

MECANISMOS DE TRANSFERÊNCIA DE ENERGIA TÉRMICA: CONDUÇÃO DE CALOR

Paulo Seleghim Jr.
Universidade de São Paulo



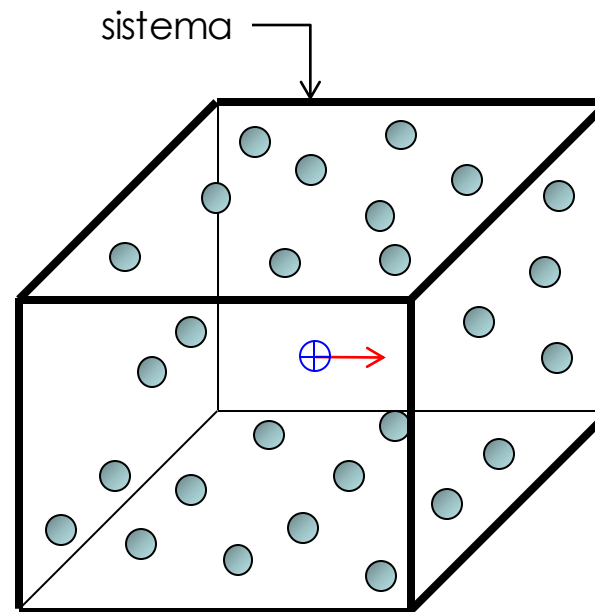
Diferentes formas de energia de um sistema

Formas macroscópicas

Energia associada ao centro de massa do sistema, relativa a um referencial inercial

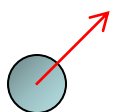
$$EC = m \frac{V^2}{2} \quad \longrightarrow \quad ec = \frac{V^2}{2}$$

$$EP = mgZ \quad \longrightarrow \quad ep = gZ$$

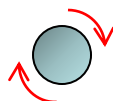


Formas microscópicas

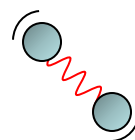
Energia associada à estrutura e ao nível de agitação molecular: energia interna "U"



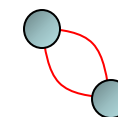
translação molecular



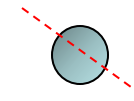
rotação molecular



vibração molecular



energia química



energia nuclear

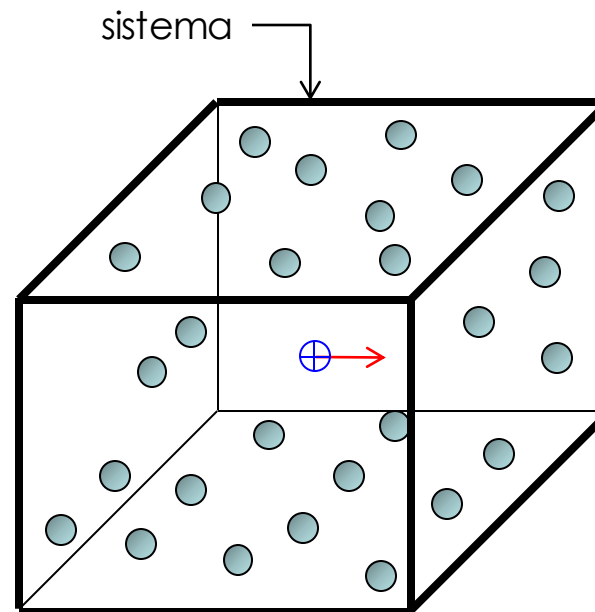
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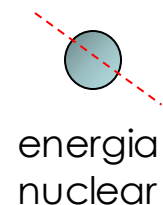
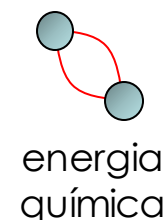
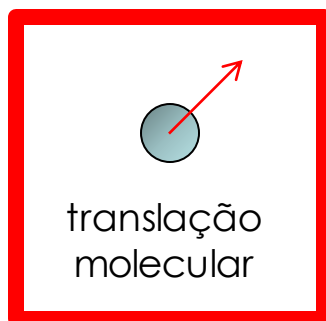
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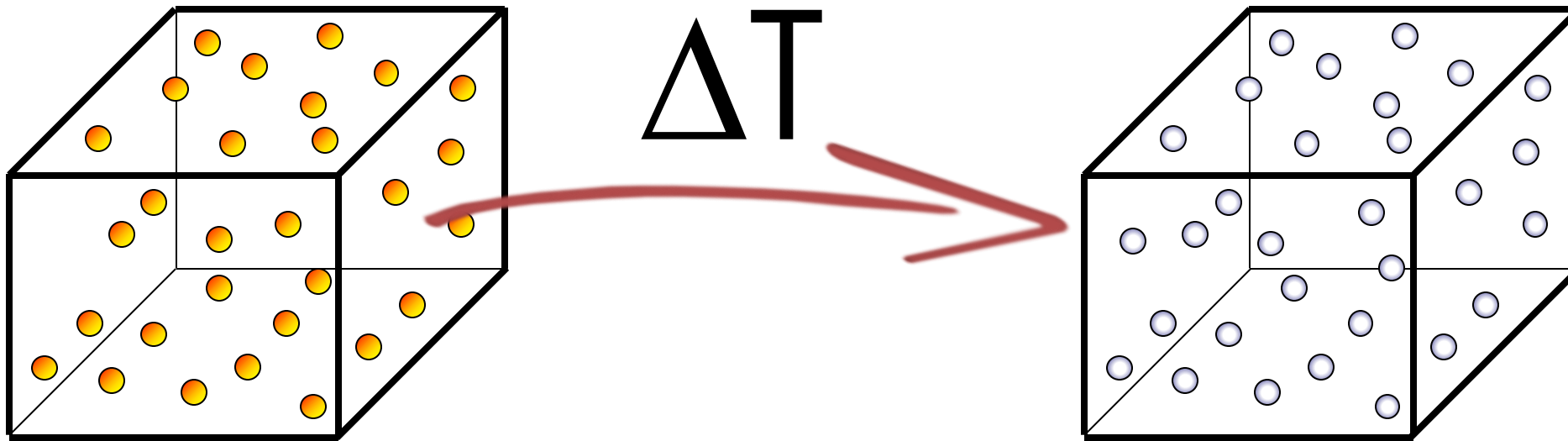


Termodinâmica →

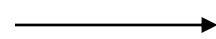
efeitos da transferência de calor no estado de uma substância

Transf. de Calor →

taxas de transferência de calor entre dois sistemas

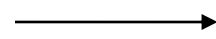


CONDUÇÃO



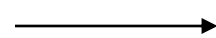
Lei de Fourier

CONVECÇÃO



Lei de Newton

RADIAÇÃO

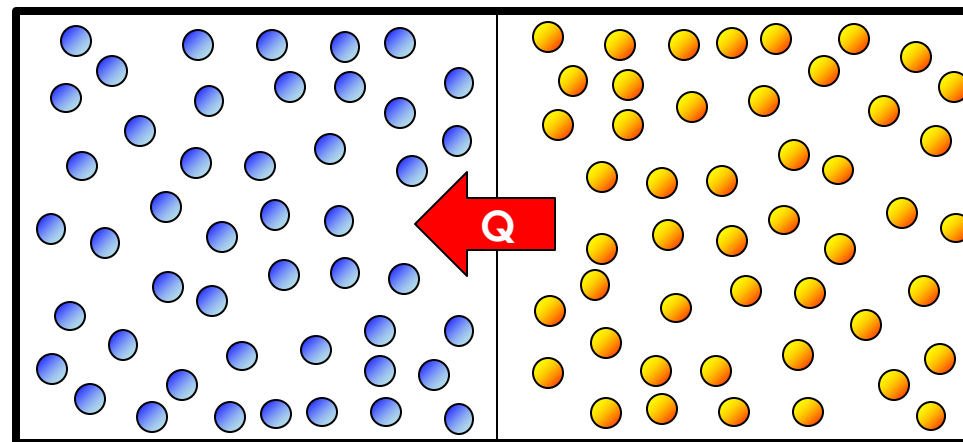


Lei de Stefan-Boltzmann

CONDUÇÃO DE CALOR

Transferência de energia via calor

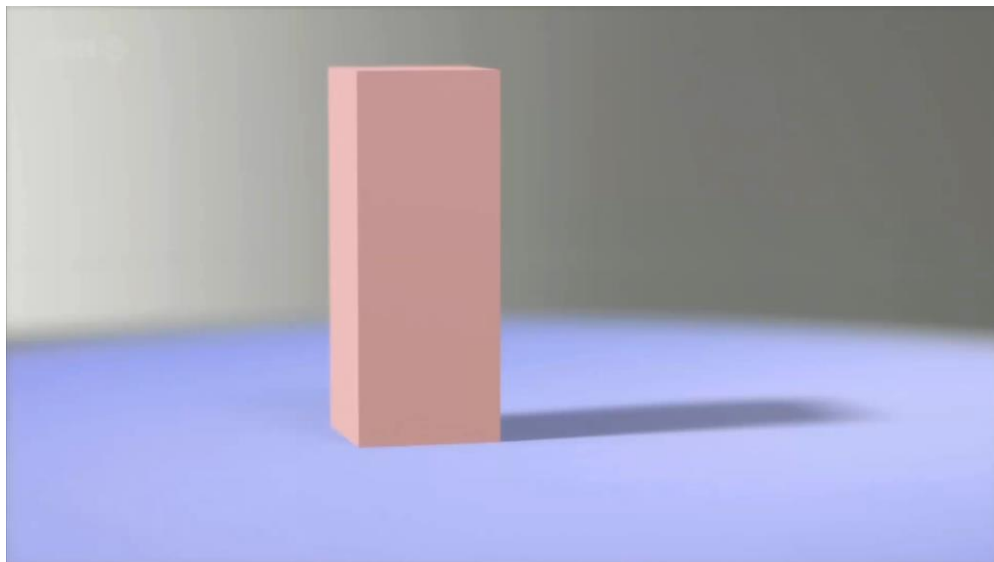
Dois sistemas em contato térmico a temperaturas diferentes tendem a equalizar seus níveis de agitação molecular...



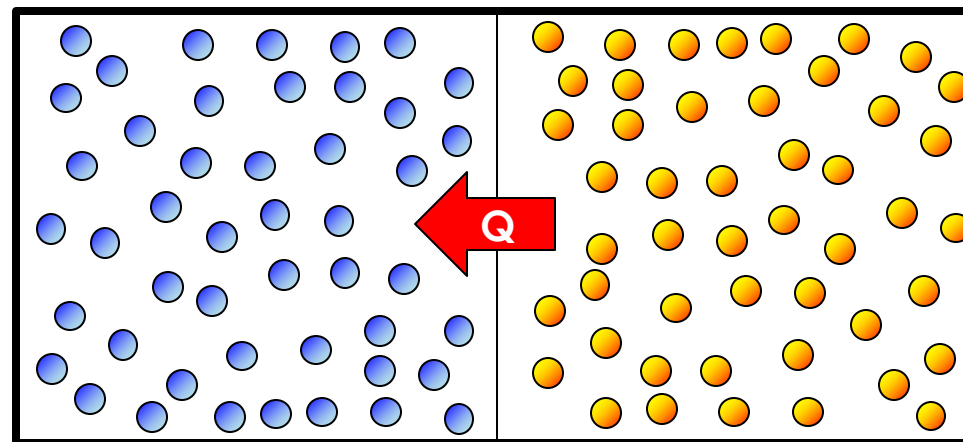
$$T_1 < T_2$$

Transferência de energia via calor

Dois sistemas em contato térmico a temperaturas diferentes tendem a equalizar seus níveis de agitação molecular...



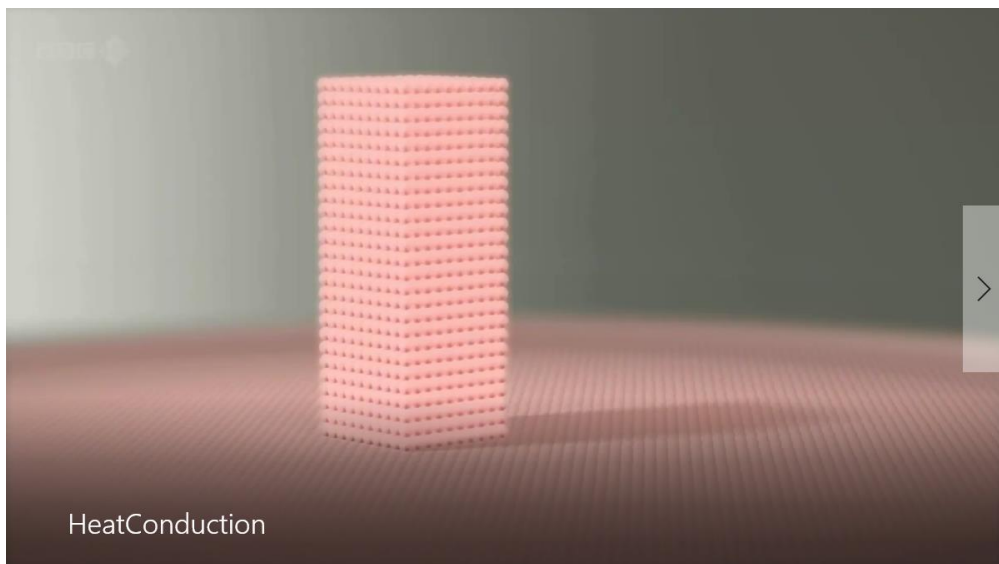
Jim Al-Khalili



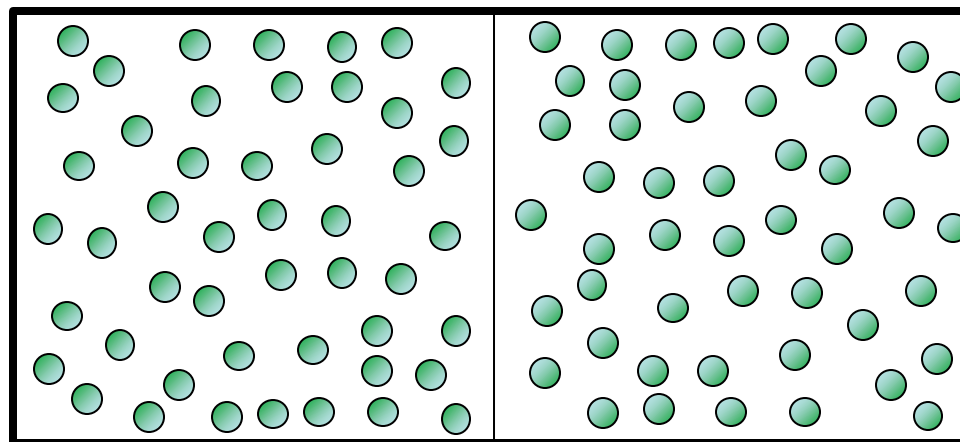
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Transferência de energia via calor

Dois sistemas em contato térmico a temperaturas diferentes tendem a equalizar seus níveis de agitação molecular...

