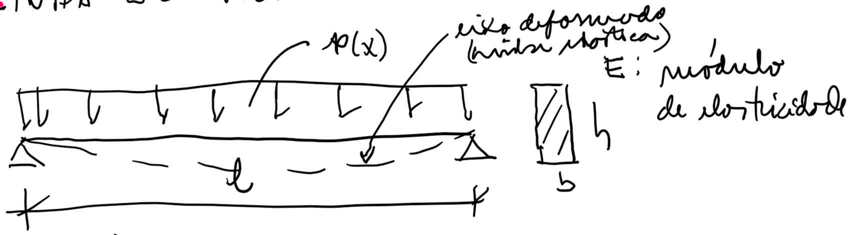
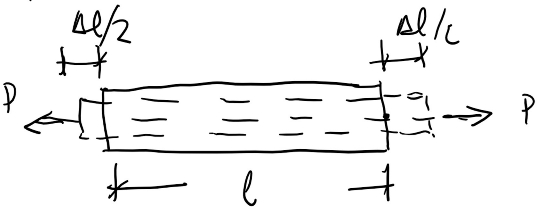
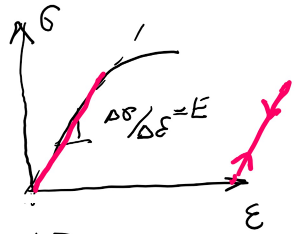
Linha Elástica
Friday, June 19, 2020
7:32 AM

LINHA ELÁSTICA





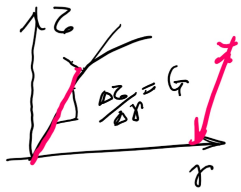
$$\varepsilon = \frac{\Delta l}{l} = \frac{l_f - l_i}{l_i}$$

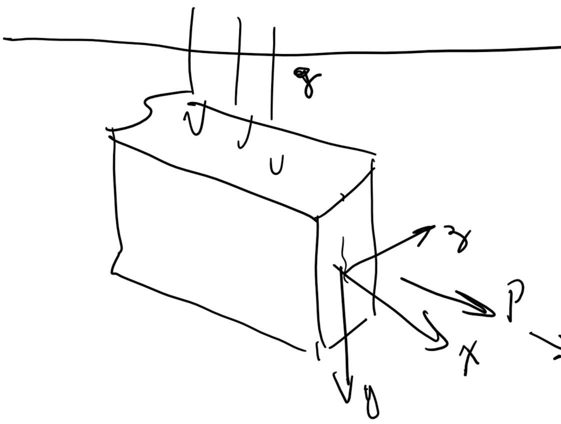


$$\sigma \propto \varepsilon \Rightarrow \sigma = E \cdot \varepsilon$$

Tensões normais

$$\tau \propto \gamma \Rightarrow \tau = G \cdot \gamma$$



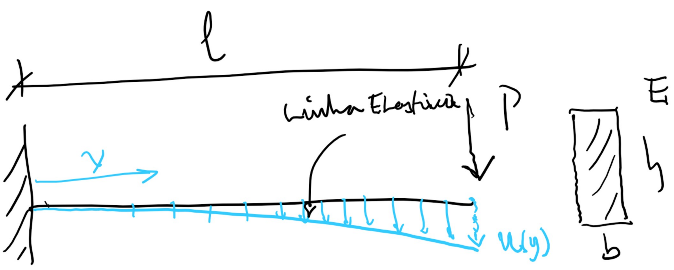


Δl — (N)

