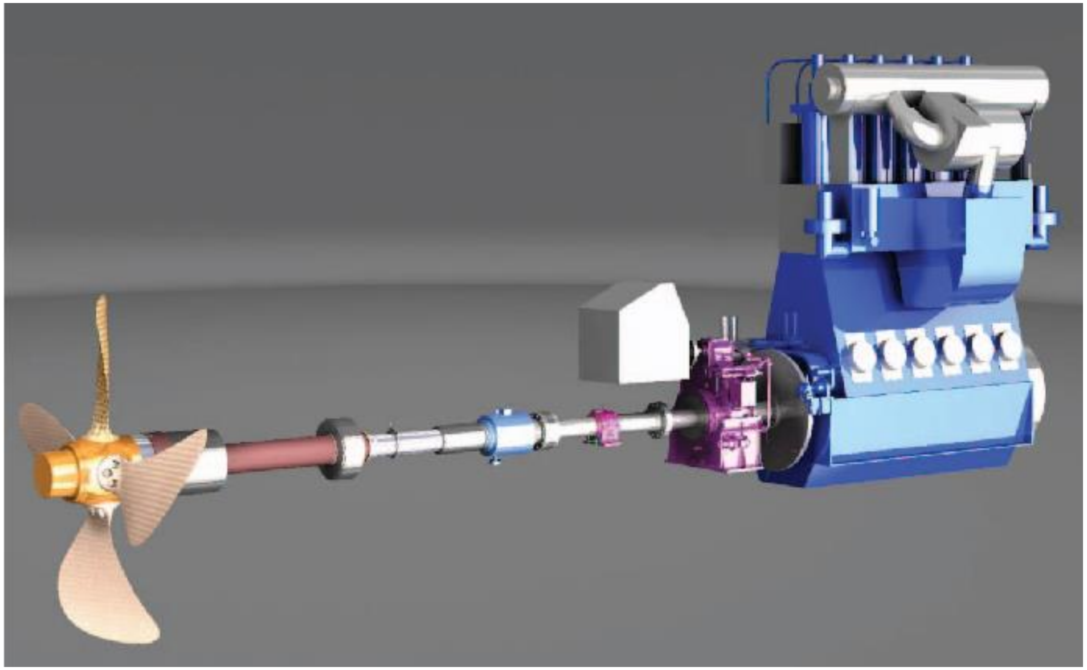
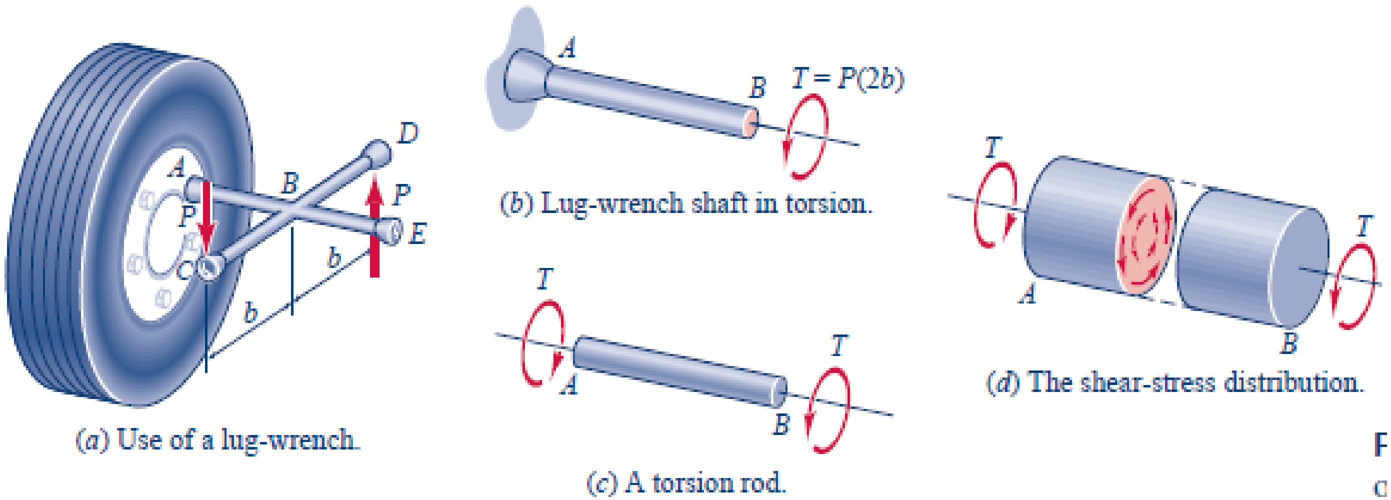
DEPARTAMENTO DE ENGEHARIA NAVAL E OCEÂNICA
ESCOLA POLITÉCNICA DA USP

