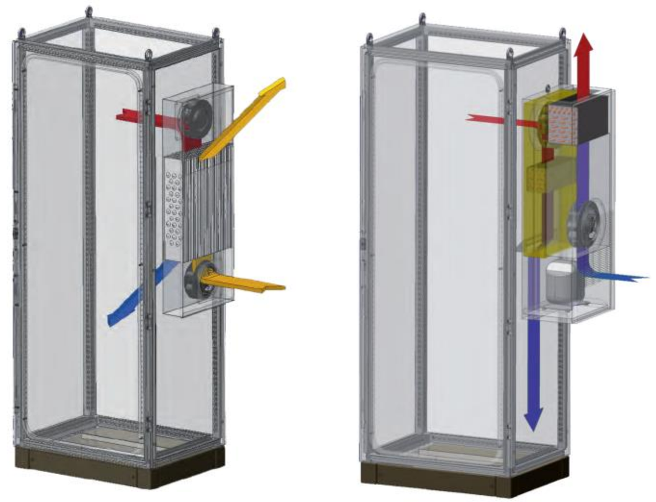
Universidade de São Paulo
Escola de Engenharia de São Carlos

SEM0551 Fenômenos de Transporte

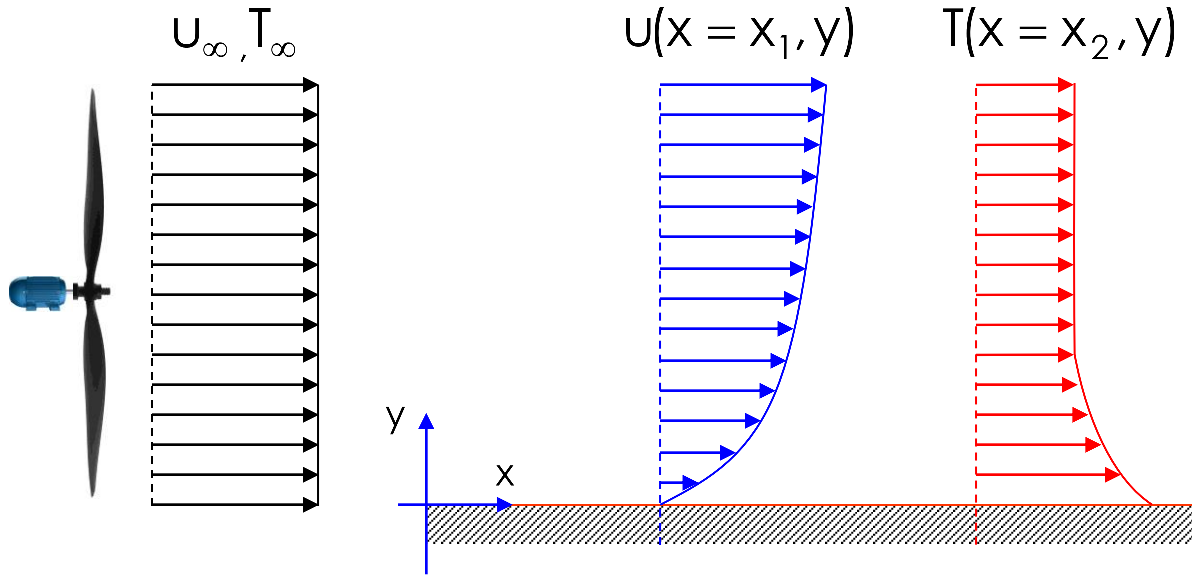
CONVECÇÃO DE CALOR EM ESCOAMENTOS EXTERNOS

Paulo Seleghim Jr.
Universidade de São Paulo

Convecção forçada / escoamentos externos



Adimensionalização das equações de escoamento sobre uma placa plana...



$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \leftarrow \text{massa}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{Re}\frac{\partial^2 u}{\partial y^2} \qquad \leftarrow \text{q. de movimento}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{Re \cdot Pr}\frac{\partial^2 T}{\partial y^2} \qquad \leftarrow \text{energia } \textbf{(desacoplada)}$$

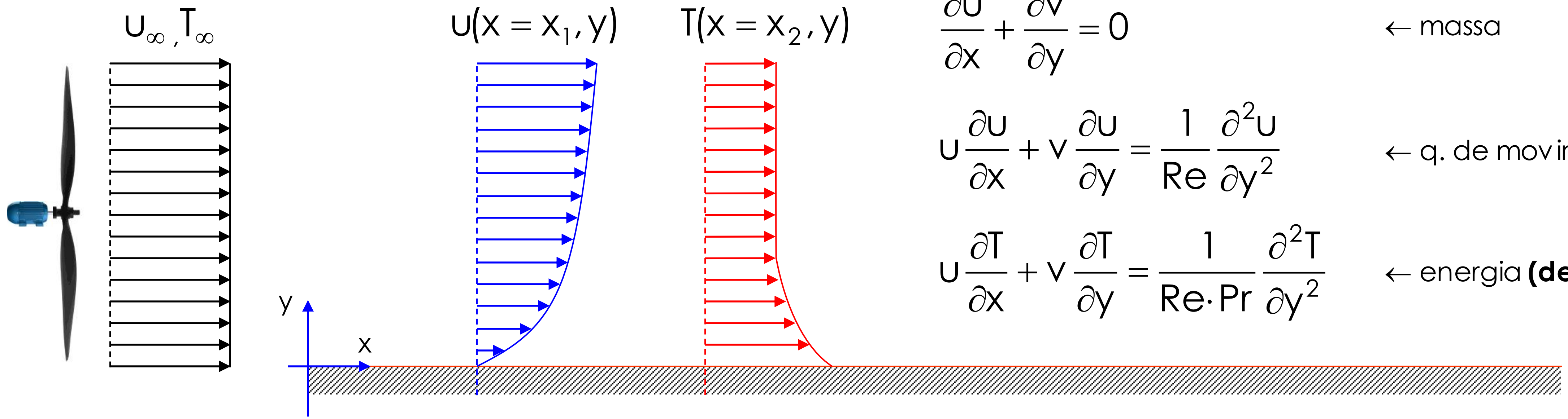
$$Nu \overset{def}{=} \frac{h_q}{k/D}$$

$$Nu = f\left(Re, Pr\right)$$

$$Re = \frac{\rho u_0 D}{\mu}$$ caracteriza o escoamento

caracteriza o fluido $$Pr = \frac{C_p \mu}{k}$$

Adimensionalização das equações de escoamento sobre uma placa plana...



$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \leftarrow \text{massa}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{1}{Re}\frac{\partial^2 u}{\partial y^2} \quad \leftarrow \text{q. de movimento}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{1}{Re \cdot Pr}\frac{\partial^2 T}{\partial y^2} \quad \leftarrow \text{energia } \textbf{(desacoplada)}$$

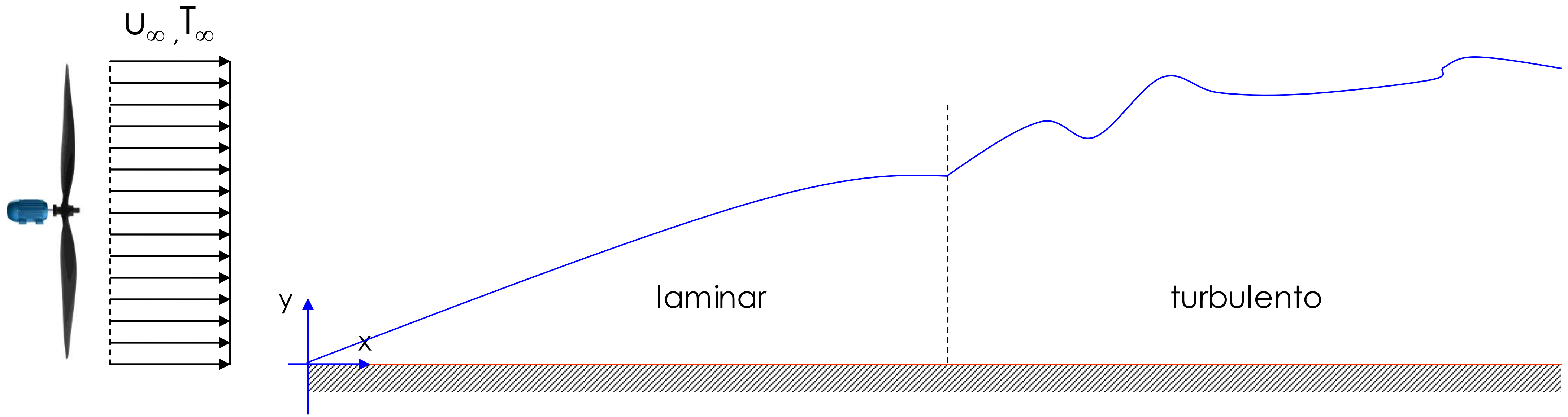
$$Nu \overset{def}{=} \frac{h_q}{k/D}$$

$$Nu = f\left(Re, Pr\right)$$

$$\rightarrow Nu = C \cdot Re^m \cdot Pr^n$$

p/ eq. convecção natural

Adimensionalização das equações de escoamento sobre uma placa plana...



$$Nu \overset{def}{=} \frac{h_q}{k/D}$$

$$Nu_x = 0.332 \cdot Re_x^{0.5} \cdot Pr^{1/3}$$

$$Pr > 0.60$$

entire plate laminar ⇓

$$Nu_L = 0.664 \cdot Re_L^{0.5} \cdot Pr^{1/3}$$

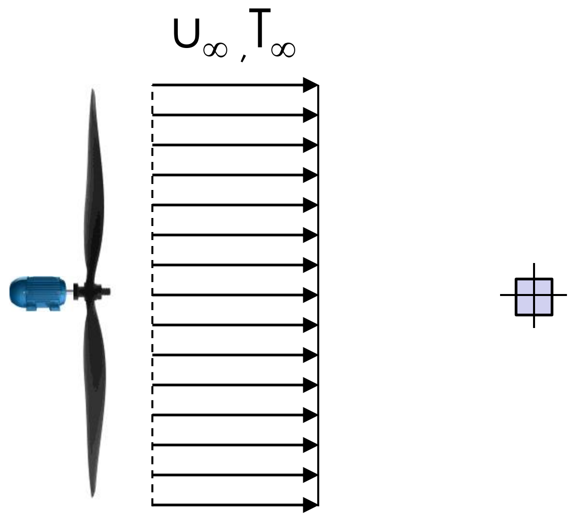
$$Nu_x = 0.0296 \cdot Re_x^{0.8} \cdot Pr^{1/3}$$