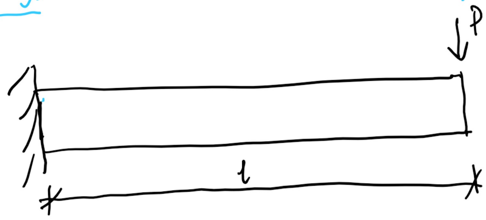
γ — T (V)

u(y) — M

Efeitos decorrentes do momento fletor



u(y): deslocamento transversal

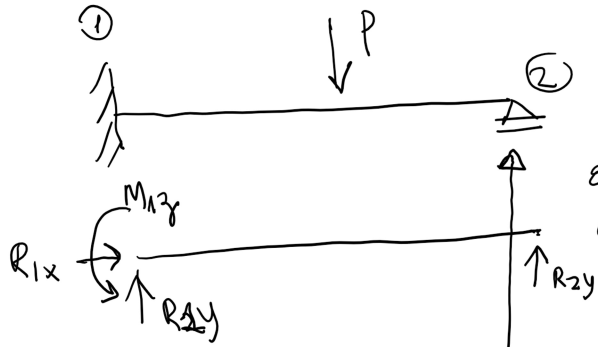


$\sigma_{act} \leq \sigma_{adm}$

$u_{max} \leq u_{limite}$ especificado

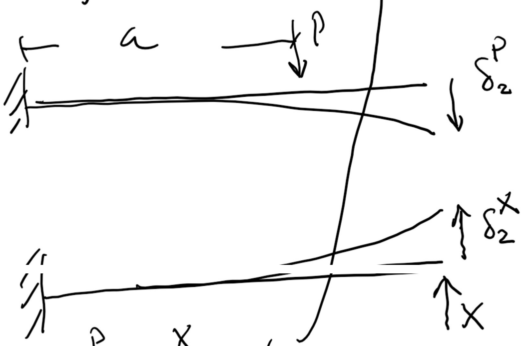
$\frac{L}{200}$

problemas hiperestáticos



Eq. de equil.
compatibilidade
constitutivas

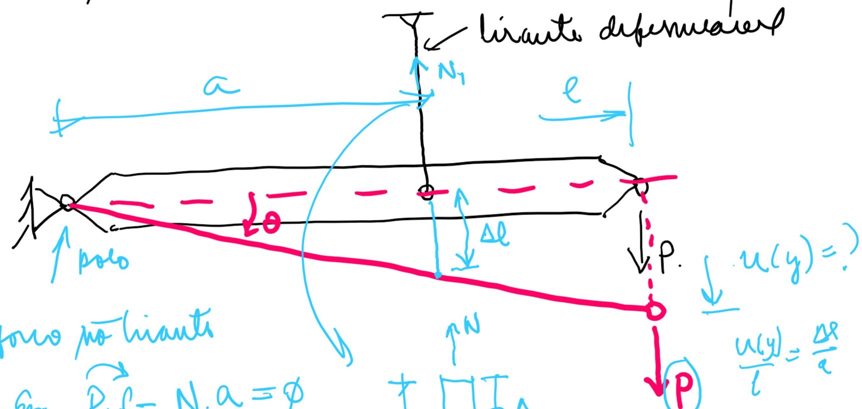
Solução



$\delta_2^P + \delta_2^X = 0 \Rightarrow$ dá o valor X

M; N; V; T → σ; e τ ← para projeto ou verificação da estrutura

deformações (efeitos) decorrentes das tensões normais σ



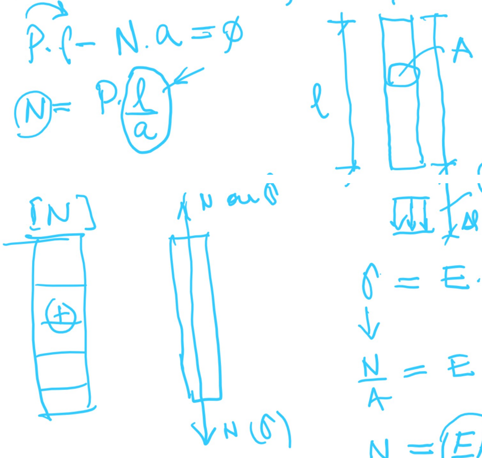
Esforço no tirante

