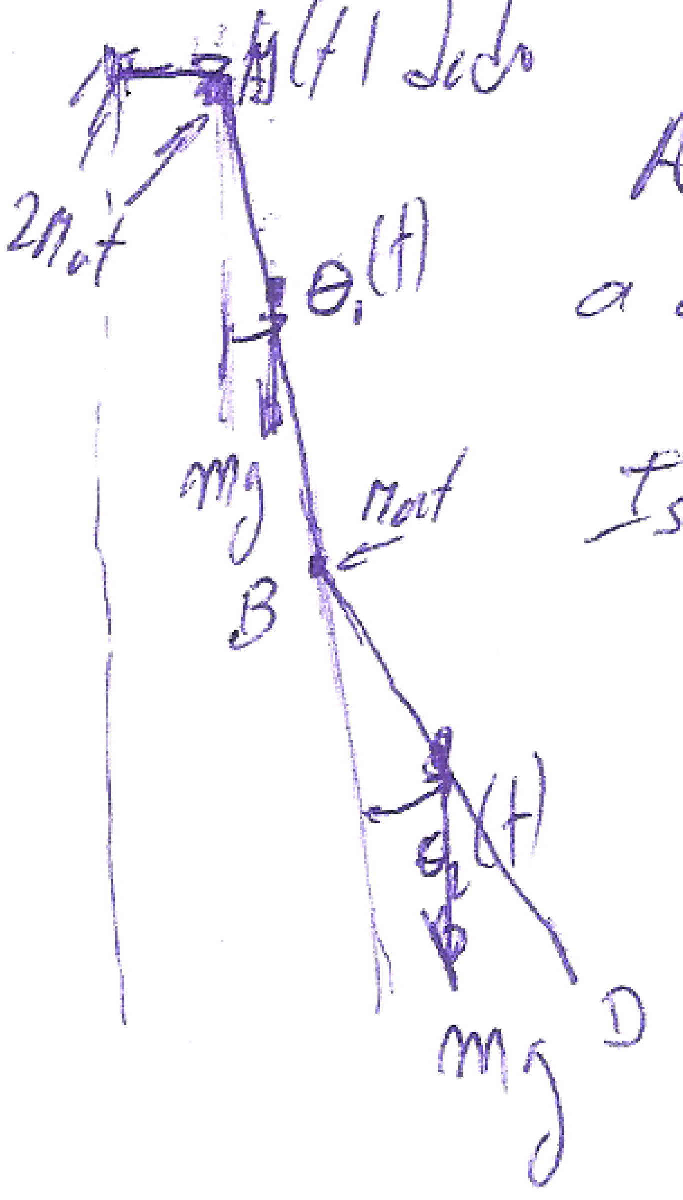
PME 3400 — P3 — 23/07/2020

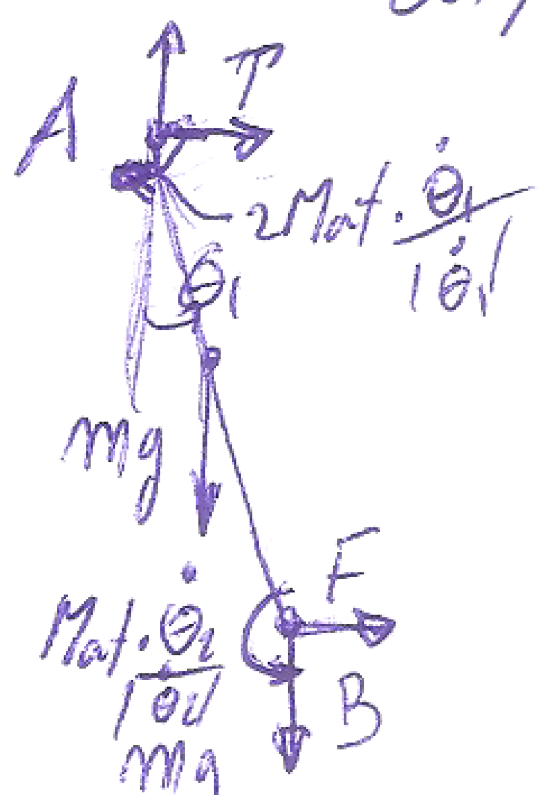
1ª Q)

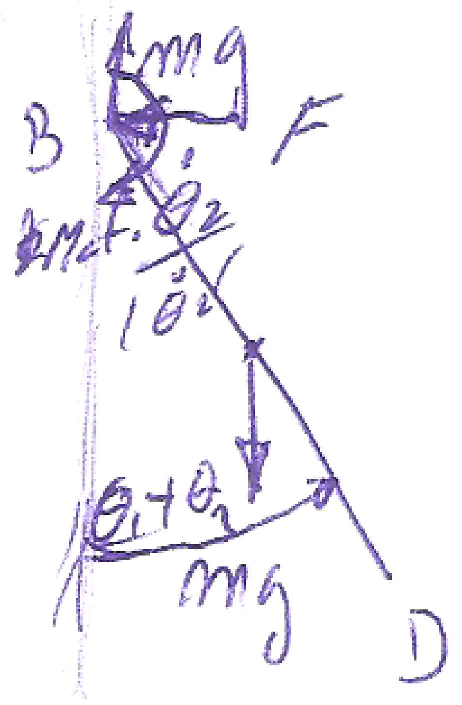


$\boxed{L = l}$

A escolha de $\theta_2(t)$ a partir do $\theta_1(t)$ é facilitar a aplicação de Mat na articulação B $\left(Mat \cdot \frac{\dot{\theta}_2}{|\dot{\theta}_2|}\right)$