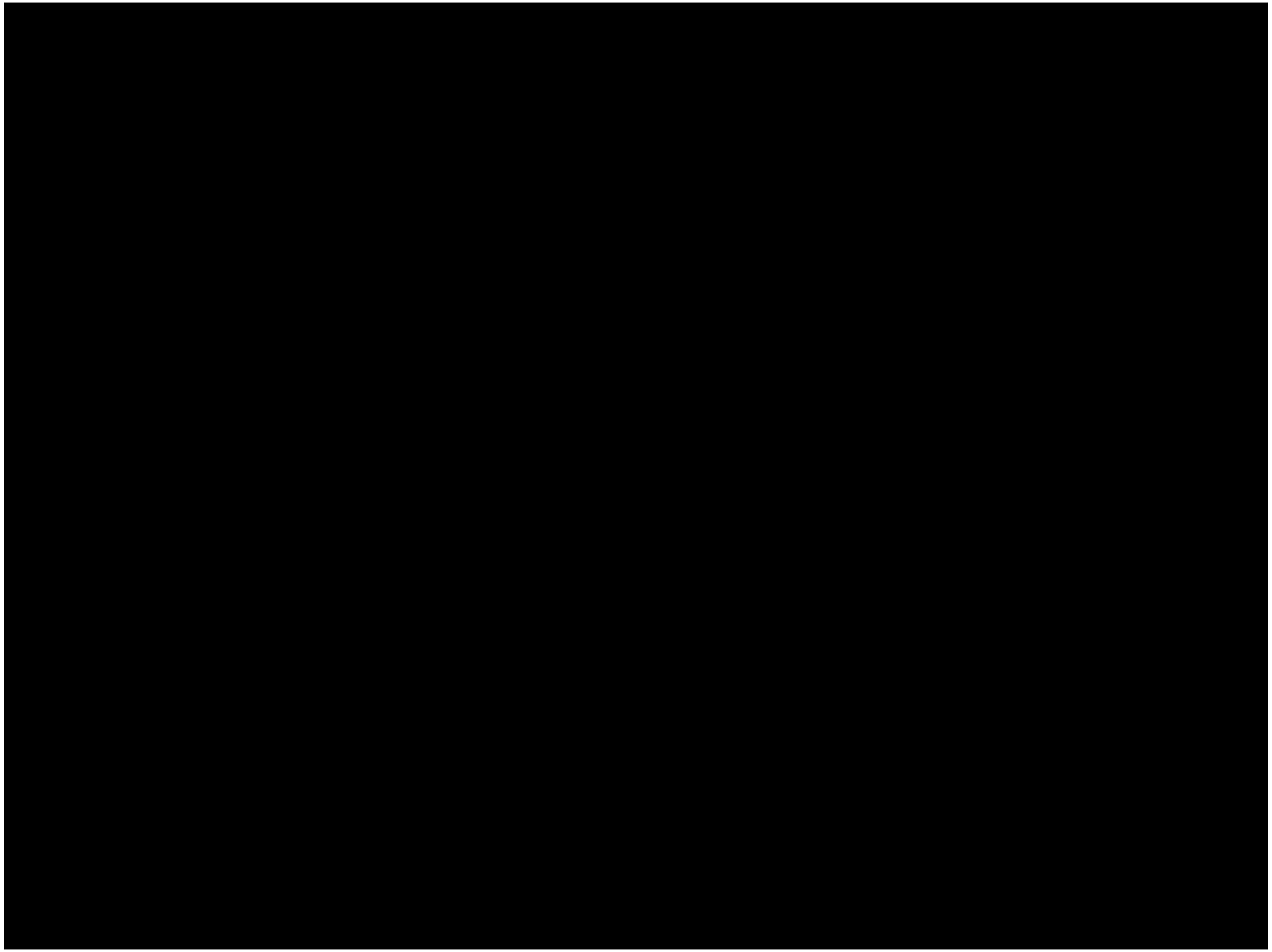
$$0.6 < Pr < 60 \qquad 5 \cdot 10^5 < Pr < 10^7$$

⇓ entire plate turbulent

$$Nu_L = 0.037 \cdot Re_L^{0.8} \cdot Pr^{1/3}$$

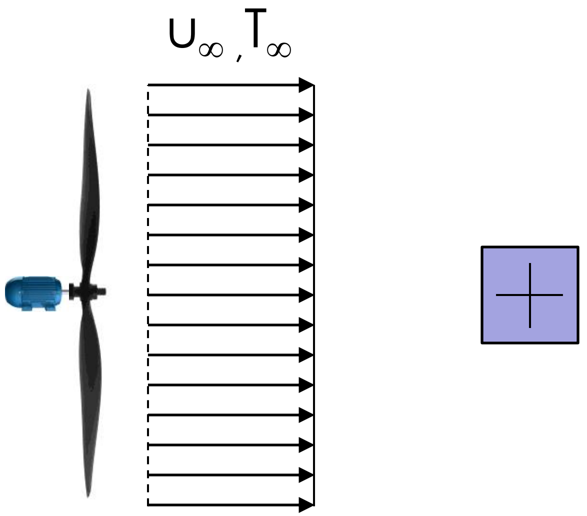
Escoamento ao redor de corpos prismáticos: a esteira de Von Karman





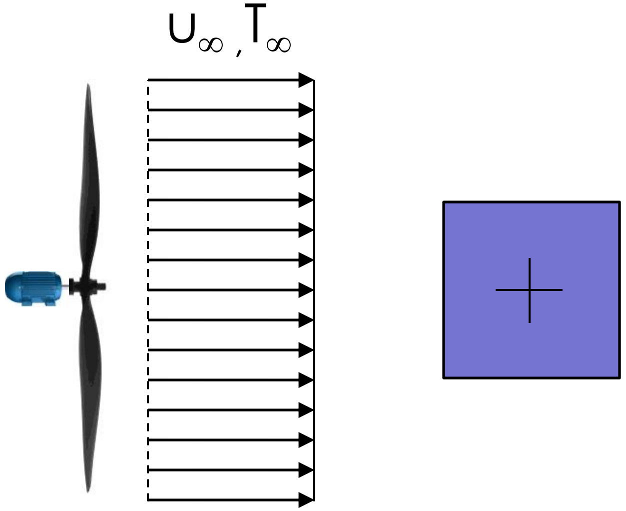
$$D_H \overset{def}{=} \frac{4A}{p} \rightarrow Re = \frac{\rho U D_H}{\mu}$$

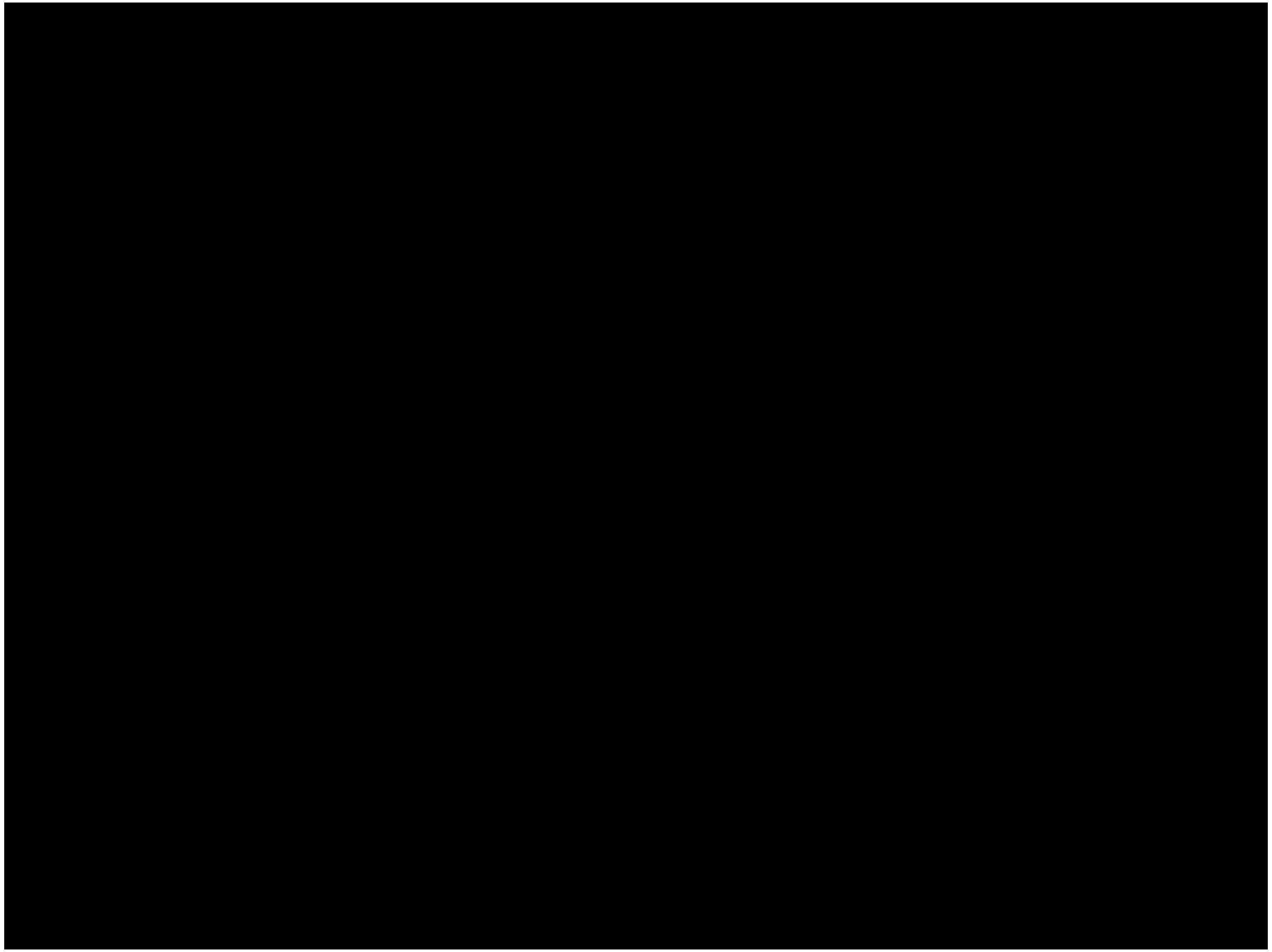
Escoamento ao redor de corpos prismáticos: a esteira de Von Karman



$$D_H \stackrel{def}{=} \frac{4A}{p} \rightarrow Re = \frac{\rho U D_H}{\mu}$$

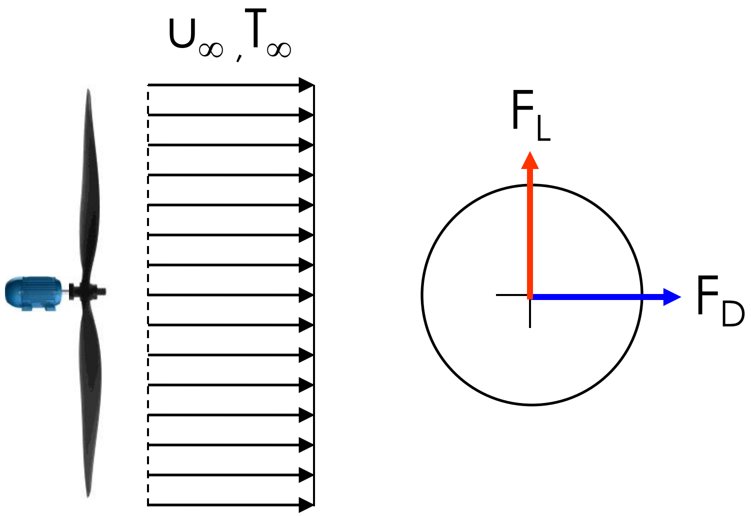
Escoamento ao redor de corpos prismáticos: a esteira de Von Karman



