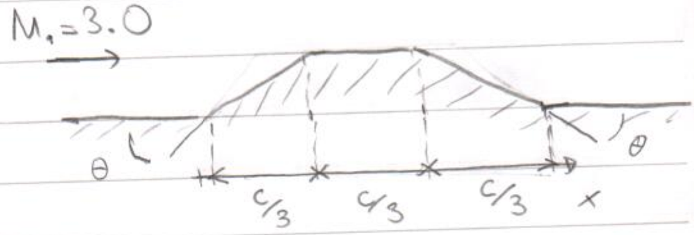
Aerodynamics Class. PME-EPUSP

Prof. Ernani V. Volpe.

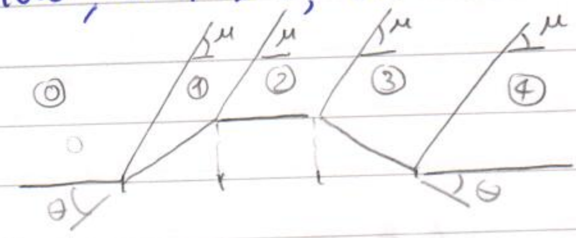
2-D supersonic flow over a "bump". Find the pressure distribution and compute the wave drag. First do it by using the linearized potential flow theory, and then do it with the shock expansion theory.



$\theta = 4° \cong 0.07$ rd

From the linearized potential flow, for $M > 1$, we have:

All F waves propagate in the same direction, the same Mach angle, which is given by:

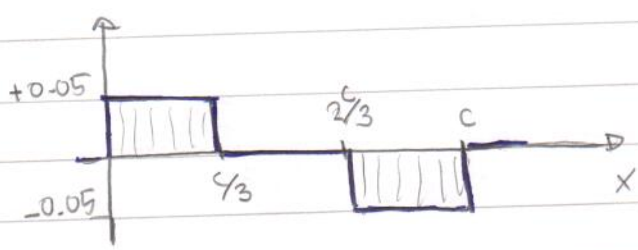


$$\mu = \sin^{-1}\left(\frac{1}{M}\right) = \sin^{-1}\left(\frac{1}{3}\right) \Rightarrow \mu \cong 19.47°$$

Moreover, the turning angle is unique: $\Theta = 4° \cong 0.07$ rd. Hence we can compute the magnitude of ΔC_p as

$$|\Delta C_p| = \frac{2\theta}{\sqrt{M^2 - 1}} \cong 0.05$$

then, on accounting for the geometry compression and expansion turns, we can draw the profile:



spiral

The drag coefficient is given by: (per unit span):

$$c_D = \int_0^1 \frac{c_p}{c}\, dy = \int_0^1 c_p \frac{dy}{dx}\Big|_w \frac{dx}{c} \quad \text{where } \frac{dy}{dx}\Big|_w = \tan\theta \cong \theta$$

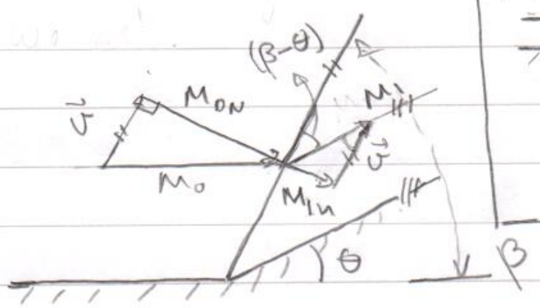
since $\theta << 1$ rd

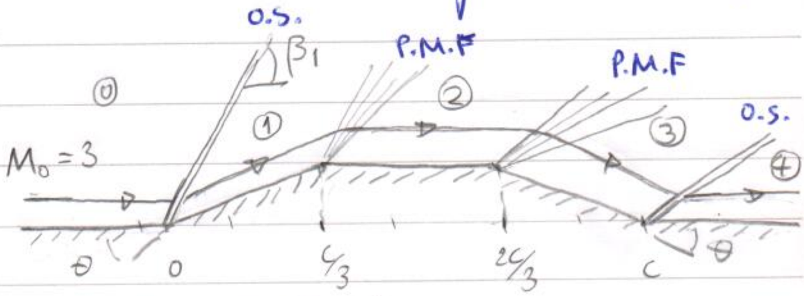
$$c_D = \int_0^{1/3} \frac{2\,\theta^2}{\sqrt{M^2-1}}\, d\left(\frac{x}{c}\right) + \int_{2/3}^{1} \frac{2(-\theta)(-\theta)}{\sqrt{M^2-1}}\, d\left(\frac{x}{c}\right)$$

$$c_D = \frac{2\theta^2}{2\sqrt{2}}\left\{\frac{1}{3} + \frac{1}{3}\right\} = \frac{0.07^2}{\sqrt{2}}\left(\frac{2}{3}\right) \Rightarrow \boxed{c_D \cong 2.3 \times 10^{-3}}$$

