V.5. Neste caso temos $p_1 = p_2 = p$ e

$$B_\phi = \frac{\omega^2}{c^2}\frac{1}{R}\,\mathrm{sen}\theta\,\exp(ikR)\left\{p_1\exp\left(\frac{-ik\Delta}{2}\cos\psi\right) + p_2\exp\left(\frac{ik\Delta}{2}\cos\psi\right)\right\}$$

$$= \frac{\omega^2}{c^2}\frac{2p}{R}\,\mathrm{sen}\theta\,\exp(ikR)\,\cos\left(\frac{k\Delta}{2}\cos\psi\right)$$

$$= \frac{\omega^2}{c^2}\frac{2p}{R}\,\mathrm{sen}\theta\,\exp(ikR)\,\cos\left(\frac{k\Delta}{2}\,\mathrm{sen}\theta\cos\phi\right)$$

$\mathbb{B} = B_\phi\, e_\phi$, $e_\phi = -\mathrm{sen}\phi\, e_x + \cos\phi\, e_y$

$\mathbb{E} = \mathbb{B}\wedge\mathbb{n}$, $\mathbb{n} = \mathrm{sen}\theta\cos\phi\, e_x + \mathrm{sen}\theta\,\mathrm{sen}\phi\, e_y + \cos\theta\, e_z$

$\Rightarrow \mathbb{E} = B_\phi\,[-\mathrm{sen}\theta\, e_z + \cos\theta\,\mathrm{sen}\phi\, e_y + \cos\theta\cos\phi\, e_x]$

$$\frac{d\bar{P}}{d\Omega} = \frac{c}{8\pi}\,\mathrm{Re}\,[R^2\,\mathbb{n}\cdot(\mathbb{E}\wedge\mathbb{B}^*)]$$

$$= \frac{c}{8\pi}\,\mathrm{Re}\,[R^2\,\mathbb{n}\cdot\mathbb{n}\, B_\phi B_\phi^*]$$

$$= \frac{\omega^4}{2\pi c^3}\,p^2\,\mathrm{sen}^2\theta\,\cos^2\left(\frac{k\Delta}{2}\,\mathrm{sen}\theta\cos\phi\right)$$

Para dipolos de meia-onda $(k\Delta = \pi)$:

$$\frac{d\bar{P}}{d\Omega} = \frac{\omega^4 p^2}{2\pi c^3}\,\mathrm{sen}^2\theta\,\cos^2\left(\frac{\pi}{2}\,\mathrm{sen}\theta\cos\phi\right)$$

V.5

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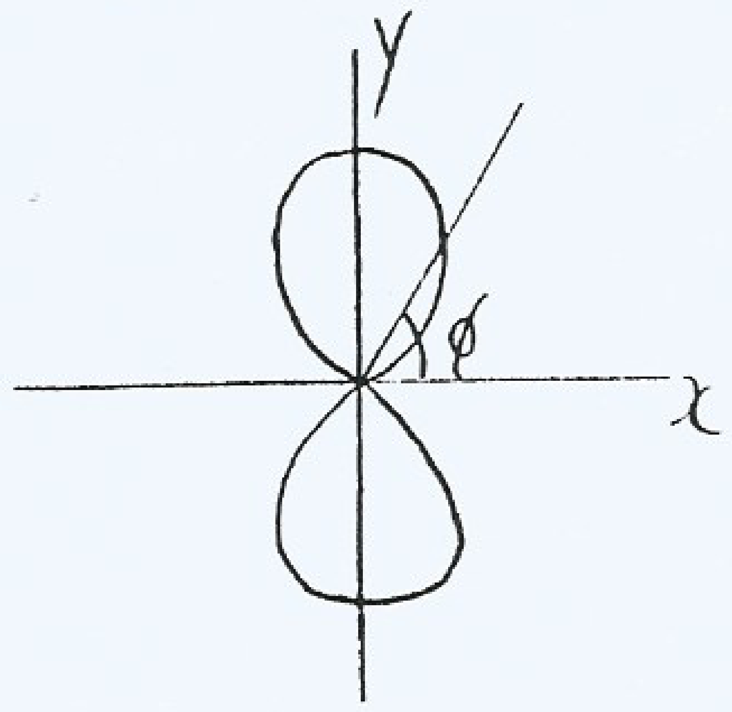
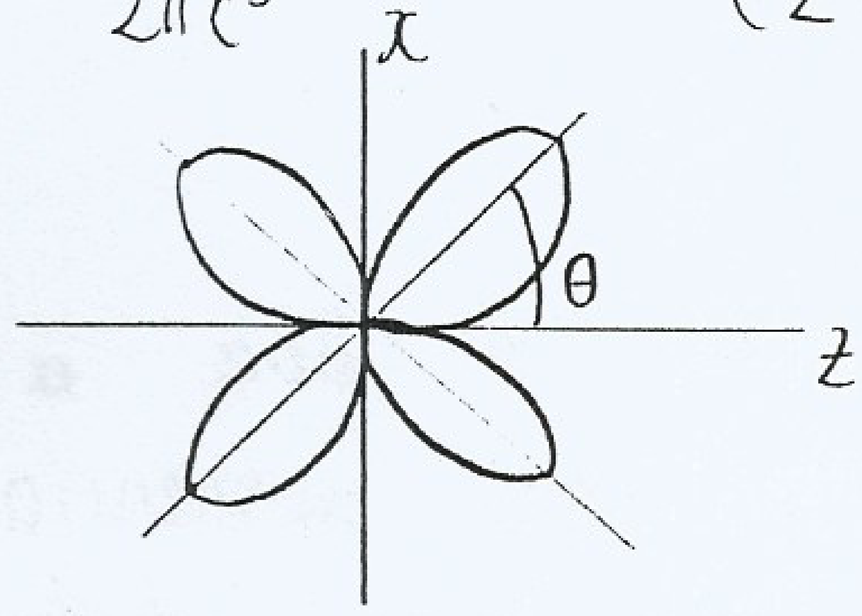
• No plano xy, $\theta = \pi/2$:

