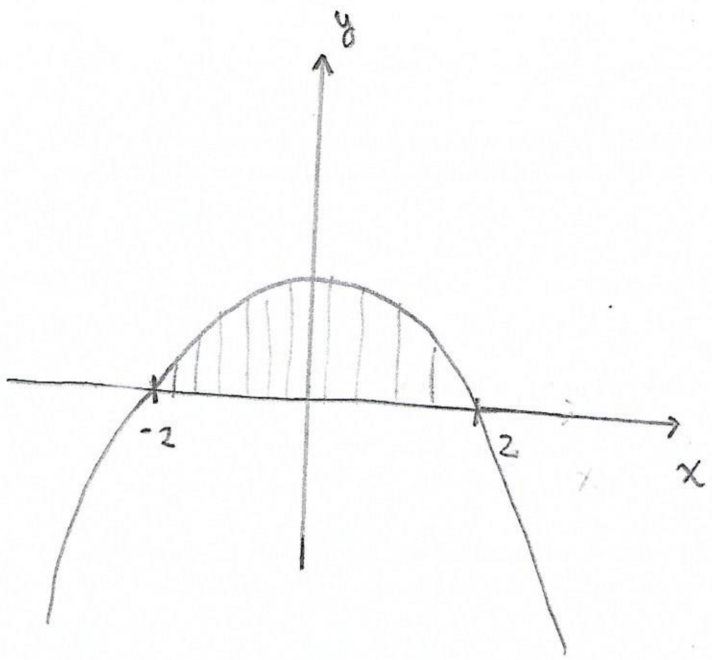
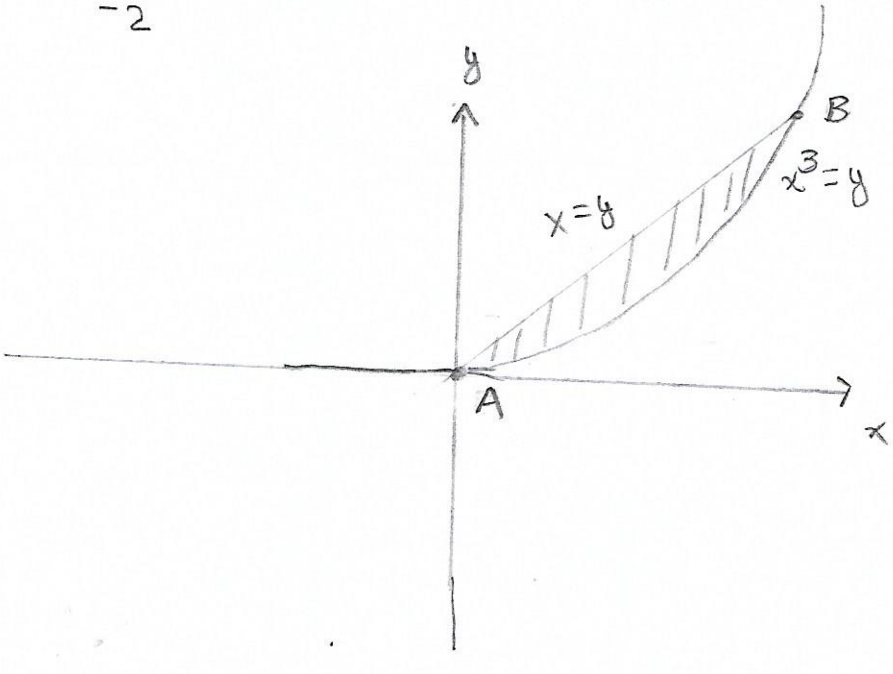
3.



$$\text{Área} = \int_{-2}^{2} (4-x^2)\,dx = \left[4x - \frac{x^3}{3}\right]_{-2}^{2} = 16 - \frac{16}{3} = \frac{32}{3}.$$

6.



Determinando os pontos de encontro:

$x^3 = x \iff x(x+1)(x-1) = 0 \iff x=0$ ou $x=1$ ou $\cancel{x=-1}$ $x \geq 0$

$\therefore A = (0,0)$, $B = (1,1)$.

$$\therefore \text{Área} = \int_{0}^{1} (x - x^3)\,dx = \left[\frac{x^2}{2} - \frac{x^4}{4}\right]_0^1 = \frac{1}{4}.$$ (Em $(0,1)$, $x > x^3$)

9.

