AD e DA

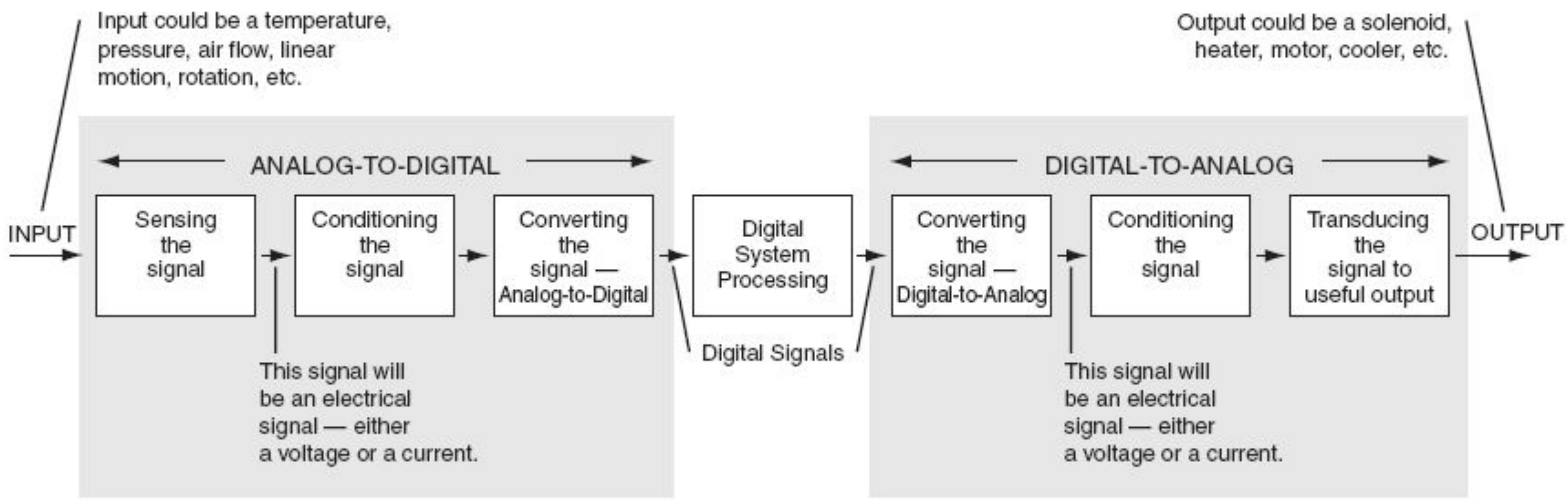
Conversão de Sinais

Aliasing e Reconstrução de Sinais

Daniel Varela Magalhães

Amostrar e Reconstruir?

- Armazenagem, transmissão, chaveamento, processamento e síntese digital;



Por que preocupar-se?

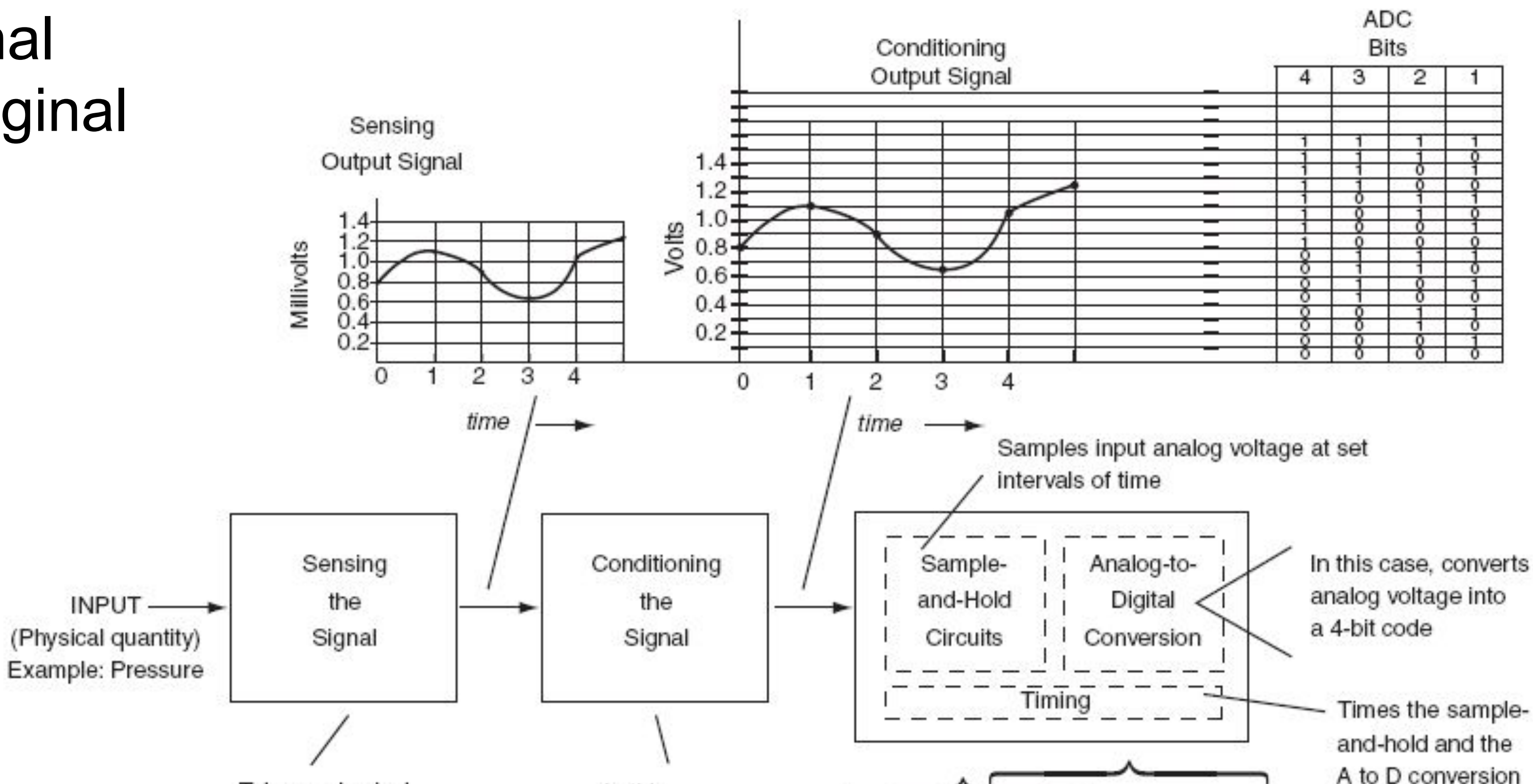
- Placas de aquisição de dados mais acessíveis;
- Conversores rápidos e menos custosos;
- Suavização em programas de visualização;

Por outro lado…

- Grande expansão de uC de baixo custo com AD's menos poderosos (...inos ou mesmo outros);
- Velocidade implica, normalmente, em maior consumo de energia;
- Demandas de IoT;

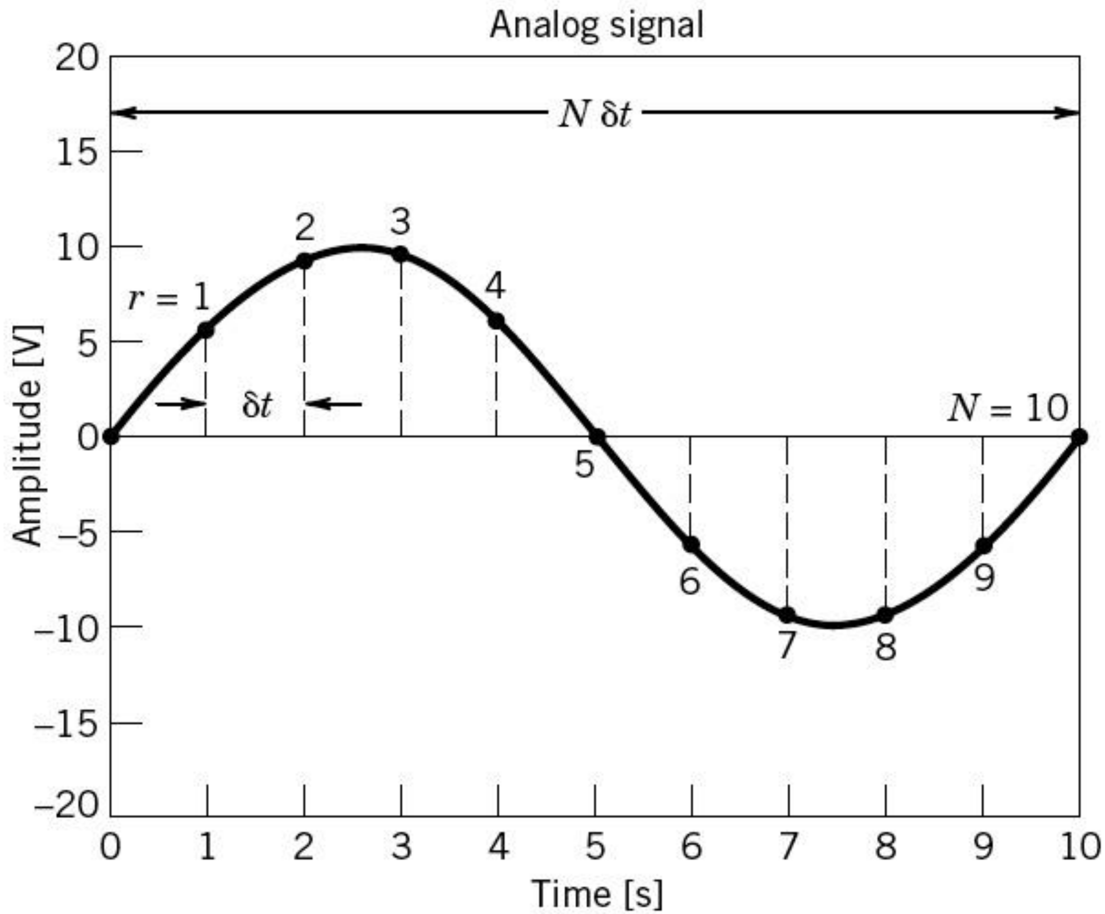
Como um sinal analógico é visto pelo AD

Sinal
Original



Sample	Value	Digital Code
0	0.8V	1000
1	1.1V	1011
2	0.9V	1001
3	0.65V	0110
4	1.05V	1010
5	1.25V	1100

