PSI3211 – CIRCUITOS ELÉTRICOS I

Solução da Lista 2: Funções de Excitação e Números Complexos

Funções de Excitação

1 –

a) $i(t) = 2t\,[H(t) - H(t-10)] + 20\,[H(t-10) - H(t-25)] +$
$+ (60 - 1{,}2t)\,[H(t-25) - H(t-50)]$ (mA, μs)

ou

$i(t) = 2t\,H(t) + (20 - 2t)\,H(t-10) + (40 - 1{,}2t)\,H(t-25) +$
$+ (1{,}2t - 60)\,H(t-50)$ (mA, μs)

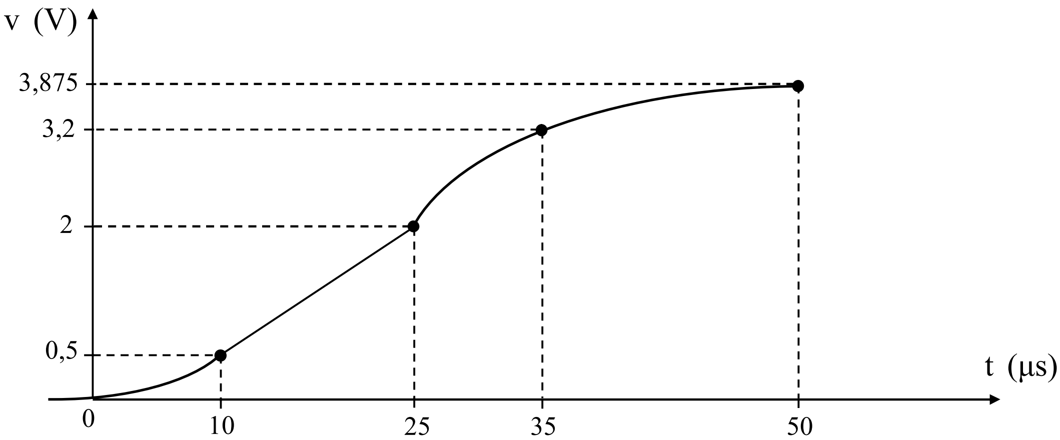
b) carga armazenada $= \int i\,dt$ = área do gráfico

$$q = \left[\frac{20\cdot 10}{2} + 15\cdot 20 + \frac{30\cdot 25}{2}\right]\cdot 10^{-9} = 775\cdot 10^{-9}\,\mathrm{C} = 0{,}775\ \mu\mathrm{C}$$

$$\text{Mas } q = Cv \rightarrow v_{final} = \frac{q}{C} = \frac{0{,}775}{0{,}2} = 3{,}875\ \mathrm{V}$$

$$W = \frac{1}{2}Cv^2 - \frac{1}{2}C\underbrace{\cancel{v_0^2}}_{0} \cong 1{,}5\ \mu\mathrm{J}$$

c) $v(t) = \dfrac{1}{C}\displaystyle\int i\,dt + \overbrace{\cancel{v_0}}^{0}$



$$i(35) = 2\cdot 35 + (20 - 2\cdot 35) + (40 - 1{,}2\cdot 35) = 18\ \mathrm{mA}$$

$$q(35) = \left[\frac{10\cdot 20}{2} + 15\cdot 20 + \frac{10\cdot 12}{2} + 10\cdot 18\right]\cdot 10^{-9} = 6{,}4\cdot 10^{-7}\,\mathrm{C}$$

$$v(35) = \frac{q(35)}{C} = 3{,}2\ \mathrm{V}$$

2 –

a) $v(t) = \dfrac{1}{C}\displaystyle\int_{t_0}^{t} i(\tau)\, d\tau + v(0) \quad \rightarrow \quad v(t) = \dfrac{1}{C}\displaystyle\int_{t_0}^{t} i\left(\tau\right)d\tau + v(0)$

(a tensão de saída é proporcional à integral da tensão de entrada)

b) $v(t) = 5t + 5$ (V, s), $t > 0$

$p = v \cdot i = (5\,t + 5) \cdot 5$ (W, s), $t > 0 \quad \rightarrow \quad p_C(t) = 25t + 25$ (W, s), $t > 0$