(e) $\displaystyle\int_0^1 x e^{x^2} dx = \frac{1}{2}\int_0^1 e^{\overset{u}{\widetilde{x^2}}} \cdot \overset{du}{\widetilde{(2x\,dx)}} = \frac{1}{2}\int_0^1 e^u\, du = \frac{1}{2}\left[e^u\right]_0^1 = \frac{1}{2}(e-1)$

$u = x^2$, $du = 2x\,dx$. $\left(\begin{array}{l} x=0 : u=0 \\ x=1 : u=1 \end{array}\right)$

(g) $\displaystyle\int_0^1 \frac{x^2}{(1+x^3)^2}\, dx$. Se $u = x^3+1$, $du = 3x^2 dx$
$\left(\begin{array}{l} x=0 : u=1 \\ x=1 : u=2 \end{array}\right)$

$\therefore \displaystyle\int_0^1 \frac{x^2}{(1+x^3)^2}\, dx = \frac{1}{3}\int_1^2 \frac{1}{u^2}\, du = \frac{1}{3}\left[-\frac{1}{u}\right]_1^2 = \frac{1}{6}$

(h) $\displaystyle\int_{-1}^1 x(1+x)^{100}\, dx$. Se $u = 1+x$, i.e., $x = u-1$, $dx = du$
$\left(\begin{array}{l} x=-1 : u=0 \\ x=1 : u=2 \end{array}\right)$

$\therefore \displaystyle\int_0^2 (u-1)\, u^{100}\, du = \int_0^2 u^{101} - u^{100}\, du = \left[\frac{1}{102}u^{102} - \frac{u^{101}}{101}\right]_0^2 =$

$= \dfrac{2^{102}}{102} - \dfrac{2^{101}}{101}$

(n) $\displaystyle\int_0^{\pi/3} \operatorname{sen}(x)\cos^2(x)\, dx$. Se $u = \cos(x)$, $du = -\operatorname{sen}(x)dx$
$\left[\begin{array}{l} x=0 : u=1 \\ x=\pi/3 : u=1/2 \end{array}\right]$

$\therefore -\displaystyle\int_0^{\pi/3} \cos^2(x)\,(-\operatorname{sen}(x)\,dx) = -\int_1^{1/2} u^2\, du = \int_{1/2}^1 u^2\, du = \left[\frac{u^3}{3}\right]_{1/2}^1 = \frac{7}{24}$.

13.

5. $\displaystyle\int e^x \sqrt{1+e^x}\, dx$. Fazendo $u = e^x + 1$, $du = e^x dx$, e assim:

$\displaystyle\int e^x \sqrt{1+e^x}\, dx = \int \sqrt{1+e^x}\; e^x dx = \int \sqrt{u}\, du = \frac{2}{3}\sqrt{(e^x+1)^3} + K$

10. $\int \frac{x+2}{x-1}\,dx$. Fazendo $u = x-1$, $du = dx$, e assim:

$$\int \frac{x+2}{x-1}\,dx = \int \frac{u+3}{u}\,du = \int 1 + \frac{3}{u}\,du.$$ Como

$$\int \left(1+\frac{3}{u}\right)du = u + 3\ln|u| + K, \quad \int \frac{x+2}{x-1}\,dx = (x-1)+3\ln|x-1| + K$$
$$(\text{OU} \quad x+3\ln|x-1|+K)$$

---

15. $\int \frac{3x+2}{1+x^2}\,dx$. Vamos calcular, separadamente:

$\int \frac{3x}{1+x^2}\,dx$ e $\int \frac{2}{1+x^2}\,dx$. Temos que:

$\int \frac{2}{1+x^2}\,dx = 2\arctan(x) + K_1$ e, fazendo $u = x^2+1$, $du = 2x\,dx$

$$\int \frac{3x}{1+x^2}\,dx = \frac{3}{2}\int \frac{1}{u}\,du = \frac{3}{2}\ln|x^2+1| + K_2.$$ Portanto:

$$\int \frac{3x+2}{1+x^2}\,dx = 2\arctan(x) + \frac{3}{2}\ln(x^2+1) + K.$$

---

16. $\int \frac{1}{x\ln(x)}\,dx$. Fazendo $u = \ln(x)$, $du = \frac{1}{x}\,dx$, e assim:

$$\int \frac{1}{x\ln(x)}\,dx = \int \frac{1}{u}\,du = \ln(|\ln(x)|) + K.$$

---

20. $\int \frac{e^x}{\sqrt{1-e^{2x}}}\,dx$. Fazendo $u = e^x$, $du = e^x\,dx$, e assim:

$$\int \frac{e^x}{\sqrt{1-(e^x)^2}}\,dx = \int \frac{1}{\sqrt{1-u^2}}\,du = \arcsen(e^x) + K$$

24. $\int \frac{x^3}{1+x^8}\,dx$. Fazendo $u = x^4$, $du = 4x^3\,dx$, e assim:

$$\frac{1}{4}\int \frac{(4x^3)}{1+x^8}\,dx = \frac{1}{4}\int \frac{1}{1+u^2}\,du = \frac{1}{4}\arctan(x^4) + K.$$

15 (b) $\int_1^2 \underset{\substack{\downarrow \\ f}}{\ln(x)} \cdot \underset{\substack{\downarrow \\ g'}}{1}\,dx = [f(x)\,g(x)]_1^2 - \int_1^2 f'(x)\,g(x)\,dx =$

$$= [\ln(x)\cdot x]_1^2 - \int_1^2 \frac{1}{x}\cdot x\,dx = 2\ln(2) - (2-1) = 2\ln(2) - 1.$$

16. (h) $\int \sqrt{-x^2+2x+2}\,dx = \int \sqrt{3-(x-1)^2}\,dx = \sqrt{3}\int \sqrt{1-\left(\frac{x-1}{\sqrt{3}}\right)^2}\,d$

Fazendo $\mathrm{sen}(u) = \frac{x-1}{\sqrt{3}} \iff x = \sqrt{3}\,\mathrm{sen}(u) + 1 = \varphi(u)$;

$$\left(\theta(x) = u = \mathrm{arcsen}\left(\frac{x-1}{\sqrt{3}}\right) \quad -\frac{\pi}{2} < u < \frac{\pi}{2}\right)$$

$dx = \sqrt{3}\cos(u)\,du$. Então:

$$\sqrt{3}\int \sqrt{1-\left(\frac{x-1}{\sqrt{3}}\right)^2}\,dx = \sqrt{3}\int \underset{\substack{\| \\ |\cos(u)| = \cos(u)}}{\sqrt{\cos^2(u)}}\cdot\sqrt{3}\cos(u)\,du = 3\int \cos^2(u)\,du$$

$|\cos(u)| = \cos(u)$, pois $-\frac{\pi}{2} < u < \frac{\pi}{2}$

Como $\cos^2(u) = \frac{\cos(2u)+1}{2}$, temos que:

$$3\int \cos^2(u)\,du = 3\left[\frac{\mathrm{sen}(2u)}{4} + \frac{u}{2}\right] + K.$$

Agora, tendo em vista que $\mathrm{sen}(2u) = 2\,\mathrm{sen}(u)\cos(u)$

e $\underset{-\frac{\pi}{2} < u < \frac{\pi}{2}}{\cos(u) = \sqrt{1-\mathrm{sen}^2(u)}}$, concluímos que:

$$\int \sqrt{-x^2+2x+2}\,dx = 3\left[\frac{1}{2}\left(\frac{x-1}{\sqrt{3}}\right)\sqrt{\frac{-x^2+2x+2}{3}} + \frac{1}{2}\mathrm{arcsen}\left(\frac{x-1}{\sqrt{3}}\right)\right] + K$$

18.

