Derivação da expressão para o campo elétrico a partir dos potenciais de Liénard-Wiechert

$$\phi(\vec r,t) = \frac{q}{4\pi\epsilon_0}\left[\frac{1}{s}\right]_{ret} \; ; \; \vec A(\vec r,t) = \frac{\mu_0 q}{4\pi}\left[\frac{\vec v_q}{s}\right]_{ret}$$

$$\vec E(\vec r,t) = -(\nabla\phi)_t - \left(\frac{\partial \vec A}{\partial t}\right)_{\vec r}$$

$$\therefore \; \vec E(\vec r,t) = \frac{q}{4\pi\epsilon_0 [s^2]_{ret}}(\nabla s)_t + \frac{\mu_0 q}{4\pi [s^2]_{ret}}\vec v_q\left(\frac{\partial s}{\partial t}\right)_{\vec r} + \frac{\mu_0 q}{4\pi [s]_{ret}}\left(\frac{\partial \vec v_q}{\partial t}\right)_{\vec r}$$

$$(\nabla s)_t = (\nabla s)_{t'} - \frac{\vec R}{cs}\left(\frac{\partial s}{\partial t'}\right)_{\vec r} \; ; \; \left(\frac{\partial s}{\partial t}\right)_{\vec r} = \frac{R}{s}\left(\frac{\partial s}{\partial t'}\right)_{\vec r}$$

$$\therefore \; \cancel{(\nabla s)_t} \; \frac{q}{4\pi\epsilon_0 s^2}(\nabla s)_t + \frac{\mu_0 q}{4\pi s^2}\vec v_q\left(\frac{\partial s}{\partial t}\right)_{\vec r} =$$

$$= \frac{q}{4\pi\epsilon_0 s^2}(\nabla s)_{t'} + \frac{q}{4\pi\epsilon_0 s^2}\left[-\frac{\vec R}{cs} + \mu_0\epsilon_0\frac{R}{s}\vec v_q\right]\left(\frac{\partial s}{\partial t'}\right)_{\vec r}$$

$$= \frac{q}{4\pi\epsilon_0 s^2}(\nabla s)_{t'} - \frac{q}{4\pi\epsilon_0 s^2}\frac{R}{cs}\left[\frac{\vec R}{R} - \vec\beta\right]\left(\frac{\partial s}{\partial t'}\right)_{\vec r}$$

$$\therefore \; \frac{q}{4\pi\epsilon_0 s^2}(\nabla s)_t + \frac{\mu_0 q}{4\pi s^2}\vec v_q\left(\frac{\partial s}{\partial t}\right)_{\vec r} = \frac{q}{4\pi\epsilon_0 s^2}\left[\left(\frac{\vec R}{R} - \vec\beta\right) \cancel{+\frac{R}{cs}\left(\frac{\vec R}{R} - \vec\beta\right)} - \frac{R}{cs}\left(\frac{\vec R}{R} - \vec\beta\right)\left(\frac{\partial s}{\partial t'}\right)_{\vec r}\right]$$

$$\therefore \; \frac{4\pi\epsilon_0 s^2}{q}\vec E(\vec r,t) = \left(\frac{\vec R}{R} - \vec\beta\right) - \frac{R}{sc}\left[-\frac{\vec R\cdot\vec v_q}{R} + \frac{v_q^2}{c} - \frac{\vec R\cdot\dot{\vec v}_q}{c}\right]\left(\frac{\vec R}{R} - \vec\beta\right) - \frac{R}{c^2}\dot{\vec v}_q$$

$$\therefore \frac{4\pi\epsilon_0 s^2}{q}\vec{E}(\vec{r},t) = \frac{1}{s}(\vec{R}-R\vec{\beta})\left[\frac{s}{R}+\frac{\vec{R}\cdot\vec{\beta}}{R}-\frac{v_q^2}{c^2}\right]$$

$$+\frac{1}{s}\left[(\vec{R}-R\vec{\beta})\frac{\vec{R}\cdot\dot{\vec{v}}_q}{c^2}-Rs\frac{\dot{\vec{v}}_q}{c^2}\right]$$

$$=\frac{1}{s}(\vec{R}-R\vec{\beta})\left[1-\frac{v_q^2}{c^2}\right]+\frac{1}{sc^2}\left[(\vec{R}\cdot\dot{\vec{v}}_q)(\vec{R}-R\vec{\beta})\right.$$

$$\left.-\vec{R}\cdot(\vec{R}-R\vec{\beta})\dot{\vec{v}}_q\right]$$

$$\therefore \quad \vec{E}(\vec{r},t)=\frac{q}{4\pi\epsilon_0}\left[\frac{1}{s^3}(\vec{R}-R\vec{\beta})(1-\beta^2)\right]_{\mathrm{rt}}+$$

$$+\frac{q}{4\pi\epsilon_0 c^2}\left[\frac{1}{s^3}\vec{R}\times[(\vec{R}-R\vec{\beta})\times\dot{\vec{v}}_q\right]_{\mathrm{rt}}$$