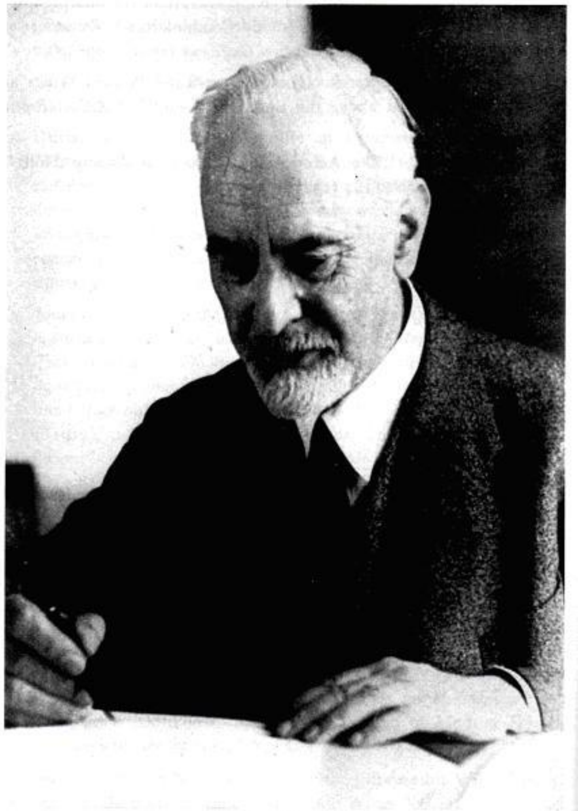
- Fenômenos de Transporte I - 2020

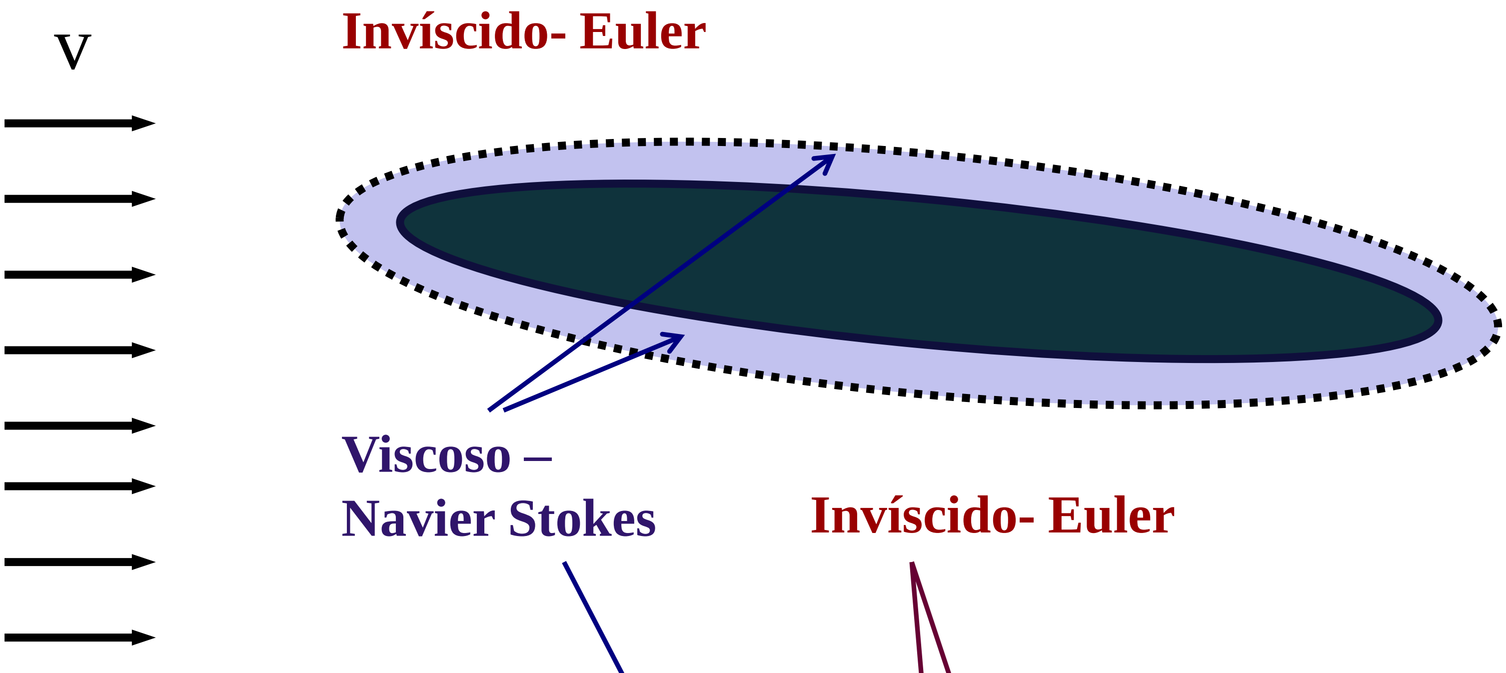
CAMADA LIMITE LAMINAR

- EXERCÍCIOS

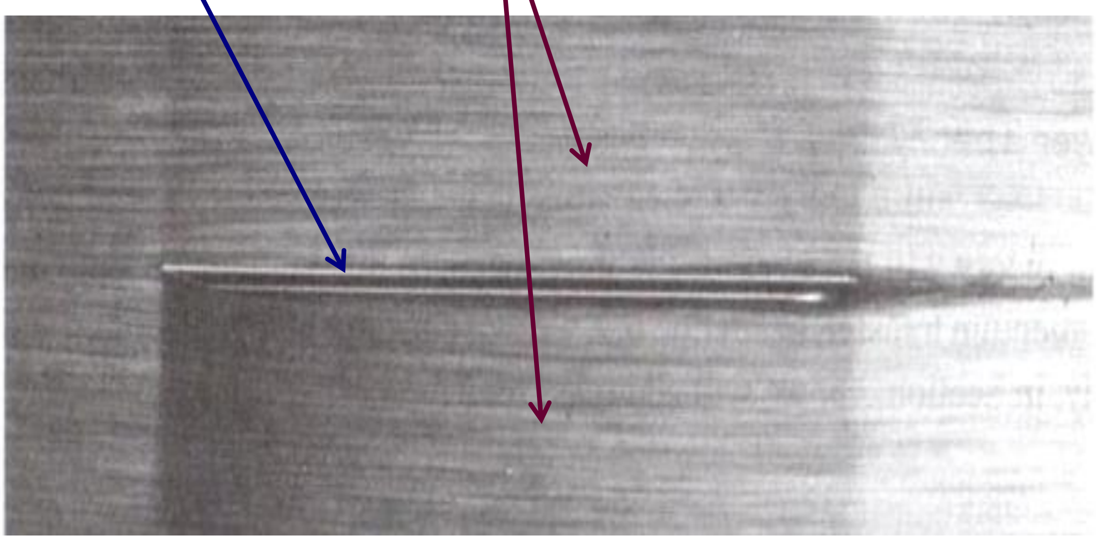


Ludwig Prandtl

Modelo - Prandtl



Placa Plana



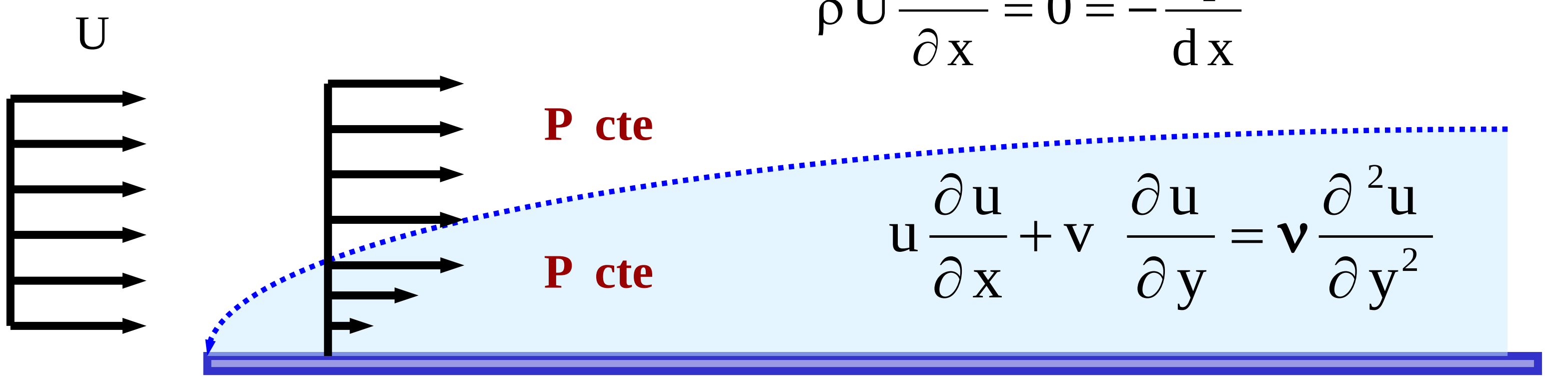
Equações – Camada Limite Laminar – Placa Plana

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2}$$

P cte

$$\rho U \frac{\partial U}{\partial x} = 0 = -\frac{dp}{dx}$$



$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2}$$

Camada Limite Laminar – Placa Plana – Equações Aproximadas e Condições de Contorno

Continuidade:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Quantidade de Movimento:

$$\boxed{u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}}$$

Condições de Contorno:

parede, $y = 0 \rightarrow u = 0$

$v = 0$

borda, $y = \infty \rightarrow u = U$

Solução de Blasius

$$u = \frac{\partial \Psi}{\partial y} \quad ; \quad v = -\frac{\partial \Psi}{\partial x}$$

$$\eta = y\sqrt{\frac{U}{\nu x}}$$

$$f(\eta) = \frac{\Psi}{\sqrt{x \nu U}}$$

$$f(\eta)\frac{d^2 f(\eta)}{d\eta^2} + 2\frac{d^3 f(\eta)}{d\eta^3} = 0$$

parede, $\eta = 0 \rightarrow f = 0$ e $f' = 0$

borda, $\eta = \infty \rightarrow f' = 1$

Solução de Blasius

Solução :

$$u = U\frac{d\,f(\eta)}{d\eta}$$

$$v = \frac{1}{2}\sqrt{\frac{\nu U}{x}}\left(\eta\frac{d\,f(\eta)}{d\eta} - f(\eta)\right)$$

$$\frac{u}{U} = 0{,}99 = \frac{df}{d\eta} = 0{,}99 \Rightarrow \eta = 5 = \delta\sqrt{\frac{U}{\nu x}}$$

