# Teoria Quântica de Campos - SFI 5892

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**Horário: Segundas-Feiras e Quartas-Feiras às 16:00 Hs**
**no Google Meet: meet.google.com/pmw-mszp-ekg**

## Programa resumido

1. **Revisão de Teoria Clássica de Campos:** simetrias de Lorentz e Poincaré, simetrias internas, teorema de Noether
2. **Quantização Canônica dos Campos:** campo escalar, campo de Dirac, campo eletromagnético (método covariante de Gupta-Bleuler)
3. **Expansão da Matriz S:** representação de interação, teorema de Wick, regras de Feynman para a QED
4. **Cálculo de processos:** espalhamento Compton, espalhamento Moller, espalhamento Bhabha, auto energia do elétron, auto energia do fóton, espalhamento por campo externo.

### Bibliografia

1. *Quantum Field Theory*, F. Mandl e G. Shaw
2. *The Quantum Theory of Fields*, S. Weinberg, vols 1 e 2
3. *Quantum Fields*, N.N. Bogoliubov e D.V. Shirkov
4. *Field Theory: A Modern Primer*, Pierre Ramond
5. *Quantum Field Theory*, C. Itzykson e J.B. Zuber
6. *An Introduction to Quantum Field Theory*, M. E. Peskin e D.V. Schroeder
7. *Quantum Field Theory in a Nutshell*, A. Zee
8. *Quantum Field Theory for the Gifted Amateur*, Tom Lancaster e Stephen J. Blundell

9. *Teoria Quântica de Campos*, Marcelo Gomes
10. *A First Book of Quantum Field Theory*, Amitabha Lahiri e Palash B. Pal
11. *Teoria Quântica de Campos*, L. A. Ferreira (notas de aula)

**Avaliação**

Dois trabalhos ao longo do semestre e um trabalho substitutivo. A nota final é a média aritmética de dois trabalhos. As datas dos trabalhos são:

1. Trabalho I: 5 de Outubro (Segunda-feira)
2. Trabalho II: 23 de Novembro (Segunda-feira)
3. Trabalho Substitutivo: 2 de Dezembro (Quarta-feira)

As listas de exercícios, bem como qualquer material relacionado ao curso, podem ser obtidas na plataforma e-disciplinas no endereço:
`https://edisciplinas.usp.br/course/view.php?id=80152`

# Why Quantum Field Theory?

Electromagnetic phenomena were first described by a classical field theory

Phenomena involving (stable) massive particles, like electrons, were first described as a theory (classical or quantum) of a single particle

High energy phenomena (high speed) needs relativistic description where mass and energy can transform one into another: particle creation and annihilation

First shell of the onion: Field Theory is in fact a formalism to treat systems where the number of particles is not constant.

Field Theory does not have to be relativistic: condensed matter, phonons in a crystal.

There is more to it: locality, causality, renormalisation, etc

# 1 Maxwell Theory of Electromagnetism

We shall work with the Heaviside-Lorentz units (rationalised Gaussian CGS) where Maxwell equations read

$$\nabla \cdot \vec{B} = 0 \qquad \vec{\nabla} \wedge \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} \tag{1.1}$$

and

$$\nabla \cdot \vec{E} = \rho \qquad \vec{\nabla} \wedge \vec{B} = \frac{1}{c}\frac{\partial \vec{E}}{\partial t} + \frac{1}{c}\vec{J} \tag{1.2}$$

Use the identity

$$\vec{\nabla} \wedge \left(\vec{\nabla} \wedge \vec{V}\right) = -\nabla^2 \vec{V} + \vec{\nabla}\left(\vec{\nabla} \cdot \vec{V}\right) \tag{1.3}$$

with

$$\rho = 0 \qquad \vec{J} = 0 \tag{1.4}$$

to get

$$\frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} - \nabla^2 \vec{E} = 0 \qquad \frac{1}{c^2}\frac{\partial^2 \vec{B}}{\partial t^2} - \nabla^2 \vec{B} = 0 \tag{1.5}$$

These are the wave equations for the electric and magnetic fields.