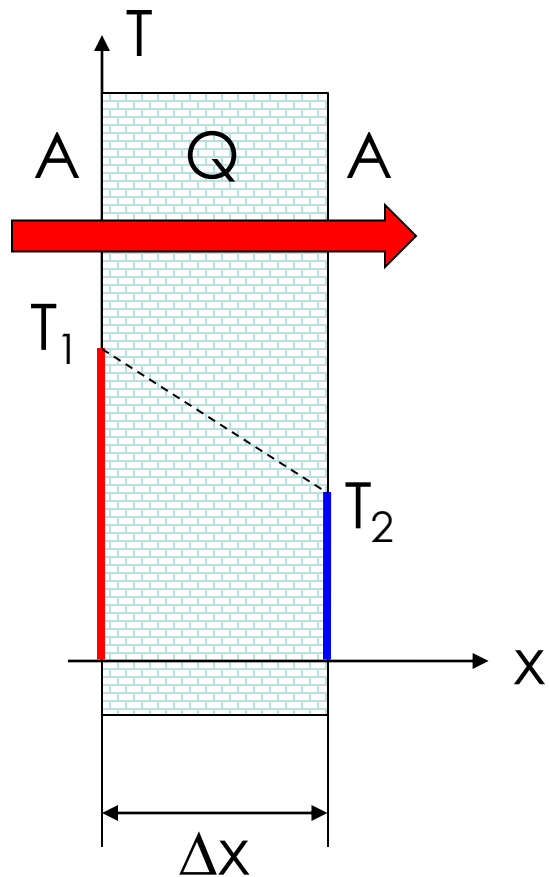


Jim Al-Khalili



$$T_1 \rightarrow T_3 = T_3 \leftarrow T_2$$

Lei de Fourier (condução de calor)



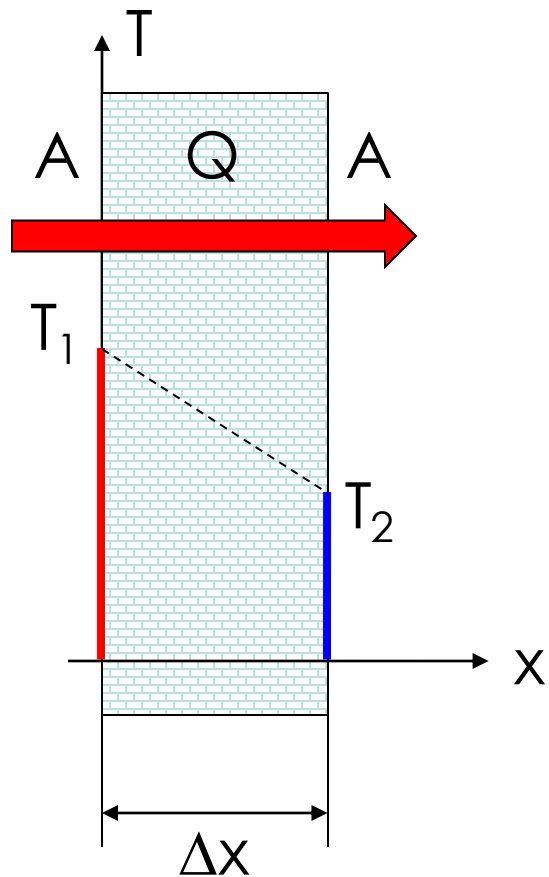
$$Q = k \cdot \frac{A}{\Delta X} \cdot (T_1 - T_2) \rightarrow Q = -kA \frac{dT}{dx}$$

K = condutibilidade térmica (W/m/°C)

$$q = \frac{Q}{A} = -k \frac{dT}{dx}$$

q = fluxo de calor (W/m²)

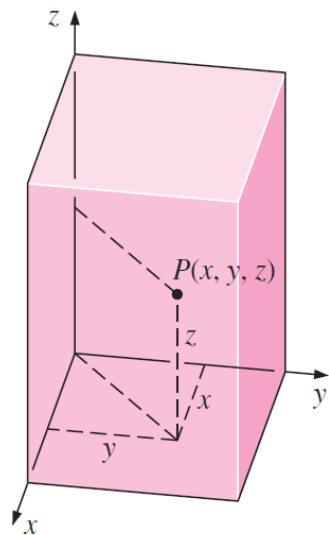
Lei de Fourier (condução de calor)



$$q = \frac{Q}{A} = -k \frac{dT}{dx}$$

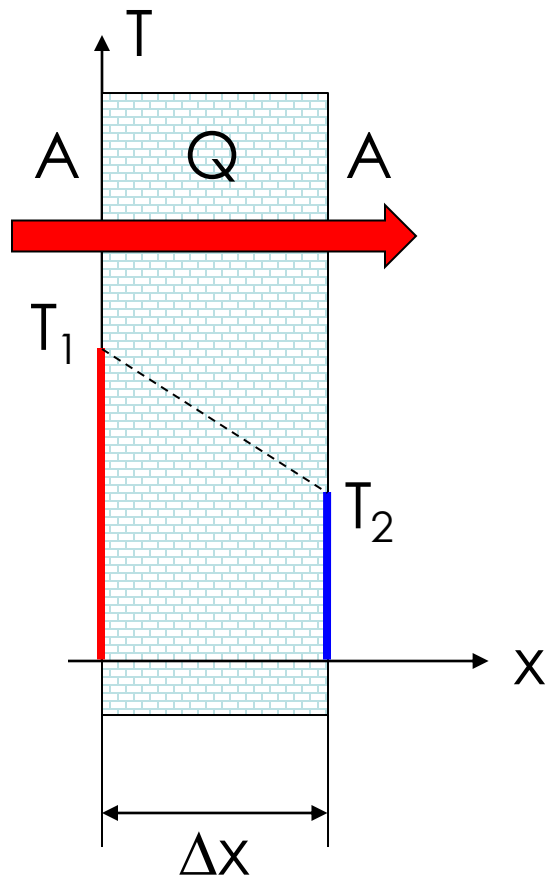
condução de calor em meio 3D

$$\vec{q}^{\text{ Fourier}} = -k \cdot \vec{\nabla} T$$



$$\vec{\nabla} T = \frac{\partial T}{\partial x} \hat{i} + \frac{\partial T}{\partial y} \hat{j} + \frac{\partial T}{\partial z} \hat{k}$$

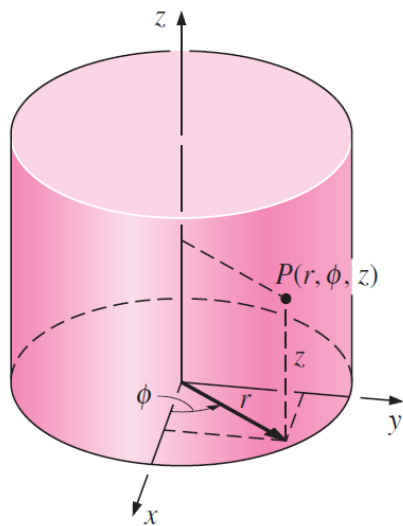
Lei de Fourier (condução de calor)



$$q = \frac{Q}{A} = -k \frac{dT}{dx}$$

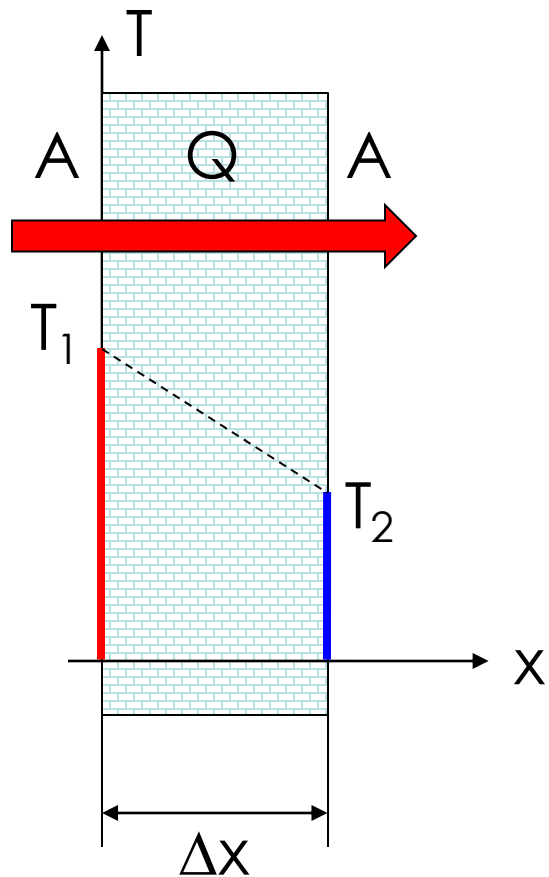
condução de calor em meio 3D

$$\vec{q}^{\text{ Fourier}} = -k \cdot \vec{\nabla} T$$



$$\vec{\nabla} T = \frac{\partial T}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial T}{\partial \phi} \hat{\phi} + \frac{\partial T}{\partial z} \hat{z}$$

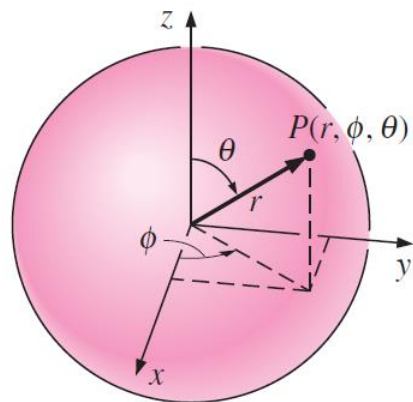
Lei de Fourier (condução de calor)



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condução de calor em meio 3D

$$\vec{q}^{\text{ Fourier}} = -k \cdot \vec{\nabla} T$$

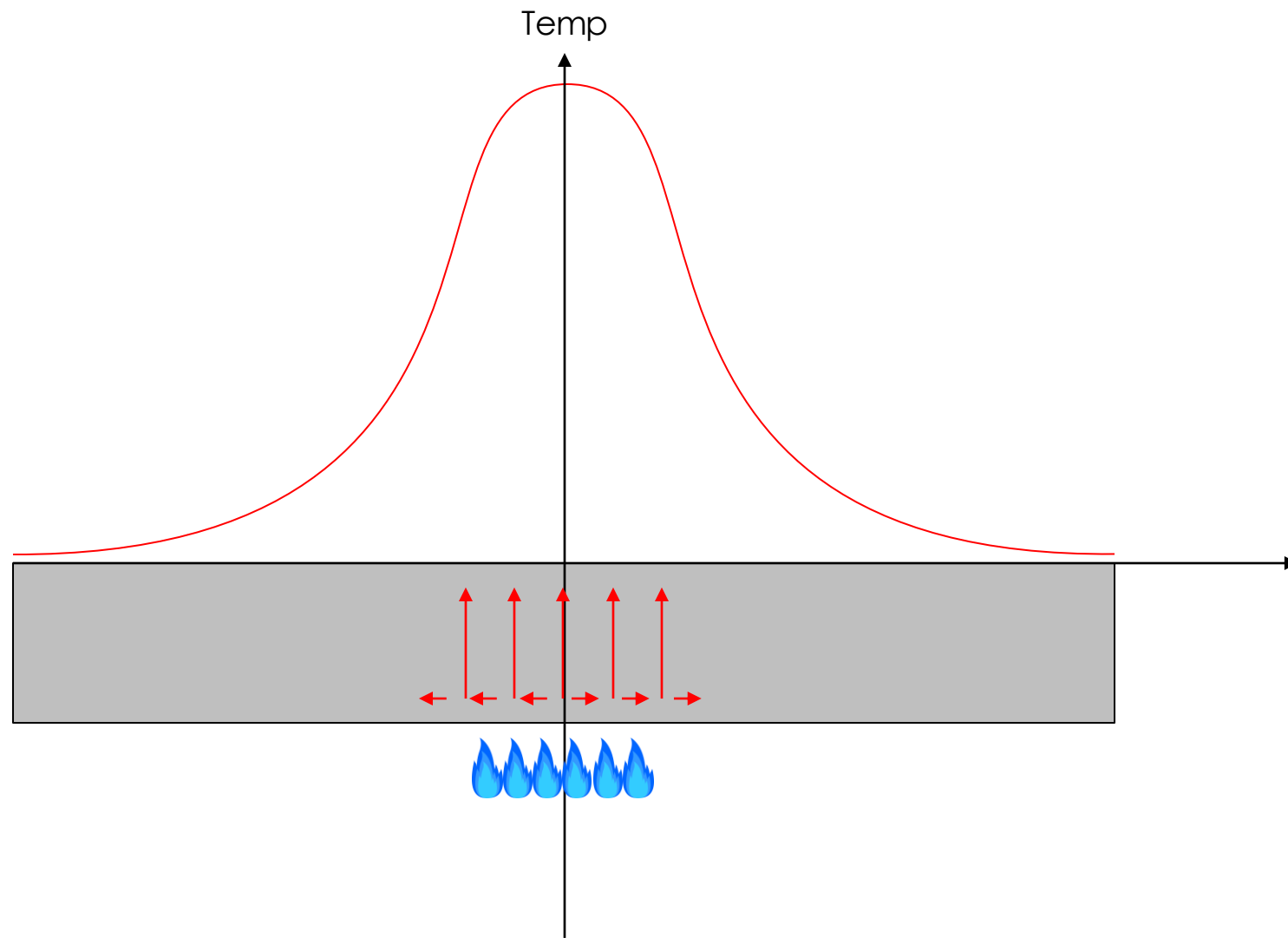


$$\vec{\nabla} T = \frac{\partial T}{\partial r} \hat{r} + \frac{1}{r} \frac{\partial T}{\partial \phi} \hat{\phi} + \frac{1}{r \sin \phi} \frac{\partial T}{\partial \theta} \hat{\theta}$$