(a) $\displaystyle\int \frac{1}{1+\sqrt{x}}\,dx$. Fazendo $u = 1+\sqrt{x} \Leftrightarrow x = \overbrace{(u-1)^2}^{\varphi(u)}$, $u \in [1, \infty)$

$dx = 2(u-1)\,du$

$$\int \frac{1}{1+\sqrt{x}}\,dx = \int \frac{1}{u}\cdot 2(u-1)\,du = \int \left(2-\frac{2}{u}\right)du = 2u - 2\ln\overbrace{|u|}^{u} + k =$$

$$= 2(1+\sqrt{x}) - 2\ln(1+\sqrt{x}) + k \quad \left(\text{ou } 2\sqrt{x} - 2\ln(1+\sqrt{x}) + k\right)$$

(b) $\displaystyle\int \sqrt{1-e^{-x}}\,dx$. Fazendo $u = \sqrt{1-e^{-x}} \Leftrightarrow x = \ln(1-u^2)$

$0 \leq u < 1$

$dx = \dfrac{1}{(1-u^2)}\cdot(-2u)\,du$. Portanto:

$$\int \sqrt{1-e^{-x}}\,dx = \int \frac{u\cdot(-2u)}{(1-u^2)}\,du = \int \frac{-2u^2}{(1-u^2)}\,du$$

Note que $\dfrac{u^2}{(1-u^2)} = \dfrac{1-(1-u^2)}{(1-u^2)} = \dfrac{1}{(1-u^2)} - 1$;

e $\dfrac{1}{(1-u^2)} = \dfrac{1}{(1+u)(1-u)} = \left[\dfrac{1}{(1+u)} + \dfrac{1}{(1-u)}\right]\cdot\dfrac{1}{2}$. Portanto:

$$-2\int \frac{u^2}{(1-u^2)}\,du = -2\int \left[\frac{1}{2}\left(\frac{1}{(1+u)} + \frac{1}{(1-u)}\right) - 1\right] du =$$

$$= -2\left[\frac{1}{2}\ln(1+u) - \frac{1}{2}\ln(1-u) - u\right] + k =$$

$$= -\ln\left(1+\sqrt{1-e^{-x}}\right) + \ln\left(1-\sqrt{1-e^{-x}}\right) + 2\sqrt{1-e^{-x}} + k$$

1e)

$$\int \frac{\arctan(e^{x})}{e^{x}}\,dx.$$ Fazendo $u = e^{x} \iff x = \ln(u)$; $dx = \frac{1}{u}\,du$

$$\therefore \int \frac{\arctan(e^{x})}{e^{x}}\,dx = \int \frac{\arctan(u)}{u^2}\,du = \int \underbrace{\frac{1}{u^2}}_{f'} \cdot \underbrace{\arctan(u)}_{g}\,du$$

$$= -\frac{1}{u}\arctan(u) + \int \frac{1}{u(1+u^2)}\,du = -\frac{\arctan(u)}{u} + \int \left[\frac{1}{u} - \frac{u}{1+u^2}\right]du$$

$$= -\frac{\arctan(u)}{u} + \ln(u) - \frac{\ln(1+u^2)}{2} + k =$$

$$= -\frac{\arctan(e^{x})}{e^{x}} + \ln(e^{x}) - \frac{\ln(1+e^{2x})}{2} + k$$

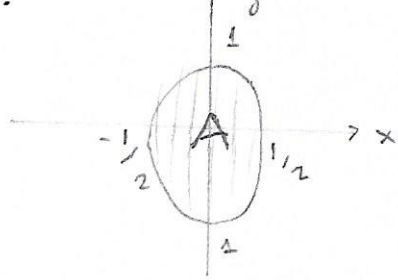
$$\int \frac{2x+1}{x^2+4x+5}\,dx \quad ; \quad x^2+4x+5 = (x+2)^2 + 1$$

$$u = (x+2) \iff x = u-2 \; ; \; dx = du$$

$$\therefore \int \frac{2(u-2)+1}{u^2+1}\,du = \int \frac{2u}{u^2+1}\,du + \int \frac{1}{u^2+1}\,du =$$

$$= \ln(u^2+1) + \arctan(u) + K = \ln(x^2+4x+5) + \arctan(x+2) + k.$$

17.



$$A = \int_{-1/2}^{1/2} \sqrt{1-4x^2}\,dx - \int_{-1/2}^{1/2} -\sqrt{1-4x^2}\,dx$$

$$= 2\int_{-1/2}^{1/2} \sqrt{1-4x^2}\,dx$$

Para resolver a última integral, faça $\operatorname{sen}(u) = 2x$.