Now let's try to use the shock expansion theory:

From the oblique shock diagram, we get:





$\theta = 4° \cong 0.07$ rd.

($0 \to 1 \Rightarrow$ oblique shock)

$$\theta = 4°,\ M_0 = 3.0 \Rightarrow \beta_1 \cong 22.5° \Rightarrow M_{0n} = M_0 \sin\beta_1 \cong 1.15 > 1$$

Then, we use R-H relations in the direction normal to the shock wave:

$$M_{1n}^2 = \frac{M_{0n}^2(\gamma-1)+2}{2\gamma M_{0n}^2 - (\gamma-1)} \Rightarrow \boxed{M_{1n} \cong 0.88} \qquad M_{1n} = M_1 \sin(\beta_1 - \theta)$$

$$\frac{p_1}{p_0} = 1 + \frac{2\gamma}{(\gamma+1)}(M_{0n}^2 - 1) \Rightarrow \boxed{\frac{p_1}{p_0} \cong 1.37} \qquad \boxed{M_1 \cong 2.77}$$

(1 → 2 ⇒ Prandtl-Meyer fan) isentropic expansion

$M_1 \cong 2.77 \Rightarrow \nu(M_1) \cong 45.11°$

$\nu(M_2) = \nu(M_1) + \theta \Rightarrow \nu(M_2) \cong 49.11° \Rightarrow \boxed{M_2 \cong 2.97}$

$$P_{01} = P_{02} \Rightarrow P_{01} = P_1\left[1 + \frac{(\gamma-1)}{2}M_1^2\right]^{\gamma/(\gamma-1)} = P_{02} = P_2\left[1 + \frac{(\gamma-1)}{2}M_2^2\right]^{\gamma/(\gamma-1)}$$

$$\frac{P_2}{P_1} = \left[\frac{1 + (\gamma-1)M_1^2/2}{1 + (\gamma-1)M_2^2/2}\right]^{\gamma/(\gamma-1)} \Rightarrow \boxed{\frac{P_2}{P_1} \cong 0.74}$$

$$\frac{P_2}{P_0} = \frac{P_2}{P_1}\frac{P_1}{P_0} \cong 1.01 \Rightarrow \boxed{\frac{P_2}{P_0} \cong 1.01}$$

(2 → 3 ⇒ Prandtl-Meyer fan) second isentropic expansion

$M_2 \cong 2.97 \Rightarrow \nu(M_2) \cong 49.11°$

$\nu(M_3) = \nu(M_2) + \theta = 53.11° \Rightarrow \boxed{M_3 \cong 3.18}$

$$\frac{P_3}{P_2} = \left[\frac{1 + (\gamma-1)M_2^2/2}{1 + (\gamma-1)M_3^2/2}\right]^{\gamma/(\gamma-1)} \Rightarrow \boxed{\frac{P_3}{P_2} \cong 0.73}$$

$$\frac{P_3}{P_0} = \frac{P_3}{P_2}\frac{P_2}{P_1}\frac{P_1}{P_0} \Rightarrow \boxed{\frac{P_3}{P_0} \cong 0.74}$$

spiral

(3→4 second oblique shock)

$\theta = 4°, M_3 = 3.18 \Rightarrow \beta_3 \cong 21.5° \Rightarrow M_{3n} = M_3 \sin\beta_3$

$M_{3n} \cong 1.17 > 1$

Here again we apply R-H relations to compute conditions downstream of the shock in its normal direction:

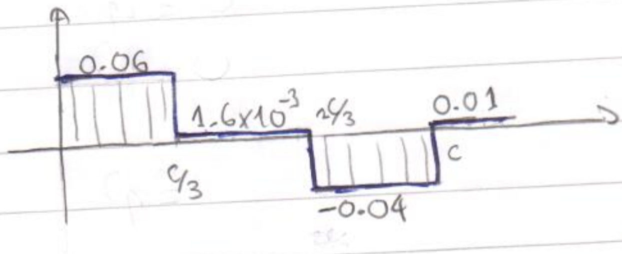
$$M_{4n}^2 = \frac{M_{3n}^2(\gamma-1)+2}{2\gamma M_{3n}^2 - (\gamma-1)} \Rightarrow \boxed{M_{4n} \cong 0.74} \qquad M_{4n} = M_4 \sin(\beta_3 - \theta)$$

$$\frac{P_4}{P_3} = 1 + \frac{2\gamma}{(\gamma+1)}(M_{3n}^2 - 1) \Rightarrow \boxed{\frac{P_4}{P_3} \cong 1.43} \qquad \boxed{M_4 \cong 2.46}$$

$$\frac{P_4}{P_0} = \frac{P_4}{P_3}\frac{P_3}{P_2}\frac{P_2}{P_1}\frac{P_1}{P_0} \Rightarrow \boxed{\frac{P_4}{P_0} \cong 1.06}$$

Now we can compute the corresponding cp distribution by using the formula:

$$c_{p_i} = \frac{2}{\gamma M_0^2}\left(\frac{P_i}{P_0} - 1\right)$$



$c_{p_0} = 0$; $c_{p_1} \cong 0.06$; $c_{p_2} \cong 1.6 \times 10^{-3}$

$c_{p_3} \cong -0.04$; $c_{p_4} \cong 0.01$

$$c_D = \int_0^1 c_p \left.\frac{dy}{dx}\right|_w dx = \frac{0.06 \times 0.07}{3} + \frac{(-0.04)(-0.07)}{3} \Rightarrow \boxed{c_D \cong 2.3 \times 10^{-3}}$$

Owing to the fact that the "bump" is very thin, the results are very close to those for linearized potential flow.

Prandtl-Meyer Flow

APPENDIX D

TABLE D-1 PRANDTL-MEYER FUNCTIONS (PERFECT GAS, $\gamma = 1.4$)*

M	ν	μ
1.00	0.0000	9.0000+01
1.02	0.1257	7.8635+01
1.04	0.3510	7.4058+01
1.06	0.6367	7.0630+01
1.08	0.9680	6.7808+01
1.10	1.3362	6.5380+01
1.12	1.7350	6.3235+01
1.14	2.1600	6.1306+01
1.16	2.6073	5.9550+01
1.18	3.0743	5.7936+01
1.20	3.5582	5.6443+01
1.22	4.0572	5.5052+01
1.24	4.5694	5.3751+01
1.26	5.0931	5.2528+01
1.28	5.6272	5.1375+01
1.30	6.1703	5.0285+01
1.32	6.7213	4.9251+01
1.34	7.2794	4.8268+01
1.36	7.8435	4.7332+01
1.38	8.4130	4.6439+01
1.40	8.9870	4.5585+01
1.42	9.5650	4.4767+01
1.44	1.0146+01	4.3983+01
1.46	1.0731+01	4.3230+01
1.48	1.1317+01	4.2507+01
1.50	1.1905+01	4.1810+01
1.52	1.2495+01	4.1140+01
1.54	1.3086+01	4.0493+01
1.56	1.3677+01	3.9868+01
1.58	1.4269+01	3.9265+01
1.60	1.4860+01	3.8682+01
1.62	1.5452+01	3.8118+01
1.64	1.6043+01	3.7572+01
1.66	1.6633+01	3.7043+01
1.68	1.7222+01	3.6530+01
1.70	1.7810+01	3.6032+01
1.72	1.8396+01	3.5549+01
1.74	1.8981+01	3.5080+01
1.76	1.9565+01	3.4624+01
1.78	2.0146+01	3.4180+01
1.80	2.0725+01	3.3749+01
1.82	2.1302+01	3.3329+01
1.84	2.1877+01	3.2921+01
1.86	2.2449+01	3.2523+01
1.88	2.3019+01	3.2135+01
1.90	2.3586+01	3.1757+01
1.92	2.4151+01	3.1388+01
1.94	2.4712+01	3.1028+01
1.96	2.5271+01	3.0677+01
1.98	2.5827+01	3.0335+01
2.00	2.6380+01	3.0000+01
2.05	2.7748+01	2.9196+01
2.10	2.9097+01	2.8437+01
2.15	3.0425+01	2.7718+01
2.20	3.1733+01	2.7036+01
2.25	3.3018+01	2.6388+01
2.30	3.4283+01	2.5771+01
2.35	3.5526+01	2.5184+01
2.40	3.6747+01	2.4624+01
2.45	3.7946+01	2.4090+01