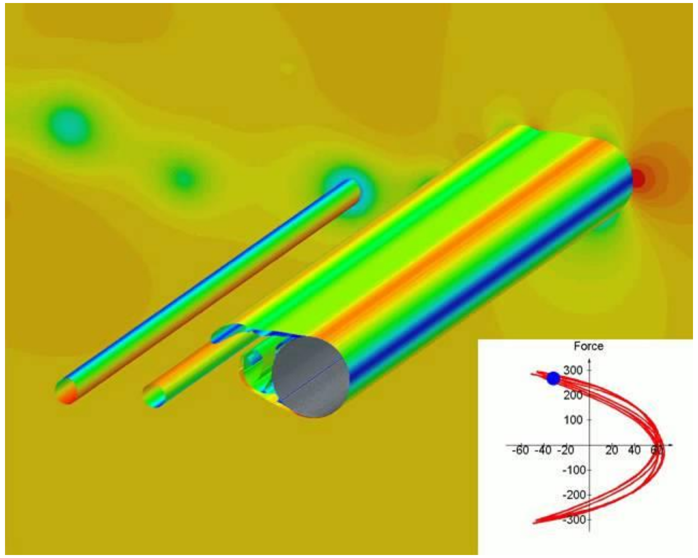


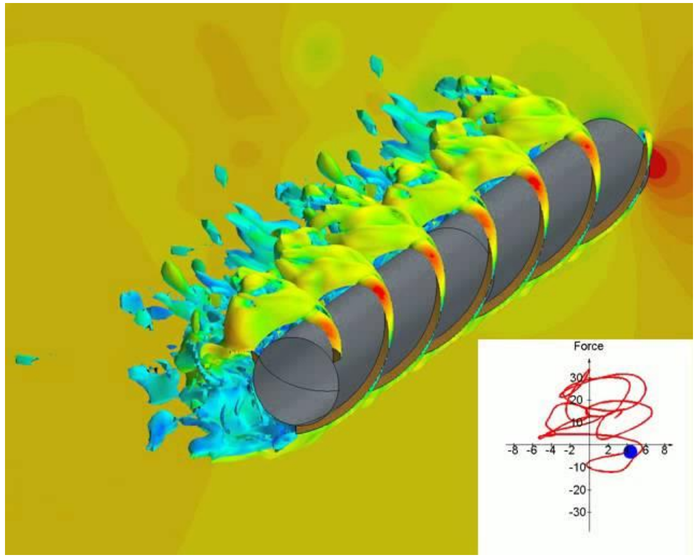
$$D_H \overset{def}{=} \frac{4A}{p} \rightarrow Re = \frac{\rho U D_H}{\mu}$$

Escoamento ao redor de corpos prismáticos: a esteira de Von Karman

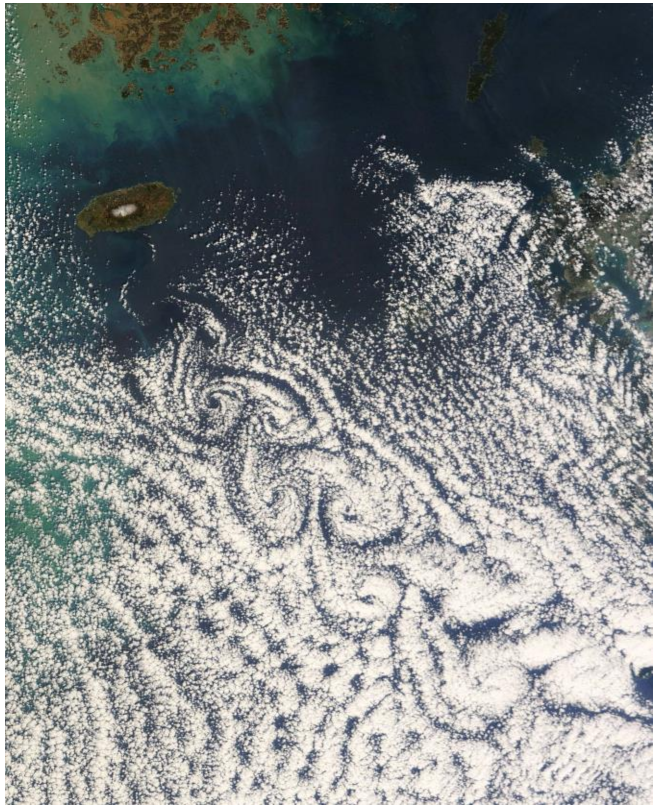


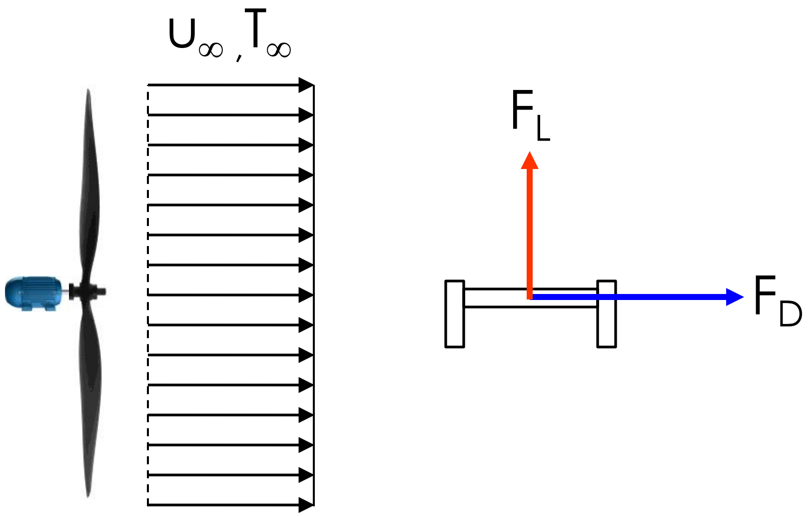
$$D_H \overset{def}{=} \frac{4A}{p} \rightarrow Re = \frac{\rho U D_H}{\mu}$$