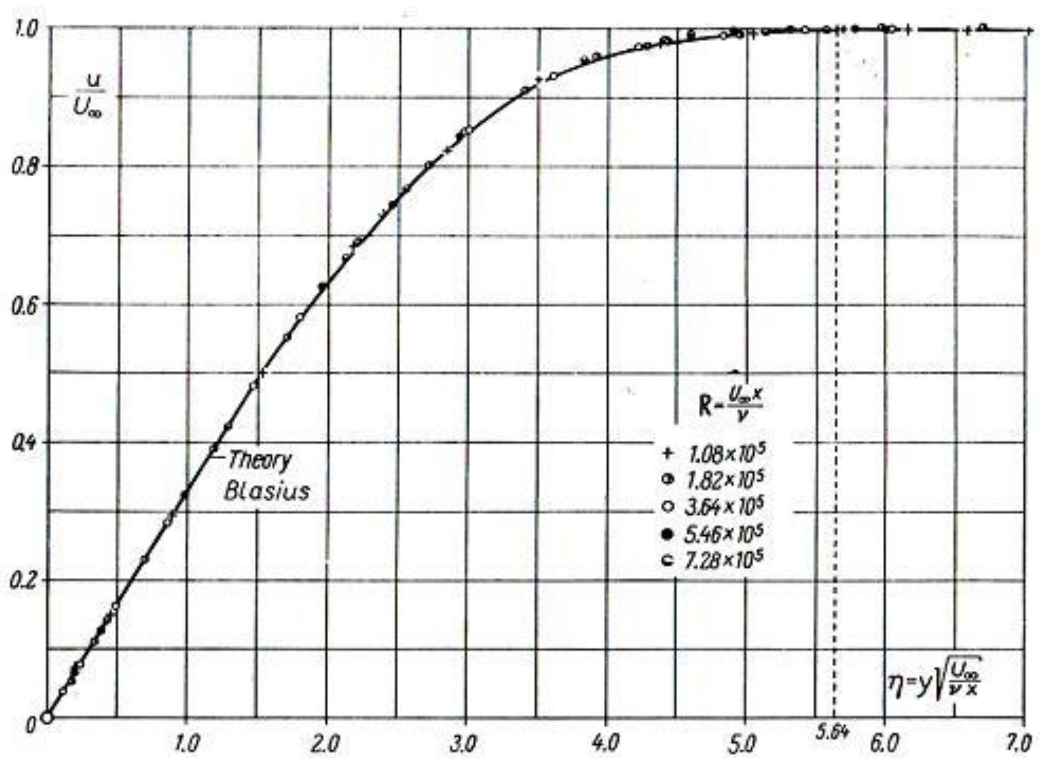


Fig. 7.9. Velocity distribution in the laminar boundary layer on a flat plate at zero incidence, as measured by Nikuradse [20]

$$\boxed{\delta = 5\sqrt{\frac{\nu x}{U}} = \frac{5x}{\sqrt{Re_x}}}$$

$$\boxed{\mathbf{Re_{crítico} = 5.10^5}}$$

Solução de Blasius

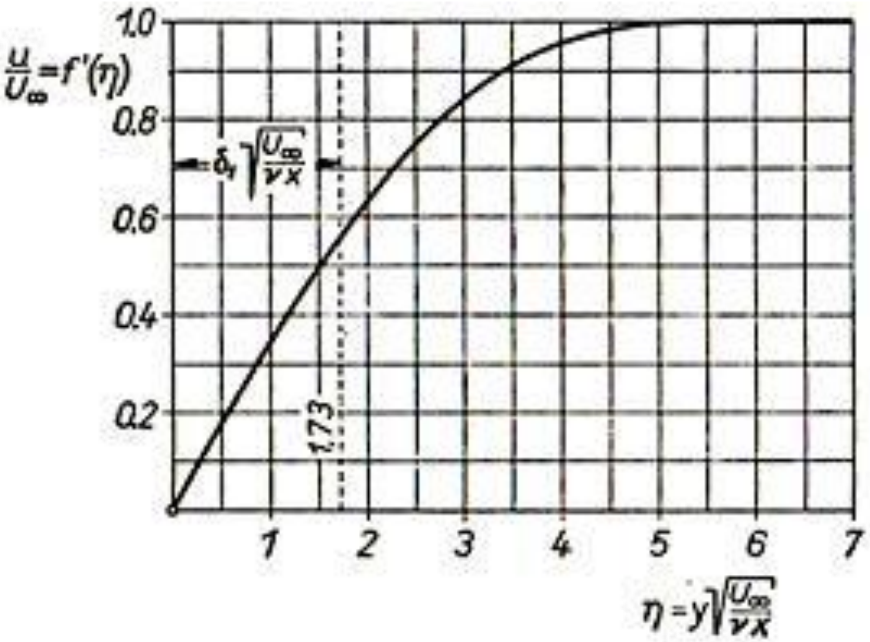


Fig. 7.7. Velocity distribution in the boundary layer along a flat plate, after Blasius [2]

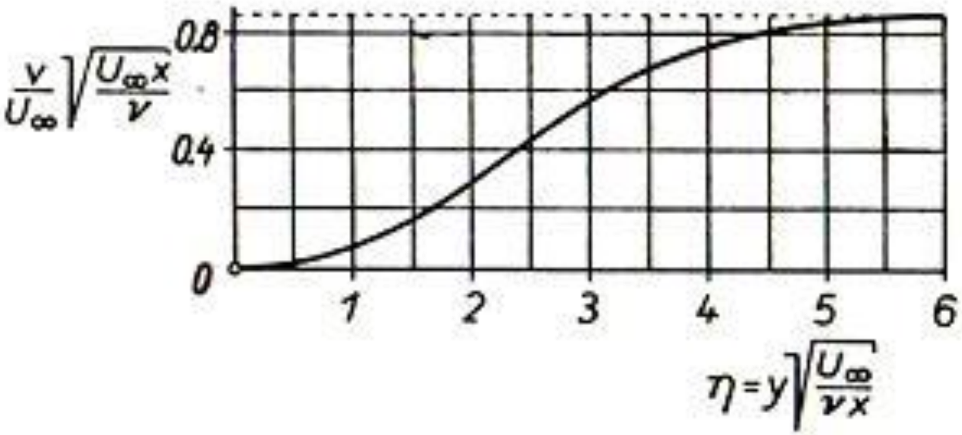


Fig. 7.8. The transverse velocity component in the boundary layer along a flat plate

$$u = U \frac{d\,f(\eta)}{d\eta}$$

$$v = \frac{1}{2}\sqrt{\frac{\nu\, U}{x}} \left(\eta \frac{d\,f(\eta(}{d\eta} - f(\eta(\right)$$

