatrix} 21 & -63 & 42 \\ -63 & 0 & 84 \\ 42 & 84 & -2 \end{bmatrix} \begin{bmatrix} 3/7 \\ 6/7 \\ -2/7 \end{bmatrix}$$

$\vec{t} \parallel \hat{n}$

$$t_x = 21 \cdot \frac{3}{7} - 63 \cdot \frac{6}{7} - 42 \cdot \frac{2}{7}$$

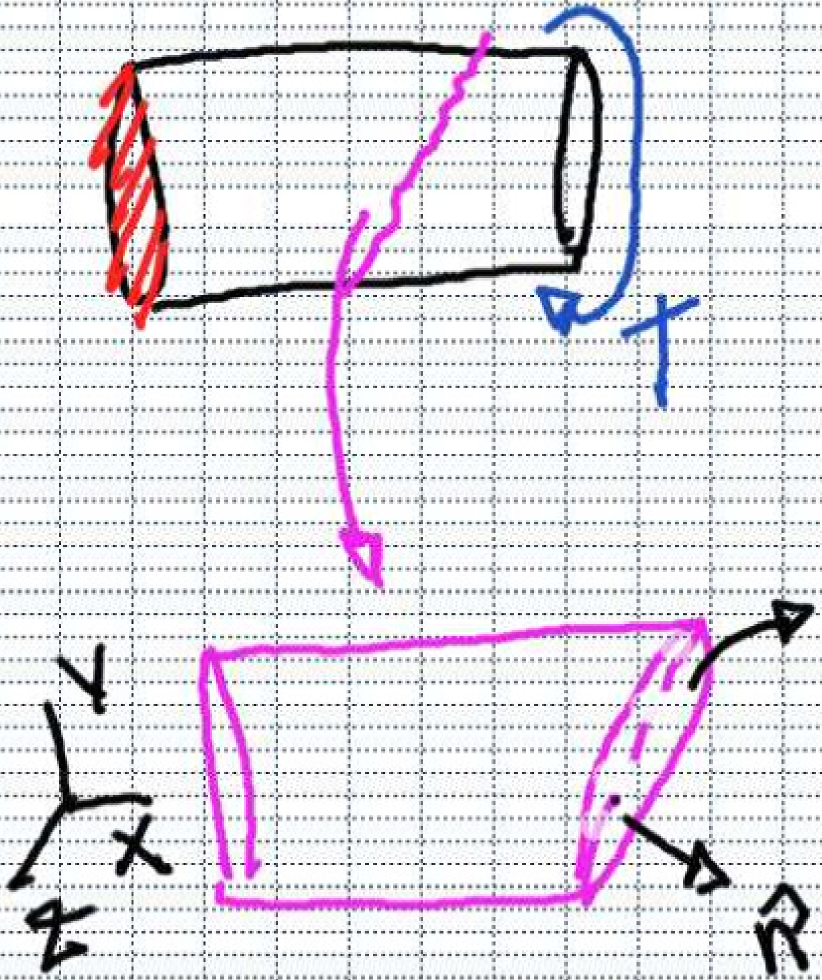


$$\left. \begin{aligned} t_x &= -57 \\ t_y &= -51 \\ t_z &= 96 \end{aligned} \right\} [MPa]$$



CISALHAMENTO NULO!

TENSÕES PRINCIPAIS / DIREÇÕES PRINCIP.



$T \leq \beta$ [N-m]

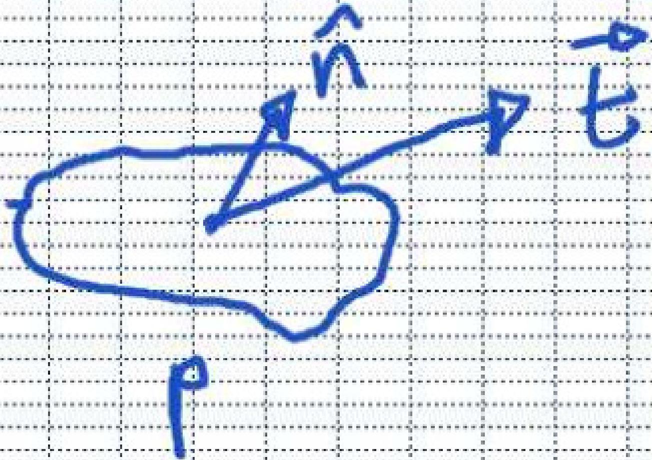
$r \geq \alpha$ [mm]