Isolando os corpos e supondo pequenas amplitudes





$J_A = \frac{ml^2}{3}$

TMQA)$_A$ para a barra AB

$$J_A \cdot \ddot{\theta}_1 = -2Mat \cdot \frac{\dot{\theta}_1}{|\dot{\theta}_1|} - mg\frac{l}{2}\theta_1 - mg \cdot l \cdot \theta_1 + Mat\frac{\dot{\theta}_2}{|\dot{\theta}_2|} + F \cdot l - m\frac{l}{2} \cdot \ddot{y}$$

Para a barra BD

$$TMB \rightarrow m \cdot \left(\ddot{y} + \frac{3}{2}l\ddot{\theta}_1 + \frac{l}{2}\ddot{\theta}_2\right) = -F$$

$$TMQA)_B \quad \frac{ml^2}{3} \cdot (\ddot{\theta}_1 + \ddot{\theta}_2) = -Mat\frac{\dot{\theta}_2}{|\dot{\theta}_2|} - mg\frac{l}{2}(\theta_1+\theta_2) - \frac{ml}{2} \cdot (\ddot{y} + l\ddot{\theta}_1)$$

$$\begin{cases} \frac{ml^2}{3} \cdot \ddot{\theta}_1 + ml\left(\ddot{y} + \frac{3}{2}l\ddot{\theta}_1 + \frac{l}{2}\ddot{\theta}_2\right) + 2Mat\frac{\dot{\theta}_1}{|\dot{\theta}_1|} + \frac{3}{2}mgl\theta_1 - Mat\frac{\dot{\theta}_2}{|\dot{\theta}_2|} = -\frac{ml}{2}\ddot{y} \\ \frac{ml^2}{3}\ddot{\theta}_1 + \frac{ml^2}{3}\ddot{\theta}_2 + Mat\frac{\dot{\theta}_2}{|\dot{\theta}_2|} + mg\frac{l}{2}\theta_1 + mg\frac{l}{2}\theta_2 + \frac{ml^2}{2}\ddot{\theta}_1 = -\frac{ml}{2}\ddot{y} \end{cases}$$

$$\begin{cases} \frac{11ml^2}{6} \cdot \ddot{\theta}_1 + \frac{ml^2}{2}\ddot{\theta}_2 + 2Mat\frac{\dot{\theta}_1}{|\dot{\theta}_1|} - Mat\frac{\dot{\theta}_2}{|\dot{\theta}_2|} + \frac{3}{2}mgl\theta_1 = -\frac{3}{2}ml\ddot{y} \\ \frac{5ml^2}{6}\ddot{\theta}_1 + \frac{ml^2}{3}\ddot{\theta}_2 + Mat\frac{\dot{\theta}_2}{|\dot{\theta}_2|} + \frac{mgl}{2}\theta_1 + \frac{mgl}{2}\theta_2 = -\frac{ml}{2} \cdot \ddot{y} \end{cases}$$

$$\begin{cases} \frac{8}{3}\cancel{m}l\ddot{\theta}_1 + \frac{5}{6}\cancel{m}l\ddot{\theta}_2 + \frac{2Mat}{ml}\frac{\dot{\theta}_1}{|\dot{\theta}_1|} + 2\cancel{m}g\cancel{l}\theta_1 + \frac{\cancel{m}g\cancel{l}}{2}\theta_2 = -2\cancel{m}\cancel{l}\ddot{y} \\ \frac{5}{6}\cancel{m}l\ddot{\theta}_1 + \frac{\cancel{m}l^{\cancel{2}}}{3}\ddot{\theta}_2 + \frac{Mat}{ml}\frac{\dot{\theta}_2}{|\dot{\theta}_2|} + \frac{\cancel{m}gl}{2l}\theta_1 + \frac{\cancel{m}gl}{2l}\theta_2 = -\frac{\cancel{m}\cancel{l}}{2}\ddot{y} \end{cases}$$

$$\frac{l}{g}\begin{bmatrix} 8/3 & 5/6 \\ 5/6 & 1/3 \end{bmatrix}\begin{pmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \end{pmatrix} + \frac{Mat}{mgl} \cdot \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}\begin{pmatrix} \frac{\dot{\theta}_1}{|\dot{\theta}_1|} \\ \frac{\dot{\theta}_2}{|\dot{\theta}_2|} \end{pmatrix} + \begin{bmatrix} 2 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}\begin{pmatrix} \theta_1 \\ \theta_2 \end{pmatrix} = -\frac{\ddot{y}}{g}\begin{pmatrix} 2 \\ 1/2 \end{pmatrix} \quad (a)$$

Como já vimos o atrito seco não afeta as frequências naturais do sistema, portanto vamos ignorar as parcelas de atrito seco. Quanto aos modos de vibrar, se ajustarmos a dissipação de energia por atrito seco para um modo, para que ele se mantenha durante o decaimento linear até parar, um modo diferente de vibrar, com outras proporções de amplitude vai requerer outros fatores de atrito.

Ignoraremos o atrito e vamos calcular os modos sem dissipação.