Conversão para Sinal Digital



Discrete time signal

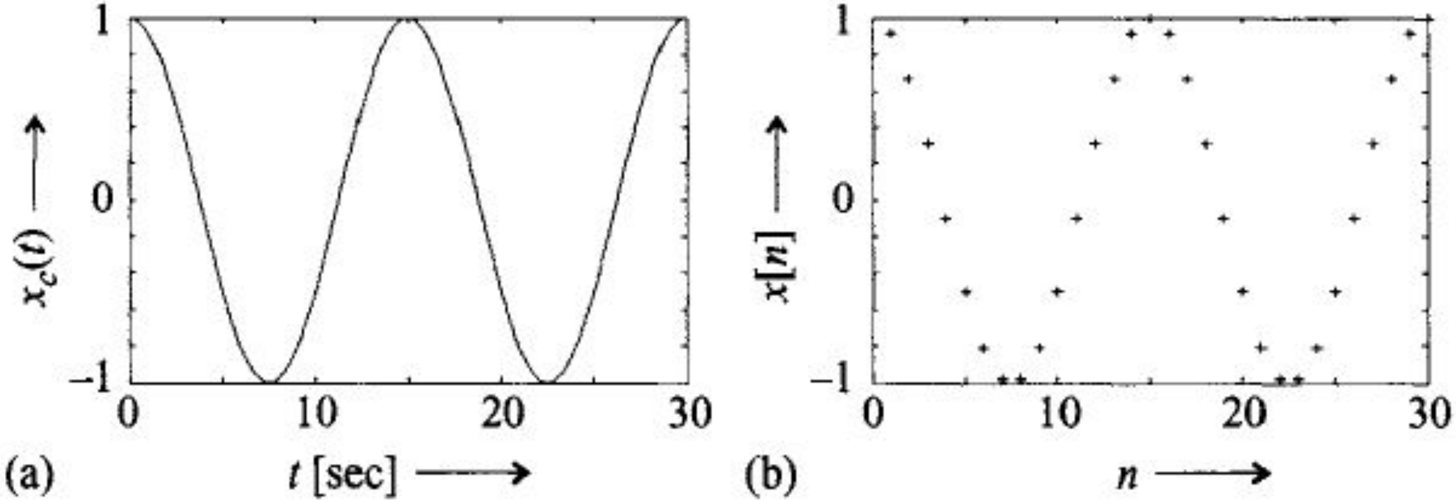
$\{y(r\delta t)\}$

r	Discrete data
0	0
1	5.9
2	9.5
3	9.5
4	5.9
5	0
6	–5.9
7	–9.5
8	–9.5
9	–5.9
10	0

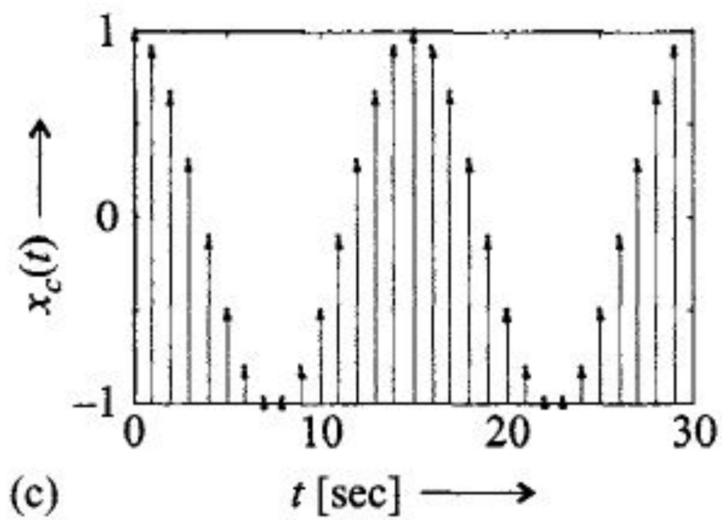
Figure 7.1 Analog and discrete representations of a time-varying signal.

Conversão para Sinal Digital

Formas de representar matematicamente após a amostragem

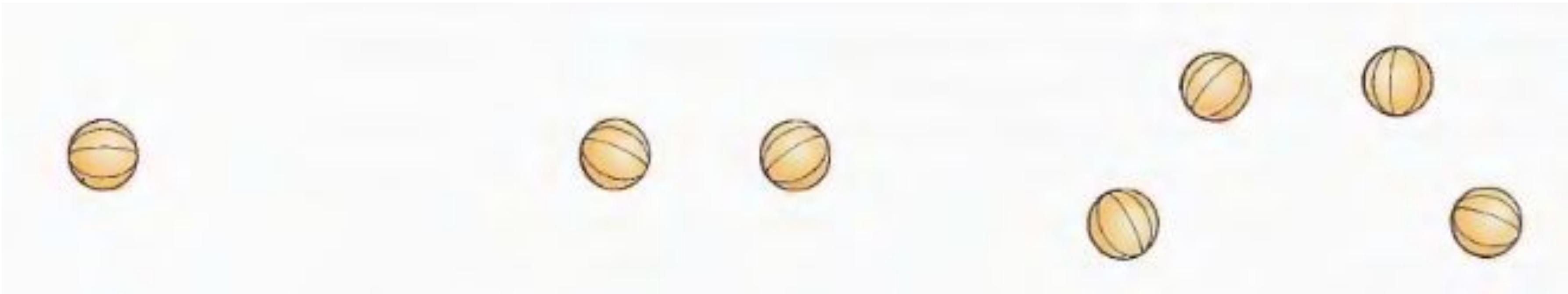


$$x[n] = x(nT), \; n \text{ integer}$$



$$x_P(t) = \sum_{n=-\infty}^{+\infty} x(nT)\, \delta(t - nT)$$

Diferentes tempos de amostragem para um mesmo sinal de entrada



Diferentes tempos de amostragem para um mesmo sinal de entrada

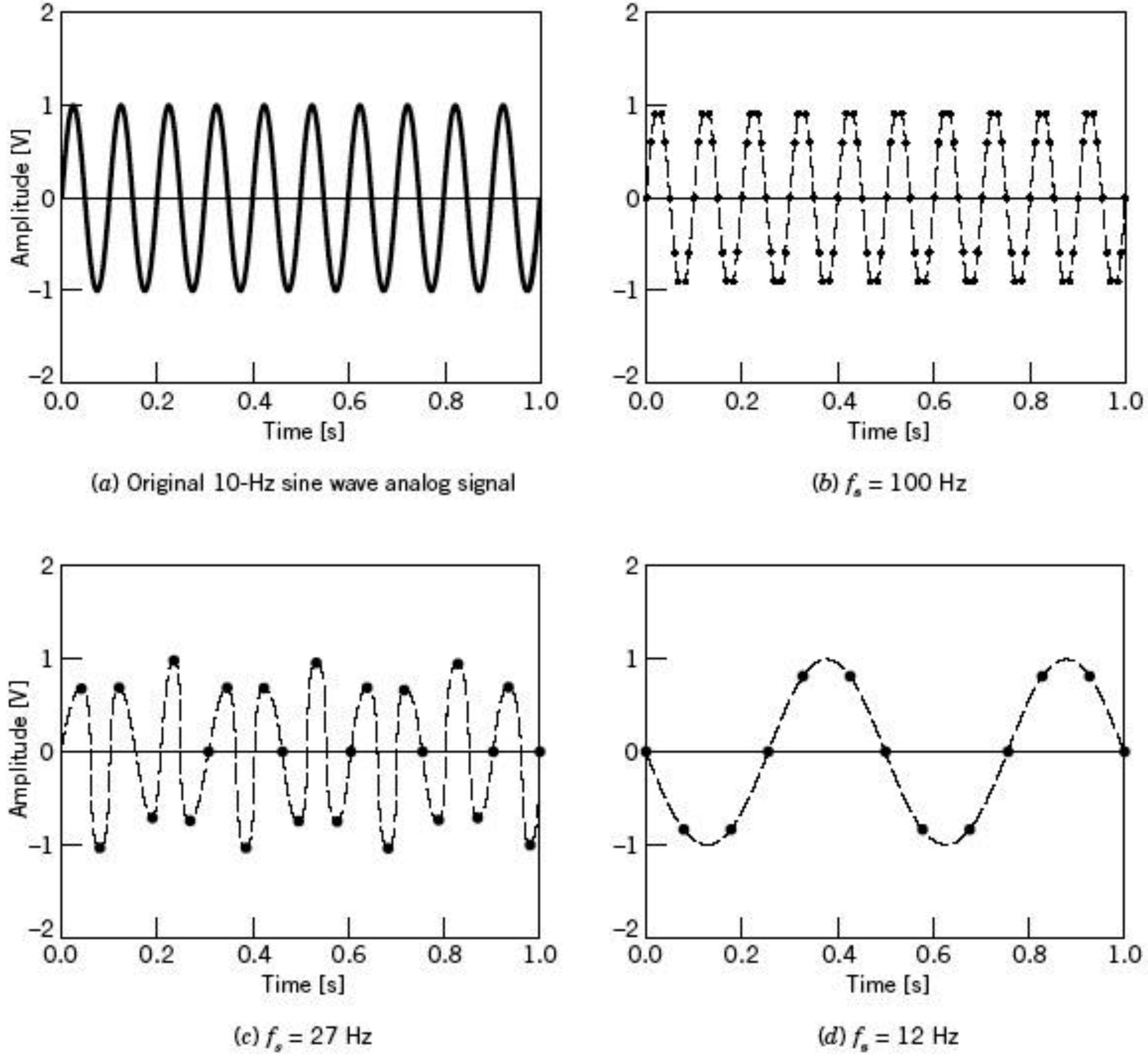


Figure 7.2 The effect of sample rate on signal frequency and amplitude interpretation.