## 1.1 Potentials

Define

$$\vec{B} = \vec{\nabla} \wedge \vec{A} \qquad \vec{E} = -\vec{\nabla}\phi - \frac{1}{c}\frac{\partial \vec{A}}{\partial t} \tag{1.6}$$

Then (1.1) are automatically satisfied.

The equations (1.2) lead to (assuming (1.4))

$$\begin{aligned} \vec{\nabla} \cdot \vec{E} &= -\nabla^2 \phi - \frac{1}{c}\frac{\partial\left(\vec{\nabla} \cdot \vec{A}\right)}{\partial t} \\ \vec{\nabla} \wedge \vec{B} - \frac{1}{c}\frac{\partial \vec{E}}{\partial t} &= -\nabla^2 \vec{A} + \vec{\nabla}\left(\vec{\nabla} \cdot \vec{A}\right) + \frac{1}{c}\frac{\partial\left(\vec{\nabla}\phi\right)}{\partial t} + \frac{1}{c^2}\frac{\partial^2 \vec{A}}{\partial t^2} \end{aligned} \tag{1.7}$$

Impose the gauge condition (Lorentz gauge)

$$\frac{1}{c}\frac{\partial \phi}{\partial t} + \vec{\nabla} \cdot \vec{A} = 0 \tag{1.8}$$

to get

$$\frac{1}{c^2}\frac{\partial^2 \vec{\phi}}{\partial t^2} - \nabla^2 \vec{\phi} = 0 \qquad \frac{1}{c^2}\frac{\partial^2 \vec{A}}{\partial t^2} - \nabla^2 \vec{A} = 0 \tag{1.9}$$

Again the wave equations.

## 2 A simple example

In 1926, Born, Heisenberg and Jordan ignored the polarization of electromagnetic waves and worked in one spatial dimension, confined to a box. The Hamiltonian is[1]

$$H = \frac{1}{2} \int_0^L dx \left[ \left( \frac{\partial u}{\partial t} \right)^2 + c^2 \left( \frac{\partial u}{\partial x} \right)^2 \right] \tag{2.10}$$

with the boundary condition

$$u(t, 0) = u(t, L) = 0 \tag{2.11}$$

Taking the Fourier components

$$u(t, x) = \sum_{k=1}^{\infty} q_k(t) \sin\left( \frac{\omega_k x}{c} \right) \qquad \omega_k = \frac{k \pi c}{L} \tag{2.12}$$

and using the identity

$$\int_0^L dx \sin\left( \frac{k \pi x}{L} \right) \sin\left( \frac{k' \pi x}{L} \right) = \frac{L}{2} \delta_{k, k'} \tag{2.13}$$

we get

$$H = \frac{L}{2} \sum_{k=1}^{\infty} \left[ \frac{1}{2} \dot{q}_k^2 + \frac{1}{2} \omega_k^2 q_k^2 \right] \tag{2.14}$$

An infinite number os oscillators with frequencies $\omega_k$ and mass $L/2$. The momentum is

$$p_k = \frac{L}{2} \dot{q}_k \tag{2.15}$$

and so

$$H = \sum_{k=1}^{\infty} \left[ \frac{2}{L} \frac{p_k^2}{2} + \frac{L}{2} \omega_k^2 q_k^2 \right] \tag{2.16}$$

We QUANTISE such oscillators canonically

$$[q_k, q_{k'}] = [p_k, p_{k'}] = 0 \qquad [q_k, p_{k'}] = i \hbar \delta_{k, k'} \tag{2.17}$$

Introduce

$$\tilde{a}_k = \sqrt{\frac{L \omega_k}{4 \hbar}} \left[ q_k + \frac{2 i}{L \omega_k} p_k \right] \qquad \tilde{a}_k^\dagger = \sqrt{\frac{L \omega_k}{4 \hbar}} \left[ q_k - \frac{2 i}{L \omega_k} p_k \right] \tag{2.18}$$

and get

$$[\tilde{a}_k, \tilde{a}_{k'}] = \left[ \tilde{a}_k^\dagger, \tilde{a}_{k'}^\dagger \right] = 0 \qquad \left[ \tilde{a}_k, \tilde{a}_{k'}^\dagger \right] = \delta_{k, k'} \tag{2.19}$$