Torção Eixos Circulares: τ_{xy}



PNV 3212 – Mecânica Dos Sólidos I
2020

Shear Stress

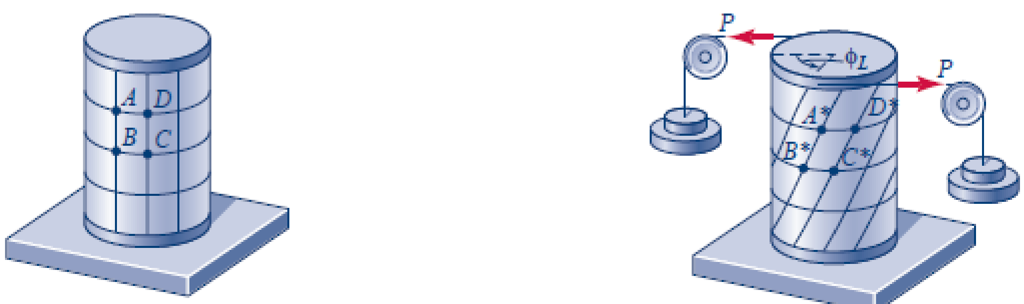


(*a*) Use of a lug-wrench.
(*b*) Lug-wrench shaft in torsion.
(*c*) A torsion rod.
(*d*) The shear-stress distribution.

Axial Deformation	Torsion
Axial Force (F)	Torque (T)
Elongation (e)	Twist angle (ϕ)
Normal stress (σ)	Shear stress (τ)
Extensional strain (ϵ)	Shear strain (γ)
Modulus of elasticity (E)	Shear modulus (G)

Tensões de Cisalhamento

- **Hipóteses (torsional-deformation assumptions)**
  - Problema é independente do tempo.
  - Eixo constituído de material **linear-elástico**.
  - *The axis remains straight and remains inextensible*.
  - *Every cross section remains plane and remains perpendicular to the axis*.
  - *Radial lines remain straight and radial as the cross section rotates about the axis*

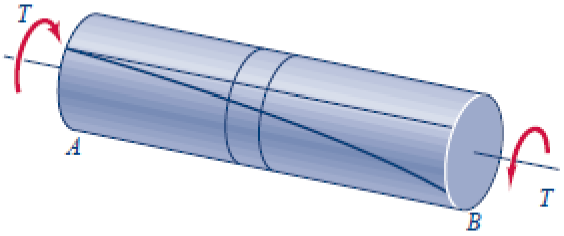


(a) Before deformation.

(b) After deformation.

Tensões de Cisalhamento

- Fórmula



$$\tau = \frac{T}{J} r$$

Tensões de Cisalhamento

- **Caminho**
  1. Relação rotação-deformação
  2. Lei de Hooke
  3. Equilíbrio da Seção (Momentos)

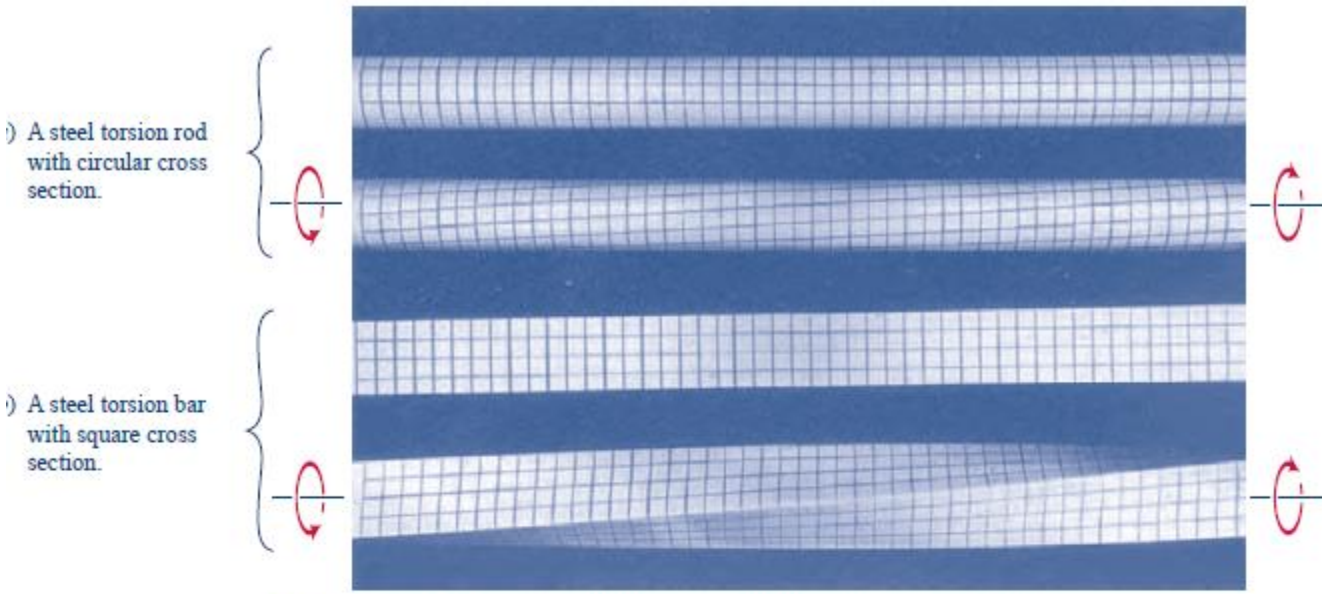
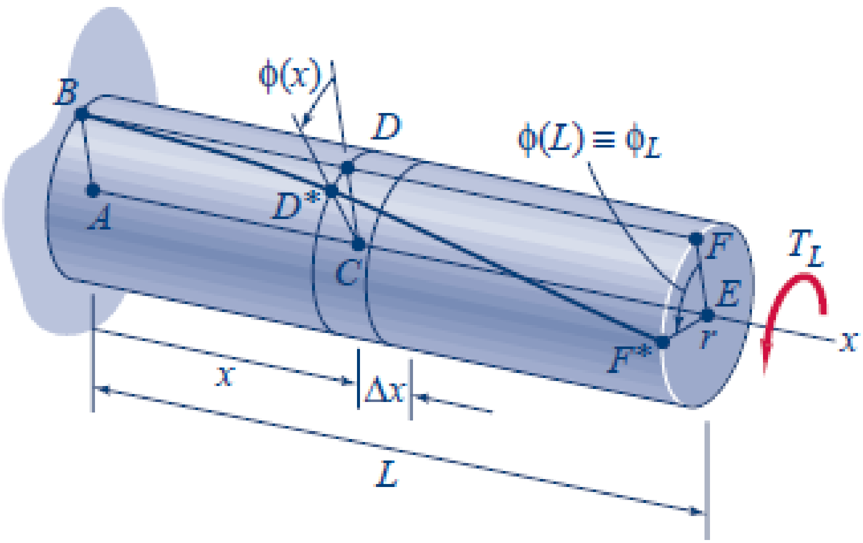