$$\frac{d\bar{P}}{d\Omega} = \frac{\omega^4 p^2}{2\pi c^3} \, 1 \cdot \cos^2\left(\frac{\pi}{2}\cos\phi\right)$$



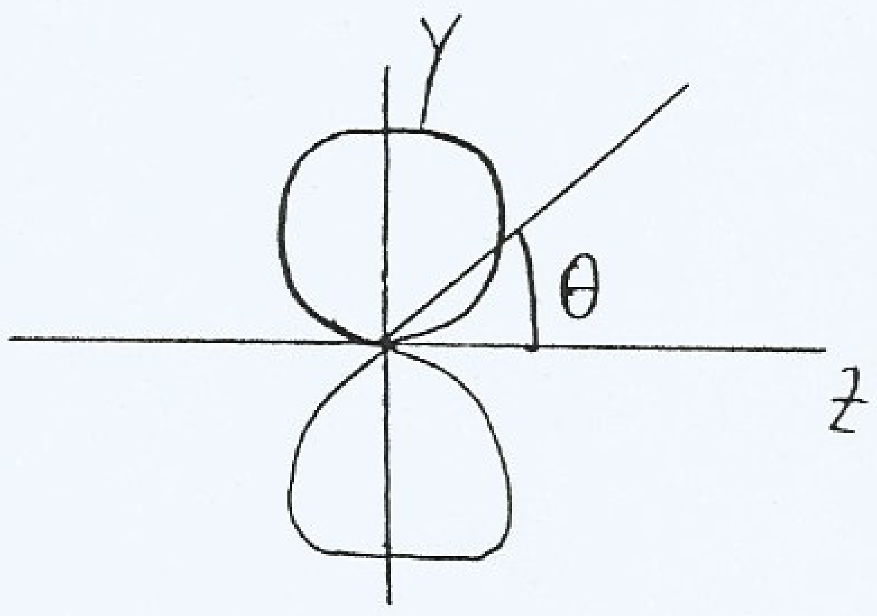
• No plano xz, $\phi = 0$:

$$\frac{d\bar{P}}{d\Omega} = \frac{\omega^4 p^2}{2\pi c^3} \, \mathrm{sen}^2\theta \, \cos^2\left(\frac{\pi}{2}\,\mathrm{sen}\,\theta\right)$$



• No plano yz, $\phi = \pi/2$:

$$\frac{d\bar{P}}{d\Omega} = \frac{\omega^4 p^2}{2\pi c^3} \, \mathrm{sen}^2\theta$$



Apesar de não parecer todos estes gráficos possuem as simetrias que, muito levemente, sugerem ter.

