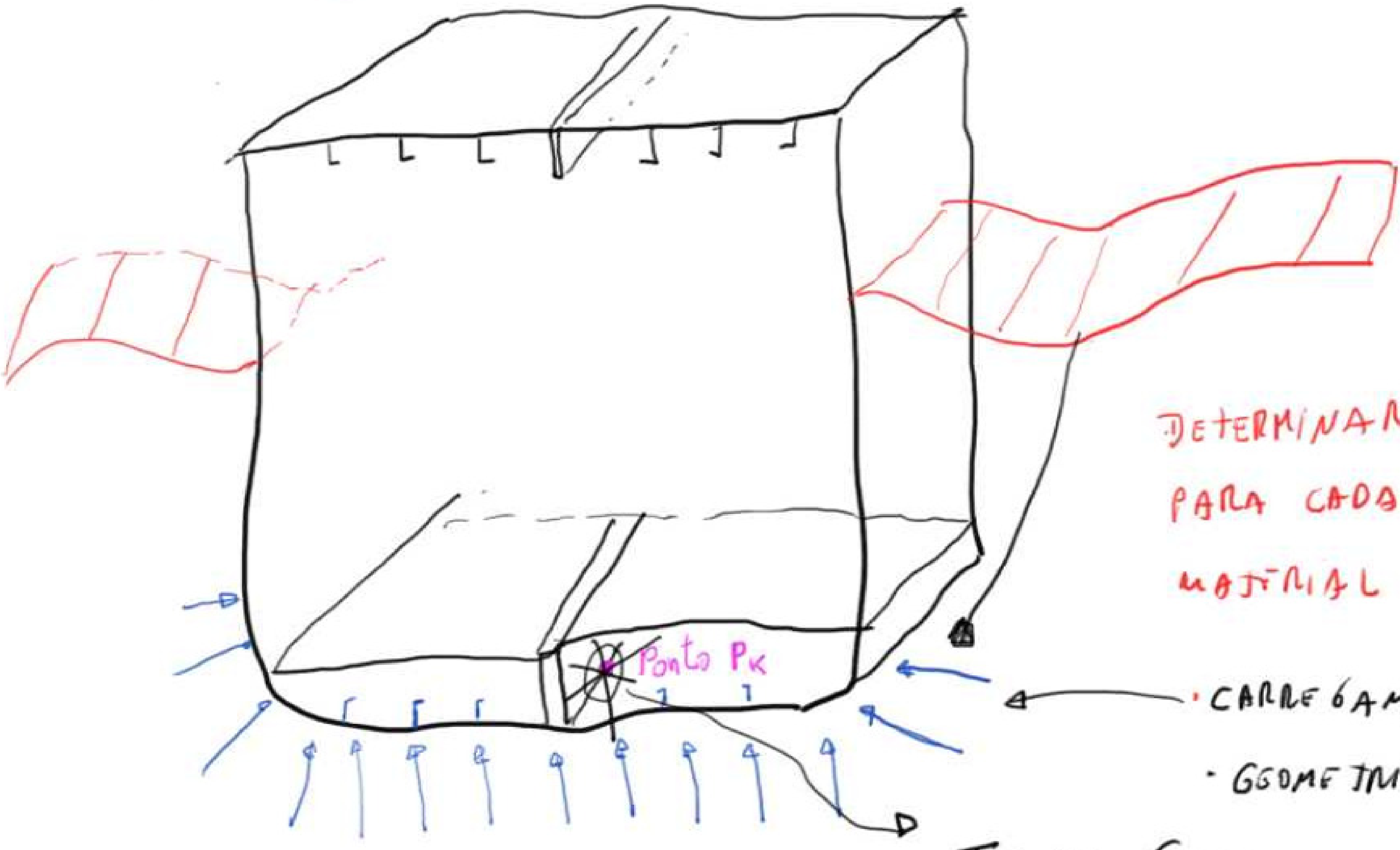
2. TENSÕES

MOTIVAÇÃO

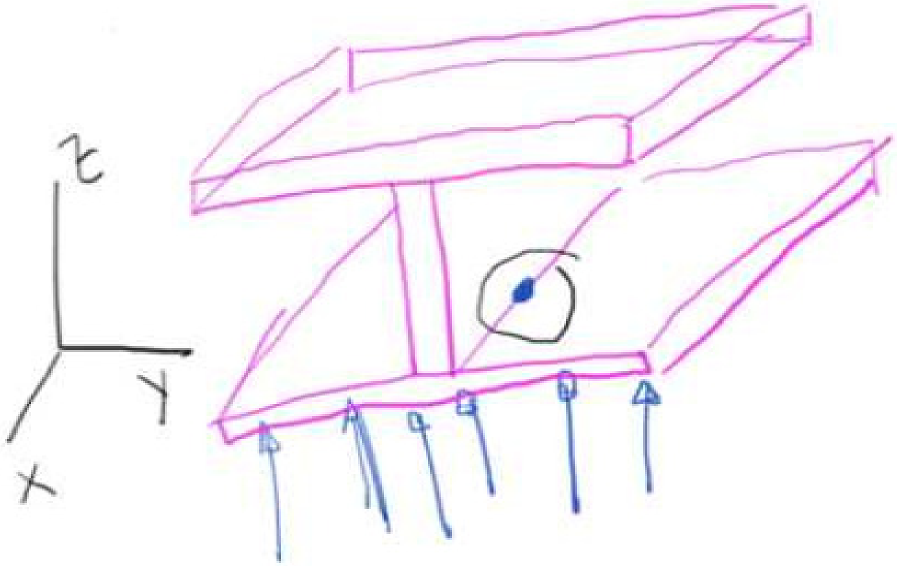


DETERMINAR PARA CADA PONTO MATERIAL

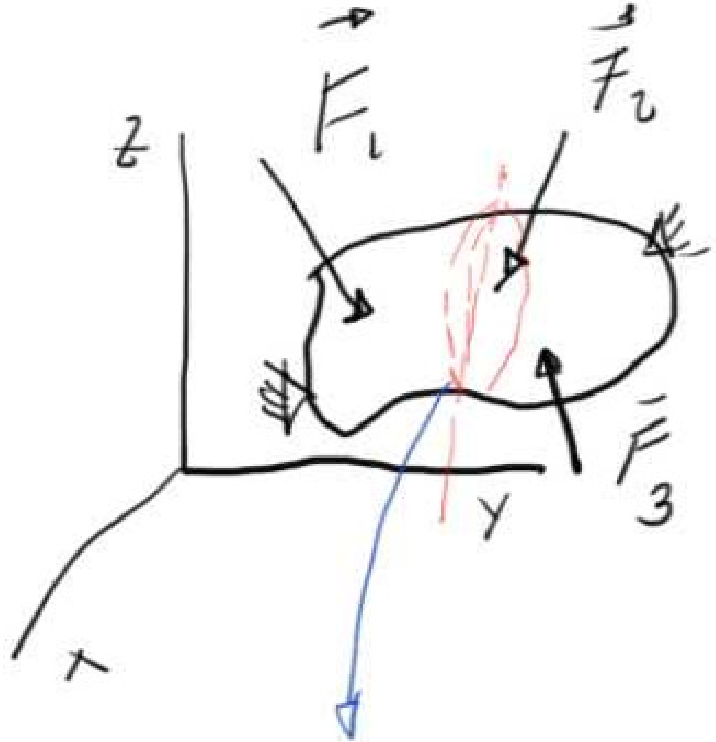
- CARREGAMENTOS
- GEOMETRIA

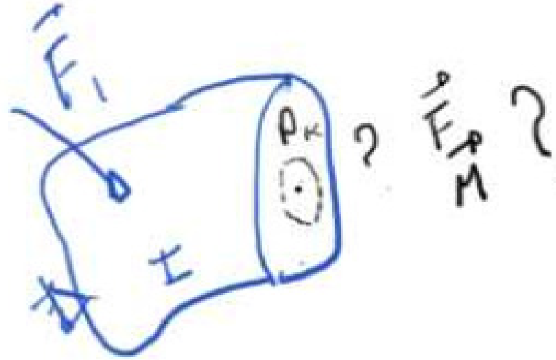
σ_{ij} vs. σ_{MAT}

PONTO P_K

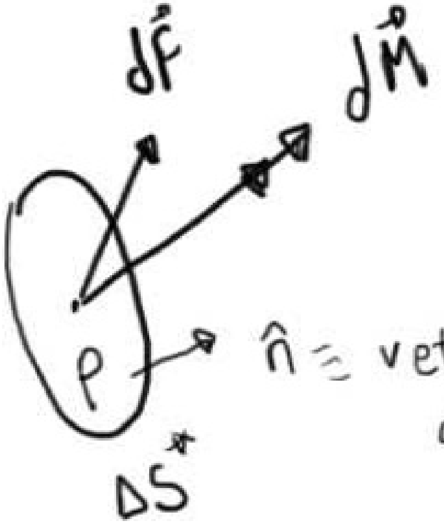


X





PRINCÍPIO DE ESFORÇOS DE CAUCHY