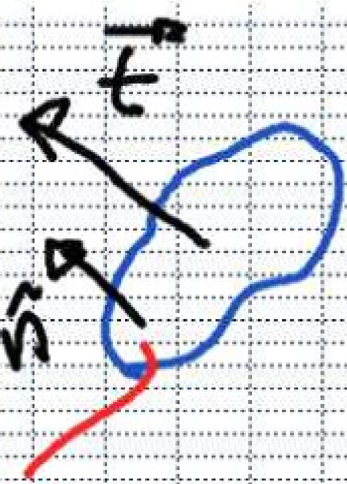
PLANO COM MÁX. TENSÕES

$\left.\begin{array}{l} \hat{n} \\ \sigma_{max} \end{array}\right\} ?$

$\hat{n} = (n_x, n_y, n_z)$

$\sigma_{max} \overset{<}{\text{vs.}} \sigma_c^{MAT}$



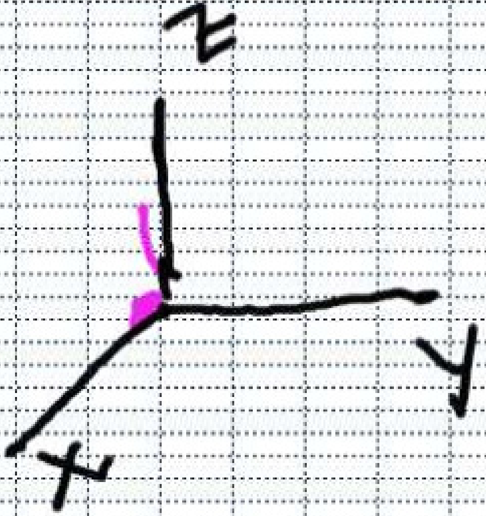


$$\underline{\vec{t} = \lambda \hat{n}}$$

cisalhamento NULO

$\pi \equiv$ plano principal

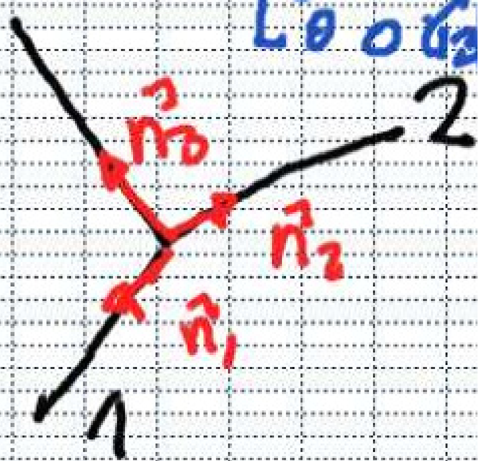
$\lambda \equiv$ tensão principal

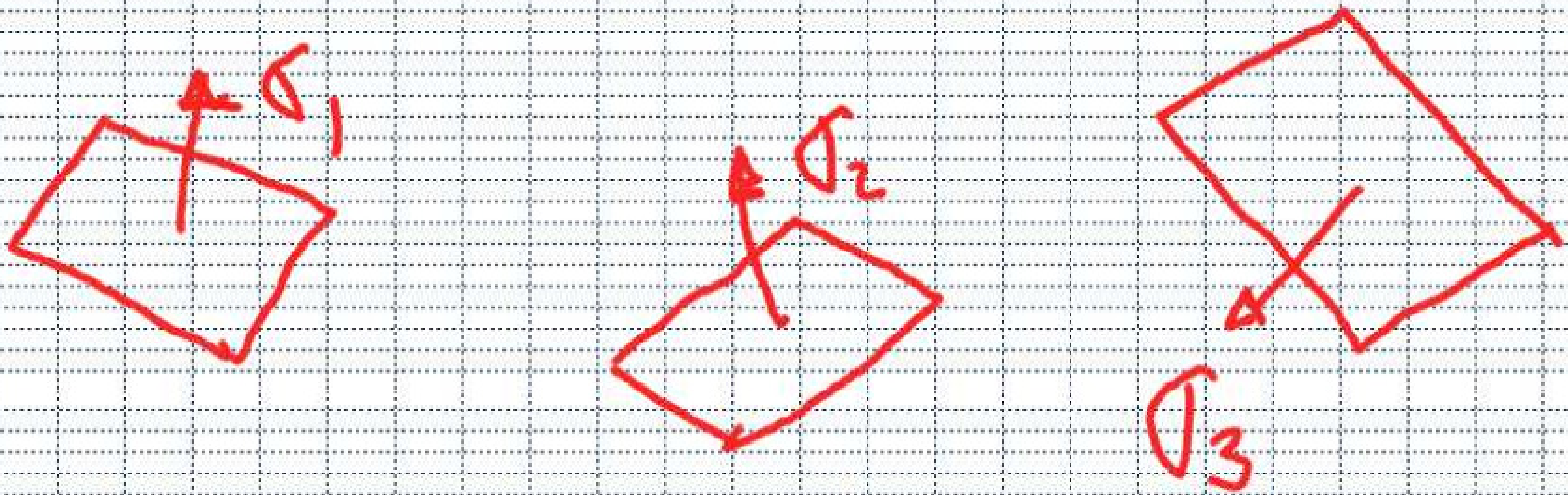


$$\sigma_{ij} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ & \sigma_y & \tau_{yz} \\ & & \sigma_z \end{bmatrix}$$

$$\sigma_{ij} = \sigma_{ji}$$

$$\sigma_{ij} = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}$$





$$\sigma_{ij} = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix} = \underline{\text{base}}\,!$$

$$\vec{t}^{(\vec{n})} = \lambda \hat{n} \quad (1)$$

$$\vec{t}^{(\vec{n})} = \sigma_{ij} \cdot \hat{n} \quad (2)$$

$$\downarrow$$

$$\sigma_{ij} \hat{n} = \lambda \hat{n}$$

EXISTE λ ?

3x1 3x1