Diferentes tempos de amostragem para um mesmo sinal de

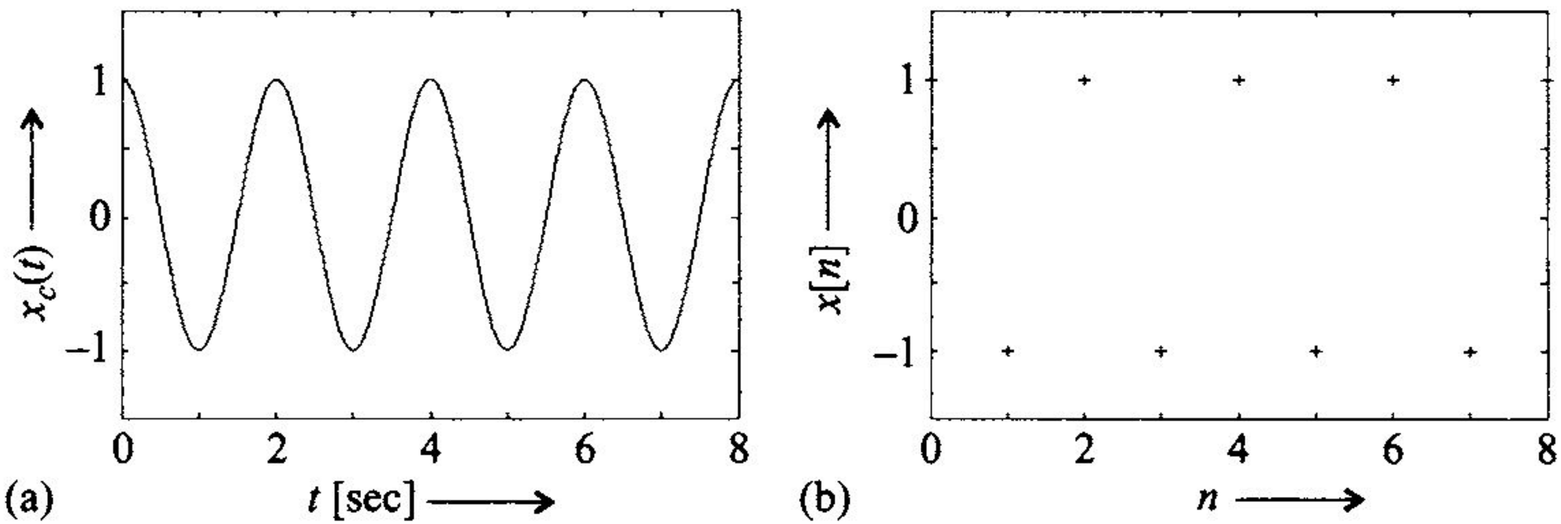
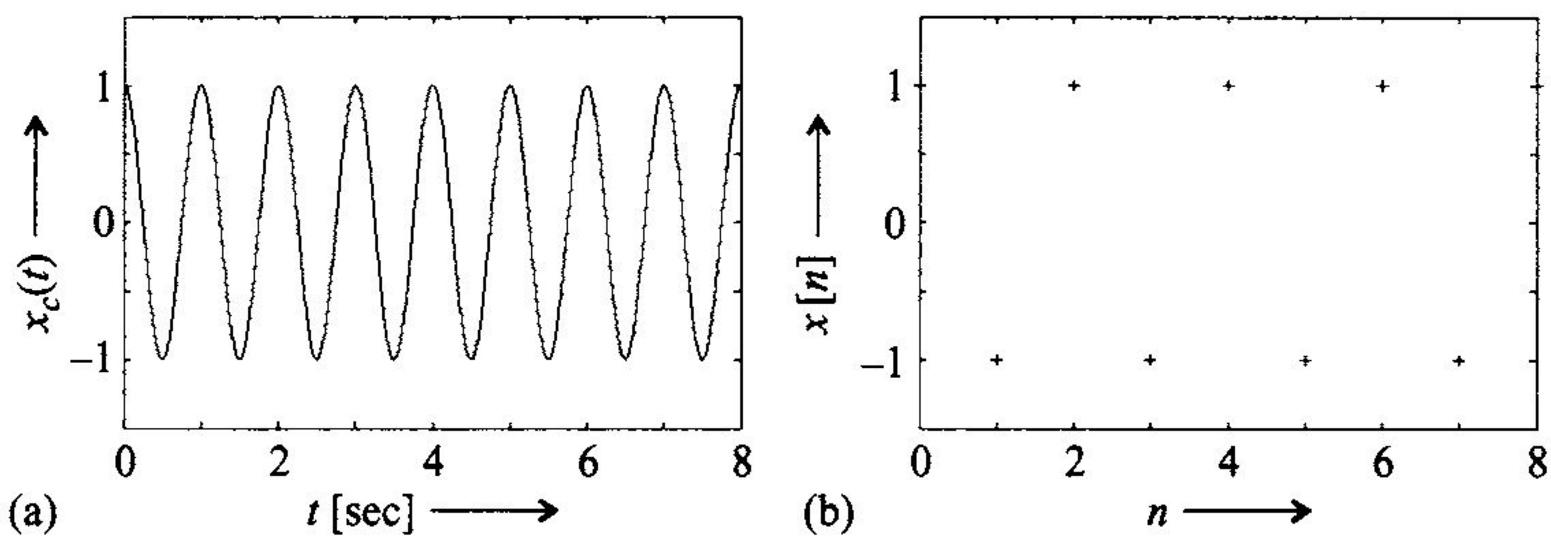
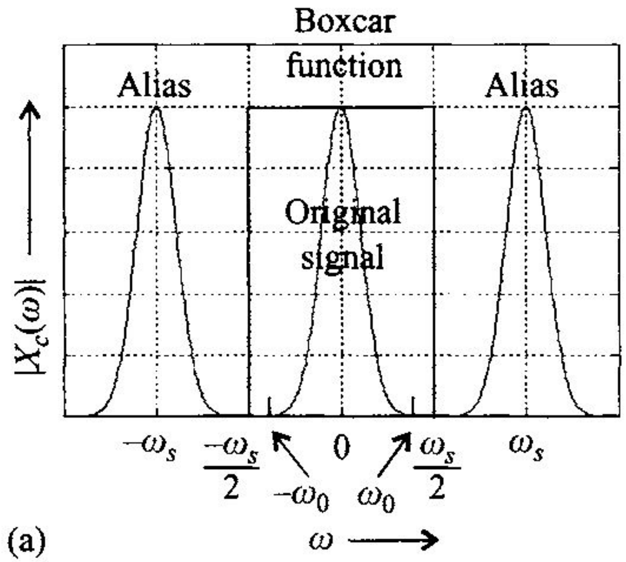


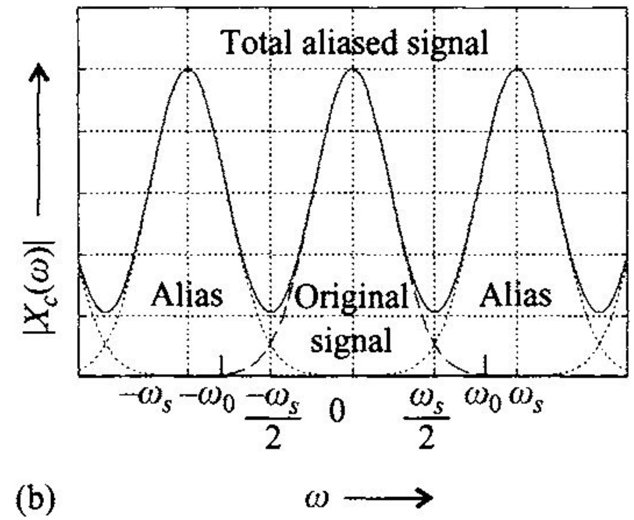
Figure 5.3 *(a)* $x_c(t) = \cos \pi t$; *(b) sampled signal with* $\Delta t = 1$