Vamos buscar soluções harmônicas $\begin{pmatrix}\theta_1(t)\\ \theta_2(t)\end{pmatrix}=\begin{pmatrix}A_1\\ A_2\end{pmatrix}\text{sen}(\omega t+\phi)$ para o sistema homogêneo. Portanto:

$$\left\{-\underbrace{\frac{l}{g}\cdot\omega^2}_{\lambda}\begin{bmatrix}8/3 & 5/6\\ 5/6 & 1/3\end{bmatrix}+\begin{bmatrix}2 & 1/2\\ 1/2 & 1/2\end{bmatrix}\right\}\cdot\begin{pmatrix}A_1\\ A_2\end{pmatrix}=\begin{pmatrix}0\\ 0\end{pmatrix} \quad \text{ou}$$

$$\begin{bmatrix}2-\frac{8}{3}\lambda & \frac12-\frac56\lambda\\ \frac12-\frac56\lambda & \frac12-\frac{\lambda}{3}\end{bmatrix}\begin{pmatrix}A_1\\ A_2\end{pmatrix}=\begin{pmatrix}0\\ 0\end{pmatrix}$$

solução ≠ da trivial det = 0

$$\left(2-\frac83\lambda\right)\left(\frac12-\frac{\lambda}{3}\right)-\left(\frac12-\frac56\lambda\right)^2=0$$

$$\frac89\lambda^2-2\lambda+\underbrace{1-\frac14}-\frac{25}{36}\lambda^2+\frac56\lambda=0$$

$$\frac{7}{36}\lambda^2-\frac76\lambda+\frac34=0 \therefore \lambda^2-6\lambda+\frac{27}{7}=0$$

$$\lambda_{1,2}=3\mp\sqrt{9-\frac{27}{7}}=3\mp2,268$$

p/ $\lambda=\lambda_1=0,732 \Rightarrow \omega_1^2=0,732\,g/l \therefore \boxed{\omega_1=0,856\sqrt{g/l}}$ (b)

$+\cancel{3,856}\,0,048\,A_{11}-0,11A_{21}=0 \therefore \boxed{A_{21}=(2,3)A_{11}=0,43A_{11}}$

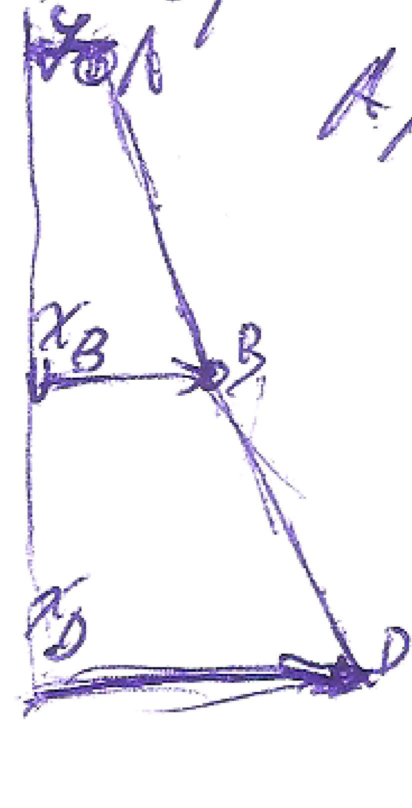
p/ $\lambda=\lambda_2=5,268 \Rightarrow \omega_2^2=5,268\,g/l \therefore \boxed{\omega_2=2,295\sqrt{g/l}}$

$-12,05\cdot A_{12}-3,89\cdot A_{22}=0$ $\boxed{A_{22}=-3,1A_{12}}$

Solução particular para $x_B(t)$ e $x_D(t)$ (valores absolutos)

Podemos calcular θ_{1p} & θ_{2p} e fazer $x_B(t)=y(t)+l\theta_{1p}(t)$ e $x_D(t)=y(t)+2l\theta_{1p}(t)+l\theta_{2p}(t)$

Vamos aqui redefinir as variáveis a título de exercício



Aproveitando o equacionamento anterior das barras e desprezando o atrito

$$\frac{ml^2}{3}\cdot\left(\frac{\ddot x_B-\ddot y}{l}\right)=-\frac{mg}{2}(x_B-y)-mg(x_B-y)+F\cdot l-\frac{ml}{2}\cdot\ddot y$$

$$m\left(\frac{\ddot x_B+\ddot x_D}{2}\right)=-F$$

$$\frac{ml^2}{3}\cdot\left(\frac{\ddot x_D-\ddot x_B}{l}\right)=-\frac{mg}{2}(x_D-x_B)-\frac{ml}{2}\cdot\ddot x_B$$

$$\frac{\cancel{m}l}{3}\cdot(\ddot{x}_B-\ddot{y})+\frac{3}{2}\cancel{m}g(x_B-y)+\frac{\cancel{m}l}{2}(\ddot{x}_B+\ddot{x}_D)=-\frac{\cancel{m}l}{2}\ddot{y}$$

$$\rightarrow\left(\frac{1}{3}+\frac{1}{2}\right)\ddot{x}_B+\frac{1}{2}\ddot{x}_D+\frac{3}{2}\frac{g}{l}x_B=-\frac{1}{6}\ddot{y}+\frac{3}{2}g/l\cdot y \quad (I)$$