$\hat n \equiv$ vetor normal ao plano

$|\hat n| = 1$

(1) $\lim\limits_{\Delta S^* \to 0} \dfrac{d\vec F}{\Delta S^*} \equiv \vec t_i^{\,\boxed{(\hat n)}}$ → plano onde atua $\vec t$

↳ VETOR TENSÃO

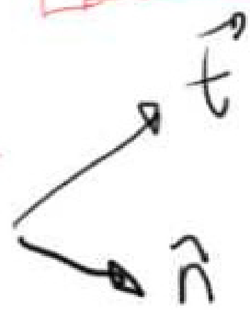
VETOR ESFORÇO

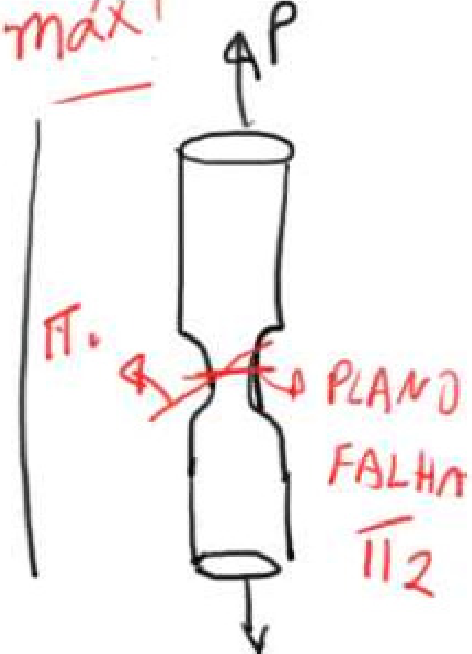
$\vec t_i^{\,(\hat n)} = \underbrace{\overline{(t_1, t_2, t_3)}^{\,(P,\, \hat n)}}$ [MPa]

máx!