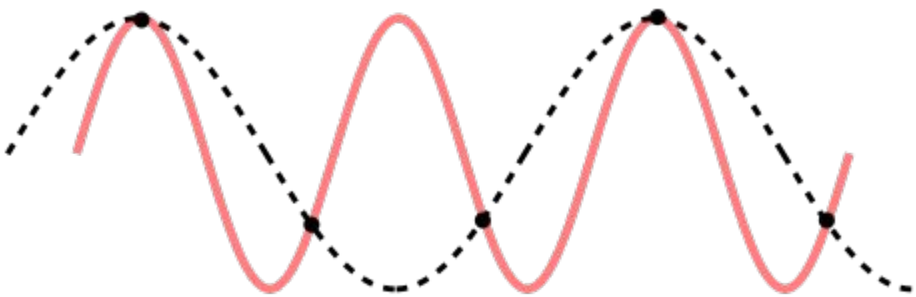
Definindo uma Taxa de Amostragem

Critério de Nyquist $f_{sampl} \geq 2 f_{cut}$

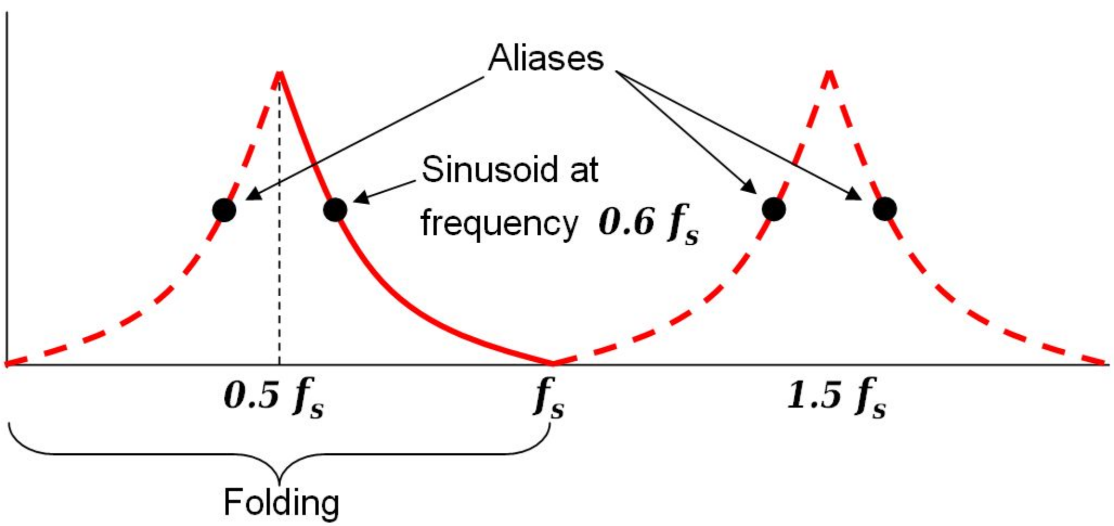


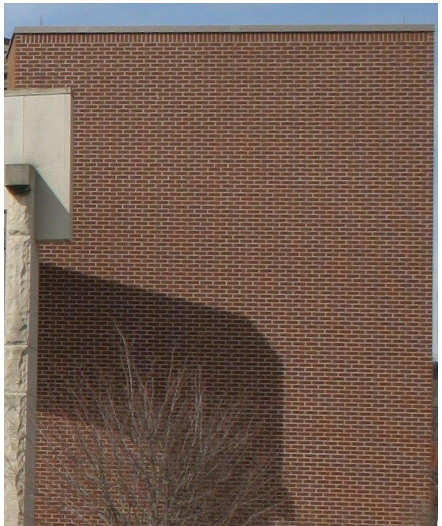


Cuidados com Aliasing



$$f_{\text{alias}}(N) \stackrel{\text{def}}{=} |f - N f_s|,$$





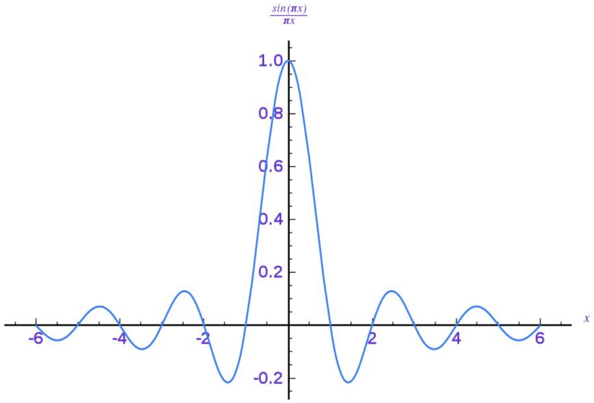
Evitando “Falsos” Sinais

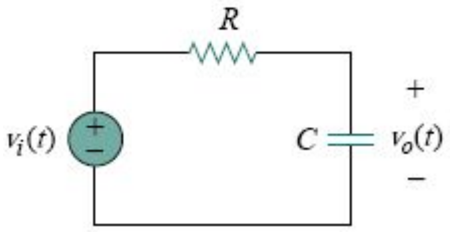
Sobre-amostragem

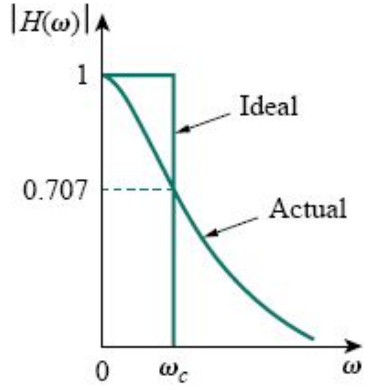
Filtros anti-aliasing

Tratamento com filtros reais

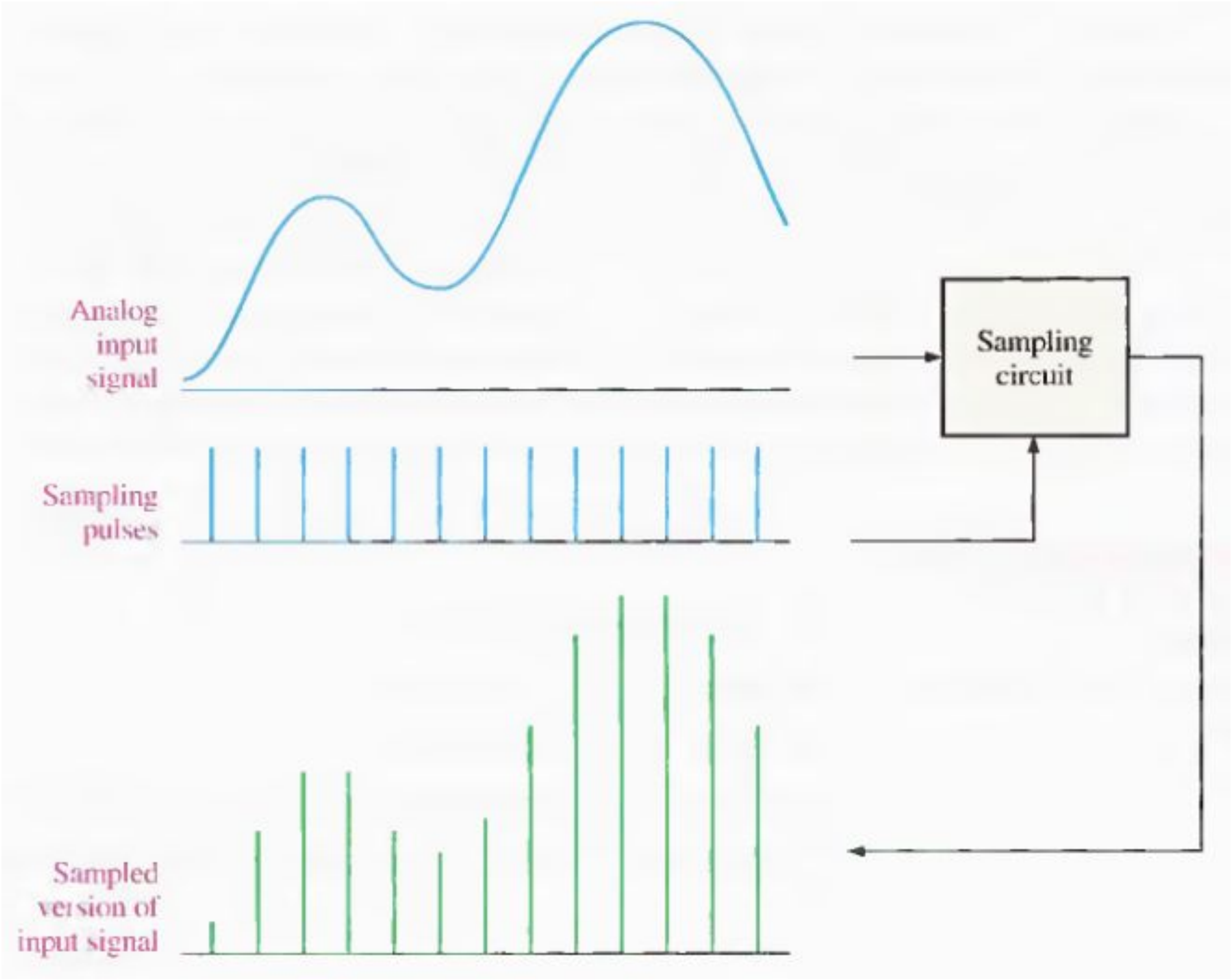
Custo-benefício para ordem dos filtros



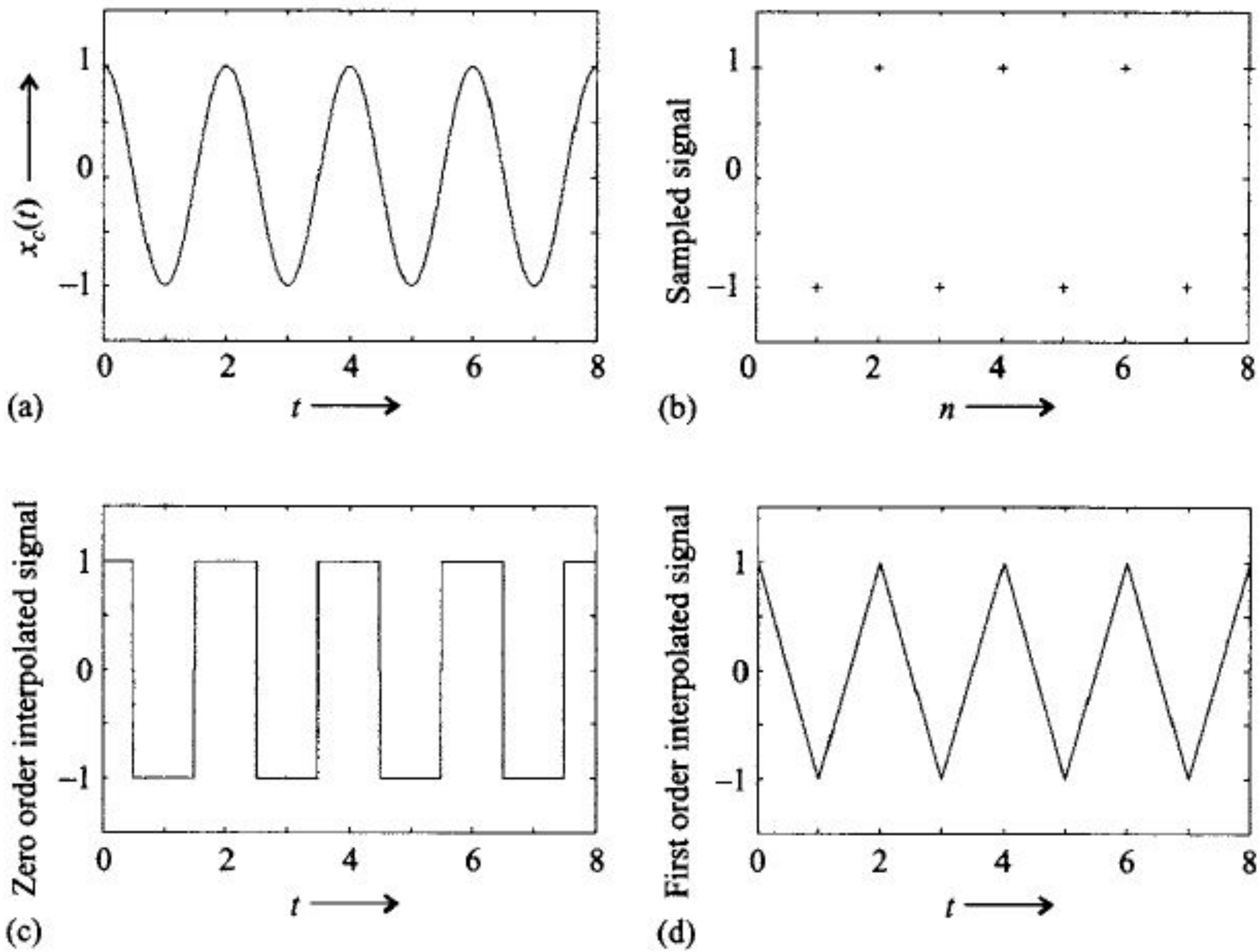




“Cara” do sinal amostrado



Maneiras de Visualizar o Sinal Interpolacão