$$(\sigma_{ij} - \lambda I) \hat{n} = \vec{0}$$

$n_1^2 + n_2^2 + n_3^2 = 1$ $\oplus$

- 3 EQUAÇÕES
- 4 INCOGNITAS (n_1, n_2, n_3, λ)

$(\sigma_{ij} - \lambda I)\,\hat{n} = \vec{0}$

→ $\hat{n} = (0, 0, 0)$ é solução

→ $\det[A] \neq 0$ → sol. única

↳ $\det[\sigma_{ij} - \lambda I] = 0$

↳ INFINITAS SOLUÇÕES

$|\hat{n}| = 1$

$$\det \begin{bmatrix} \sigma_x - \lambda & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y - \lambda & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z - \lambda \end{bmatrix} = 0$$

\# Real / escalar

$\sigma_{ij} \equiv$ conhecidos

$$\sigma_x, \sigma_y, \sigma_z, \tau_{xy}, \tau_{xz}, \tau_{yz}$$

$$\underline{\lambda = ?}$$

- TENSÕES PRINCIPAIS / DIREÇÕES PRINCIPAIS

CONHECIDO σ_{ij} no ponto $\longrightarrow \begin{cases} \sigma_1, \sigma_2, \sigma_3 \\ \pi_1, \pi_2, \pi_3 \end{cases}$

$$\Big\downarrow$$

$$\det \begin{bmatrix} \sigma_x - \lambda & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y - \lambda & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z - \lambda \end{bmatrix} = 0$$