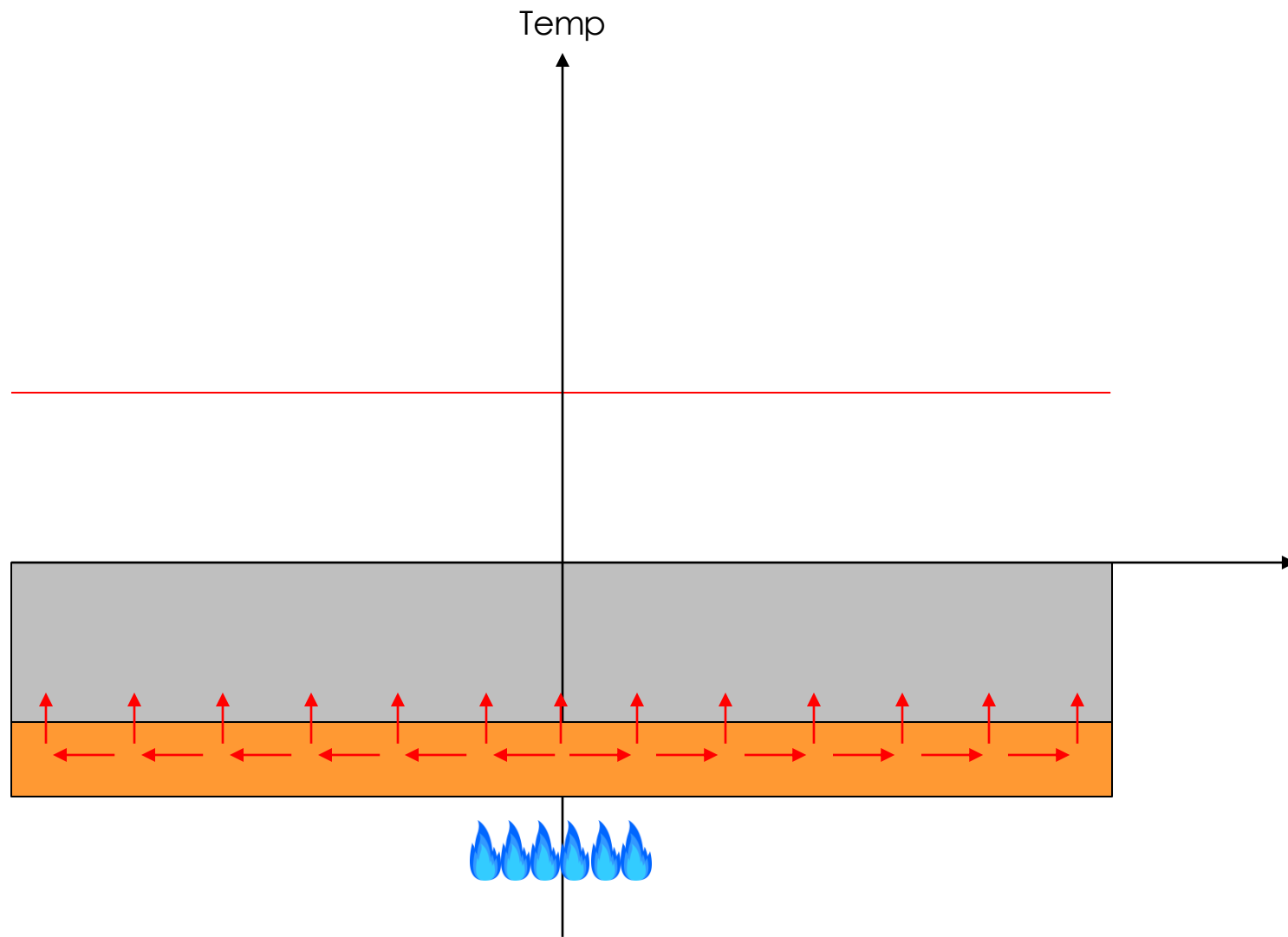
$k = 15 - 25 \text{ W/m/K}$

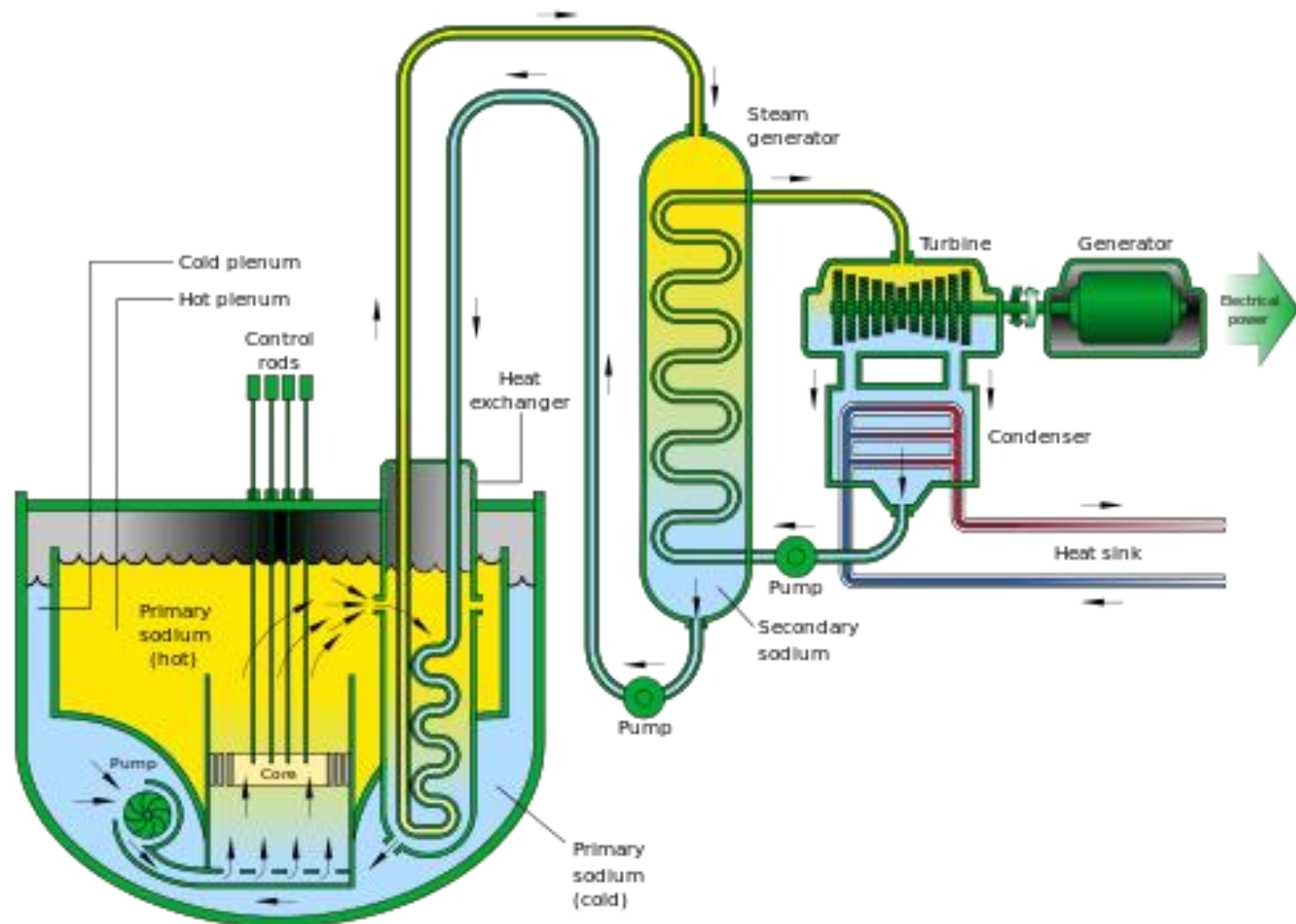
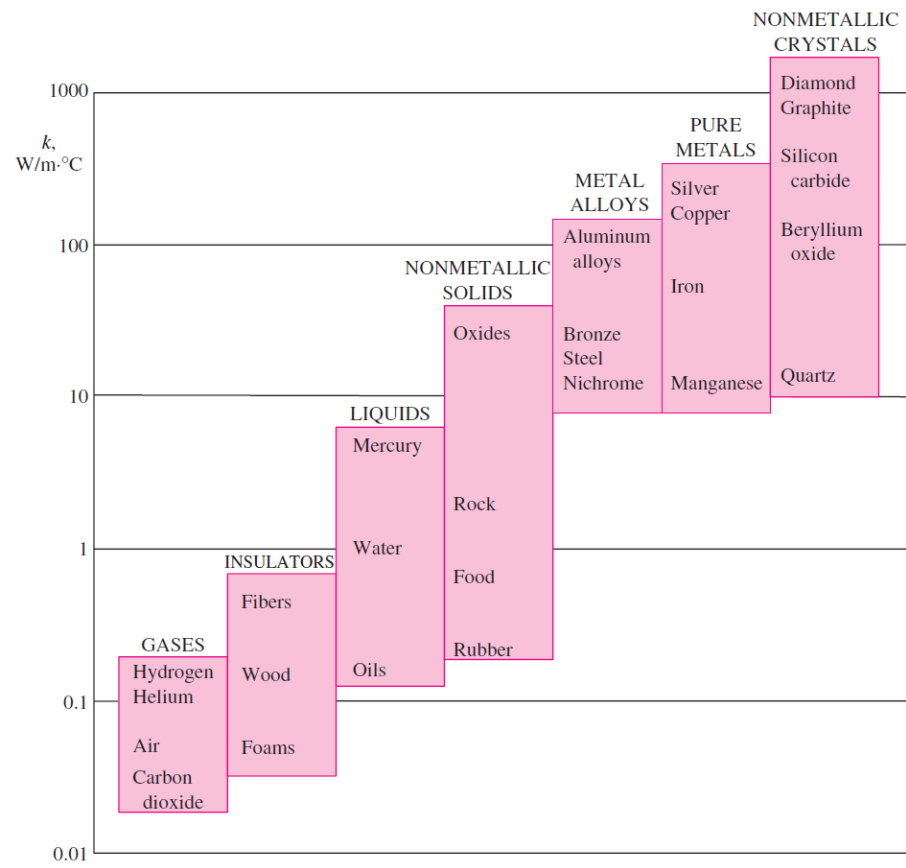




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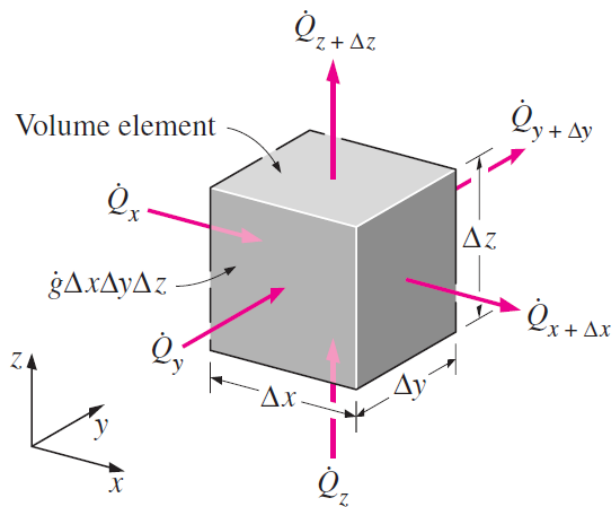
$k = 401 \text{ W/m/K}$





Inventário de energia: eq. de condução

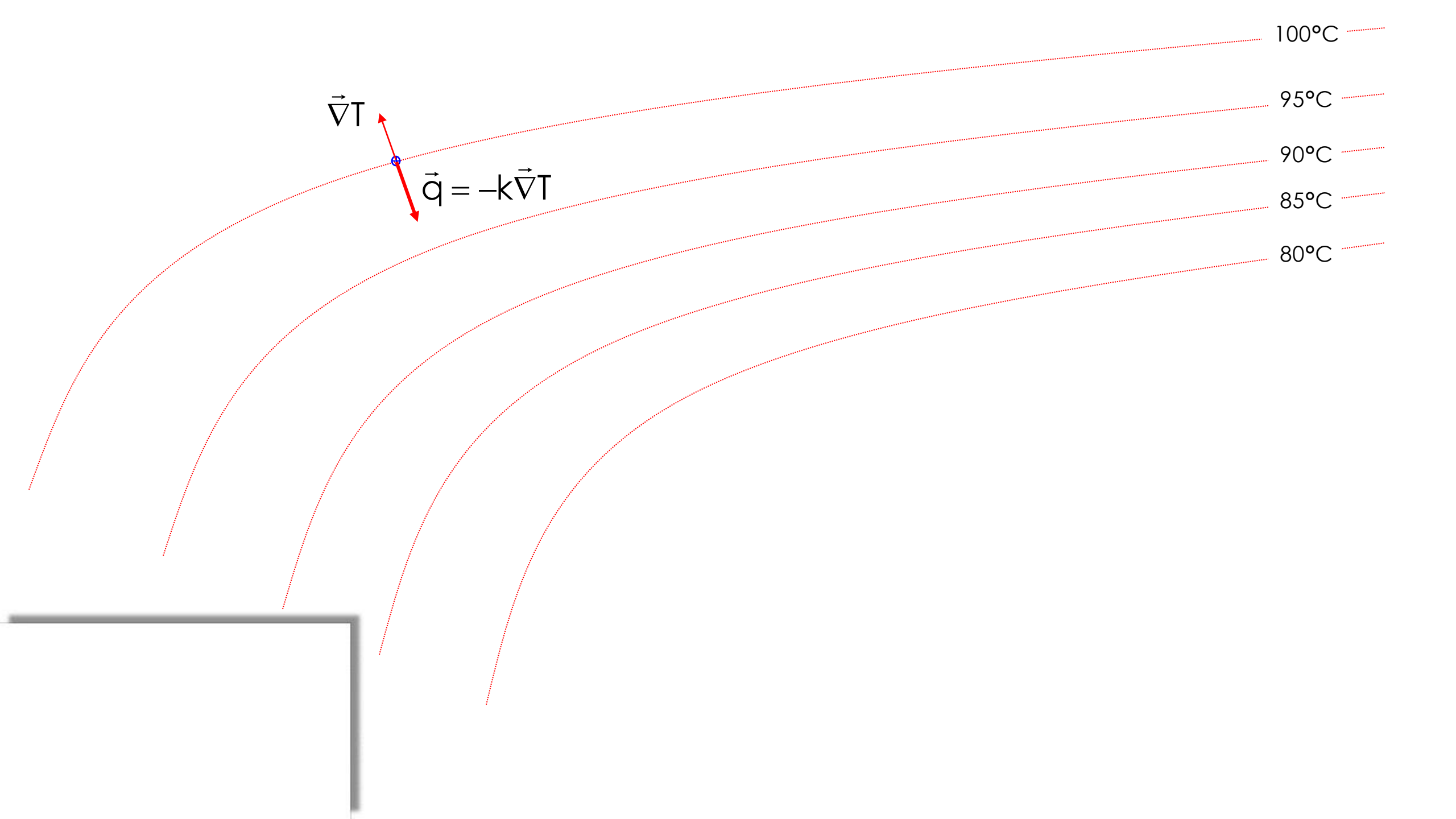
$$+ \left[\begin{array}{c} \text{taxa líquida de} \\ \text{condução de} \\ \text{calor entrando em} \\ \text{x, y e z} \end{array} \right] - \left[\begin{array}{c} \text{taxa líquida de} \\ \text{condução de} \\ \text{calor saindo em} \\ \text{x+dx, y+dy e z+dz} \end{array} \right] + \left[\begin{array}{c} \text{taxa de geração} \\ \text{de calor no} \\ \text{volume de} \\ \text{controle} \end{array} \right] = \left[\begin{array}{c} \text{taxa de variação} \\ \text{da energia no} \\ \text{volume de} \\ \text{controle} \end{array} \right]$$



$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{g}}{k} = \frac{\rho C_P}{k} \frac{\partial T}{\partial t}$$

$$\underbrace{\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}}_{\nabla^2 T} = \nabla^2 T$$

$$\nabla^2 T + \frac{\dot{g}}{k} = \frac{\rho C_P}{k} \frac{\partial T}{\partial t} \quad \alpha = \frac{k}{\rho C_P}$$



