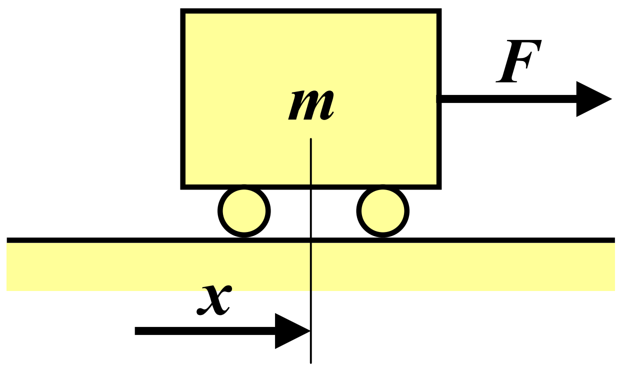
Control for DUMMIES

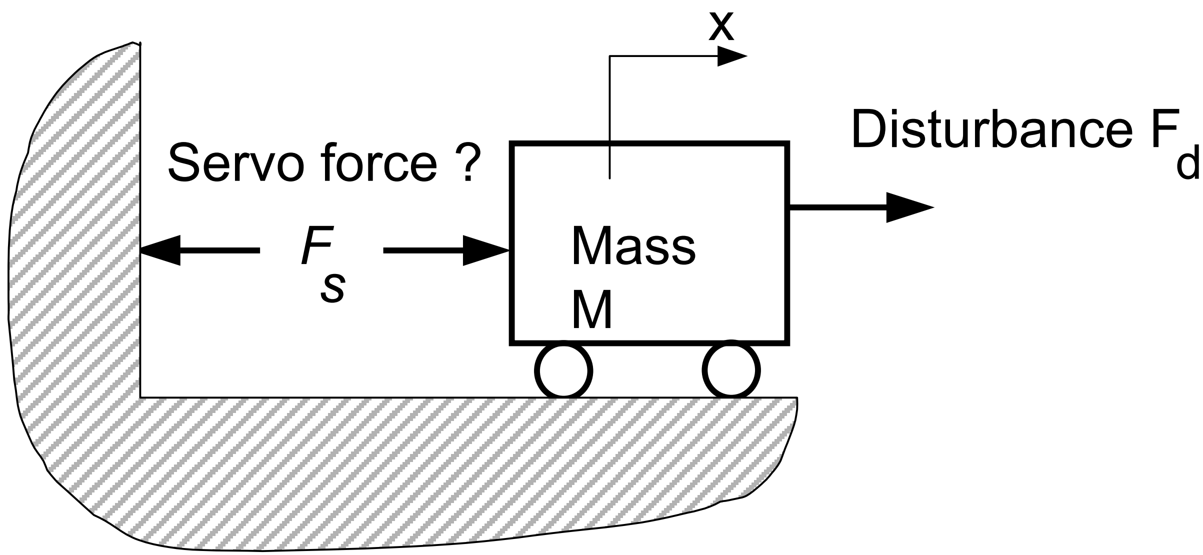
Maarten Steinbuch
Dept. Mechanical Engineering
Control Systems Technology Group
TU/e

Motion Systems

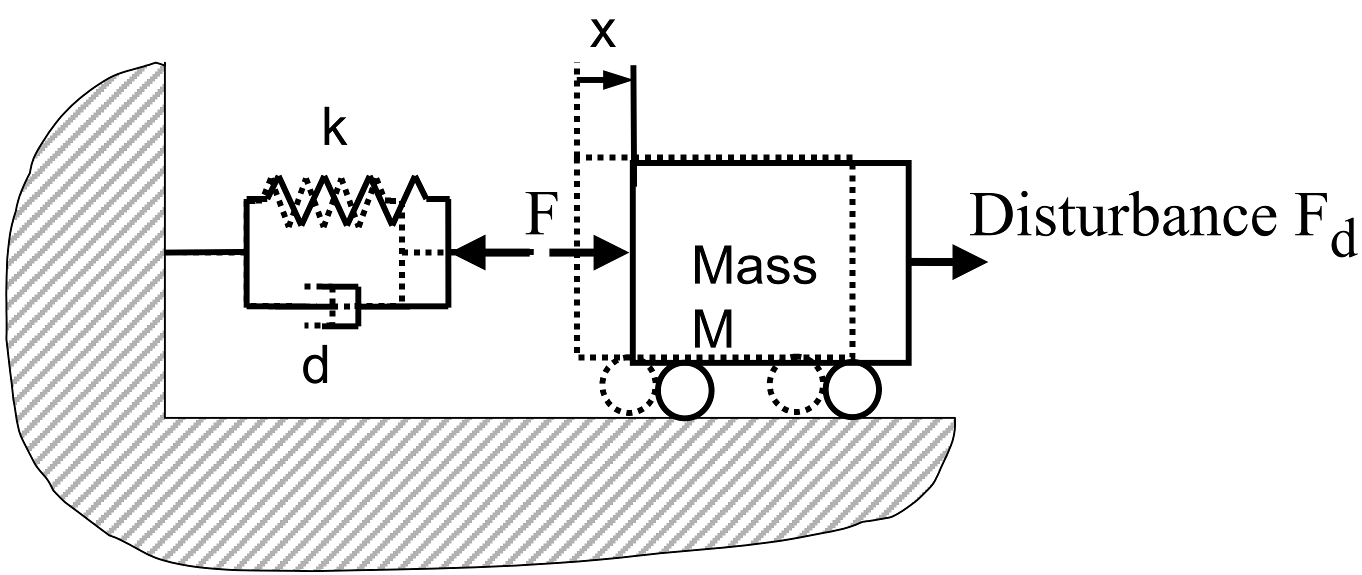


- **Introduction**
- **Timedomain tuning**
- **Frequency domain & stability**
- **Filters**
- **Feedforward**
- **Servo-oriented design of mechanical systems**

2. Time Domain Tuning


x
Disturbance F
d
Servo force ?
F
s
Mass
M

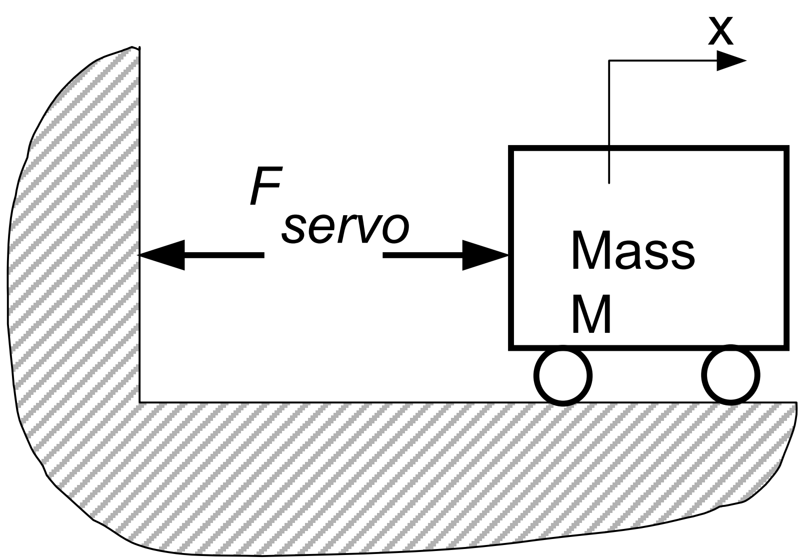
Mechanical solution:



Force spring-damper: $F = -k \cdot x - d \cdot \dot{x}$

Eigenfrequency: $f = \frac{1}{2\pi}\sqrt{\frac{k}{M}}$

Servo analogon:



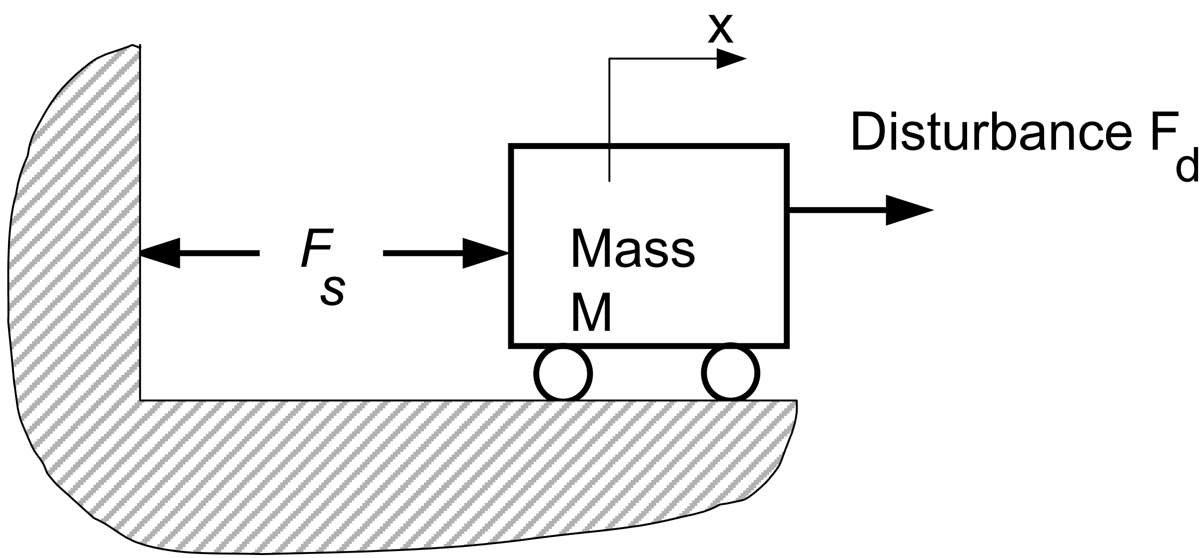
Servo Force: $F_s = -k_p \cdot x - k_v \cdot \dot{x}$

Eigenfrequency: $f = \frac{1}{2\pi}\sqrt{\frac{k_p}{M}}$

k_p : servo stiffness

k_v : servo damping

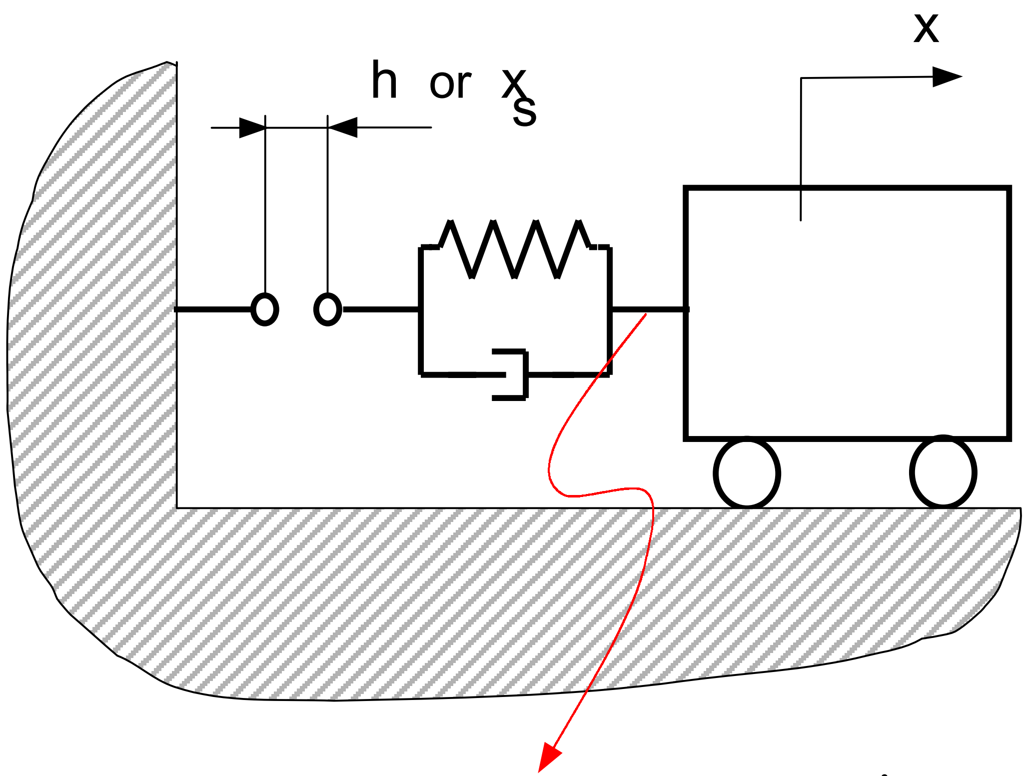
Example:



Slide: mass = 5 kg
Required accuracy 10 μm at all times
Disturbance (f.e. friction) = 3 N

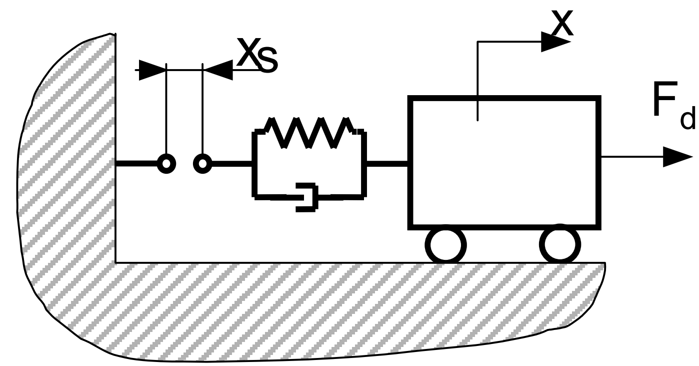
1. Required servo stiffness?
2. Eigenfrequency?

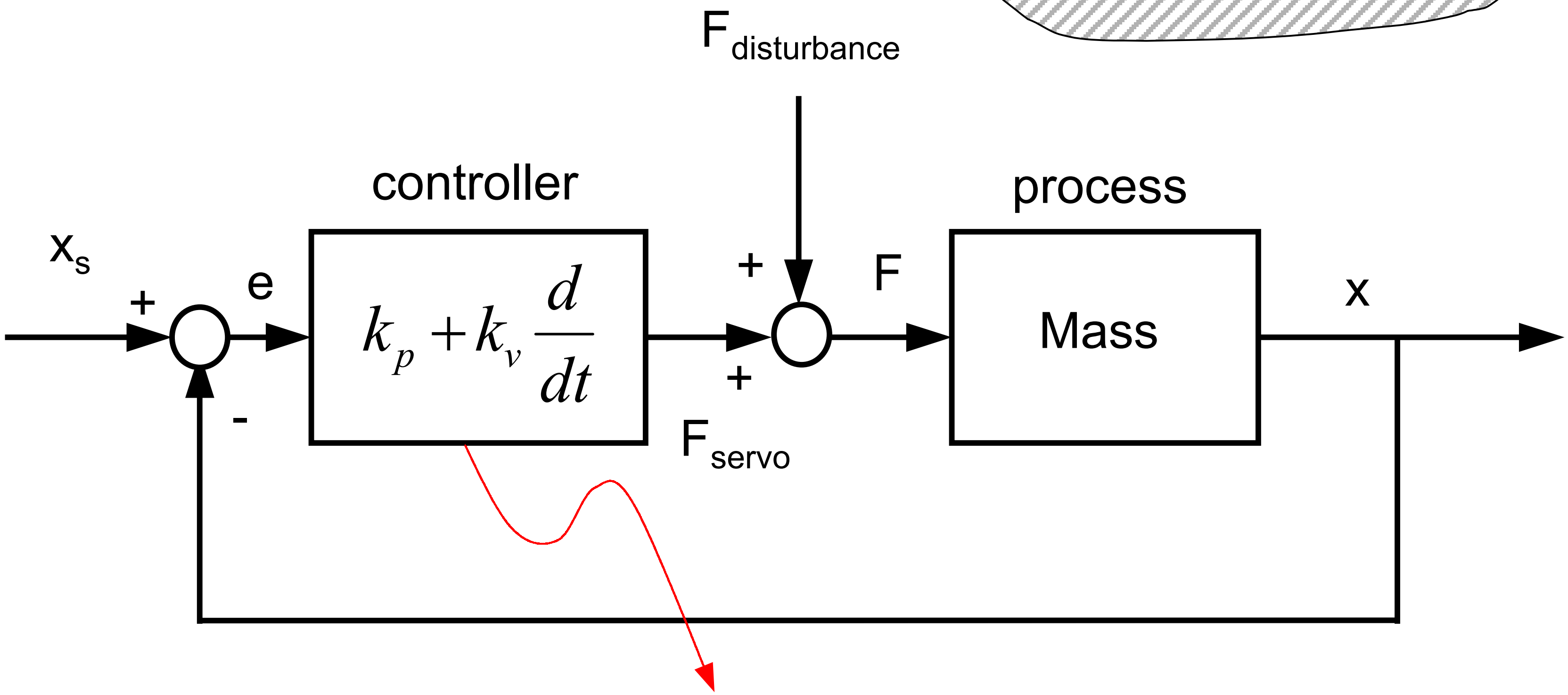
How to move to / follow a setpoint:



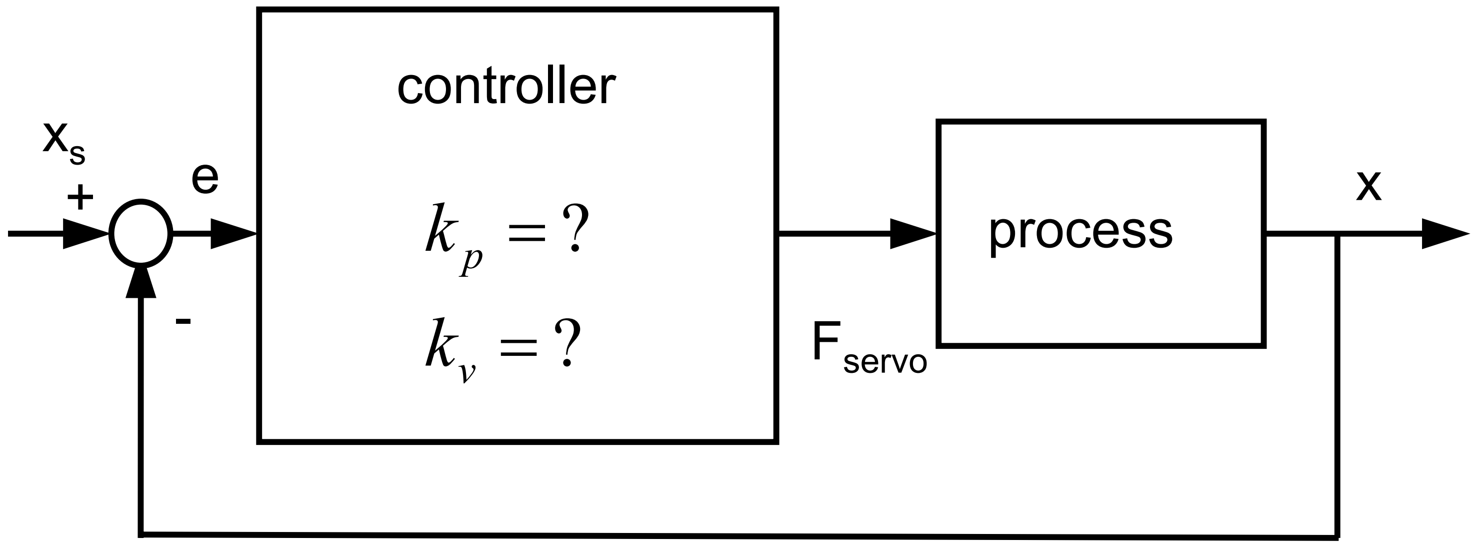
Spring-damper: $F = k(h - x) + d(\dot{h} - \dot{x})$

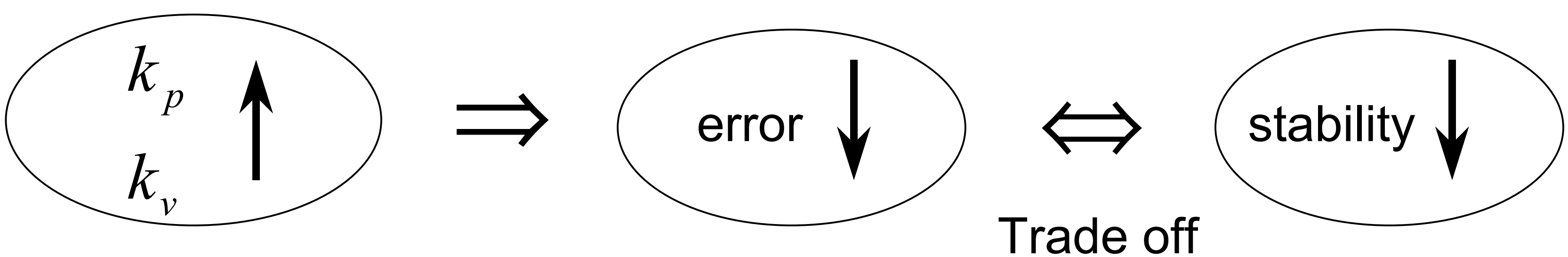
Controller $F_s = k_p(x_s - x) + k_v(\dot{x}_s - \dot{x})$





K_p/k_v-controller or PD-controller

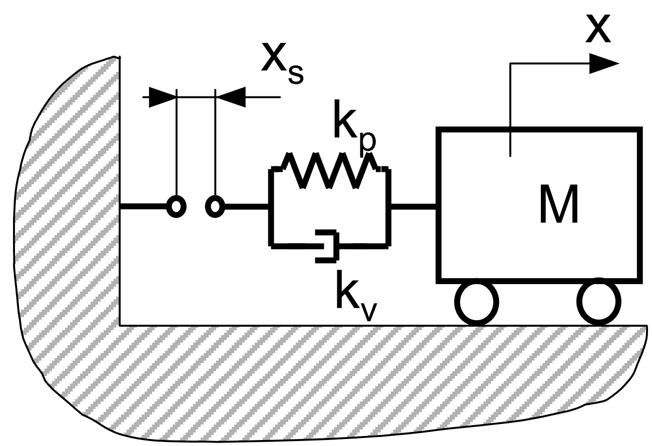

controller
x_s
+
e
$k_p = ?$
$k_v = ?$
-
F_{servo}
process
x


k_p
k_v
$\Rightarrow$
error
$\Leftrightarrow$
stability
Trade off

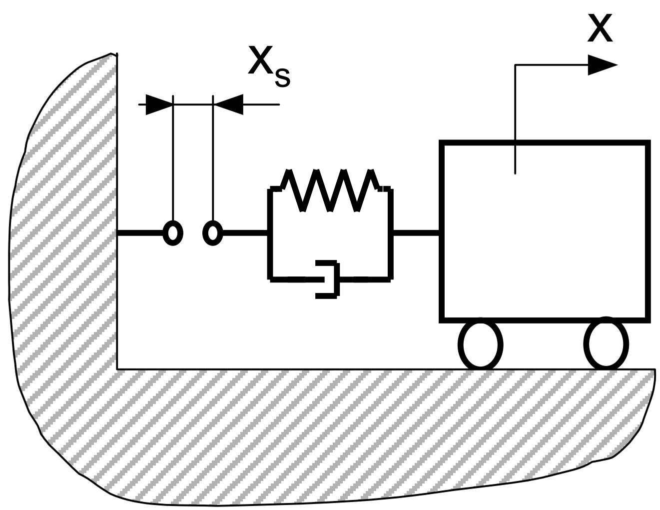
Concluding remarks time domain tuning

A control system, consisting of only a single mass m and a k_p/k_v controller (as depicted below), is *always* stable.
k_p will act as a spring; k_v will act as a damper

As a result of this: when a control system is unstable, it *cannot* be a pure single mass + k_p/k_v controller
(With positive parameters m, k_p and k_v)



Setpoints:

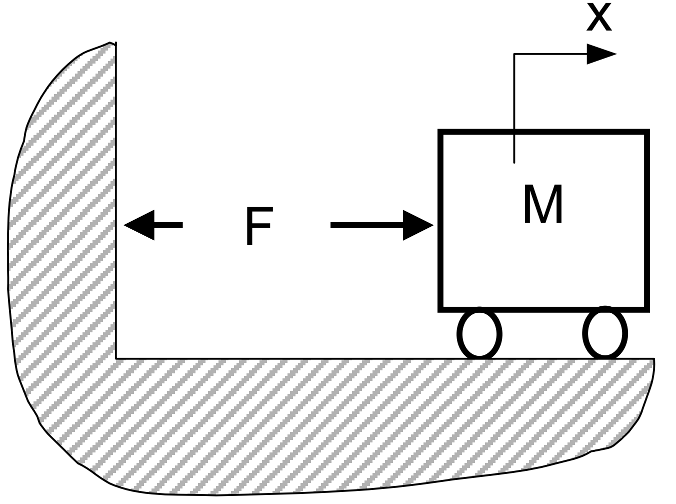


What should x_s look like as a function of time, when moving the mass?

(first order, second order, third order,….?)

Apply a force *F* (step profile):

$$F(t) = M\ddot{x}(t)$$



$$\Downarrow$$

x(t) is second order, when *F* constant

Second order profile requires following information:

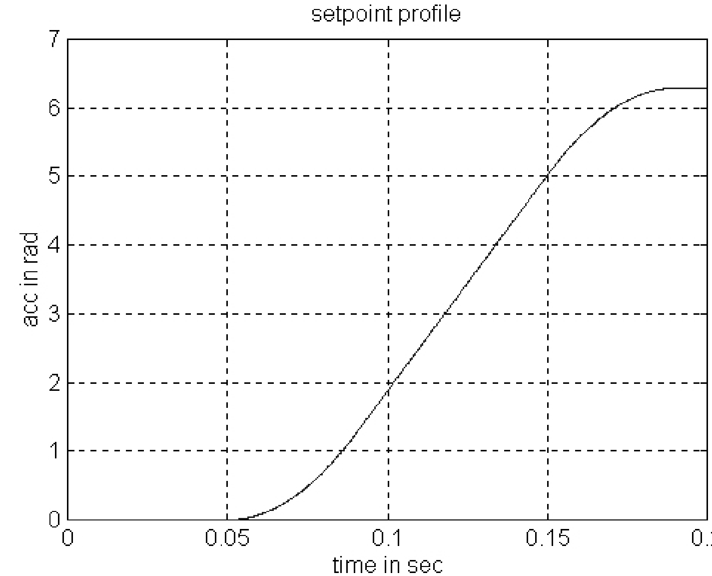
- maximum acceleration
- maximum velocity
- travel distance

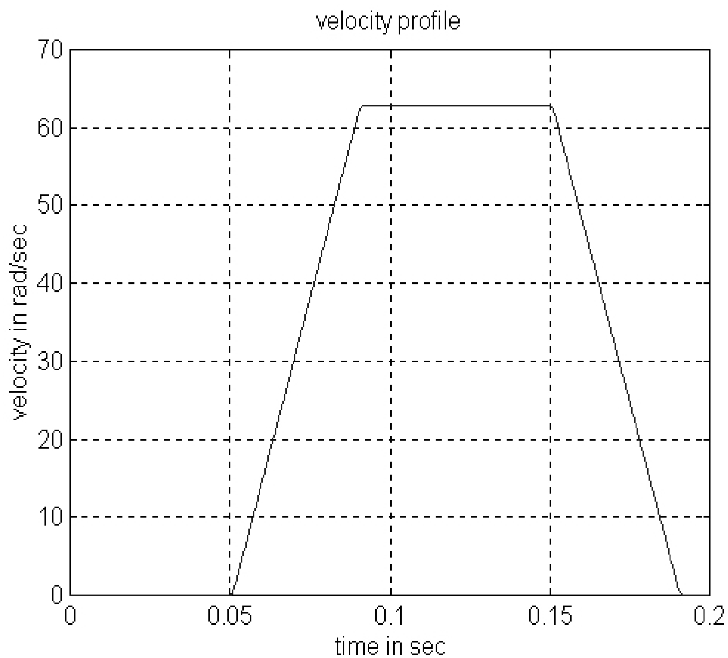
Example

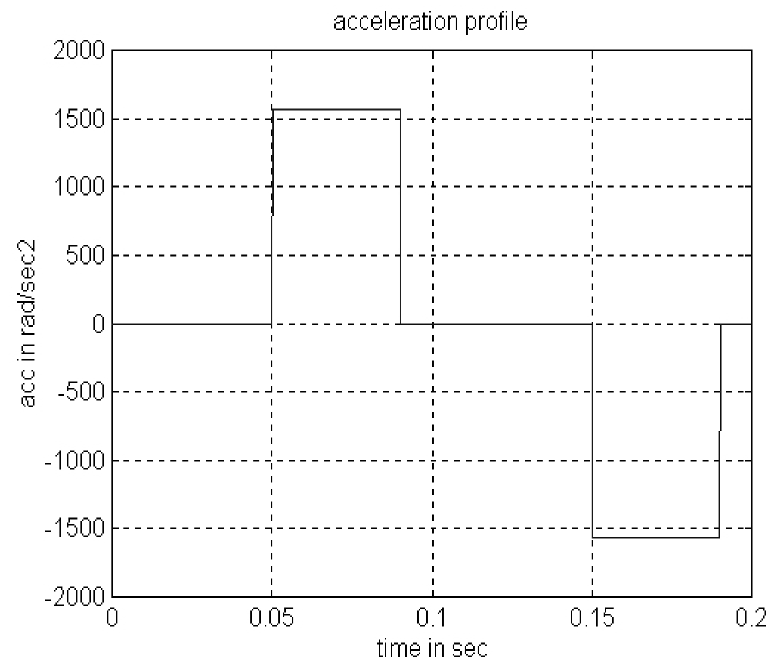
$$Pos = 2\pi \approx 6.3 rad$$

$$Vel_{\max} = 20\pi \approx 63 rad/\sec$$

$$Acc_{\max} = 500\pi \approx 1.6e3 rad/\sec^2$$







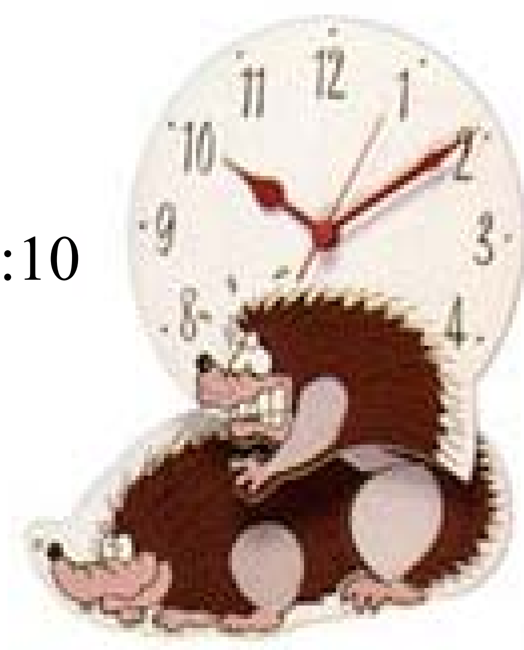
3 Frequency domain

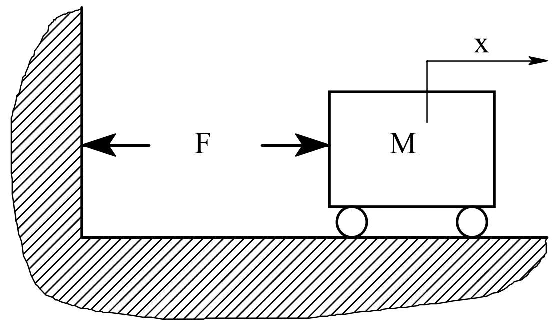
Time domain:

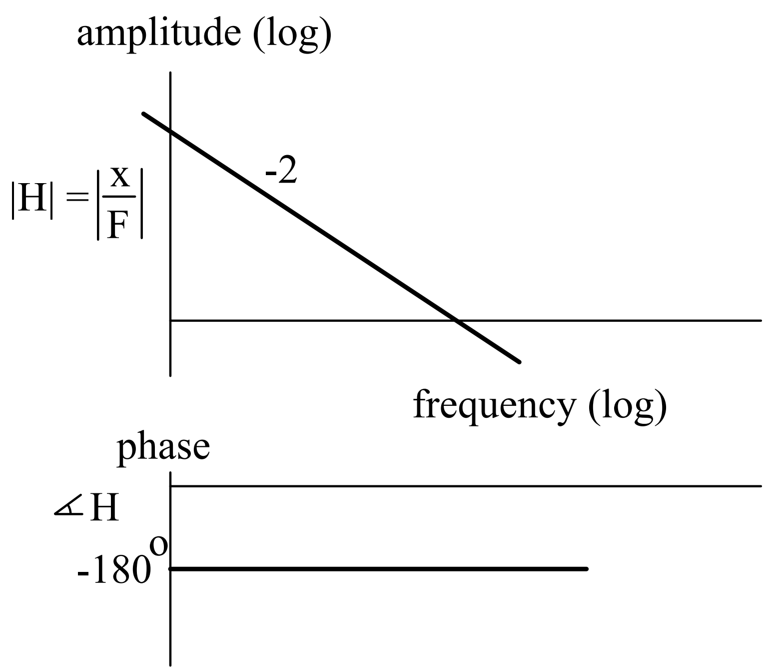
Monday and Thursday at 22:10

Frequency domain:

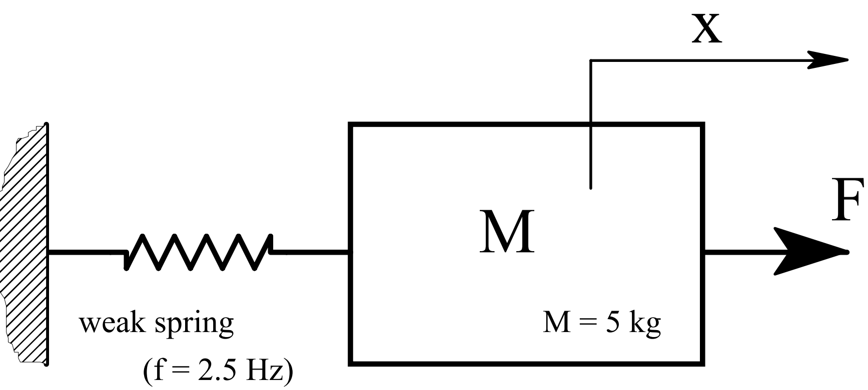
twice a week

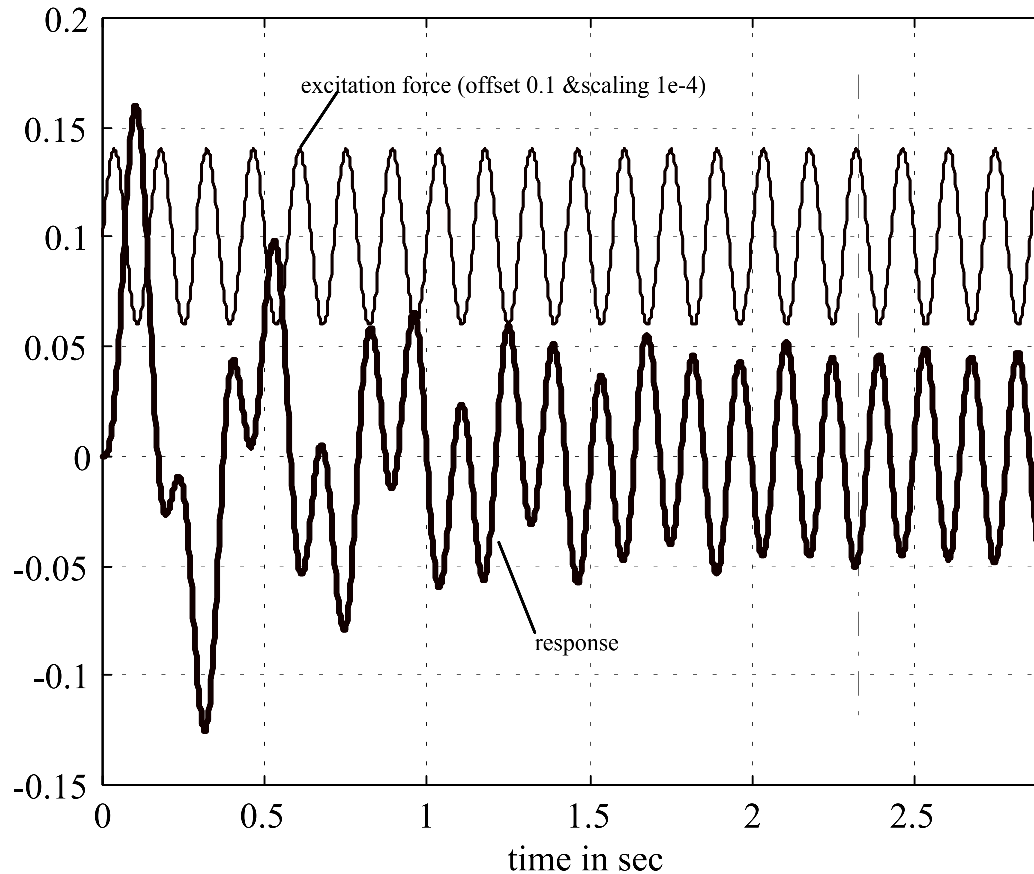



x
F
M


amplitude (log)
-2
|H| = |x/F|
frequency (log)
phase
∡H
-180°

going from Time-domain to the Frequency-domain



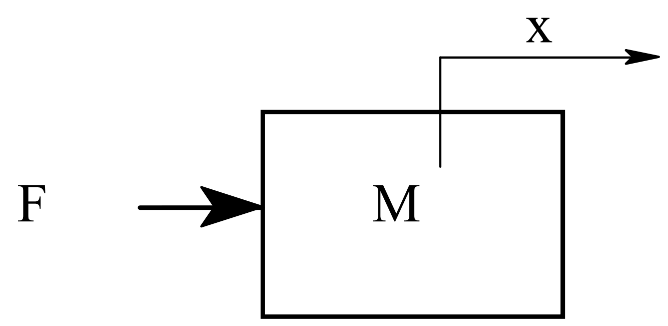


$$F(t) = 400\sin(2\pi 7t)$$

$$\left|H(7\,Hz)\right| \approx 0.045/400 = 1e-4\ m/N$$

$$\angle H(7\,Hz) \approx -180^{0}$$

finding a solution of the equation of motion:



$$F = M\ddot{x}$$

choose input:

$$F = \hat{F}\sin(\omega t)$$

then:

$$x = \hat{x}\sin(\omega t + \varphi) \qquad \hat{x} = ?;\varphi = ?$$

solution:

$$x(t) = -\frac{\hat{F}}{M\omega^2}\sin(\omega t) + c_1 t + c_2$$

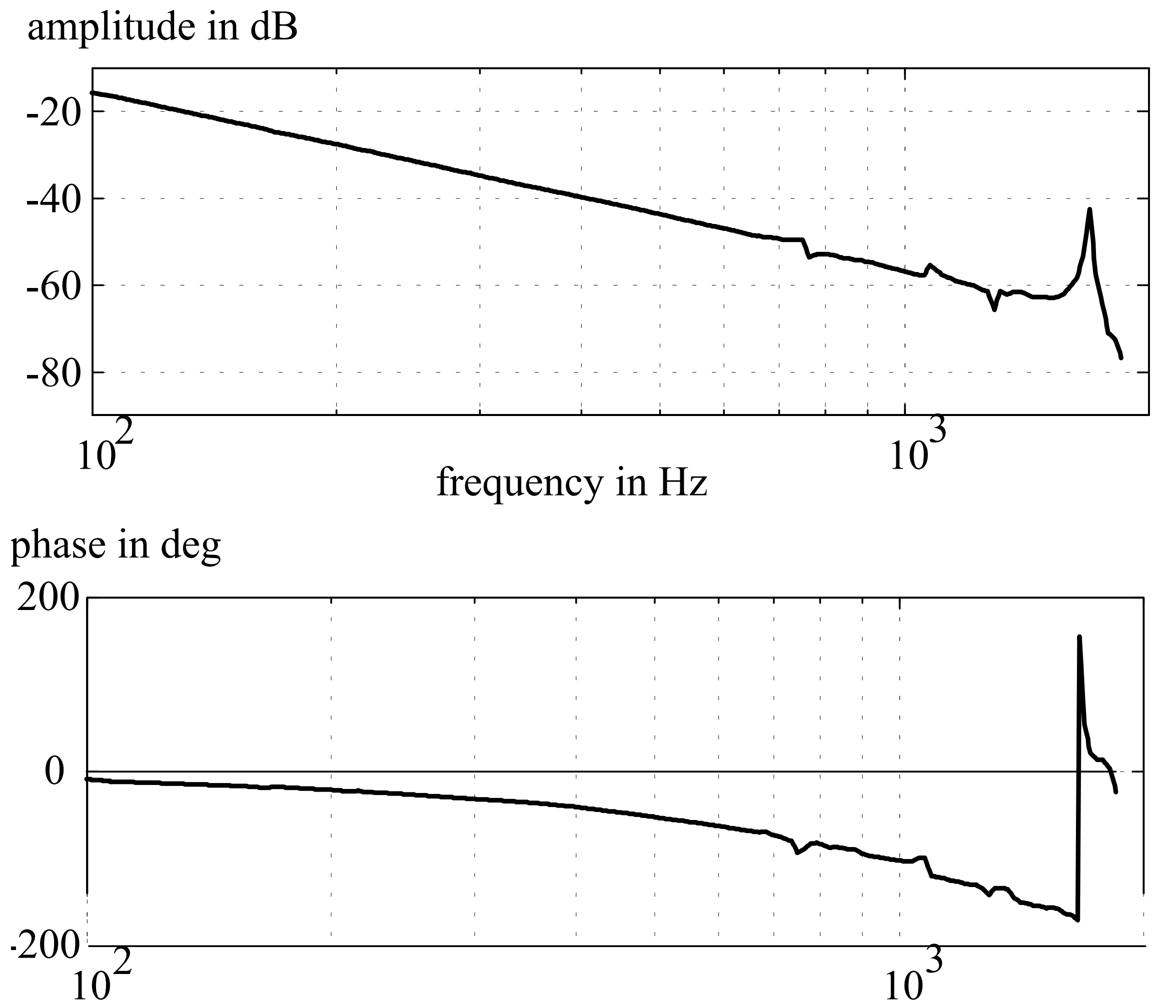
$$\boxed{H = \frac{x}{F} = -\frac{1}{M\omega^2}}$$

$$|H| = \frac{\hat{x}}{\hat{F}} = \frac{1}{M\omega^2}$$

$$\log(|\mathrm{H}|) = \log\frac{1}{\mathrm{M}} - 2\log\omega$$

$$\angle H = \varphi = -180^\circ$$

measurement mechanics stage

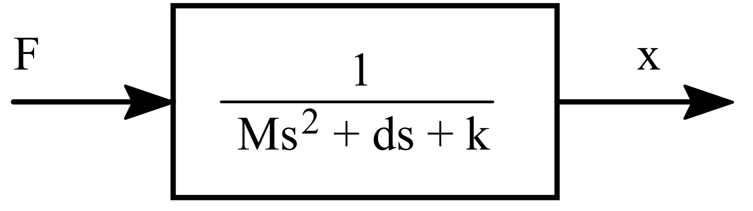


Derivation of transfer function

- make a model of the dynamics: differential equations
- substitute $s=d./dt$
- rearrange the equations and get the transfer function e.g. $H(s)$
- for sinusoids make a 'Bode' plot using $s=j\omega$

Transfer function:

$$H(s) = \frac{x(s)}{F(s)} = \frac{1}{Ms^2 + ds + k}$$



consider sinusoidal signals ('Euler notation'):

$$x(t) = \hat{x}(\cos\omega t + j\sin\omega t) = \hat{x}e^{j\omega t}$$

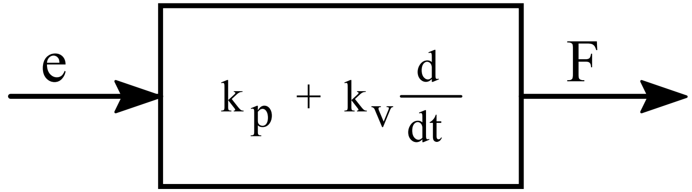
$$\dot{x}(t) = \omega\hat{x}(-\sin\omega t + j\cos\omega t) = j\omega\hat{x}e^{j\omega t}$$

apparently: $s = j\omega$ for sinusoidal signals

Frequency Response Function:

$$s \rightarrow j\omega$$

$$H(j\omega) = \frac{1}{-M\omega^2 + jd\omega + k}$$



$$F = k_p e + k_v \dot{e}$$

$$F(s) = (k_p + k_v s)e(s)$$

transfer function:

$$C(s) = \frac{F}{e}(s) = (k_p + k_v s)$$

frequency response:

$$C = k_p + jk_v\omega$$

Amplitude: $|C| = \sqrt{k_p^2 + k_v^2\omega^2}$

$$\omega \to 0 \quad \Rightarrow \quad |C| \to k_p$$

$$\angle C \to 0°$$

$$\omega \to \infty \quad \Rightarrow \quad |C| \to k_v\omega$$

$$\angle C \to 90°$$

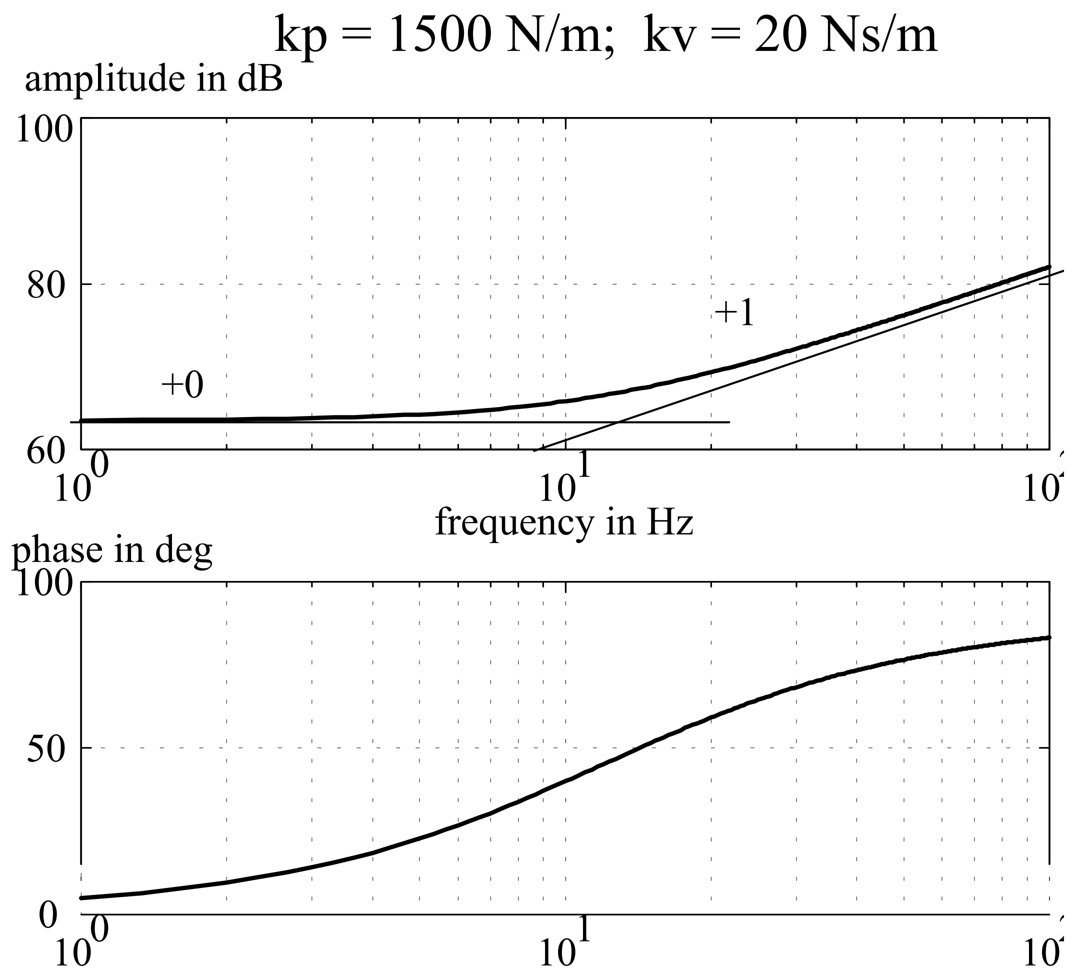
$$\omega \to \infty \qquad \log(|C|) = \log k_v + \log\omega$$

break point:

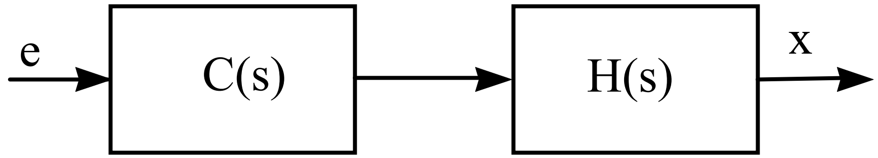
$$\log k_p = \log k_v + \log\omega$$

$$\omega = \frac{k_p}{k_v}$$

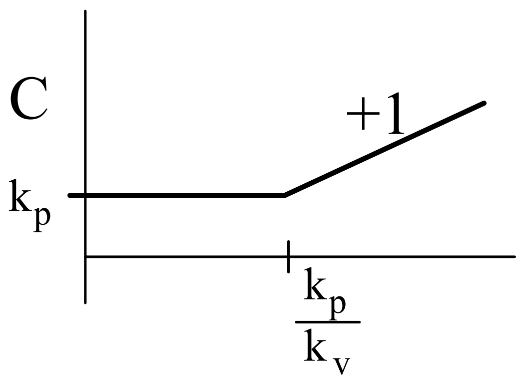
Bode plot of the PD-controller:

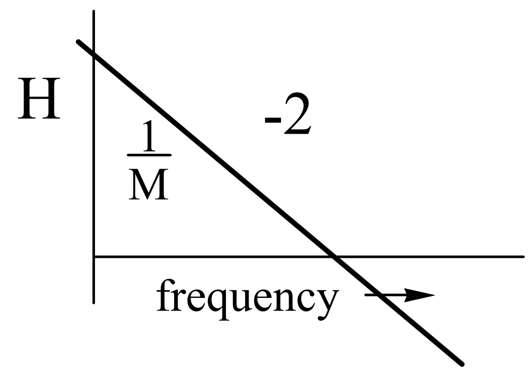


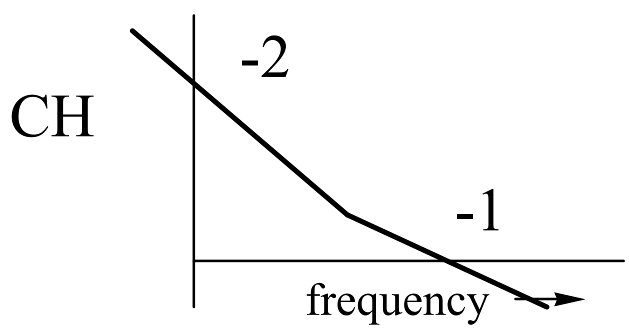
Block manipulation

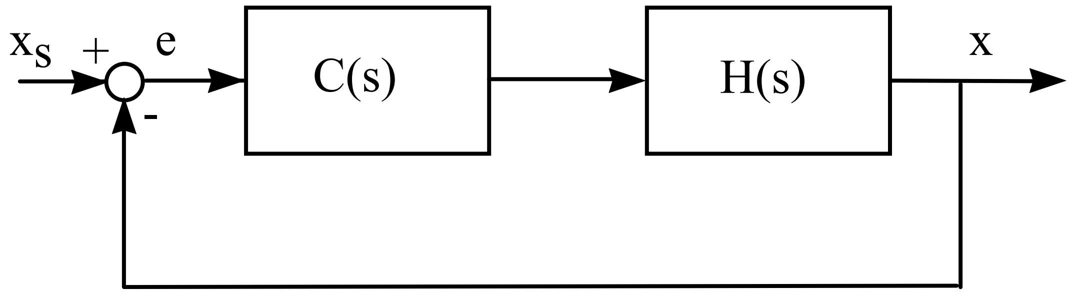


$$\frac{x(s)}{e(s)} = C(s)H(s)$$



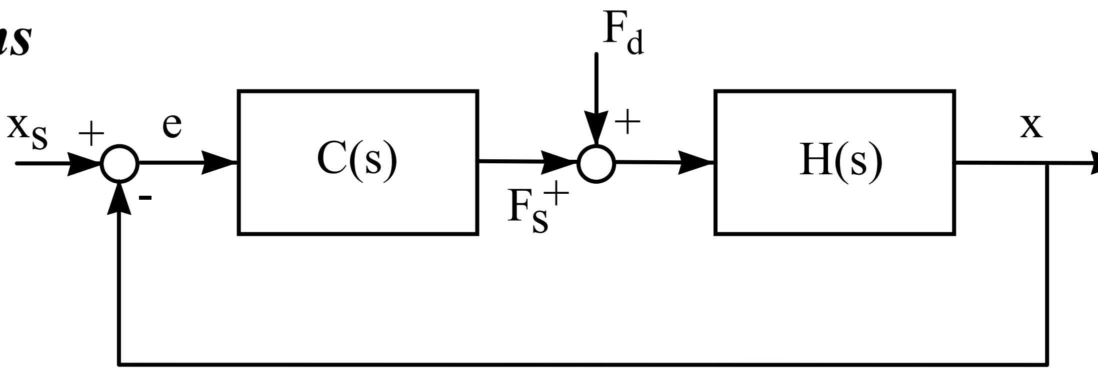






$$H_c = \frac{x}{x_s} = \frac{CH}{1 + CH}$$

Four important transfer functions



1. open loop:

$$H_o(s) = C(s)H(s)$$

2. closed loop:

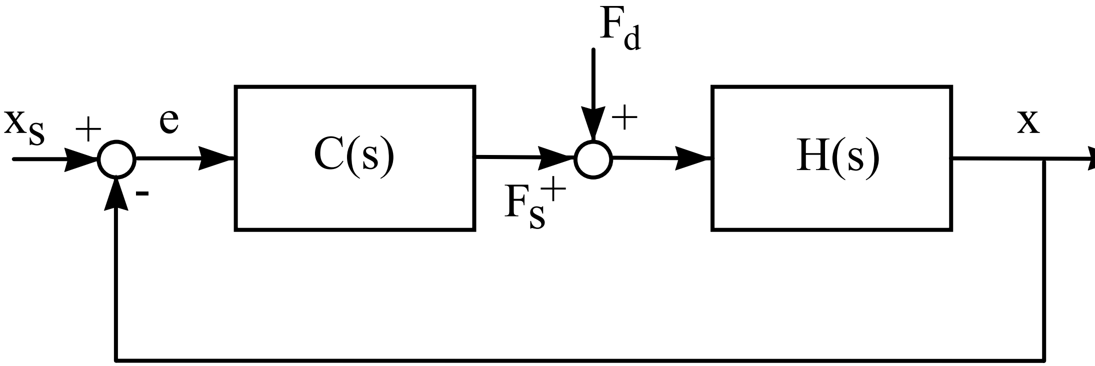
$$H_c(s) = \frac{x}{x_s}(s) = \frac{C(s)H(s)}{1 + C(s)H(s)}$$

3. sensitivity:

$$S(s) = \frac{e}{x_s}(s) = \frac{1}{1 + C(s)H(s)}$$

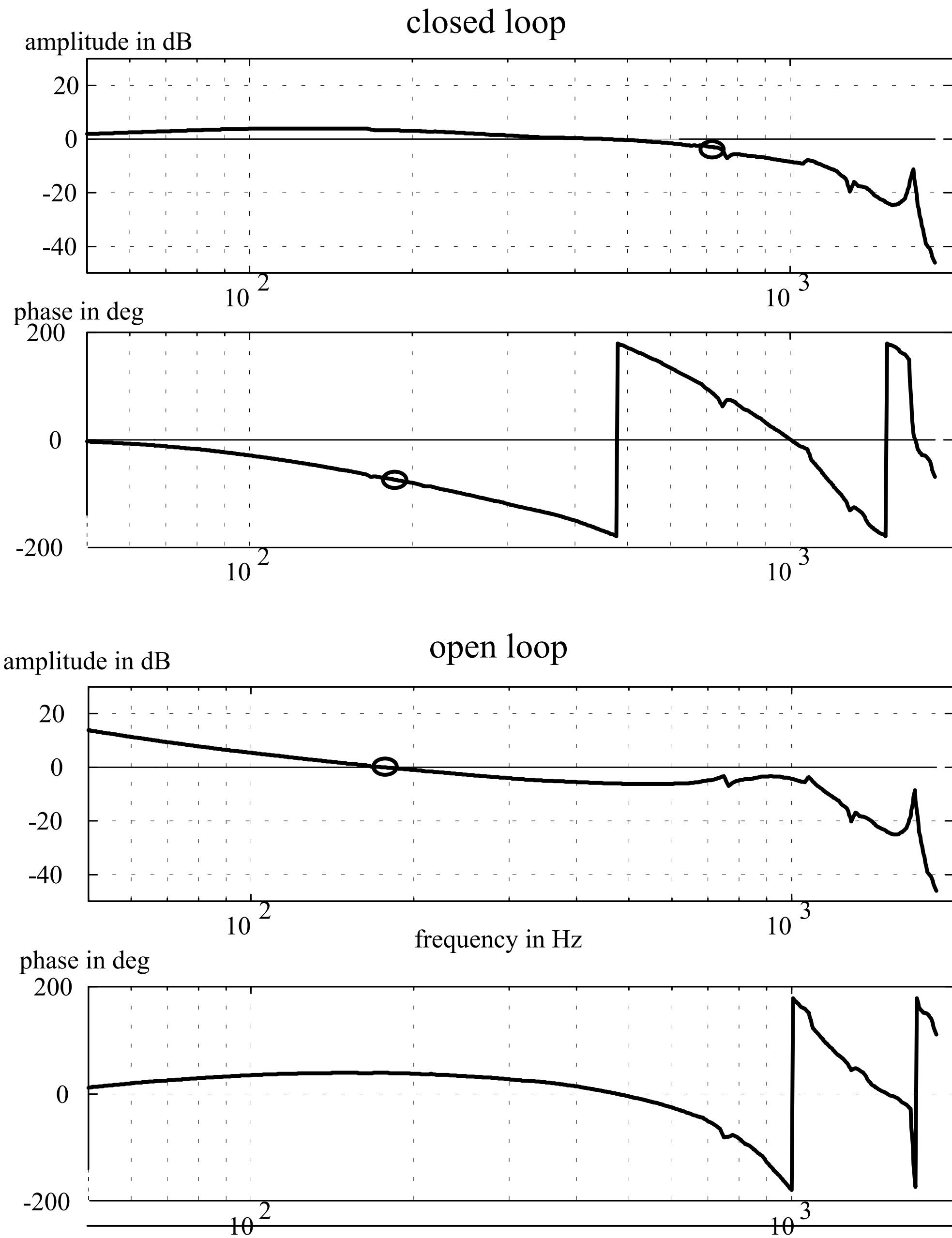
4. process sensitivity:

$$H_{ps}(s) = \frac{x}{F_d}(s) = \frac{H(s)}{1 + C(s)H(s)}$$



Derivation of closed-loop transfer functions:

- start with the output variable of interest
- go back in the loop, against the signal flow
- write down the relations, using intermediate variables
- stop when arrived at the relevant input variable
- eliminate the intermediate variables

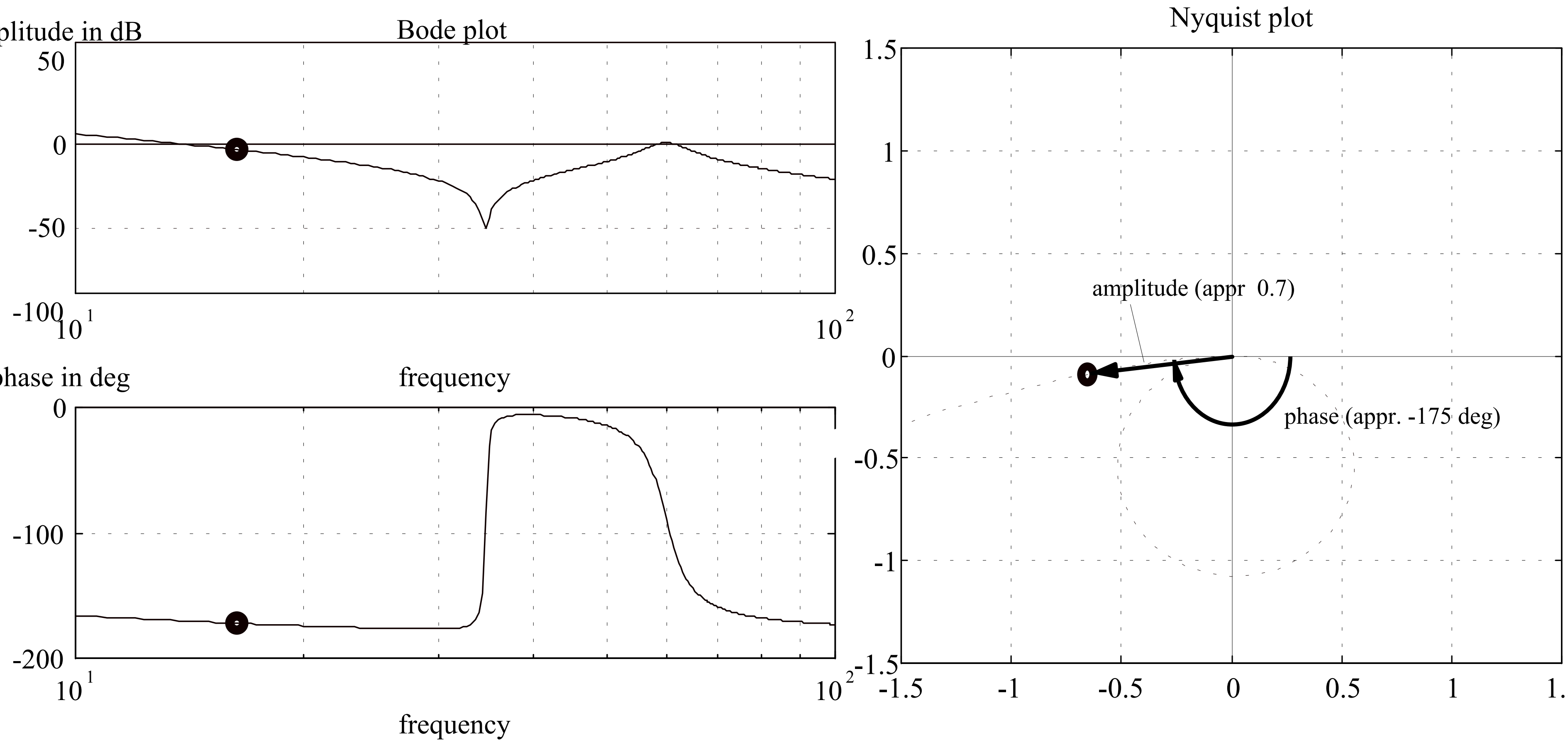


Experimental results:

stage servo

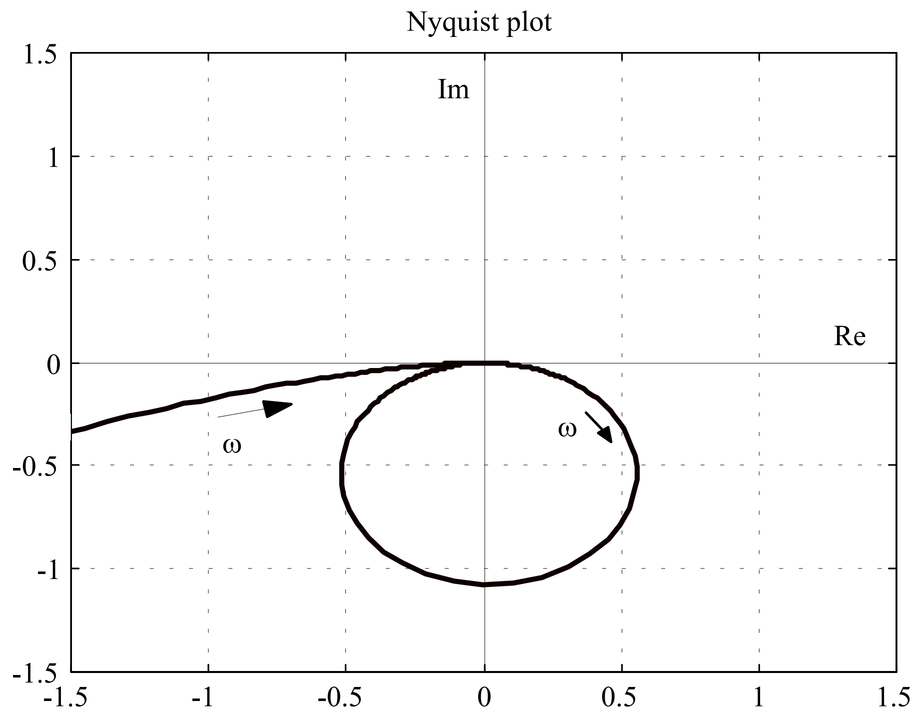
bandwidth: 0 dB crossing open loop
(cross-over frequency)

The Nyquist curve



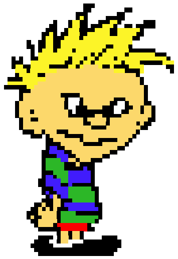
Stability:

The **open-loop** FRF CH(jω) should have the (-1,0) point at left side



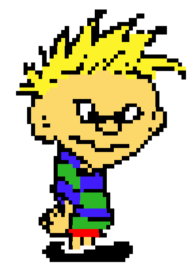
4. Filters

Mimo

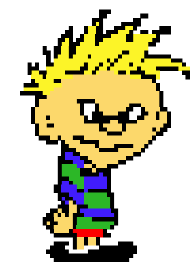


- Integral action
- Differential action
- Low-pass
- High-pass
- Band-pass
- Notch (‘sper’) filter

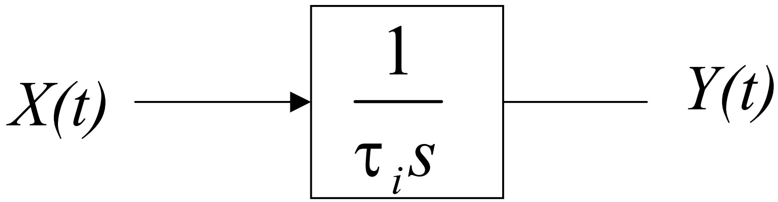
PeeDee



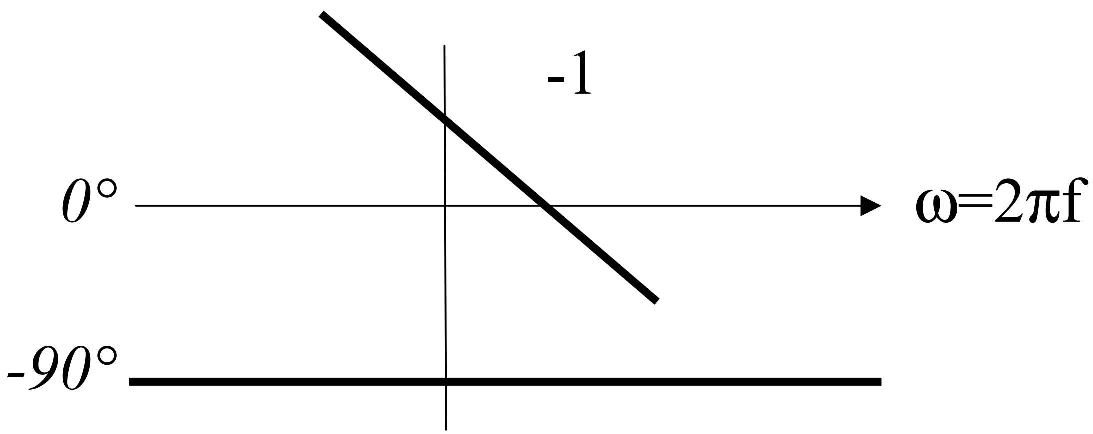
PeeEye



Integral action :

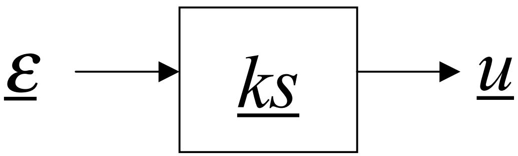


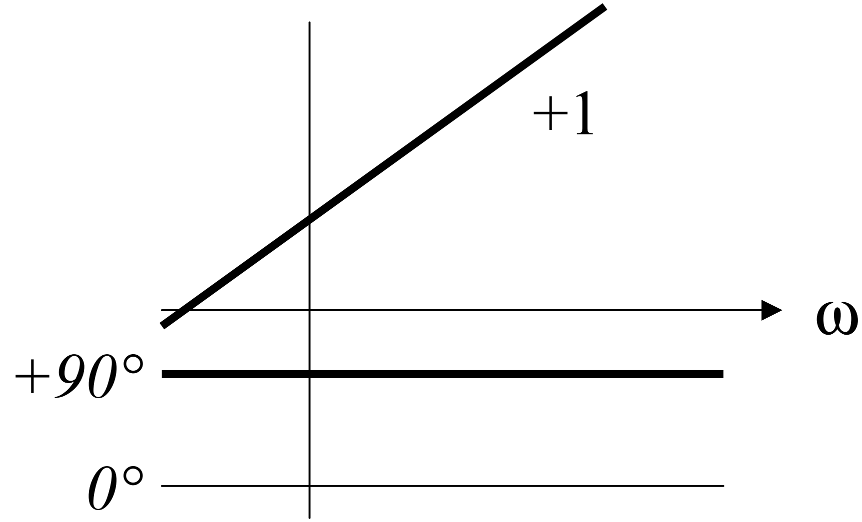
τ_I integral time constant $\tau_I = 1/k_i$