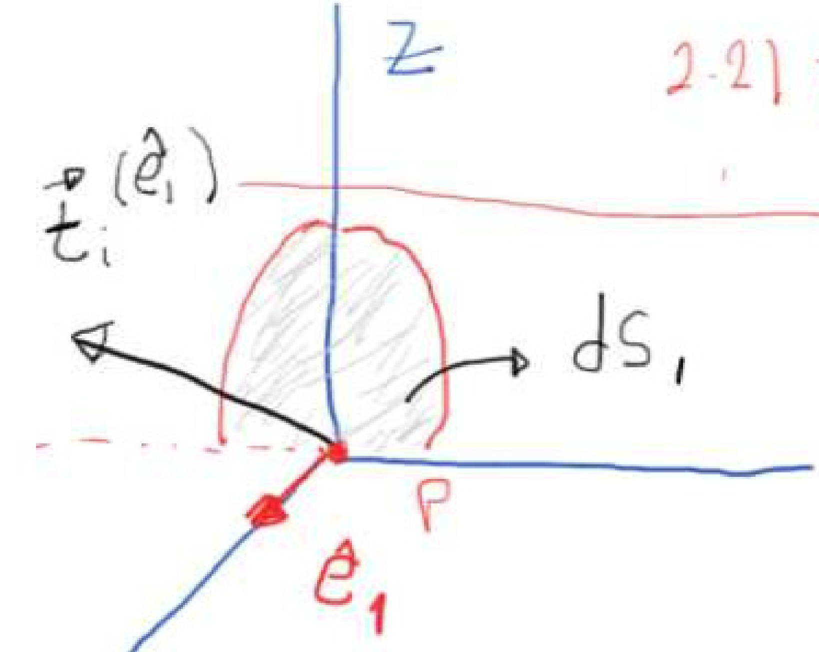
(2) $\lim\limits_{\boxed{\Delta S^* \to 0}} \dfrac{d\vec M}{\Delta S^*} = 0$

$\vec t^{\,(\hat n)} \nparallel \hat n$

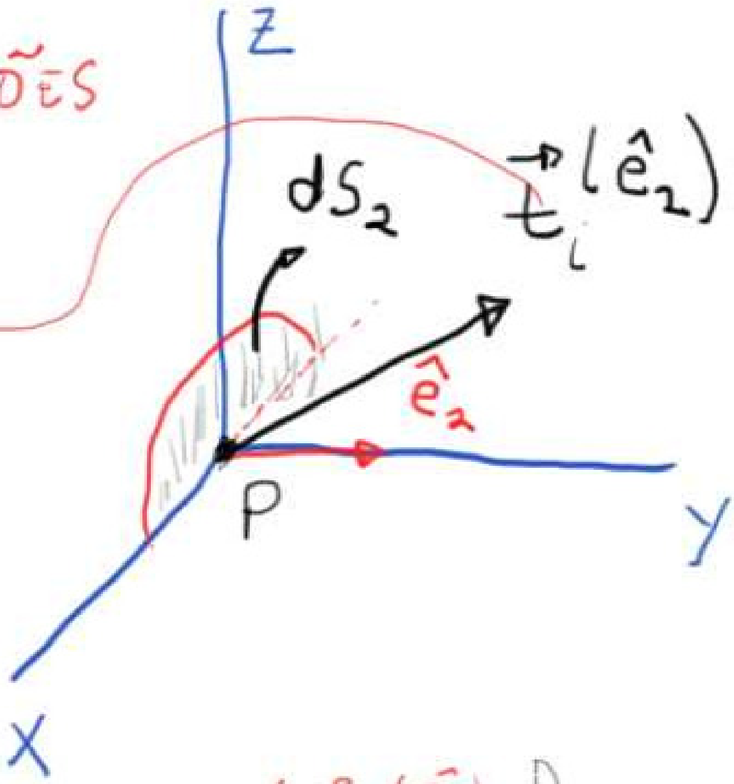




2.2) TENSOR DAS TENSÕES

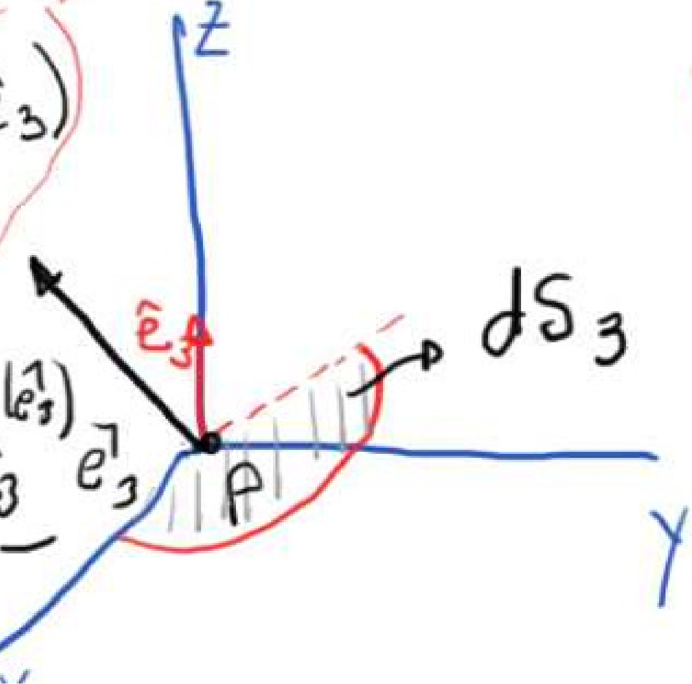


$$\sigma_{ij} = \begin{bmatrix} 0 & 0 & 0 \\ & & \\ & & \end{bmatrix}$$



$$\vec{t}^{(\hat e_1)} = \underbrace{t_1^{(\hat e_1)}}\,\hat e_1 + \underbrace{t_2^{(\hat e_1)}}\,\hat e_2 + \underbrace{t_3^{(\hat e_1)}}\,\hat e_3$$