3 – $i_{máx} = 10^{-12}\,(e^{32} - 1) \cong 78{,}9$ mA $\qquad T = \dfrac{2\pi}{10} \cong 0{,}6283$ s

$i_{min} = 10^{-12}\,(e^{-32} - 1) \cong -10^{-12}$ mA

$i_{médio} = \dfrac{3{,}513}{T}\,(\text{mA}), \left(\begin{array}{l}\text{por solução numérica}\\ \text{por computador}\end{array}\right) \cong 5{,}5911$ mA

4 –

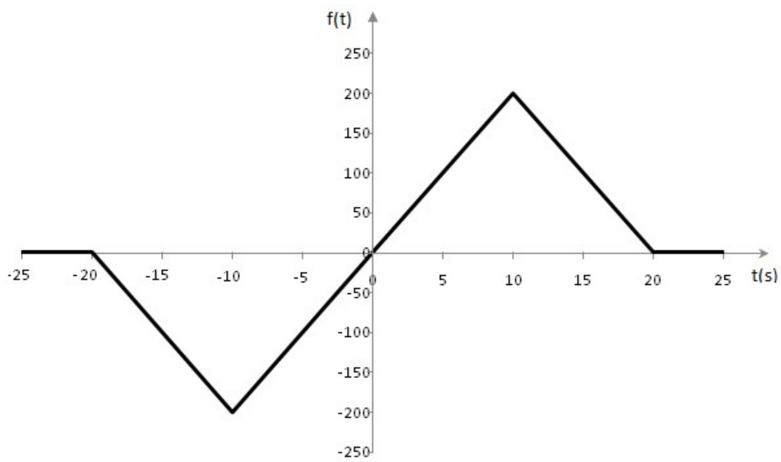
a) $f(t) = 5t\,[H(t) - H(t-2)] + 10\,[H(t-2) - H(t-6)] +$
$+ (-5t + 40)\,[H(t-6) - H(t-8)]$

b) $f(t) = 50\,\text{sen}(\pi t/2)\,[H(t) - H(t-4)]$

c) $f(t) = 4t\,[H(t) - H\,(t-5)]$

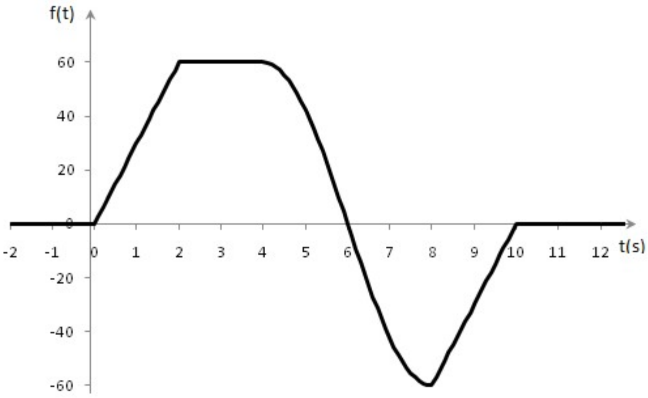
d) $f(t) = (30t + 120)\,H(t+4) - 60t\,H(t) + (60t - 480)\,H(t-8) +$
$+ (-30t + 360)\,H(t-12)$

5 –



6 –

a)



b) $f(t) = 30t\,[H(t)-H(t-2)] + 60\,[H(t-2)-H(t-4)] +$
$+\,60\cos(\pi t/4-\pi)\,[H(t-4)-H(t-8)] +$
$+\,(30\,t-300)\,[H(t-8)-H(t-10)]$