100°C

95°C

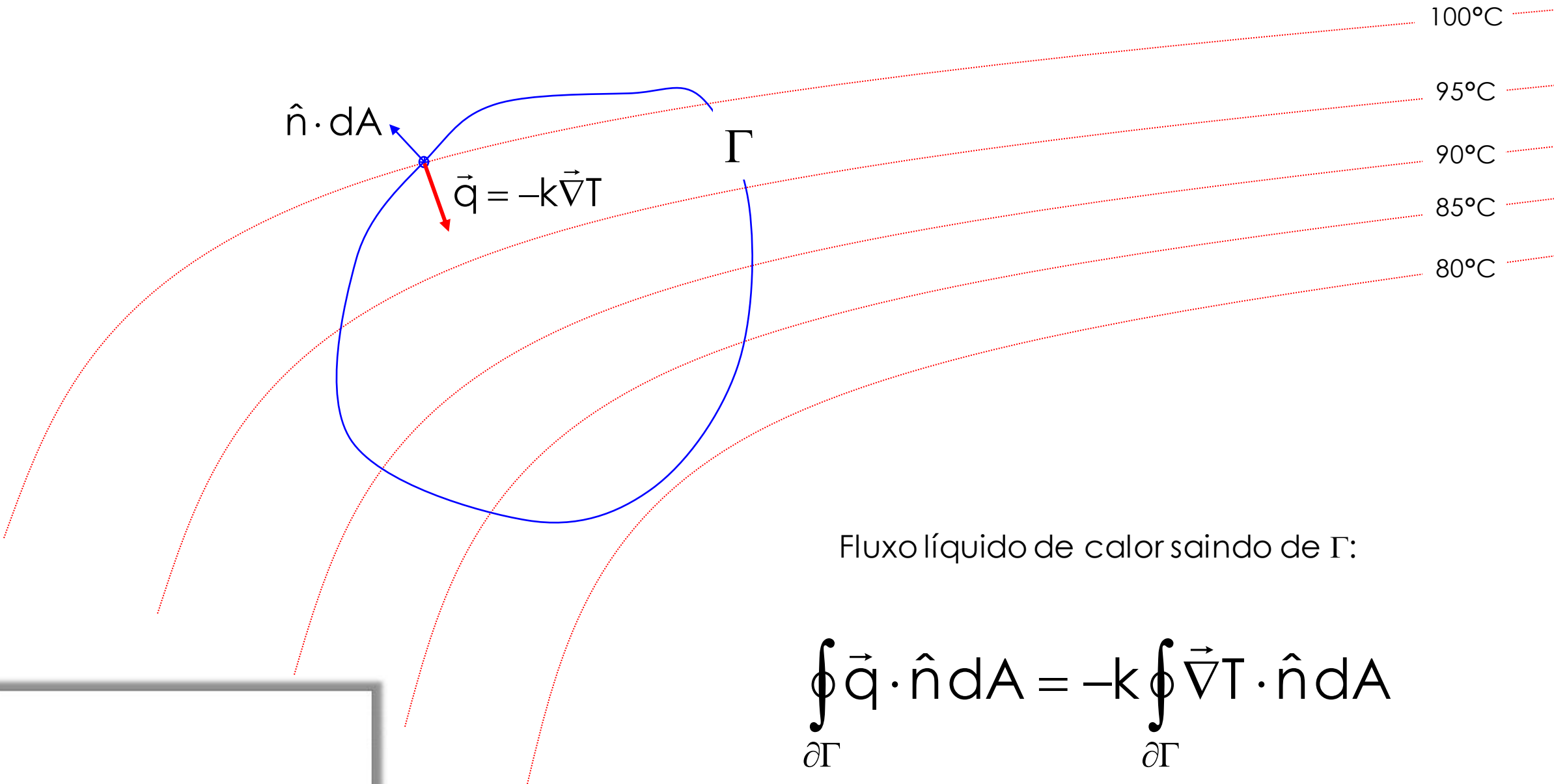
90°C

85°C

80°C

$\vec{\nabla}T$

$\vec{q} = -k\vec{\nabla}T$



Fluxo líquido de calor saindo de Γ :

$$\oint_{\partial\Gamma} \vec{q} \cdot \hat{n} dA = -k \oint_{\partial\Gamma} \vec{\nabla}T \cdot \hat{n} dA$$

$$\oint_{\partial\Gamma} \vec{q} \cdot \hat{n} dA = -k \oint_{\partial\Gamma} \vec{\nabla} T \cdot \hat{n} dA$$

Obs.: entrando em Γ

Teorema do divergente:

$$\oint_S \vec{F} \cdot \hat{n} dS \stackrel{\text{t.div}}{=} \iiint_V \vec{\nabla} \cdot \vec{F} dV$$

Obs.: saindo de S

$$\oint_{\partial\Gamma} \vec{q} \cdot \hat{n} dA = -k \int_{\Gamma} \nabla^2 T dV$$

Inventário de energia: eq. de condução

$$+ \left[\begin{array}{c} \text{taxa líquida de} \\ \text{condução de} \\ \text{calor entrando em} \\ \text{x, y e z} \end{array} \right] - \left[\begin{array}{c} \text{taxa líquida de} \\ \text{condução de} \\ \text{calor saindo em} \\ \text{x+dx, y+dy e z+dz} \end{array} \right] + \left[\begin{array}{c} \text{taxa de geração} \\ \text{de calor no} \\ \text{volume de} \\ \text{controle} \end{array} \right] = \left[\begin{array}{c} \text{taxa de variação} \\ \text{da energia no} \\ \text{volume de} \\ \text{controle} \end{array} \right]$$

$$- \oint_{\partial\Gamma} \vec{q} \cdot \hat{n} dA = k \int_{\Gamma} \nabla^2 T dV$$

$$\int_{\Gamma} \dot{g} dV$$

$$\int_{\Gamma} \rho C_p \frac{\partial T}{\partial t} dV$$

$$k \int_{\Gamma} \nabla^2 T dV + \int_{\Gamma} \dot{g} dV = \int_{\Gamma} \rho C_p \frac{\partial T}{\partial t} dV$$

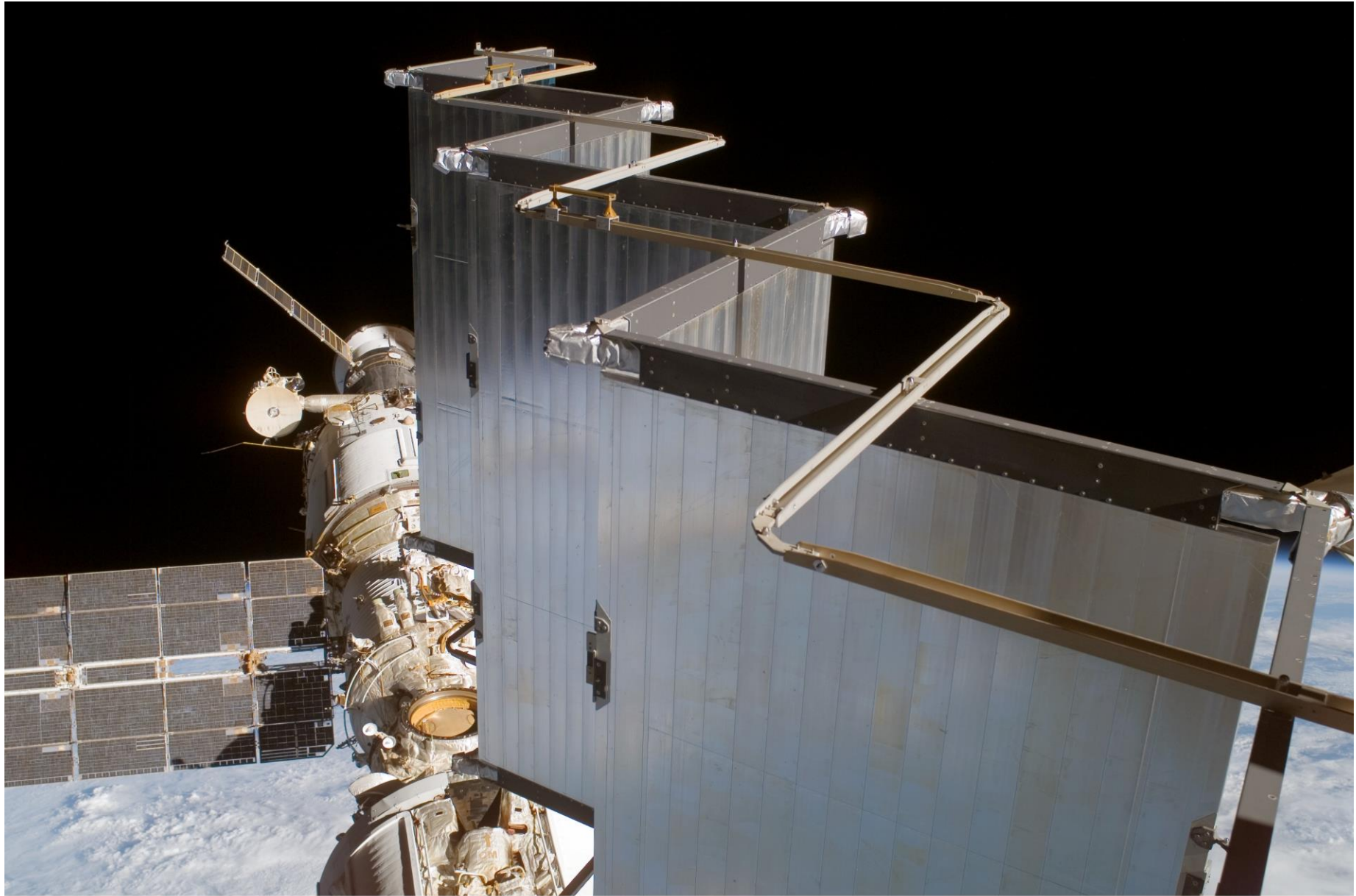
$$\nabla^2 T + \frac{\dot{g}}{k} = \frac{\rho C_p}{k} \frac{\partial T}{\partial t}$$