Tática de Reconstrução

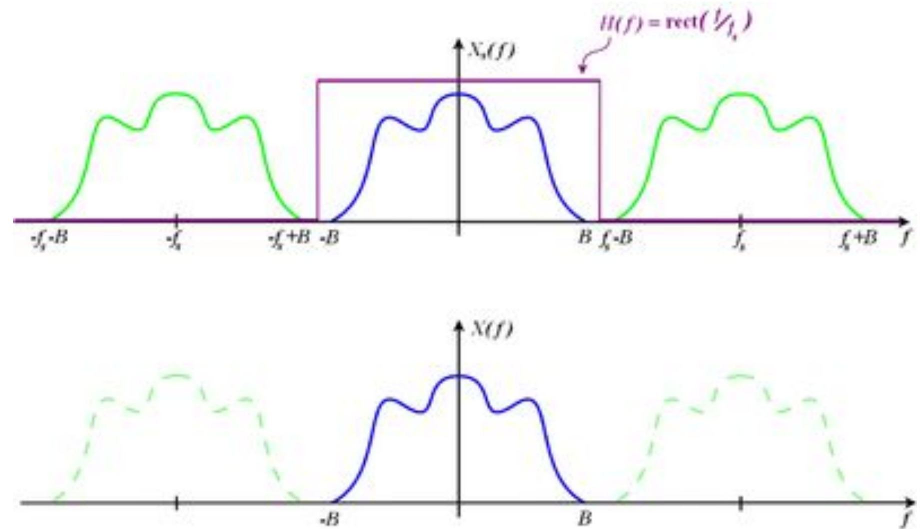
$$X(f) = H(f) \cdot X_s(f),$$

$$H(f) \stackrel{\text{def}}{=} \begin{cases} 1 & |f| < B \\ 0 & |f| > f_s - B. \end{cases}$$

$$H(f) = \mathrm{rect}\left(\frac{f}{f_s}\right) = \begin{cases} 1 & |f| < \frac{f_s}{2} \\ 0 & |f| > \frac{f_s}{2}, \end{cases}$$

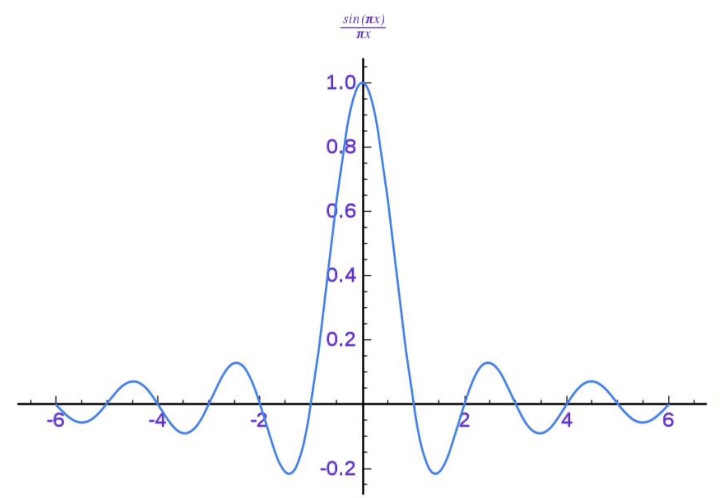
$$X(f) = \mathrm{rect}\left(\frac{f}{f_s}\right) \cdot X_s(f)$$

$$= \sum_{n=-\infty}^{\infty} x(nT) \cdot \underbrace{T \cdot \mathrm{rect}(Tf) \cdot e^{-i2\pi nTf}}_{\mathcal{F}\left\{\mathrm{sinc}\left(\frac{t-nT}{T}\right)\right\}}.$$

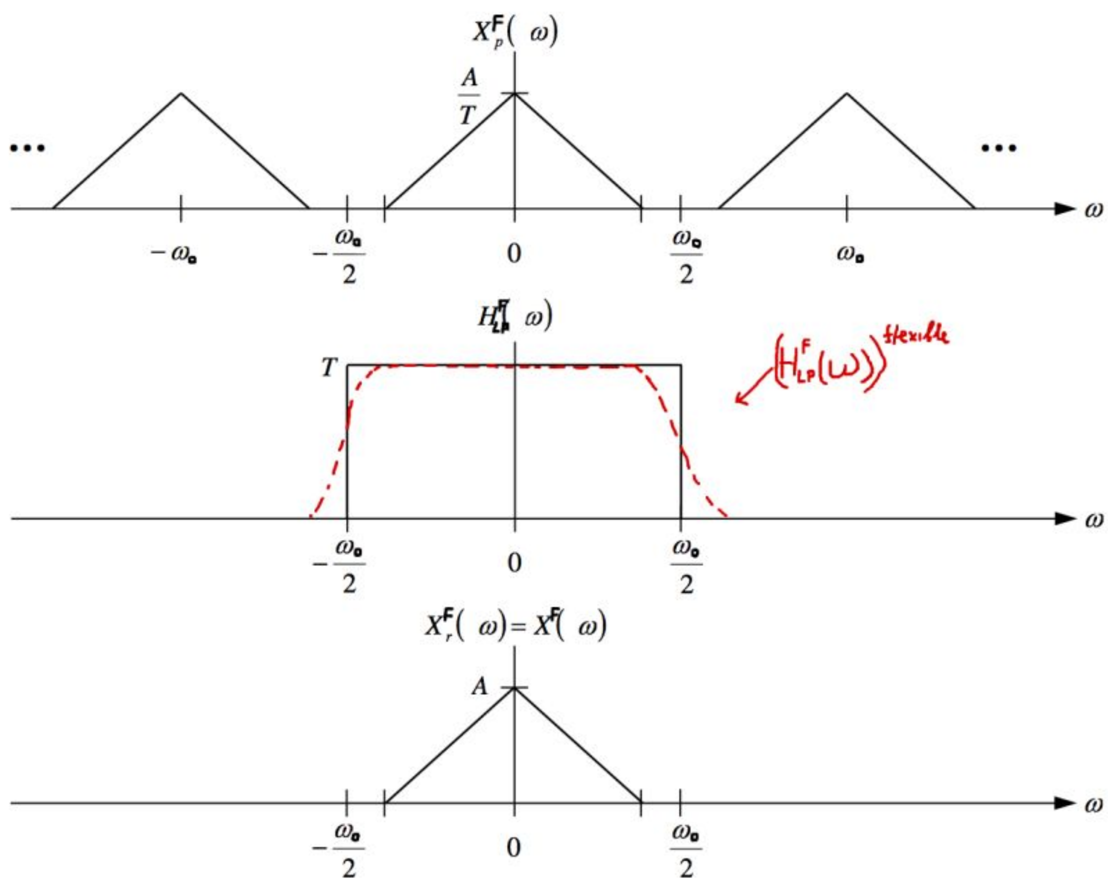


$$x(t) = \sum_{n=-\infty}^{\infty} x(nT) \cdot \mathrm{sinc}\left(\frac{t-nT}{T}\right),$$