$$\frac{\cancel{m}l}{3}\cdot(\ddot{x}_D-\ddot{x}_B)+\frac{\cancel{m}l}{2}\ddot{x}_B+\frac{\cancel{m}g}{2}x_D-\frac{\cancel{m}g}{2}x_B=0$$

$$\rightarrow\frac{1}{3}\ddot{x}_D+\frac{1}{6}\ddot{x}_B+\frac{g}{2l}x_D-\frac{g}{2l}x_B=0 \quad (\times 3)+(I)$$

$$\frac{3}{2}\ddot{x}_D+\frac{4}{3}\ddot{x}_B+\frac{3g}{2l}\cdot x_D=-\frac{1}{6}\ddot{y}+\frac{3}{2}g/l\; y \quad \left(\times\frac{3}{8}\right)$$

$$\begin{cases}\frac{5}{6}\ddot{x}_B+\frac{1}{2}\ddot{x}_D+\frac{3}{2}\frac{g}{l}x_B=-\frac{1}{6}\ddot{y}+\frac{3}{2}g/l\; y\\ \frac{9}{16}\ddot{x}_D+\frac{1}{2}\ddot{x}_B+\frac{9}{16}\cdot g/l\cdot x_D=-\frac{1}{16}\ddot{y}+\frac{9}{16}g/l\cdot y\end{cases}$$

observar que se calculássemos as frequências naturais nessas novas coordenadas, obteríamos exatamente os mesmos valores.

$$\begin{bmatrix}\frac{5}{6}&\frac{1}{2}\\ 1/2&9/16\end{bmatrix}\begin{pmatrix}\ddot{x}_B\\ \ddot{x}_D\end{pmatrix}+g/l\begin{bmatrix}3/2&0\\0&9/16\end{bmatrix}\begin{pmatrix}x_B\\x_D\end{pmatrix}=-\begin{pmatrix}1\\3/8\end{pmatrix}\frac{\ddot{y}}{6}+\begin{pmatrix}1\\3/8\end{pmatrix}\frac{3g}{2l}\cdot y$$

Vamos buscar $x_B(t)=X_B\,\mathrm{sen}(\omega_f t)$, $x_D(t)=X_D\,\mathrm{sen}(\omega_f t)$

mas $y(t)=Y\,\mathrm{sen}(\omega_f t)$, $\ddot{y}(t)=-Y\omega_f^2\,\mathrm{sen}(\omega_f t)$

$$\left\{-\omega_f^2\begin{bmatrix}5/6&1/2\\1/2&9/16\end{bmatrix}+g/l\begin{bmatrix}3/2&0\\0&9/16\end{bmatrix}\right\}\begin{pmatrix}X_B\\X_D\end{pmatrix}=\begin{pmatrix}1\\3/8\end{pmatrix}\cdot\left(\frac{3g}{2l}+\frac{\omega_f^2}{6}\right)\cdot Y$$

$$\begin{bmatrix}\frac{3}{2}-\frac{l\omega_f^2}{g}\frac{5}{6}&-\frac{l\omega_f^2}{g}\frac{1}{2}\\-\frac{l\omega_f^2}{g}\frac{1}{2}&\frac{9}{16}\left(1-\frac{l\omega_f^2}{g}\right)\end{bmatrix}\begin{pmatrix}X_B\\X_D\end{pmatrix}=\begin{pmatrix}1\\3/8\end{pmatrix}\cdot\left(\frac{3}{2}+\frac{l\omega_f^2}{6g}\right)\cdot Y$$

