Um modelo analítico de um Motor de Corrente Contínua sem Escovas

O problema

- Texto original em:

Analytical model for the optimal design of a brushless DC wheel motor

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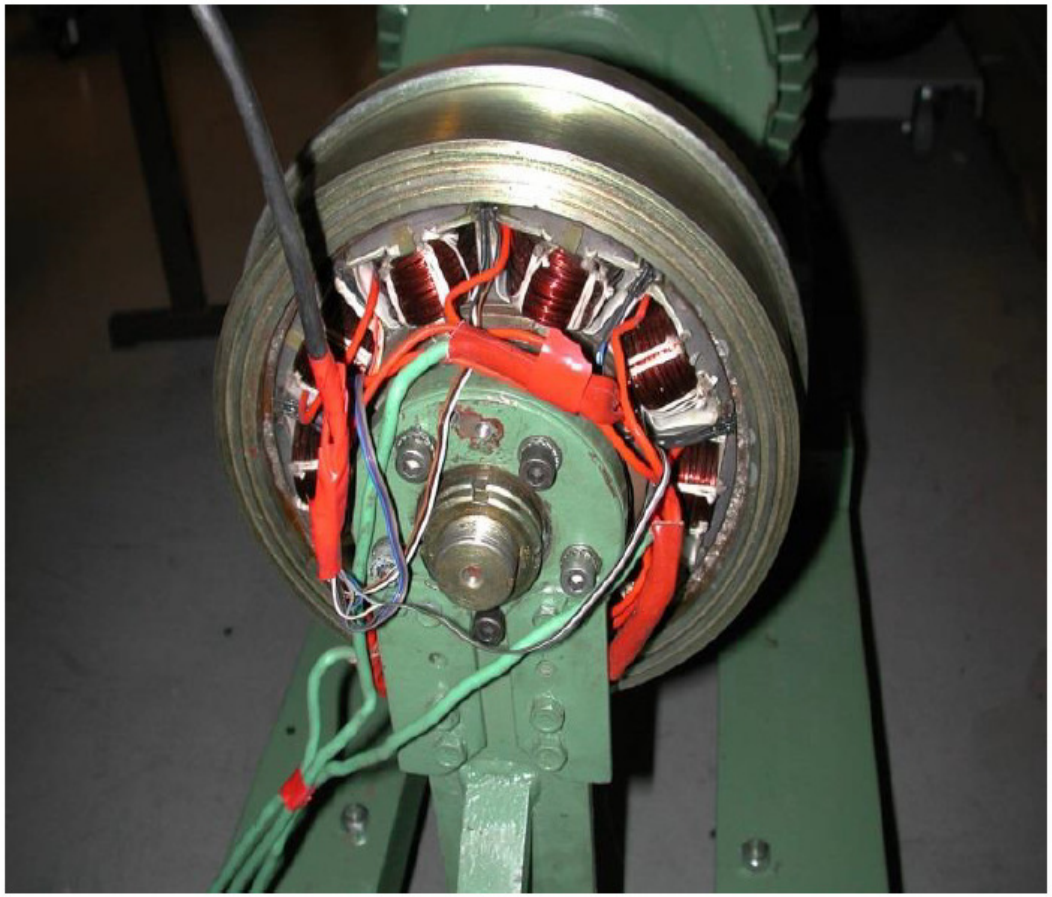
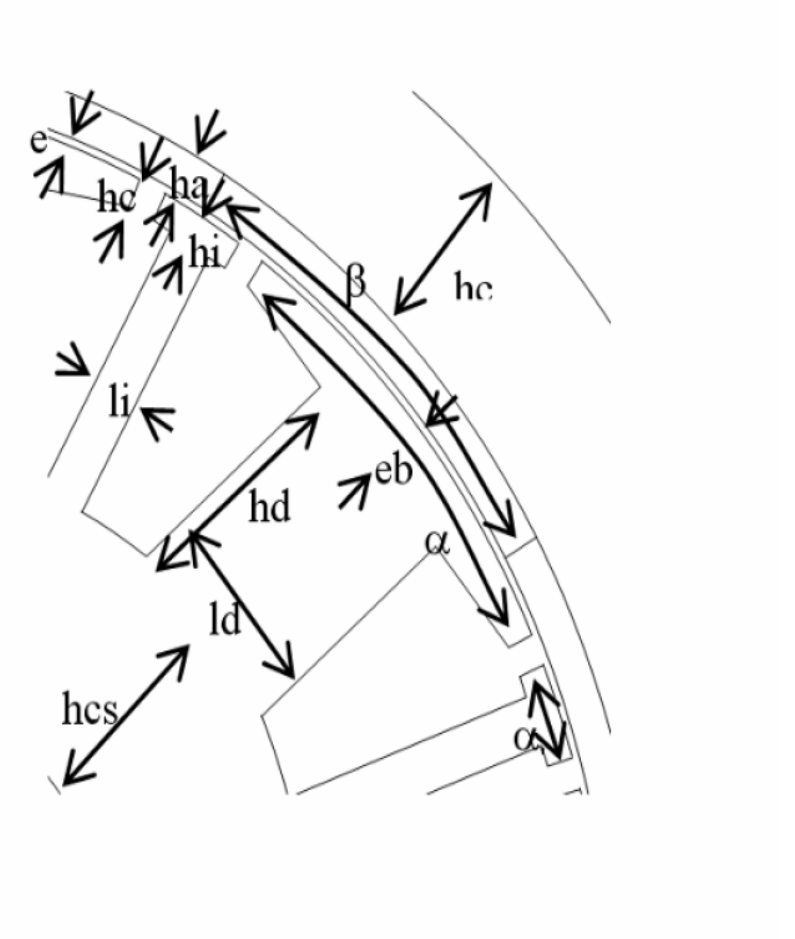
O problema do anteprojeto do motor