Eq. Eq. $P \cdot \ell - N \cdot a = 0$

$$N = P \cdot \frac{\ell}{a}$$

$$\frac{u(y)}{\ell} = \frac{\Delta\ell}{a}$$

$$u(y) = \frac{\ell}{a} \cdot \Delta\ell$$



$$\sigma = \frac{N}{A}$$

$$\sigma = E \cdot \varepsilon$$

$$\frac{N}{A} = E \cdot \frac{\Delta\ell}{\ell}$$

$$N = \frac{EA}{\ell} \cdot \Delta\ell$$

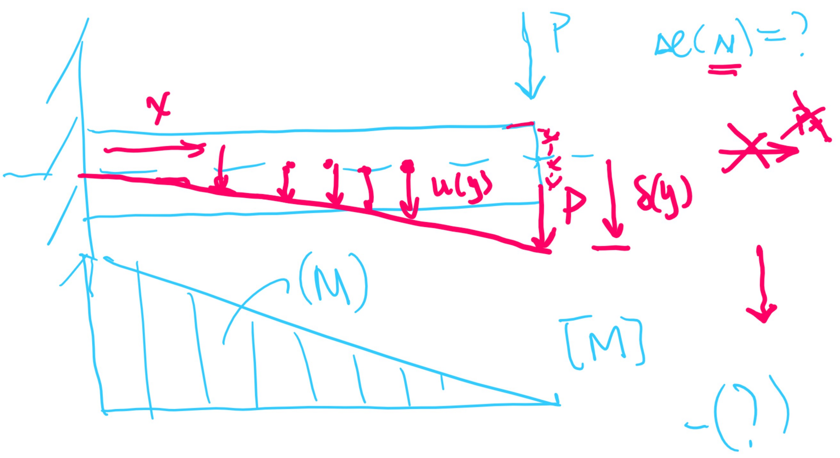
$$N = k \cdot \Delta\ell$$

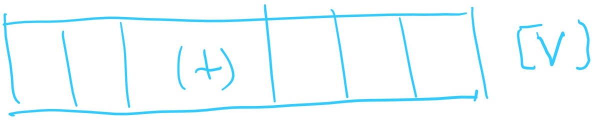
rigidez

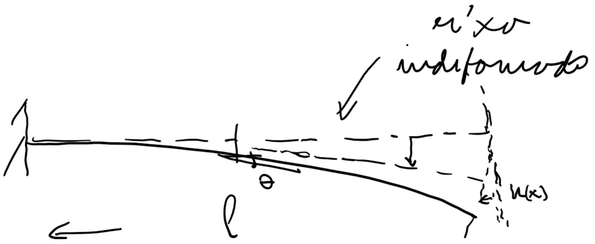
$$k = \frac{EA}{\ell}$$

$$\frac{\ell}{EA}$$

$$\Delta\ell = \frac{N \cdot \ell}{E A}$$

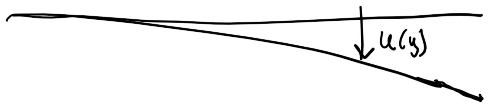






θ: peq. rotações
u peq. deslocam.

$\theta \cong tg\theta$.



