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Prova: 1) c) $\lim_{x\to 0} \frac{x^2 \operatorname{sen}(\frac{1}{x})}{\operatorname{sen} x}$

$$\lim_{x\to 0} \frac{x^2 \operatorname{sen}(\frac{1}{x})}{\operatorname{sen} x} = \lim_{x\to 0} \underbrace{\frac{x}{\operatorname{sen} x}}_{\to 1} \cdot \underbrace{x}_{\to 0} \cdot \underbrace{\operatorname{sen}\left(\frac{1}{x}\right)}_{\text{limitada}} = 0$$

Resultados: 1) $\lim_{x\to 0} \frac{\operatorname{sen} x}{x} = 1$ (limite fundamental)

2) $\lim_{x\to 0} \frac{x}{\operatorname{sen} x} = \lim_{x\to 0} \frac{1}{\frac{\operatorname{sen} x}{x}} = \frac{1}{1} = 1$

3) $\lim_{x\to x_0} f(x) = 0$, $g(x)$ limitada em torno do x_0

$\Rightarrow \lim_{x\to x_0} f(x)g(x) = 0$

g é limitada em torno do x_0 se existe $r>0$ e $M>0$ tq. $|g(x)| \leq M$, $\forall x \in D_g$ com $0<|x-x_0|<r$

$|g(x)| \leq M \Rightarrow -M \leq g(x) \leq M$

se $g(x) = \operatorname{sen} x$, então $-1 \leq \operatorname{sen} x \leq 1$, $\forall x \in \mathbb{R}$

3) Derivar: d) $f(x) = (1+\operatorname{arctg}(x^2))^{10}$

$y = \operatorname{arctg} x \Rightarrow y' = \frac{1}{1+x^2}$

$y = f(g(x)) \Rightarrow y' = f'(g(x)) \cdot g'(x)$ (Regra da cadeia)

$f'(x) = 10(1+\operatorname{arctg}(x^2))^9 (1+\operatorname{arctg}(x^2))'$

$= 10(1+\operatorname{arctg}(x^2))^9 (0 + \operatorname{arctg}'(x^2) \cdot (x^2)')$

$= 10(1+\operatorname{arctg}(x^2))^9 \frac{1}{1+(x^2)^2} \cdot 2x$

$= 20x(1+\operatorname{arctg}(x^2))^9$

$$= \frac{20x(1+\operatorname{arctg}(x^2))^9}{1+x^4}$$

INTEGRAIS INDEFINIDAS (PRIMITIVAÇÃO)

Dizemos que F é uma primitiva da f se $F' = f$

Se F e G têm mesma derivada no intervalo aberto I ($F'(x) = G'(x)$, $\forall x \in I$), então existe $k \in \mathbb{R}$ tq.

$$G(x) - F(x) = k, \ \forall x \in I$$

Notação: $\int f(x)\,dx = F(x) + k$, onde F é uma primitiva da f

chamada de integral indefinida

$$\boxed{\int F'(x)\,dx = F(x) + k}$$

Integrais imediatas:

1) $\int x^n\,dx = \frac{x^{n+1}}{n+1} + k$, se $n \neq -1$, pois

$$\left(\frac{x^{n+1}}{n+1}\right)' = \frac{(n+1)x^{n+1-1}}{n+1} = x^n$$

2) $\int x^{-1}\,dx = \int \frac{1}{x}\,dx = \ln x + k$, $(\ln x)' = \frac{1}{x}$

3) $\int \cos x\,dx = \operatorname{sen} x + k$, $(\operatorname{sen} x)' = \cos x$

4) $\int \operatorname{sen} x\,dx = -\cos x + k$ $\quad (-\cos x)' = +\operatorname{sen} x$

5) $\int e^x\,dx = e^x + k$

6) $\int a^x\,dx = \frac{a^x}{\ln a} + k$, $\left(\frac{a^x}{\ln a}\right)' = \frac{a^x \ln a}{\ln a} = a^x$, $a > 0$ e $a \neq 1$

$$y = a^x \Rightarrow y' = a^x \ln a$$

7) $\int \sqrt[n]{x^m}\,dx = \int x^{\frac{m}{n}}\,dx = \frac{x^{\frac{m}{n}+1}}{\frac{m}{n}+1} + k$, $\frac{m}{n} \neq -1$