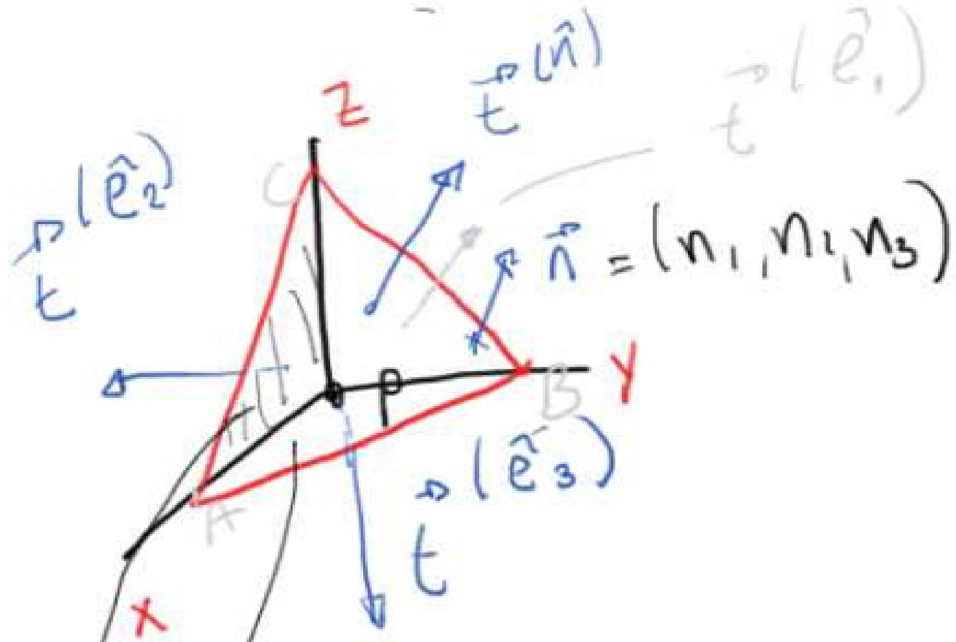
$\vec{t}_i^{(\hat e_3)}$



$$\vec{t}^{(\hat e_3)} = \underbrace{t_1^{(\hat e_3)}}\,\hat e_1 + \underbrace{t_2^{(\hat e_3)}}\,\hat e_2 + \underbrace{t_3^{(\hat e_3)}}\,\hat e_3$$

base $\left\{ \begin{array}{l} \vec{t}_i^{(\hat e_1)} \\ \vec{t}_i^{(\hat e_2)} \\ \vec{t}_i^{(\hat e_3)} \end{array} \right\} \rightarrow \vec{t}^{(\hat n)}$

• EQUILIBRIO $\rightarrow$ $\vec{t}^{(\hat{n})} \leftarrow t_i^{(\vec{e}_i)}$



$$\frac{d}{dt}(\vec{p}) = F_R \qquad (2^{\circ} \text{ NEWTON})$$

$\vec{F}_R \Rightarrow$ F. CORPO $\rightarrow$ F. GRAVITACIONAL

F. SUP

dS

• $F_{corpo} = (f_m^1, f_m^2, f_m^3)$

?

• $F_{sup} = \vec{t} \cdot dS$

$dS_i = dS \cos(\hat{n}, \hat{e}_i)$, $i = 1, 2, 3 \rightarrow$

$dS_1 = n_1 \, dS$

$dS_2 = n_2 \, dS$

$dS_3 = n_3 \, dS$

$$F_s = \vec{t}^{(n)} ds - \vec{t}_i^{(\vec{e}_1)} ds_1 - \vec{t}_i^{(\vec{e}_2)} ds_2 - \vec{t}_i^{(\vec{e}_3)} ds_3$$

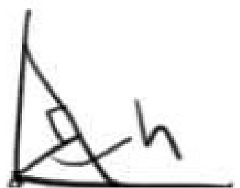
$$= \vec{t}^{(n)} ds - \underbrace{\vec{t}_i^{(\vec{e}_1)} n_1 ds - \vec{t}_i^{(\vec{e}_2)} n_2 ds - \vec{t}_i^{(\vec{e}_3)} n_3 ds}_{\vec{t}_i^{(\vec{e}_j)} n_j ds}$$

$$F_s = \vec{t}^{(n)} ds - \vec{t}^{(\vec{e}_j)} n_j ds$$

$$F_c = (f_m^1, f_m^2, f_m^3) \rightarrow F_n = \rho dV \frac{d\vec{v}}{dt}$$

$$F_c = \rho dV b_i$$

$$\underline{\vec{t}^{(n)} ds - \vec{t}^{(\vec{e}_j)} n_j ds} + \rho dV b_i = \rho dV \frac{d}{dt}$$