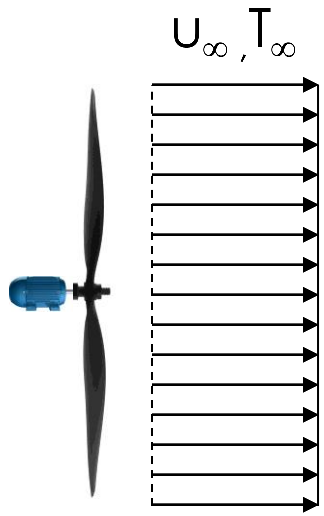
Yakushima Island

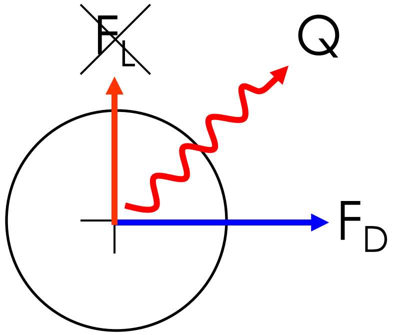


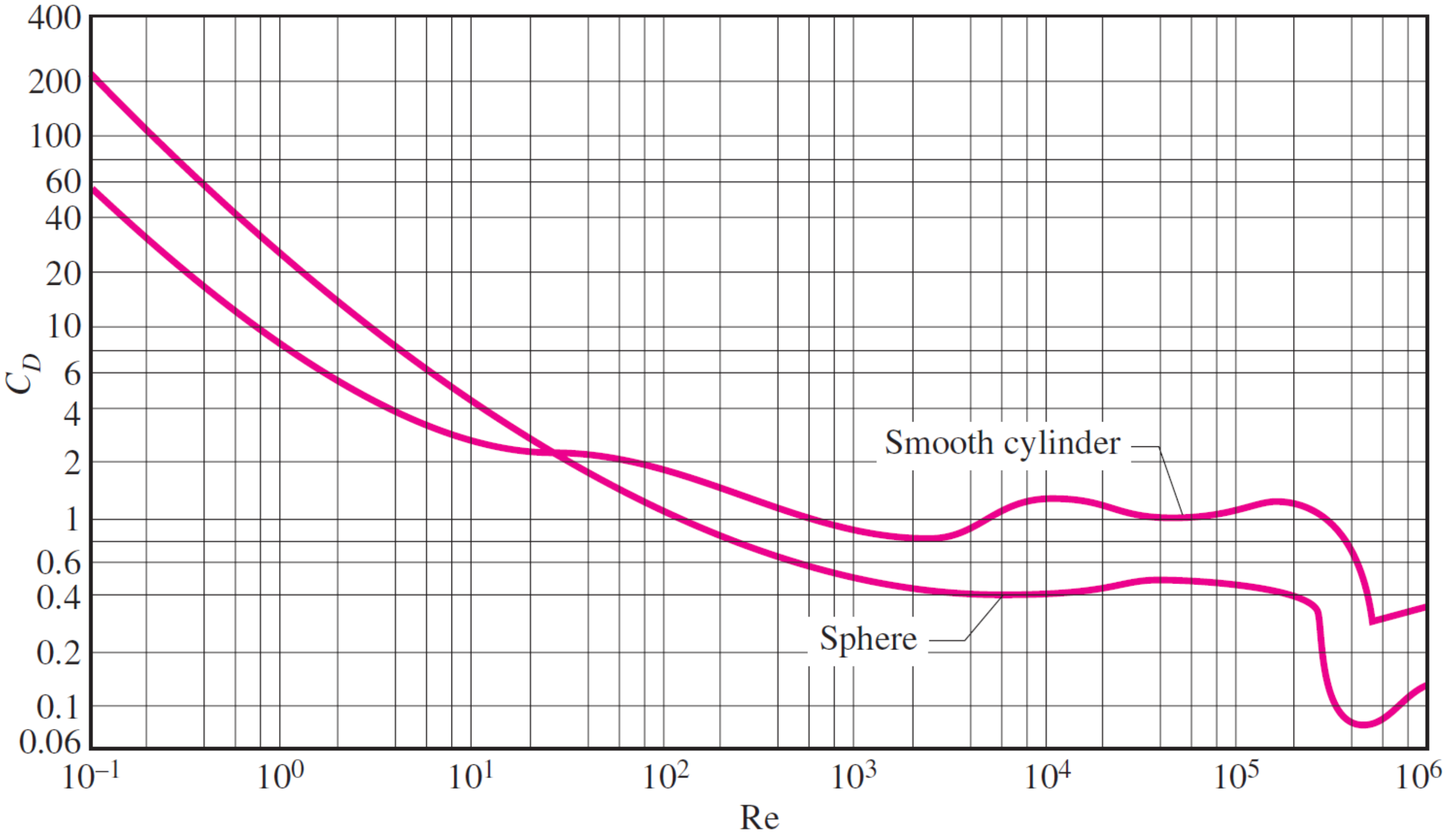

U_∞ , T_∞
F_L
F_D


GALE CAUSES
BRIDGE
TO SWAY

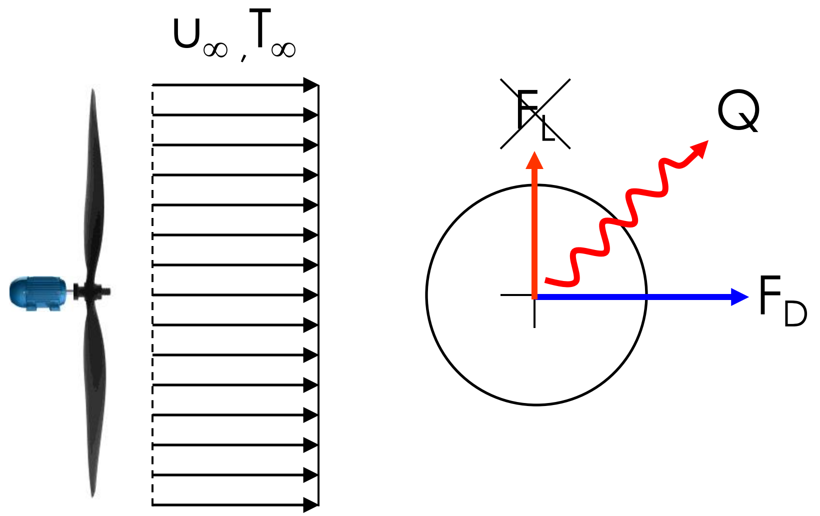
$$F_D = C_D \cdot \frac{\rho A_{frontal} u_\infty^2}{2} \qquad \rightarrow C_D = \frac{r}{2\pi} \int_0^{2\pi} C_D(\theta) \cdot d\theta$$

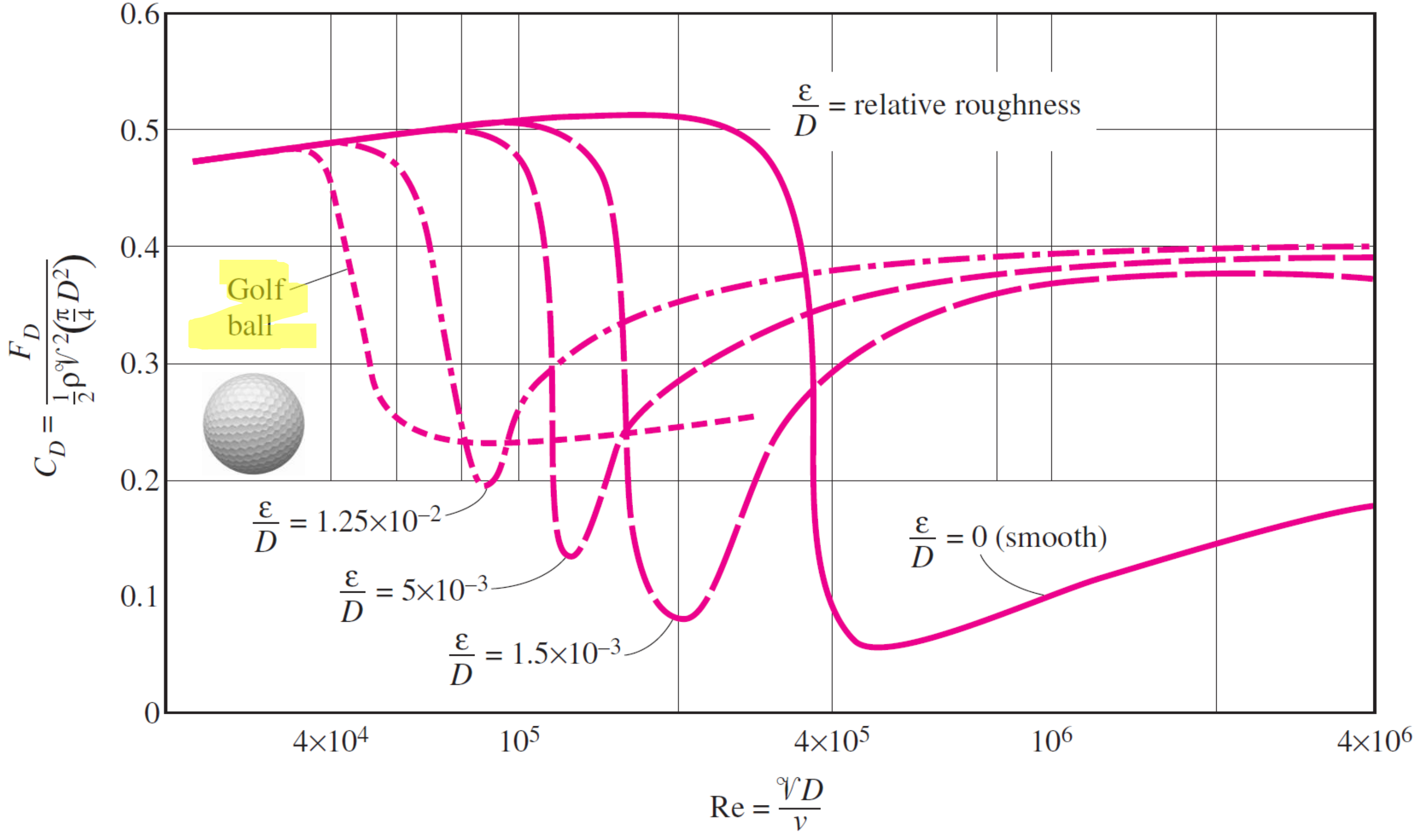


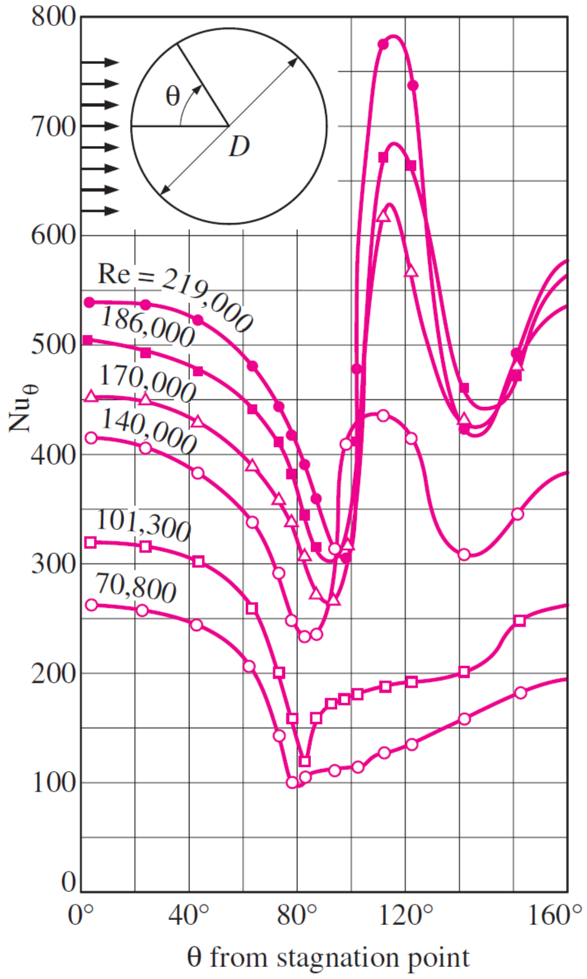




$$F_D = C_D \cdot \frac{\rho A_{frontal} u_\infty^2}{2} \qquad \rightarrow C_D = \frac{r}{2\pi}\int_0^{2\pi} C_D(\theta)\cdot d\theta$$







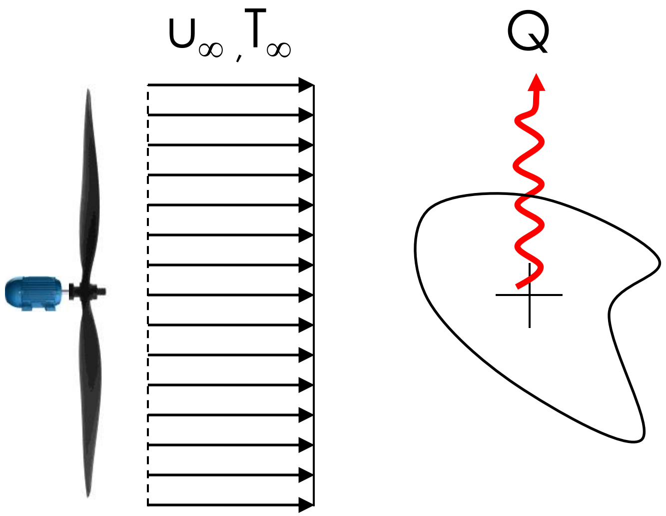
$$Q = hA_{lateral} \cdot (T - T_\infty) \qquad \rightarrow h = \frac{r}{2\pi}\int_0^{2\pi} h(\theta)\cdot d\theta$$