FIGURE 4.2 Examples of torsional deformation. (Roy Craig)

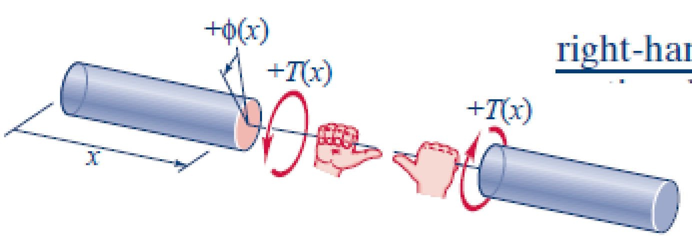
Relação rotação-deformação angular



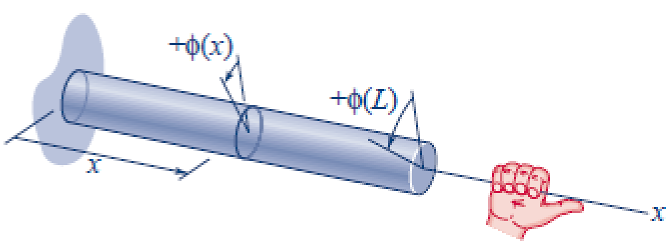
Cada seção transversal gira um ângulo

$\phi(x)$ angle of twist

right-hand-rule

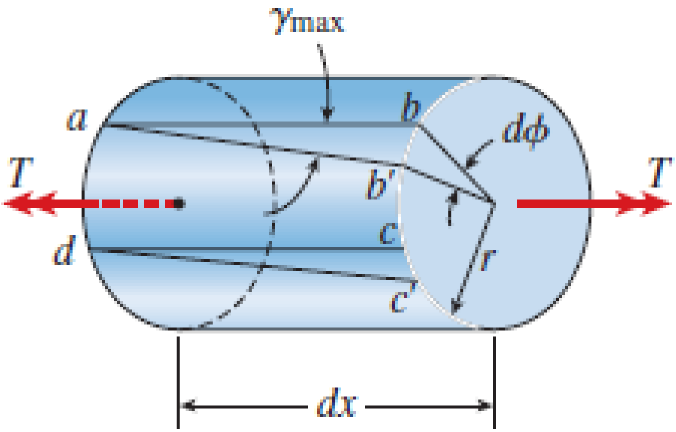


(*b*) Sign convention for internal (resisting) torque $T(x)$.



(*c*) Sign convention for angle of rotation $\phi(x)$.

