Lista de exercícios (extraída do livro de Boyce e Di Prima, seção 5.2, pag. 195)

Problems

In each of Problems 1 through 6, determine the radius of convergence of the given power series.

1. $\sum_{n=0}^{\infty}(x-3)^n$

2. $\sum_{n=0}^{\infty}\frac{n}{2^n}x^n$

3. $\sum_{n=0}^{\infty}\frac{x^{2n}}{n!}$

4. $\sum_{n=0}^{\infty}2^n x^n$

5. $\sum_{n=1}^{\infty}\frac{(x-x_0)^n}{n}$

6. $\sum_{n=1}^{\infty}\frac{(-1)^n n^2(x+2)^n}{3^n}$

In each of Problems 7 through 13, determine the Taylor series about the point x_0 for the given function. Also determine the radius of convergence of the series.

7. $\sin x, \quad x_0 = 0$

8. $e^x, \quad x_0 = 0$

9. $x, \quad x_0 = 1$

10. $x^2, \quad x_0 = -1$

11. $\ln x, \quad x_0 = 1$

12. $\frac{1}{1-x}, \quad x_0 = 0$

13. $\frac{1}{1-x}, \quad x_0 = 2$

14. Let $y = \sum_{n=0}^{\infty} nx^n$.

a. Compute y' and write out the first four terms of the series.
b. Compute y'' and write out the first four terms of the series.

15. Let $y = \sum_{n=0}^{\infty} a_n x^n$.

a. Compute y' and y'' and write out the first four terms of each series, as well as the coefficient of x^n in the general term.
b. Show that if $y'' = y$, then the coefficients a_0 and a_1 are arbitrary, and determine a_2 and a_3 in terms of a_0 and a_1.
c. Show that $a_{n+2} = \frac{a_n}{(n+2)(n+1)}$, $n = 0, 1, 2, 3, \ldots$.

In each of Problems 16 and 17, verify the given equation.

16. $\sum_{n=0}^{\infty} a_n(x-1)^{n+1} = \sum_{n=1}^{\infty} a_{n-1}(x-1)^n$

17. $\sum_{k=0}^{\infty} a_{k+1}x^k + \sum_{k=0}^{\infty} a_k x^{k+1} = a_1 + \sum_{k=1}^{\infty}(a_{k+1}+a_{k-1})x^k$

In each of Problems 18 through 22, rewrite the given expression as a single power series whose generic term involves x^n.

18. $\sum_{n=2}^{\infty} n(n-1)a_n x^{n-2}$

19. $x\sum_{n=1}^{\infty} na_n x^{n-1} + \sum_{k=0}^{\infty} a_k x^k$

20. $\sum_{m=2}^{\infty} m(m-1)a_m x^{m-2} + x\sum_{k=1}^{\infty} ka_k x^{k-1}$

21. $\sum_{n=1}^{\infty} na_n x^{n-1} + x\sum_{n=0}^{\infty} a_n x^n$

22. $x\sum_{n=2}^{\infty} n(n-1)a_n x^{n-2} + \sum_{n=0}^{\infty} a_n x^n$

23. Determine the a_n so that the equation

$$\sum_{n=1}^{\infty} na_n x^{n-1} + 2\sum_{n=0}^{\infty} a_n x^n = 0$$

is satisfied. Try to identify the function represented by the series $\sum_{n=0}^{\infty} a_n x^n$.