$\frac{m}{n}+1$

8) $y = \operatorname{tg} x \Rightarrow y' = \sec^2 x$

$$\Rightarrow \int \sec^2 x \, dx = \operatorname{tg} x + k$$

9) $y = \sec x \Rightarrow y' = \sec x \operatorname{tg} x$

$$\Rightarrow \int \sec x \operatorname{tg} x \, dx = \sec x + k$$

10) $\displaystyle \int \sqrt{x} \, dx = \int x^{\frac{1}{2}} dx = \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} + k = \frac{2}{3} x^{\frac{3}{2}} + k = \frac{2\sqrt{x^3}}{3} + k$

Integrais trancendentes, são integrais de funções onde não é possível escrever por meio de funções conhecidas, como polinômio, seno, cosseno, funcao exponencial, funçao logaritmo, etc

Exemplo: $\int e^{x^2} dx$ é uma integral transcendente

MÉTODOS DE INTEGRAÇÃO

1) Substituição

$$y = f(g(x)) \Rightarrow y' = f'(g(x))\, g'(x)$$

$$\int f(g(x))\, g'(x)\, dx$$

pensar como um quociente

$$u = g(x) \Rightarrow u' = g'(x) \Rightarrow \frac{du}{dx} = g'(x) \Rightarrow du = g'(x)\, dx$$

$$\int f(g(x))\, g'(x)\, dx = \int f(u)\, du = F(u) + k = F(g(x)) + k,$$

onde $F' = f$

Exemplos: 1) $\displaystyle \int x^2 \sqrt{1+x^3}\, dx$

$$u = 1 + x^3 \Rightarrow \frac{du}{dx} = 3x^2 \Rightarrow \frac{du}{3} = x^2 dx$$

$$\int x^2 \sqrt{1+x^3}\, dx = \int \frac{1}{3} \sqrt{u}\, du = \frac{1}{3} \int \sqrt{u}\, du = \frac{1}{3} \cdot \frac{2}{3} \sqrt{u^3} + k$$

$$= \frac{2}{9} \sqrt{(1+x^3)^3} + k$$

Verificação: $y = \frac{2}{9}\sqrt{(1+x^3)^3} \Rightarrow y' = x^2\sqrt{1+x^3}$

$$y = \frac{2}{9}(1+x^3)^{\frac{3}{2}} \Rightarrow y' = \frac{2}{9}\cdot\frac{3}{2}(1+x^3)^{\frac{3}{2}-1}(1+x^3)'$$

$$= \frac{1}{3}(1+x^3)^{\frac{1}{2}}\cdot 3x^2 = x^2(1+x^3)^{\frac{1}{2}} = x^2\sqrt{1+x^3}$$

PROPRIEDADES: 1) $\int (f(x) \pm g(x))\,dx = \int f(x)\,dx \pm \int g(x)\,dx$

2) $\int k f(x)\,dx = k\int f(x)\,dx$

Seja F uma primitiva da f ($F' = f$)

$(kF)' = kF' = kf \Rightarrow kF$ é uma primitiva de kf

Exemplos: 1) Mostre que $\boxed{\int \frac{1}{x}\,dx = \ln|x| + k}$, $x \in \mathbb{R}$, $x \neq 0$

$x > 0 \Rightarrow \int \frac{1}{x}\,dx = \ln x + k = \ln|x| + k$

$|x| = x$, se $x \geq 0$
$|x| = -x$, se $x < 0$

$x < 0 \Rightarrow y = \ln|x| = \ln(-x)$

$$y' = \frac{1}{-x}\cdot(-x)' = \frac{1}{-x}\cdot(-1) = \frac{1}{x}$$

$$\Rightarrow \int \frac{1}{x}\,dx = \ln(-x) + k = \ln|x| + k$$

2) $\int \operatorname{tg} x\,dx = \int \frac{\operatorname{sen} x}{\cos x}\,dx$

$u = \cos x \Rightarrow \frac{du}{dx} = u' = -\operatorname{sen} x \Rightarrow -du = \operatorname{sen} x\,dx$

$$\int \operatorname{tg} x\,dx = \int \frac{\operatorname{sen} x}{\cos x}\,dx = -\int \frac{du}{u} = -\ln|u| + k$$

$$\Rightarrow \int \operatorname{tg} x\,dx = -\ln|\cos x| + k$$

3) $\int \frac{\ln x}{x}\,dx$

$u = \ln x \Rightarrow u' = \frac{1}{x} \Rightarrow \frac{du}{dx} = \frac{1}{x} \Rightarrow du = \frac{1}{x}\,dx$

$$\int \frac{\ln x}{x}\,dx = \int u\,du = \frac{u^2}{2} + k = \frac{(\ln x)^2}{2} + k$$