Solução de Blasius

$\eta = y\sqrt{\frac{U_\infty}{\nu x}}$	f	$f' = \frac{u}{U_\infty}$	f''
0	0	0	0·33206
0·2	0·00664	0·06641	0·33199
0·4	0·02656	0·13277	0·33147
0·6	0·05974	0·19894	0·33008
0·8	0·10611	0·26471	0·32739
1·0	0·16557	0·32979	0·32301
1·2	0·23795	0·39378	0·31659
1·4	0·32298	0·45627	0·30787
1·6	0·42032	0·51676	0·29667
1·8	0·52952	0·57477	0·28293
2·0	0·65003	0·62977	0·26675
2·2	0·78120	0·68132	0·24835
2·4	0·92230	0·72899	0·22809
2·6	1·07252	0·77246	0·20646
2·8	1·23099	0·81152	0·18401
3·0	1·39682	0·84605	0·16136
3·2	1·56911	0·87609	0·13913
3·4	1·74696	0·90177	0·11788
3·6	1·92954	0·92333	0·09809
3·8	2·11605	0·94112	0·08013
4·0	2·30576	0·95552	0·06424
4·2	2·49806	0·96696	0·05052
4·4	2·69238	0·97587	0·03897
4·6	2·88826	0·98269	0·02948
4·8	3·08534	0·98779	0·02187
5·0	3·28329	0·99155	0·01591
5·2	3·48189	0·99425	0·01134
5·4	3·68094	0·99616	0·00793
5·6	3·88031	0·99748	0·00543
5·8	4·07990	0·99838	0·00365
6·0	4·27964	0·99898	0·00240
6·2	4·47948	0·99937	0·00155
6·4	4·67938	0·99961	0·00098
6·6	4·87931	0·99977	0·00061
6·8	5·07928	0·99987	0·00037
7·0	5·27926	0·99992	0·00022
7·2	5·47925	0·99996	0·00013
7·4	5·67924	0·99998	0·00007
7·6	5·87924	0·99999	0·00004
7·8	6·07923	1·00000	0·00002
8·0	6·27923	1·00000	0·00001
8·2	6·47923	1·00000	0·00001
8·4	6·67923	1·00000	0·00000
8·6	6·87923	1·00000	0·00000
8·8	7·07923	1·00000	0·00000

Fator de atrito

Coeficiente de arraste (fator de atrito) na placa:

$$C_D = \frac{\left(\tau_{yx}\right)_{y=0}}{\frac{1}{2}\rho U^2}$$

$$\mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

$$C_D = \frac{\mu}{\frac{1}{2}\rho U^2}\left(\frac{\partial u}{\partial y} + \cancel{\frac{\partial v}{\partial x}}\right)_{y=0} = \frac{\mu}{\frac{1}{2}\rho U^2}\, U \sqrt{\frac{U}{x\,\nu}}\, \frac{d^2 f}{d\eta^2}(0)$$

Para y = 0 (η=0) ⇒ f″(0) = 0,332, resulta:

$$C_D = 0{,}664\sqrt{\frac{\nu}{U\,x}}$$

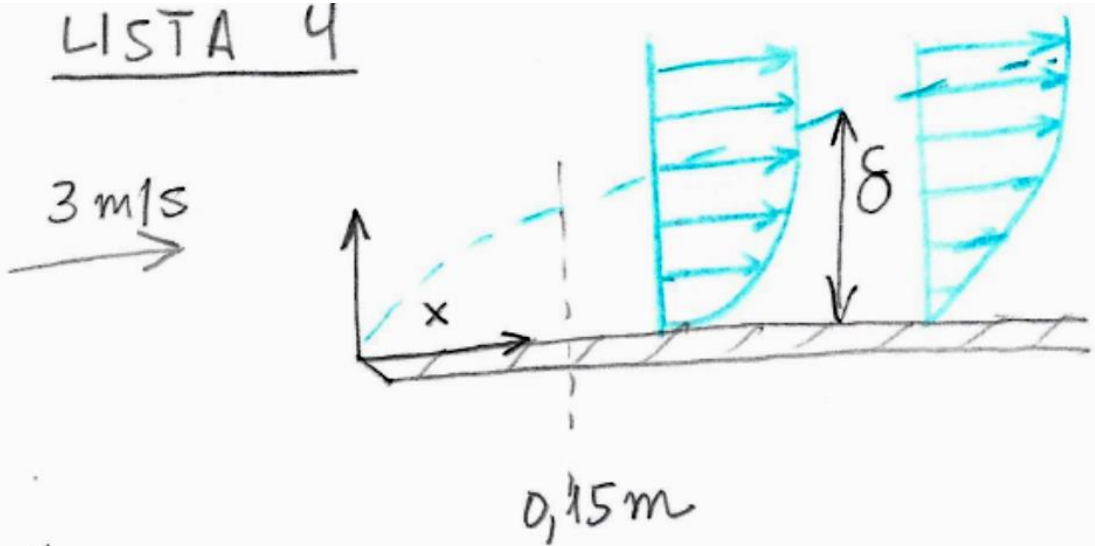
$$\boxed{C_D = 0{,}664\ \mathrm{Re}_x^{-1/2}}$$

(1) Estimar a espessura da camada limite para o escoamento sobre uma placa plana horizontal, com velocidade de aproximação constante de 3 m/s, na posição distantede0,15 m da borda de ataque, admitida com incidência nula, considerando-se os seguintes casos:

Caso	Fluido	Densidade (kg/m^3)	Viscosidade (cP)	Difusividade de quant. de mov. (m^2/s)
a	Água	1000	1	1×10^{-6}
b	Ar	1	0,02	2×10^{-5}
c	Glicerina	1260	1500	$1{,}19 \times 10^{-3}$

Resposta: (a) $\delta_a = 0{,}0011$ m; (b) $\delta_b = 0{,}005$ m; (c) $\delta_c = 0{,}0386$ m.