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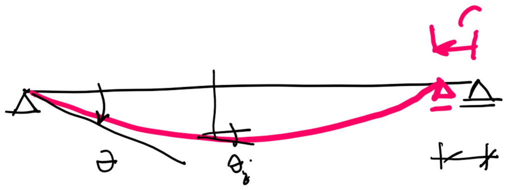
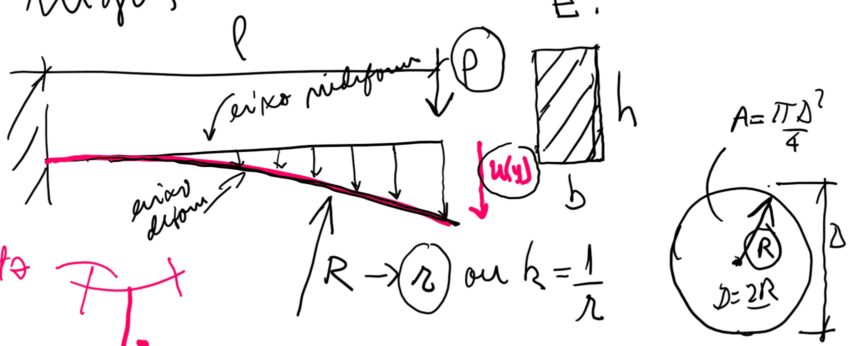
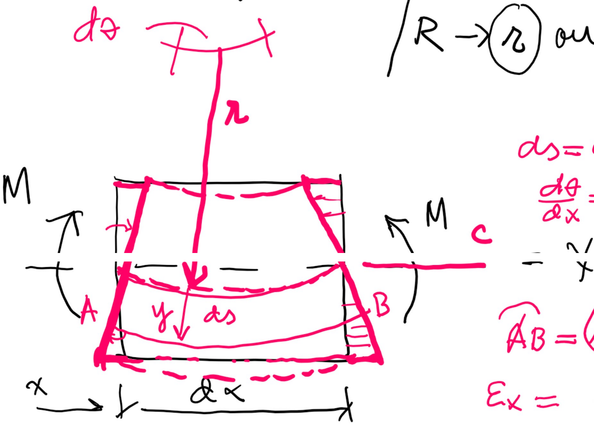


Simplificações

Cálculo das deformações transversais das vigas



$R \to r$ ou $k = \frac{1}{r}$



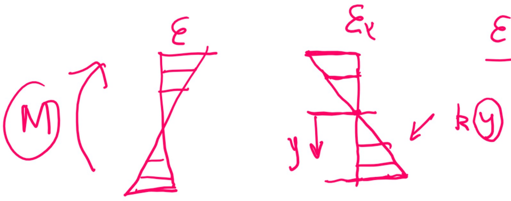
$ds = dx = r\,d\theta$

$\frac{d\theta}{dx} = \frac{1}{r} = k$ (curvatura)

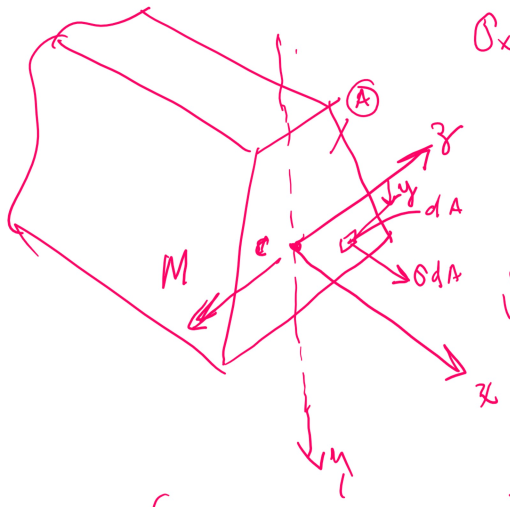
$\widehat{AB} = (r+y)\,d\theta$

$\varepsilon_x = \frac{\widehat{AB} - dx}{dx} = \frac{(r\,d\theta + y\,d\theta) - dx}{dx}$

$\varepsilon_x = y\,\frac{d\theta}{dx} = \frac{y}{r} = ky$



$\sigma_x = E \cdot \varepsilon_x$ (ky)



$\sigma_x = k E y$

$dM_z = \sigma dA \cdot y$

$M_z = \int_A \sigma \cdot y \cdot dA$

mo. de inércia de área

$M_z = \int_A k E \cdot y \cdot y \cdot dA = k \cdot E \cdot \int y^2 \cdot dA$

$M(z) = k (E I_z)$ ← produto de rigidez à flexão

curvatura do eixo da barra

$$M(z) = \frac{1}{r} \cdot E \cdot I_z$$

$M_z = k E$

I

ou

$\frac{d\theta}{dx} = \frac{1}{r}$ com $\theta \approx tg\theta$ $\quad tg\theta = \frac{dy}{dx}$