INTEGRAÇÃO POR PARTES

Regra do produto

$$\big(f(x)\,g(x)\big)' = f'(x)\,g(x) + f(x)\,g'(x)$$

$$\Longrightarrow \quad f'(x)\,g(x) = \big(f(x)\,g(x)\big)' - f(x)\,g'(x)$$

$$\Longrightarrow \quad \int f'(x)\,g(x)\,dx = \int \big(f(x)\,g(x)\big)'\,dx - \int f(x)\,g'(x)\,dx$$

$$\Longrightarrow \quad \int f'(x)\,g(x)\,dx = f(x)\,g(x) - \int f(x)\,g'(x)\,dx$$

$u = g(x) \Longrightarrow du = g'(x)\,dx$

$dv = f'(x)\,dx \Longrightarrow v = \int f'(x)\,dx = f(x)$

$$\boxed{\int u\,dv = uv - \int v\,du}$$

Exemplos: 1) $\int x \operatorname{sen} x\,dx$

$u = x \Rightarrow du = dx$

$dv = \operatorname{sen} x\,dx \Rightarrow v = \int \operatorname{sen} x\,dx = -\cos x$

$$\int x \operatorname{sen} x\,dx = -x\cos x + \int \cos x\,dx = -x\cos x + \operatorname{sen} x + k$$

verificação: $(-x\cos x + \operatorname{sen} x)' = x \operatorname{sen} x$

$(-x\cos x + \operatorname{sen} x)' = -\cos x + x\operatorname{sen} x + \cos x = x \operatorname{sen} x$

2) $\int x^r \ln x\,dx, \quad r \in \mathbb{R}$

1) $r = 0 \Rightarrow \int \ln x\,dx =$

$u = \ln x \Rightarrow du = \frac{1}{x}\,dx$ $\qquad \int u\,dv = uv - \int v\,du$

$dv = dx \Rightarrow v = x$

$$\int \ln x\,dx = x\ln x - \int x\,\frac{1}{x}\,dx = x\ln x - \int dx = x\ln x - x + k$$

2) $r = -1 \Rightarrow \int x^{-1}\ln x\,dx = \int \frac{\ln x}{x}\,dx = (\ln x)^2 \ldots$

$\int \ln x \, dx = x \ln x - \int x \frac{1}{x} dx = x \ln x - \int dx = x \ln x - x + k$

2) $r = -1 \Rightarrow \int x^{-1} \ln x \, dx = \int \frac{\ln x}{x} dx = \frac{(\ln x)^2}{2} + k$

3) $r \neq 0$ e $r \neq -1$

$u = \ln x \Rightarrow du = \frac{1}{x} dx$

$dv = x^r dx \Rightarrow v = \frac{x^{r+1}}{r+1}$

$$\int x^r \ln x \, dx = \frac{x^{r+1} \ln x}{r+1} - \int \frac{x^{r+1}}{r+1} \cdot \frac{1}{x} dx$$

$$= \frac{x^{r+1} \ln x}{r+1} - \frac{1}{r+1} \int x^r dx$$

$$= \frac{x^{r+1} \ln x}{r+1} - \frac{1}{r+1} \cdot \frac{x^{r+1}}{r+1} + k$$

$$= \frac{x^{r+1} \ln x}{r+1} - \frac{x^{r+1}}{(r+1)^2} + k$$

$$\int x^r \ln x \, dx = \begin{cases} \frac{(\ln x)^2}{2} + k, & \text{se } r = -1 \\ \frac{x^{r+1} \ln x}{r+1} - \frac{x^{r+1}}{(r+1)^2} + k, & \text{se } r \neq -1 \end{cases}$$

4) $\int e^x \operatorname{sen} x \, dx$

$u = e^x \Rightarrow du = e^x dx$

$dv = \operatorname{sen} x \, dx \Rightarrow v = -\cos x$

$$\int e^x \operatorname{sen} x \, dx = -e^x \cos x + \int e^x \cos x \, dx \quad (*)$$