Relação rotação-deformação angular



Movimento relativo entre as seções em x e x+dx

$$\phi(x) < \phi(x + dx)$$

Deformação angular

$$\gamma_{max} = \frac{bb'}{dx}$$

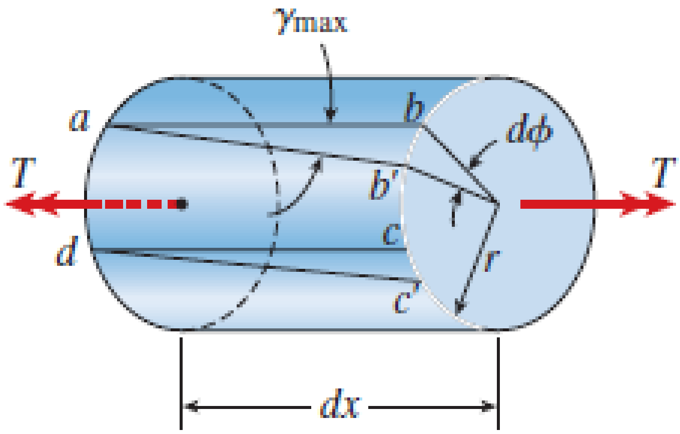
+

$$bb' = r d\phi$$

↓

$$\gamma_{max} = r\frac{d\phi}{dx}$$

Relação rotação-deformação angular



Deformação angular

$$\gamma_{max} = r\frac{d\phi}{dx}$$

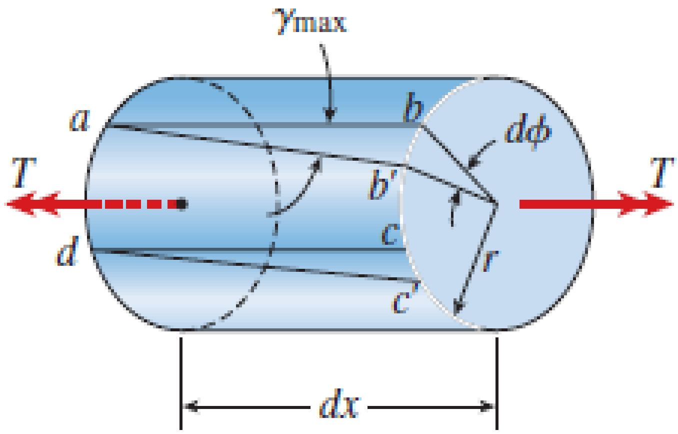
Razão de torção (razão de cambio do ângulo de torção)

$$\theta = \frac{d\phi}{dx}$$

↓

$$\gamma_{max} = r\theta$$ (rate of twist)

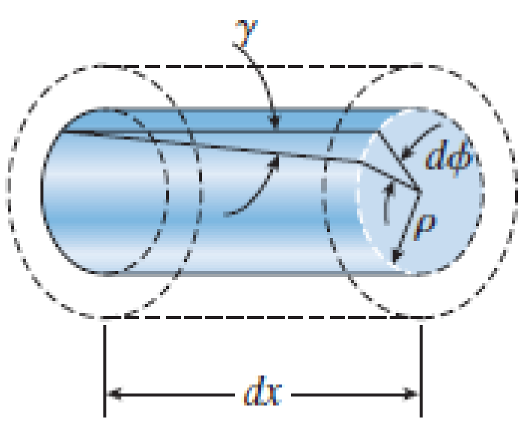
Relação rotação-deformação angular



Deformação angular na superfície

$$\gamma_{max} = r\frac{d\phi}{dx}$$

Deformação angular no interior



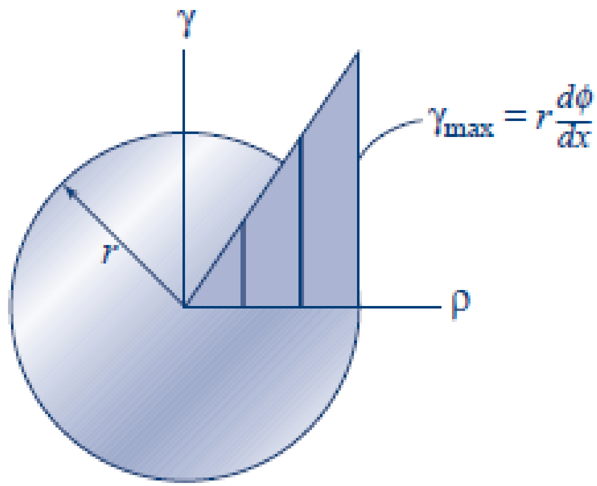
$$\gamma = \rho\theta \quad + \quad \frac{\gamma_{max}}{r} = \theta$$