Exemplo de Aplicação



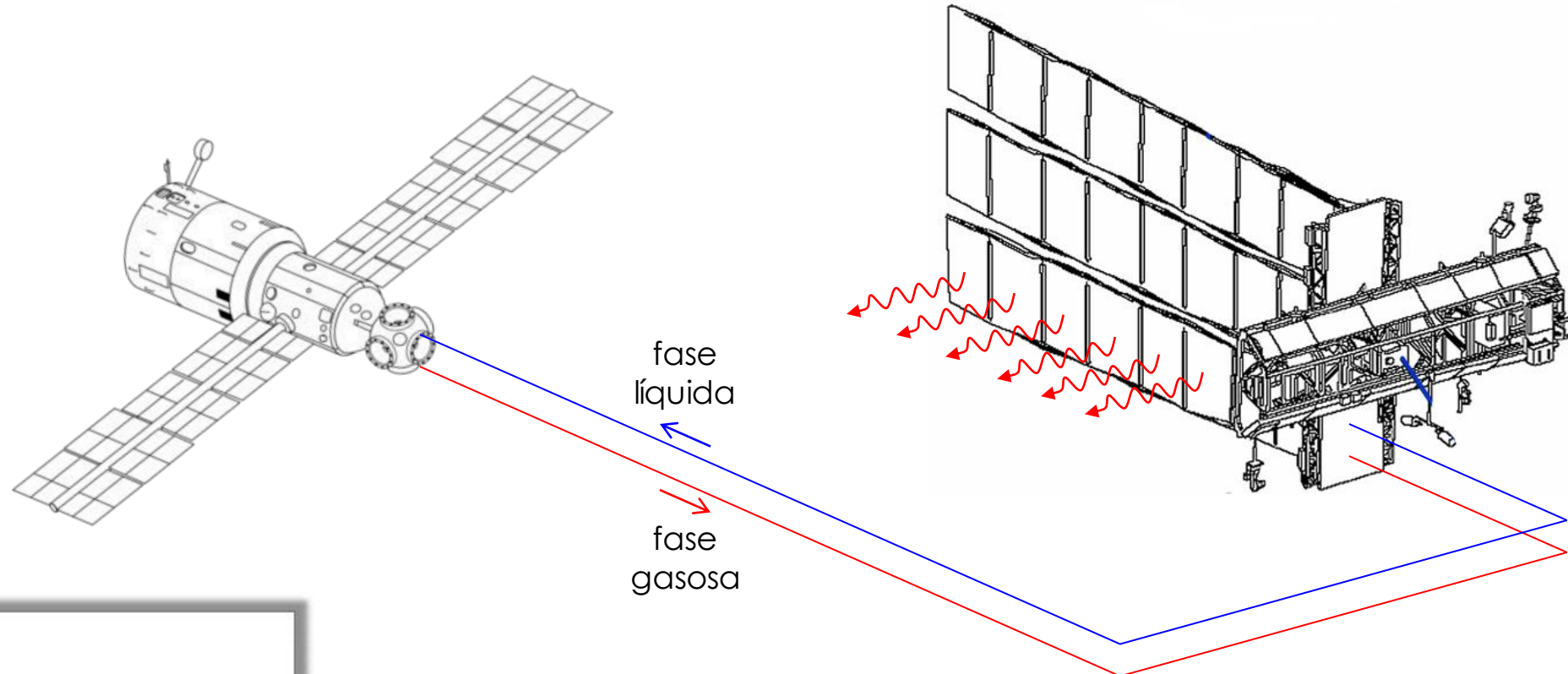


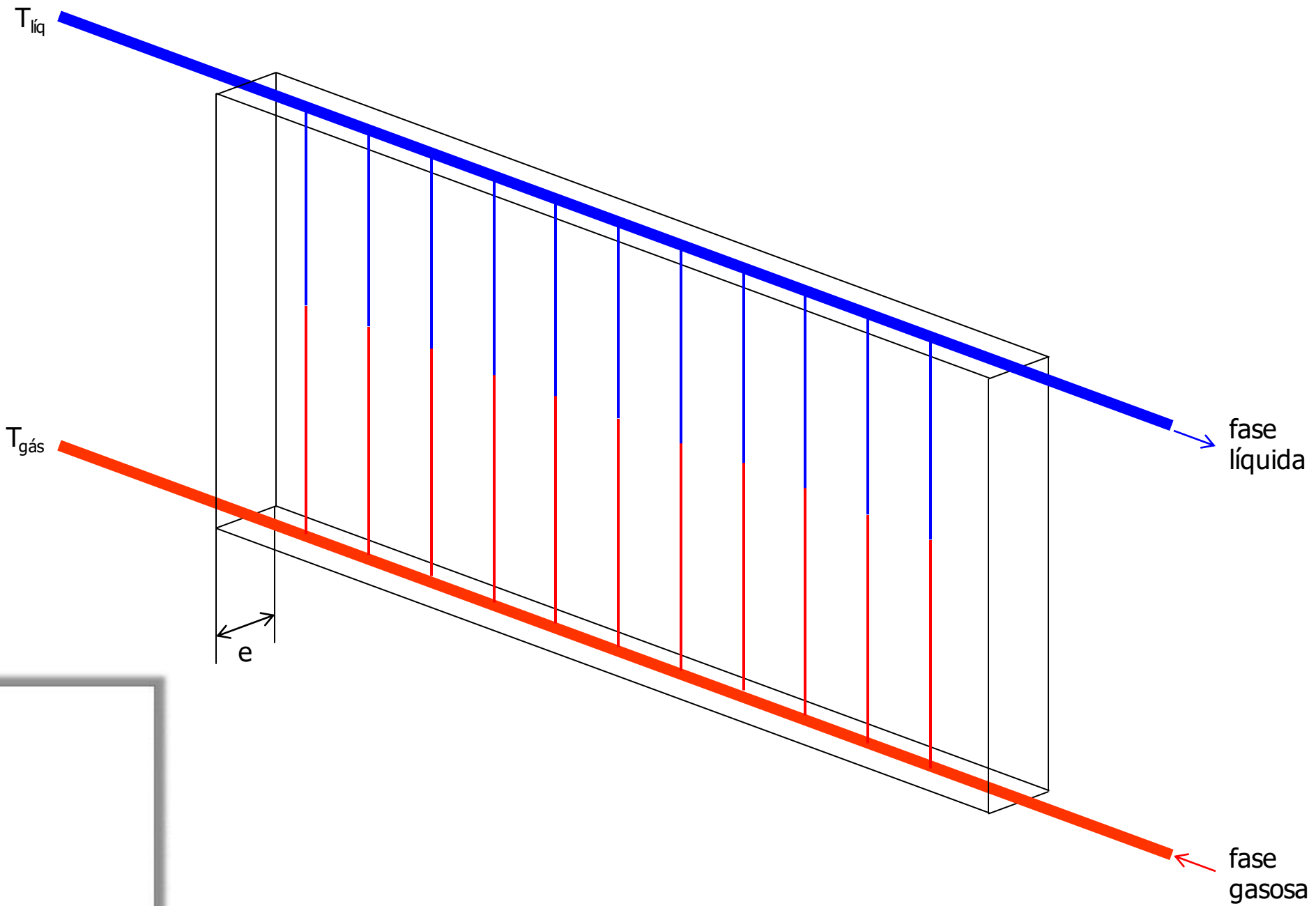


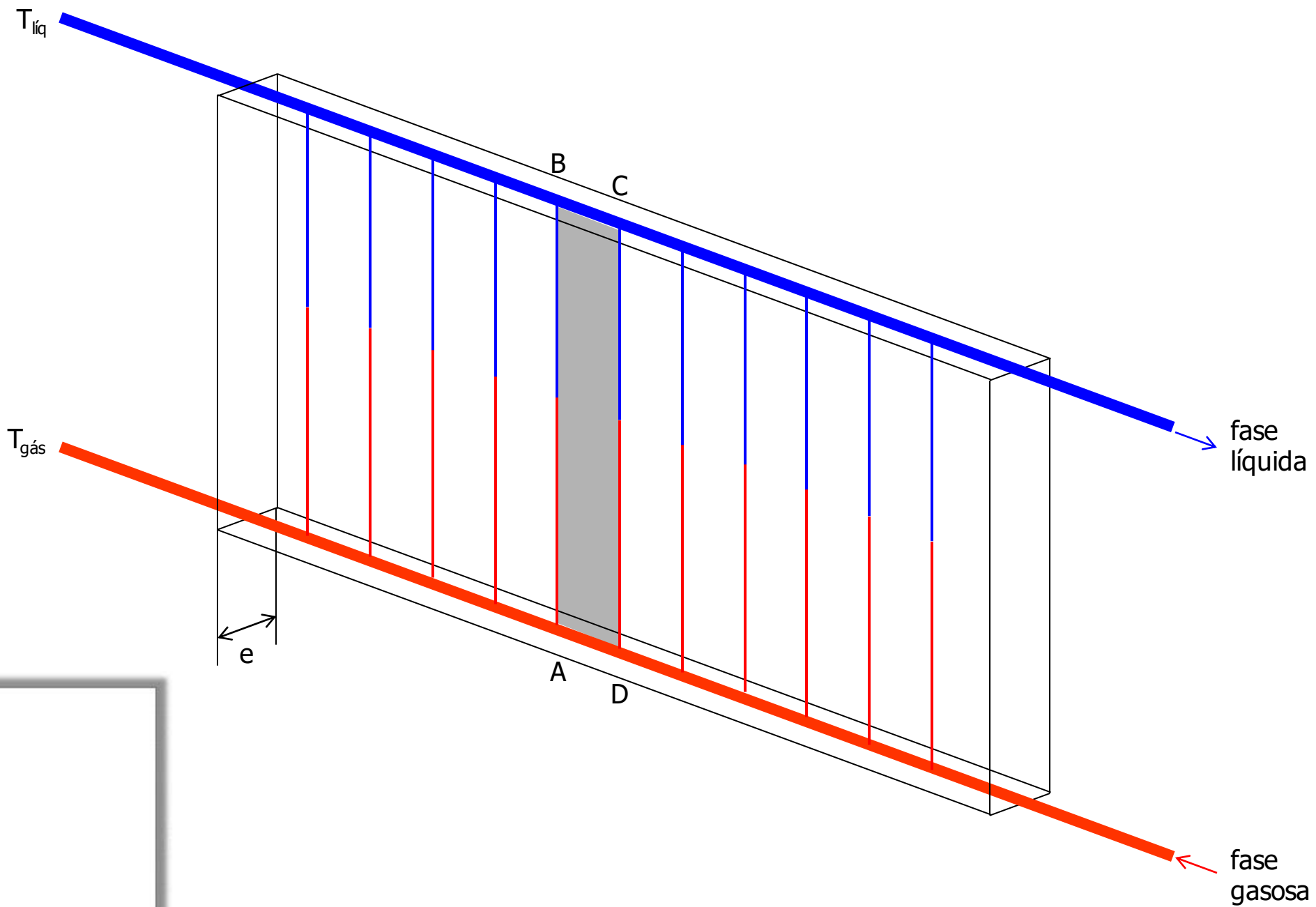


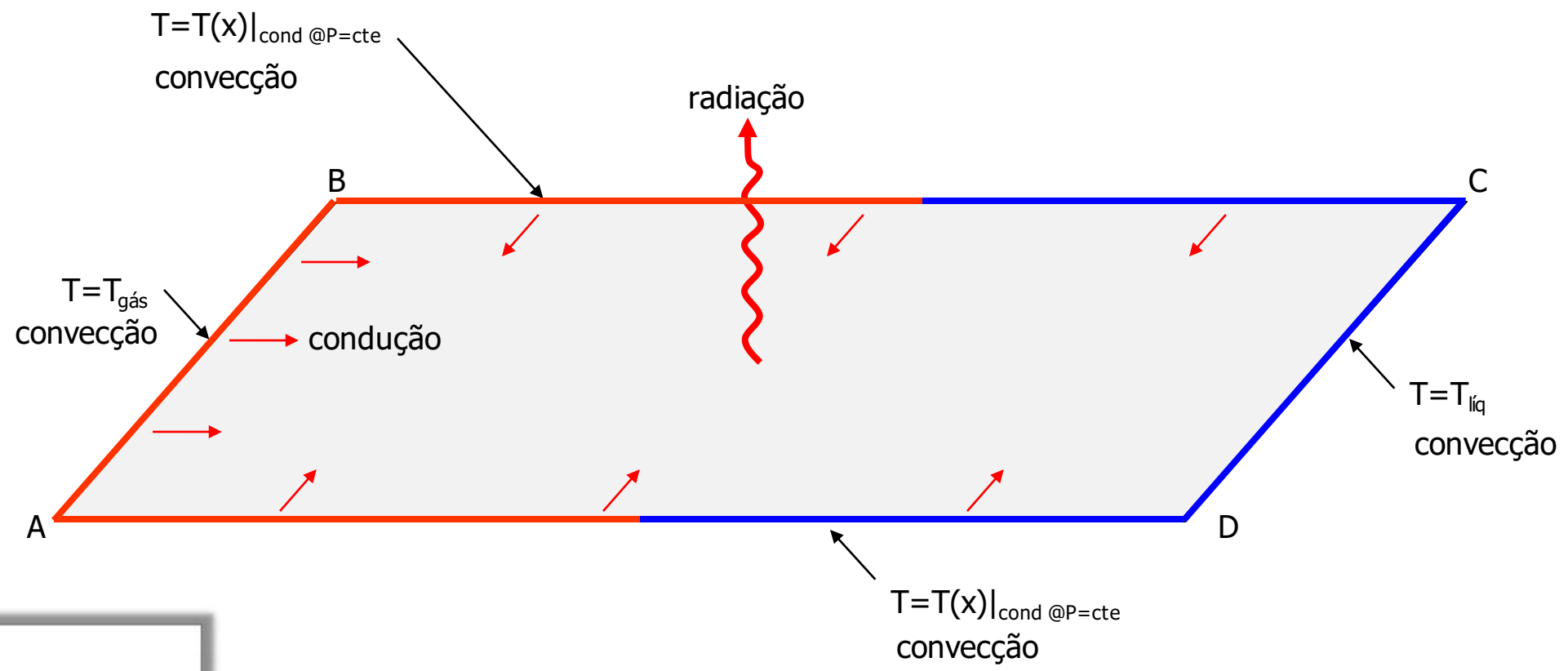
ISS014E13293

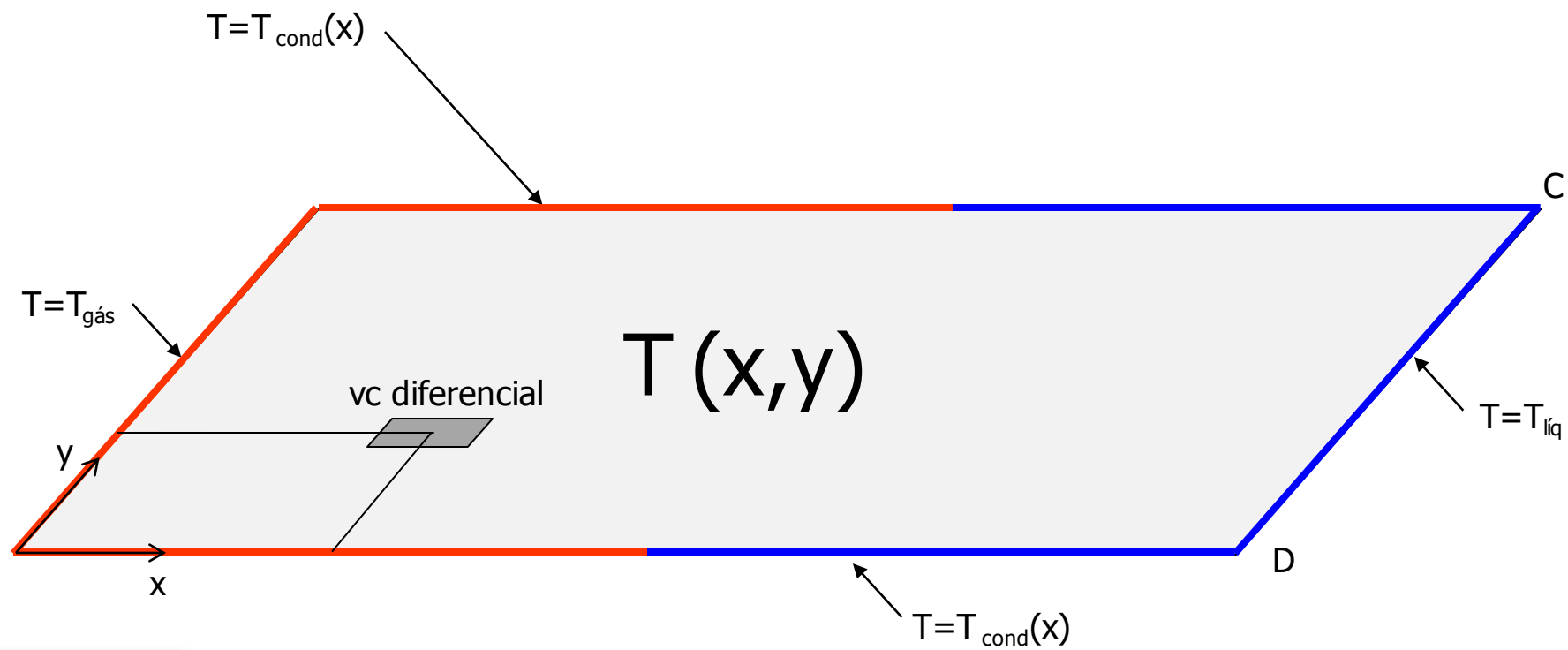
Formulação do problema aplicado:

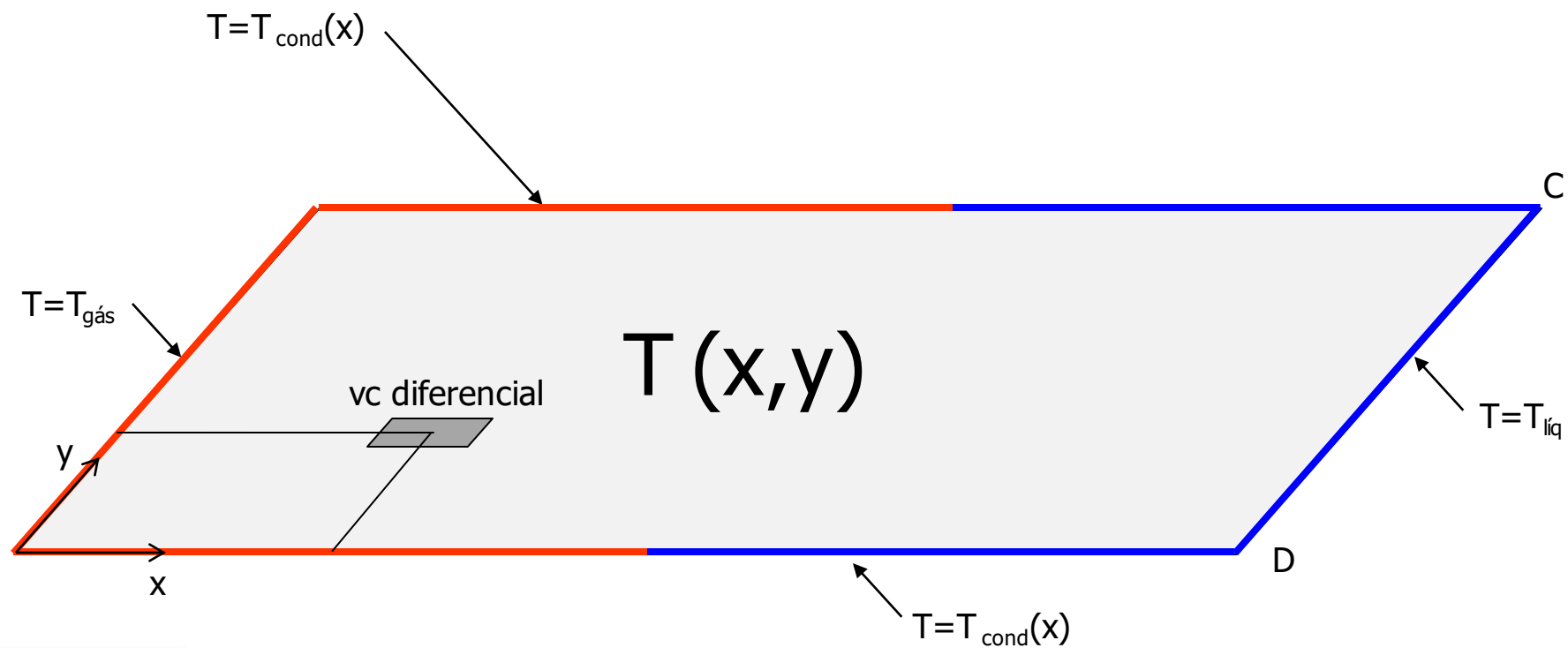












$$\mathbf{k} \cdot \nabla^2 \mathbf{T} + \dot{q}_{\text{rad}} = 0$$

+ Condições de contorno

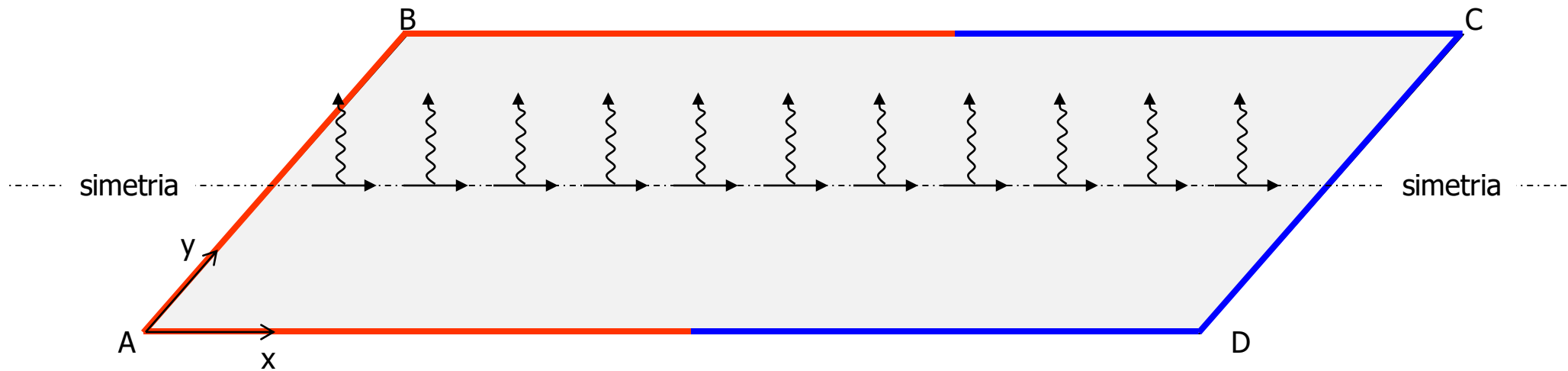


Diferenças finitas...
Elementos finitos...
Etc...