The Hamiltonian becomes

$$H = \sum_{k=1}^{\infty} \hbar \omega_k \left( \tilde{a}_k^\dagger \tilde{a}_k + \frac{1}{2} \right) \tag{2.20}$$

[1]based on section 1.2 of Weinberg's book, vol. 1

## 2.1 Hilbert space

The Hilbert space of such a system is the direct product of the Hilbert space of an infinite number of harmonic oscillators with frequency $\omega_k$. The states are labelled by the occupation number, and the number operator is

$$N_k = \widetilde{a}_k^\dagger \, \widetilde{a}_k \tag{2.21}$$

Indeed

$$N_k \ \mid n_k\rangle_k = n_k \ \mid n_k\rangle_k \tag{2.22}$$

Note that

$$\left[ N_k \, , \, \widetilde{a}_{k'}^\dagger \right] = \widetilde{a}_k^\dagger \, \delta_{k\,,\,k'} \qquad\qquad [\, N_k \, , \, \widetilde{a}_{k'} \,] = -\widetilde{a}_k \, \delta_{k\,,\,k'} \tag{2.23}$$

and so

$$\widetilde{a}_k^\dagger \ \mid n_k\rangle_k \sim \mid n_k + 1\rangle_k \qquad\qquad \widetilde{a}_k \ \mid n_k\rangle_k \sim \mid n_k - 1\rangle_k \tag{2.24}$$

Given a generic state

$$\mid \psi\rangle = \prod_{k=1}^{\infty} \otimes \mid n_k\rangle_k \tag{2.25}$$

one has

$$H \ \mid \psi\rangle = \sum_{k=1}^{\infty} \hbar \, \omega_k \, \left( n_k + \frac{1}{2} \right) \ \mid \psi\rangle \tag{2.26}$$

The vacuum state

$$\mid 0\rangle = \prod_{k=1}^{\infty} \otimes \mid 0\rangle_k \tag{2.27}$$

has infinite energy!!!

$$H \ \mid 0\rangle = \sum_{k=1}^{\infty} \frac{1}{2} \, \hbar \, \omega_k \ \mid 0\rangle \tag{2.28}$$

RENORMALISE it!!!

## 2.2 The field

Note that

$$q_k\,(t) = \sqrt{\frac{\hbar}{L\,\omega_k}}\,\left[a_k\,e^{-i\,\omega_k\,t} + a_k^\dagger\,e^{i\,\omega_k\,t}\right] \qquad \widetilde{a}_k = a_k\,e^{-i\,\omega_k\,t} \qquad \widetilde{a}_k^\dagger = a_k^\dagger\,e^{i\,\omega_k\,t} \tag{2.29}$$

satisfies the harmonic oscillator equation

$$\frac{d^2\,q_k}{d\,t^2} = -\omega_k^2\,q_k \tag{2.30}$$

In addition, $a$'s satisfy the same commutation relations as $\widetilde{a}$'s.

$$[\,a_k\,,\;a_{k'}\,] = \left[\,a_k^\dagger\,,\;a_{k'}^\dagger\,\right] = 0 \qquad \left[\,a_k\,,\;a_{k'}^\dagger\,\right] = \delta_{k\,,\,k'} \tag{2.31}$$

Therefore the field becomes an operator

$$u\,(t\,,\,x) = \sum_{k=1}^{\infty}\sqrt{\frac{\hbar}{L\,\omega_k}}\,\left[a_k\,e^{-i\,\omega_k\,t} + a_k^\dagger\,e^{i\,\omega_k\,t}\right]\,\sin\left(\frac{\omega_k\,x}{L}\right) \qquad \omega_k = \frac{k\,\pi\,c}{L} \tag{2.32}$$

## 2.3 Moral of the story

We have a theory of a massless particle. The states can have any number of such identical particles with momentum and energy

$$e_k = \hbar\,\omega_k \qquad p_k = \frac{\hbar\,k\,\pi}{L} \tag{2.33}$$

satisfying the (dispersion) relation

$$e_k^2 - p_k^2\,c^2 = 0 \tag{2.34}$$

The number of particles can vary. So, we have a formalism to treat a quantum system with a varying number of particles (excitations of the fields).