$\lambda = ?$

$|\vec{n}| = 1$

$\tau_{xy} = \tau_{yx}$

$\tau_{yz} = \tau_{zy}$

$= \tau_{xz}$

$$\lambda^3 - I_1 \lambda^2 + I_2 \lambda - I_3 = 0$$

$\hookrightarrow p(\lambda) \equiv$ polin. característico

$$I_1, I_2, I_3 \equiv \text{Invariantes do tensor das tensões de Cauchy}$$

$$I_1 = \sigma_x + \sigma_y + \sigma_z$$

$$I_2 = \sigma_x \sigma_y + \sigma_y \sigma_z + \sigma_x \sigma_z - \tau_{xy}^2 - \tau_{xz}^2 - \tau_{yz}^2$$

$$I_3 = \det[\sigma_{ij}]$$

$$I_3 = \sigma_x \sigma_y \sigma_z + 2 \tau_{xy} \tau_{xz} \tau_{yz} - \tau_{xy}^2 \sigma_z - \tau_{yz}^2 \sigma_x - \tau_{xz}^2 \sigma_y$$

$$\sigma_{ij} = \begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ & \sigma_y & \tau_{yz} \\ & & \sigma_z \end{bmatrix} \quad \text{vs} \quad \sigma'_{ij} = \begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}$$

$$\sigma_{ij} = \sigma_{ji}$$

$$I_1 = \sigma_x + \sigma_y + \sigma_z \qquad I_1 = \sigma_1 + \sigma_2 + \sigma_3$$

$$\sigma_x + \sigma_y + \sigma_z = \sigma_1 + \sigma_2 + \sigma_3$$

$p(\lambda) = 0 \rightarrow$ raízes $\rightarrow$ $\lambda_1 = \sigma_1$

$\lambda_2 = \sigma_2$

$\lambda_3 = \sigma_3$

USUALMENTE

$\sigma_1 \geq \sigma_2 \geq \sigma_3$

$\downarrow$

$\Pi_1 \ , \Pi_2 \ , \Pi_3$

$\downarrow$ σ_i no sistema Eq.

$\underline{n_x^2 + n_y^2 + n_z^2 = 1}$ $\oplus$ $\underline{[\sigma_{ij} - \lambda_i I]}\, \hat{n}_i = \vec{0} \rightarrow \hat{n} = \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix}$

$\sigma_1 \rightarrow \hat{n}_1 = (n_x, n_y, n_z)$

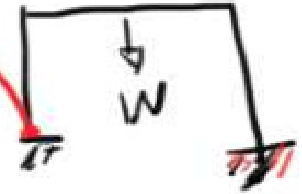
$\sigma_2 \rightarrow \hat{n}_2 = (n_x, n_y, n_z)$

$\downarrow$

$\hat{n}_3 = \hat{n}_1 \times \hat{n}_2$

EXEMPLO

$$\sigma_{ij} = \begin{bmatrix} -1 & 2\sqrt{2} & 0 \\ 2\sqrt{2} & -3 & 0 \\ 0 & 0 & 2 \end{bmatrix} \quad [MPa)$$