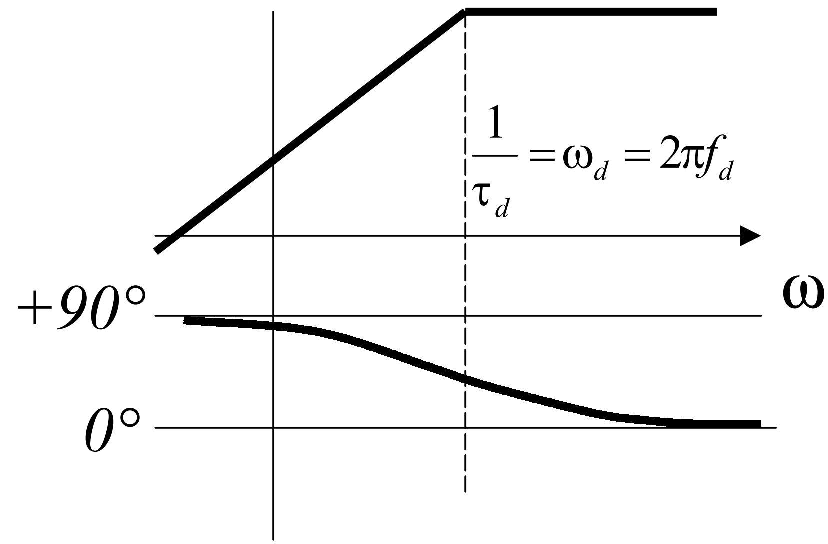
Differential action

$$H = ks = \frac{u}{\varepsilon}; \quad s = j\omega; \quad \left|\frac{u}{\varepsilon}\right| = k\omega$$





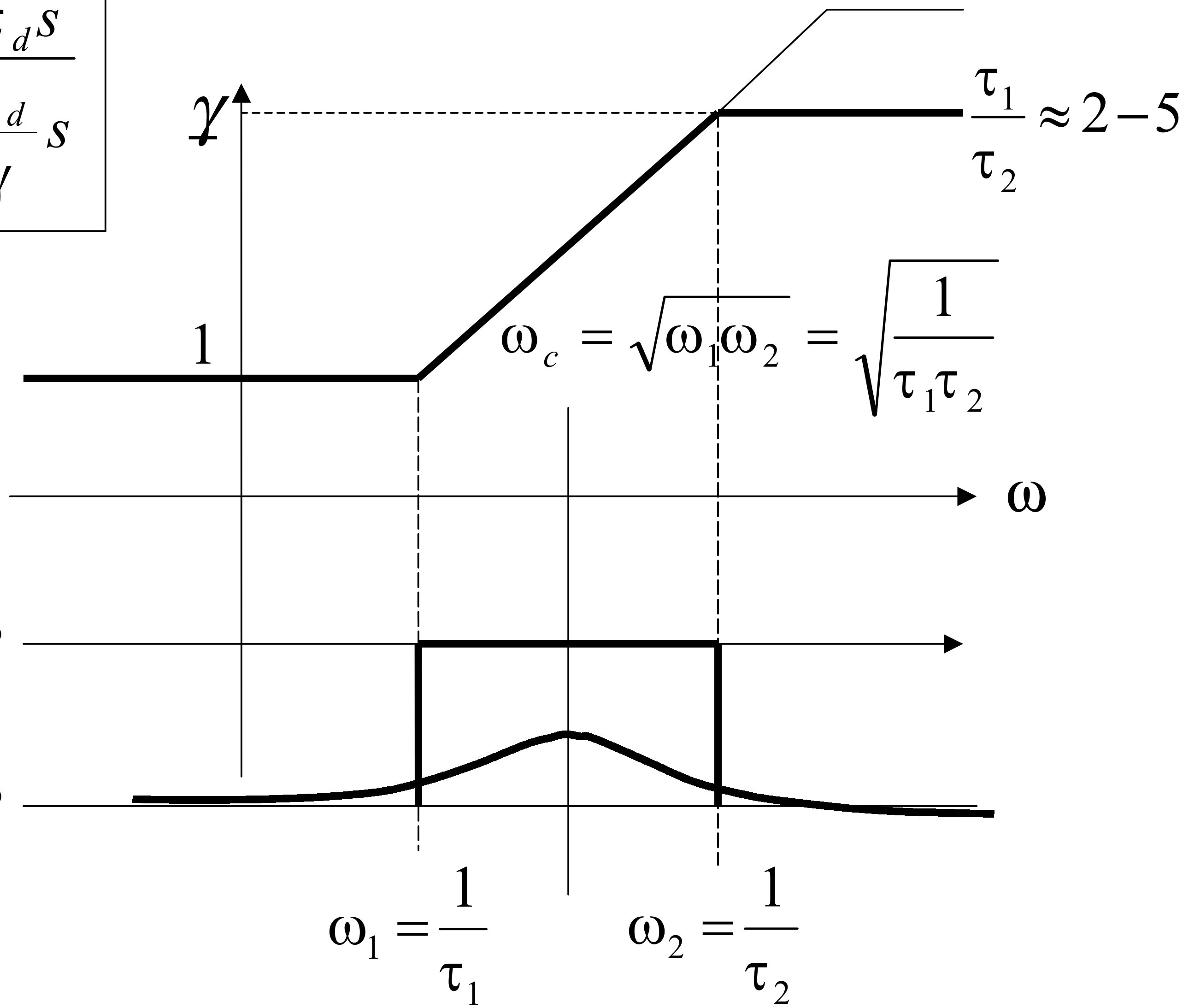
“tamme” differentiator $\frac{u}{\varepsilon} := \frac{ks}{\tau_d s + 1}$



“lead” filter

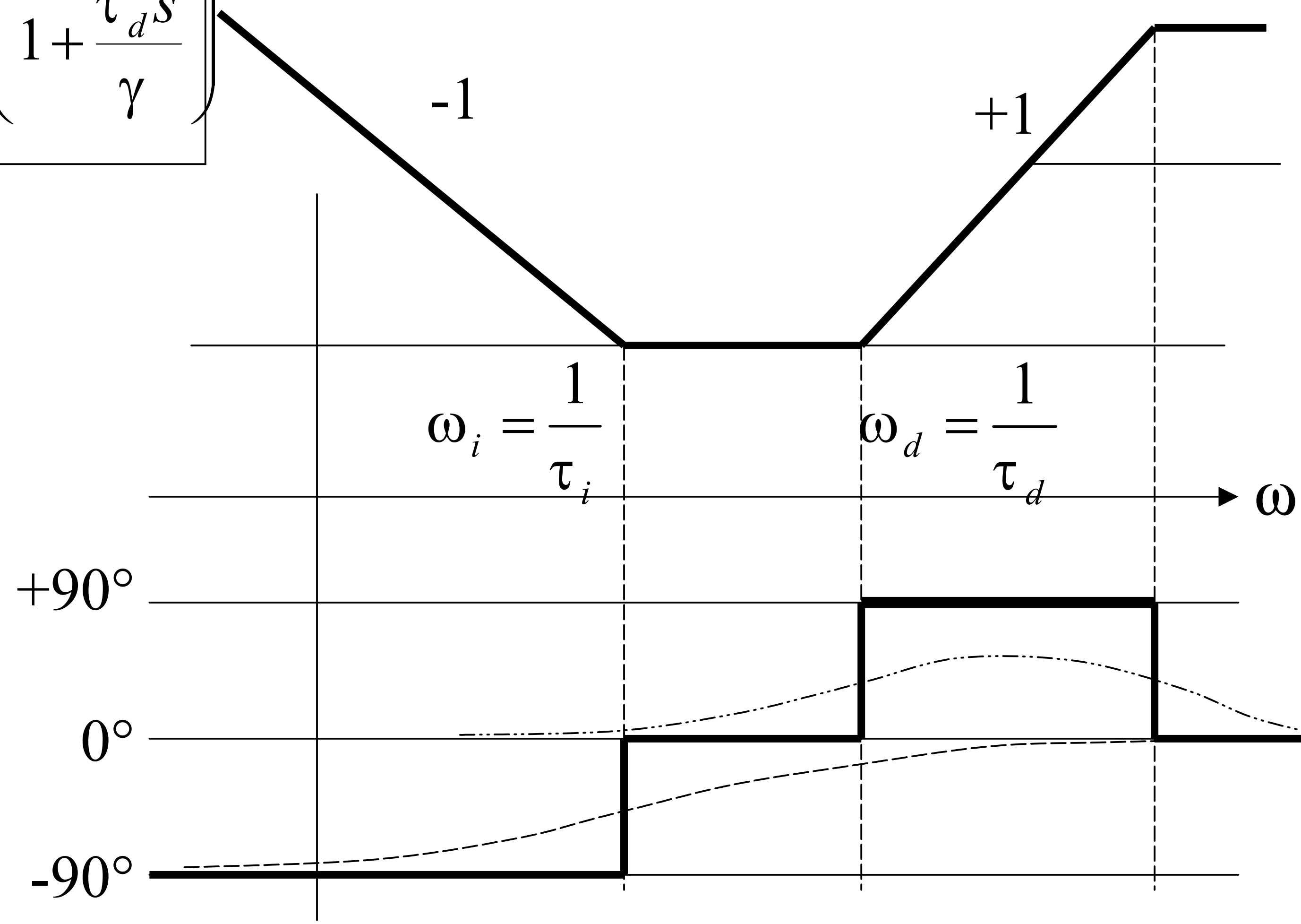
$$H = \frac{u}{\varepsilon} = \frac{1+\tau_1 s}{1+\tau_2 s} = \frac{1+\tau_d s}{1+\frac{\tau_d}{\gamma} s}$$

$\gamma > 1$



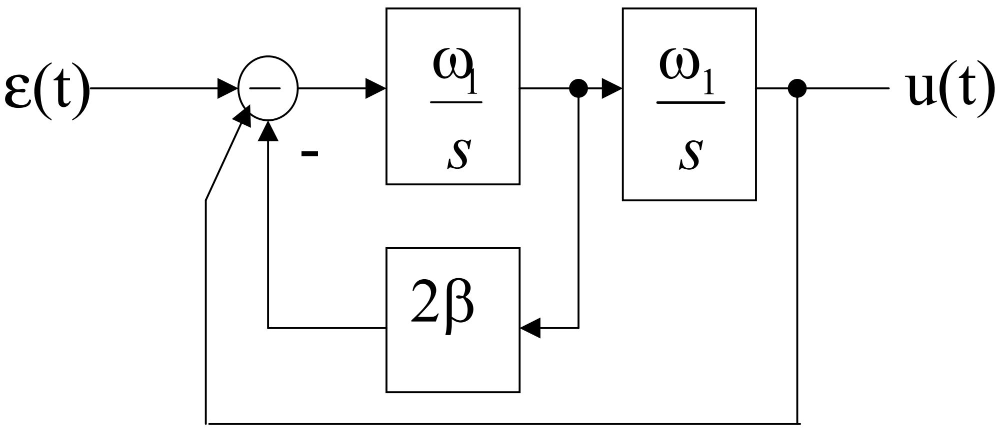
P+I+D

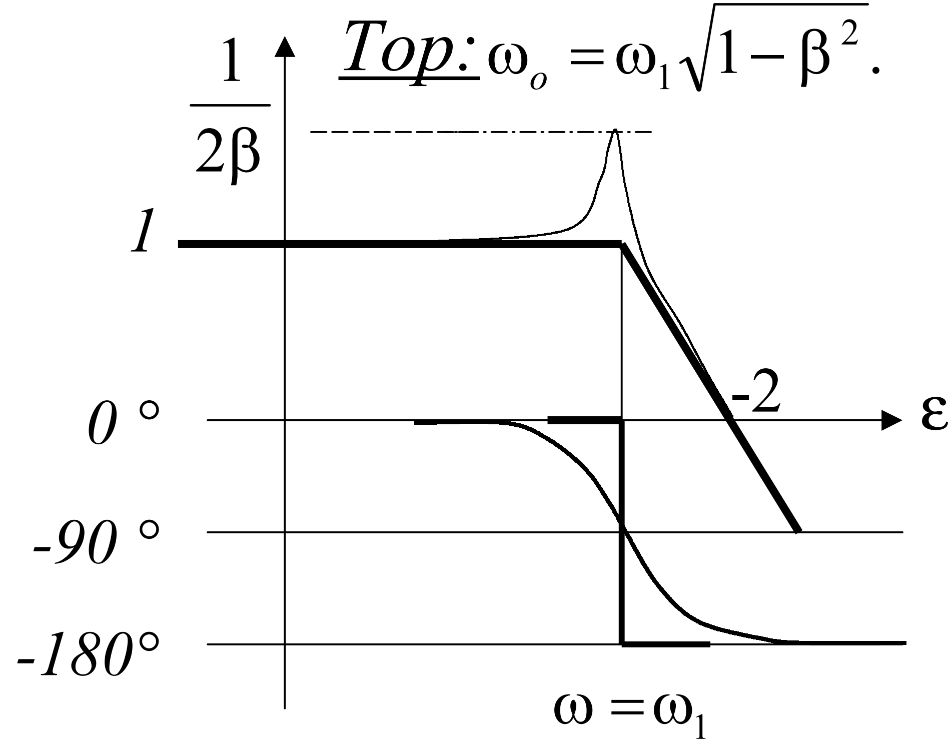
$$H = \frac{u}{\varepsilon} = k\left(1 + \frac{1}{\tau_i s}\right)\left(\frac{1+\tau_d s}{1+\frac{\tau_d s}{\gamma}}\right)$$



2nd order filter

$$H = \frac{u}{\varepsilon} = \frac{k}{\dfrac{s^2}{\omega_1^2} + 2\beta\,\dfrac{s}{\omega_1} + 1}$$

