4) $x(t) = \begin{bmatrix} t \\ 1 \end{bmatrix}$ $y(t) = \begin{bmatrix} t^2 \\ 2t \end{bmatrix}$

a) $W[x(t), y(t)](t) = \begin{vmatrix} t & t^2 \\ 1 & 2t \end{vmatrix} = 2t^2 - t^2 = t^2$

b) x e y são Linearmente independentes para todo $t \in \mathbb{R} \setminus \{0\}$. (TODOS OS REAIS, EXCETO NO ZERO).

c) Podemos concluir que os coeficientes são todos constantes

d)

3)

$$A = \begin{bmatrix} 2 & 3 \\ -1 & -2 \end{bmatrix}$$

$$\det(A-\lambda I) = \begin{bmatrix} 2-\lambda & 3 \\ -1 & -2-\lambda \end{bmatrix} = (2-\lambda)(-2-\lambda) + 3 =$$

$$= \underbrace{-4 - 2\lambda + 2\lambda + \lambda^2 + 3 = 0}_{\text{Equação característica}} \Rightarrow \lambda^2 = 1 \Rightarrow \lambda = 1 \quad \lambda = -1$$

$\therefore$ Autovalores de A são 1 e -1

$$B = \begin{bmatrix} 1 & -4 \\ 3 & 1 \end{bmatrix}$$

$$\det(B-\lambda I) = \begin{bmatrix} 1-\lambda & -4 \\ 3 & 1-\lambda \end{bmatrix} = (1-\lambda)(1-\lambda) + 12 = 0$$

$$\underbrace{1 - 2\lambda + \lambda^2 + 12 = 0}_{\text{EQUAÇÃO CARACTERÍSTICA}} \Rightarrow \lambda^2 - 2\lambda + 13 = 0$$

$$\Delta = -48 < 0$$

$\therefore$ B NÃO possui Autovalores

$$C = \begin{bmatrix} -2 & -3 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

matriz Não-quadrada!

$\therefore$ Não possui autovalores.

2b) $\ddot{x} + 4x = sen^2(2t)$ $\boxed{\text{Trocando } x \text{ por } y \text{ temos :}}$

$y'' + 4y = sen^2(2t)$

Eq. característica $\lambda^2 + 4\lambda = 0 = \lambda(\lambda+4) = 0$ $\lambda_1 = 0$, $\lambda_2 = -4$

$\underline{\lambda_1 \neq \lambda_2}$: $y_h = c_1 e^{\lambda_1 t} + c_2 e^{\lambda_2 t}$

$y_h = \underbrace{c_1 e^0}_{y_1} + \underbrace{c_2 e^{-4t}}_{y_2} = c_1 + c_2 e^{-4t}$ (sol. Homogênea

* $y_p = v_1 y_1 + v_2 y_2$ (solução particular)

$\begin{cases} v_1' y_1 + v_2' y_2 = 0 \\ v_1' y_1' + v_2' y_2' = f(x) \end{cases} \Rightarrow \begin{cases} v_1' 1 + v_2' e^{-4t} = 0 \\ v_1' 0 + v_2'(-4e^{-4t}) = sen^2(2t) \end{cases} \sim$

$\sim \begin{cases} v_1' + v_2' e^{-4t} = 0 \\ v_2'(-4e^{-4t}) = sen^2(2t) \end{cases} \Rightarrow v_2' = \dfrac{sen^2(2t)}{-4e^{-4t}} = -\dfrac{1}{4} sen^2(2t)\, e^{4t}$

$\Rightarrow v_2 = \dfrac{1}{4}\int sen^2(2t) \cdot e^{4t} = -\dfrac{1}{4} \cdot \left(-\dfrac{1}{16} e^{4t} (sen(4t) + cos(4t) - 2\right) =$

$= \boxed{\dfrac{1}{64} e^{4t} (sen(4t) + cos(4t) - 2 = v_2}$

$\Rightarrow v_1' = -v_2' e^{-4t}$

$v_1' = -\left(-\dfrac{1}{4} sen^2(2t)\, e^{4t}\right) e^{-4t} \Rightarrow v_1' = \dfrac{1}{4} sen^2(2t) \Rightarrow$

$v_1 = \dfrac{1}{4}\int sen^2(2t) = \boxed{\dfrac{1}{32}(4x - sen\, 4x)}$

$Y_G = \dfrac{1}{32}(4x - sen(4x)) \cdot 1 + \dfrac{e^{4t}}{64}(sen(4t) + cos(4t) - 2) \cdot e^{-4t} + c_1 + c_2 e^{-4t}$

onde $y_G = y_h + y_p$ = solução Geral

1d) $y''' - 6y'' + 11y' - 6y = 2xe^{-x}$

$y_p = (Ax+B)e^{-x} =$

$\Rightarrow y_p = Ax.e^{-x} + Be^{-x}$

$y'_p = Ae^{-x} - Ax.e^{-x} - Be^{-x}$

$y''_p = -Ae^{-x} - A.e^{-x} + Ax.e^{-x} + Be^{-x}$

$y'''_p = Ae^{-x} + Ae^{-x} + Ae^{-x} - Axe^{-x} - Be^{-x}$

subst.