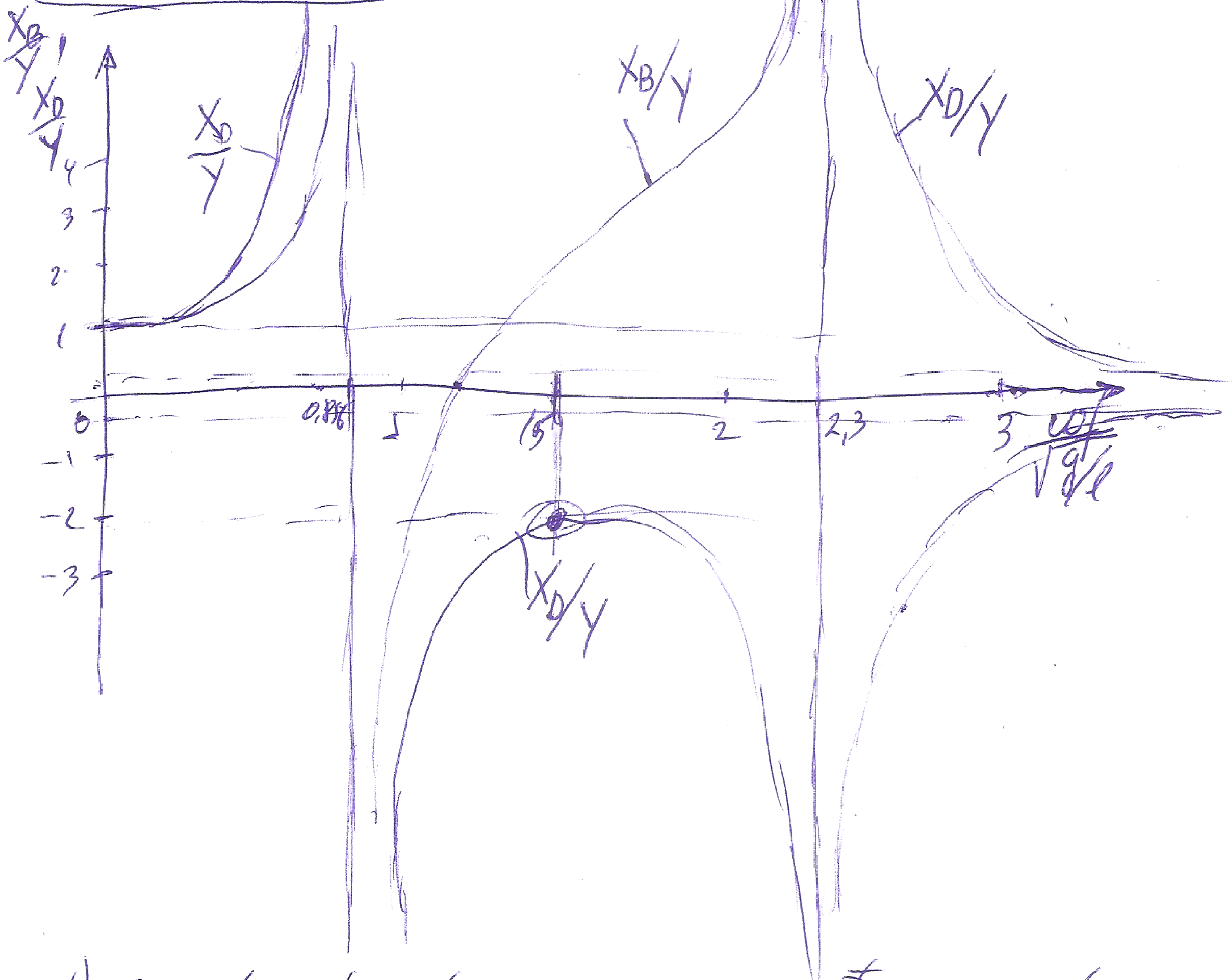
$$\frac{X_B}{Y}=\frac{\left[\frac{9}{16}\left(1-\frac{l\omega_f^2}{g}\right)+\frac{3}{16}\frac{l\omega_f^2}{g}\right]\left(\frac{3}{2}+\frac{l\omega_f^2}{6g}\right)}{\frac{9}{16}\left(1-\frac{l\omega_f^2}{g}\right)\left(\frac{3}{2}-\frac{5l\omega_f^2}{6g}\right)-\left(\frac{l\omega_f^2}{2g}\right)^2}=\frac{\left(\frac{9}{16}-\frac{6}{16}\frac{l\omega_f^2}{g}\right)\left(\frac{3}{2}+\frac{l\omega_f^2}{6g}\right)}{\frac{27}{32}\left(1-\left(\frac{\omega_f}{\omega_1}\right)^2\right)\left(1-\left(\frac{\omega_f}{\omega_2}\right)^2\right)}$$

frequências naturais

$$\boxed{\frac{X_B}{Y}=\frac{\left(1-\frac{2l\omega_f^2}{3g}\right)\cdot\left(1+\frac{l\omega_f^2}{9g}\right)}{\left(1-\left(\frac{\omega_f}{\omega_1}\right)^2\right)\left(1-\left(\frac{\omega_f}{\omega_2}\right)^2\right)}}\quad (C)$$

$$\frac{X_D}{Y} = \frac{\left(\frac{3}{2} - \frac{5l\omega_f^2}{6g}\right)\cdot\frac{3}{8} + \frac{l\omega_f^2}{2g}\left(\frac{3}{2} + \frac{l\omega_f^2}{6g}\right)}{\frac{27}{32}\cdot\left(1-\left(\frac{\omega_f}{\omega_1}\right)^2\right)\cdot\left(1-\left(\frac{\omega_f}{\omega_2}\right)^2\right)} = \frac{\frac{27}{32}\left(1 - \frac{5l\omega_f^2}{9g} + \frac{8l\omega_f^2}{9g}\right)\left(1+\frac{l\omega_f^2}{9g}\right)}{\frac{27}{32}\left(1-\left(\frac{\omega_f}{\omega_1}\right)^2\right)\left(1-\left(\frac{\omega_f}{\omega_2}\right)^2\right)}$$

$$\boxed{\frac{X_D}{Y} = \frac{\left(1+\frac{3l\omega_f^2}{9g}\right)\left(1+\frac{l\omega_f^2}{9g}\right)}{\left(1-\left(\frac{\omega_f}{\omega_1}\right)^2\right)\left(1-\left(\frac{\omega_f}{\omega_2}\right)^2\right)}} \quad (c)$$