$\hookrightarrow$ EPD

Determine

a) Invariantes I_1, I_2, I_3 ✓ $\quad \varepsilon_z = 0 \rightarrow \sigma_z$

b) tensões principais ✓

c) Direções principais

$I_3 = -10$ MPa

solução

$$I_1 = \sigma_x + \sigma_y + \sigma_z$$

$$I_1 = -1 - 3 + 2$$

$$I_1 = -2 \text{ MPa}$$

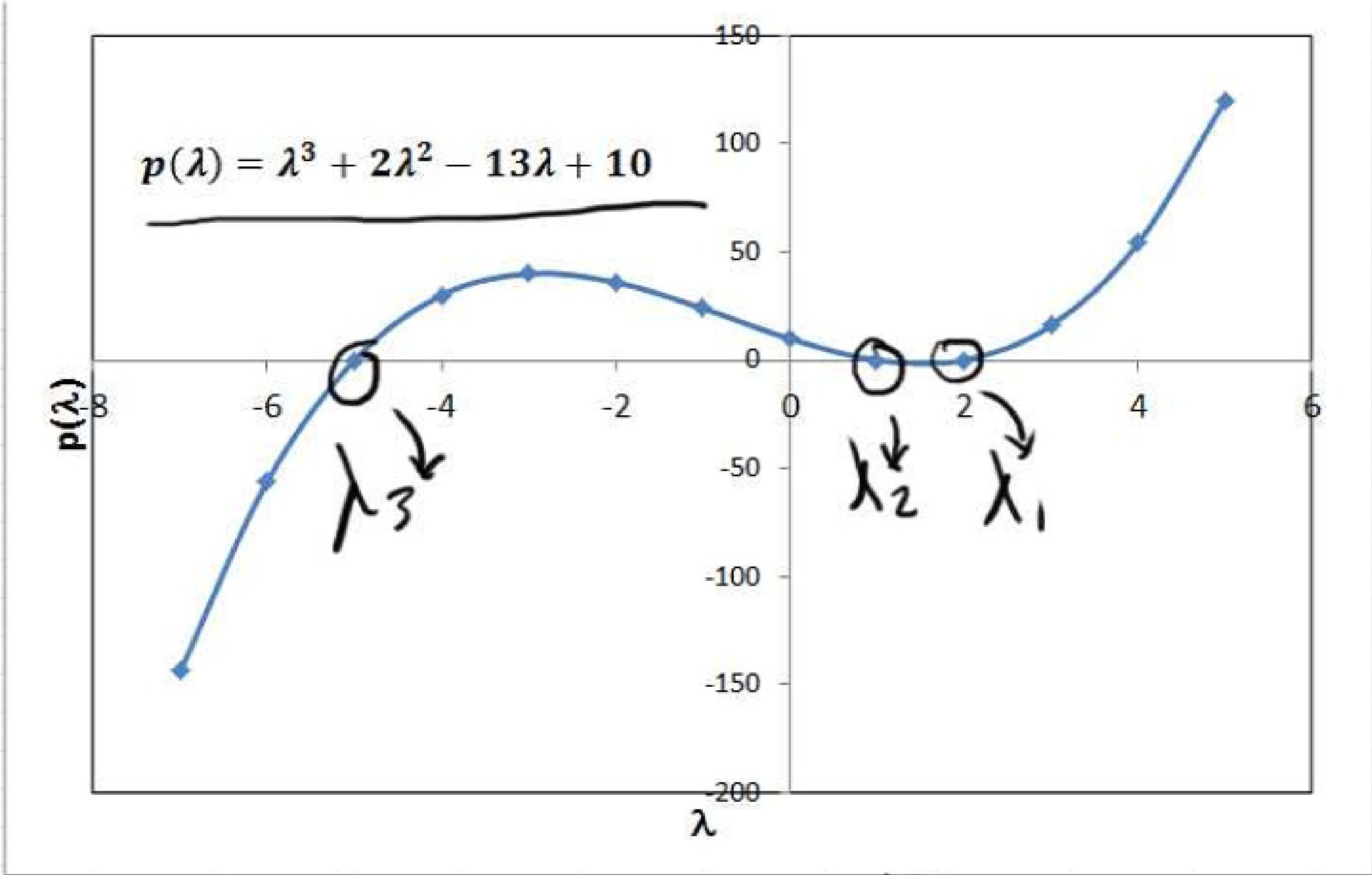
$$I_2 = (-1)(-3) + (-3)(2) + (-1)(2) - (2\sqrt{2})^2 - 0 - 0$$

$$I_2 = 3 - 6 - 2 - 8$$

$$I_2 = -13 \text{ MPa}^2$$

roots $p(\lambda)$

λ	$p(\lambda)$
-7	-144
-6	-56
-5	0
-4	30
-3	40
-2	36
-1	24
0	10
1	0
2	0
3	16
4	54
5	120