$\int e^x \cos x \, dx$

$u = e^x \Rightarrow du = e^x dx$

$dv = \cos x \, dx \Rightarrow v = \operatorname{sen} x$

$$\int e^x \cos x \, dx = e^x \operatorname{sen} x - \int e^x \operatorname{sen} x \, dx$$

$(*) \int e^x \operatorname{sen} x \, dx = -e^x \cos x + e^x \operatorname{sen} x - \int e^x \operatorname{sen} x \, dx$

(*) $\int e^x \operatorname{sen} x\,dx = -e^x \cos x + e^x \operatorname{sen} x - \int e^x \operatorname{sen} x\,dx$

$$\Rightarrow 2\int e^x \operatorname{sen} x\,dx = e^x(\operatorname{sen} x - \cos x)$$

$$\Rightarrow \int e^x \operatorname{sen} x\,dx = \frac{e^x}{2}(\operatorname{sen} x - \cos x) + k$$

Verificação: $y = \frac{e^x}{2}(\operatorname{sen} x - \cos x) \Rightarrow y' = e^x \operatorname{sen} x$

$$y' = \frac{e^x}{2}(\operatorname{sen} x - \cos x) + \frac{e^x}{2}(\cos x + \operatorname{sen} x)$$

$$= \frac{e^x}{2}(\operatorname{sen} x - \cancel{\cos x} + \cancel{\cos x} + \operatorname{sen} x) = \frac{e^x}{2}(2\operatorname{sen} x)$$

$$= e^x \operatorname{sen} x$$

5) $\int \sec x\,dx = \int \sec x \frac{\sec x + \operatorname{tg} x}{\sec x + \operatorname{tg} x}\,dx = \int \frac{\sec^2 x + \sec x \operatorname{tg} x}{\sec x + \operatorname{tg} x}\,dx$

$u = \sec x + \operatorname{tg} x$

$du = (\sec x \operatorname{tg} x + \sec^2 x)\,dx$

$$\int \sec x\,dx = \int \frac{du}{u} = \ln|u| + k = \ln|\sec x + \operatorname{tg} x| + k$$

$$\Rightarrow \int \sec x\,dx = \ln|\sec x + \operatorname{tg} x| + k$$

6) $\int \sec^3 x\,dx = \int \sec x \sec^2 x\,dx$

$\cos^2 x + \operatorname{sen}^2 x = 1 \Rightarrow \frac{\cos^2 x}{\cos^2 x} + \frac{\operatorname{sen}^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} \Rightarrow$

$$\Rightarrow \boxed{1 + \operatorname{tg}^2 x = \sec^2 x}$$

$$\int \sec^3 x\,dx = \int \sec x(1 + \operatorname{tg}^2 x)\,dx = \int \sec x\,dx + \int \sec x \operatorname{tg}^2 x\,dx$$

$$= \ln|\sec x + \operatorname{tg} x| + \int \sec x \operatorname{tg}^2 x\,dx$$

$$\int \sec x \operatorname{tg}^2 x\,dx = \int \sec x \operatorname{tg} x \operatorname{tg} x\,dx$$

$u = \operatorname{tg} x \Rightarrow du = \sec^2 x\,dx$

$dv = \sec x \operatorname{tg} x \, dx \Rightarrow v = \sec x$

$$\int \sec x \operatorname{tg}^2 x \, dx = \sec x \operatorname{tg} x - \int \sec x \sec^2 x \, dx$$

$$= \sec x \operatorname{tg} x - \int \sec^3 x \, dx$$

$$\int \sec^3 x \, dx = \ln|\sec x + \operatorname{tg} x| + \int \sec x \operatorname{tg}^2 x \, dx$$

$$= \ln|\sec x + \operatorname{tg} x| + \sec x \operatorname{tg} x - \int \sec^3 x \, dx$$

$$\Rightarrow 2\int \sec^3 x \, dx = \ln|\sec x + \operatorname{tg} x| + \sec x \operatorname{tg} x$$

$$\Rightarrow \int \sec^3 x \, dx = \frac{1}{2}\left[\ln|\sec x + \operatorname{tg} x| + \sec x \operatorname{tg} x\right] + k$$