V.8.

$$\mathbb{A}(\mathbf{r},t) = \mathbb{A}(\mathbf{r})\exp(-i\omega t)$$

$$\mathbb{A}(\mathbf{r}) = \frac{ik}{r}\exp(ikr)\ \mathbf{n}\wedge\mathbf{m}$$

$$e \quad \mathbf{m} = \frac{I_0}{c}\pi R^2\, e_z\,.$$

Assim

$$\mathbb{A}(\mathbf{r},t) = \frac{ik}{r}\exp(ikr)\exp(-i\omega t)\,\frac{I_0\pi R^2}{c}\ \mathbf{n}\wedge e_z$$

$$= \frac{I_0\pi R^2\, ik}{cr}\exp(ikr)\exp(-i\omega t)\,\{\operatorname{sen}\theta\operatorname{sen}\phi\, e_x - \operatorname{sen}\theta\cos\phi\, e_y\}$$

$$= -\frac{I_0\pi R^2\, ik}{cr}\exp(ikr)\exp(-i\omega t)\operatorname{sen}\theta\, e_\phi$$

$$\mathbb{E} = -\frac{1}{c}\frac{\partial\mathbb{A}}{\partial t} = I_0\pi R^2\,\frac{\omega^2}{rc^3}\exp(ikr)\exp(-i\omega t)\operatorname{sen}\theta\, e_\phi\,.$$

$$\mathbb{B} = \operatorname{rot}\mathbb{A} \simeq \frac{i\omega}{c}\,\mathbf{n}\wedge\mathbb{A} = -I_0\pi R^2\frac{\omega^2}{rc^3}\exp(ikr)\exp(-i\omega t)\operatorname{sen}\theta\, e_\theta$$

$$\bar{\mathbb{S}} = \frac{c}{8\pi}\,\mathcal{Re}\,(\mathbb{E}\wedge\mathbb{B}^*)$$

$$= \frac{c}{8\pi}\left(I_0\pi R^2\frac{\omega^2}{rc^3}\operatorname{sen}\theta\right)^2 e_r$$

$$= \frac{(I_0\pi R^2)^2}{8\pi}\,\frac{\omega^4}{r^2c^5}\operatorname{sen}^2\theta\, e_r$$

V.10. $$|Q| = \begin{bmatrix} -2qb^2 & & 0 \\ & -2qb^2 & \\ 0 & & 4qb^2 \end{bmatrix}$$ (v. eq. (2.18))

$$Q_i(n) = \sum_j Q_{ij} n_j \quad , \quad n = \operatorname{sen}\theta\cos\phi\, e_x + \operatorname{sen}\theta \operatorname{sen}\phi\, e_y + \cos\theta\, e_z$$

$$Q(n) = -2qb^2 [n_x e_x + n_y e_y - 2 n_z e_z]$$

$$n \wedge Q(n) = 6qb^2 (n_y e_x - n_x e_y)\, n_z$$

$$\| n \wedge Q(n) \|^2 = 36 q^2 b^4 n_z^2 (n_x^2 + n_y^2)$$
$$= 36 q^2 b^4 \cos^2\theta \operatorname{sen}^2\theta$$

$$\frac{d\bar{P}}{d\Omega} = \frac{c}{288\pi} k^6 \, \| n \wedge Q(n) \|^2$$

$$= \frac{c k^6 q^2 b^4}{8\pi} \operatorname{sen}^2\theta \cos^2\theta$$

$$\bar{P} = \frac{c k^6 q^2 b^4}{8\pi} \int d\Omega \operatorname{sen}^2\theta \cos^2\theta = \frac{c k^6 q^2 b^4}{2} \int_0^{\pi} d\theta \operatorname{sen}^3\theta \cos^2\theta$$

$$= \frac{c k^6 q^2 b^4}{2} \int_{-1}^{1} dx\, (1 - x^2)\, x^2 = \frac{c k^6 q^2 b^4}{15}$$

Ou $$\bar{P} = \frac{c k^6}{360} \sum_{i,j} |Q_{ij}|^2 = \frac{c k^6 q^2 b^4}{15}$$

