PMT3306 - Módulo “Elasticidade” - Material de apoio

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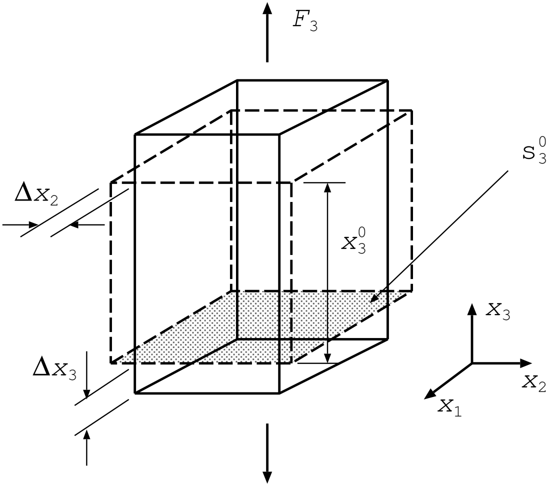
Deformação normal



Deformação normal

$$\sigma_{33} = E\varepsilon_{33} \Rightarrow \frac{F_3}{s_3^0} \approx E\frac{\Delta x_3}{x_3^0} \quad (1a)$$

$$\varepsilon_{22} = \varepsilon_{11} = -\nu\varepsilon_{33} \Rightarrow \frac{\Delta x_2}{x_2^0} \approx -\nu\frac{\Delta x_3}{x_3^0} \quad (1b)$$

Deformação normal

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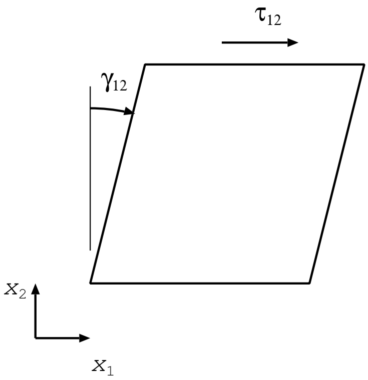
Módulo de flexibilidade?

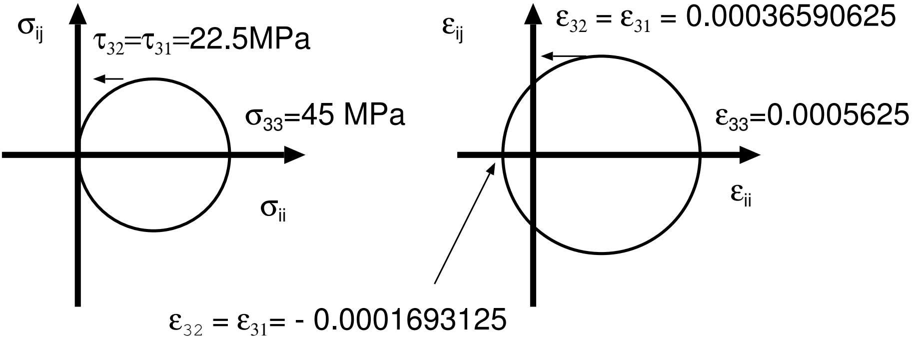
Deformação angular



Deformação angular

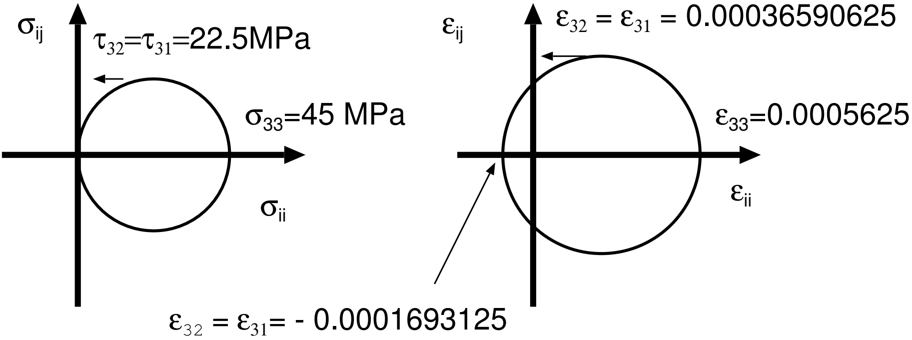
$$\tau_{12} = G\gamma_{12} = 2G\varepsilon_{12} \qquad (2)$$

Relação entre os módulos



$G = 30{,}74$ GPa.