$$dV = \frac{1}{3} h dS$$

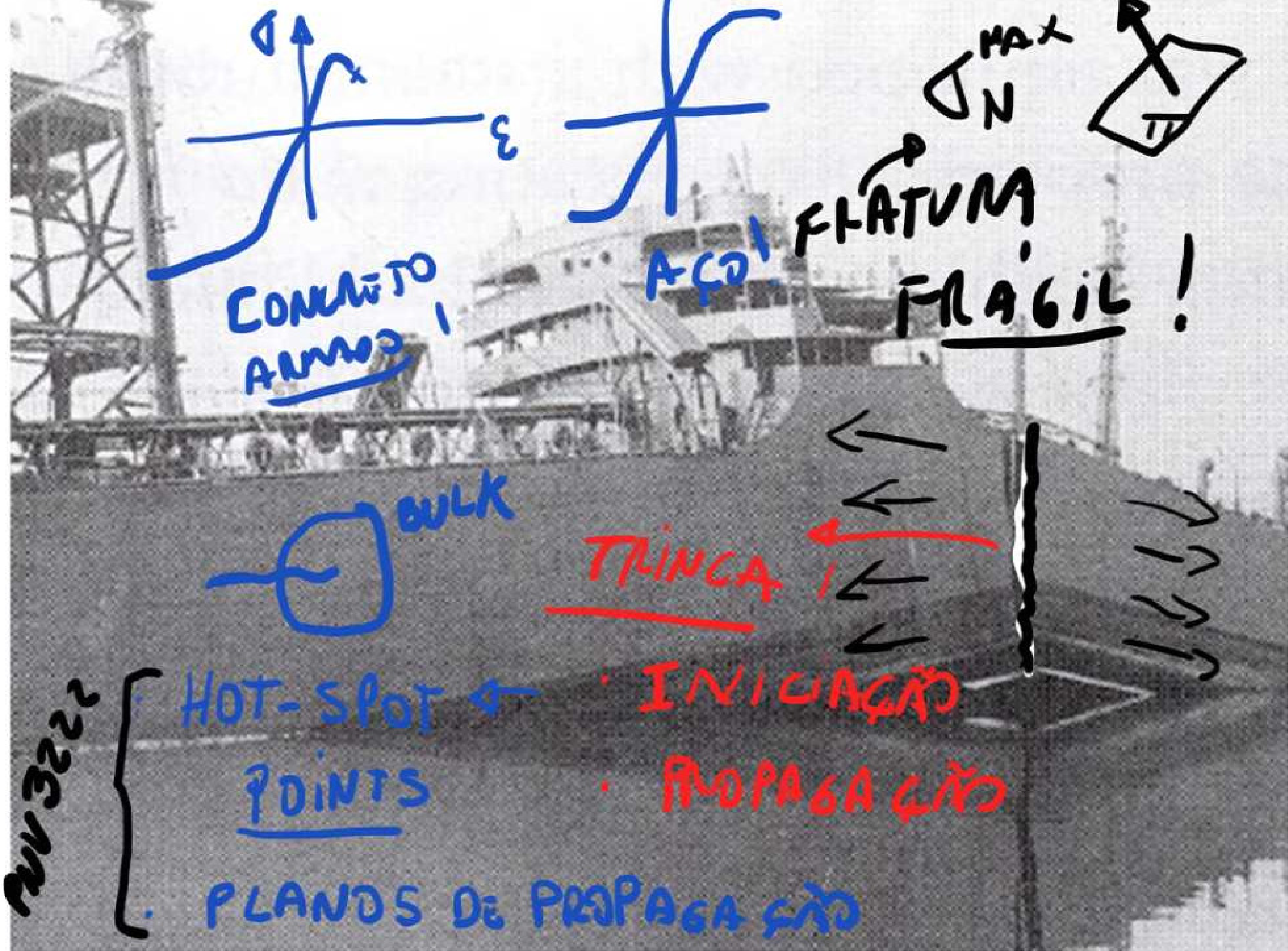
$$\vec{t}^{(n)} = \boxed{t^{(\hat{e}_j)}} n_j$$

RELAÇÃO DE CAUCHY

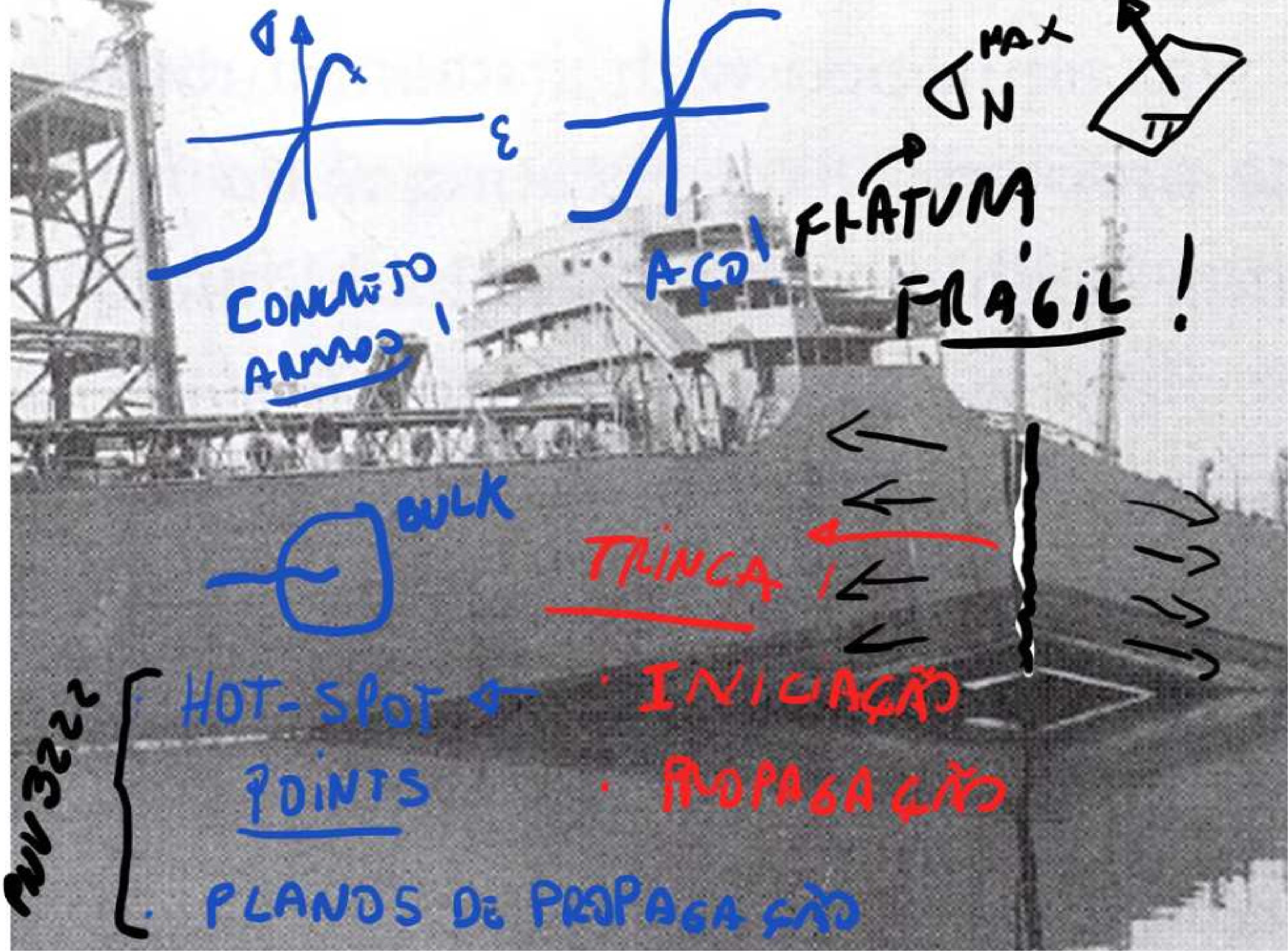
$\sigma_{ij} \equiv$ tensor das tensões !