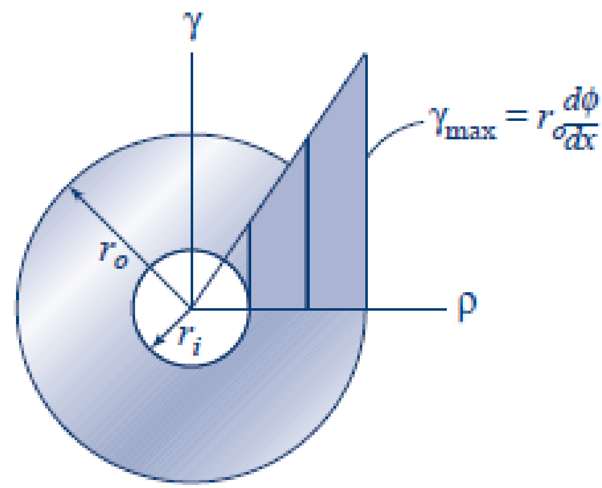
↓

$$\gamma = \frac{\rho}{r}\gamma_{max}$$ (variação linear com ρ)

Relação rotação-deformação angular

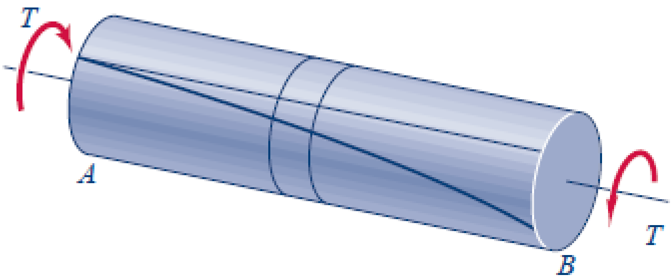
Deformação angular na superfície $\gamma = \frac{\rho}{r}\gamma_{max}$ (variação linear com ρ)



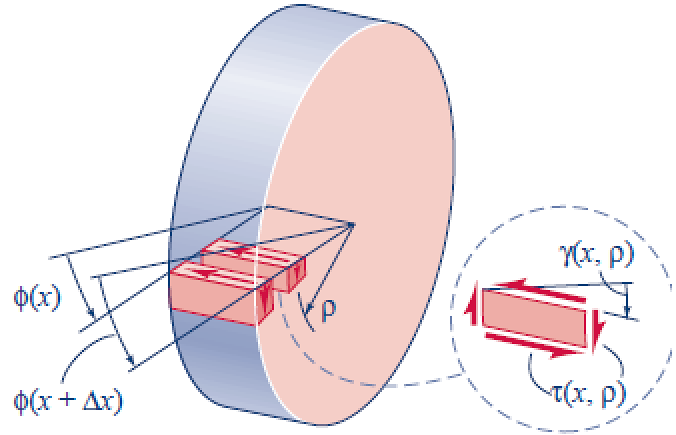


$$\gamma_{max} = r\frac{d\phi}{dx}$$

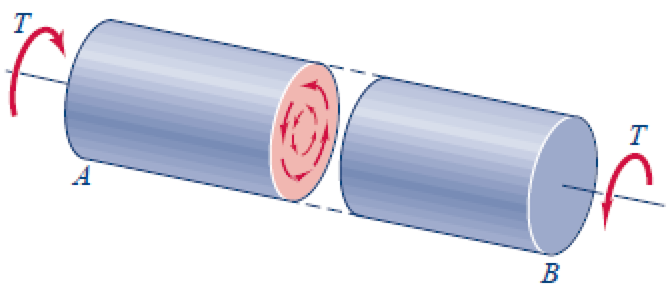
Relação tensão-deformação angular



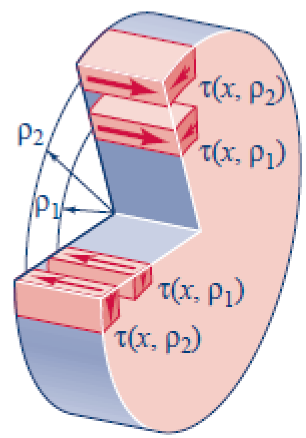
(*a*) Torsional deformation.



(*c*) Shear stress and shear strain at typical points.

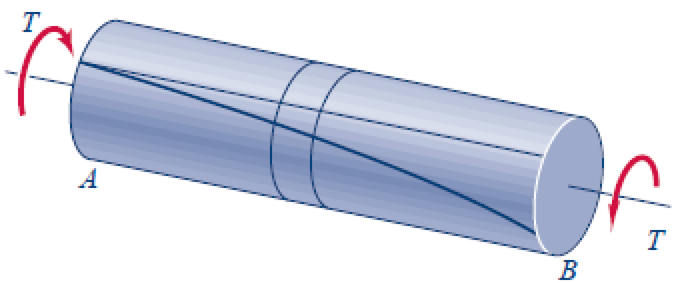


(*b*) Shear stress due to torsion.