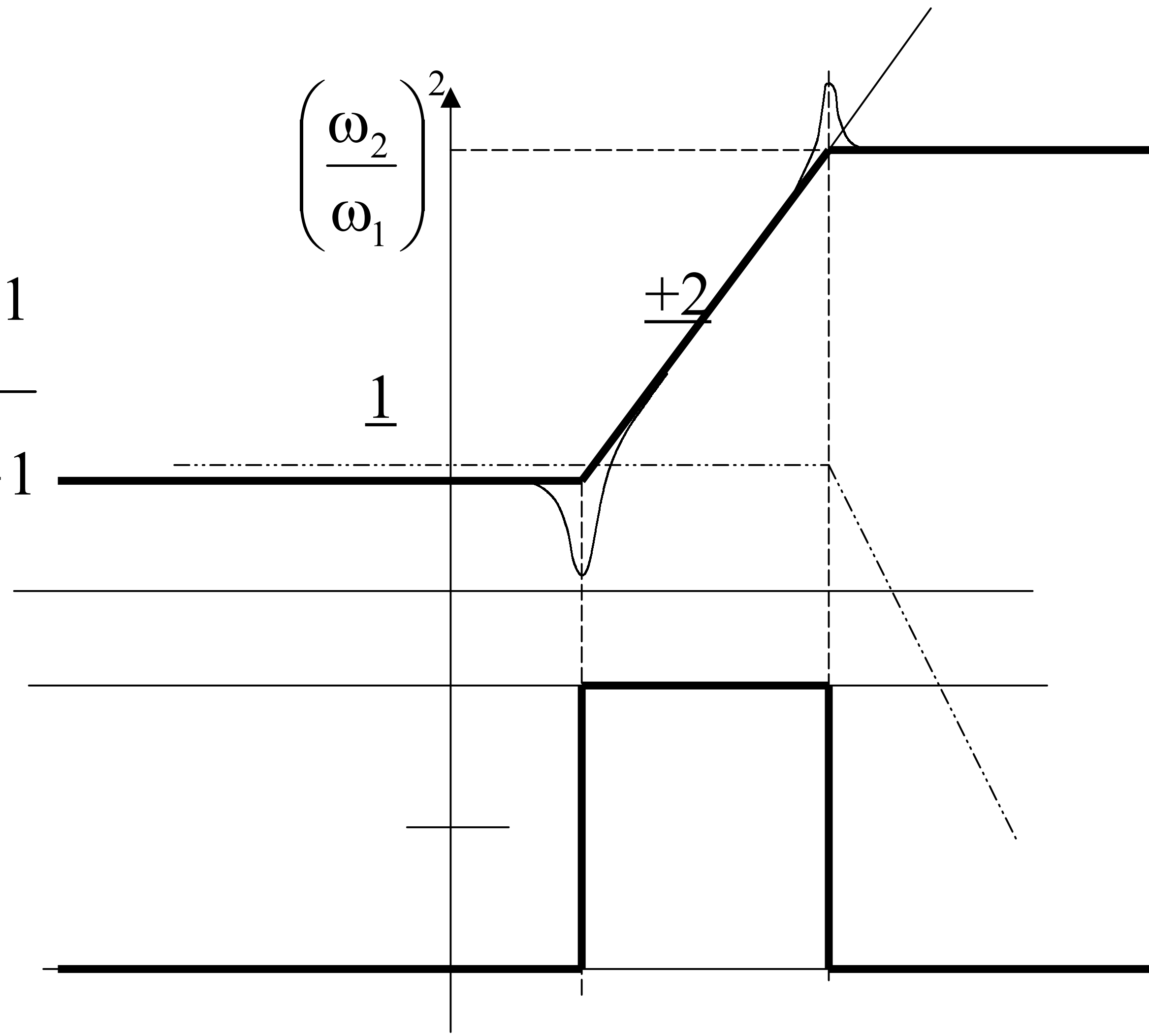


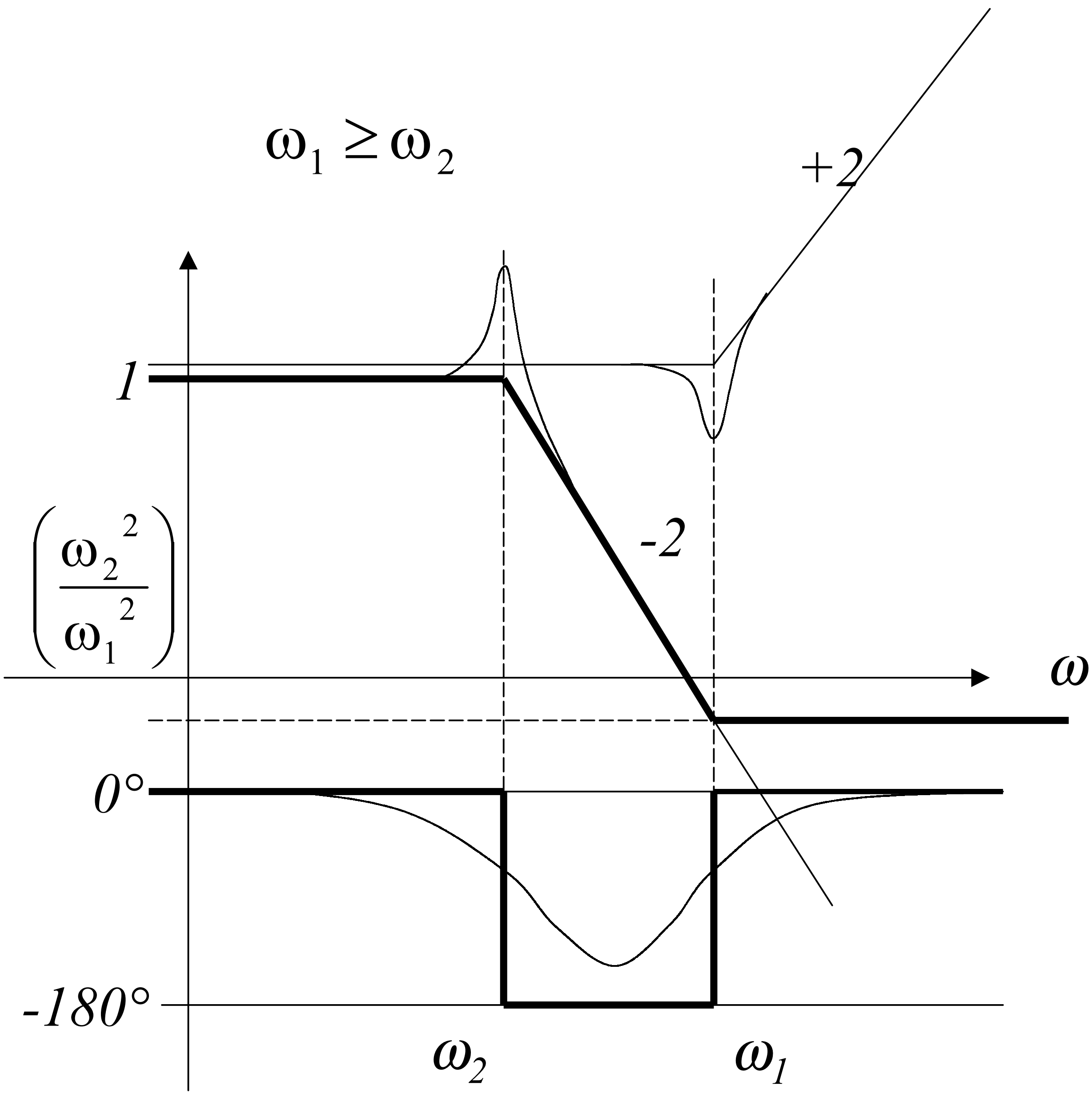


General 2nd order filters

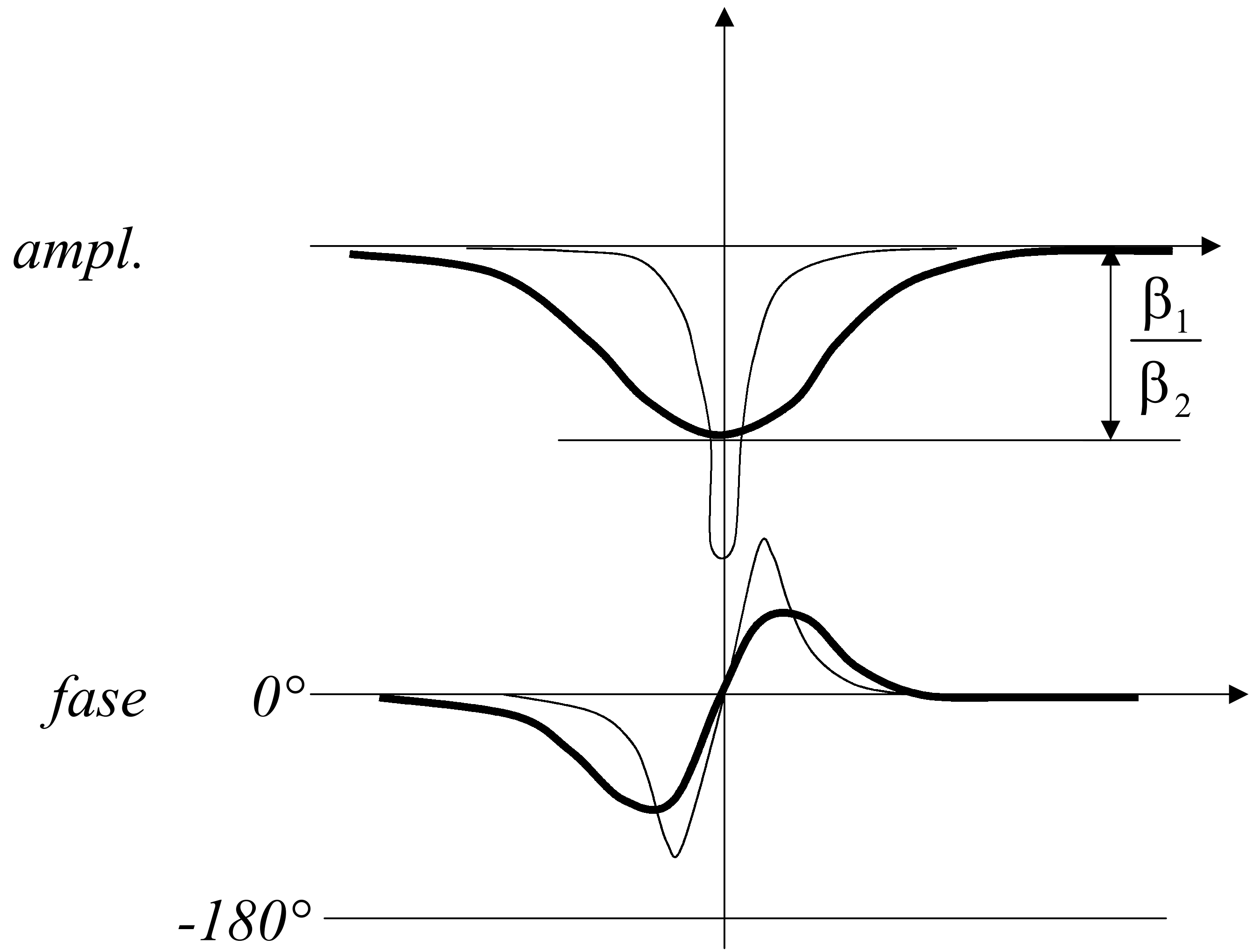
General: $\omega_1 \neq \omega_2$

$$H = \frac{u}{\varepsilon} = \frac{\frac{s^2}{\omega_1^2} + 2\beta_1 \frac{s}{\omega_1} + 1}{\frac{s^2}{\omega_2^2} + 2\beta_2 \frac{s}{\omega_2} + 1}$$

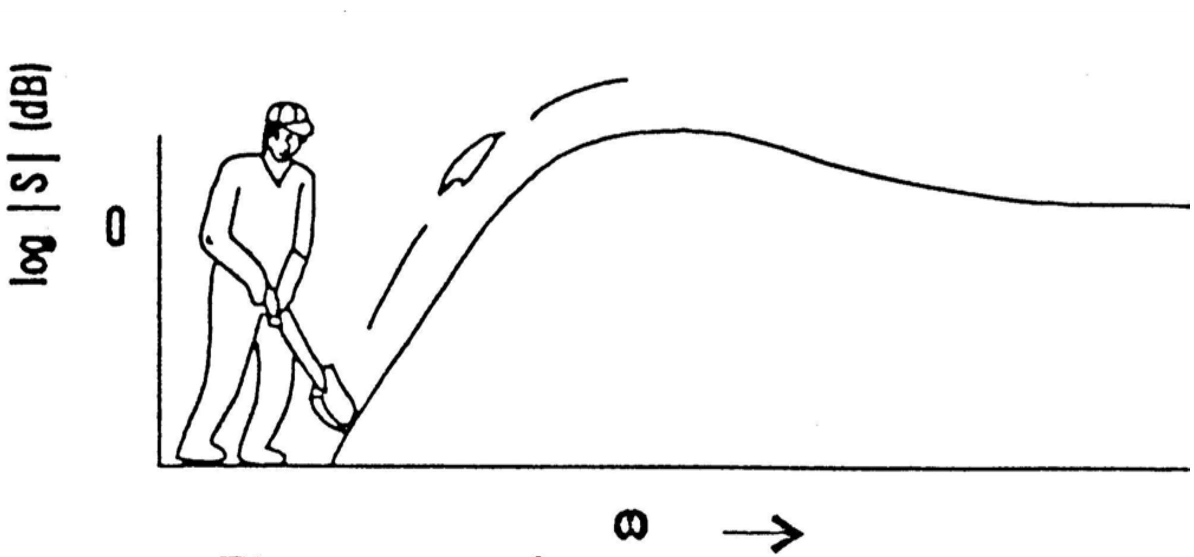



ω1 ≥ ω2
+2
1
-2
(ω2^2 / ω1^2)
ω
0°
-180°
ω2
ω1

“Notch”-filter : $\omega_1 = \omega_2$



W.B.E.



Loop shaping procedure

1. *stabilize the plant:*
   add lead/lag with zero = bandwidth/3 and pole = bandwidth*3, adjust gain to get stability; or add a pure PD with break point at the bandwidth
2. *add low-pass filter:*
   choose poles = bandwidth*6
3. *add notch if necessary, or apply any other kind of first or second order filter and shape the loop*
4. *add integral action:*
   choose zero = bandwidth/5
5. *increase bandwidth:*
   increase gain and zero/poles of integral action, lead/lag and other filters

during steps 2-5: check all relevant transfer functions, and relate to disturbance spectrum

Implementation issues

1. sampling = delay: linear phase lag

for example: sampling at 4 kHz gives phase lag due to Zero-Order-Hold of:

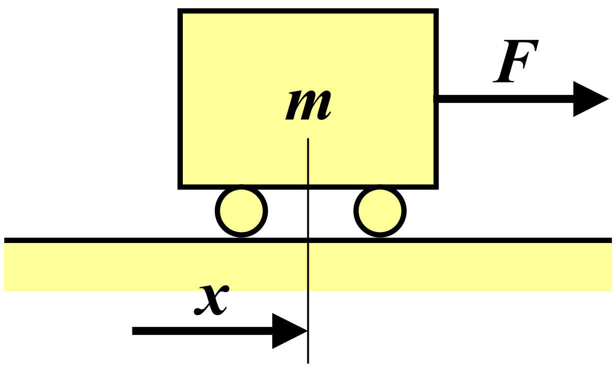
180º	@ 4 kHz
18º	@ 400 Hz
9º	@ 200 Hz

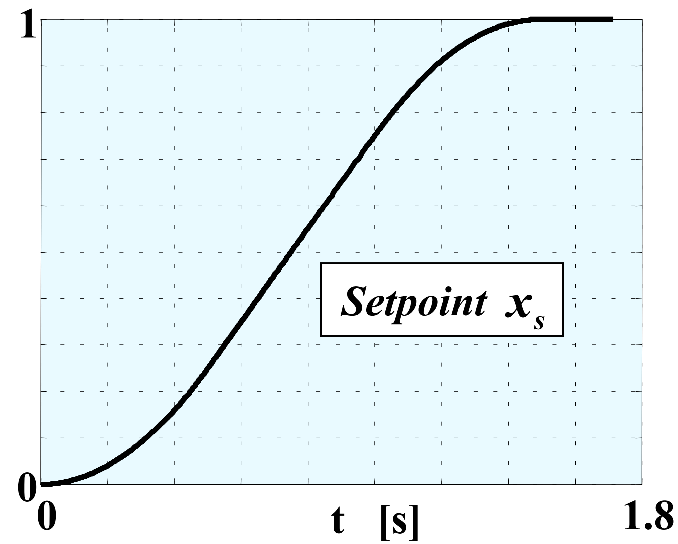
2. Delay due to calculations
3. Quantization (sensors, digital representation)

5 Feedforward design

Why feedforward?

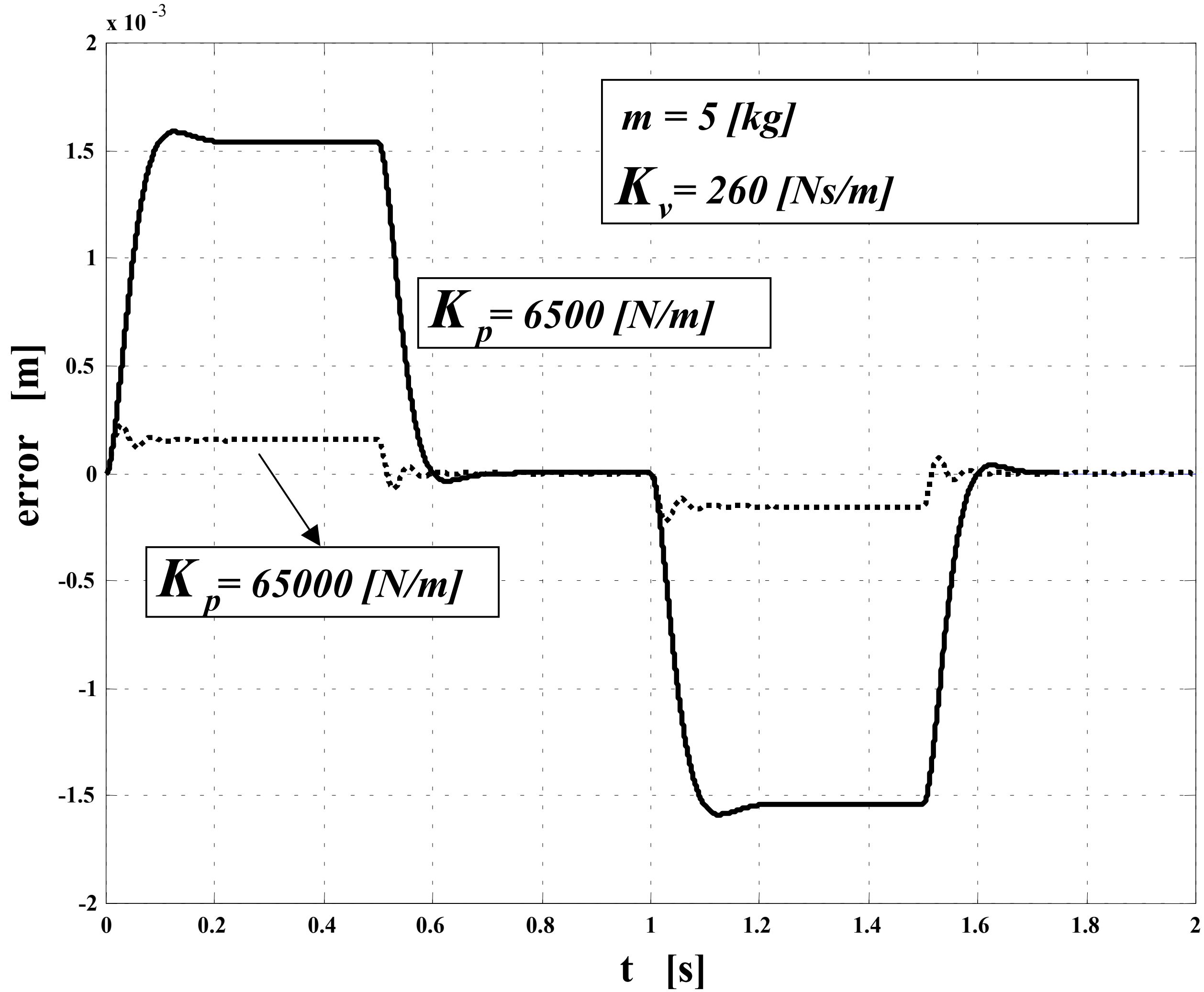
- ***Consider the simple motion system***