$$\vec{t}^{(n)} = \sigma_{ij} n_j \rightarrow \hat{n}_{crítico} ?$$

$$[\quad]_{3\times3}[\quad]_{3\times1}$$



σ
ε
CONCRETO ARMADO !
AÇO !
σ_N^{max}
FRATURA
FRÁGIL !
BULK
TRINCA !
· INICIAÇÃO
· PROPAGAÇÃO
· HOT-SPOT
POINTS
· PLANOS DE PROPAGAÇÃO
PNV3222



σ
ε
CONCRETO ARMADO!
AÇO!
σ_N^{max}
FRATURA
FRÁGIL!
BULK
TRINCA!
HOT-SPOT
POINTS
· INICIAÇÃO
· PROPAGAÇÃO
· PLANOS DE PROPAGAÇÃO
PNV3222

PREVER FALHAS !

?

σ_{ij}

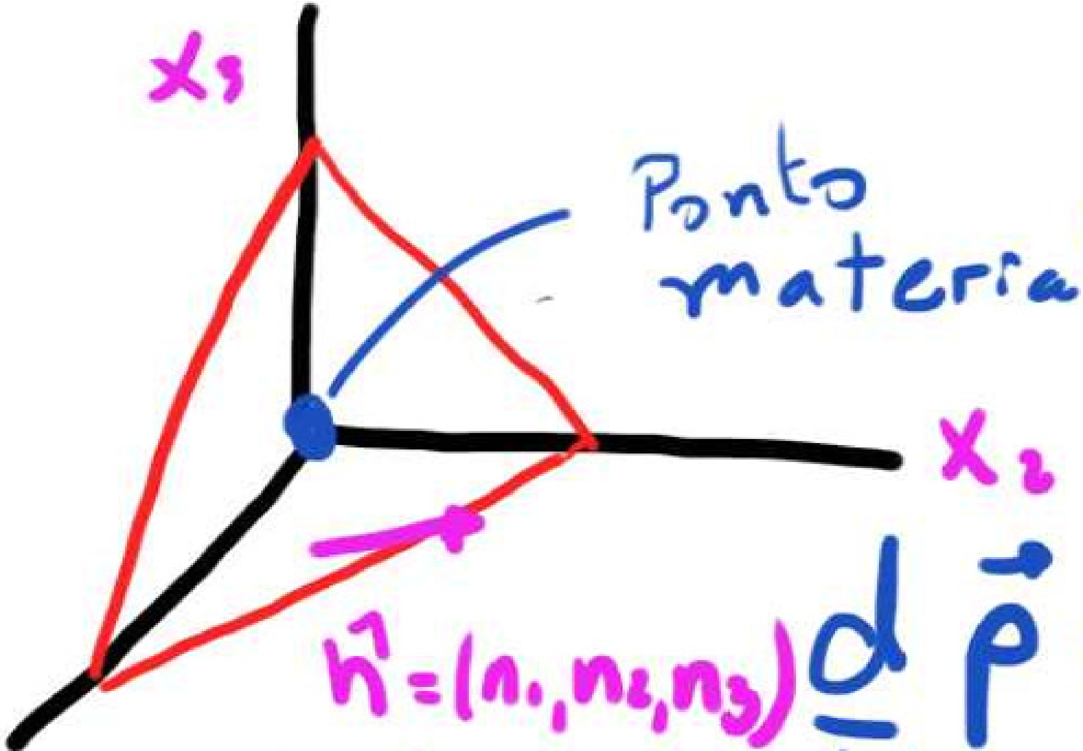
$\sigma_i \,,\ i=1,2,3$

$\Pi_i \,,$

vs. σ_{MAT}

MODOS FALHA

- RESISTÊNCIA
- FLAMBAGEM
- FADIGA
- FLUÊNCIA / CORROSÃO

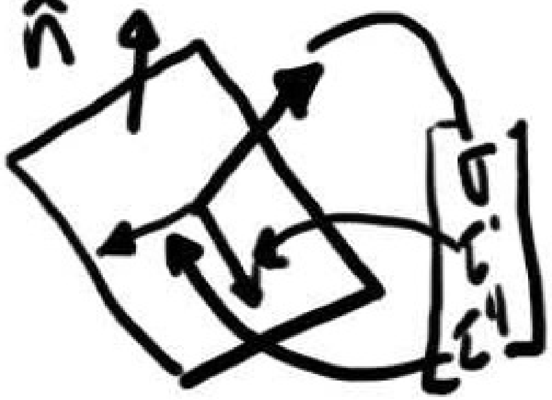


$\vec{n} = (n_1, n_2, n_3)$

$|\hat{n}| = 1$

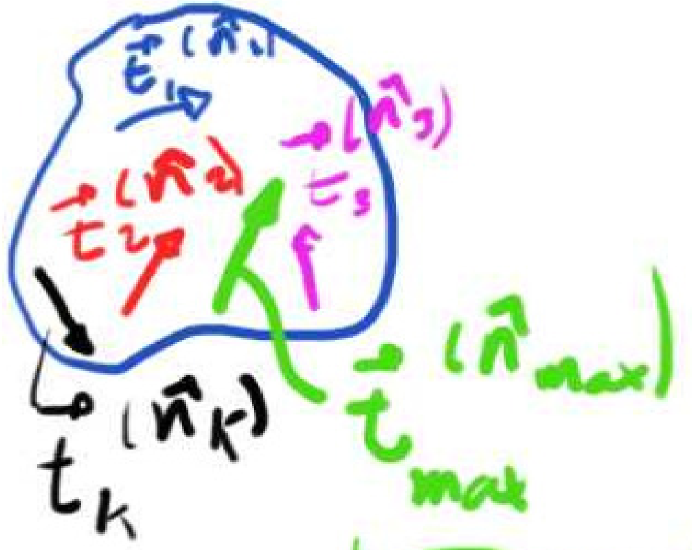
$$\frac{d}{dt}\vec{p} = \vec{F}_R \quad ; \quad \vec{p} = m\vec{v}$$

