SEM 536 - Sistemas de Controle I

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Exemplos de Transformada de Laplace:

1) Função impulso unitário: $\delta(t)$

$$F(s) = \int_0^\infty \delta(t) e^{-st} dt$$

Considere a função pulso:

$$F(s) = \int_0^{t_0} \frac{1}{t_0} e^{-st} dt + \int_{t_0}^\infty 0 e^{-st} dt$$

$$F(s) = \left. \frac{-e^{-st}}{t_0 s} \right|_0^{t_0}$$

$$F(s) = \frac{-e^{-st_0}}{t_0 s} + \frac{1}{t_0 s} = \frac{1 - e^{-st_0}}{t_0 s}$$

Para o impulso:

$$F(s) = lim_{t_0 \to 0} \frac{1 - e^{-st_0}}{t_0 s} = lim_{t_0 \to 0} \frac{s e^{-st_0}}{s} = lim_{t_0 \to 0} e^{-st_0} = 1.$$

2) Função degrau com amplitude a: $f(t) = a$, para $t \geq 0$

$$F(s) = \int_0^\infty a e^{-st} dt = \left. \frac{-a e^{-st}}{s} \right|_0^\infty = 0 - \frac{-a}{s} = \frac{a}{s}$$

3) Função rampa: $f(t) = bt$

$$F(s) = \int_0^\infty bt e^{-st} dt = \left. \left[\frac{-bt e^{-st}}{s} - \frac{b e^{-st}}{s^2} \right] \right|_0^\infty = \frac{b}{s^2}$$

$$\int u dv = uv - \int v du, \qquad u = bt, \quad dv = e^{-st} dt$$

4) Função seno: $f(t) = sen(wt)$

$$F(s) = \int_0^\infty sen(wt)e^{-st}dt$$

$$sen(wt) = \frac{e^{jwt} - e^{-jwt}}{2j}$$

$$F(s) = \frac{1}{2j}\int_0^\infty (e^{-(s-jw)t} - e^{-(s+jw)t})dt$$

$$F(s) = \frac{1}{2j}\left[\frac{-e^{-(s-jw)t}}{s-jw} + \frac{e^{-(s+jw)t}}{s+jw}\right]\Bigg|_0^\infty = \frac{1}{2j}\left[\frac{1}{s-jw} + \frac{-1}{s+jw}\right]$$

$$F(s) = \frac{1}{2j}\frac{2jw}{s^2+w^2} = \frac{w}{s^2+w^2}$$

Exemplos de Transforma de Laplace Inversa

1)

$$F(s) = \frac{(s+2)(s+4)}{s(s+1)(s+3)} = \frac{C_1}{s} + \frac{C_2}{s+1} + \frac{C_3}{s+3}$$

$$C_1 = \frac{(s+2)(s+4)}{(s+1)(s+3)}\Bigg|_{s=0} = \frac{8}{3}$$

$$C_2 = \frac{(s+2)(s+4)}{s(s+3)}\Bigg|_{s=-1} = \frac{-3}{2}$$

$$C_3 = \frac{(s+2)(s+4)}{s(s+1)}\Bigg|_{s=-3} = \frac{-1}{6}$$

$$F(s) = \frac{8}{3}\frac{1}{s} - \frac{3}{2}\frac{1}{s+1} - \frac{1}{6}\frac{1}{s+3} \quad \Rightarrow \quad f(t) = \frac{8}{3} - \frac{3}{2}e^{-t} - \frac{1}{6}e^{-3t}$$

Teorema do Valor Final

$$F(s) = \frac{(s+2)(s+4)}{s(s+1)(s+3)}$$

$$f(\infty) = lim_{s \to 0} sF(s) = lim_{s \to 0} \frac{s(s+2)(s+4)}{s(s+1)(s+3)} = \frac{8}{3}$$

Teorema do Valor Inicial

$$F(s) = \frac{(s+2)(s+4)}{s(s+1)(s+3)}$$

$$f(0^+) = lim_{s \to \infty} sF(s) = lim_{s \to \infty} \frac{s(s^2+6s+8)}{s(s^2+4s+3)}$$

$$f(0^+) = lim_{s \to \infty} \frac{(s^2+6s+8)}{(s^2+4s+3)} = lim_{s \to \infty} \frac{(2s+6)}{(2s+4)} = lim_{s \to \infty} \frac{2}{2} = 1$$

$$f(t) = \frac{8}{3} - \frac{3}{2}e^{-t} - \frac{1}{6}e^{-3t}$$

$$f(0^+) = \frac{8}{3} - \frac{3}{2} - \frac{1}{6} = \frac{16}{6} - \frac{9}{6} - \frac{1}{6} = \frac{6}{6} = 1$$