Relação entre os módulos



$G = 30{,}74$ GPa.

$$G = \frac{E}{2(1+\nu)} \tag{3}$$

Lei de Hooke generalizada

$$\left\{\begin{array}{rl} \varepsilon_{11} = & \frac{1}{E}\sigma_{11} - \frac{\nu}{E}\sigma_{22} - \frac{\nu}{E}\sigma_{33} \\ \varepsilon_{22} = & -\frac{\nu}{E}\sigma_{11} + \frac{1}{E}\sigma_{22} - \frac{\nu}{E}\sigma_{33} \\ \varepsilon_{33} = & -\frac{\nu}{E}\sigma_{11} - \frac{\nu}{E}\sigma_{22} + \frac{1}{E}\sigma_{33} \\ \varepsilon_{23} = & \frac{1}{2G}\sigma_{23} \\ \varepsilon_{13} = & \frac{1}{2G}\sigma_{13} \\ \varepsilon_{12} = & \frac{1}{2G}\sigma_{12} \end{array}\right. \tag{4}$$

Lei de Hooke generalizada

$$\begin{vmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{vmatrix} \equiv \begin{vmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \varepsilon_{23} \\ \varepsilon_{13} \\ \varepsilon_{12} \end{vmatrix} \qquad \text{e} \qquad \begin{vmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{vmatrix} \equiv \begin{vmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{23} \\ \sigma_{13} \\ \sigma_{12} \end{vmatrix}$$

Lei de Hooke generalizada

$$\left|\begin{array}{ccccc}
\sigma_{11} & & \sigma_{12} & \leftarrow & \sigma_{13} \\
 & \searrow & & & \uparrow \\
 & & \sigma_{22} & & \sigma_{23} \\
 & \cdots & & \searrow & \uparrow \\
 & & & & \sigma_{33}
\end{array}\right|$$

Forma matricial

$$\begin{vmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \\ \varepsilon_4 \\ \varepsilon_5 \\ \varepsilon_6 \end{vmatrix} = \begin{vmatrix} S_{11} & S_{12} & S_{13} & S_{14} & S_{15} & S_{16} \\ S_{21} & S_{22} & S_{23} & S_{24} & S_{25} & S_{26} \\ S_{31} & S_{32} & S_{33} & S_{34} & S_{35} & S_{36} \\ S_{41} & S_{42} & S_{43} & S_{44} & S_{45} & S_{46} \\ S_{51} & S_{52} & S_{53} & S_{54} & S_{55} & S_{56} \\ S_{61} & S_{62} & S_{63} & S_{64} & S_{65} & S_{66} \end{vmatrix} \begin{vmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \\ \sigma_4 \\ \sigma_5 \\ \sigma_6 \end{vmatrix}$$

Forma matricial

$$\left|S_{ij}\right| = \left|\begin{matrix} \frac{1}{E} & -\frac{\nu}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ -\frac{\nu}{E} & \frac{1}{E} & -\frac{\nu}{E} & 0 & 0 & 0 \\ -\frac{\nu}{E} & -\frac{\nu}{E} & \frac{1}{E} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2G} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2G} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2G} \end{matrix}\right| \quad (4)$$

Compressibilidades

$$B\frac{\Delta V}{V} = -P \Rightarrow B = \frac{E}{3\left(1 - 2\nu\right)} = -\frac{1}{V}\left(\frac{\partial P}{\partial V}\right) \tag{5}$$

Constantes de Lamé

$$\sigma_{11} = \lambda\theta + 2\mu\varepsilon_{11} \quad (6a)$$

$$\sigma_{22} = \lambda\theta + 2\mu\varepsilon_{22} \quad (6b)$$

$$\sigma_{33} = \lambda\theta + 2\mu\varepsilon_{33} \quad (6c)$$

$$\sigma_{12} = 2\mu\varepsilon_{12} \quad (6d)$$

$$\sigma_{23} = 2\mu\varepsilon_{23} \quad (6e)$$

$$\sigma_{31} = 2\mu\varepsilon_{31} \quad (6f)$$

onde $\theta = (\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33})$.