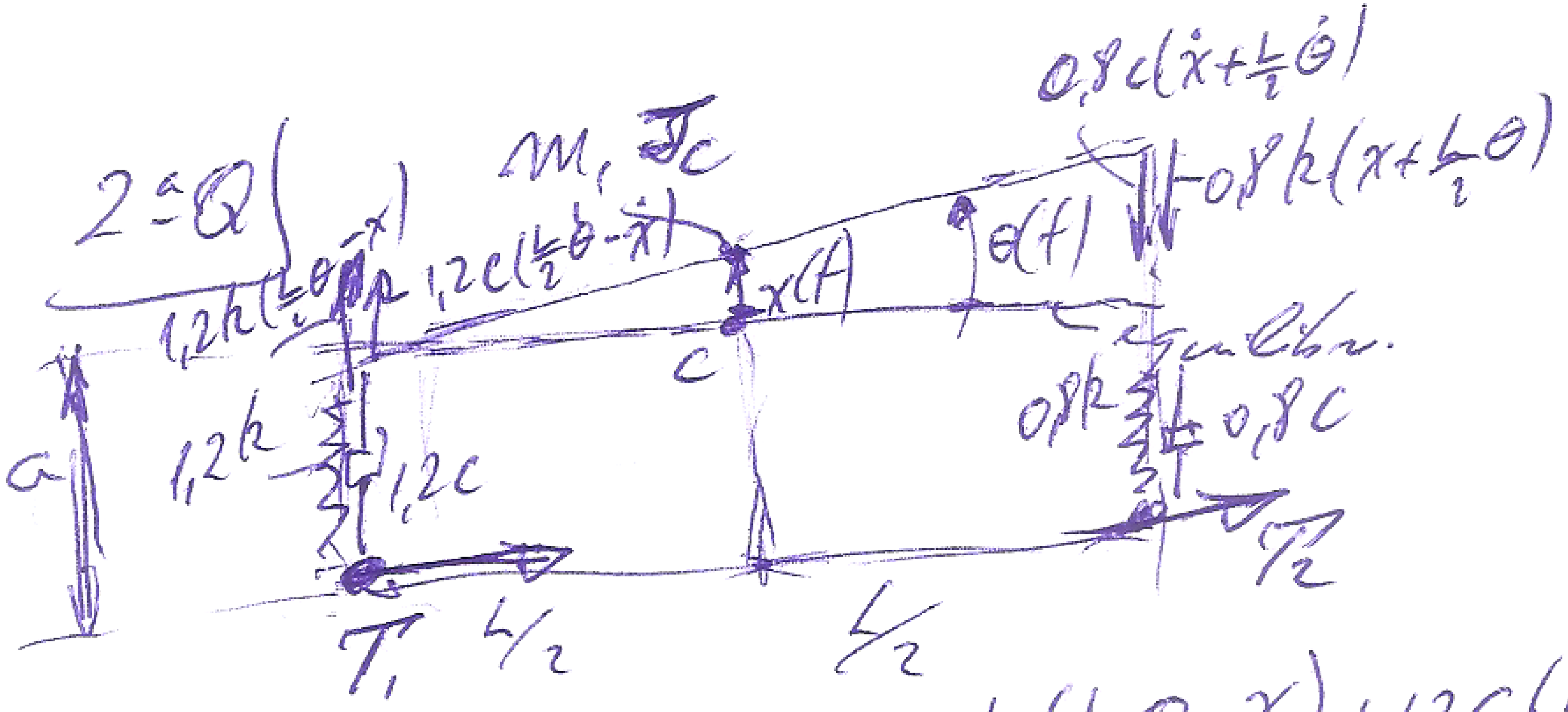


d) Fixando o absorvedor em D, com o sistema operando com $\omega_f = 1{,}5\sqrt{g/L}$, (L maiúsculo como no enunciado) levaríamos X_D para 0

Para o absorvedor funcionar $\sqrt{g/l}$ do absorvedor $= 1{,}5\sqrt{g/L}$ da barra.

$$\boxed{l = \frac{1}{1{,}5^2}\cdot L}$$

A massa do absorvedor deveria ser suficientemente grande para que a amplitude do movimento do absorvedor fosse limitada.



TMB → $m\ddot{x} = 1{,}2k\left(\frac{L}{2}\theta - x\right) + 1{,}2c\left(\frac{L}{2}\dot\theta - \dot x\right) - 0{,}8k\left(x+\frac{L}{2}\theta\right) + -0{,}8c\left(\dot x + \frac{L}{2}\dot\theta\right)$

$$TMA)_C \rightarrow J_c\ddot\theta = -0{,}8c\left(\dot x+\frac{L}{2}\dot\theta\right)\frac{L}{2} - 0{,}8k\left(x+\frac{L}{2}\theta\right)\frac{L}{2} + -1{,}2c\left(\frac{L}{2}\dot\theta-\dot x\right)\frac{L}{2} - 1{,}2k\left(\frac{L}{2}\theta - x\right)\frac{L}{2} + (T_1+T_2)\cdot a$$

Observar que as forças de tração ou frenagem do veículo aplicam momento em torno do eixo pelo centro de massa, e que $x \ll a$

$$\begin{bmatrix} m & 0 \\ 0 & \frac{mL^2}{4}\end{bmatrix}\begin{pmatrix}\ddot x\\ \ddot\theta\end{pmatrix} + c\begin{bmatrix} 1{,}2+0{,}8 & (0{,}8-1{,}2)\cdot\frac{L}{2} \\ -0{,}4\frac{L}{2} & (1{,}2+0{,}8)\frac{L^2}{4}\end{bmatrix}\begin{pmatrix}\dot x\\ \dot\theta\end{pmatrix} + k\begin{bmatrix} 2 & -0{,}4\frac{L}{2} \\ -0{,}4\frac{L}{2} & 2\frac{L^2}{4}\end{bmatrix}\begin{pmatrix}x\\ \theta\end{pmatrix} = \begin{pmatrix}0\\ a\end{pmatrix}(T_1(t)+T_2(t))$$