- Motor para carro movido a energia solar
- O problema deve elevar em conta, ao menos os seguintes aspectos:
  - Equacionamento de Conversão Eletromecânica
  - Equacionamento Magnético
  - Equacionamento do Acionamento Eletrônico
  - Equacionamento Térmico
- No anteprojeto, o equacionamento analítico é preferível ao de elementos finitos: menor precisão, mas maior flexibilidade

A EC Lille já o constriu



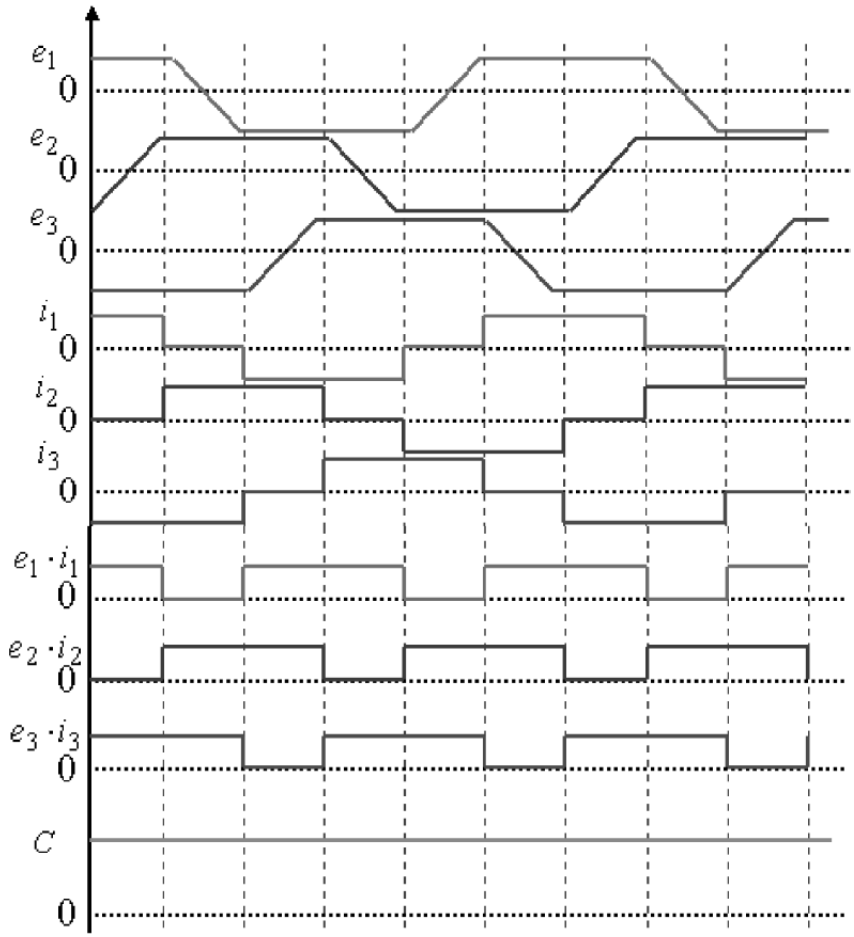
O motor em corte



O equacionamento eletromecânico

$$P_{\text{em}}(t) = C(t)\Omega(t) = \sum_{i=1}^{m} e_i(t) i_i(t)$$

$$C\Omega = 2EI$$



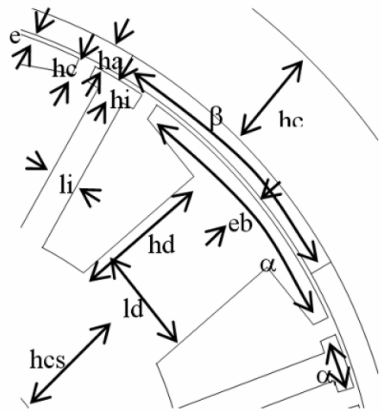
Lei de Lenz