A vida-média é definida por (5.30)

$$\tau = \frac{\hbar\omega}{\langle P \rangle} .$$

a. Para o dipolo elétrico (5.29)

$$\langle P \rangle = \frac{\omega^4 p^2}{3c^3} \quad \text{e}$$

$$\tau_p = \frac{3\hbar c^3}{\omega^3 p^2}$$

$$\cong \frac{3\hbar c^3}{\omega^3 e^2 d^2}$$, pois $p \sim ed$, sendo d uma típica distância atômica, $d \sim 1 Å$,

$$= \frac{3c^2}{\omega^3 d^2} \cdot \frac{\hbar c}{e^2} = \frac{3}{(2\pi)^3} \cdot \frac{\hbar c}{e^2} \cdot \frac{\lambda^3}{cd^2}$$ pois $\omega = \frac{2\pi c}{\lambda}$.

$$\tau_p = \frac{3}{(2\pi)^3} \cdot \frac{\hbar c}{e^2} \cdot \frac{\lambda^3}{cd^2} \cong \frac{3}{(2\pi)^3} \cdot 137 \cdot \frac{(5{,}5\times 10^{-5})^3}{3\times 10^{10} \cdot (10^{-8})^2} = 9{,}2 \times 10^{-8}\,s .$$

b. Para o dipolo magnético (5.43)

$$\langle P \rangle = \frac{\omega^4 m^2}{3c^3} \quad \text{e}$$

$$\tau_m = \frac{3\hbar c^3}{\omega^3 m^2} \cong \frac{3\hbar c^3}{\omega^3}\left(\frac{2Mc}{e\hbar}\right)^2$$, pois $m \sim \frac{e\hbar}{2Mc}$,

$$= 4\left(\frac{Mcd}{\hbar}\right)^2 \tau_p = 4\left(\frac{e^2}{\hbar c} \cdot \frac{Mc^2}{e^2} \cdot d\right)^2 \tau_p ,$$

assim

$$\tau_m = 4\left(\frac{e^2}{\hbar c}\cdot\frac{Mc^2}{e^2}\cdot d\right)^2 \tau_p \cong 4\cdot\left(\frac{137\cdot 2{,}8\times 10^{-13}}{10^{-8}}\right)^{-2}\cdot 9{,}2\times 10^{-8}$$

$$= 2{,}5\times 10^{-2}\,s\,.$$

c. Para o quadrupolo elétrico (3.63)

$$\langle P\rangle = \frac{\omega^6 Q_0^2}{240\, c^5} \qquad e$$

$$\tau_Q = \frac{240\,\hbar c^5}{Q_0^2\,\omega^5} \cong \frac{240\,\hbar c^5}{e^2 d^4 \omega^5} \quad , \text{pois } Q_0 \sim ed^2,$$

$$= \frac{20}{\pi^2}\left(\frac{\lambda}{d}\right)^2 \tau_p \cong \frac{20}{\pi^2}\left(\frac{5{,}5\times 10^{-5}}{10^{-8}}\right)^2\cdot 9{,}2\times 10^{-8} = 5{,}6\ s$$

VIII.8

$$\mathbb{F} = q\left(\mathbb{E} + \frac{\mathbb{v}}{c}\wedge\mathbb{B}\right) = q\left(\mathbb{E} + \beta\wedge\mathbb{B}\right) \quad , \quad \beta = \frac{\mathbb{v}}{c}$$

$$\mathbb{E} = -\operatorname{grad}\Phi - \frac{1}{c}\frac{\partial\mathbb{A}}{\partial t} \quad , \quad \mathbb{B} = \operatorname{rot}\mathbb{A}$$

$$\Phi = \frac{q}{R - \frac{\mathbb{R}\cdot\mathbb{v}}{c}} = \frac{q}{R - \mathbb{R}\cdot\beta} = \frac{q}{S}$$

$$\mathbb{A} = \frac{\mathbb{v}}{c}\frac{q}{R - \frac{\mathbb{R}\cdot\mathbb{v}}{c}} = \beta\frac{q}{S}$$

