

1a) idem prova PAR

1b)  $\binom{n}{0} + \binom{n+1}{1} + \binom{n+2}{2} = 81$

$$\frac{n!}{0!n!} + \frac{(n+1)!}{n! 1!} + \frac{(n+2)!}{n! 2!} = 81$$

$$1 + (n+1) + \frac{(n+2)(n+1)}{2} = 81$$

$n^2 + 5n - 156 = 0$  has positive solution in  $\mathbb{N}$ .

2) a)  $f(x) = \frac{3x}{\sqrt{1-e^x}}$

$$1-e^x > 0 \Rightarrow e^x < 1 \Rightarrow \ln(e^x) < \ln 1$$

$$\Rightarrow x < 0$$

$$\therefore \text{dom } f = \{x \in \mathbb{R} : x < 0\}$$

$$2b) e^x + e^{-x} = 3$$

$$\frac{e^x + 1}{e^x} = 3$$

$$\frac{e^{2x} + 1}{e^x} = 3$$

$$e^{2x} - 3e^x + 1 = 0$$

$$\text{faga } y = e^x$$

$$y^2 - 3y + 1 = 0$$

$$y = \frac{3 \pm \sqrt{5}}{2}$$

$$\therefore e^x = \frac{3 \pm \sqrt{5}}{2} > 0 \text{ em ambos os casos}$$

$$\therefore x = \ln\left(\frac{3 + \sqrt{5}}{2}\right) \text{ ou } x = \ln\left(\frac{3 - \sqrt{5}}{2}\right)$$

5)

a) FALSO

$$\ln p - \ln q = \ln\left(\frac{p}{q}\right) \neq \frac{p}{q}$$

b) FALSO

$$3 \ln x = \ln(x^3) \neq x^3$$

c) VERDADEIRO

$$\frac{a}{b} \ln(N^{\frac{b}{a}}) = \frac{b}{a} \cdot \frac{a}{b} \cdot \ln(N) = \ln(N)$$

d) VERDADEIRO

$$3 \ln(\ln x) = \ln(\ln x)^3$$

e) FALSO

$$\begin{aligned} 3 \ln a + 4 \ln b &= \ln(a^3) + \ln(b^4) = \\ &= \ln(a^3 \cdot b^4) \neq \ln(a^3 + b^4) \end{aligned}$$

4)  $(1-i)^2 = 1 - 2i + (i^2) = -2i.$

$$\therefore (1-i)^{20} = ((1-i)^2)^5 = (-2i)^5 = -32i^5 = -32i$$

$$\therefore w = -3^{20} = -(-32i) = 32i.$$

4b) idem prova PAR