RESOLVENDO UM PROBLEMA SIMPLIFICADO...

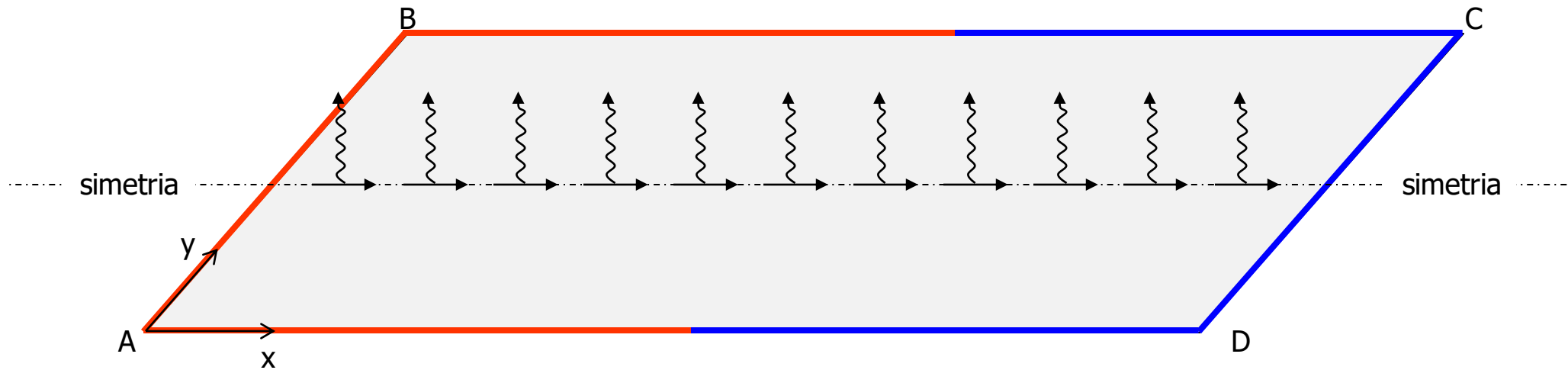


$$\left. \frac{dT}{dy} \right|_{y=L_{AB}/2} = 0 \rightarrow k \cdot \nabla^2 T + \dot{q}_{\text{rad}} = 0 \rightarrow k \cdot \frac{d^2 T}{dx^2} + \dot{q}_{\text{rad}} = 0$$



$$\dot{q}_{\text{rad}} = \varepsilon \sigma (T^4 - T_{\infty}^4) / (e/2)$$

$$\left. \frac{dT}{dy} \right|_{y=L_{AB}/2} = 0 \rightarrow k \cdot \nabla^2 T + \dot{q}_{\text{rad}} = 0 \rightarrow k \cdot \frac{d^2 T}{dx^2} + \dot{q}_{\text{rad}} = 0$$



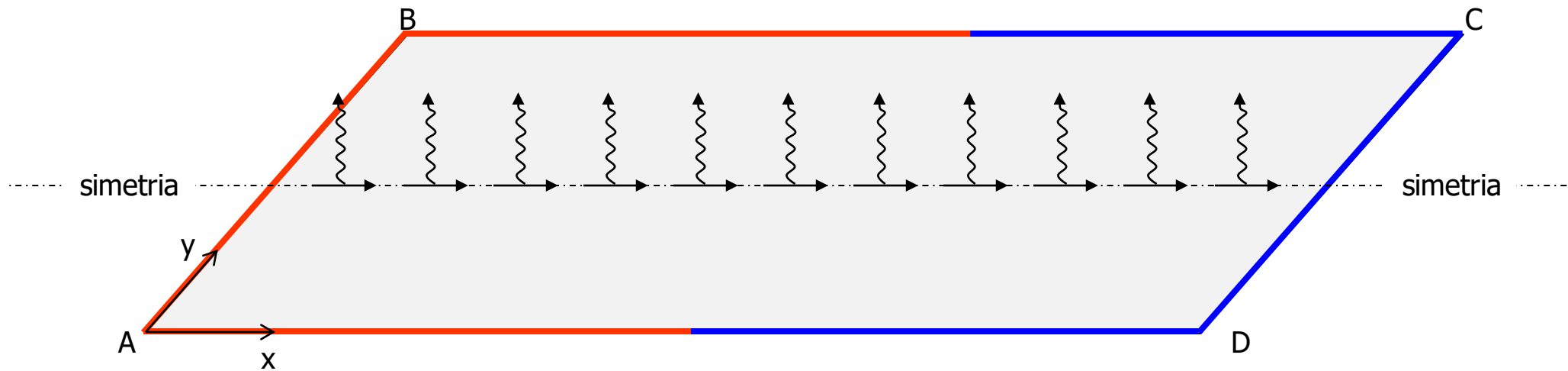
$$\dot{q}_{\text{rad}} = \varepsilon \sigma (T^4 - T_{\infty}^4) / (e/2)$$

← 50% do volume irradia na direção indicada

↑
radiação emitida para o do espaço @ T

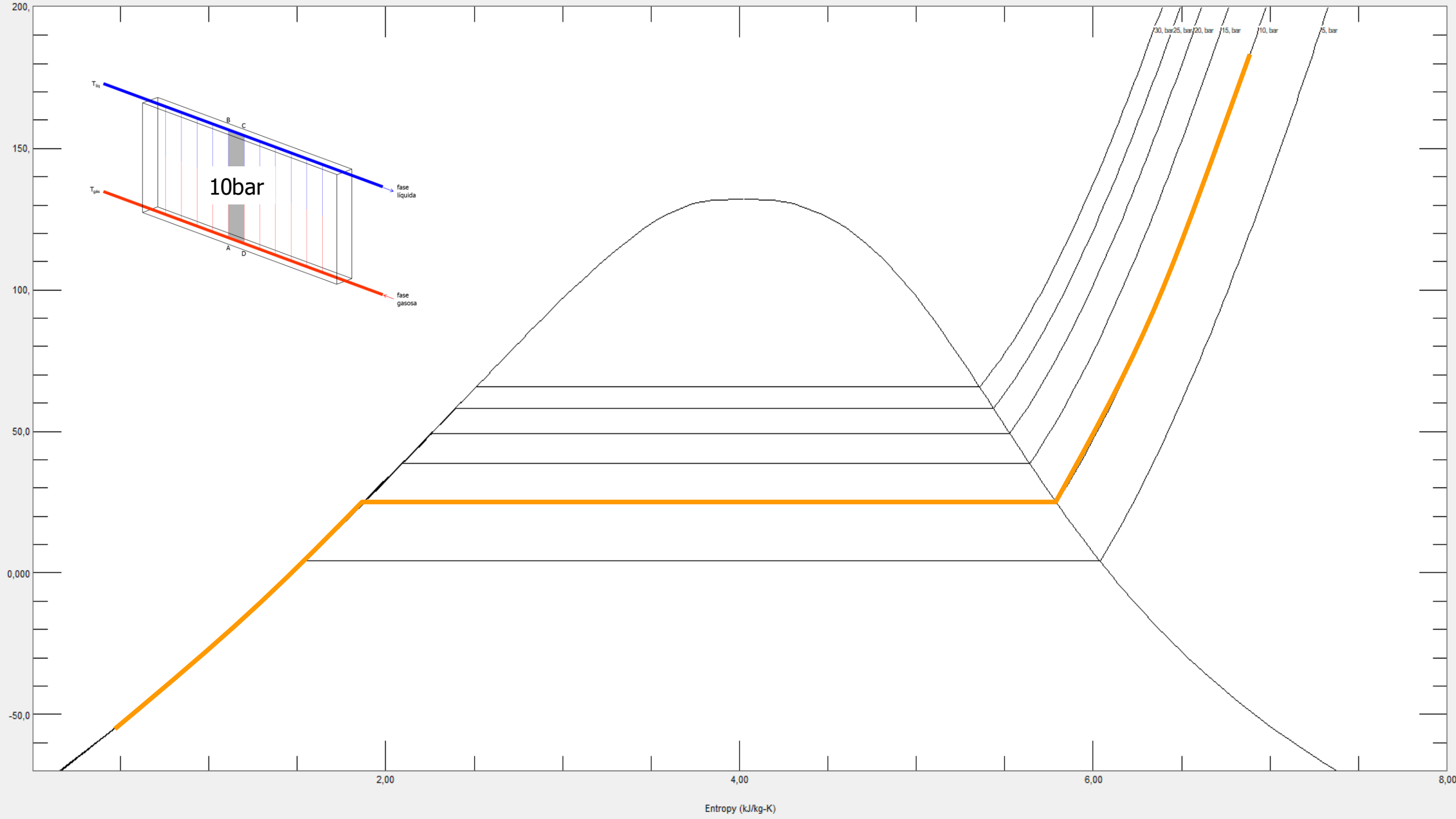
↑
radiação recebida do espaço @ ~4K

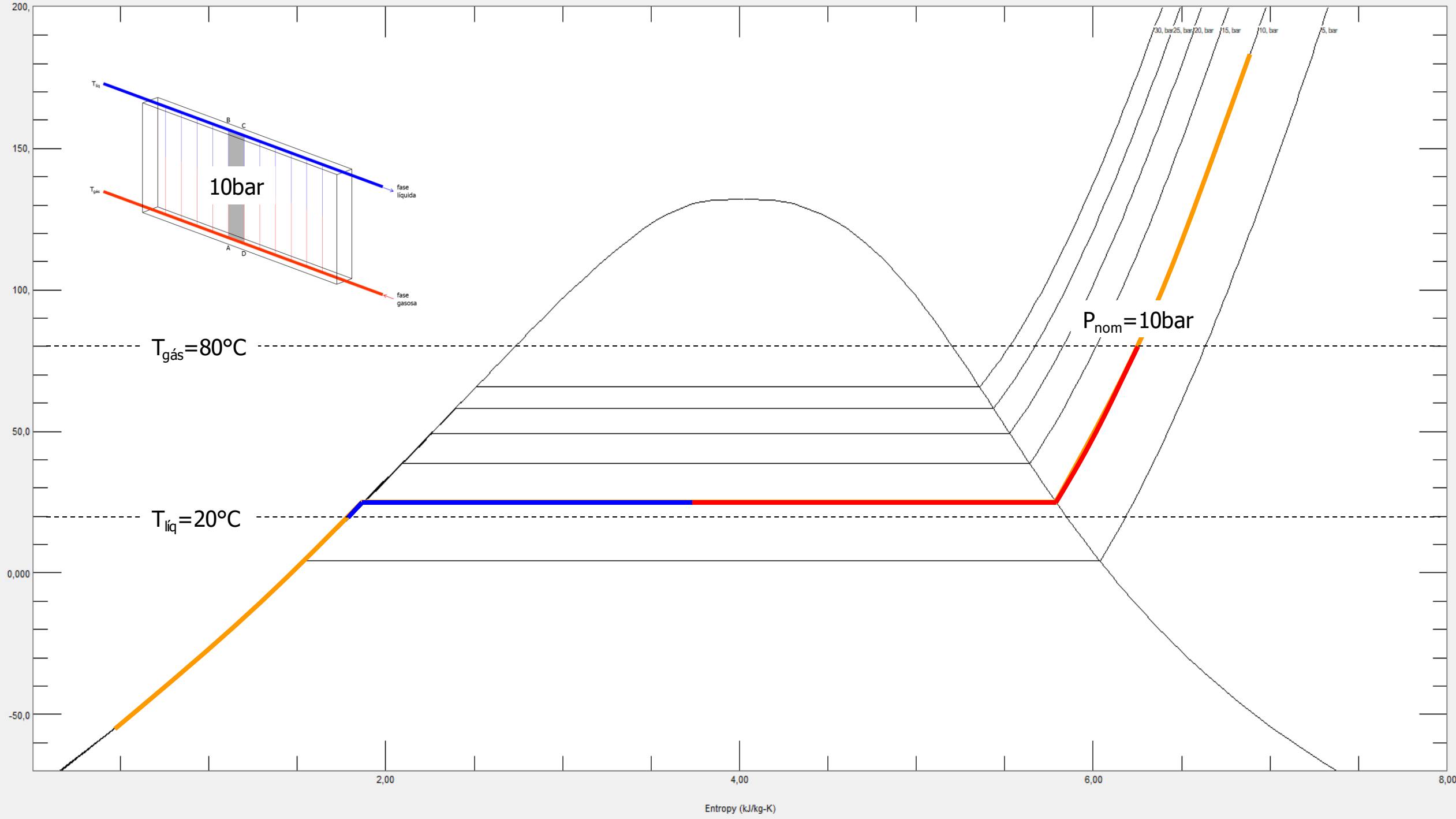
$$k \cdot \frac{d^2 T}{dx^2} + 2\varepsilon\sigma / e(T^4 - T_\infty^4) \rightarrow k \cdot \frac{d^2 T}{dx^2} + 2h_{\text{rad}} / e(T - T_\infty) = 0$$