$3Ae^{-x} - Axe^{-x} - Be^{-x} - 6(-2Ae^{-x} + Axe^{-x} + Be^{-x}) + 11(Ae^{-x} - Axe^{-x} - Be^{-x})$

$-6(Axe^{-x} + Be^{-x}) =$

$= 3Ae^{-x} - Axe^{-x} - Be^{-x} + 12Ae^{-x} - 6Axe^{-x} - 6Be^{-x} + 11Ae^{-x} - 11Axe^{-x} -$

$- 11Be^{-x} - 6Axe^{-x} - 6Be^{-x} =$

$= 26Ae^{-x} - 24Axe^{-x} - 24Be^{-x} = 2xe^{-x}$

$26A - 24B = 0$

$-24A = 2 \Rightarrow \boxed{A = \frac{-1}{12}}$; $B =$

$24B = 26A$

$24B = -26.\frac{1}{12} = \frac{-13}{6} \Rightarrow B = \frac{-13}{6} \cdot \frac{1}{24} = \frac{-13}{144}$

$\therefore \boxed{y_p = \left(-\frac{x}{12} - \frac{13}{144}\right)e^{-x}} = \left(\frac{-12x-13}{144}\right)e^{-x}$

1a) $y'' - y' - 2y = 4x^2$

$y_p = Ax^2 + Bx + C \longrightarrow$ $y'_p = 2AX + B$

$y''_p = 2A$

SUBSTITUINDO NA EQUAÇÃO, TEMOS:

$\cancel{2A} - \cancel{2AX} - \cancel{B} - \cancel{2AX^2} - \cancel{2BX} - \cancel{2C} = 4x^2$

$-2AX^2 = 4x^2 \Rightarrow -2A = 4 \rightarrow \boxed{A = -2}$

$-2AX - 2BX = 0X \Rightarrow -2A - 2B = 0 \Rightarrow -2B = -4$

$\Rightarrow \boxed{B = 2}$

$2A - B - 2C = 0 \Rightarrow -4 - 2 - 2C = 0$

$-2C = 6 \Rightarrow \boxed{C = -3}$

$\boxed{y_p = -2x^2 + 2X - 3}$

2a) $y'' - y' - 2y = e^{3x}$ (Métodos: coef. A dete. ou, var. dos parâmetr.

Eq. característica:

$\lambda^2 - \lambda^1 - 2 = 0 \rightarrow \lambda_1 = 2$

$\rightarrow \lambda_2 = -1$

$y_h = C_1 e^{2x} + C_2 e^{-1x}$

$y_p = V_1 e^{2x} + V_2 e^{-x}$

$\begin{cases} V_1' e^{2x} + V_2' e^{-x} = 0 \longrightarrow V_1' e^{2x} = -V_2' e^{-x} \Rightarrow V_1' = \dfrac{-V_2' e^{-x}}{e^{2x}} \\ V_1' 2e^{2x} + V_2'(-e^{-x}) = e^{3x} \end{cases}$

$V_1' = -V_2' e^{-3x}$

$\downarrow$

$2(-V_2' e^{-3x}) e^{2x} - V_2 e^{-x} = e^{3x}$

$(-2V_2' e^{-3x}) e^{2x} - V_2 e^{-x} = e^{3x}$

$-2V_2' e^{-x} - V_2' e^{-x} = e^{3x}$

$e^{-x}(-2V_2' - V_2') = e^{3x}$

$-3V_2' = \dfrac{e^{3x}}{e^{-x}} \Rightarrow -3V_2' = e^{4x}$

$\boxed{V_2' = -\dfrac{e^{4x}}{3}}$

$V_1' = \dfrac{e^{4x}}{3} \cdot e^{-3x} \Rightarrow \boxed{V_1' = \dfrac{e^{x}}{3}}$

$4x = u \Rightarrow du = 4dx$
$dx = \frac{1}{4}du$

$$V_2 = \int -\frac{e^{4x}}{3}\,dx = -\frac{1}{3}\int e^{4x} = -\frac{1}{3}\int e^{u}\frac{1}{4}\,du$$

$$= -\frac{1}{3}\cdot\frac{1}{4}e^{u} = \boxed{-\frac{1}{12}e^{u}} \Rightarrow \boxed{-\frac{1}{12}e^{4x} = V_2}$$

$$V_1' = \frac{e^{x}}{3} \Rightarrow V_1 = \frac{1}{3}\int e^{x} = \boxed{\frac{e^{x}}{3} = V_1}$$

$$y_p = V_1 e^{2x} + V_2 e^{-x}$$

$$y_p = \frac{e^{x}}{3}\cdot e^{2x} + \left(-\frac{1}{12}\right)e^{4x}e^{-x}$$

$$y_p = \frac{1}{3}e^{3x} - \frac{1}{12}e^{3x}$$

$$y_G = y_h + y_p \quad (\text{sol. Geral})$$

$$y_G = C_1 e^{2x} + C_2 e^{-x} + \frac{e^{3x}}{4}$$