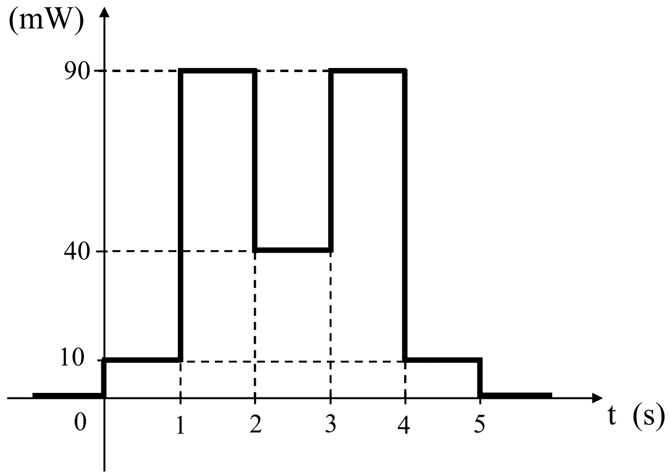
7 –

a) $e_g(t) = [H(t)-H(t-1)] + 3\,[H(t-1) - H(t-2)] +$
$+\ 2\,[H(t-2) - H(t-3)] + 3\,[H(t-3) - H(t-4)] +$
$+\ [H(t-4) - H(t-5)]$ (V, s)

ou, juntando os termos comuns:

$$e_g(t) = H(t) + 2H(t-1) - H(t-2) + H(t-3) - 2H(t-4) - H(t-5) \quad \text{(V, s)}$$

b)



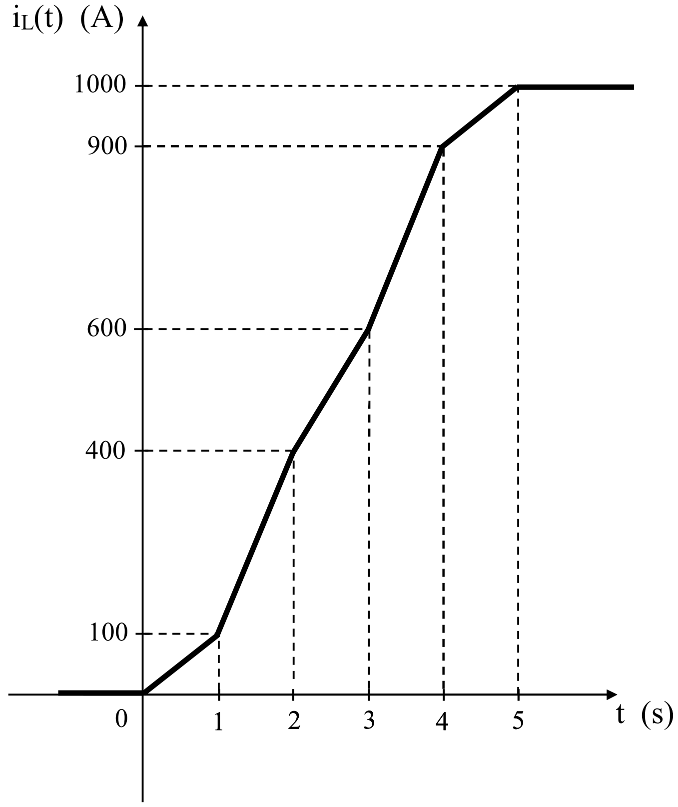
$$p_R(t) = \frac{e_g^2(t)}{R}$$

c) $w_C = \frac{1}{2}\,C\,v_C^2(2{,}5) - \frac{1}{2}\,C\,v_C^2(0{,}5)$

$\Rightarrow\ w_C = \frac{1}{2}\cdot 100\cdot 10^{-6}\left[e_g^2(2{,}5) - e_g^2(0{,}5)\right] = \frac{1}{2}\cdot 100\cdot 10^{-6}\left(2^2-1^2\right) \quad \Rightarrow$

$w_C = 1{,}5\cdot 10^{-4}$ J

d)



$$i_L(t) = \frac{1}{L}\int e_g(t)dt + \cancel{i_L(0)}^{\,0} = 100\int e_g(t)dt$$

Calcula-se a integral graficamente (área).

e) $\hat{I}_L = \dfrac{\hat{E}_g}{j\omega L} = \dfrac{2\angle -30^o}{j\cdot 5\cdot 10\cdot 10^{-3}} = \dfrac{2\angle -30^o}{0{,}05\angle 90^o}$

$\hat{I}_L = 40\angle -120^o$ A

$i_L(t) = 40\cos(5t - 120^o)$ (A, s)

Operação com números complexos

4 –

a) $\hat{I} = 8\angle 60^o$ $\quad\omega = 10$ rad/s

b) $\hat{V} \cong 12{,}581\angle -78{,}54^o$ $\quad\omega = 10$ rad/s

c) Não existe (soma de duas senoides de frequências diferentes)

d) $\hat{P} = 25\angle -90^o$ $\quad\omega = 40$ rad/s

e) Não existe.

