

FIGURE 5.19
Lift and pitching moment of a NACA 0009 airfoil.

increases beyond the limits of the small angle of attack assumption, the streamlines do not follow the airfoil surface shape (Fig. 5.20) and the flow is considered to be *separated*. This results in loss of lift, as indicated by the experimental data in Fig. 5.19 (for $\alpha > 10^\circ$) and this condition is called *airfoil stall*.

3. Airfoil camber does not change the lift slope and can be viewed as an additional angle of attack effect (α_{L_0} in Eq. (5.66)). This is shown schematically by Fig. 5.21. The symmetric airfoil will have zero lift at $\alpha = 0$ while the airfoil with camber has an “effective” angle of attack that is larger by α_{L_0} .
4. The trailing edge section has a larger effect on the above camber effect. Therefore, if the airfoil lift needs to be changed without changing its angle

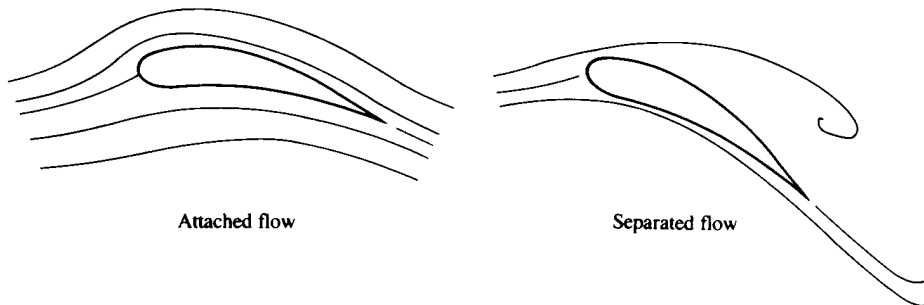


FIGURE 5.20
Streamlines of the attached (a) and separated (b) flow over an airfoil.

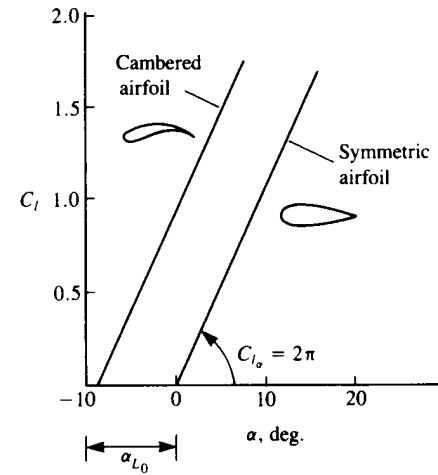


FIGURE 5.21.
Schematic description of airfoil camber effect on the lift coefficient.

of attack, then changing the chordline geometry (e.g., by flaps, or slats) at the trailing edge region is more effective than at the leading edge region.

5. The effect of thickness on the airfoil lift is not treated in a satisfactory manner by the small-disturbance approach, but will be calculated more accurately in the following two chapters.
6. The two-dimensional drag coefficient obtained by this model is zero and there is no drag associated with the generation of two-dimensional lift. Experimental airfoil data, however, includes drag due to the viscous boundary layer on the airfoil, which should be included in engineering calculations. The experimental drag coefficient values for the NACA 0009 airfoil are also plotted in Fig. 5.19 and for example the “zero-lift” drag coefficient is close to $C_d = 0.0055$.

REFERENCES

- 5.1. Moran, J., *An Introduction to Theoretical and Computational Aerodynamics*, Wiley, New York, 1984.
- 5.2. Glauert, H., *The Elements of Aerofoil and Airscrew theory*, 2d edn., Cambridge University Press, 1959.
- 5.3. Van Dyke, M., *Perturbation Methods in Fluid Mechanics*, The Parabolic Press, Stanford, Calif. 1975.
- 5.4. Lighthill, M. J., “A New Approach to Thin Airfoil Theory,” *The Aeronautical Quarterly*, vol. 3, pp. 193–210, 1951.

PROBLEMS

- 5.1. Find the camberline shape that leads to a constant pressure jump along the airfoil chordline for zero angle of attack.
- 5.2. Consider the flow of a uniform stream of speed Q_∞ at angle of attack α past a thin

airfoil whose camberline is given by

$$\eta_c = h \left(\left(1 - \frac{x}{c} \right) \left(1 - \frac{\lambda x}{c} \right) \right)$$

where $h \ll 1$ and λ is a constant. Show that

$$C_l = 2\pi(\alpha + \epsilon) \quad C_{m_0} = 2 \left(\mu - \frac{\pi\epsilon}{4} \right) - \frac{C_l}{4}$$

where

$$\epsilon = \frac{h}{8}(4 - 3\lambda) \quad \text{and} \quad \mu = \frac{\pi}{64} h\lambda$$

Find the value of λ for the zero-lift angle to be zero.

- 5.3. Find the hinge moment for the flapped airfoil of Eq. (5.90).
- 5.4. Consider the flow of a uniform stream of speed Q_∞ at angle of attack α past a thin airfoil whose upper surface is given by the parabola in Eq. (5.80) and whose lower surface is $z = 0$. Find the lift coefficient and moment coefficient about the leading edge.
- 5.5. Consider the flow of a uniform stream of speed Q_∞ at angle of attack α past a biplane consisting of two flat-plate airfoils of chord c a distance h apart (no stagger). Find the lift coefficient for each airfoil using a single vortex to represent each one.

CHAPTER 6

EXACT SOLUTIONS WITH COMPLEX VARIABLES

Approximate solutions to the exact potential flow problem are obtained in this book using both classical small-disturbance methods and numerical modeling. It is important to have exact solutions available to test the accuracy of the approximations and to assess their applicability. In this chapter complex variables will be used to obtain the solution to three model problems: the flat plate, the circular arc, and a symmetrical airfoil.

6.1 SUMMARY OF COMPLEX VARIABLE THEORY

Prior to applying complex variable methods to potential flow problems, some of the principles are discussed briefly (for more details about the mathematics of complex variables, see Churchill^{6.1}). To begin, first define the imaginary unit i by

$$i^2 = -1 \tag{6.1}$$

Then any complex number Y can be written as

$$Y = a + ib \tag{6.2}$$

where a and b are real and are called the real and imaginary parts of Y ,