(a) ou

$$m\begin{bmatrix} 1 & 0 \\ 0 & \frac{1}{2}\end{bmatrix}\begin{pmatrix}\ddot x\\ \frac{L\ddot\theta}{2}\end{pmatrix} + c\begin{bmatrix} 2 & -0{,}4 \\ -0{,}2 & 2\cdot\frac{1}{2}\end{bmatrix}\begin{pmatrix}\dot x\\ \frac{L}{2}\dot\theta\end{pmatrix} + k\begin{bmatrix} 2 & -0{,}4 \\ -0{,}2 & 2/2\end{bmatrix}\begin{pmatrix}x\\ \frac{L}{2}\theta\end{pmatrix} = \begin{pmatrix}0\\ 0{,}2\end{pmatrix}(T_1(t)+T_2(t))$$

Observando que a matriz de amortecimento é proporcional à de rigidez, vamos resolver o sistema homogêneo não amortecido

$$\begin{pmatrix}x(t)\\ \frac{L\theta(t)}{2}\end{pmatrix} = \begin{pmatrix}A_1\\ A_2\end{pmatrix}\operatorname{sen}(\omega t+\phi) \Rightarrow \left\{-\underbrace{\frac{m\omega^2}{k}}_{\lambda}\cdot\begin{bmatrix}1 & 0\\ 0 & 1/2\end{bmatrix} + \begin{bmatrix}2 & -0{,}4\\ -0{,}2 & 1\end{bmatrix}\right\}\begin{pmatrix}A_1\\ A_2\end{pmatrix} = \begin{pmatrix}0\\ 0\end{pmatrix}$$

$$\begin{bmatrix}2-\lambda & -0{,}4\\ -0{,}2 & 1-\frac{\lambda}{2}\end{bmatrix}\begin{pmatrix}A_1\\ A_2\end{pmatrix} = \begin{pmatrix}0\\ 0\end{pmatrix}$$

solução ≠ da trivial ⇒ det = 0

$(2-\lambda)(1-\frac{\lambda}{2}) - 0{,}08 = 0$

$(2-\lambda)^2 = 0{,}16$ $\begin{cases}\lambda_1 = 1{,}6\\ \lambda_2 = 2{,}4\end{cases}$ (b)

p/ $\lambda = \lambda_1 = 1{,}434 \Rightarrow \boxed{\omega_1^2 = 1{,}434\frac{k}{m}}$

$0{,}566A_{11} - 0{,}4A_{21} = 0 \therefore \boxed{A_{21} = 1{,}415\cdot A_{11}}$

$$\begin{pmatrix}x_1(t)\\ \frac{L\theta_1(t)}{2}\end{pmatrix} = \begin{pmatrix}1\\ 1{,}415\end{pmatrix}(A_1\operatorname{sen}(\omega_1 t+\phi_1))$$

Para $\lambda = 1{,}6$, substituindo nas equações das amplitudes,

$(2-1{,}6)\cdot A_{11} - 0{,}4\cdot A_{21} = 0 \quad \therefore \boxed{A_{21} = A_{11}} \quad \boxed{\omega_1 = \sqrt{\frac{1{,}6\cdot k}{m}}}$

(b)

Para $\lambda = 2{,}4 = \lambda_2 \rightarrow \boxed{\omega_2 = \sqrt{\frac{2{,}4\cdot k}{m}}}$

$(2-2{,}4)\cdot A_{12} - 0{,}4 A_{21} = 0 \quad \therefore \boxed{A_{22} = -A_{12}}$

Portanto

$$\begin{pmatrix} x_1(t) \\ \frac{L\theta_1(t)}{2} \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \cdot [A\,\mathrm{sen}(\omega_1 t) + B\cos(\omega_1 t)]$$

$$\begin{pmatrix} x_2(t) \\ \frac{L\theta_2(t)}{2} \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix} [C\,\mathrm{sen}(\omega_2 t) + D\cos(\omega_2 t)]$$

A solução geral do sistema homogêneo não amortecido fica

$$\begin{pmatrix} x(t) \\ \frac{L\theta(t)}{2} \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} [A\,\mathrm{sen}(\omega_1 t) + B\cos(\omega_1 t)] + \begin{pmatrix} 1 \\ -1 \end{pmatrix} (C\,\mathrm{sen}\,\omega_2 t + D\cos(\omega_2 t))$$

Para obtermos os amortecimentos modais, uma vez que a matriz de amortecimento é proporcional e os modos do sistema não amortecido são os mesmos do sistema amortecido, basta utilizarmos os modos em qualquer das equações diferenciais.

Assim, para o 1º modo

$$m\ddot{x}_1 + 2c\dot{x}_1 - 0{,}4c\dot{x}_1 + 2kx_1 - 0{,}4kx_1 = 0$$

$$m\ddot{x}_1 + 1{,}6c\dot{x}_1 + 1{,}6kx_1 = 0 \qquad \zeta_1 = \frac{1{,}6c}{2\sqrt{1{,}6k\cdot m}} = \frac{1{,}6\sqrt{km}}{2\sqrt{1{,}6km}} = 0{,}63$$

(c)

Para o 2º modo $\frac{L\theta_2(t)}{2} = -x_2(t)$

$$m\ddot{x}_2 + 2c\dot{x}_2 + 0{,}4c\dot{x}_2 + 2kx_2 + 0{,}4kx_2 = 0$$

$$m\ddot{x}_2 + 2{,}4c\dot{x}_2 + 2{,}4k\,x_2 = 0 \rightarrow \zeta_2 = \frac{2{,}4c}{2\sqrt{2{,}4\cdot k\cdot m}} = 0{,}77$$