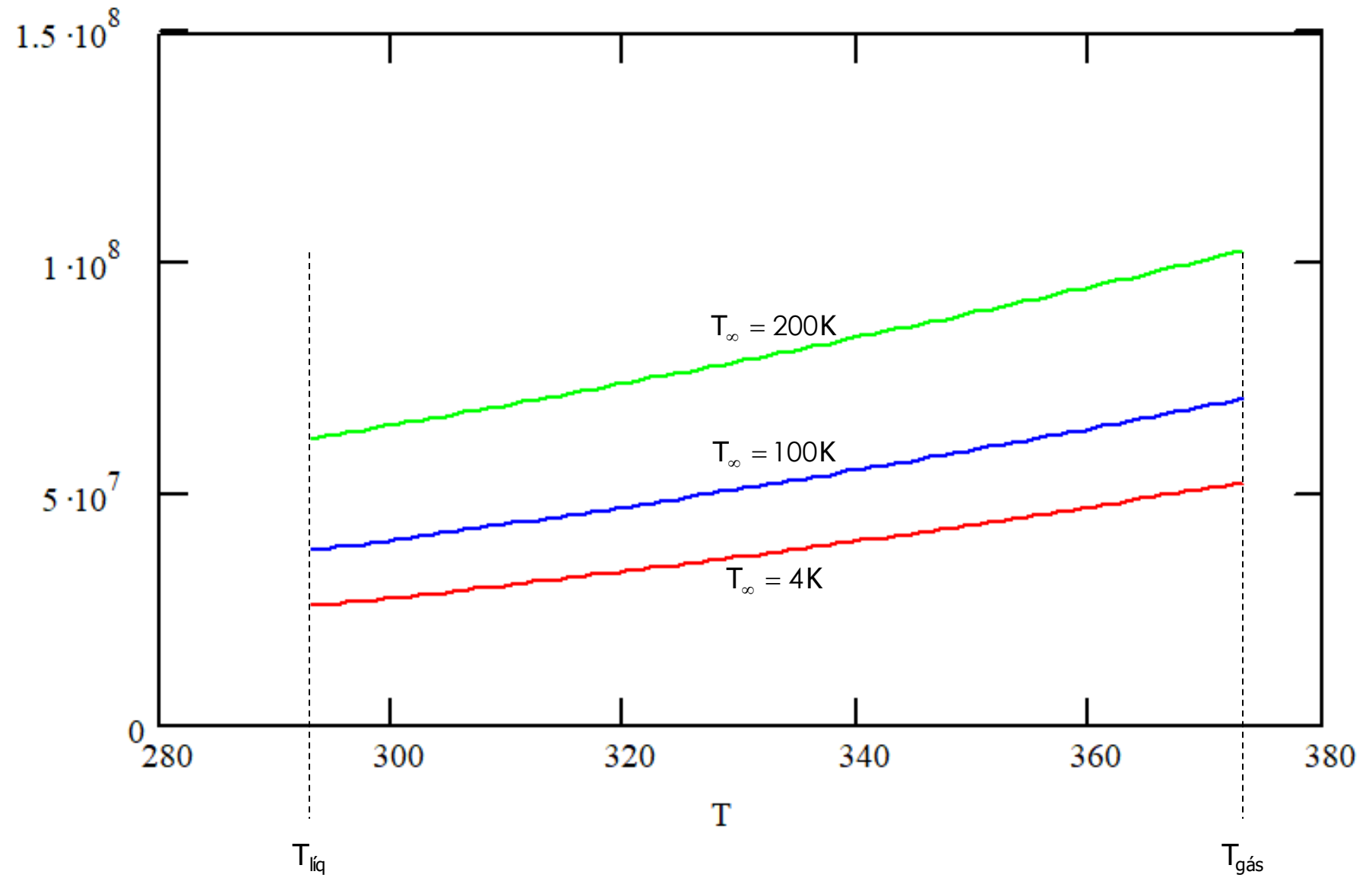
$$h_{\text{rad}} = \varepsilon\sigma (T_\infty + T)(T_\infty^2 + T^2)$$

$$\bar{h}_{\text{rad}} = \frac{\varepsilon\sigma}{T_{\text{gás}} - T_{\text{líq}}} \int_{T_{\text{gás}}}^{T_{\text{líq}}} (T_\infty + T)(T_\infty^2 + T^2) dT$$

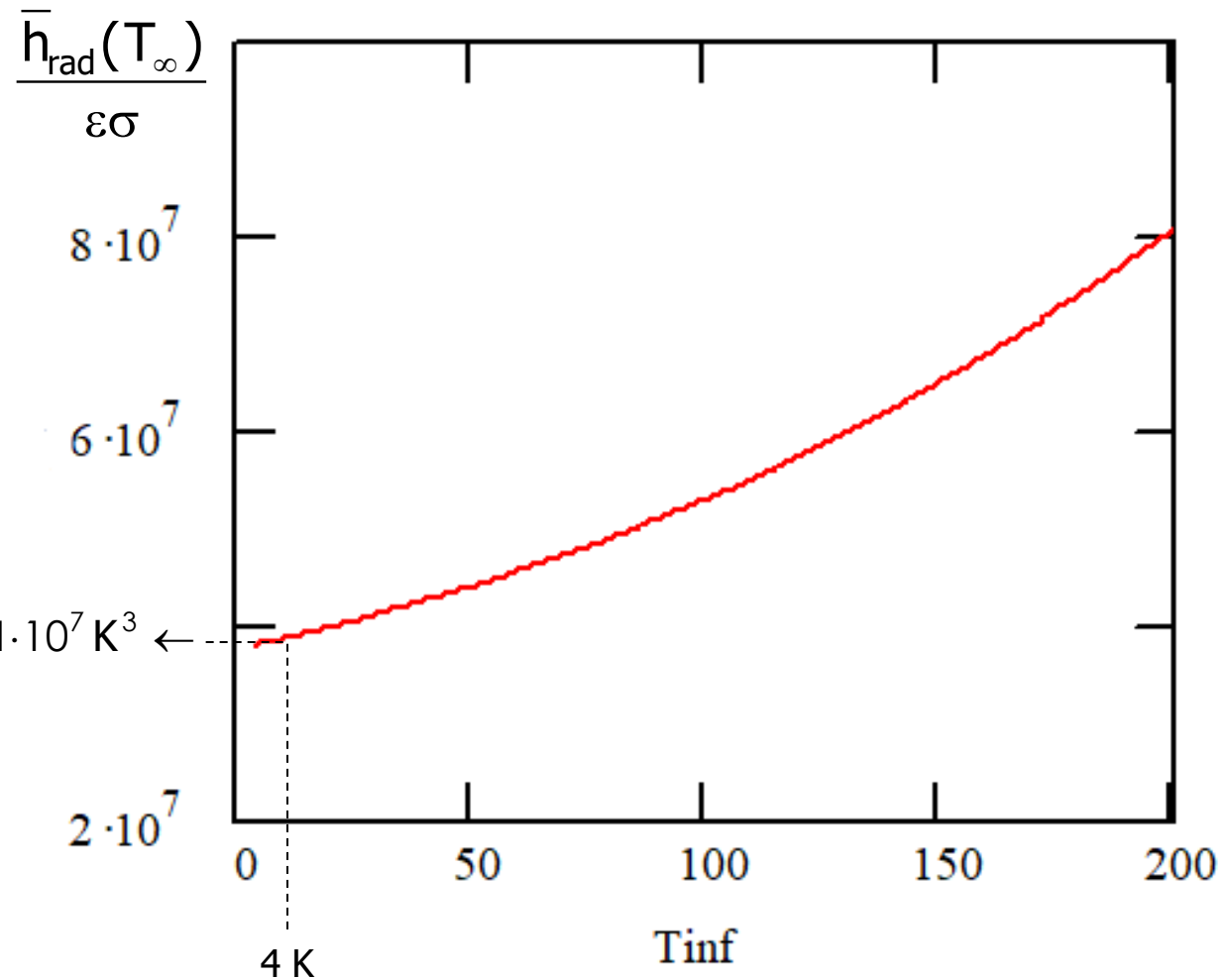




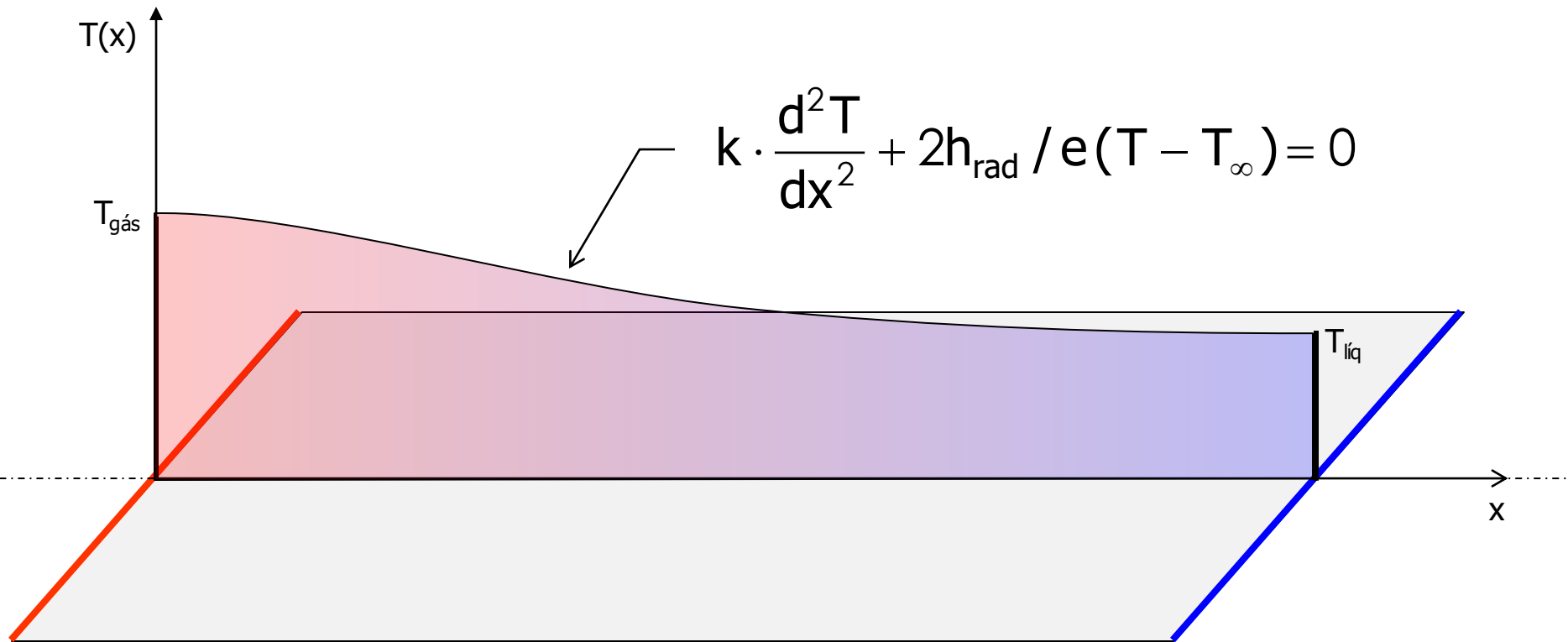
$$\frac{h_{\text{rad}}}{\epsilon\sigma} = (T_{\infty} + T)(T_{\infty}^2 + T^2)$$



$$\bar{h}_{\text{rad}} = \frac{\varepsilon\sigma}{T_{\text{gás}} - T_{\text{líqu}} \int_{T_{\text{gás}}}^{T_{\text{líqu}}} (T_{\infty} + T)(T_{\infty}^2 + T^2) dT \rightarrow \frac{\bar{h}_{\text{rad}}(T_{\infty})}{\varepsilon\sigma} = \frac{1}{373 - 293} \int_{373}^{293} (T_{\infty} + T)(T_{\infty}^2 + T^2) dT$$



$$\frac{1}{373 - 293} \int_{373}^{293} (4 + T)(4^2 + T^2) dT = 3.791 \cdot 10^7 \text{ K}^3 \leftarrow$$

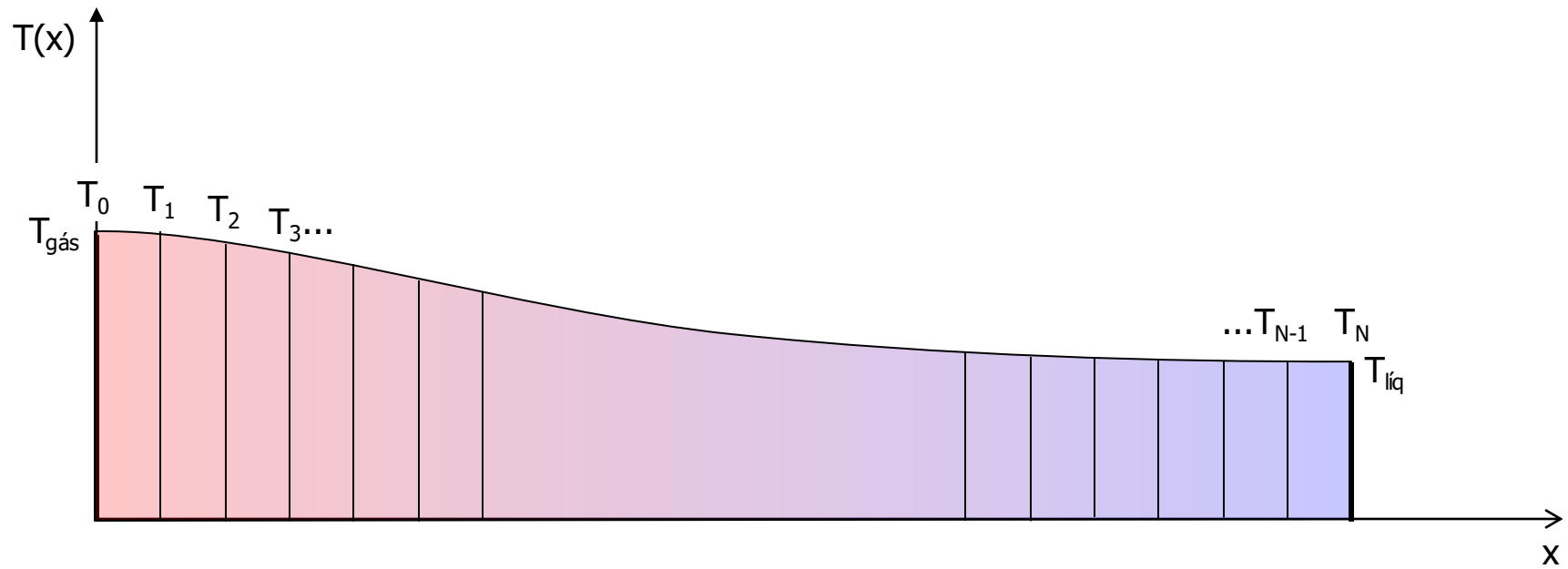


$$\bar{h}_{\text{rad}} = \epsilon \sigma \cdot 3.791 \cdot 10^7 \rightarrow \bar{h}_{\text{rad}} = 2.153 \frac{\text{W}}{\text{m}^2 \text{K}}$$

emissividade próxima de 1 por construção... $\uparrow \uparrow$ constante de Stephan-Boltzman = $5.67 \cdot 10^{-8} \text{ W/m}^2/\text{K}^4$

$$k_{\text{Al}} = 237 \frac{\text{W}}{\text{mK}} \quad e = 2 \text{ cm}$$

Conditions of heat transfer	$W/(m^2K)$
Gases in free convection	5-37
Water in free convection	100-1200
Oil under free convection	50-350
Gas flow in tubes and between tubes	10-350
Water flowing in tubes	500-1200
Oil flowing in tubes	300-1700
Molten metals flowing in tubes	2000-45000
Water nucleate boiling	2000-45000
Water film boiling	100-300
Film-type condensation: of water vapor	4000-17000
Dropsize condensation of water vapor	30000-140000
Condensation of organic liquids	500-2300



$$\left. \frac{dT}{dx} \right|_{@x_n} \cong \frac{T_{n+1} - T_n}{\Delta x}$$

$$\left. \frac{d^2T}{dx^2} \right|_{@x_n} \cong \frac{dT}{dx} \left(\frac{T_{n+1} - T_n}{\Delta x} \right) = \frac{1}{\Delta x} \left(\frac{dT_{n+1}}{dx} - \frac{dT_n}{dx} \right) = \frac{1}{\Delta x} \left(\frac{T_{n+2} - T_{n+1}}{\Delta x} - \frac{T_{n+1} - T_n}{\Delta x} \right)$$

$$\left. \frac{d^2T}{dx^2} \right|_{@x_n} \cong \frac{T_{n+2} - 2T_{n+1} + T_n}{\Delta x^2}$$

$$k \cdot \frac{d^2T}{dx^2} + 2h_{\text{rad}} / e(T - T_{\infty}) = 0$$

$$k \cdot \frac{d^2T}{dx^2} + (2h_{\text{rad}} / e) \cdot T = (2h_{\text{rad}} / e) \cdot T_{\infty}$$