1) LISTA 4



a) água

$\rho = 1000\ kg/m^3 \qquad \mu = 1cP \qquad \nu = 10^{-6}$

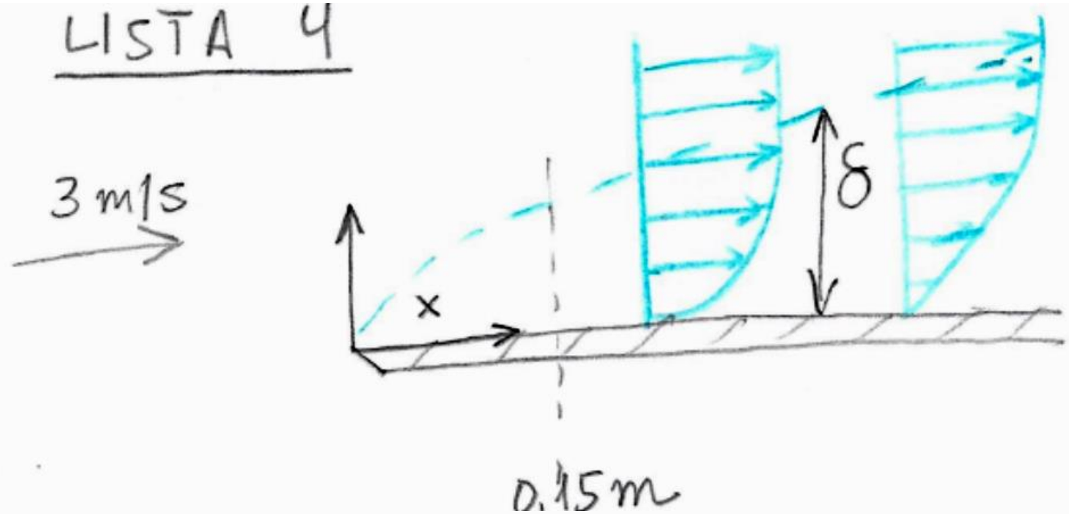
$$Re = \frac{\rho V x}{\mu} = \frac{Vx}{\nu} = \frac{3 \times 0{,}15}{10^{-6}} = 4{,}5 \cdot 10^{5}$$

$$\frac{\delta}{x} = \frac{5}{\sqrt{Re_x}} = \frac{5}{\sqrt{4{,}5 \cdot 10^5}} = 7{,}45 \cdot 10^{-3}$$

$$\delta = 7{,}45 \cdot 10^{3} \cdot 0{,}15 = \underline{1{,}1 \cdot 10^{-3}\ m}$$

LISTA 4

1)



b) ar

$Re = 2{,}25 \cdot 10^{4}$

$\delta = 5 \cdot 10^{-3}\ m$

c) glicerina

$Re = 378$

$\delta = 38{,}6 \cdot 10^{-3}\ m$

$\delta_{glicerina} > \delta_{ar} > \delta_{água}$

1) Extra - $C_D = ?$

$$C_D = \frac{0,664}{\sqrt{Rex}}$$

$$C_D\ \text{água} = \frac{0,664}{\sqrt{4,5 \cdot 10^{+5}}} = 9,9 \cdot 10^{-4}$$

$$C_D\ \text{ar} = \frac{0,664}{\sqrt{2,25 \cdot 10^{4}}} = 4,4 \cdot 10^{-3}$$

$$C_D\ \text{glic} = \frac{0,664}{\sqrt{378}} = 3,41 \cdot 10^{-2}$$

C_D glicerina $> C_{D\,ar} > C_D$ água

τ_w ?

$$\tau_w = \frac{1}{2} \rho U^2 \cdot C_D$$

$$\tau_{w\,\text{água}} = \frac{1}{2} \cdot 1000 \cdot 3^2 \cdot 9{,}9 \cdot 10^{-4} = 4{,}45 \text{ Pa}$$

$$\tau_{ar} = \frac{1}{2} \cdot 1 \cdot 3^2 \cdot 4{,}4 \cdot 10^{-3} = 0{,}02 \text{ Pa}$$

$$\tau_{glicerina} = \frac{1}{2} \cdot 1260 \cdot 3^2 \cdot 3{,}41 \cdot 10^{-2} = 193{,}3 \text{ Pa}$$

$$\tau_{glicerina} > \tau_{w\,\text{água}} >> \tau_{ar}$$

(8) Ar a 50°C e 20 psig (viscosidade = 0,02 cP) escoa sobre uma placa horizontal de incidência nula, com uma velocidade de aproximação uniforme de 10 m/s.

(a) Determinar, usando a solução exata de Blasius, as componentes de velocidade v_x e v_y no ponto dado por x = 0,05 m e y = δ/2, onde x é a direção de escoamento, y é a direção normal à placa e δ é a espessura da camada limite nessa posição. (b) Determinar a força de atrito numa placa de comprimento igual a 10 m (medido na direção de escoamento a partir da borda de ataque) e largura também de 10 m.