$\theta = \frac{dy}{dx}$

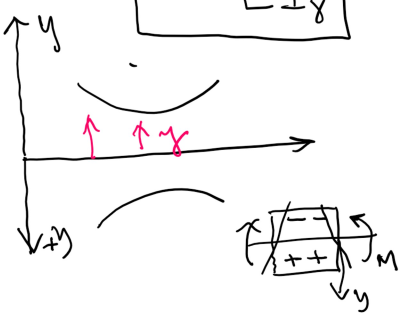
$$\frac{d^2y}{dx^2} = \frac{1}{r}$$

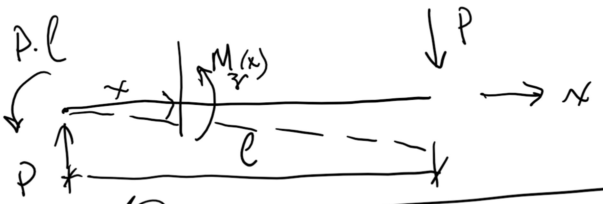
$$\frac{d^2y}{dx^2} = k$$

$$k = \frac{M(z)}{E I_z}$$

$$\frac{d^2y}{dx^2} = (-) \frac{M_z}{E I_z}$$

y ← M





$M_{(x)} + P\cdot\ell - P\cdot x = 0$ $\boxed{M(x) = -P\ell + Px}$

$\frac{d^2y}{dx^2} = -\frac{M(x)}{EI}$ ∴ $EI\cdot y''$

$EIy'' = +P\ell - Px$

$EIy' = +P\ell x - \frac{Px^2}{2} + C_1$

(x)

$= - M$

$p(x)$

$= \frac{dy}{dx}$

θ

$\rightarrow$

$EIy = +\frac{P\ell x^2}{2} - \frac{Px^3}{6} + C_1 x + C_2$

rotação (x)

$\theta(x) = \frac{P\ell x}{EI} - \frac{Px^2}{2EI}$

$\frac{P\ell(x=0)}{EI} - \frac{P(x=0)^2}{2EI} + C_1 = 0$ ∴ $C_1 = 0$

$+\frac{P\ell(x=0)^2}{2EI} - \frac{P(x=0)^3}{6EI} + (C_1=0) + C_2 = 0 \Rightarrow C_2 = 0$

$\boxed{y(x) = \frac{P\ell x^2}{2EI} - \frac{Px^3}{6EI}}$ ← linha elástica da viga

$$y(x=l) = \frac{Pl \cdot l^2}{2EI} - \frac{Pl^3}{6EI} = \frac{3Pl^3}{6EI} - \frac{Pl^3}{6EI} =$$

$$\boxed{y(x=l) = \frac{Pl^3}{3EI}}$$ deslocamento máximo do eixo da barra

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$y(x=\ell) \Rightarrow u(y)_{max} = \frac{P\ell^3}{3EI}$

20 GPa
20.000 MPa

$P = 20kN$ $\ell = 400cm$ $E = 2000 \frac{kN}{cm^2}$

$I = \frac{bh^3}{12} = \frac{10 \times 20^3}{12} =$

$u_{y\,max} = \frac{20kN \cdot (400cm)^3 \times 12}{3 \times 2000 \frac{kN}{cm^2} \cdot 10cm \times (20cm)^3} = 32 \; cm$