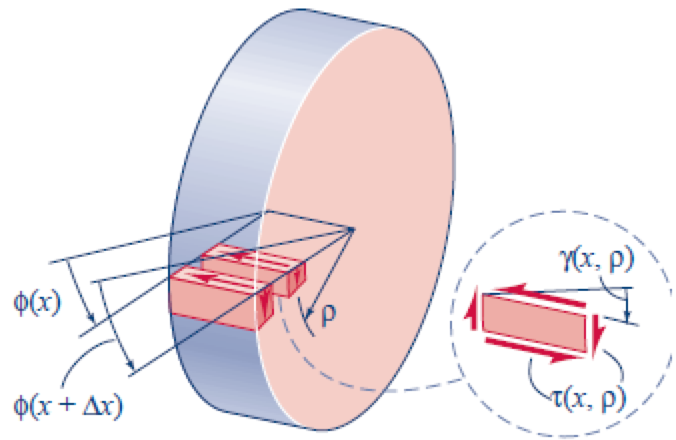


(*d*) Shear stresses along two typical radial lines in a cross section, and shear stress on radial planes.

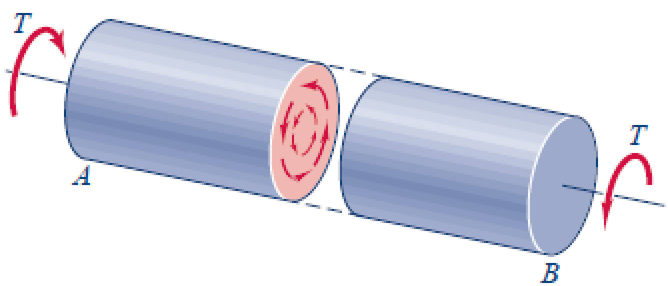
Relação tensão-deformação angular



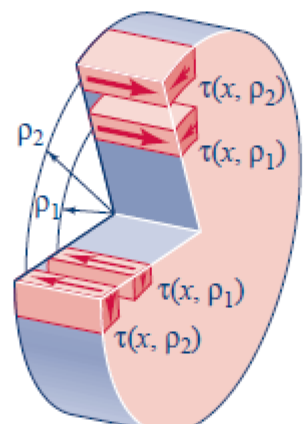
(a) Torsional deformation.



(c) Shear stress and shear strain at typical points.

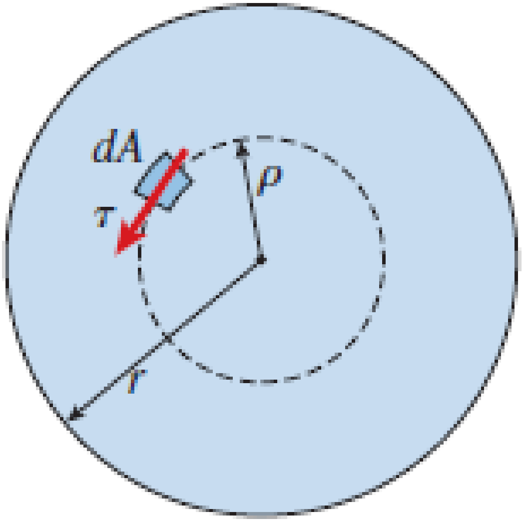


(b) Shear stress due to torsion.



(d) Shear stresses along two typical radial lines in a cross section, and shear stress on radial planes.