TABLE D-1 (concluded)

M	ν	μ
2.50	3.9124+01	2.3578+01
2.55	4.0280+01	2.3089+01
2.60	4.1415+01	2.2620+01
2.65	4.2529+01	2.2170+01
2.70	4.3621+01	2.1738+01
2.75	4.4694+01	2.1324+01
2.80	4.5746+01	2.0925+01
2.85	4.6778+01	2.0541+01
2.90	4.7790+01	2.0171+01
2.95	4.8783+01	1.9815+01
3.00	4.9757+01	1.9471+01
3.05	5.0713+01	1.9139+01
3.10	5.1650+01	1.8819+01
3.15	5.2569+01	1.8509+01
3.20	5.3470+01	1.8210+01
3.25	5.4355+01	1.7920+01
3.30	5.5222+01	1.7640+01
3.35	5.6073+01	1.7368+01
3.40	5.6908+01	1.7105+01
3.45	5.7726+01	1.6849+01
3.50	5.8530+01	1.6602+01
3.55	5.9318+01	1.6361+01
3.60	6.0091+01	1.6128+01
3.65	6.0850+01	1.5901+01
3.70	6.1595+01	1.5680+01
3.75	6.2326+01	1.5466+01
3.80	6.3044+01	1.5258+01
3.85	6.3748+01	1.5055+01
3.90	6.4440+01	1.4857+01
3.95	6.5118+01	1.4665+01
4.00	6.5785+01	1.4478+01
4.05	6.6439+01	1.4295+01
4.10	6.7082+01	1.4117+01
4.15	6.7713+01	1.3943+01
4.20	6.8333+01	1.3774+01
4.25	6.8942+01	1.3609+01
4.30	6.9541+01	1.3448+01
4.35	7.0129+01	1.3290+01
4.40	7.0706+01	1.3137+01
4.45	7.1274+01	1.2986+01
4.50	7.1832+01	1.2840+01
4.55	7.2380+01	1.2696+01
4.60	7.2919+01	1.2556+01
4.65	7.3449+01	1.2419+01
4.70	7.3970+01	1.2284+01
4.75	7.4482+01	1.2153+01
4.80	7.4986+01	1.2025+01
4.85	7.5482+01	1.1899+01
4.90	7.5969+01	1.1776+01
4.95	7.6449+01	1.1655+01
5.00	7.6920+01	1.1537+01
5.10	7.7841+01	1.1308+01
5.20	7.8732+01	1.1087+01
5.30	7.9596+01	1.0876+01
5.40	8.0433+01	1.0672+01
5.50	8.1245+01	1.0476+01
5.60	8.2032+01	1.0287+01
5.70	8.2796+01	1.0104+01
5.80	8.3537+01	9.9282
5.90	8.4256+01	9.7583
6.00	8.4955+01	9.5941
6.10	8.5635+01	9.4353
6.20	8.6295+01	9.2818
6.30	8.6937+01	9.1332
6.40	8.7561+01	8.9893
6.50	8.8168+01	8.8499
6.60	8.8759+01	8.7147
6.70	8.9335+01	8.5837
6.80	8.9895+01	8.4565
6.90	9.0441+01	8.3331
7.00	9.0973+01	8.2132
7.10	9.1491+01	8.0968
7.20	9.1997+01	7.9836
7.30	9.2490+01	7.8735
7.40	9.2970+01	7.7664
7.50	9.3440+01	7.6623
7.60	9.3898+01	7.5608
7.70	9.4345+01	7.4621
7.80	9.4781+01	7.3659
7.90	9.5208+01	7.2721
8.00	9.5625+01	7.1808
9.00	9.9318+01	6.3794
10.00	1.0232+02	5.7392
11.00	1.0480+02	5.2159
12.00	1.0688+02	4.7802
13.00	1.0865+02	4.4117
14.00	1.1018+02	4.0960
15.00	1.1151+02	3.8226
16.00	1.1268+02	3.5833
17.00	1.1371+02	3.3723
18.00	1.1463+02	3.1847
19.00	1.1545+02	3.0170
20.00	1.1620+02	2.8660
22.00	1.1748+02	2.6053
24.00	1.1856+02	2.3880
26.00	1.1947+02	2.2042
28.00	1.2025+02	2.0467
30.00	1.2092+02	1.9102
32.00	1.2152+02	1.7908
34.00	1.2204+02	1.6854
36.00	1.2251+02	1.5918
38.00	1.2292+02	1.5080
40.00	1.2330+02	1.4325
42.00	1.2364+02	1.3643
44.00	1.2395+02	1.3023
46.00	1.2423+02	1.2457
48.00	1.2449+02	1.1937
50.00	1.2473+02	1.1460