$$E = \frac{n}{4}\frac{\mathrm{d}\phi}{\mathrm{d}\theta}\frac{\mathrm{d}\theta}{\mathrm{d}t} = \frac{n}{4}\frac{2\phi}{\pi/p}\Omega$$

S_e área no entreferro
S_p área do polo (ímã)

$$\phi = B_e S_p \qquad S_e = 2pS_p$$

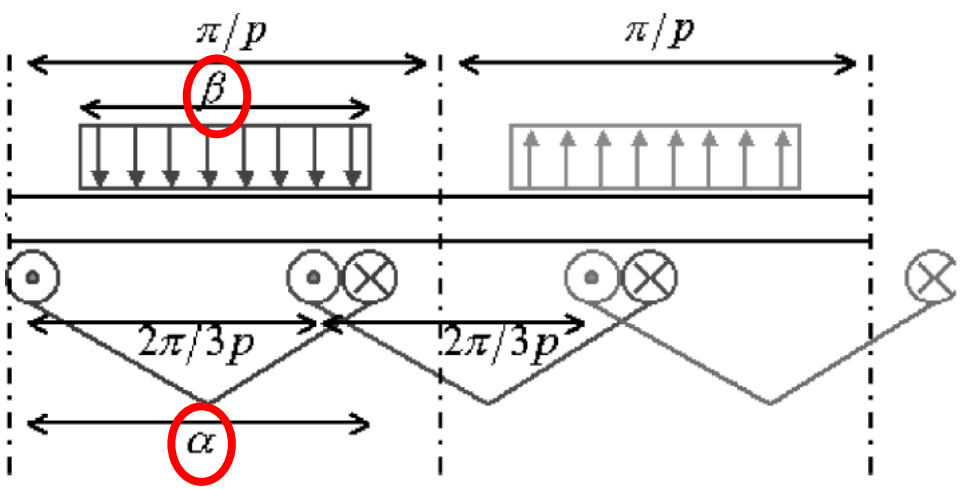
$$C\Omega = 2EI \quad \Rightarrow \quad C = nIB_e\frac{S_e}{2\pi}$$



Lei de Lenz

$$S_e = 2pS_p \qquad S_e = \pi D_s L_m$$

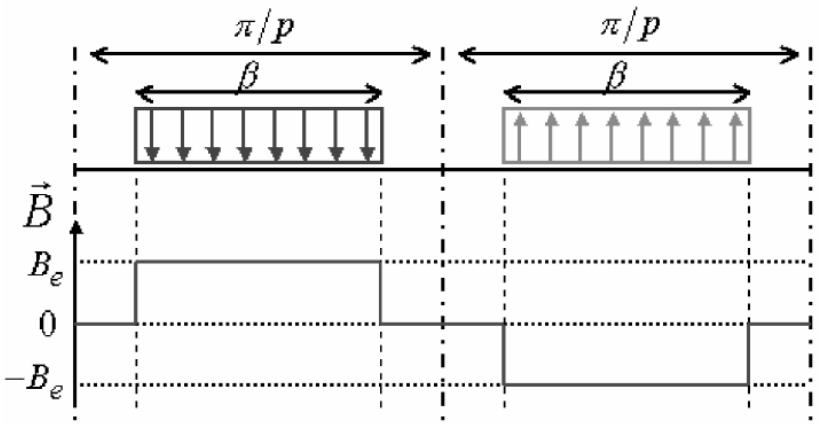
$$E = \frac{n}{4} B_e D_s L_m \Omega$$



Indução Magnética ao longo do Entreferro

- Condição para fluxo máximo:

$$\alpha = \frac{\pi}{p} \qquad \beta = \frac{\pi}{p}$$



Relações Geométricas

$$S_{\text{enc}} \approx hd\left[2\pi\left(\frac{D_{\text{s}}}{2} - eb\right) - \pi hd - N_{\text{e}}(li + ld)\right]$$