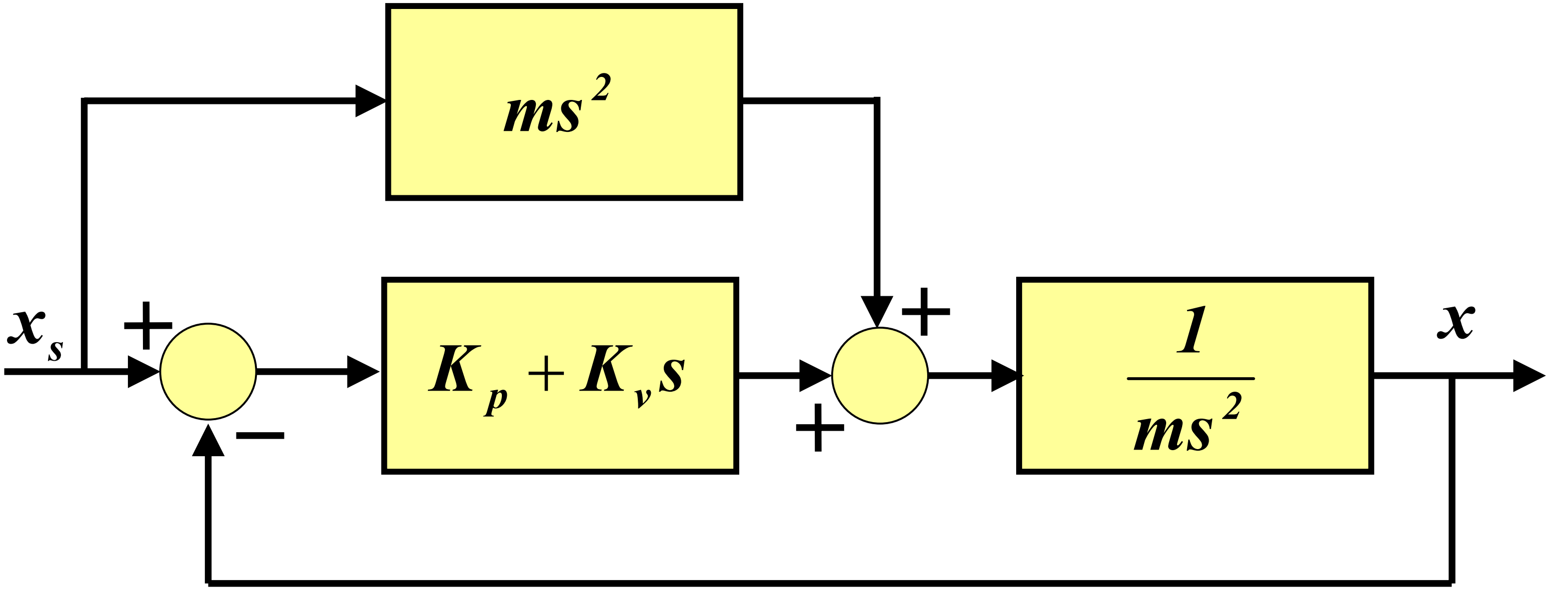
- ***Control problem: track setpoint x_s***
- ***Is this possible with a PD-controller?***

Analysis (IV)

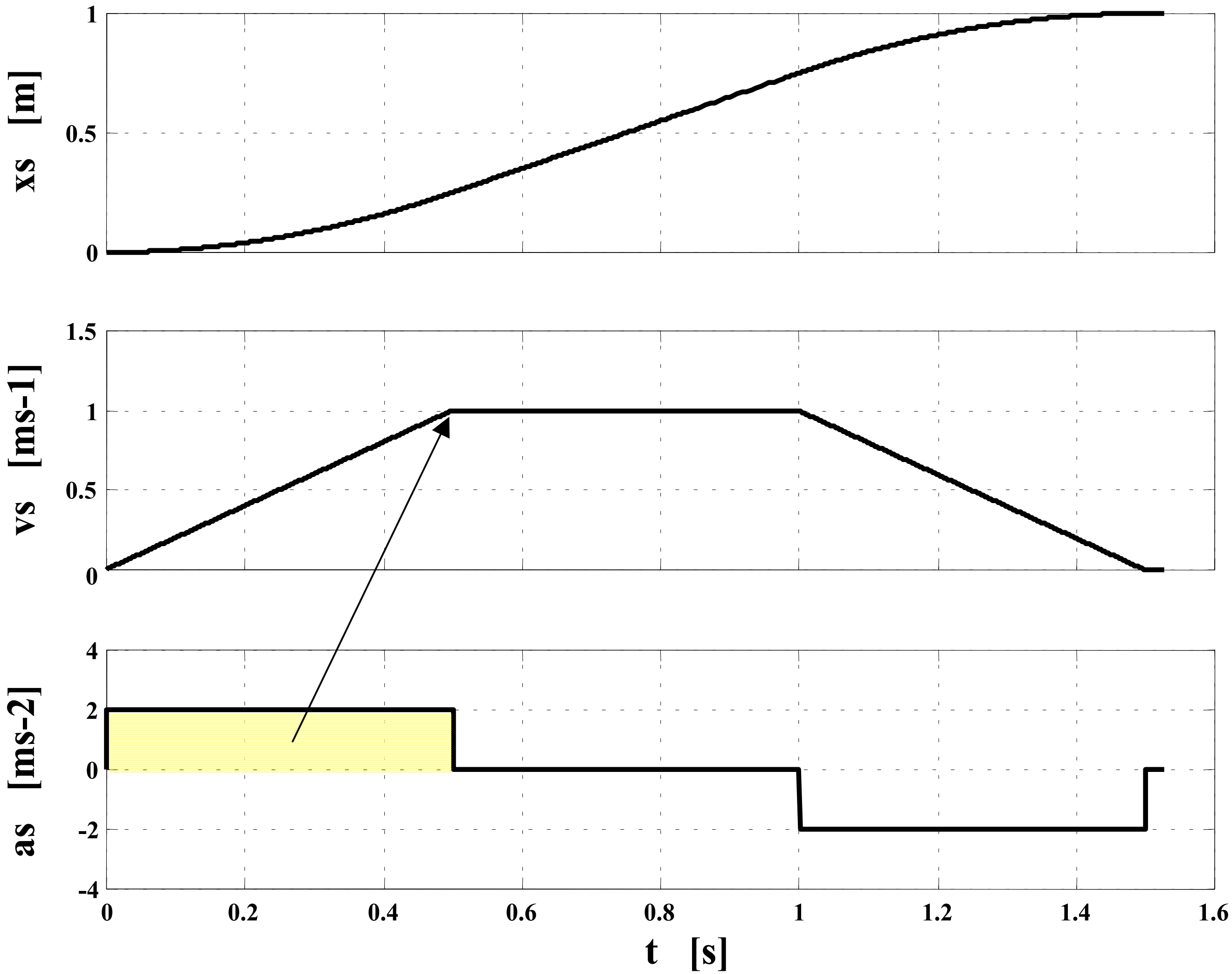


x 10 -3
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
0
0.2
0.4
0.6
0.8
1
1.2
1.4
1.6
1.8
2
error [m]
t [s]
m = 5 [kg]
K v= 260 [Ns/m]
K p= 6500 [N/m]
K p= 65000 [N/m]

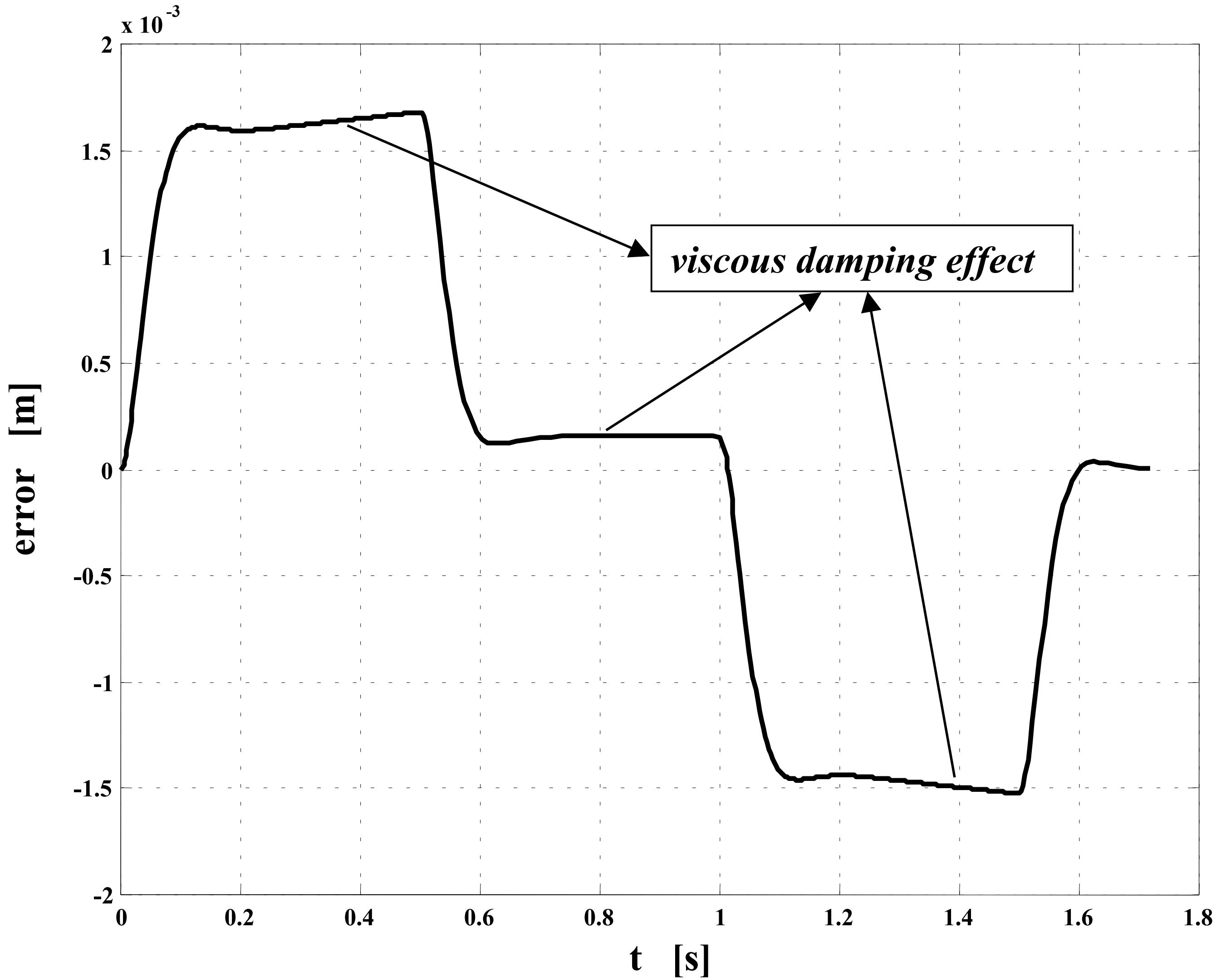
Feedforward based on inverse model



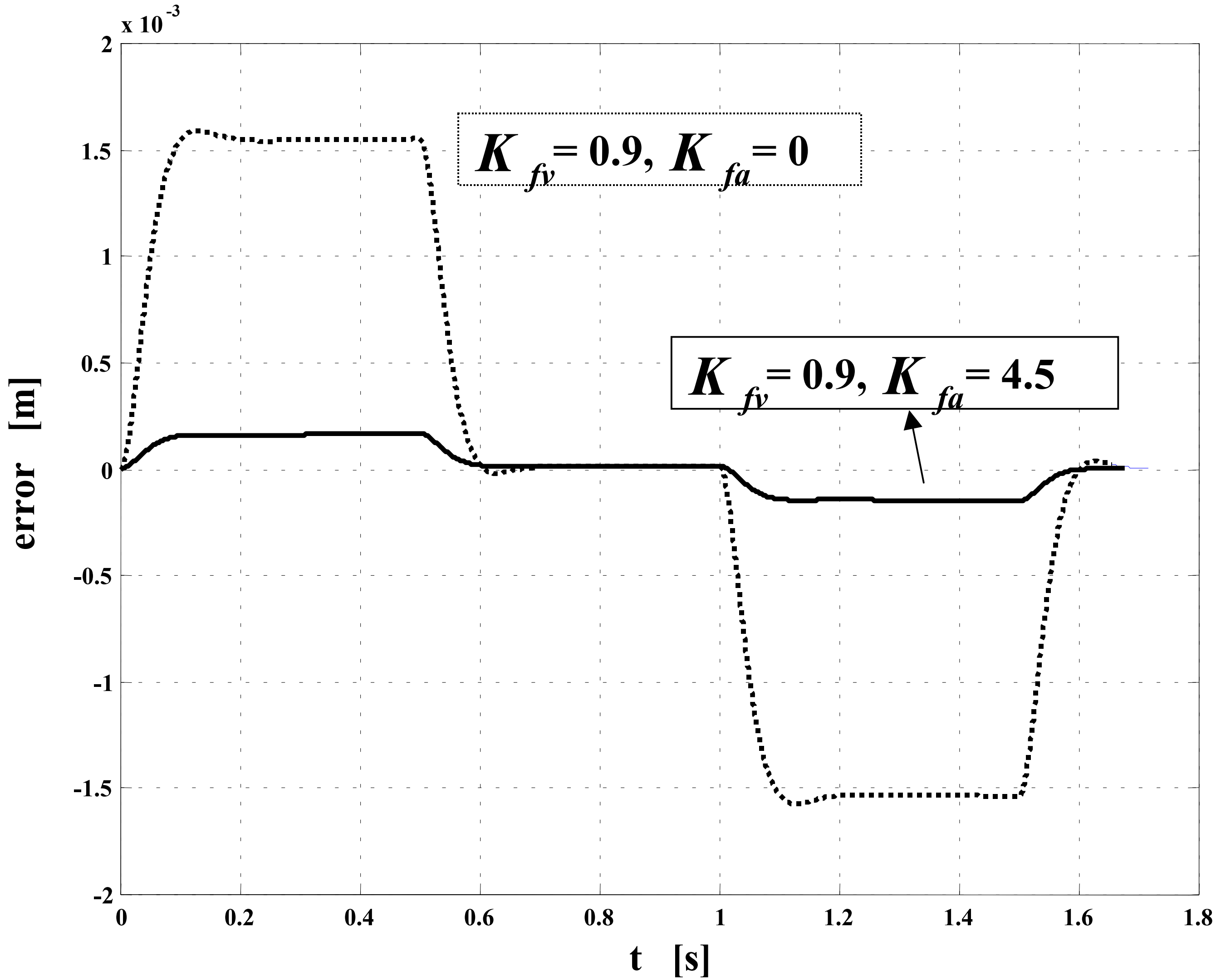
Example: m=5 [kg], b=1 [Ns/m], 2nd degree setpoint