Constantes de Lamé

$$G = \mu \tag{6a}$$

$$E = \frac{\mu\left(3\lambda + 2\mu\right)}{\lambda + \mu} \tag{6b}$$

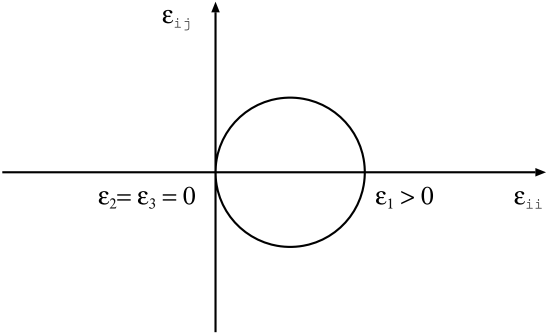
$$\nu = \frac{\lambda}{2\left(\lambda + \mu\right)} \tag{6c}$$

e

$$B = \lambda + \frac{2}{3}\mu \tag{7}$$

Efeito de restrições

Constraints



Efeito de restrições

Constraints

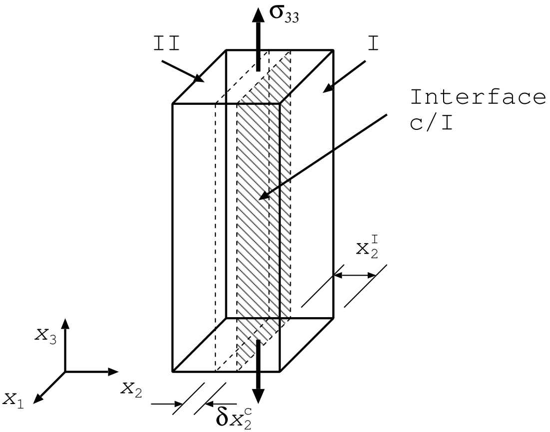
$$\sigma_1 = E\left(1 - \frac{\nu^2}{1-\nu}\right)^{-1} \varepsilon_1 \tag{8a}$$

e

$$\sigma_3 = \sigma_2 = \frac{\nu}{1-\nu}\sigma_1 \tag{8b}$$

Restrições e o EPD

Gedankenexperiment



Cristais cúbicos

$$\left|C_{ij}\right| = \begin{vmatrix} C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{12} & C_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{44} \end{vmatrix} \tag{9}$$

$$A \equiv \frac{2C_{44}}{(C_{11} - C_{12})} \tag{10}$$

Cristais cúbicos

$$\begin{cases} S_{11} = & \frac{C_{11}+C_{12}}{(C_{11}-C_{12})(C_{11}+2C_{12})} \\ S_{12} = & \frac{-C_{12}}{(C_{11}-C_{12})(C_{11}+2C_{12})} \\ S_{44} = & \frac{1}{C_{44}} \end{cases} \tag{9}$$

$$A \equiv \frac{2C_{44}}{(C_{11}-C_{12})} \tag{10}$$

Cristais monoclínicos

$$\left|C_{ij}\right| = \left|\begin{array}{cccccc} C_{11} & C_{12} & C_{13} & 0 & 0 & C_{16} \\ & C_{22} & C_{23} & 0 & 0 & C_{26} \\ & & C_{33} & 0 & 0 & C_{36} \\ & & & C_{44} & C_{45} & 0 \\ & \ddots & & & C_{55} & 0 \\ & & & & & C_{66} \end{array}\right| \quad (11)$$

Módulos direcionais em monocristais

$$\frac{1}{E_{hkl}} = S_{11} - 2\left(S_{11} - S_{12} - \tfrac{1}{2}S_{44}\right)\ell^{[hkl]}$$
$$\frac{1}{G_{hkl}} = S_{44} + 4\left(S_{11} - S_{12} - \tfrac{1}{2}S_{44}\right)\ell^{[hkl]}$$

com

$$\ell^{[hkl]} \equiv \cos^2(\theta_1^{hkl})\cos^2(\theta_2^{hkl}) + \cos^2(\theta_2^{hkl})\cos^2(\theta_3^{hkl}) + \cos^2(\theta_1^{hkl})\cos^2(\theta_3^{hkl})$$

$\cos\left(\theta_i^{hkl}\right)$ é o cosseno diretor do eixo i na direção $[hkl]$, para simetrias cúbicas

$$\cos(\theta_{uvw}^{hkl}) = \frac{hu + kv + lw}{\left(h^2 + k^2 + l^2\right)^{\frac{1}{2}}\left(u^2 + v^2 + w^2\right)^{\frac{1}{2}}}$$

Materiais ortotrópicos

$$\begin{vmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ & C_{22} & C_{23} & 0 & 0 & 0 \\ & & C_{33} & 0 & 0 & 0 \\ & & & C_{44} & 0 & 0 \\ & \cdots & & & C_{55} & 0 \\ & & & & & C_{66} \end{vmatrix} \tag{12}$$