Além disto é preciso lembrar da derivada convectiva

$$\frac{\partial}{\partial t} = -\mathbb{v}\cdot\nabla \qquad (\text{eq. } 8.12) \,,$$

assim

$$\mathbb{F} = q\left\{-\operatorname{grad}\left(\frac{q}{S}\right) - \frac{1}{c}\frac{\partial}{\partial t}\left(\frac{\beta q}{S}\right) + \beta\wedge\operatorname{rot}\left(\frac{\beta q}{S}\right)\right\}$$

$$= q^2\left\{-\operatorname{grad}\left(\frac{1}{S}\right) - \frac{1}{c}(-\mathbb{v}\cdot\nabla)\left(\frac{\beta}{S}\right) + \beta\wedge\operatorname{rot}\left(\frac{\beta}{S}\right)\right\}$$

$$= q^2\left\{-\operatorname{grad}\left(\frac{1}{S}\right) + (\beta\cdot\nabla)\left(\frac{\beta}{S}\right) + \beta\wedge\operatorname{rot}\left(\frac{\beta}{S}\right)\right\}$$

Agora $\operatorname{rot}\left(\frac{\beta}{S}\right) = \operatorname{grad}\left(\frac{1}{S}\right)\wedge\beta + \frac{1}{S}\operatorname{rot}\beta = \operatorname{grad}\left(\frac{1}{S}\right)\wedge\beta$

pois a) $\operatorname{rot}(f\mathbb{A}) = (\operatorname{grad} f)\wedge\mathbb{A} + f(\operatorname{rot}\mathbb{A})$ e
b) $\operatorname{rot}\beta = 0$ pois β é constante,

xi.

então

$$\mathbb{F} = q^2 \left\{ -\mathrm{grad}\left(\frac{1}{s}\right) + (\beta . \nabla)\left(\frac{\beta}{s}\right) + \beta \wedge \left(\mathrm{grad}\left(\frac{1}{s}\right) \wedge \beta\right) \right\}$$

$$= q^2 \left\{ -\mathrm{grad}\left(\frac{1}{s}\right) + (\beta . \nabla)\left(\frac{\beta}{s}\right) + \mathrm{grad}\left(\frac{1}{s}\right)\beta^2 - \beta\left(\beta . \mathrm{grad}\left(\frac{1}{s}\right)\right)\right\}$$

, pois $\mathbb{A} \wedge (\mathbb{B} \wedge \mathbb{C}) = \mathbb{A}(\mathbb{B}.\mathbb{C}) - \mathbb{C}(\mathbb{A}.\mathbb{B})$,

$$= q^2 \left\{ -\mathrm{grad}\left(\frac{1}{s}\right) . (1-\beta^2) + (\beta . \nabla)\left(\frac{\beta}{s}\right) - \beta\left(\beta . \mathrm{grad}\left(\frac{1}{s}\right)\right)\right\},$$

mas

$$(\beta . \nabla)\left(\frac{\beta}{s}\right) = e_x\left\{\beta_x \frac{\partial}{\partial x}\left(\frac{\beta_x}{s}\right) + \beta_y \frac{\partial}{\partial y}\left(\frac{\beta_x}{s}\right) + \beta_z \frac{\partial}{\partial z}\left(\frac{\beta_x}{s}\right)\right\} +$$
$$+ e_y\left\{\beta_x \frac{\partial}{\partial x}\left(\frac{\beta_y}{s}\right) + \beta_y \frac{\partial}{\partial y}\left(\frac{\beta_y}{s}\right) + \beta_z \frac{\partial}{\partial z}\left(\frac{\beta_y}{s}\right)\right\} +$$
$$+ e_z\left\{\beta_x \frac{\partial}{\partial x}\left(\frac{\beta_z}{s}\right) + \beta_y \frac{\partial}{\partial y}\left(\frac{\beta_z}{s}\right) + \beta_z \frac{\partial}{\partial z}\left(\frac{\beta_z}{s}\right)\right\} =$$

$$= e_x \beta_x \left(\beta . \mathrm{grad}\,(1/s)\right) + e_y \beta_y \left(\beta . \mathrm{grad}\,(1/s)\right) +$$
$$+ e_z \beta_z \left(\beta . \mathrm{grad}\,(1/s)\right)$$, pois $\frac{\partial}{\partial x_i}\left(\frac{\beta_i}{s}\right) = \beta_i \frac{\partial}{\partial x_i}\left(\frac{1}{s}\right)$,

$$= \beta\left(\beta . \mathrm{grad}\,(1/s)\right),$$

sendo assim

$$\mathbb{F} = -\mathrm{grad}\left(\frac{q^2(1-\beta^2)}{s}\right) = -\mathrm{grad}\left(\frac{q^2(1-v^2/c^2)}{R - \frac{\mathbb{R}.\mathbb{V}}{c}}\right)$$