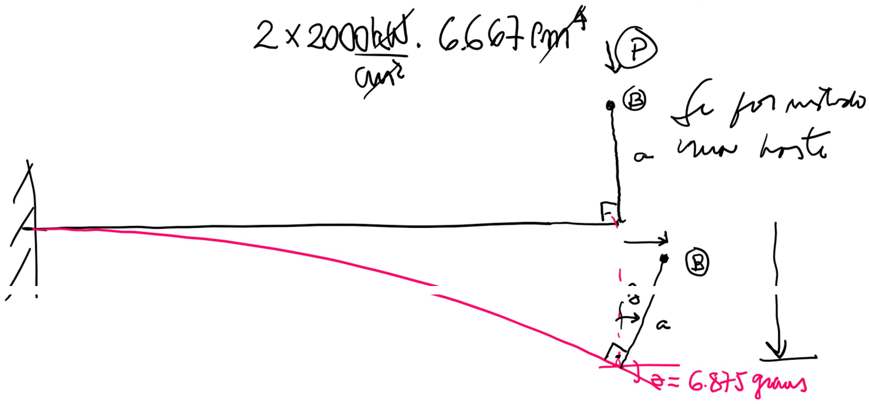
→ $u_{y\,max}$ → flecha do eixo elástico

Calculo de θ em $x=\ell$

$\theta(x) = \frac{P\ell x}{EI} - \frac{Px^2}{2EI}$

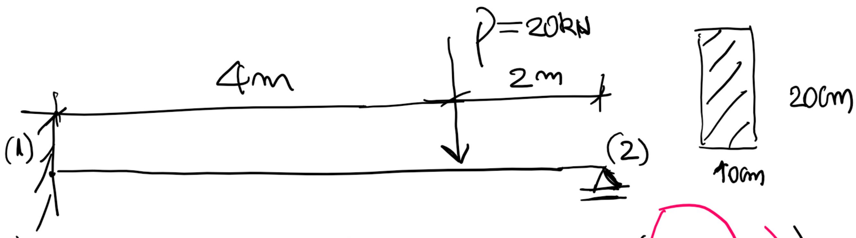
$\theta(x=\ell) = \frac{P\ell^2}{EI} - \frac{P\ell^2}{2EI} = \frac{P\ell^2}{2EI}$

$\theta_{max}(\text{em } x=\ell) = \frac{20kN \cdot (400cm)^2}{2 \times 2000 \frac{kN}{cm^2} \cdot 6667 cm^4} = 6.875$ graus.



PROBLEMA 4

$E = 2000kN/cm^2$



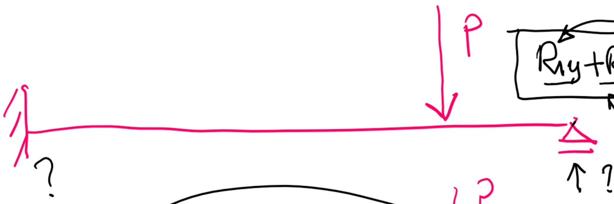
Det. os esforços solicitantes da viga (M; V e ~~N~~)