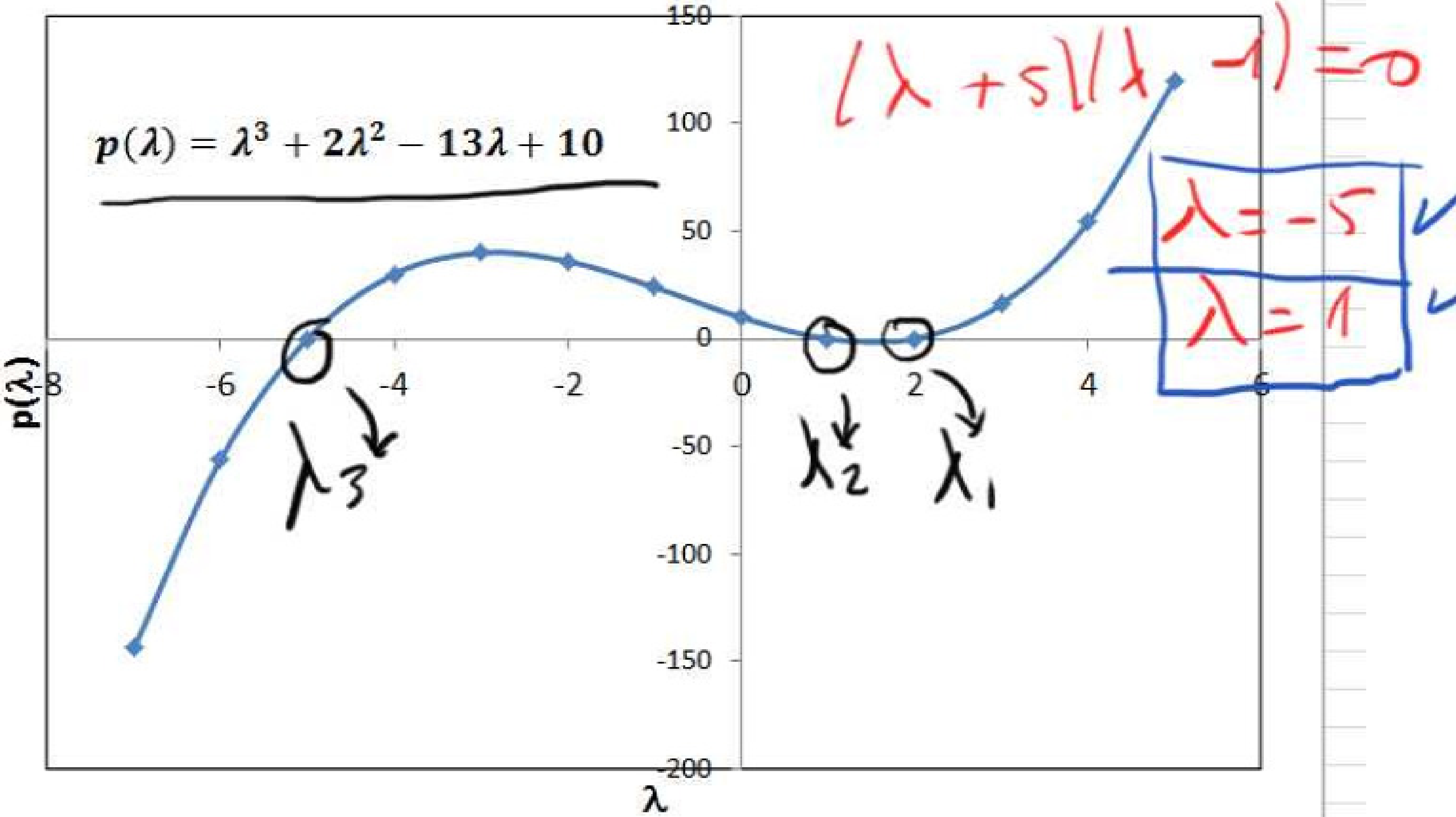
$$\det \begin{bmatrix} -1-\lambda & 2\sqrt{2} & 0 \\ 2\sqrt{2} & -3-\lambda & 0 \\ 0 & 0 & 2-\lambda \end{bmatrix} = 0$$

