Exercícios Derivada - Regra da cadeia

1- $y(x) = (3x^2 - 5x - 3)^{15}$ $y(u) = u^{15}$

$u = 3x^2 - 5x - 3$

$y'(u) = 15u^{14}$ $u'(x) = 6x - 5$

Love Flowers

$u'(x) = 15(3x^2 - 5x - 3)^{14}(6x - 5)$

2- $y(x) = \sqrt[3]{x^2+3x} = (x^2+3x)^{\frac{1}{3}}$

$y(u) = u^{\frac{1}{3}}$ $u(x) = x^2+3x$

$y'(u) = \frac{1}{3} u^{-\frac{2}{3}}$ $u'(x) = 2x+3$

$= \frac{1}{3u^{\frac{2}{3}}}$

Então $y'(x) = y'(u) \cdot u'(x)$

$y'(x) = \frac{1}{3(x^2+3x)^{\frac{2}{3}}} \cdot 2x+3$

$y'(x) = \frac{2x+3}{3\sqrt[3]{(x^2+3x)^2}}$

3- $y(x) = \frac{1}{\sqrt[3]{3x+5}} = (3x+5)^{-\frac{1}{3}}$

$y(u) = u^{-\frac{1}{3}}$ $u(x) = 3x+5$

$y'(u) = \frac{-1}{3} u^{-\frac{4}{3}}$ $u'(x) = 3$

$y'(x) \frac{-1}{\cancel{3}} u^{-\frac{4}{3}} \cdot \cancel{3} = \frac{-1}{\sqrt[3]{(3x+5)^4}}$

4 - $y(x) = \log_{10}(x^2+3)$

$y(u) = \log_{10} u$ $\qquad$ $u(x) = x^2+3$

$u'(x) = 2x$

$y'(u) = \dfrac{1}{u \ln(10)}$

Então $y'(x) = y'(u) \cdot u'(x)$

$y'(x) = \dfrac{1}{u \ln(10)} \cdot 2x = \dfrac{2x}{(x^2+3)\ln(10)}$

5 - $y(x) = e^{(3x^2-2x+1)}$

$y(u) = e^u$ $\qquad$ $u(x) = 3x^2-2x+1$

$y'(u) = e^u$ $\qquad$ $u'(x) = 6x-2$

Então $y'(x) = y'(u) \cdot u'(x)$

$y'(x) = e^u \cdot (6x-2)$

$y'(x) = e^{3x^2-2x+1}\ (6x-2)$

$$6 - y(x) = \underbrace{(x^2+3)^5}_{w(x)} \cdot \underbrace{(2x+3)^6}_{z(x)}$$

$$y'(x) = w'(x)\,z(x) + w(x)\,z'(w)$$

$$w(x) = (x^2+3)^5$$

$w(u) = u^5$ $\quad$ $u(x) = x^2+3$

$w'(u) = 5u^4$ $\quad$ $u'(x) = 2x$

$$w'(x) = 5u^4(2x) = 5(x^2+3)^4(2x) = 10x(x^2+3)^4$$

$$z(w) = (2x+3)^6$$

$z(u) = u^6$ $\quad$ $u(x) = 2x+3$

$z'(u) = 6u^5$ $\quad$ $u'(x) = 2$

$$z'(w) = 6u^5\,2 = 12(2x+3)^5 = 12(2x+3)^5$$

Então, aplicando a regra do produto

$$y'(x) = 10x(x^2+3)^4(2x+3)^6 + (x^2+3)^5\,12(2x+3)^5$$

Exercício

$x(p) = \frac{8000}{p} = 8000\,p^{-1}$ liq. por mês

$p(t) = 0,05\sqrt{t^3} + 16,8$ reais (preço em t meses)

$p(t) = 0,05\,t^{3/2} + 16,8$

Calcular taxa variação da demanda mensal

$x'(t)$ para $t = 16$

$x'(t) = x'(p) \cdot p'(t)$

$x'(p) = (-1)\,8000\,p^{-2} = -\frac{8000}{p^2}$

$p'(t) = \frac{3}{2}\,0,05\,t^{\frac{1}{2}}$.

Então $x'(t) = -\frac{8000}{p^2} \cdot 0,075\sqrt{t} = -\frac{600\sqrt{t}}{p^2}$

substituindo $p = 0,05\sqrt{t^3} + 16,8$

$$x'(t) = \frac{-600\sqrt{t}}{(0,05\sqrt{t^3} + 16,8)^2}$$

$$x'(16) = \frac{-600\sqrt{16}}{(0,05\sqrt{16^3} + 16,8)^2} = \frac{-2400}{400} = -6$$

Derivadas de ordem superior

1- $y = \sqrt[6]{x} = x^{\frac{1}{6}}$

$y' = \frac{1}{6} x^{\frac{1}{6}-1} = \frac{1}{6} x^{-\frac{5}{6}}$

$y'' = \frac{1}{6}\left(\frac{-5}{6}\right) x^{-\frac{11}{6}} = \frac{-5x^{-\frac{11}{6}}}{36} = \frac{-5}{36\sqrt[6]{x^{11}}}$