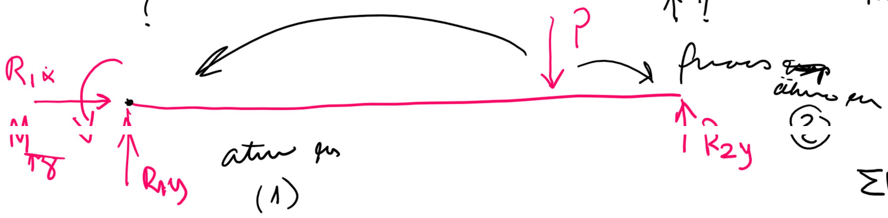
$\Sigma F_y = 0$

$R_{1y} + R_{2y} = P$

$\Sigma F_x = 0$

$R_{1x} = 0$

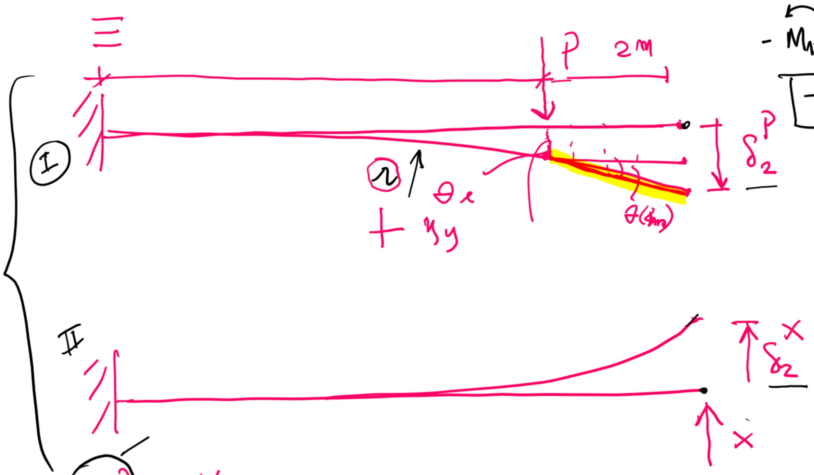




$\Sigma M_z = 0$

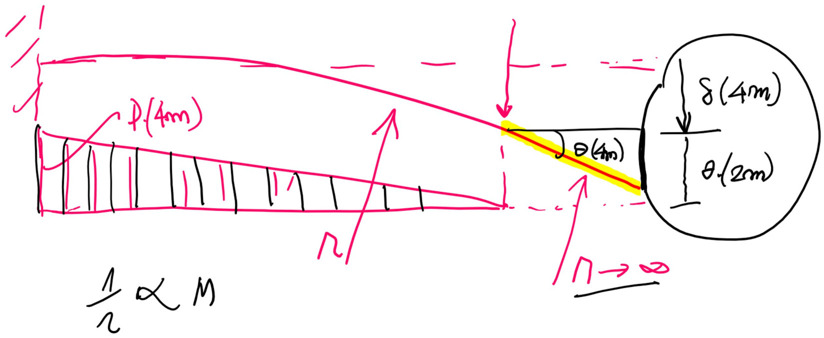
$-M_{1z} + P.4 - 6.R_{2y} = 0$

$-M_{1z} - 6R_{2y} = -4P$

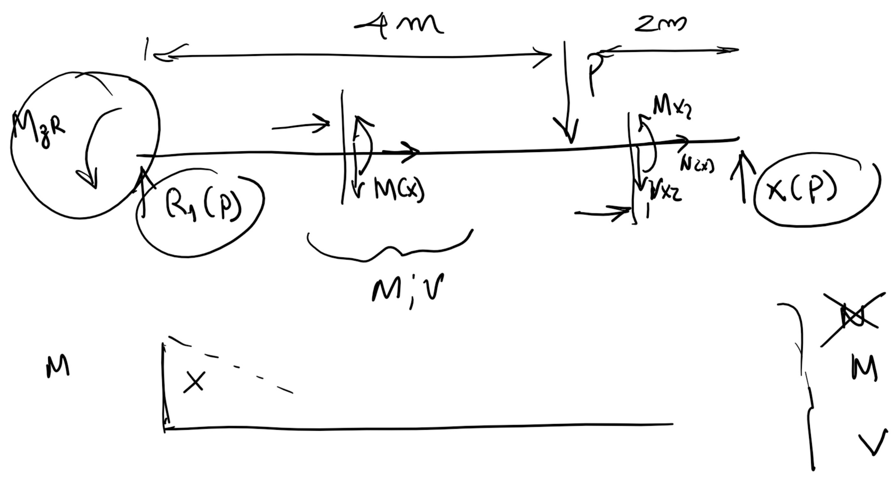


$\delta_2^P + \delta_2^X = 0 \Rightarrow$ o valor de (X) ⟸ Eq. de compatibilidade

$\delta_2^X = \dfrac{X.l^3}{3EI}$ (X)



$\frac{1}{r} \propto M$



Entregar no dia 25 às 7h.