Senc = secção das ranhuras

$$S_{\text{enc}}k_r = \frac{3}{2}n\frac{I}{\delta}$$

k_r – fator de enchimento

$$D_{\text{ext}} = D_{\text{s}} + 2(e + ha + hcr)$$

D_{ext} Diâmetro Externo

$$D_{\text{int}} = D_{\text{s}} - 2(eb + hd + hcs)$$

D_{int} Diâmetro Interno

δ Densidade de corrente

$$hc = \frac{eb}{\cos(\alpha/2)} - \frac{D_{\text{s}}}{2}\left(\frac{1}{\cos(\alpha/2)} - 1\right)$$

hc altura da coroa

$$hi = \frac{D_{\text{s}}}{2}\left[1 - \cos\left(\frac{\alpha_i}{2}\right)\right] + hc\cos\left(\frac{\alpha_i}{2}\right)$$

hi altura do ímã

N_e Número de Ranhuras

Relações Geométricas

$$R_{\mathrm{tb}} \approx \frac{ld - li}{4} + \left(\frac{D_{\mathrm{s}}}{2} - eb - \frac{hd}{2}\right)\frac{\pi}{2N_{\mathrm{e}}}$$

Raio médio da “cabeça” de uma bobina

$$L_{\mathrm{ds}} \approx \frac{L_{\mathrm{m}}}{k_{\mathrm{foi}}} + \pi R_{\mathrm{tb}}$$

Comprimento aproximado de meia volta de uma bobina

Comprimento axial e Massas

$$L_{\mathrm{tot}} \approx \frac{L_{\mathrm{m}}}{k_{\mathrm{foi}}} + 2\left[\left(\frac{D_{\mathrm{s}}}{2} - eb - \frac{hd}{2}\right)\frac{\pi}{N_{\mathrm{e}}} - \frac{li}{2}\right]$$

Massa

$$M_{\mathrm{a}} = d_{\mathrm{a}} p \beta ha\left[ha + 2\left(\frac{D_{\mathrm{s}}}{2} + e\right)\right] L_{\mathrm{m}} r_{\mathrm{rs}}$$

Ímãs

$$M_{\mathrm{cr}} = d_{\mathrm{cr}} \pi hcr\left[hcr + 2\left(\frac{D_{\mathrm{s}}}{2} + e + ha\right)\right] L_{\mathrm{m}} r_{\mathrm{rs}}$$

Coroa do rotor

$$M_{\mathrm{cs}} = d_{\mathrm{t}} \pi hcs\left[2\left(\frac{D_{\mathrm{s}}}{2} - eb - hd\right) - hcs\right] L_{\mathrm{m}}$$

Coroa do estator

$$M_{\mathrm{ds}} \approx d_{\mathrm{t}} N_{\mathrm{e}}\left[(ld + li)hd + \left(\alpha\frac{eb + hc}{2} + \alpha_i\frac{hi + hc}{2}\right)\frac{D_{\mathrm{s}}}{2}\right] L_{\mathrm{m}}$$