Resposta: (a) $v_x = 7{,}2$ *m/s,* $v_y = 0{,}017$ *m/s; (b) F = 35,1 N.*

20 psig   10 m/s $\xrightarrow[50°C]{ar}$

$\mu = 0{,}02$ cP

x = 0,05 m

$$20 \text{ psig} \rightarrow (20 + 14{,}7)\text{ psia} \rightarrow 2{,}36 \text{ atm}$$

$$\rho = \frac{PM}{RT} = \frac{2{,}36 . 29}{0{,}082 . 323} = 2{,}582 \text{ kg/m}^3$$

$$Re = \frac{\rho V x}{\mu} = \frac{2{,}582 . 10 . 0{,}05}{0{,}02 . 10^{-3}} = 64550$$

$$\delta = \frac{5x}{\sqrt{Re_x}} = \frac{0{,}25}{\sqrt{64550}} = 10^{-3} \text{ m}$$

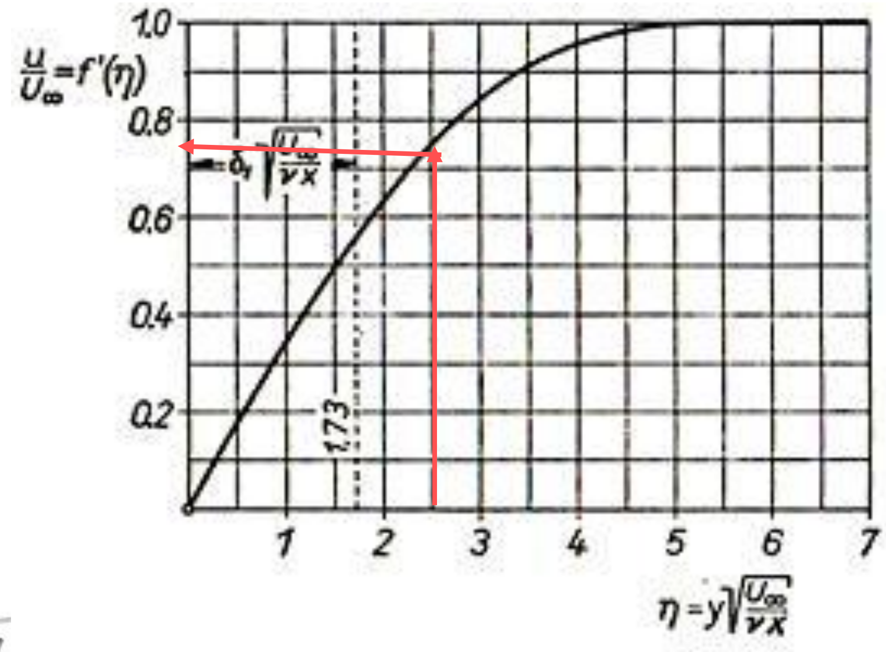
$$\delta = \frac{5x}{\sqrt{Re_x}} = \frac{0{,}25}{\sqrt{64550}} = 10^{-3}\ m$$

$$y = \delta/2 = 5.10^{-4}\ m \quad e \quad x = 0{,}05\ m$$

$$\eta = y\sqrt{\frac{U_0}{\nu x}} = 5.10^{-4}\sqrt{\frac{10}{7{,}75.10^{-6} \cdot 0{,}05}} = 2{,}5$$

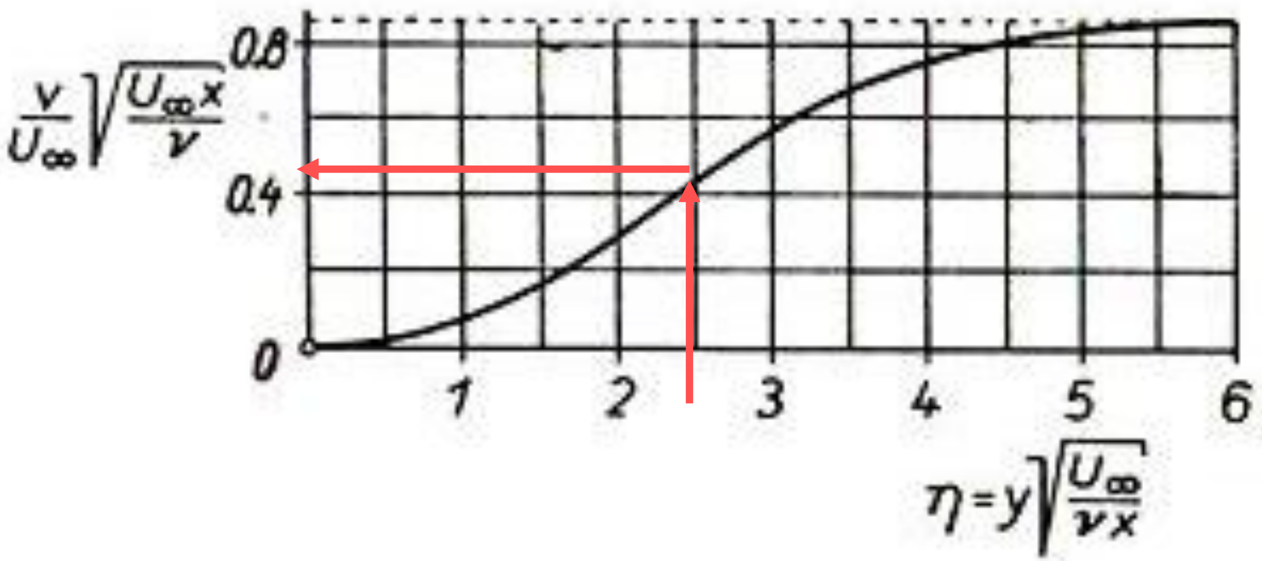
$$\nu = \frac{\mu}{\rho} = 7{,}75.10^{-6}$$