Churchill and Bernstein ⇓ properties @ $T_{fluid} = \left(\frac{T_s + T_\infty}{2}\right)$

$$Nu_{cylinder} = 0.3 + \frac{0.62Re^{1/2}Pr^{1/3}}{\left[1+(0.4/Pr)^{2/3}\right]^{1/4}}\left[1+\left(\frac{Re}{282000}\right)^{5/8}\right]^{4/5}$$

Whitaker ⇓ $3.5 < Re < 80000$ and $0.7 < Pr < 380$

$$Nu_{sphere} = 2 + \left[0.4Re^{1/2} + 0.06Re^{2/3}\right]\cdot Pr^{0.4}\cdot\left(\frac{\mu_\infty}{\mu_s}\right)^{1/4}$$



$$Q = hA_{lateral} \cdot (T - T_{\infty})$$

$$Nu = C \cdot Re^{m} \cdot Pr^{1/3}$$

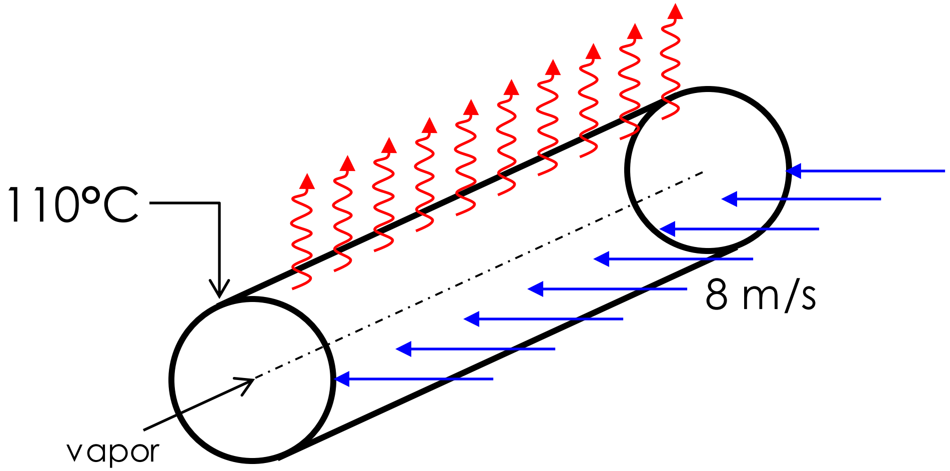
$$\text{properties @ } T_{fluid} = \left(\frac{T_s + T_{\infty}}{2} \right)$$

Empirical correlations for the average Nusselt number for forced convection over circular and noncircular cylinders in cross flow (from Zukauskas, Ref. 14, and Jakob, Ref. 6)

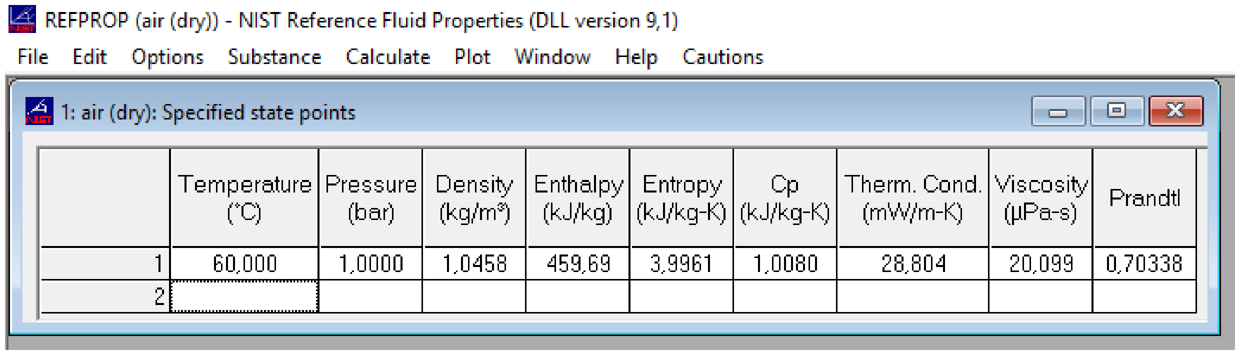
Cross-section of the cylinder	Fluid	Range of Re	Nusselt number
Circle	Gas or liquid	0.4–4 4–40 40–4000 4000–40,000 40,000–400,000	$Nu = 0.989Re^{0.330}\ Pr^{1/3}$ $Nu = 0.911Re^{0.385}\ Pr^{1/3}$ $Nu = 0.683Re^{0.466}\ Pr^{1/3}$ $Nu = 0.193Re^{0.618}\ Pr^{1/3}$ $Nu = 0.027Re^{0.805}\ Pr^{1/3}$
Square	Gas	5000–100,000	$Nu = 0.102Re^{0.675}\ Pr^{1/3}$
Square (tilted 45°)	Gas	5000–100,000	$Nu = 0.246Re^{0.588}\ Pr^{1/3}$
Hexagon	Gas	5000–100,000	$Nu = 0.153Re^{0.638}\ Pr^{1/3}$
Hexagon (tilted 45°)	Gas	5000–19,500 19,500–100,000	$Nu = 0.160Re^{0.638}\ Pr^{1/3}$ $Nu = 0.0385Re^{0.782}\ Pr^{1/3}$
Vertical plate	Gas	4000–15,000	$Nu = 0.228Re^{0.731}\ Pr^{1/3}$
Ellipse	Gas	2500–15,000	$Nu = 0.248Re^{0.612}\ Pr^{1/3}$

EXAMPLE 7–5 Heat Loss from a Steam Pipe in Windy Air

A long 10-cm-diameter steam pipe whose external surface temperature is 110°C passes through some open area that is not protected against the winds. Determine the rate of heat loss from the pipe per unit of its length when the air is at 1 atm pressure and 10°C and the wind is blowing across the pipe at a velocity of 8 m/s.



Propriedades do ar @ $T_{média}$ = (110 + 10)/2 = 60°C



REFPROP (air (dry)) - NIST Reference Fluid Properties (DLL version 9,1)

File Edit Options Substance Calculate Plot Window Help Cautions

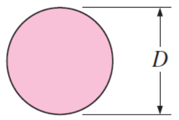
1: air (dry): Specified state points

	Temperature (°C)	Pressure (bar)	Density (kg/m³)	Enthalpy (kJ/kg)	Entropy (kJ/kg-K)	Cp (kJ/kg-K)	Therm. Cond. (mW/m-K)	Viscosity (µPa-s)	Prandtl
1	60,000	1,0000	1,0458	459,69	3,9961	1,0080	28,804	20,099	0,70338
2									

$$Re = \frac{\rho U D}{\mu} = \frac{1.0458 \cdot 8 \cdot 0.1}{20.099 \cdot 10^{-6}} = 4.163 \cdot 10^{4}$$

$$Nu = 0.027 \cdot (4.163 \cdot 10^{4})^{0.805} \cdot (0.70338)^{1/3} = \ldots$$

$$Nu = 125.617$$

Cross-section of the cylinder	Fluid	Range of Re	Nusselt number
Circle 	Gas or liquid	0.4–4 4–40 40–4000 4000–40,000 40,000–400,000	$Nu = 0.989Re^{0.330}\,Pr^{1/3}$ $Nu = 0.911Re^{0.385}\,Pr^{1/3}$ $Nu = 0.683Re^{0.466}\,Pr^{1/3}$ $Nu = 0.193Re^{0.618}\,Pr^{1/3}$ $Nu = 0.027Re^{0.805}\,Pr^{1/3}$

Correlação mais precisa: $Nu = 0.3 + \dfrac{0.62 Re^{1/2} Pr^{1/3}}{\left[1+(0.4/Pr)^{2/3}\right]^{1/4}} \left[1+\left(\dfrac{Re}{282000}\right)^{5/8}\right]^{4/5}$

$$Nu = 0.3 + \frac{0.62 \cdot (4.163 \cdot 10^4)^{1/2} (0.70338)^{1/3}}{\left[1+(0.4/0.70338)^{2/3}\right]^{1/4}} \left[1+\left(\frac{4.163 \cdot 10^4}{282000}\right)^{5/8}\right]^{4/5} = 122.266$$

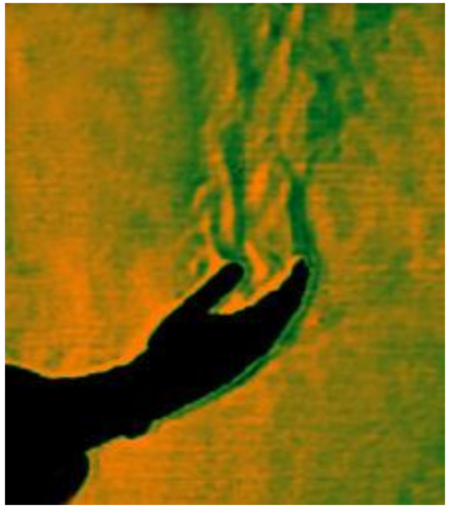
$$Nu = \frac{hD}{k} \rightarrow k = \frac{28.804 \cdot 10^{-3} \cdot 122.266}{0.1} = 35.217 \frac{W}{m^2\,^\circ C}$$

$$Q = h \cdot A \cdot (T_s - T_\infty) \quad \leftarrow A = \pi \cdot D \cdot L$$