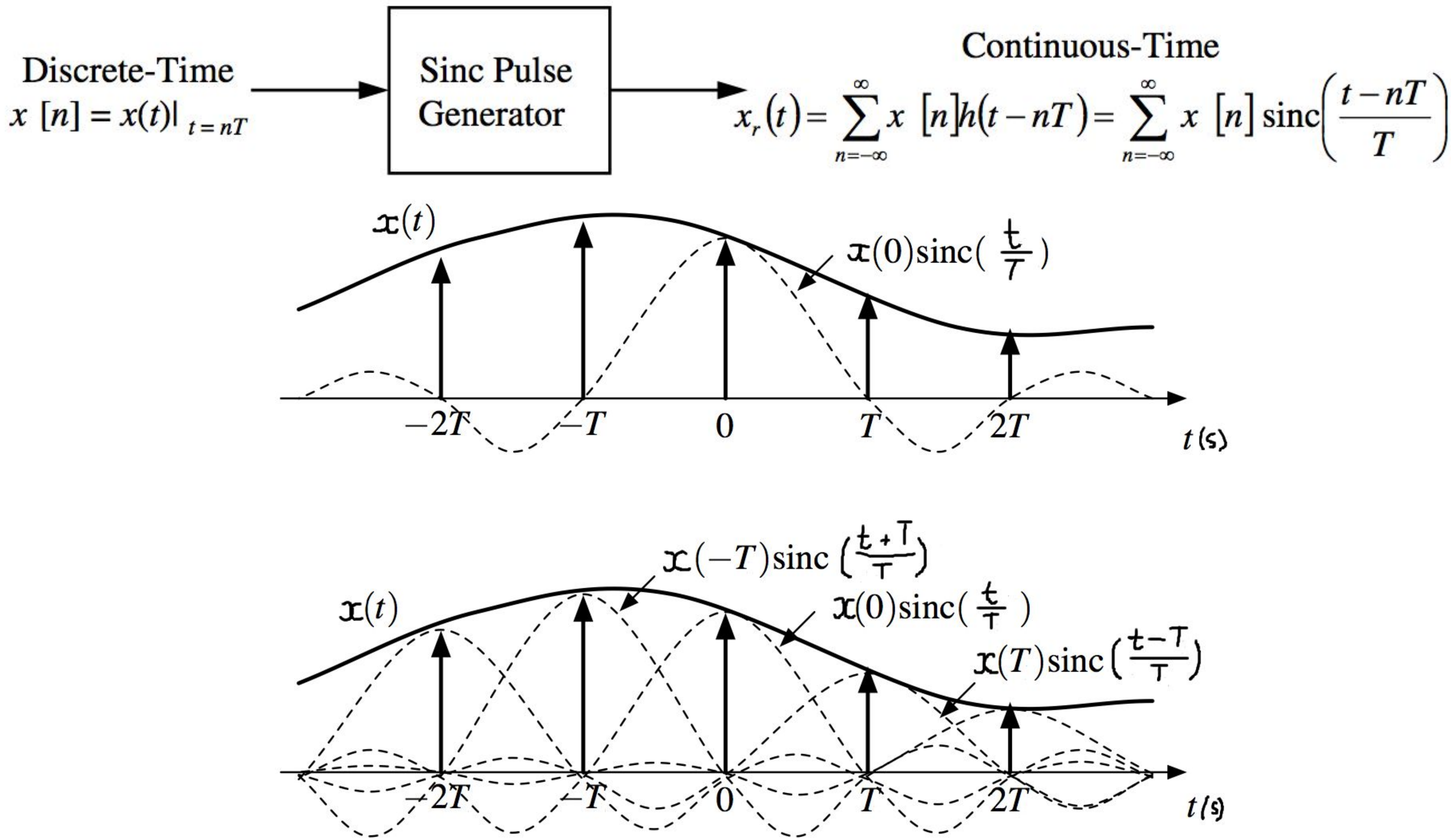
Whittaker–Shannon interpolation formula



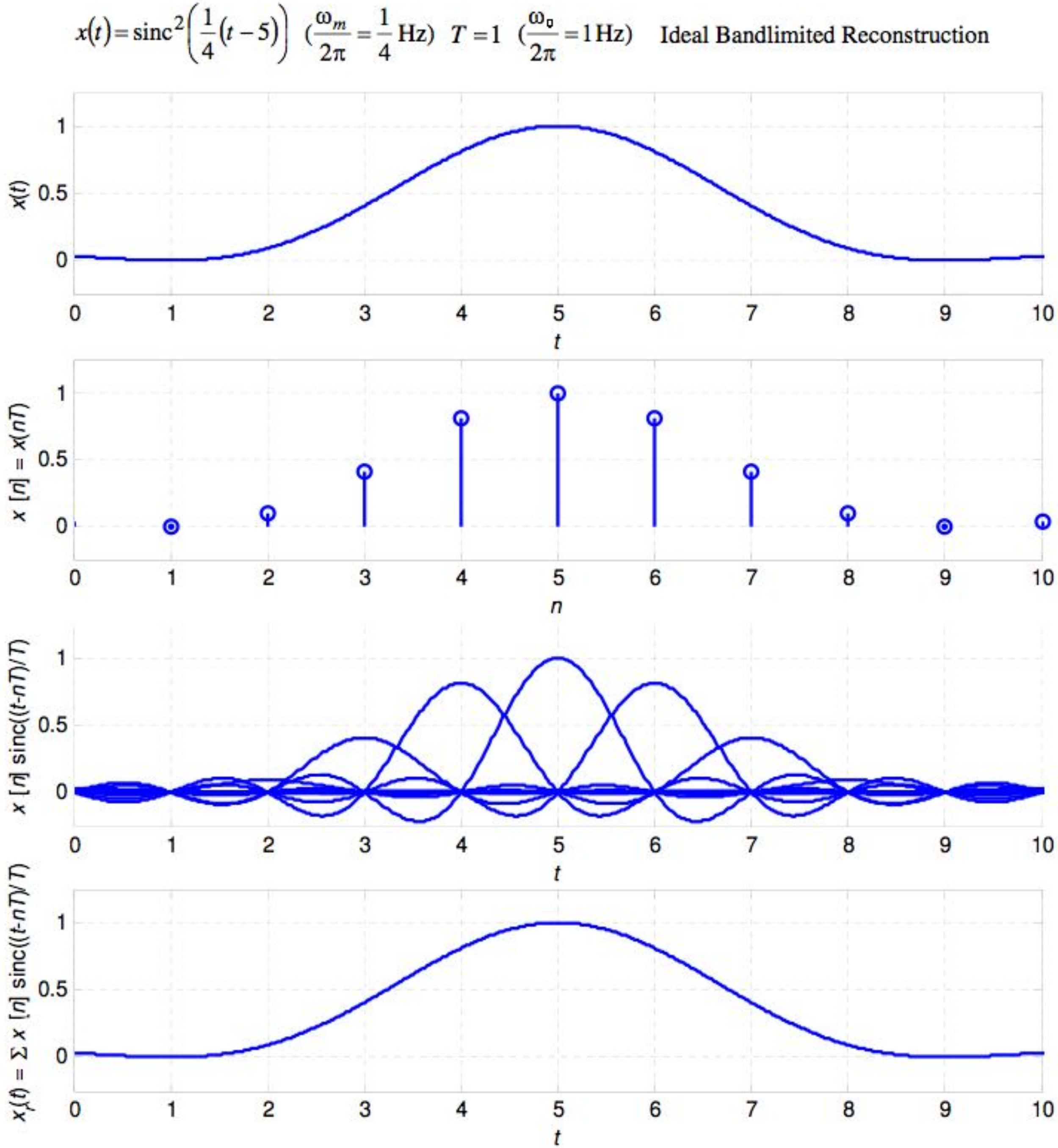
Tática de Reconstrução



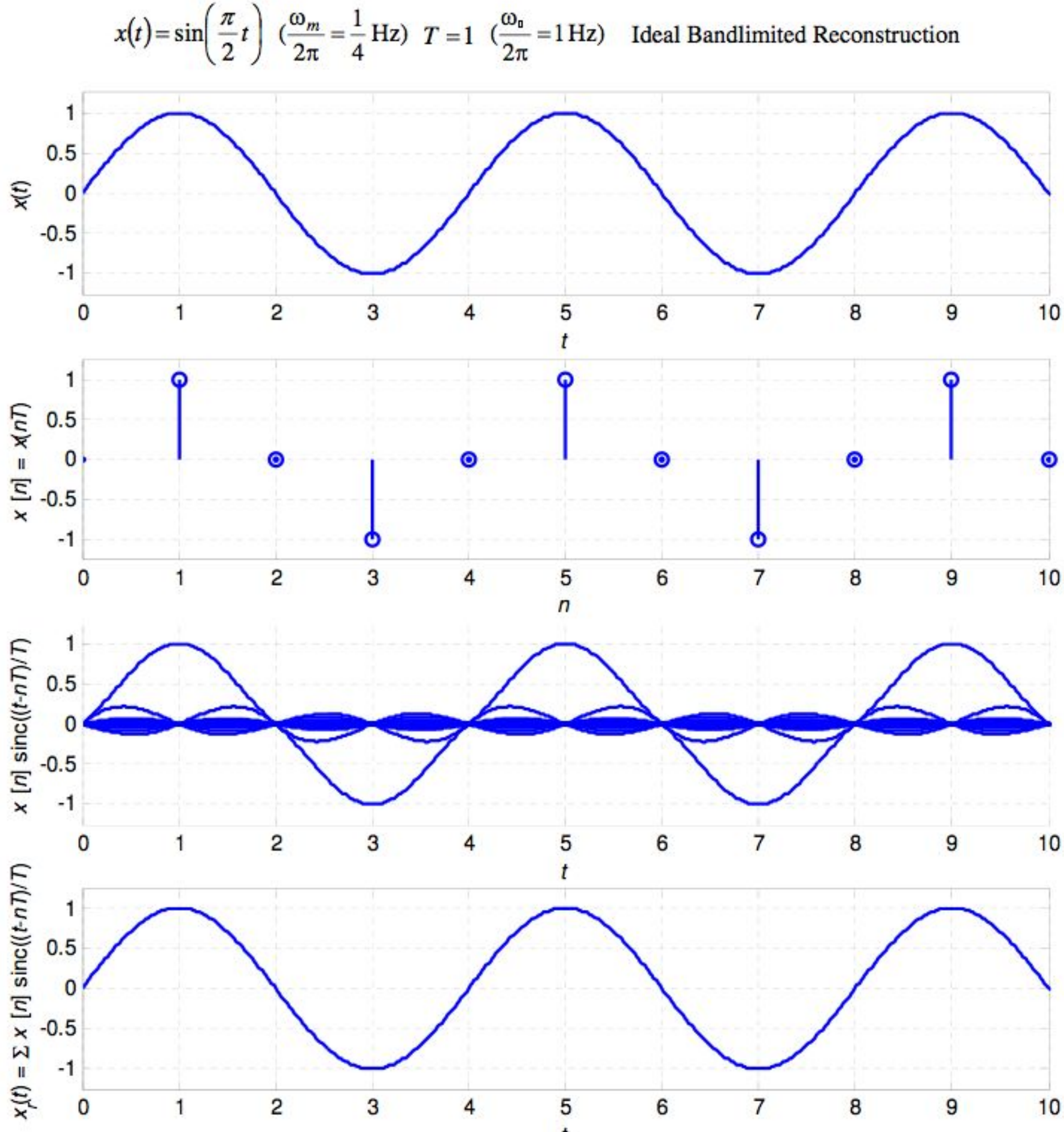
Definindo um Reconstrutor



Reconstruindo Sinais

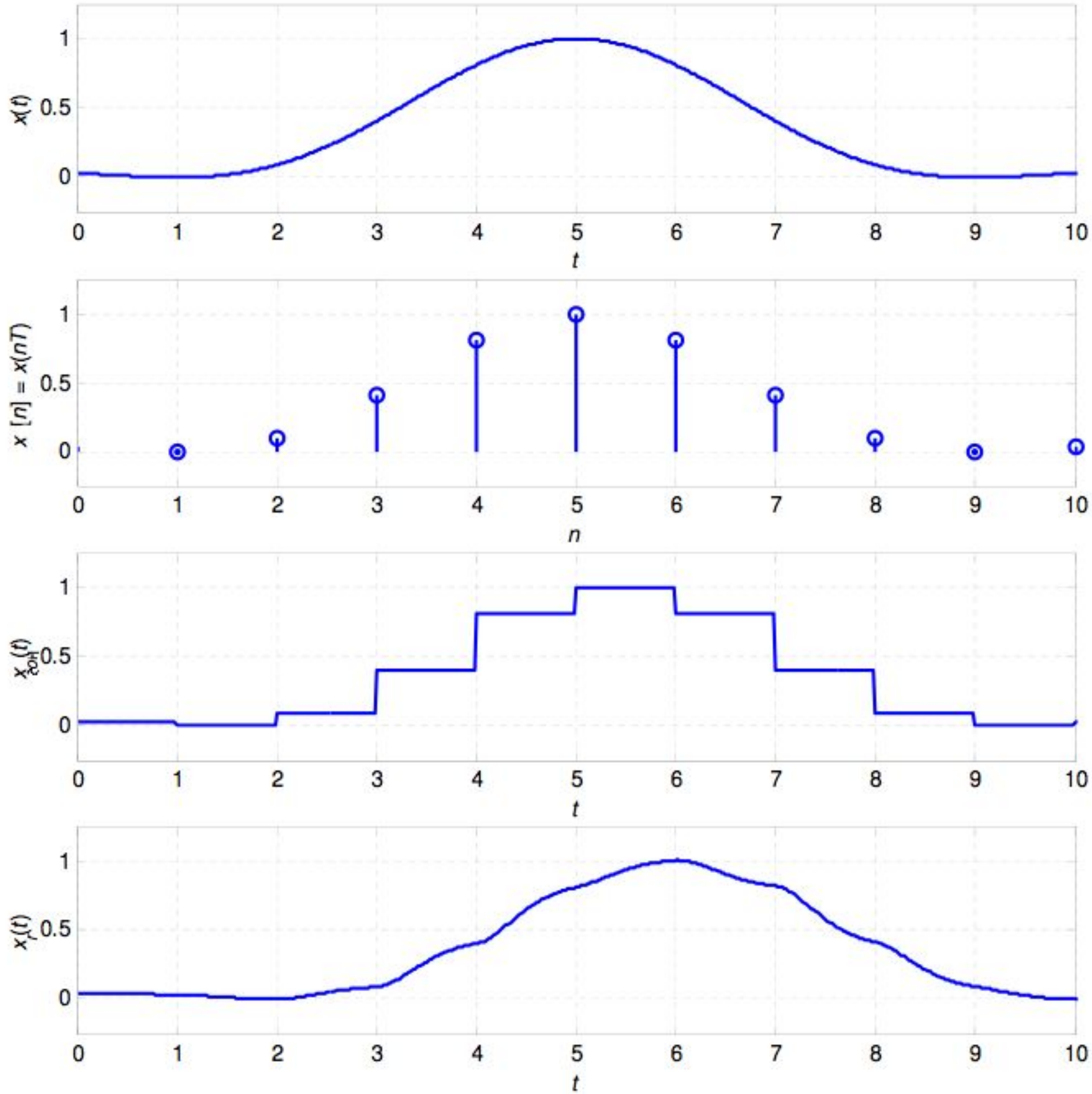


Reconstruindo Sinais



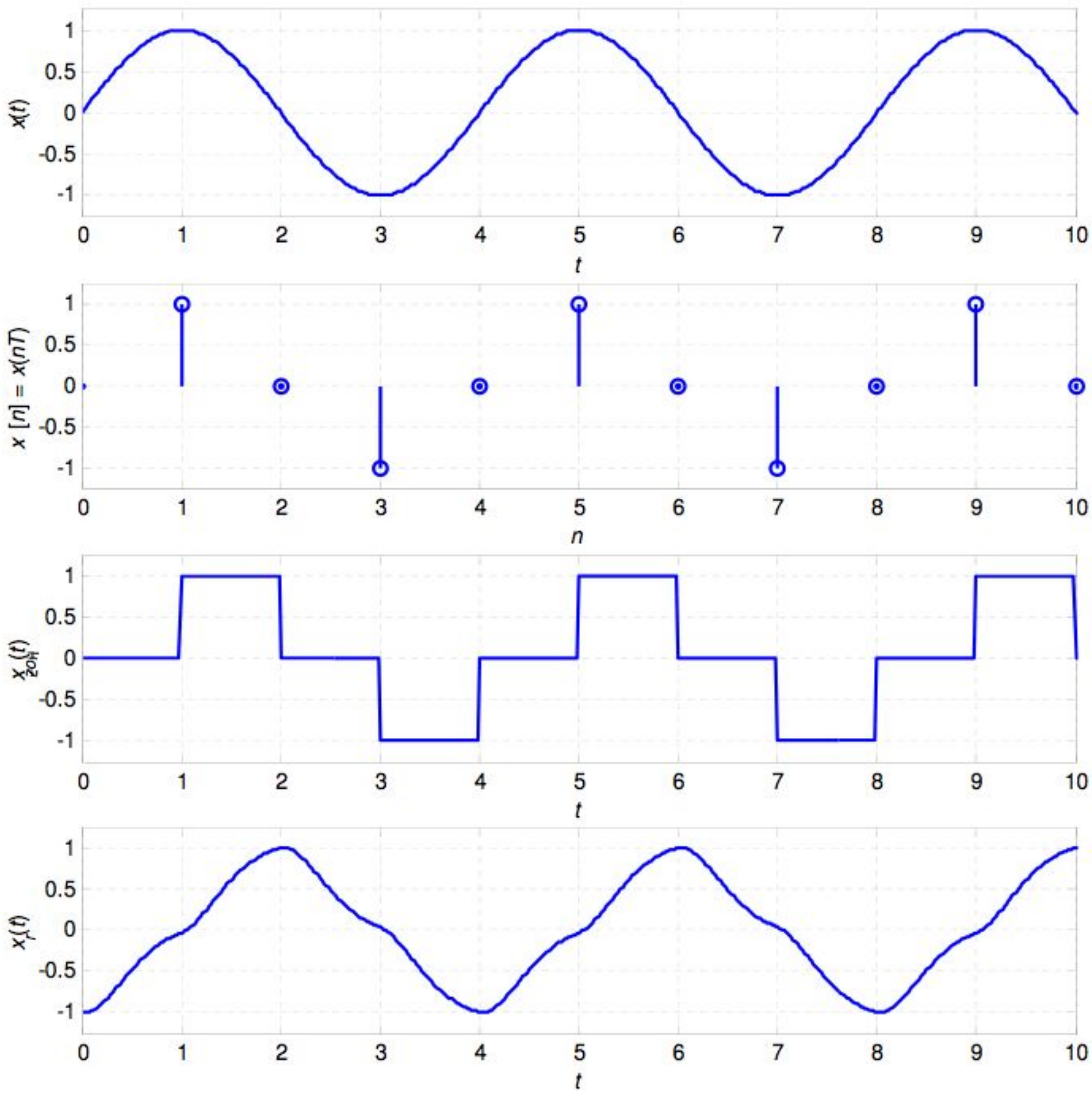
Reconstruindo Sinais

$x(t) = \text{sinc}^2\left(\frac{1}{4}(t-5)\right)$ $(\frac{\omega_m}{2\pi} = \frac{1}{4}\text{Hz})$ $T = 1$ $(\frac{\omega_s}{2\pi} = 1\text{Hz})$ Zero-Order Hold, 2nd-Order Butterworth LPF $\frac{\omega_c}{2\pi} = \frac{1}{2}\text{Hz}$



Reconstruindo Sinais