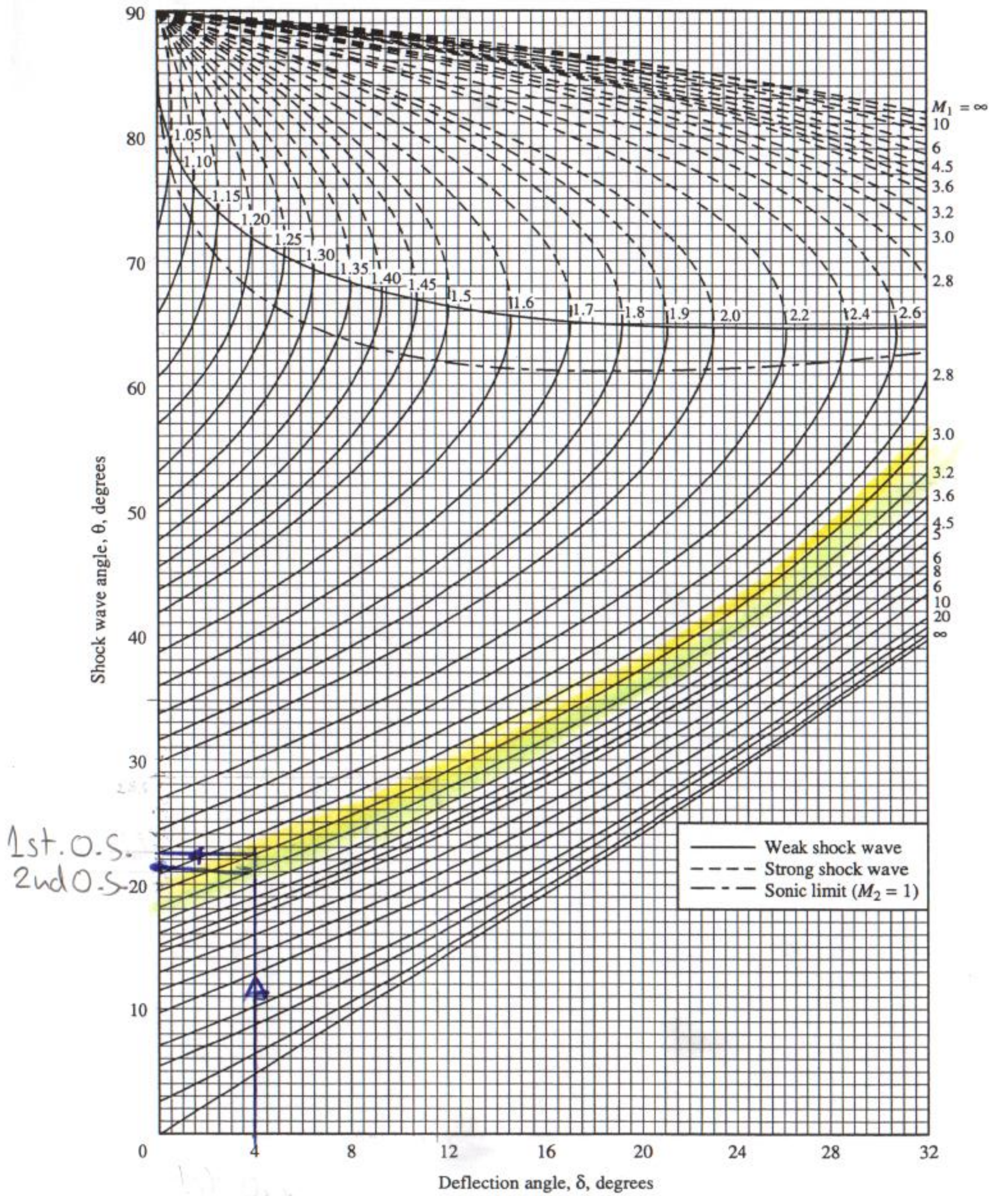


Figure C-1 Variation of Shock Wave Angle with Flow-Deflection