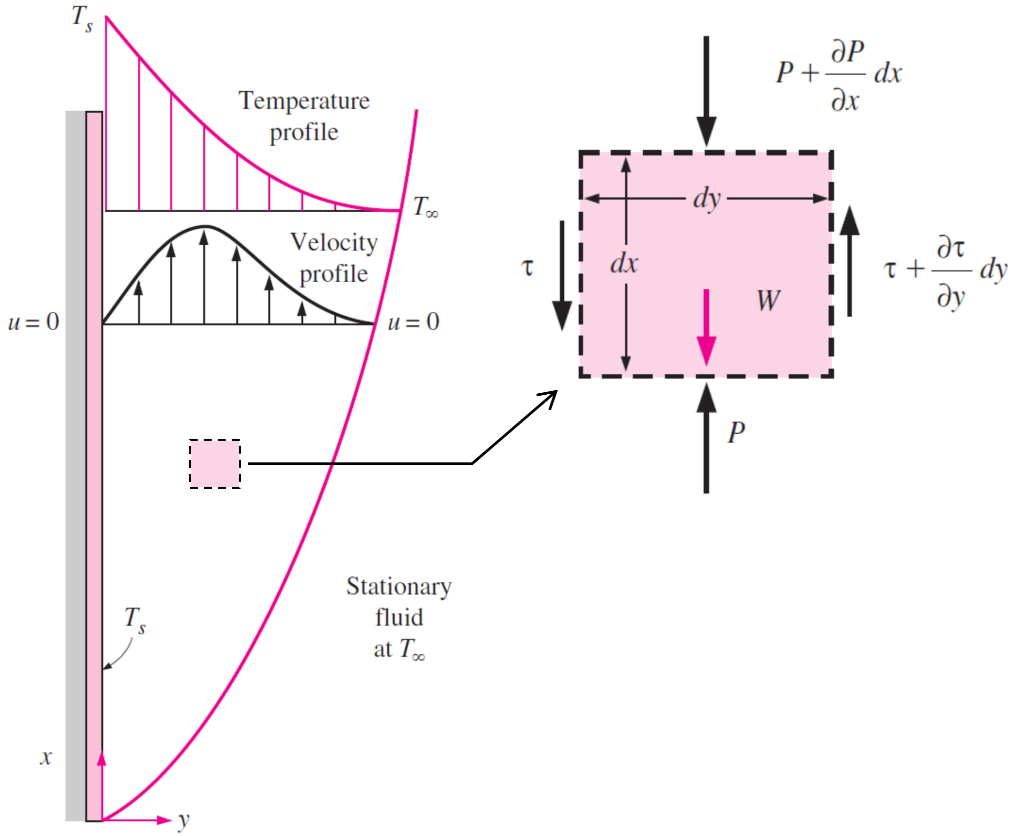
$$Q/L = h\pi D \cdot (T_s - T_\infty)$$

$$Q/L = 122.266 \cdot \pi \cdot 0.1 \cdot (110 - 10) = 1.106 kW/m$$

Convecção natural / escoamentos externos



Convecção natural: equações governantes e o número de Grashof...



$$F_x = a_x \cdot dm$$

$$a_x = \frac{du}{dt} = \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt} = u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}$$

$$\frac{\partial P}{\partial y} = 0 \rightarrow \quad \Downarrow \quad \leftarrow \frac{\partial P}{\partial x} = -\rho_\infty g$$

$$\rightarrow \rho \cdot \left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \right) = \mu \frac{\partial^2 u}{\partial y^2} + (\rho_\infty - \rho)g$$

$$\Downarrow \quad \leftarrow \Delta\rho = \rho \cdot \beta \cdot \Delta T$$

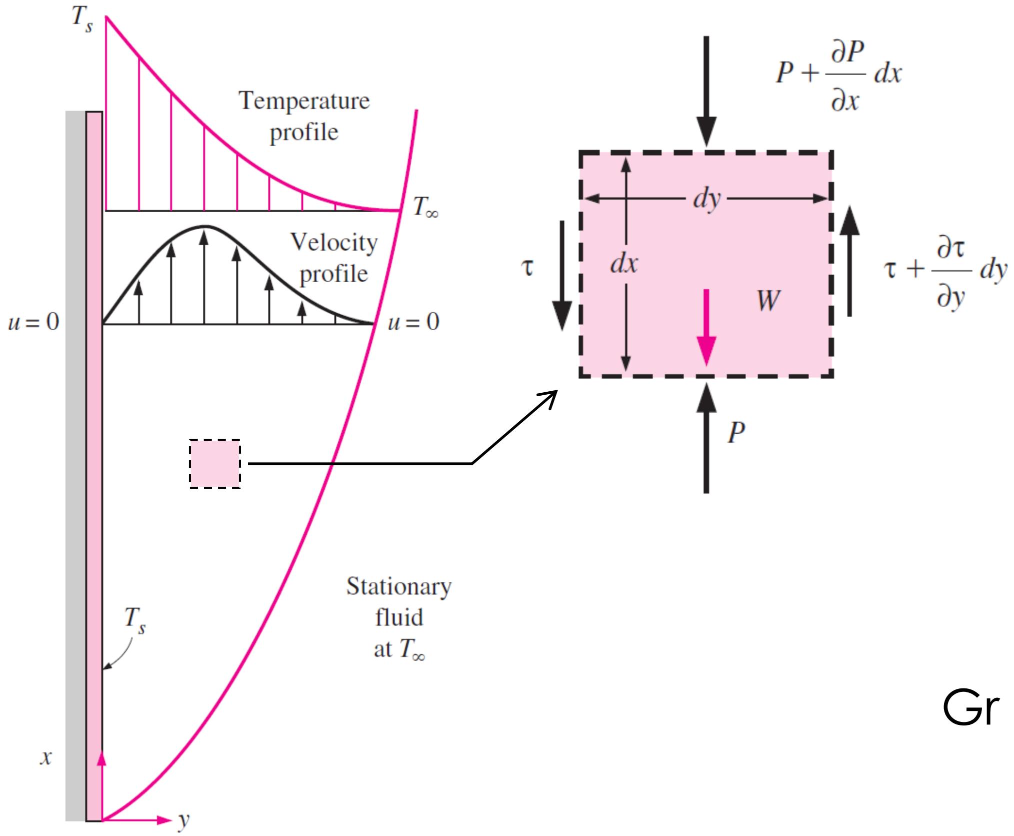
Coef. expansão volumétrica

$$\rightarrow \rho \cdot \left(u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \right) = \mu \frac{\partial^2 u}{\partial y^2} + g\beta \cdot (T - T_\infty)$$

Força de empuxo dependente da temperatura

p/ eq. convecção forçada

Convecção natural: equações governantes e o número de Grashof...



$$x^* = \frac{x}{L},\; y^* = \frac{y}{L},\; u^* = \frac{u}{U},\; v^* = \frac{v}{U},\; T^* = \frac{T - T_\infty}{T_s - T_\infty}$$

adimensionalização

$$u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial y^*} = \frac{1}{Re} \frac{\partial^2 u^*}{\partial y^{*2}} + \left(\frac{g\beta(T_s - T_\infty)L^3}{\nu^2} \right) \cdot \frac{1}{Re} T^*$$

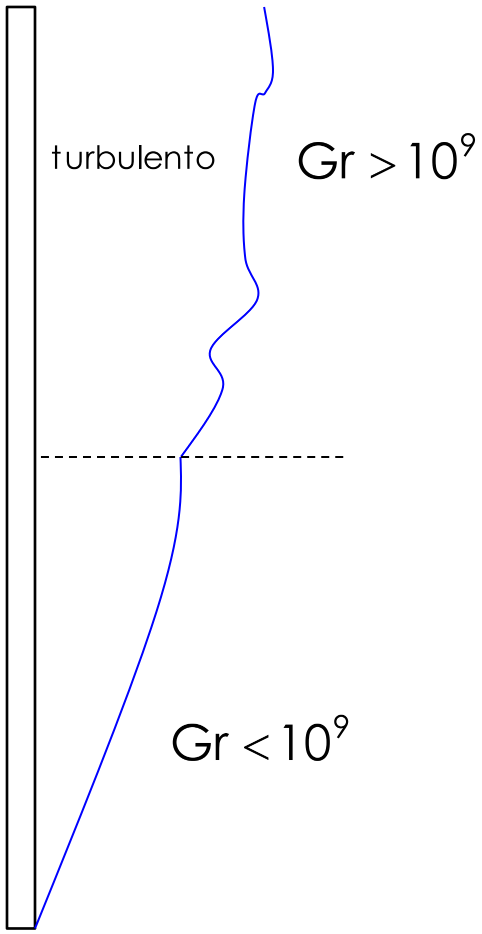
$$Gr = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2} = \frac{\text{força de empuxo}}{\text{dissipação viscosa}}$$

Análogo de Reynolds para convecção natural...

$$Ra = Gr \cdot Pr$$

Forma genérica para o Nr. Nusselt:

$$Nu = C \cdot (Gr \cdot Pr)^n = C \cdot Ra^n$$

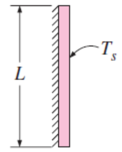
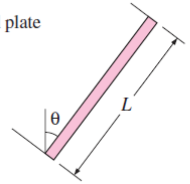
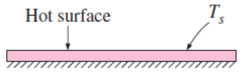
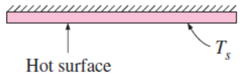
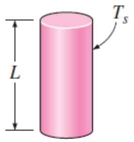
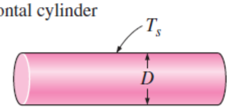
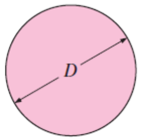


$$Q = hA_{lateral} \cdot (T - T_\infty)$$

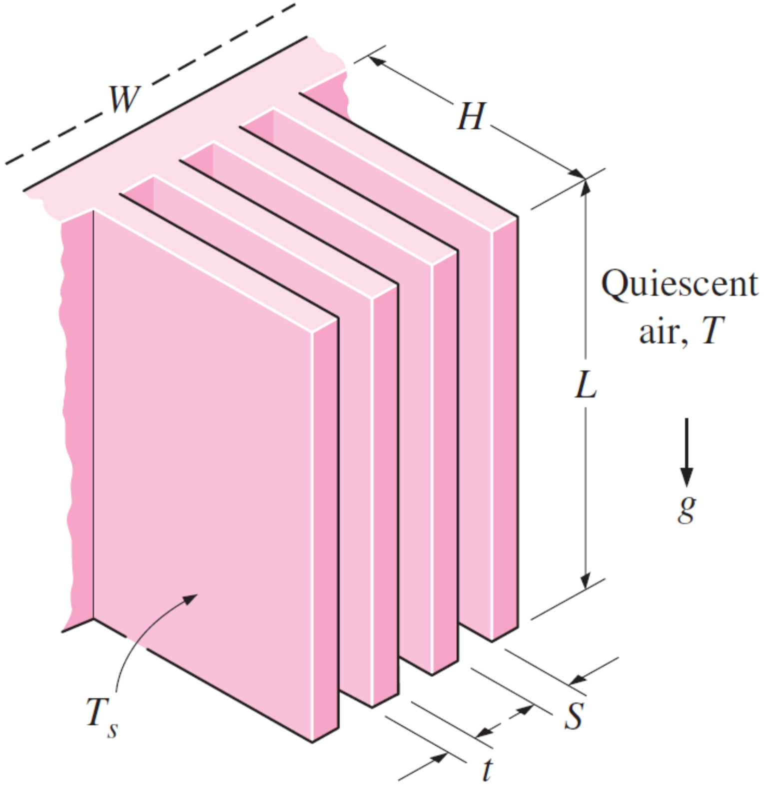
$$Nu = C \cdot Ra^n$$

$$\text{properties @ } T_{fluid} = \left(\frac{T_s + T_\infty}{2}\right)$$

Empirical correlations for the average Nusselt number for natural convection over surfaces