$\downarrow$ $v \to 0$ (volume)



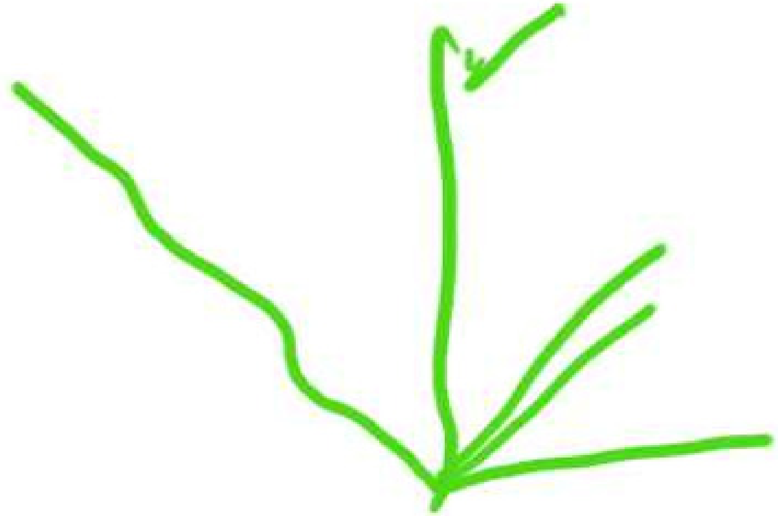
$$= \underbrace{\vec{t}^{(\hat{n})}}_{\text{VETOR TENSÃO}} = \underbrace{\sigma_{ij}}_{\text{TENSOR DAS TENSÕES}} \cdot \hat{n}$$

• ESTADO DE TENSÃO EM UM PONTO



$$\underbrace{\vec{t}_{max}^{(\hat{n}_{max})}}_{\text{tensões principais}}$$

$$\sum_{i=1}^{\infty} (\vec{t}, \hat{n})_i$$



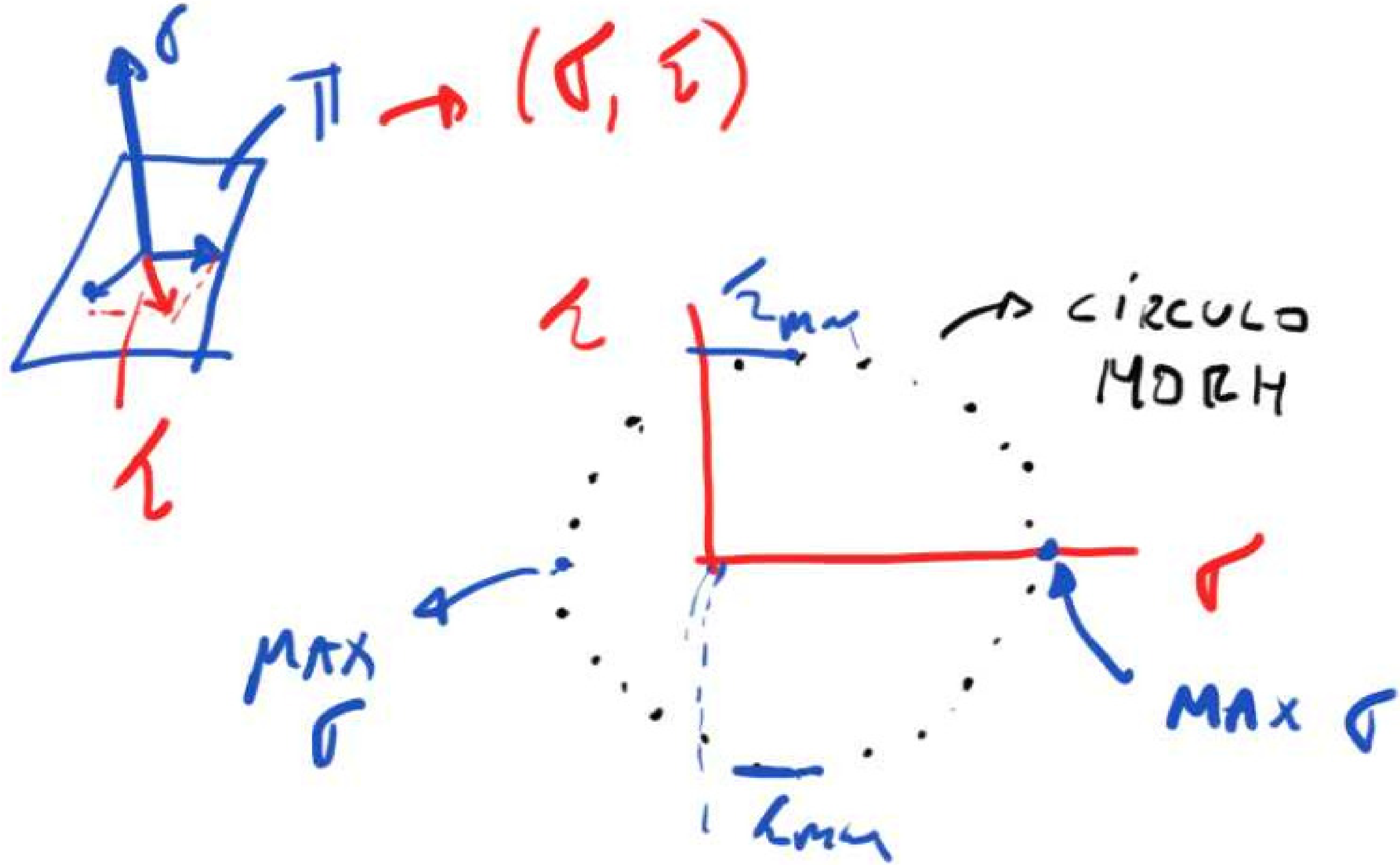
?