Angle for Various Upstream Mach Numbers

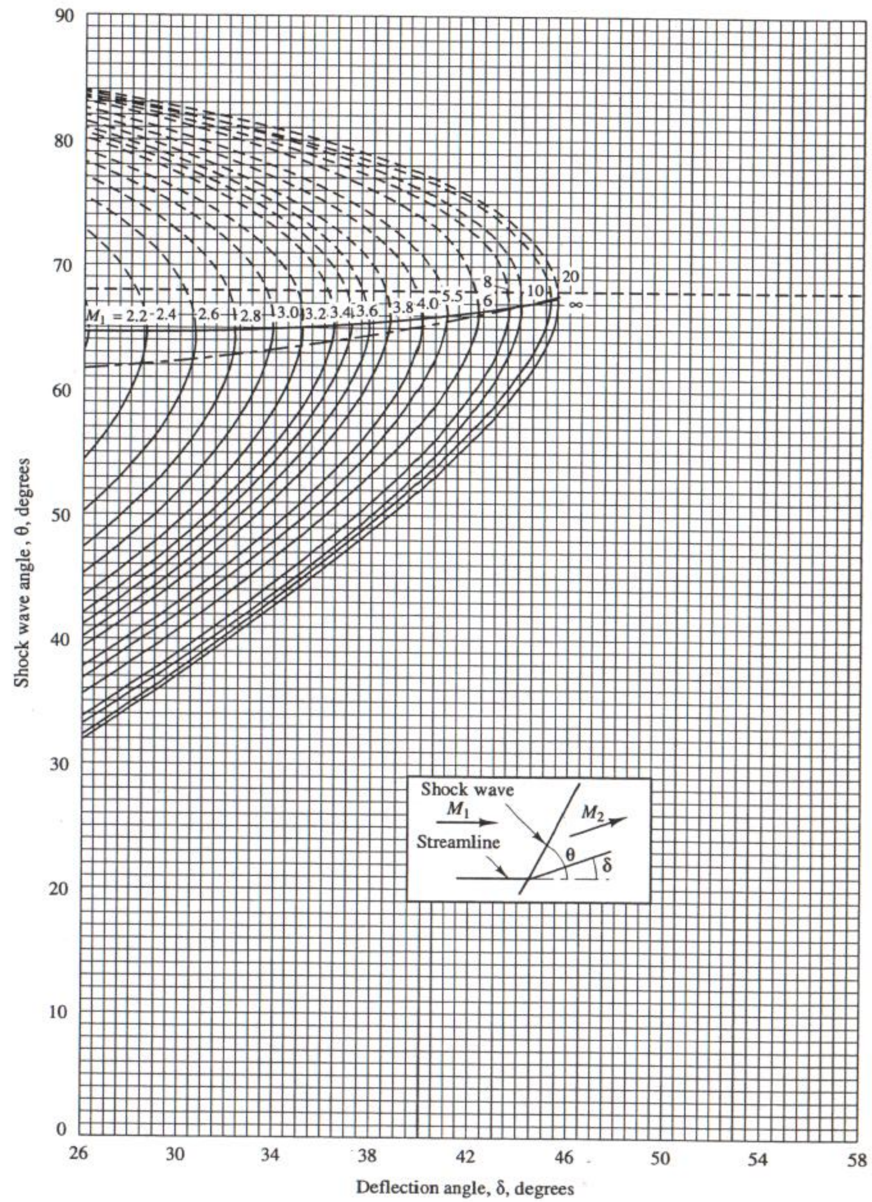


Figure C-1 (continued)