Relação tensão-deformação angular



Equilíbrio

$$\sum M_x = 0$$

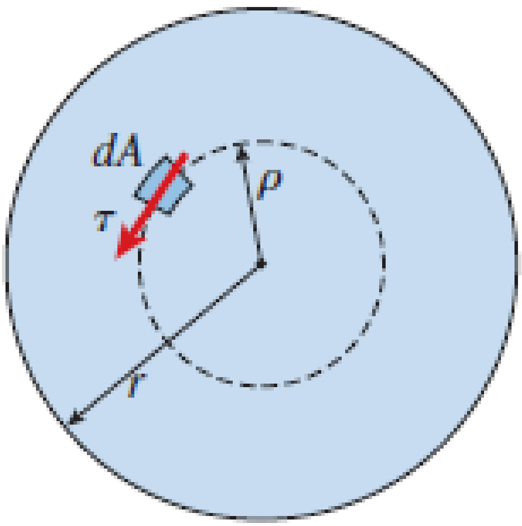
$$T_{ext} = \int_A dM$$

Lei de Hooke

$$\tau = G\gamma$$

$$T_{ext} = \int_A \tau \rho dA$$

Relação tensão-deformação angular



Lei de Hooke

$$\tau = G\gamma$$

$$\gamma_{max} = r\frac{d\phi}{dx}$$

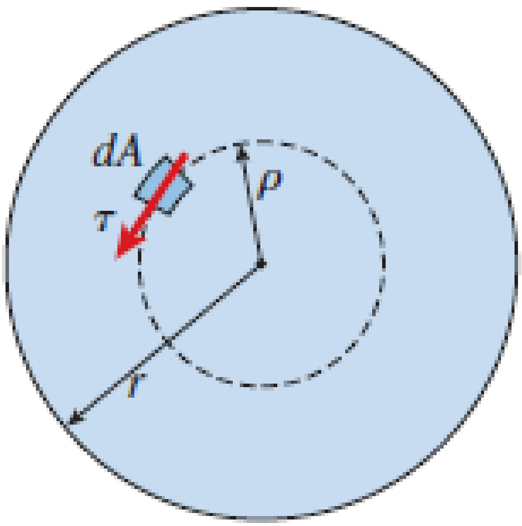
Equilíbrio

$$T_{ext} = \int_A G\gamma\,\rho dA$$

$$T_{ext} = \int_A G\frac{\rho}{r}\gamma_{max}\rho dA$$

$$T_{ext} = G\frac{\gamma_{max}}{r}\int_A \rho^2 dA$$

Relação tensão-deformação angular



Lei de Hooke

$$\tau = G\gamma$$

$$\gamma_{max} = r\frac{d\phi}{dx}$$

Equilíbrio

$$T_{ext} = G\frac{d\phi}{dx}\int_A \rho^2 dA$$

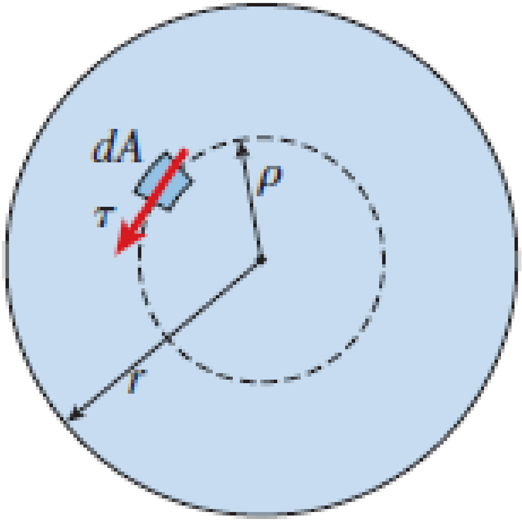
$$J \quad \frac{\pi r^4}{2} = \frac{\pi d^4}{32}$$

$$T_{ext} = G\frac{d\phi}{dx}J$$

$$\frac{T_{ext}}{GJ} = \frac{d\phi}{dx}$$

Eq. Torque-rotação

Relação tensão-deformação angular



Lei de Hooke

$$\frac{T_{ext}}{GJ} = \frac{d\phi}{dx}$$

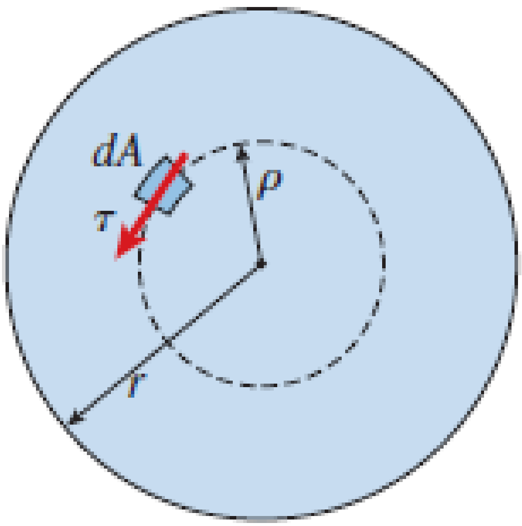
Eq. Torque-rotação

$$\tau = G\gamma \qquad \gamma = \rho\frac{d\phi}{dx}$$

$$\tau = G\rho\frac{d\phi}{dx}$$

$$\frac{\tau}{\rho} = G\frac{d\phi}{dx}$$

Relação tensão-deformação angular



$$\frac{T_{ext}}{J} = G\frac{d\phi}{dx} \qquad \frac{\tau}{\rho} = G\frac{d\phi}{dx}$$

Lei de Hooke

$$\frac{T_{ext}}{J} = \frac{\tau}{\rho}$$

Formula de torção $\tau = \frac{T_{ext}}{J}\rho$ vs $\sigma = \frac{M}{I}y$

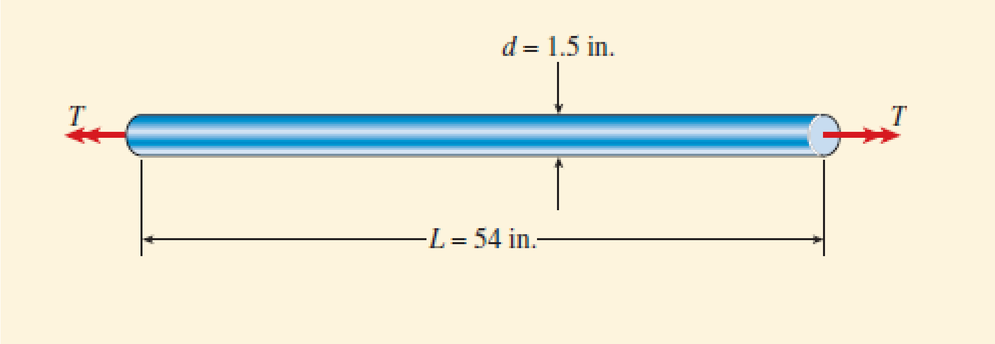
Exemplo

Eixo sólido circular de diâmetro d=1.5 in, L =54 in. Aplica-se torque na extremidade T=250 lbf-ft. Determine a tensão de cisalhamento e o ângulo de torção