Dentes do estator

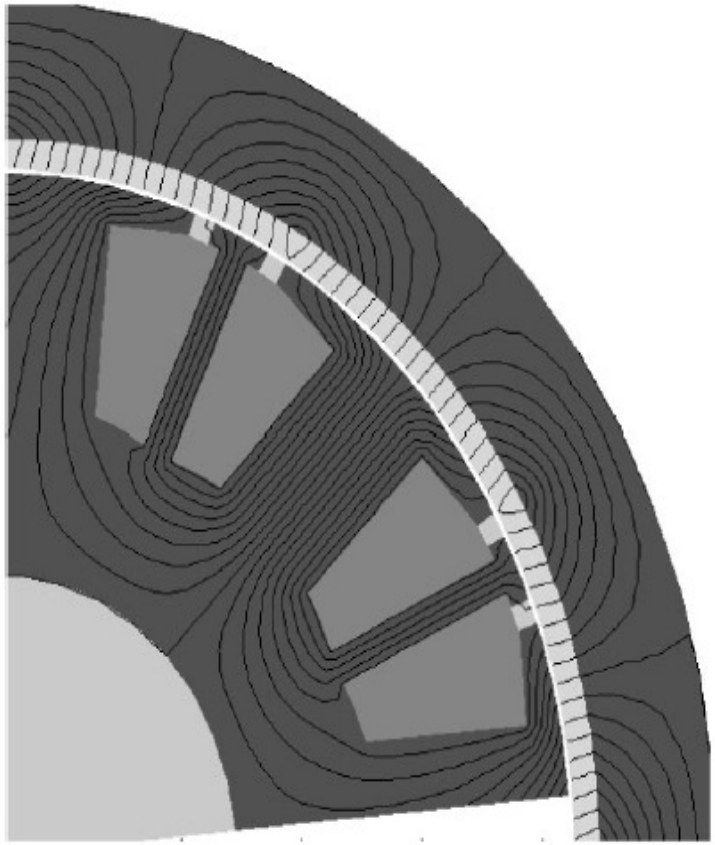
$$M_{\mathrm{cu}} = d_{\mathrm{cu}}\frac{3}{2} n \frac{I}{\delta} L_{\mathrm{ds}}$$

Cobre

$$M_{\mathrm{tot}} = M_{\mathrm{a}} + M_{\mathrm{cr}} + M_{\mathrm{cs}} + M_{\mathrm{ds}} + M_{\mathrm{cu}}$$

Massa Total

Linhas de Campo por Elementos Finitos (motor sem carga mecânica no eixo)

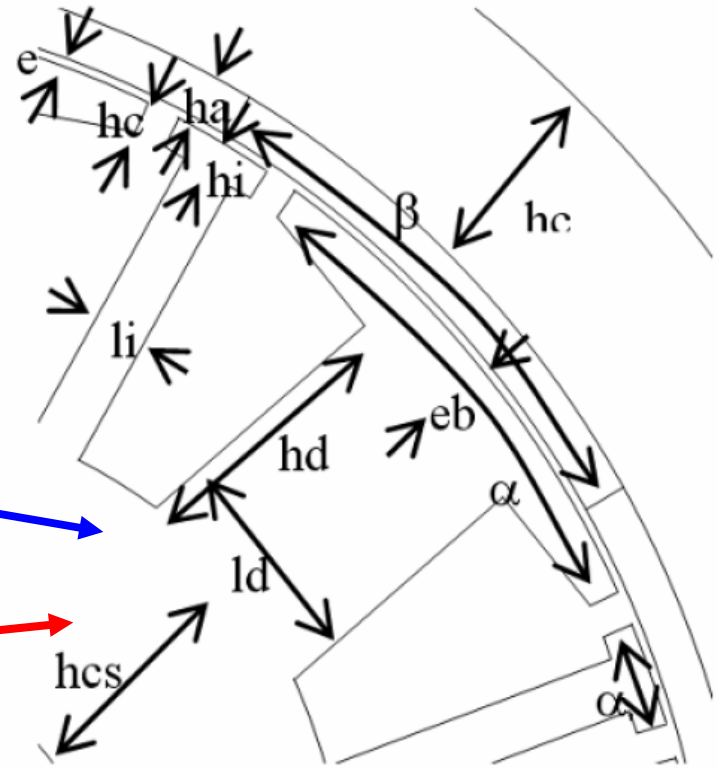


Algumas relações de Conservação do Fluxo

$$B_d li = B_e \alpha_i \frac{D_s}{2}$$

$$B_d ld = B_e \alpha \frac{D_s}{2}$$

$$\frac{1}{2} B_d ld = B_{cs} hcs$$



Lei de Ampère

- Em vazio: $$\left(\frac{B_{\mathrm{a}} - B_{\mathrm{r}}(1 + \alpha_{\mathrm{a}} T_{\mathrm{a}})}{\mu_{\mathrm{a}}}\right) ha + B_{\mathrm{e}} e = 0$$

- Em carga:

$$\left(\frac{B_{\mathrm{c}} - B_{\mathrm{r}}(1 + \alpha_{\mathrm{a}} T_{\mathrm{a}})}{\mu_{\mathrm{o}} \mu_{\mathrm{a}}}\right) ha + \frac{n I_{\max}}{4p} \quad + \frac{B_{\mathrm{c}}}{\mu_{\mathrm{o}}} \frac{\beta}{\alpha} \left(1 + \frac{2e}{D_{\mathrm{s}}}\right) r_{\mathrm{rs}} k_{\mathrm{fui}} e = 0$$