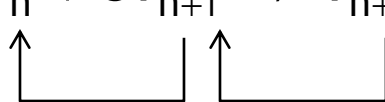
- Duas possibilidades:
- 1) Problema especificado em $x=0$ e $x=L$, resultando no fluxo de calor máximo na linha de simetria do painel (sistema de eqs.) → **sugestão...**
 - 2) Problema especificado completamente em $x = 0$, determinando o fluxo máximo de calor que resulta em uma temperatura de saída dentro do especificado (desacopla o sistema de equações)

$$k \cdot \frac{d^2 T}{dx^2} + 2h_{\text{rad}} / e(T - T_{\infty}) = 0$$

$$k \cdot \frac{d^2 T}{dx^2} + (2h_{\text{rad}} / e) \cdot T = (2h_{\text{rad}} / e) \cdot T_{\infty}$$

Duas possibilidades:

- 1) Problema especificado em $x=0$ e $x=L$, resultando no fluxo de calor máximo na linha de simetria do painel (sistema de eqs.) → **sugestão...**
- 2) Problema especificado completamente em $x = 0$, determinando o fluxo máximo de calor que resulta em uma temperatura de saída dentro do especificado (desacopla o sistema de equações)

$$\left. \begin{array}{l} T_0 = T_{\text{gás}} \\ q_0 = -k \frac{dT(0)}{dx} \end{array} \right\} \rightarrow a \cdot T_{\infty} + bT_n + cT_{n+1} \rightarrow T_{n+2}$$


$$k \cdot \frac{d^2 T}{dx^2} + 2h_{\text{rad}} / e(T - T_{\infty}) = 0$$

$$k \cdot \frac{d^2 T}{dx^2} + (2h_{\text{rad}} / e) \cdot T = (2h_{\text{rad}} / e) \cdot T_{\infty}$$

$$\frac{k}{\Delta x^2} \cdot (T_{n+2} - 2T_n + T_n) + (2h_{\text{rad}} / e) \cdot T_n = (2h_{\text{rad}} / e) \cdot T_{\infty}$$

$$T_{n+2} = \frac{2h_{\text{rad}} \Delta x^2}{ke} \cdot T_{\infty} - \left(\frac{2h_{\text{rad}} \Delta x^2}{ke} + 1 \right) \cdot T_n + 2T_{n+1}$$

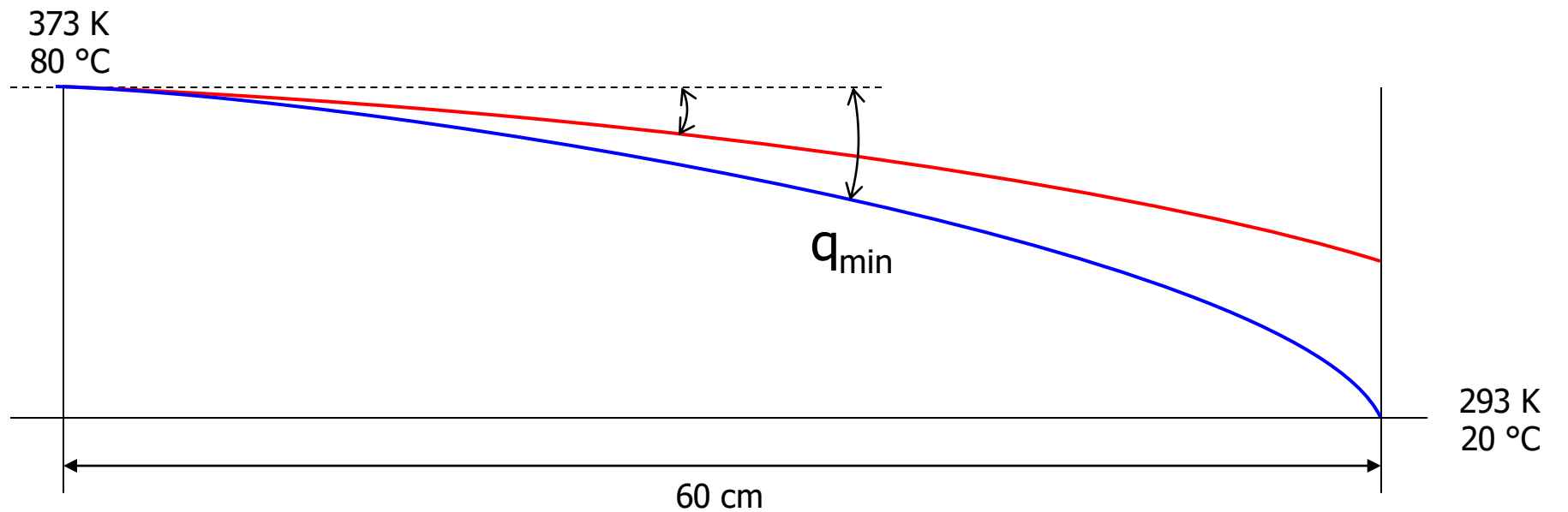
$$a = \frac{2h_{\text{rad}} \Delta x^2}{ke} \quad b = - \left(\frac{2h_{\text{rad}} \Delta x^2}{ke} + 1 \right) \quad c = 2$$

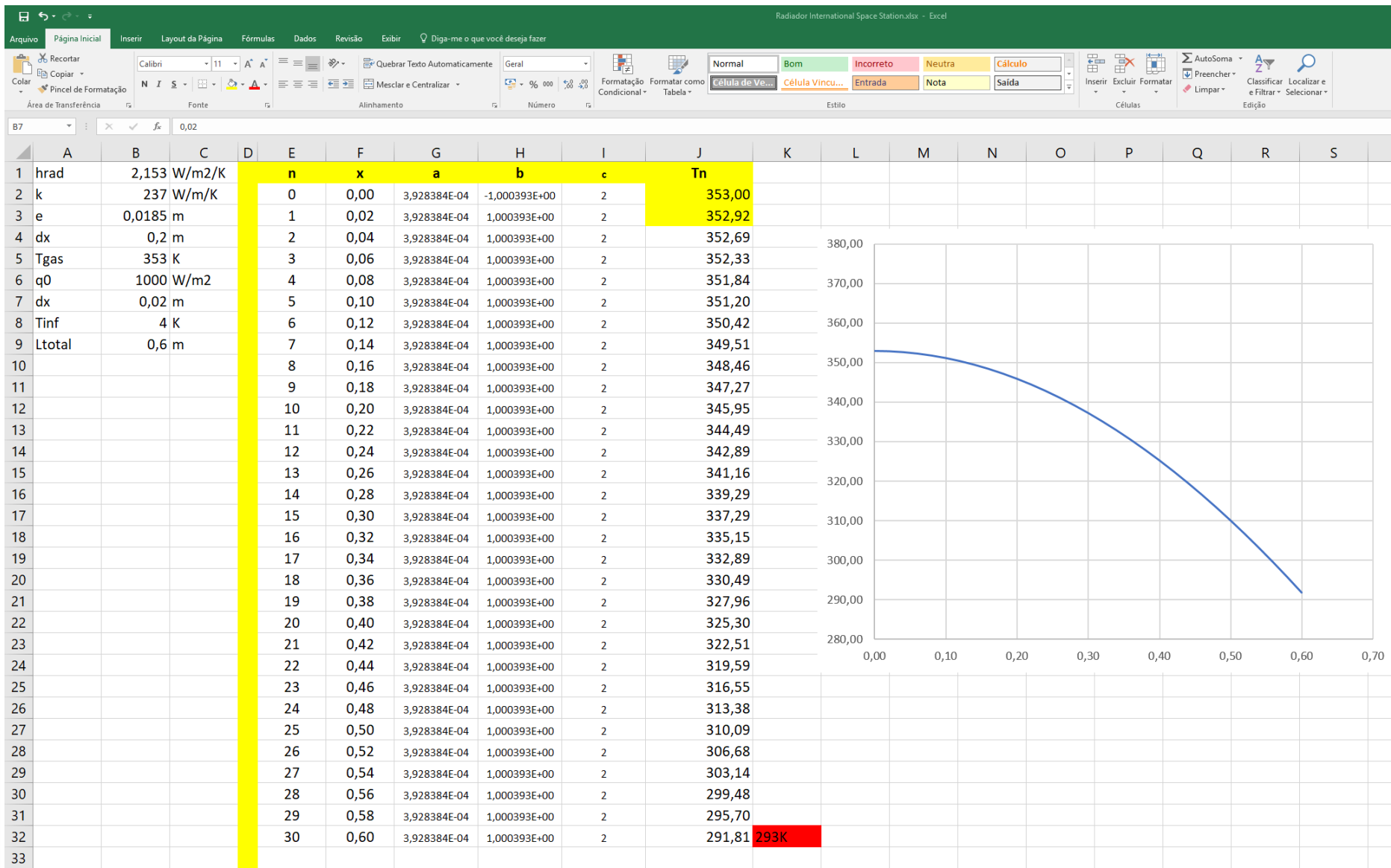
$$a \cdot T_{\infty} + bT_n + cT_{n+1} \rightarrow T_{n+2}$$

Imposição de condições de contorno em $x = 0$:

$$T_0 = T_{\text{gás}}$$

$$q_0 = -k \frac{dT(0)}{dx} \rightarrow q_0 = -k \frac{dT(0)}{dx} \cong -k \frac{T_1 - T_0}{\Delta x} \rightarrow T_1 = T_0 - \frac{\Delta x}{k} \cdot q_0$$



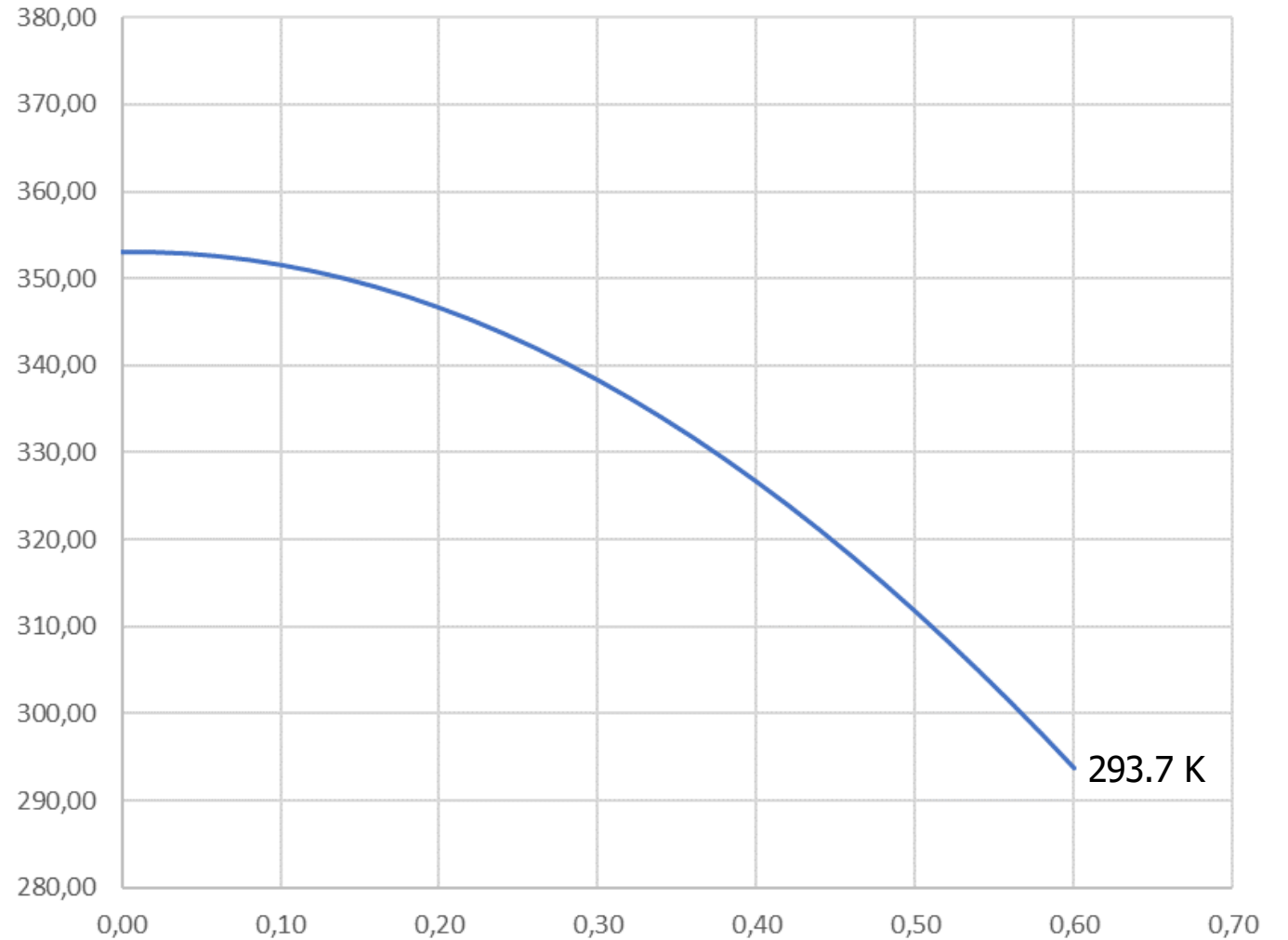


$$a = \frac{2h_{\text{rad}}\Delta x^2}{ke} = \frac{2 \cdot 2.153 \cdot (0.2)^2}{237 \cdot 0.2} = 3.634 \cdot 10^{-3}$$

$$b = -\left(\frac{2h_{\text{rad}}\Delta x^2}{ke} + 1\right) = -\frac{2 \cdot 2.153 \cdot (0.2)^2}{237 \cdot 0.2} - 1 = -1.0004$$

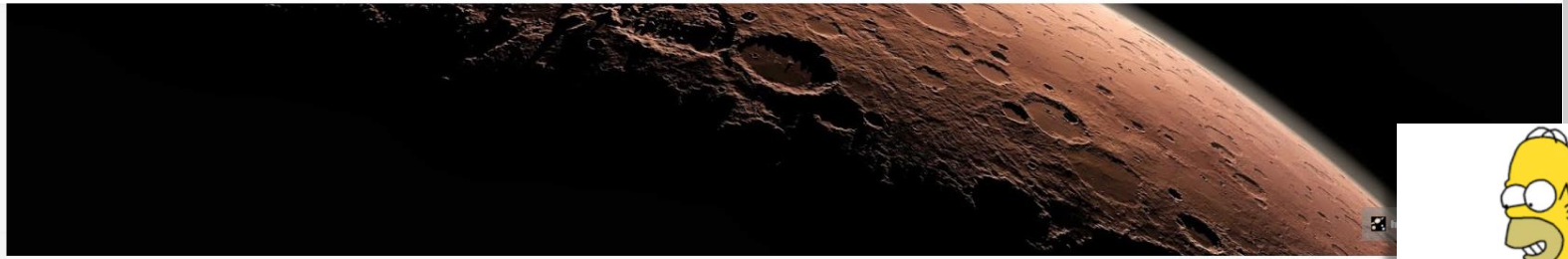
$$c = 2$$

$$a \cdot T_{\infty} + bT_n + cT_{n+1} \rightarrow T_{n+2}$$



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