Geometry	Characteristic length L_c	Range of Ra	Nu
Vertical plate 	L	10^4–10^9 10^9–10^{13} Entire range	$Nu = 0.59Ra_L^{1/4}$ (9-19) $Nu = 0.1Ra_L^{1/3}$ (9-20) $Nu = \left\{0.825 + \frac{0.387Ra_L^{1/6}}{[1 + (0.492/Pr)^{9/16}]^{8/27}}\right\}^2$ (9-21) (complex but more accurate)
Inclined plate 	L		Use vertical plate equations for the upper surface of a cold plate and the lower surface of a hot plate Replace g by $g\cos\theta$ for $Ra < 10^9$
Horiontal plate (Surface area A and perimeter p) (a) Upper surface of a hot plate (or lower surface of a cold plate)  (b) Lower surface of a hot plate (or upper surface of a cold plate) 	A_s/p	10^4–10^7 10^7–10^{11} 10^5–10^{11}	$Nu = 0.54Ra_L^{1/4}$ (9-22) $Nu = 0.15Ra_L^{1/3}$ (9-23) $Nu = 0.27Ra_L^{1/4}$ (9-24)
Vertical cylinder 	L		A vertical cylinder can be treated as a vertical plate when $D \geq \frac{35L}{Gr_L^{1/4}}$
Horizontal cylinder 	D	$Ra_D \leq 10^{12}$	$Nu = \left\{0.6 + \frac{0.387Ra_D^{1/6}}{[1 + (0.559/Pr)^{9/16}]^{8/27}}\right\}^2$ (9-25)
Sphere 	D	$Ra_D \leq 10^{11}$ $(Pr \geq 0.7)$	$Nu = 2 + \frac{0.589Ra_D^{1/4}}{[1 + (0.469/Pr)^{9/16}]^{4/9}}$ (9-26)

Convecção natural em superfícies aletadas @ T_s = cte ...



$$Ra_S = \frac{g\beta(T_s - T_\infty)\cdot S^3}{\nu^2}Pr \quad \text{ou} \quad Ra_L = \frac{g\beta(T_s - T_\infty)\cdot L^3}{\nu^2}Pr$$

$$Nu = \frac{hS_{opt}}{k} = \left[\frac{576}{(Ra_S \cdot S/L)^2} + \frac{2.873}{(Ra_S \cdot S/L)^{0.5}}\right]^{-0.5}$$ Bar-Cohen and Rohsenow

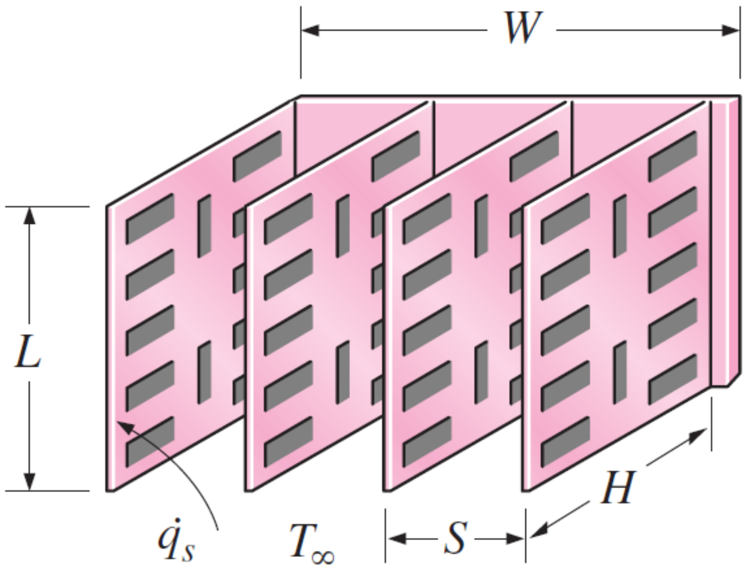
Espaçamento ótimo (trade-off área x vazão):

$$@\ T_s = cte \rightarrow S_{opt} = 2.714\frac{L}{Ra_L^{0.25}} \rightarrow Nu = 1.307$$

$$Q = h\cdot(2nLH)\cdot(T_s - T_\infty) \quad \leftarrow \ t \ll S$$

$$\text{properties } @\ T_{avg} = (T_s + T_\infty)/2$$

Convecção natural em superfícies aletadas @ q_s = cte ...



$$Ra_S = \frac{g\beta q_s \cdot S^4}{k\nu^2} Pr$$

$$Nu = \frac{hL}{k} = \left[\frac{48}{Ra_S \cdot S/L} + \frac{2.51}{(Ra_S \cdot S/L)^{0.4}} \right]^{-0.5}$$

Bar-Cohen and Rohsenow

Espaçamento ótimo (trade-off área x vazão):

$$@\ q_s = cte \rightarrow S_{opt} = 2.12 \cdot \left(\frac{S^4 L}{Ra_S} \right)^{0.2}$$

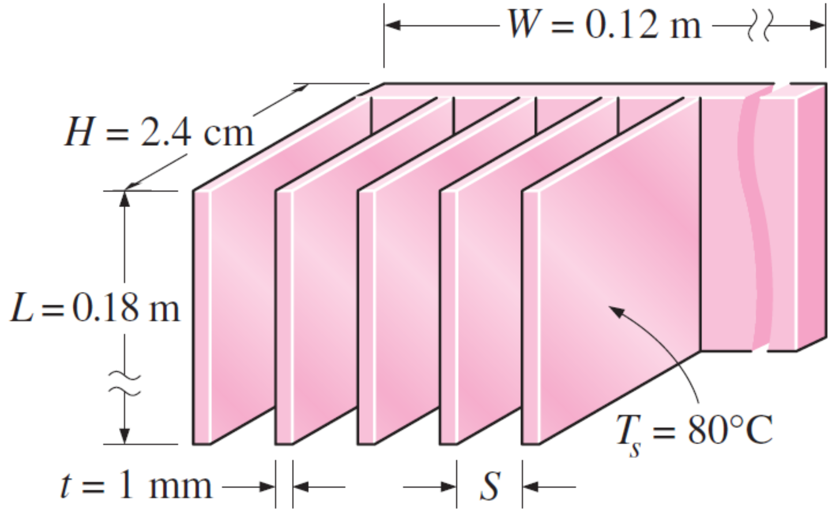
$$Q = q_s \cdot (2nLH) \quad \leftarrow t << S$$

$$\text{properties } @\ T_{avg} = (T_L + T_\infty)/2 \quad \leftarrow T_L = T_\infty + q_s/h$$

temperatura crítica ocorrendo na borda superior

EXAMPLE 9–3 Optimum Fin Spacing of a Heat Sink

A 12-cm-wide and 18-cm-high vertical hot surface in 30°C air is to be cooled by a heat sink with equally spaced fins of rectangular profile. The fins are 0.1 cm thick and 18 cm long in the vertical direction and have a height of 2.4 cm from the base. Determine the optimum fin spacing and the rate of heat transfer by natural convection from the heat sink if the base temperature is 80°C.



Propriedades do ar @ $T_{média}$ = (80 + 30)/2 = 55°C

REFPROP (air (dry)) - NIST Reference Fluid Properties (DLL version 9,1)

File Edit Options Substance Calculate Plot Window Help Cautions

5: air (dry): Specified state points

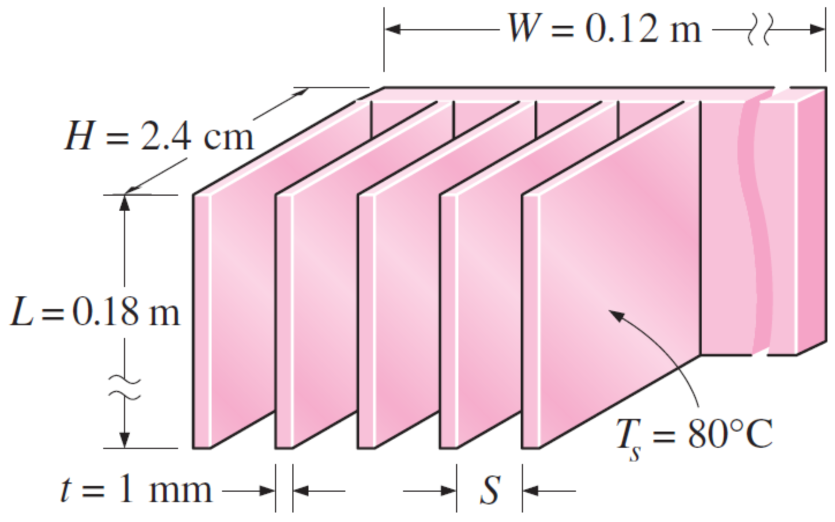
	Temperature (°C)	Pressure (bar)	Density (kg/m³)	Enthalpy (kJ/kg)	Entropy (kJ/kg-K)	Cp (kJ/kg-K)	Therm. Cond. (mW/m-K)	Viscosity (µPa-s)	Kin. Viscosity (cm²/s)	Prandtl	Vol. Expansivity (1/K)
1	55,000	1,0000	1,0617	454,65	3,9808	1,0077	28,444	19,868	0,18713	0,70386	0,0030534
2											

$$\nu = \frac{\mu}{\rho} = \frac{19.868 \cdot 10^{-6}}{1.0617} = 1.871 \cdot 10^{-5} \frac{m^2}{s} = 1.854 \cdot 10^{-5} \frac{(10^{-2} cm)^2}{s} = 0.1871 \frac{cm^2}{s}$$

EXAMPLE 9–3 Optimum Fin Spacing of a Heat Sink

erros de digitação

A 12-cm-wide and 18-cm-high vertical hot surface in 30°C air is to be cooled by a heat sink with equally spaced fins of rectangular profile. The fins are 0.1 cm thick and 18 cm long in the vertical direction and have a height of 2.4 cm from the base. Determine the optimum fin spacing and the rate of heat transfer by natural convection from the heat sink if the base temperature is 80°C.



$$Ra_L = \frac{g\beta(T_s - T_\infty)\cdot L^3}{\nu^2}Pr = \frac{9.81\cdot 3.053\cdot 10^{-3}(80-30)}{1.8713\cdot 10^{-5}}\cdot 0.70386 = 1.846\cdot 10^7$$