2- $y = x^{-5}$

$y' = -5x^{-6}$

$y'' = -5(-6)\, x^{-7} = 30x^{-7} = \frac{30}{x^7}$

3- $y = \frac{x}{2(x+1)}$ $\begin{array}{l} \rightarrow g(x) \\ \rightarrow w(x) \end{array}$

$g'(x) = 1$

$w'(x) = 2$

Regra quociente

$y'(x) = \frac{g'(x)\, w(x) - g(x)\, w'(x)}{[w(x)]^2}$

$y'(x) = \frac{1(2(x+1)) - x(2)}{[2(x+1)]^2} = \frac{\cancel{2x} + 2 - \cancel{2x}}{(2x+2)^2} = \frac{2}{(2x+2)^2}$

$= 2(2x+2)^{-2}$

Love Flowers

$$y''(x) = 2(-2)(2x+2)^{-3}(2)$$

$$y''(x) = \frac{-8}{(2x+2)^3}$$

4- $y = \left(1 + \frac{1}{x}\right)^2$

Regra cadeia $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$

$y = u^2 \qquad u = 1 + \frac{1}{x} = 1 + x^{-1}$

$\frac{dy}{du} = 2u$

$\frac{du}{dx} = -x^{-2} = \frac{-1}{x^2}$

$\frac{dy}{dx} = 2u \cdot \left(\frac{-1}{x^2}\right) = 2\left(1 + \frac{1}{x}\right)\left(\frac{-1}{x^2}\right)$

$= -\frac{2}{x^2}\left(1 + \frac{1}{x}\right) = \frac{-2}{x^2} - \frac{2}{x^3} = \frac{-2x-2}{x^3}$ $\leftarrow g(x)$ $\leftarrow w(x)$

y'' por regra do quociente $= \frac{g'(x)w(x) - g(x)w'(x)}{[w(x)]^2}$

$g'(x) = -2$
$w'(x) = 3x^2$

$y''(x) = \frac{-2(x^3) - (-2x-2)(3x^2)}{(x^3)^2}$

$y'' = \frac{-2x^3 + 6x^3 + 6x^2}{x^6} = \frac{-2x + 6x + 6}{x^4} = \frac{4x+6}{x^4}$

5- $y = \dfrac{x}{\sqrt{x^2-1}} \begin{array}{l} \to g(x) \\ \to w(x) \end{array} = (x^2-1)^{\frac{1}{2}}$

Regra cadeia

$y' = \dfrac{g'(x)\, w(x) - g(x)\, w'(x)}{(w(x))^2}$

$g'(x) = 1$

$w'(x) = \dfrac{1}{\cancel{2}} (x^2-1)^{-\frac{1}{2}} \cdot \cancel{2}x = \dfrac{x}{\sqrt{x^2-1}}$

$y' = \dfrac{\sqrt{x^2-1} - x \dfrac{x}{\sqrt{x^2-1}}}{(\sqrt{x^2-1})^2} = \dfrac{\dfrac{(\sqrt{x^2-1})^2 - x^2}{\sqrt{x^2-1}}}{x^2-1}$

$= \dfrac{\dfrac{\cancel{x^2} - 1 - \cancel{x^2}}{\sqrt{x^2-1}}}{x^2-1} = \dfrac{-1}{(x^2-1)\sqrt{x^2-1}} =$

$= \dfrac{-1}{(x^2-1)^1 (x^2-1)^{1/2}} = \dfrac{-1}{(x^2-1)^{\frac{3}{2}}} = -(x^2-1)^{-\frac{3}{2}}$

$y''(x)$ por regra da cadeia

$\dfrac{dy}{dx} = \dfrac{dy}{du} \cdot \dfrac{du}{dx}$ $\qquad y = -(u)^{-\frac{3}{2}}$

$u = x^2 - 1$

Dem

$$\frac{dy}{du} = \frac{3}{2} u^{-\frac{5}{2}}$$

$$\frac{du}{dx} = 2x$$

$$\frac{dy}{dx} = \frac{3}{\cancel{2}} (x^2 - 1)^{-\frac{5}{2}} \cdot \cancel{2}x = \frac{3x}{(x^2-1)^{5/2}}$$

6- $y = \frac{e^x}{x}$ $\leftarrow g(x)$ $\leftarrow w(x)$

$g'(x) = e^x$
$w'(x) = 1$

Regra quociente

$$y' = \frac{e^x x - e^x}{x^2} \begin{array}{l} \leftarrow g(x) \\ \leftarrow w(x) \end{array}$$

$$g'(x) = e^x x + \cancel{e^x} - \cancel{e^x} = x e^x$$

$$w'(x) = 2x$$

Regra quociente $y''(x) = \frac{xe^x \cdot x^2 - [2x(e^x x - e^x)]}{x^4}$

$$= \frac{e^x x^3 - 2x^2 e^x + 2xe^x}{x^4} = \frac{e^x x^2 - 2xe^x + 2e^x}{x^3}$$

Femmina

$$= \frac{e^x (x^2 - 2x + 2)}{x^3} = y''(x)$$