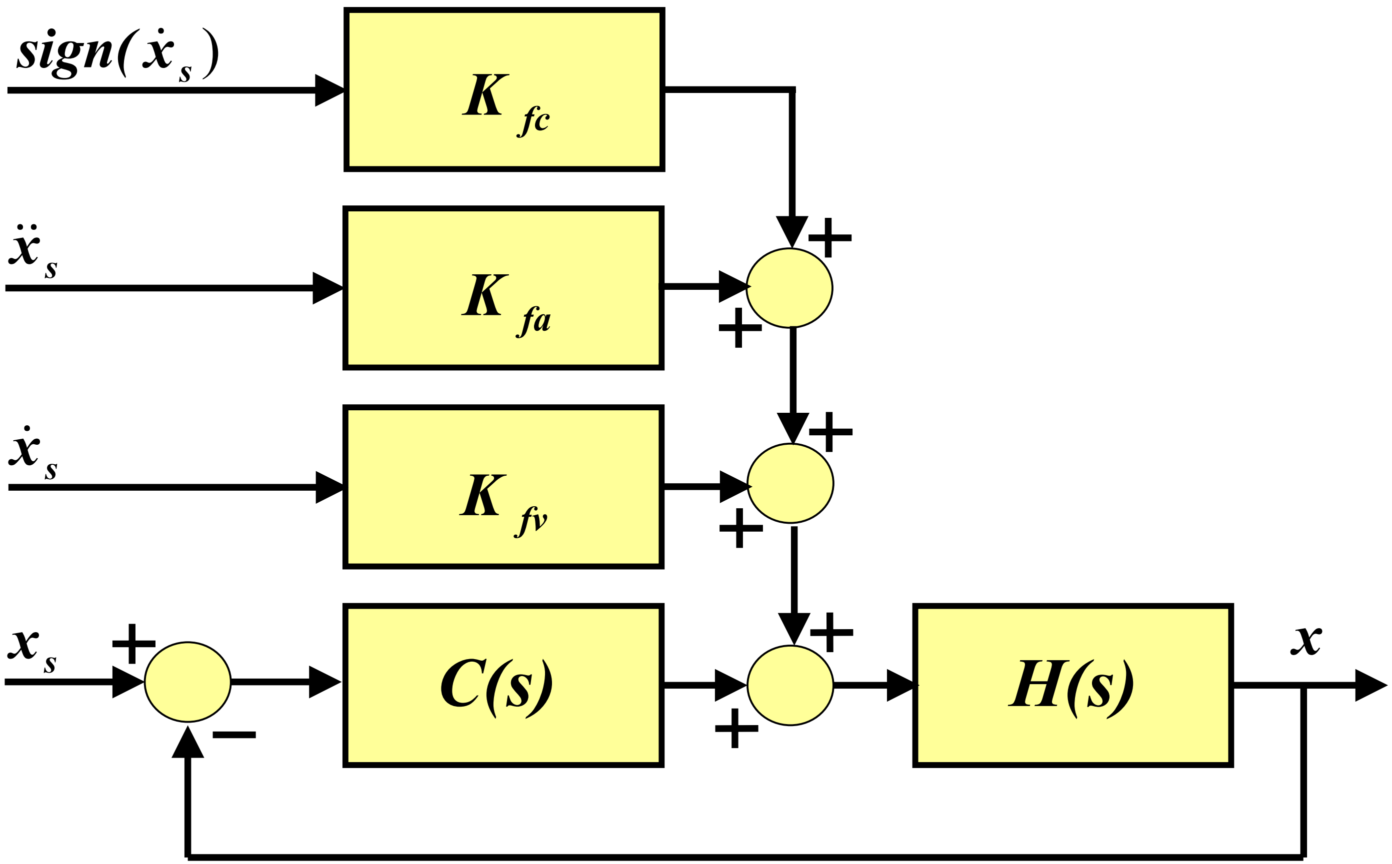
Example: tracking error, no feedforward



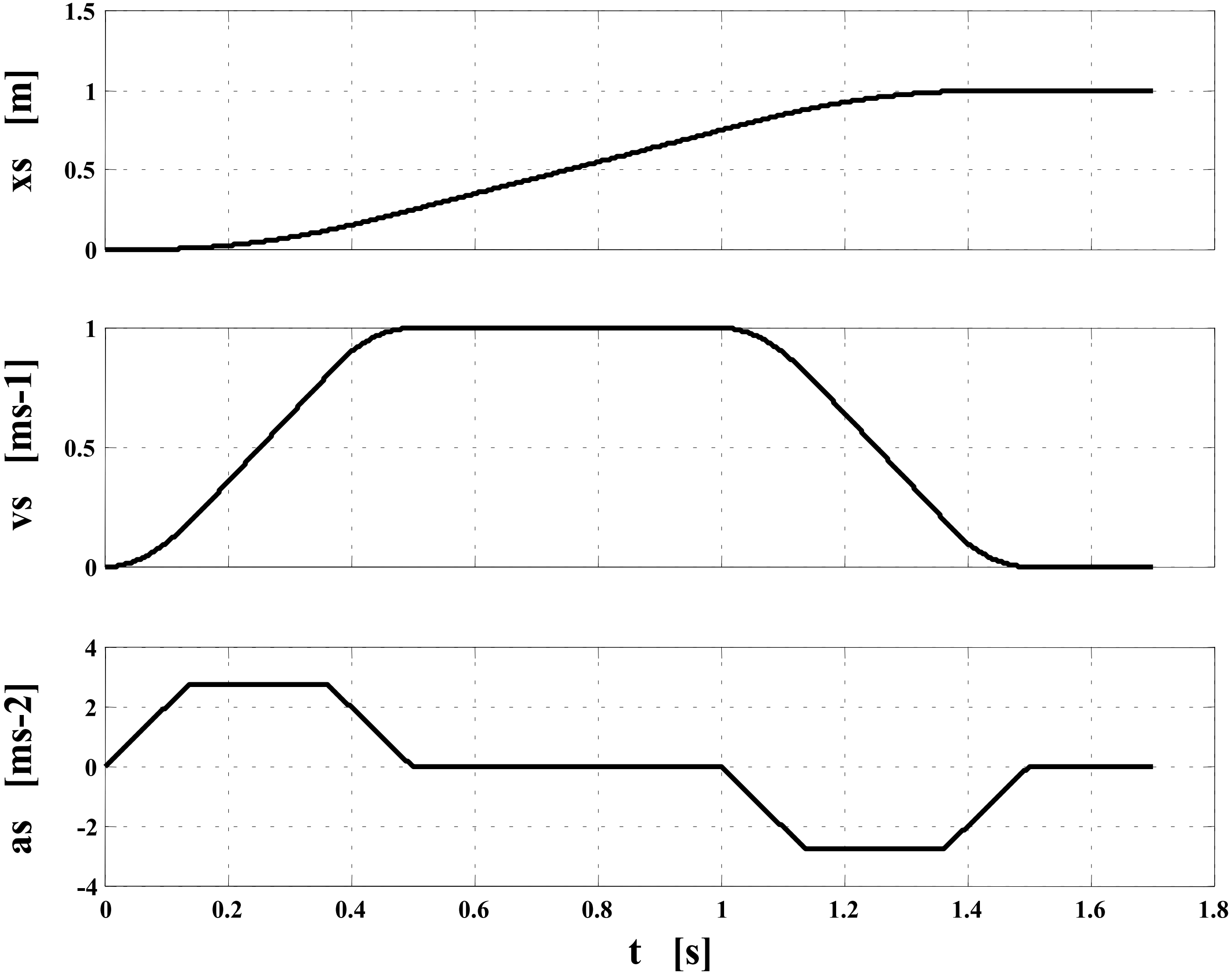
Example: tracking error, with feedforward



feedforward structure

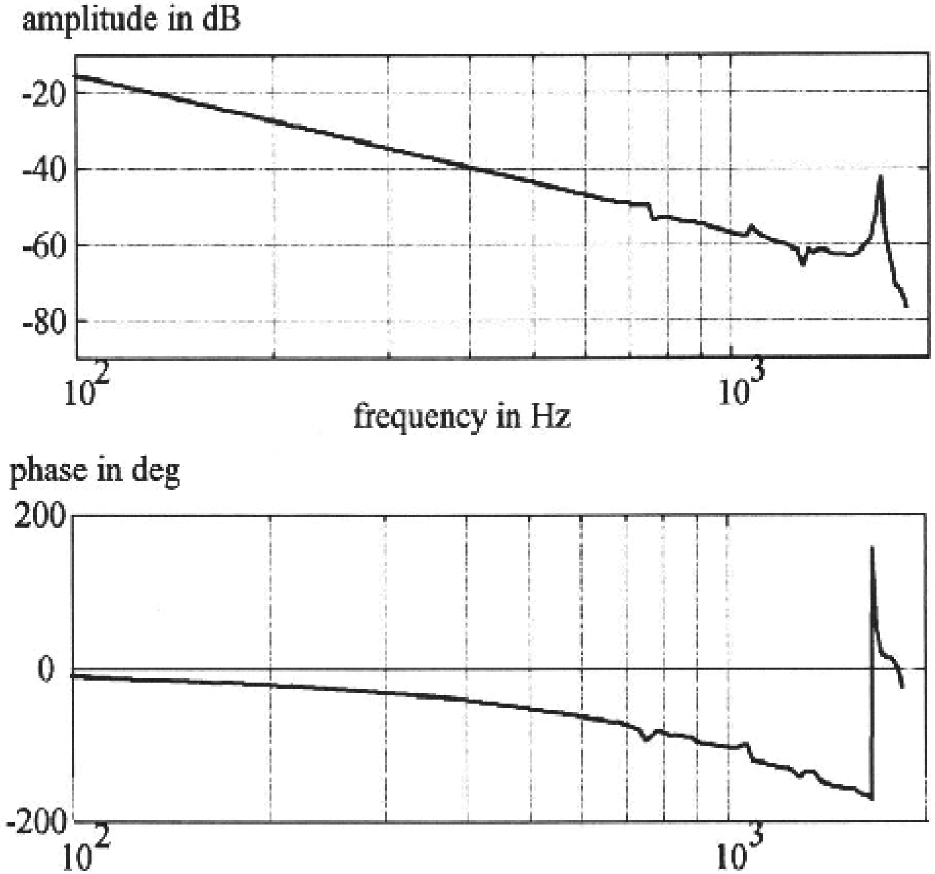


3rd degree setpoint trajectory



6. Servo-oriented design of mechanical systems

Example of measurement:
mechanical system (force to position)



modelling ⟶ understanding the dynamical behaviour

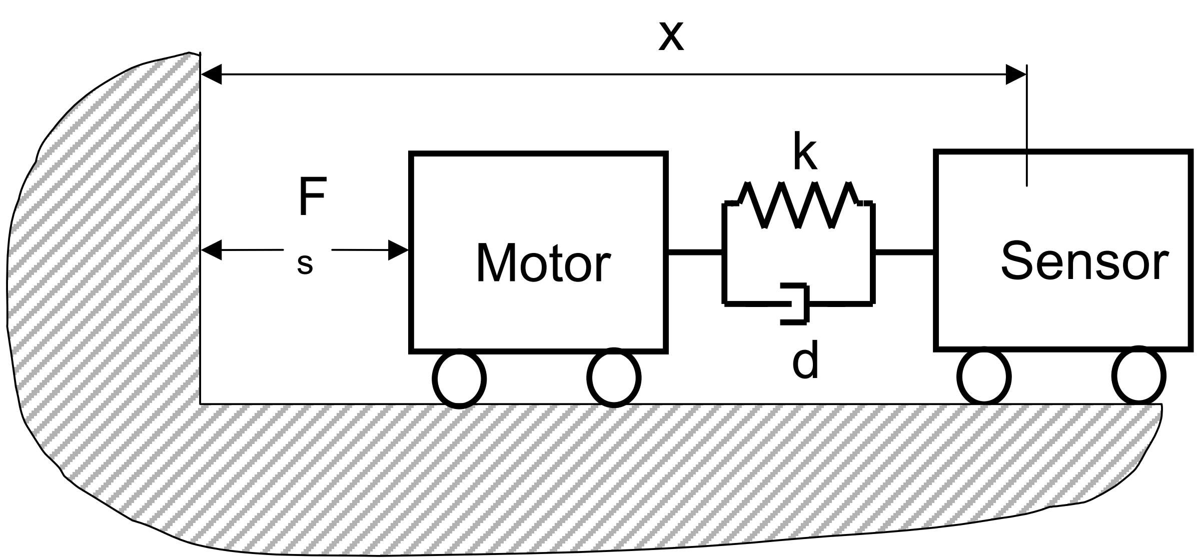
Three Types of Dynamic Effects

- Actuator flexibility

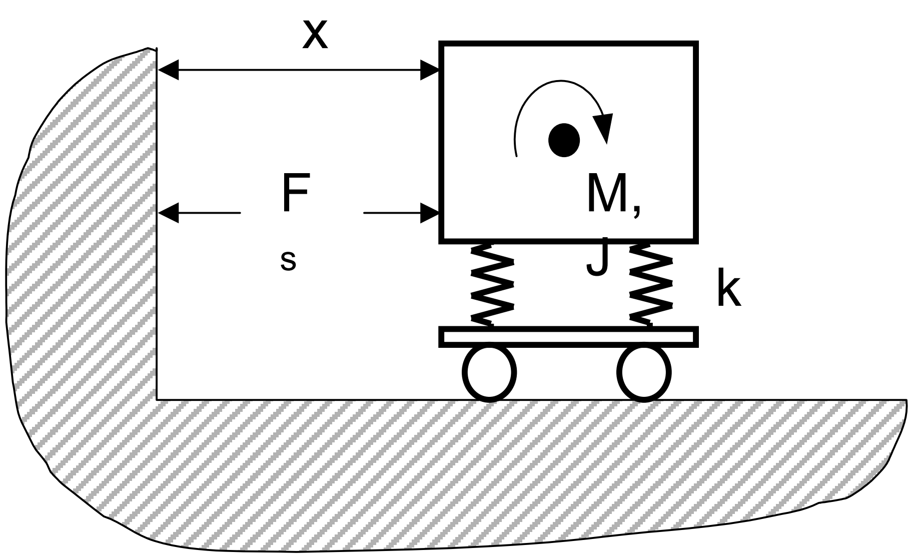
- Guidance flexibility

- Limited mass and stiffness of frame

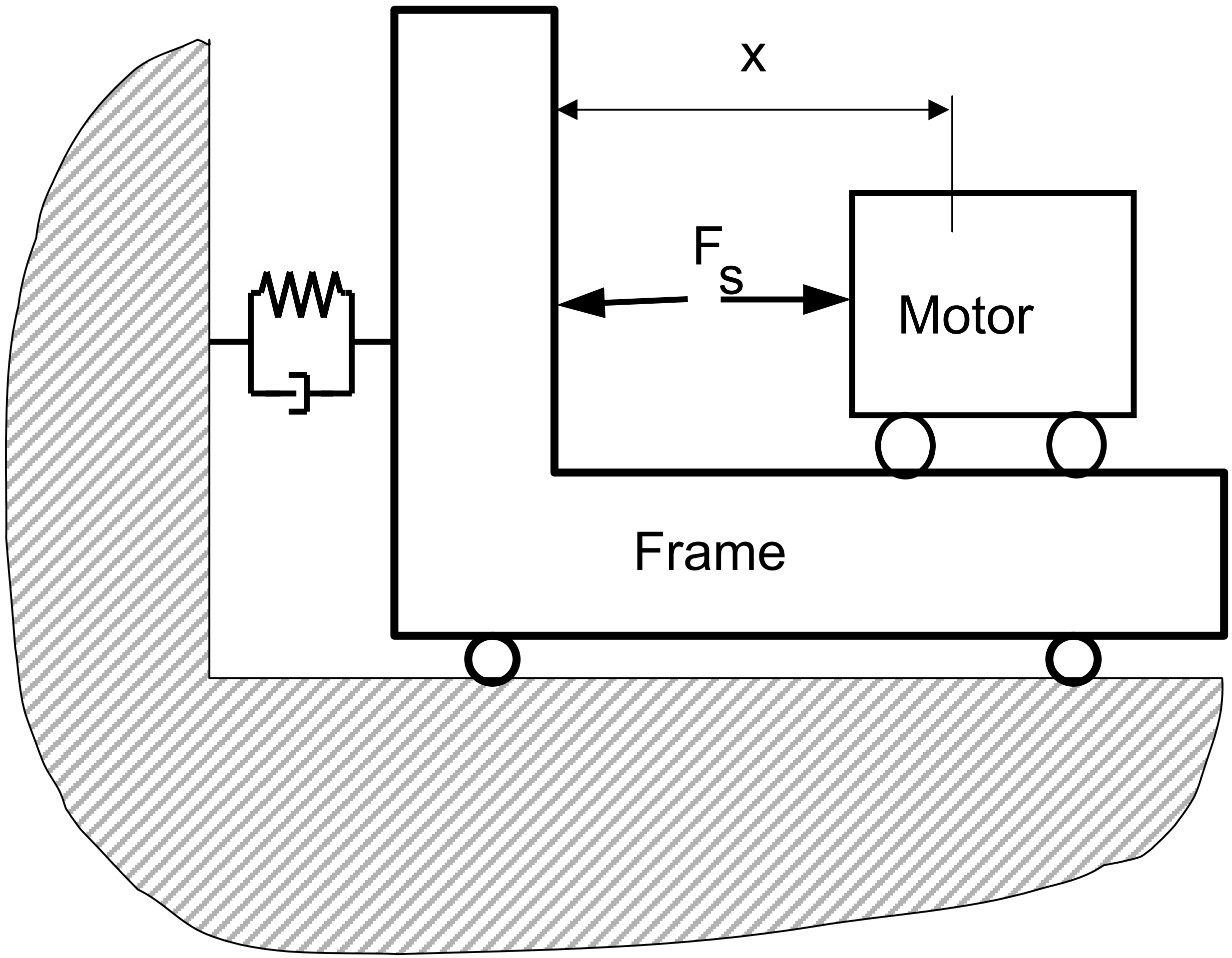
1. Actuator flexibility

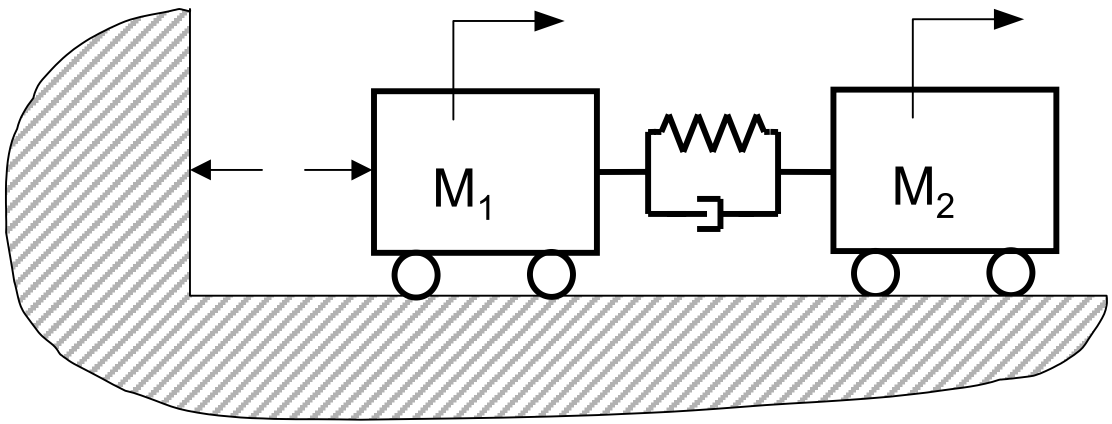


2. Guidance flexibility



3. Limited mass and stiffness of frame





Positioning the load M_2 (while using x_1 for feedback):

Rule of thumb:
Optimal bandwidth with 0 dB crossing of open loop between the antiresonance and resonance frequency of the mechanical system.

Concluding Remarks

- bit of control into mechanical design
- bit of mechanics into control design
- same language (‘mechatronics’)