$$\tau = \frac{T_{ext}}{J}\rho$$

Exemplo

Solução



$$\tau = \frac{T_{ext}}{J}\rho$$

$$J = \frac{\pi d^4}{32}$$

$$J = \frac{\pi \times 1.5^4}{32}$$

$$J = 0.497\ in^4$$

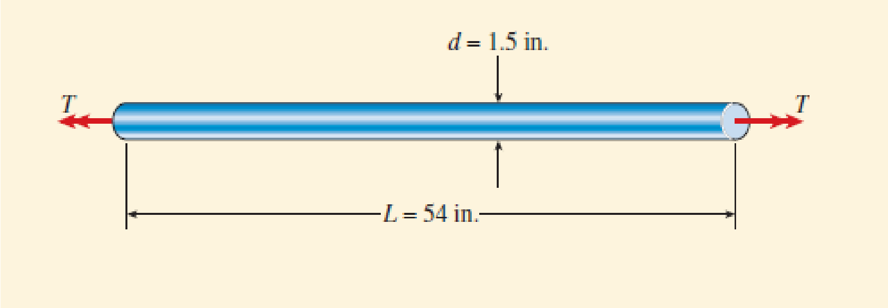
$$\tau = \frac{250}{0.497}\frac{1.5}{2} \times \left(\frac{12in}{1ft}\right)$$

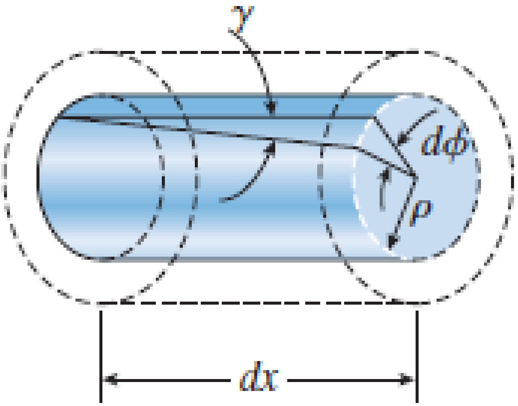
$$\tau = 4530 \text{ psi}$$ (31.2 MPa)

(1 ksi =6.894 MPa)

Exemplo

Solução





Rotação

$$\frac{T_{ext}}{GJ} = \frac{d\phi}{dx} \qquad \theta = \frac{d\phi}{dx}$$

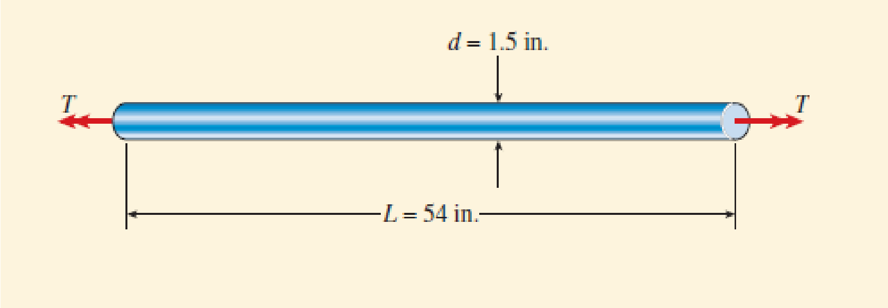
$$\theta L = \phi$$

$$\boxed{\frac{T_{ext}x}{GJ} = \phi(x)}$$

$$\frac{250 \times 12 \times 54}{\frac{206000}{1-0.3^2}0.497} = \phi$$

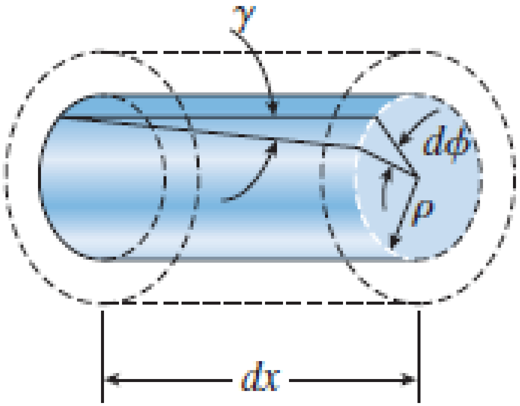
Exemplo

Solução



Rotação

$$\frac{250 \times 12 \times 54}{\frac{206000}{1 - 0.3^2} 0.497} = \phi$$



$$\phi = 0.028 \text{ rad}$$

$$\phi = 1.61°$$