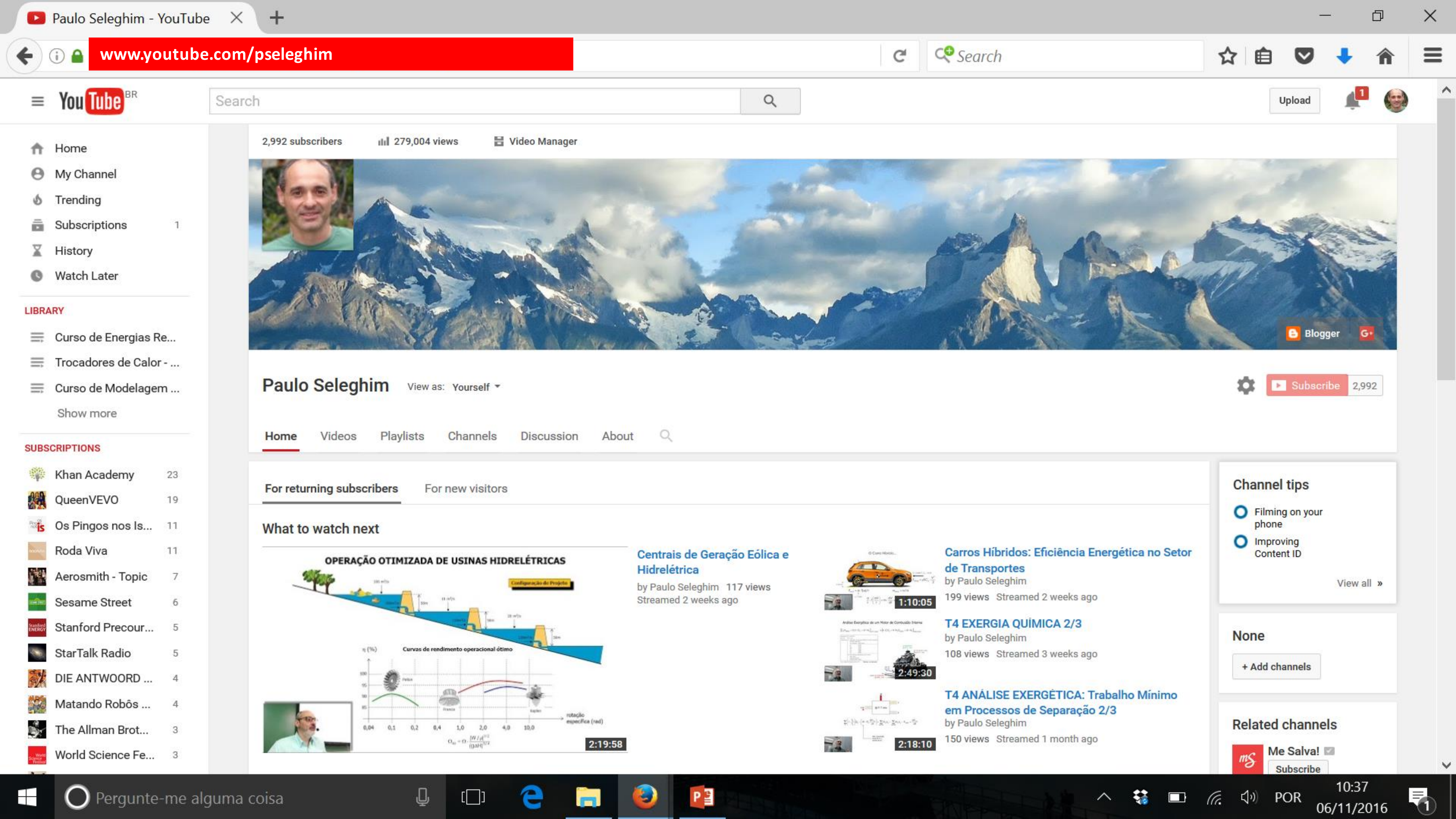
$$S_{opt} = 2.714\frac{L}{Ra_L^{0.25}} = 2.714\frac{0.8}{(1.846\cdot 10^7)^{0.25}} = 7.45\,mm$$

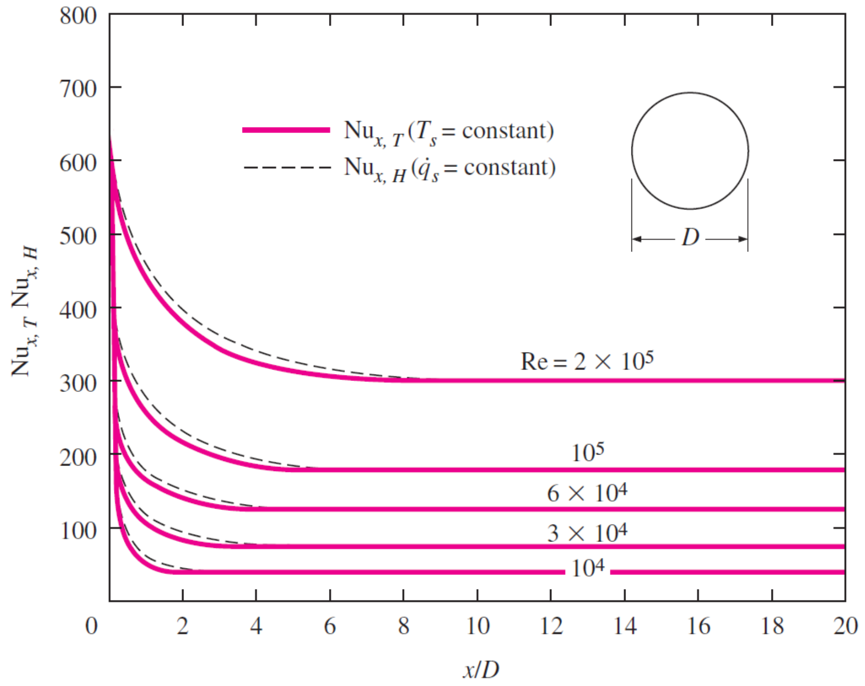
$$n = \frac{W}{S+t} = \frac{0.12}{0.00745+0.0001} \approx 15\,\text{aletas}$$

$$h = Nu_{opt}\frac{k}{S_{opt}} = 1.307\frac{0.028444}{0.00745} = 4.99\frac{W}{m^2\,°C}$$

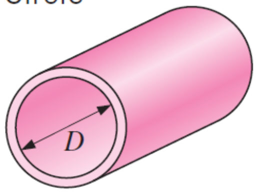
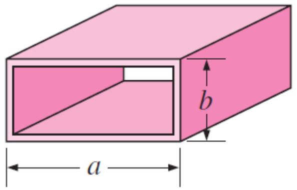
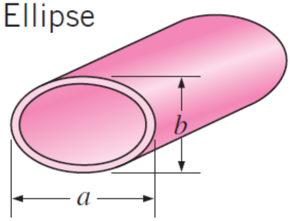
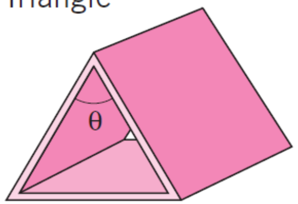
$$Q = hA(T_s - T_\infty) = 4.99\cdot(2\cdot 15\cdot 0.18\cdot 0.024)\cdot(80-30) = 32.335\,W$$

Obs.: $S_{opt} >> t$ (= 7.45·t)


Paulo Seleghim - YouTube
www.youtube.com/pseleghim
Search
Upload
Home
My Channel
Trending
Subscriptions 1
History
Watch Later
LIBRARY
Curso de Energias Re...
Trocadores de Calor - ...
Curso de Modelagem ...
Show more
SUBSCRIPTIONS
Khan Academy 23
QueenVEVO 19
Os Pingos nos Is... 11
Roda Viva 11
Aerosmith - Topic 7
Sesame Street 6
Stanford Precour... 5
StarTalk Radio 5
DIE ANTWOORD ... 4
Matando Robôs ... 4
The Allman Brot... 3
World Science Fe... 3
2,992 subscribers
279,004 views
Video Manager
Blogger
Paulo Seleghim
View as: Yourself
Subscribe 2,992
Home
Videos
Playlists
Channels
Discussion
About
For returning subscribers
For new visitors
What to watch next
OPERAÇÃO OTIMIZADA DE USINAS HIDRELÉTRICAS
Curvas de rendimento operacional ótimo
2:19:58
Centrais de Geração Eólica e Hidrelétrica
by Paulo Seleghim 117 views
Streamed 2 weeks ago
1:10:05
Carros Híbridos: Eficiência Energética no Setor de Transportes
by Paulo Seleghim
199 views Streamed 2 weeks ago
2:49:30
T4 EXERGIA QUÍMICA 2/3
by Paulo Seleghim
108 views Streamed 3 weeks ago
2:18:10
T4 ANÁLISE EXERGÉTICA: Trabalho Mínimo em Processos de Separação 2/3
by Paulo Seleghim
150 views Streamed 1 month ago
Channel tips
Filming on your phone
Improving Content ID
View all »
None
+ Add channels
Related channels
Me Salva!
Subscribe
Pergunte-me alguma coisa
POR
10:37
06/11/2016



Nusselt number and friction factor for fully developed laminar flow in tubes of various cross sections ($D_h = 4A_c/p$, $\text{Re} = \mathcal{V}_m D_h/\nu$, and $\text{Nu} = hD_h/k$)

Tube Geometry	a/b or $\theta°$	Nusselt Number T_s = Const.	Nusselt Number $\dot{q}_s$ = Const.	Friction Factor f
Circle 	—	3.66	4.36	64.00/Re
Rectangle 	a/b			
	1	2.98	3.61	56.92/Re
	2	3.39	4.12	62.20/Re
	3	3.96	4.79	68.36/Re
	4	4.44	5.33	72.92/Re
	6	5.14	6.05	78.80/Re
	8	5.60	6.49	82.32/Re
	∞	7.54	8.24	96.00/Re
Ellipse 	a/b			
	1	3.66	4.36	64.00/Re
	2	3.74	4.56	67.28/Re
	4	3.79	4.88	72.96/Re
	8	3.72	5.09	76.60/Re
	16	3.65	5.18	78.16/Re
Triangle 	θ			
	10°	1.61	2.45	50.80/Re
	30°	2.26	2.91	52.28/Re
	60°	2.47	3.11	53.32/Re
	90°	2.34	2.98	52.60/Re
	120°	2.00	2.68	50.96/Re