$$\frac{V_x}{U_0} = 0{,}75 = f'(\eta) \Rightarrow \boxed{V_x = 7{,}5\ m/s}$$

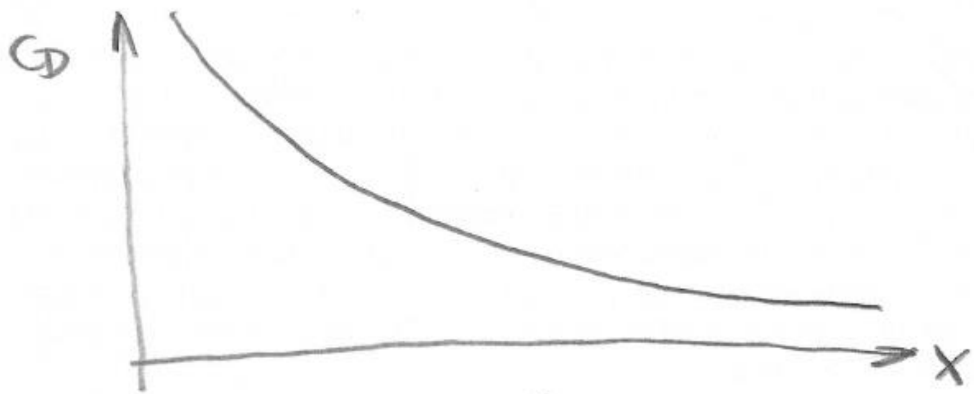


$$\frac{v_y}{u_0}\sqrt{\frac{u_0 x}{\nu}} \underset{\eta=2,5}{=} 0,42 = \frac{v_y}{10}\sqrt{\frac{10\times 0,05}{7,75\cdot 10^{-6}}}$$

$$\boxed{v_y = 0,017 \text{ m/s}}$$



$$C_D = \frac{0{,}664}{\sqrt{Re_x}}$$



$$\tau_w = C_D \times \frac{1}{2} \rho U^2$$

$$\tau_w = \frac{0{,}332\, \rho U^2}{\sqrt{\rho U x / \mu}} = 0{,}332\, \mu U \sqrt{\frac{\rho U}{\mu x}}$$

$$F_w = \int^{w} \int^{L} \tau_w \cdot dx\, dz = \iint 0{,}332\, \mu U \sqrt{\frac{\rho U}{\nu x}}\, dx\, dz$$

$$F_w = \int_0^w \int_0^L \tau_w \cdot dx\, dz = \iint 0{,}332 \mu U \sqrt{\frac{\rho U}{\mu x}}\, dx\, dz$$

$$F_w = w \cdot \left[0{,}332 \mu U \sqrt{\frac{\rho U}{\mu x}}\; 2 \cdot \sqrt{x} \right]_0^L$$

$$F_w = 0{,}664\, w \mu U \sqrt{\frac{\rho U L}{\mu}}$$

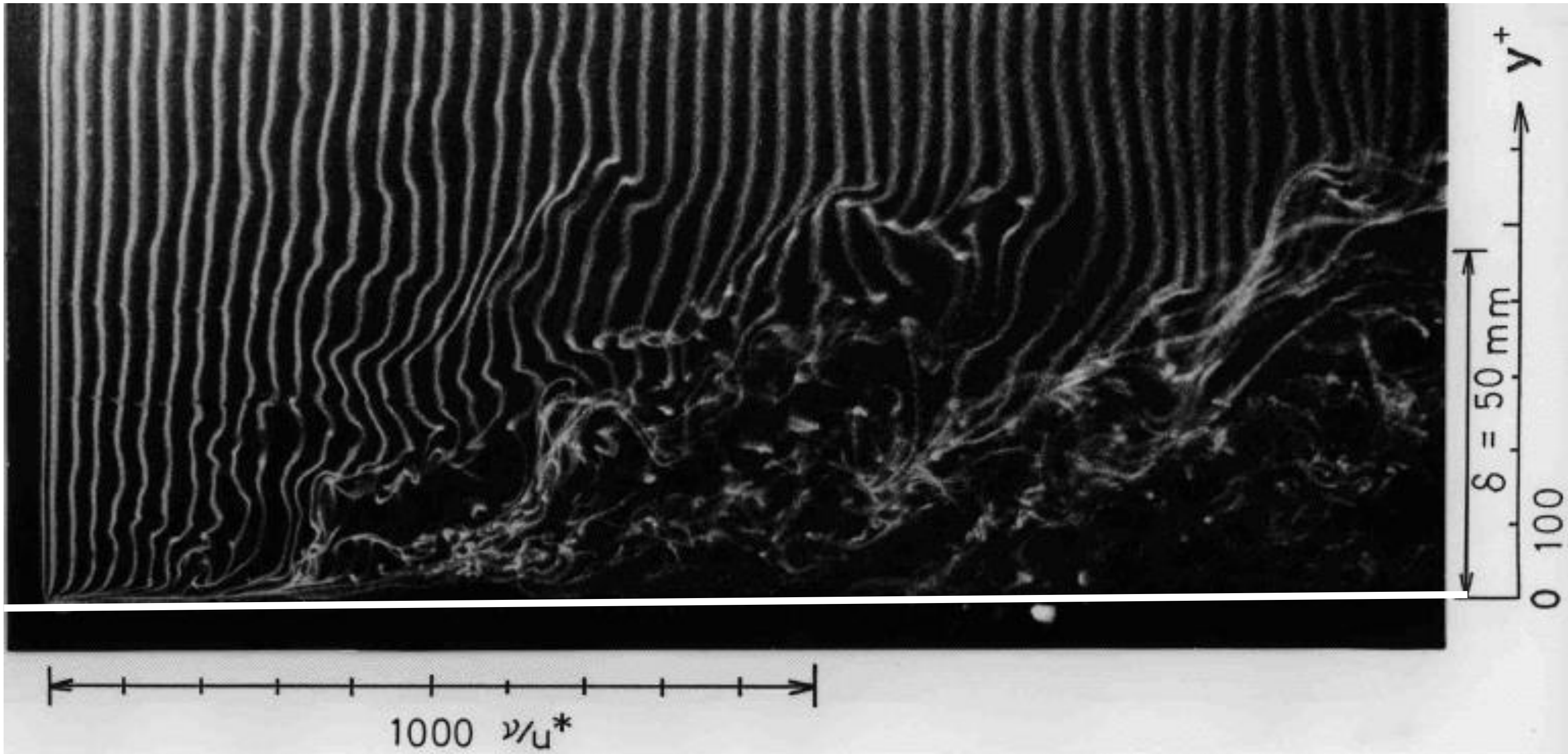
$$F_w = 0{,}664\, w \mu U \sqrt{Re_L}$$

ou $$\overline{C_D} = \frac{F_w}{w \cdot L \cdot \frac{1}{2} \rho U^2} = 1{,}328 \sqrt{\frac{\mu}{\rho U L}} =$$