$\Sigma \vec{F}$

$\Sigma \vec{M}$

- EQUILÍBRIO
- $\sigma - \varepsilon \Leftrightarrow$
-

$$\sigma_{ij} \rightarrow \begin{matrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{matrix}$$



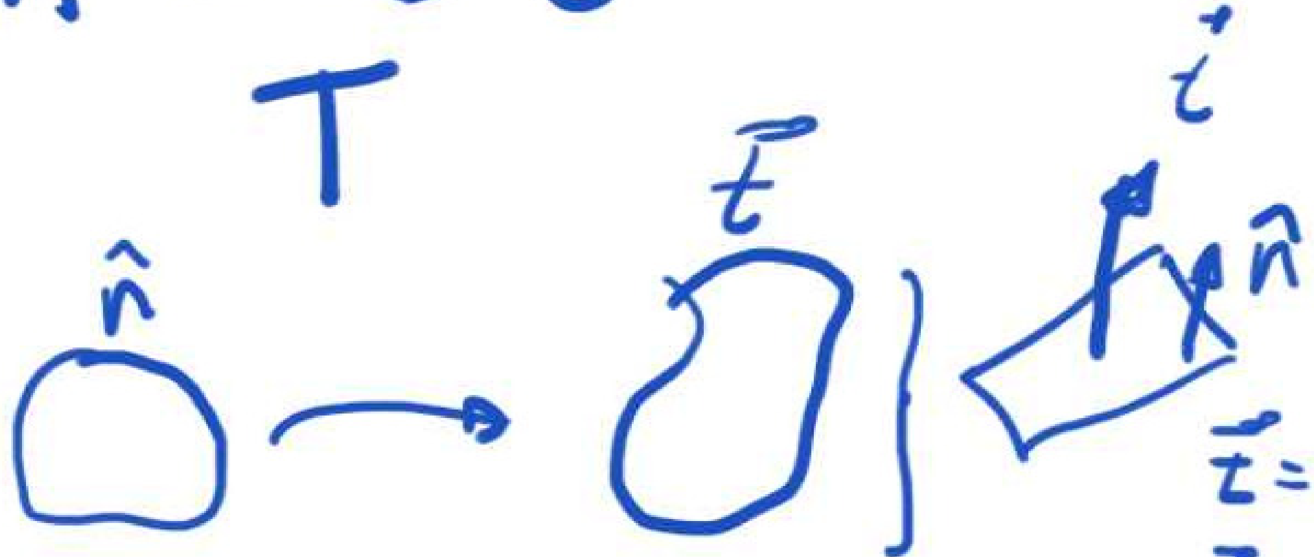
σ
π
(σ, τ)
τ
τ
τmax
CIRCULO MOHR
σ
MAX σ
MAX σ
τmin

$\sigma_{ij} = T$

↳ TENSOR DAS TENSÕES

UMA TRANSFORMAÇÃO LINEAR

$$\hat{n} \xrightarrow[T]{} \vec{t}$$

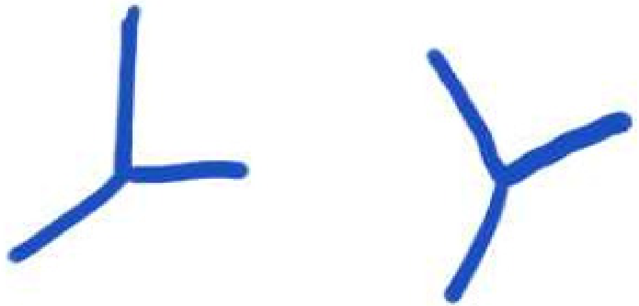


$\vec{t} = \lambda \vec{n}$

$\vec{t} = \sigma_{ij} \vec{n} \longrightarrow$

$\vec{t} = T \vec{n}$

$\sigma_{ij} = T$



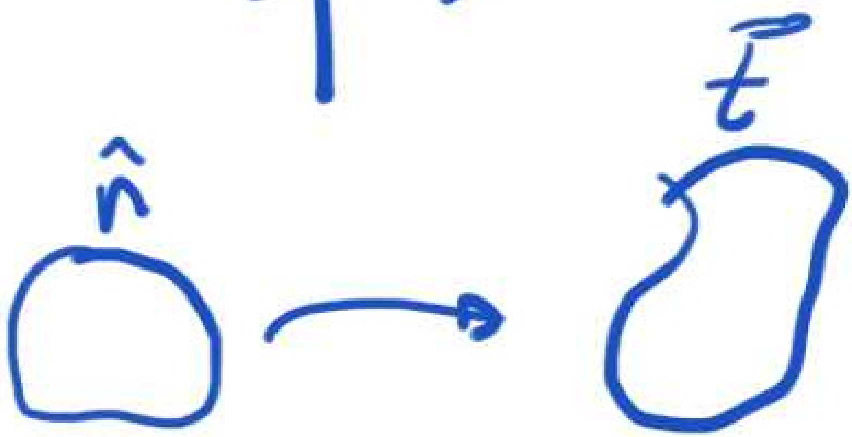
↳ TENSOR DAS TENSÕES

UMA TRANSFORMAÇÃO LINEAR

$$\hat{n} \longrightarrow \vec{t}$$

$$T$$

$$\begin{bmatrix} \sigma_x - \lambda & & \\ \tau_{yx} & \sigma_y - \lambda & \\ \tau_{zx} & \tau_{zy} & \sigma_z - \lambda \end{bmatrix} \; 3 \times 3$$



$$\lambda \hat{n} = T\vec{n}$$

$$(T - \lambda I)\hat{n} = 0$$

λ_i = tensões principais ← eq. caract. λ^3 ← $\det[T - \lambda I] = 0$ $\vec{n} \neq \vec{0}$