$x(t) = \sin\left(\frac{\pi}{2} t\right)$ $(\frac{\omega_m}{2\pi} = \frac{1}{4}$ Hz$)$ $T = 1$ $(\frac{\omega_{\square}}{2\pi} = 1$ Hz$)$ Zero-Order Hold, 2nd-Order Butterworth LPF $\frac{\omega_c}{2\pi} = \frac{1}{2}$ Hz

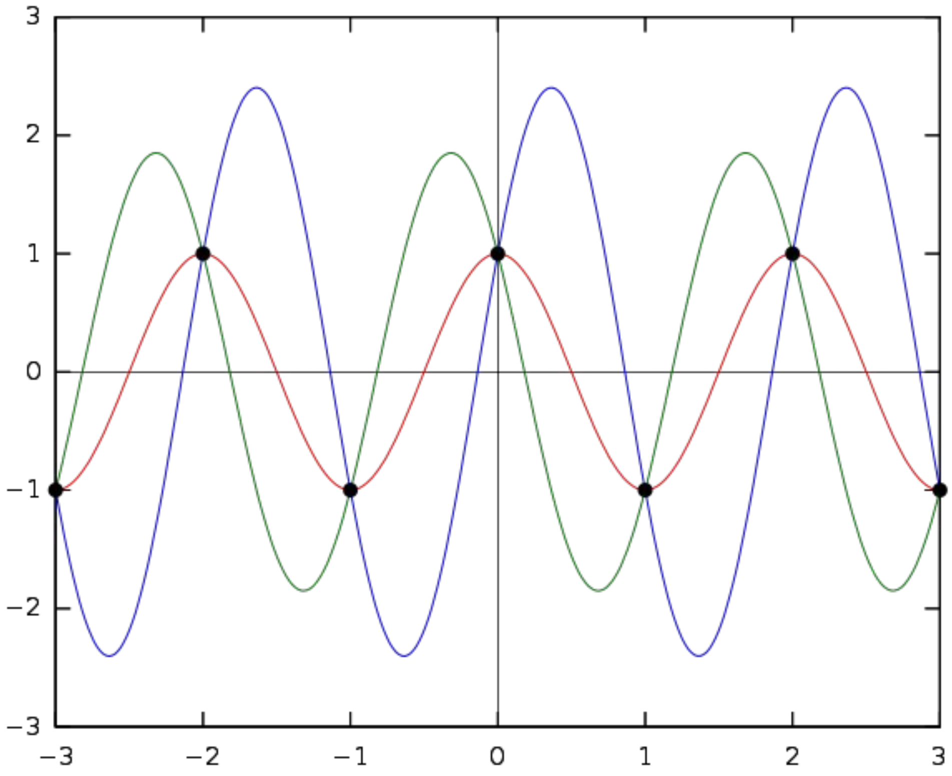


Considerações Aliasing e Reconstrução

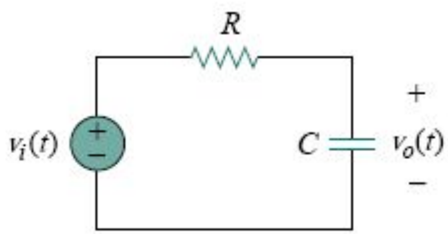
Adequações de condições ideais e reais

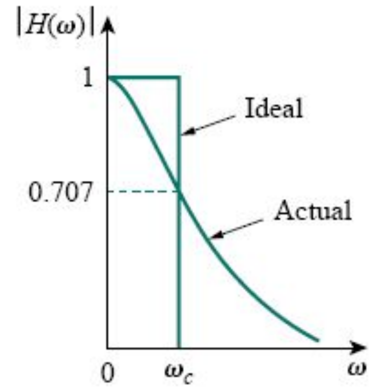
Condição limite do Critério de Nyquist

Custo-benefício de processamento dos filtros de interpolação



Filtro anti-aliasing e a reconstrução





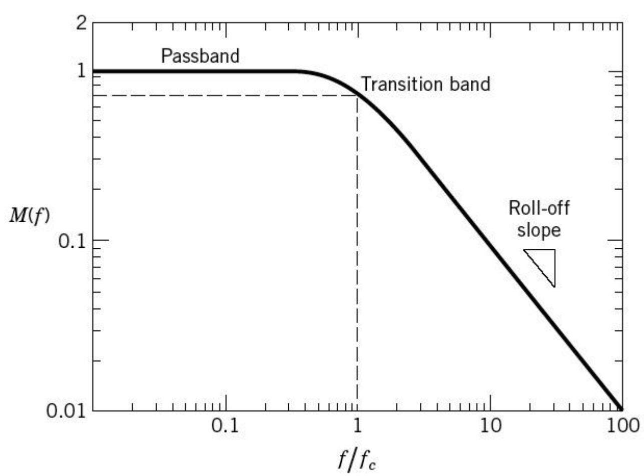


Figure 6.28 Magnitude ratio for a low-pass Butterworth filter.