$$\overline{C_D} = 1{,}328\, Re_L^{-1/2}$$

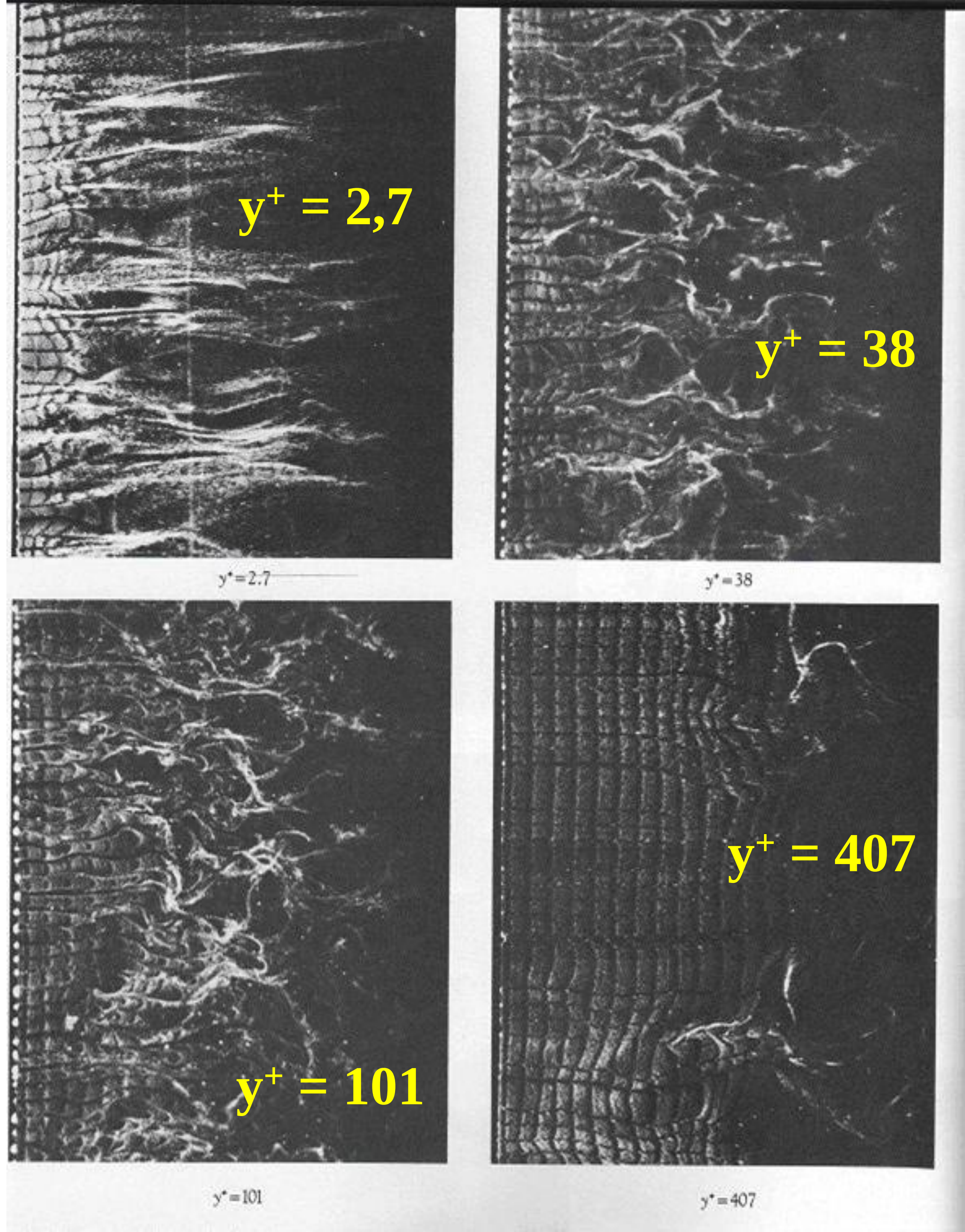
$$F_w = 0{,}664 \cdot 10 \cdot 0{,}02 \cdot 10^{-3} \cdot 10 \sqrt{\underbrace{\frac{2582 \cdot 10 \cdot 10}{902 \cdot 10^{-3}}}_{1{,}3 \cdot 10^7}}$$

$$\boxed{F_w = 4{,}77\ N}$$?



y+
δ = 50 mm
0 100
1000 ν/u*

161. Structure of a turbulent boundary layer. Successive layers of the flow near a flat plate in a water channel are shown by tiny hydrogen bubbles released periodically from a thin platinum wire seen at the left. The height $y^+ = y\,u_\tau/\nu$ of the wire above the plate is shown in wall variables, where $u_\tau = (\tau_w/\varrho)^{1/2}$ is the friction velocity. The characteristic low- and high-speed streaks shown in the viscous sublayer at $y^+ = 2.7$ become less noticeable farther away, and have disappeared in the logarithmic region at $y^+ = 101$. In the wake region at $y^+ = 407$ the turbulence is seen to be intermittent and of larger scale. *Kline, Reynolds, Schraub & Runstadler 1967*



y+