Calculo de perdas

- Resistência

$$R_{\mathrm{ph}} = \rho_{\mathrm{cu}}(1 + \alpha_{\mathrm{cu}} T_{\mathrm{cu}}) \frac{n}{2} L_{\mathrm{ds}} \frac{\delta}{I}$$

- Perdas Joule

$$P_j = 2R_{\mathrm{ph}} I^2$$

Cálculo de Perdas

- Frequência:

$$f = \frac{p\Omega}{2\pi}$$

- Perdas no Ferro:

$$P_{\mathrm{f}} = q_{\mathrm{t}} \left(\frac{f}{f_{\mathrm{t}}}\right)^{1.5} \left[M_{\mathrm{cs}} \left(\frac{B_{\mathrm{cs}}}{B_{\mathrm{t}}}\right)^{2} + M_{\mathrm{ds}} \left(\frac{B_{\mathrm{d}}}{B_{\mathrm{t}}}\right)^{2} \right]$$

Modelo Térmico

$$T_{\mathrm{a}} = T_{\mathrm{cu}}$$

$$S_{\mathrm{ext}} = \frac{\pi}{2} D_{\mathrm{ext}}^2 + \pi D_{\mathrm{ext}} L_{\mathrm{tot}}$$

$$T_{\mathrm{cu}} = T_{\mathrm{ext}} + \frac{P_{\mathrm{j}} + P_{\mathrm{f}} + P_{\mathrm{m}}}{h S_{\mathrm{ext}}}$$

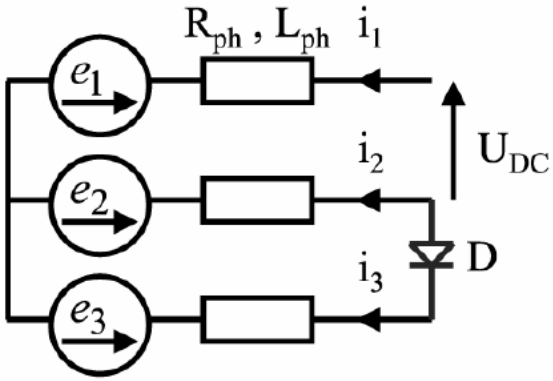
Modelo do Conversor Eletrônico

- ➔ Comutação é relevante

$$U_{DC} = R_{ph} i_1 + L_{ph} \frac{di_1}{dt} + e_1 - R_{ph} i_2 - L_{ph} \frac{di_2}{dt} + e_2$$

$$0 = R_{ph} i_2 + L_{ph} \frac{di_2}{dt} + e_2 - R_{ph} i_3 - L_{ph} \frac{di_3}{dt} + e_3$$

$$i_1 + i_2 + i_3 = 0$$



$$t_1 = -\frac{L_{ph}}{R_{ph}} \ln\left[1 - \frac{3R_{ph} I}{U_{DC}(2 - 1/k_v)}\right]$$

$k_v = U_{DC}/2E$

O problema de otimização

Maximizar η

$$150\,\mathrm{mm} \leq D_\mathrm{s} \leq 330\,\mathrm{mm}, \quad 0.9\,\mathrm{T} \leq B_\mathrm{d} \leq 1.8\,\mathrm{T}$$

$$2.0\,\mathrm{A/mm^2} \leq \delta \leq 5.0\,\mathrm{A/mm^2}, \quad 0.5\,\mathrm{T} \leq B_\mathrm{e} \leq 0.76\,\mathrm{T},$$

$$0.6\,\mathrm{T} \leq B_\mathrm{cs} \leq 1.6\,\mathrm{T}$$

$$M_\mathrm{tot} \leq 15\,\mathrm{kg}, \quad D_\mathrm{ext} \leq 340\,\mathrm{mm}, \quad D_\mathrm{int} \geq 76\,\mathrm{mm},$$

$$I_\mathrm{max} \geq 125\,\mathrm{A}, \quad \mathrm{discr}(D_\mathrm{s}, \delta, B_\mathrm{d}, B_\mathrm{e}) \geq 0, \quad T_\mathrm{a} \leq 120^\circ\mathrm{C}$$

Restrições

http://optimisation.l2ep.ec-lille.fr/benchmarks/brushless_DC_wheel_motor/brushless_DC_wheel_motor_benchmark.htm