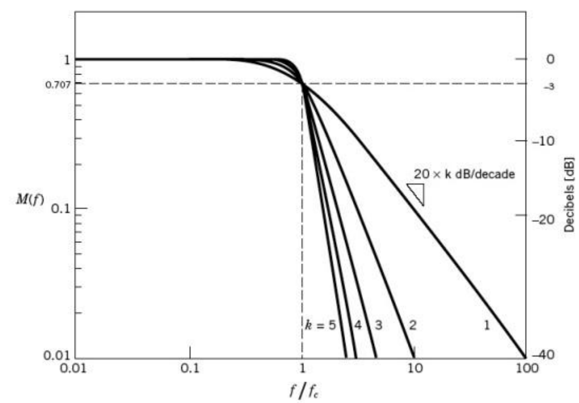
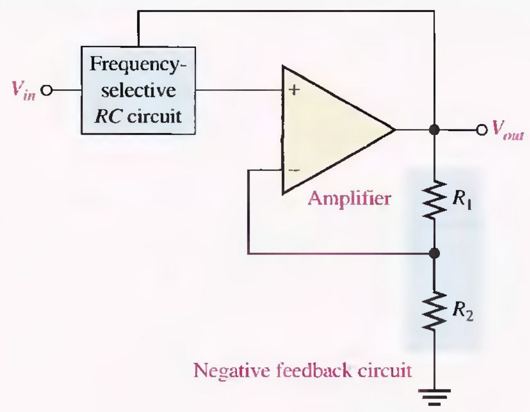


Figure 6.31 Magnitude characteristics for Butterworth low-pass filters of various stages.

Filtro anti-aliasing e a reconstrução

FIGURE 15–6

General diagram of an active filter.



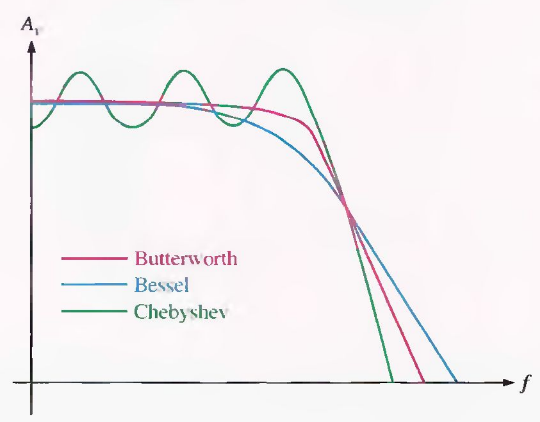
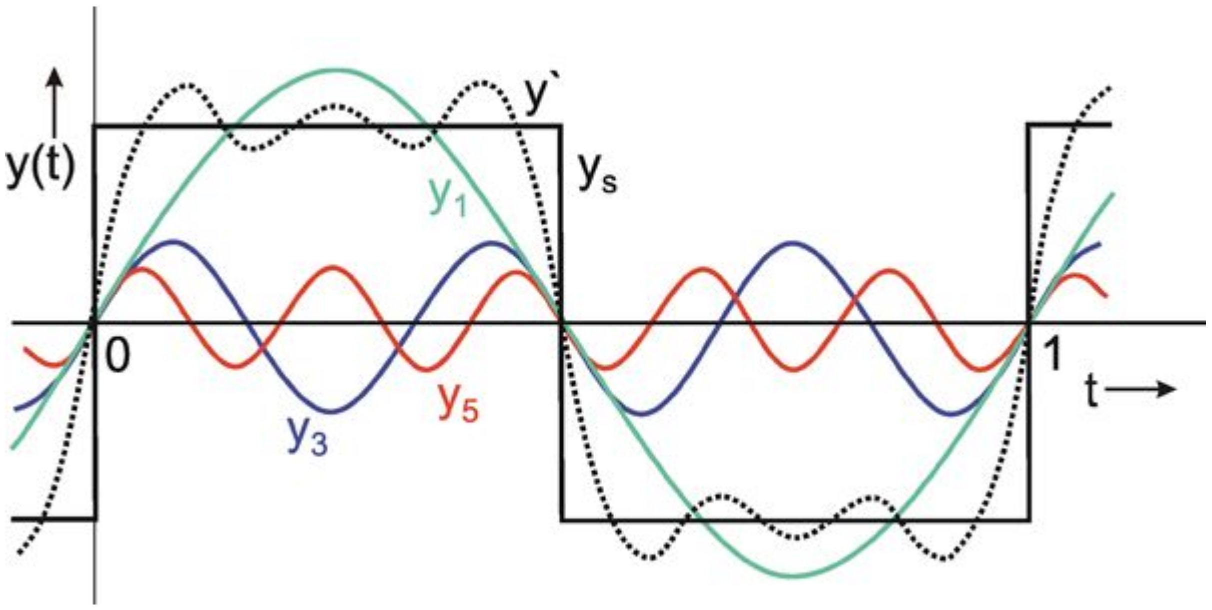


FIGURE 15–5

Comparative plots of three types of filter response characteristics.

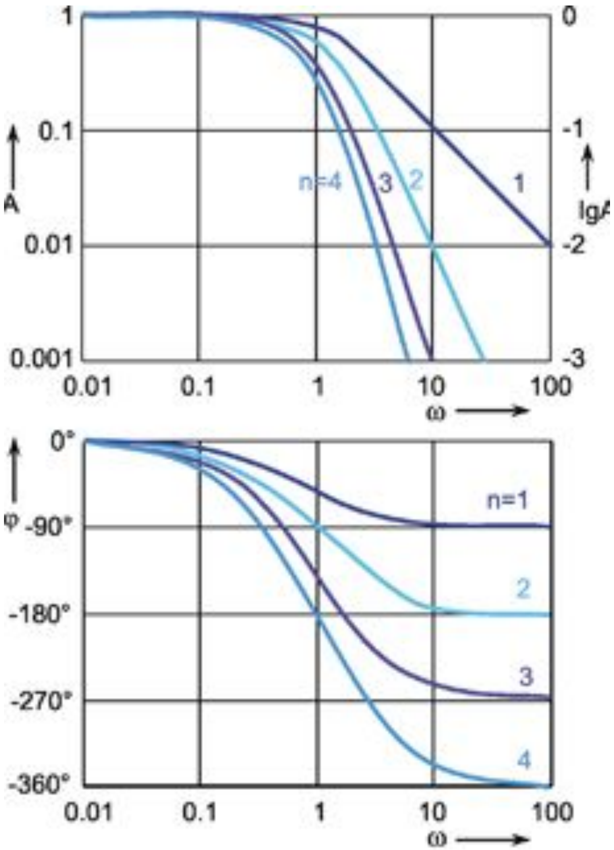
$$y(t) = a_0 + a_1 \sin(\omega t + \varphi_1) + a_2 \sin(2\omega t + \varphi_2) + \ldots + a_k \sin(k\omega t + \varphi_k) + \ldots$$

Filtro anti-aliasing e a reconstrução

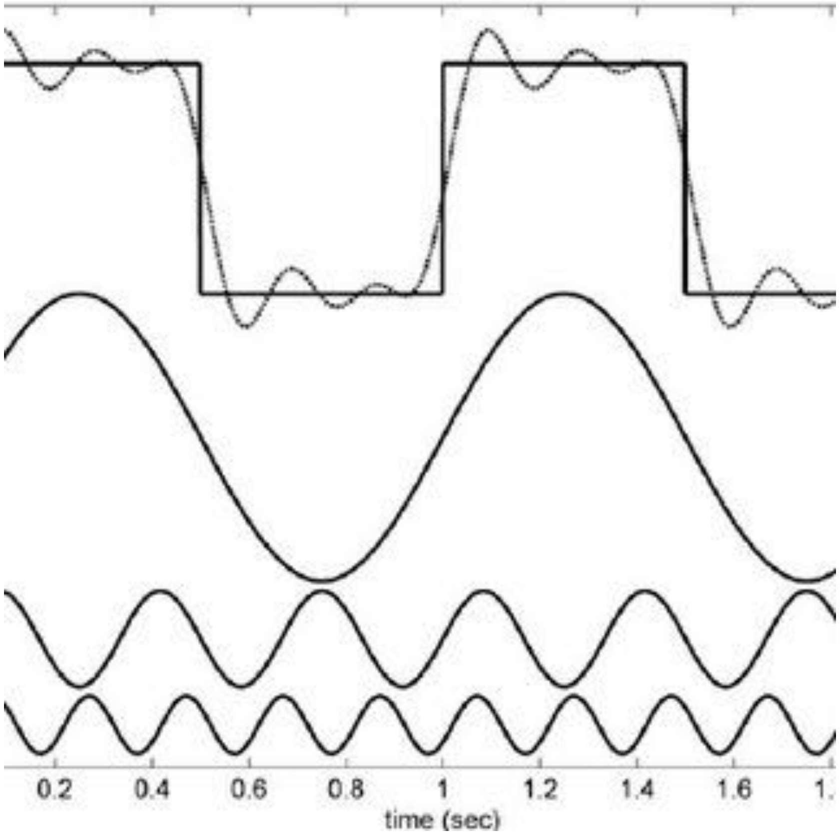


$$x_{\text{square}}(t) = \frac{4}{\pi}\sum_{k=1}^{\infty}\frac{\sin\left(2\pi(2k-1)ft\right)}{(2k-1)}$$
$$= \frac{4}{\pi}\left(\sin(2\pi ft) + \frac{1}{3}\sin(6\pi ft) + \frac{1}{5}\sin(10\pi ft) + \cdots\right)$$

Filtro anti-aliasing e a reconstrução



Filtro anti-aliasing e a reconstrução



Bibliografia

M. J. Korsten, W. Otthius, F. van der Heijden; Measurement Science for Engineers; Elsevier Science & Technology Books; 2004; ISBN: 1903996589

Signals and systems, by A.V. Oppenheim, A.S. Willsky, and I.T. Young. Prentice Hall, Englewood Cliffs, New Jersey, 1983

https://en.wikipedia.org/wiki/Nyquist%E2%80%93Shannon_sampling_theorem