A solução geral do sistema homogêneo amortecido seria com

$\omega d_1 = \omega_1\sqrt{1-\zeta_1^2}$ e $\omega d_2 = \omega_2\cdot\sqrt{1-\zeta_2^2} \quad \therefore \begin{cases} \omega d_1 = 0{,}98\sqrt{\frac{k}{m}} \\ \omega d_2 = 0{,}99\sqrt{k/m} \end{cases}$

Observar que para os valores dados, as duas frequências das vibrações amortecidas em cada um dos modos são exatamente iguais. (Mera coincidência). A solução geral seria (para o sistema homogêneo)

$$\begin{pmatrix} x(t) \\ \frac{L\,\theta(t)}{2} \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-\xi_1\omega_1 t}[A\,sen(\omega_{d_1}t)+B\cos(\omega_{d_1}t)] + \begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-\xi_2\omega_2 t}[C\,sen(\omega_{d_2}t)+D\cos(\omega_{d_2}t)]$$

Para o automóvel com tração traseira, durante a aceleração $T_2 = 0$ e T_1 deve ser suficiente para acelerar o veículo ∴ $T_1 = 0{,}6 \cdot mg$ (constante enquanto durar a aceleração) (4 seg.)
A solução particular do sistema fica com $x_p = cte$ e $\theta_p = cte$ ∴ $\dot{x}_p = 0 = \dot{\theta}_p$; $\ddot{x}_p = 0 = \ddot{\theta}_p$

$$k\cdot\begin{bmatrix} 2 & -0{,}4 \\ -0{,}2 & 1 \end{bmatrix}\begin{pmatrix} x_p \\ \frac{L\theta_p}{2} \end{pmatrix} = \begin{pmatrix} 0 \\ 0{,}2 \end{pmatrix} \times 0{,}6\,mg \;\therefore\; \left.\begin{array}{l} 2\cdot x_p = 0{,}4\left(\frac{L\theta_p}{2}\right) \\ (-0{,}04+1)\cdot\left(\frac{L\theta_p}{2}\right) = 0{,}12\frac{mg}{k} \end{array}\right\} \begin{array}{l} \frac{L\theta_p}{2} = 0{,}125\frac{mg}{k} \\ x_p = 0{,}025\frac{mg}{k} \end{array}$$

A solução geral para o período de aceleração fica:

$$\begin{pmatrix} x(t) \\ \frac{L\theta(t)}{2} \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}[A\,sen(\omega_1 t)+B\cos(\omega_1 t)] + \begin{pmatrix} 1 \\ -1 \end{pmatrix}[C\,sen(\omega_2 t)+D\cos(\omega_2 t)] + \begin{pmatrix} 0{,}025 \\ 0{,}125 \end{pmatrix}\cdot\frac{mg}{k}$$

Derivando p/ impor as condições iniciais

$$\begin{pmatrix} \dot{x}(t) \\ \frac{L\dot{\theta}(t)}{2} \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}[A\omega_1\cos(\omega_1 t)-B\omega_1 sen(\omega_1 t)] + \begin{pmatrix} 1 \\ -1 \end{pmatrix}[C\omega_2\cos(\omega_2 t)-D\omega_2 sen(\omega_2 t)]$$

Condições iniciais

$$\text{p/ } t=0 \left\{\begin{array}{l} x(0)=0 \\ \dot{x}(0)=0 \\ \frac{L\theta(0)}{2}=0 \\ \dot{\theta}(0)=0 \end{array}\right\} \xrightarrow{\text{das derivadas}} A=0,\; C=0 \qquad \begin{pmatrix} 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}\cdot B + \begin{pmatrix} 1 \\ -1 \end{pmatrix}\cdot D + \begin{pmatrix} 0{,}025 \\ 0{,}125 \end{pmatrix}\frac{mg}{k}$$

$$\left.\begin{array}{l} B+D = -0{,}025\,mg \\ B-D = -0{,}125\,mg \end{array}\right\} \begin{array}{l} B = -0{,}075\frac{mg}{k} \\ D = 0{,}050\frac{mg}{k} \end{array}$$

Mas $\frac{k}{m} = 40\,rad/s^2$ ∴ $\omega_1 = \sqrt{1{,}6\frac{k}{m}} = 8\,rad/s$; $\omega_2 = \sqrt{2{,}4\frac{k}{m}} = 9{,}8\,rad/s$

$B = -18\,mm$

$D = 12\,mm$

$$\boxed{\begin{pmatrix} x(t) \\ \frac{L\theta(t)}{2} \end{pmatrix} = -\begin{pmatrix} 1 \\ 1 \end{pmatrix}\cdot 18(mm)\cdot\cos(8t) + \begin{pmatrix} 1 \\ -1 \end{pmatrix}\cdot 12(mm)\cdot\cos(9{,}8t) + \begin{pmatrix} 6\,mm \\ -30\,mm \end{pmatrix}} \quad (d)$$

Observem que, neste caso sem amortecedores, para 2,8 segundos $\frac{L\theta}{2} \cong 55\,mm$ e para 2,6 segundos $x \cong 27\,mm$ (velocidades pico a pico > 200 mm/s)