$$(2-\lambda)\Big[\underbrace{(-1-\lambda)(-3-\lambda) - 2\sqrt{2}\cdot 2\sqrt{2}}_{\text{roots } p(\lambda)}\Big] = 0$$

$$\rightarrow \lambda^2 + 4\lambda + 3 - 8 = 0$$

$$\boxed{\lambda = 2}$$

λ	p(λ)
-7	-144
-6	-56
-5	0
-4	30
-3	40
-2	36
-1	24
0	10
1	0
2	0
3	16
4	54
5	120



$$\det\begin{bmatrix} -1-\lambda & 2\sqrt{2} & 0 \\ 2\sqrt{2} & -3-\lambda & 0 \\ 0 & 0 & 2-\lambda \end{bmatrix} = 0$$

⊛ DIREÇÕES PRINCIPAIS

$$\sigma_1 = 2 \qquad \sigma_2 = 1 \qquad \sigma_3 = -5 \qquad [\text{MPa}]$$

$$\begin{bmatrix} -1-2 & 2\sqrt{2} & 0 \\ 2\sqrt{2} & -3-2 & 0 \\ 0 & 0 & 2-2 \end{bmatrix} \begin{bmatrix} n_x \\ n_y \\ n_z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

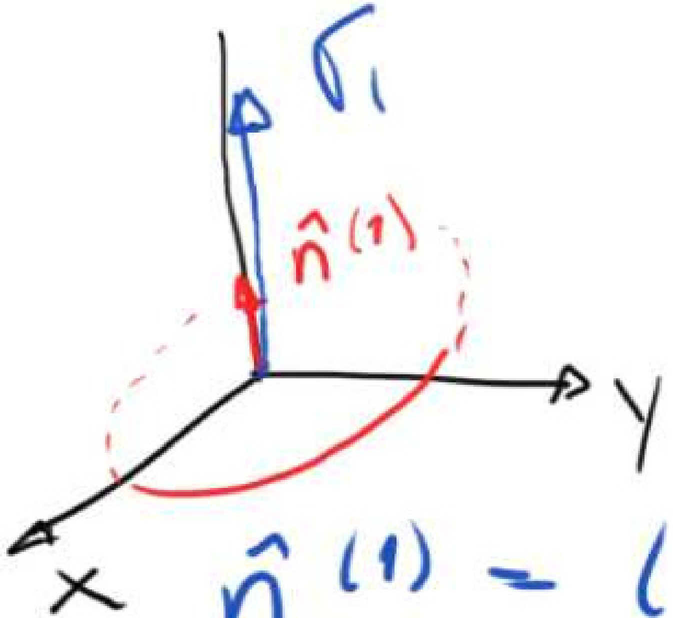
REAL $|\vec{n}| = 1$

$$\vec{t} = \lambda \hat{n} \rightarrow (\sigma_{ij} - \lambda I)\vec{n} = \vec{0}$$

$$\vec{t} = \sigma_{ij} \cdot \vec{n}$$

$$\begin{bmatrix} -3 & 2\sqrt{2} \\ 2\sqrt{2} & -5 \end{bmatrix} \begin{bmatrix} n_x \\ n_y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \rightarrow \det \begin{vmatrix} -3 & 2\sqrt{2} \\ 2\sqrt{2} & -5 \end{vmatrix} \neq 0$$

$\hat{n}^{(1)} = (0, 0, 1)$ ← sol. única



$\sigma_1 = 2 \text{ MPa}$

$$\hat{n}^{(1)} = (0, 0, 1)$$

$$\hat{n}^{(2)} = \left(\sqrt{2/3}, \sqrt{1/3}, 0\right)$$

$$\hat{n}^{(3)} = \hat{n}^{(1)} \times \hat{n}^{(2)}$$

$$\hat{n}^{(3)} = \left(-\sqrt{1/3}, \sqrt{2/3}, 0\right)$$