5 –

a) $v(t) = 100\cos(10t + 90^o)$

b) $i(t) = 5\cos(10t + 82^o)$

c) $v(t) \cong 70{,}71\cos(10t + 75^o)$

d) $v(t) \cong 0{,}51\cos(10t - 191{,}31^o)$

7 – Seja $s(t) = \sum_{k=1}^{n} A_k \cos(\omega t + \theta_k)$

$\hat{S}$ = fasor correspondente a s(t),

$\hat{A}_k$ = fasor correspondente a cada senoide.

Então

$$s(t) = A_1 \cos(\omega t + \theta_1) + A_2 \cos(\omega t + \theta_2) + ... + A_n \cos(\omega t + \theta_n)$$

$$= \mathrm{Re}\left\{\left(\underbrace{A_1 e^{j\theta_1}}_{\hat{A}_1} + \underbrace{A_2 e^{j\theta_2}}_{\hat{A}_2} + ... + \underbrace{A_n e^{j\theta_n}}_{\hat{A}_n}\right) e^{j\omega t}\right\}$$

$$= \mathrm{Re}\left\{\hat{A}_1 + \hat{A}_2 + ... + \hat{A}_n\right\} e^{j\omega t}$$

Portanto,

$$\hat{S} = \hat{A}_1 + \hat{A}_2 + ... + \hat{A}_n .$$

8 – $v(t) = A_1\cos(\omega t + \theta_1) \cdot A_2\cos(\omega t + \theta_2)$

$$= \frac{A_1 A_2}{2} \cos(\theta_1 - \theta_2) + \frac{A_1 A_2}{2} \cdot \cos(2\omega t + \theta_1 + \theta_2)$$

$\Rightarrow$ não existe $\hat{V}$ que represente v(t), já que o sinal não é uma cossenoide unicamente.

Já $\hat{V}_1 \cdot \hat{V}_2 = A_1 e^{j\theta_1} \cdot A_2 e^{j\theta_2} = A_1 A_2 e^{j(\theta_1 + \theta_2)}$ representa a função

$$A_1A_2 \cos(\omega t + \theta_1 + \theta_2)$$

no domínio do tempo, que é diferente da v(t) representada acima.

9 –

a) $\hat{V} = R\hat{I} = 1{,}2 - j0{,}4$ V

b) $\hat{V} = j\omega L\hat{I} = 0{,}3 + j0{,}9$ V

c) $\hat{V} = \dfrac{\hat{I}}{j\omega C} = -0{,}25 - j0{,}75$ V

10 –

a) $v(t) \cong 1{,}2649 \cos(\omega t - 18{,}43^o)$ (V, ms) $\rightarrow$ $v(1) \cong 0{,}9849$ V

b) $v(t) \cong 0{,}9487 \cos(\omega t + 71{,}56^o)$ (V, ms) $\rightarrow$ $v(1) \cong -0{,}5952$ V

c) $v(t) \cong 0{,}7906 \cos(\omega t - 108{,}43^o)$ (V, ms) $\rightarrow$ $v(1) \cong 0{,}4960$ V

11 – $\hat{I}_R = 2 \angle 30^o$ A , $\hat{I}_C = 20 \angle 120^o$ A

$\hat{I} = \hat{I}_R + \hat{I}_C \longrightarrow \hat{I} \cong 20{,}1 \angle 114{,}28^o$ A

$i(t) \cong 20{,}1 \cos(10t + 114{,}28^o)$ (A, s)