

Aula 11

Interação hiperfina no hidrogênio

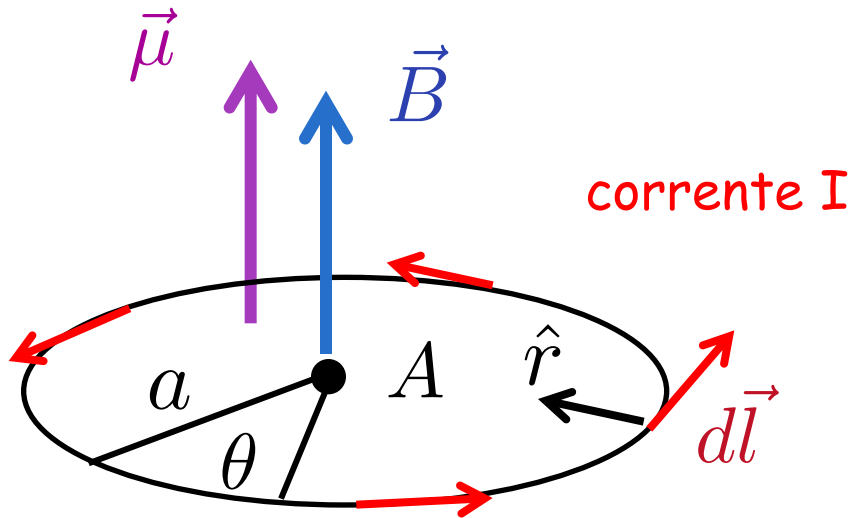
A linha de 21 cm

A rotação das galáxias

Interação hiperfina no átomo de hidrogênio

Modelo:
interação magnética
entre duas espiras

Campo magnético de uma espira



Biot-Savart :

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{l} \times \hat{r}}{r^2}$$

$$d\vec{l} \times \hat{r} = |d\vec{l} \times \hat{r}| \hat{k} = dl \hat{k}$$

$$d\vec{l} \times \hat{r} = r d\theta \hat{k}$$

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{r}{r^2} d\theta \hat{k}$$

$$\vec{B} = \frac{\mu_0 I}{4\pi a} \int_0^{2\pi} d\theta \hat{k}$$

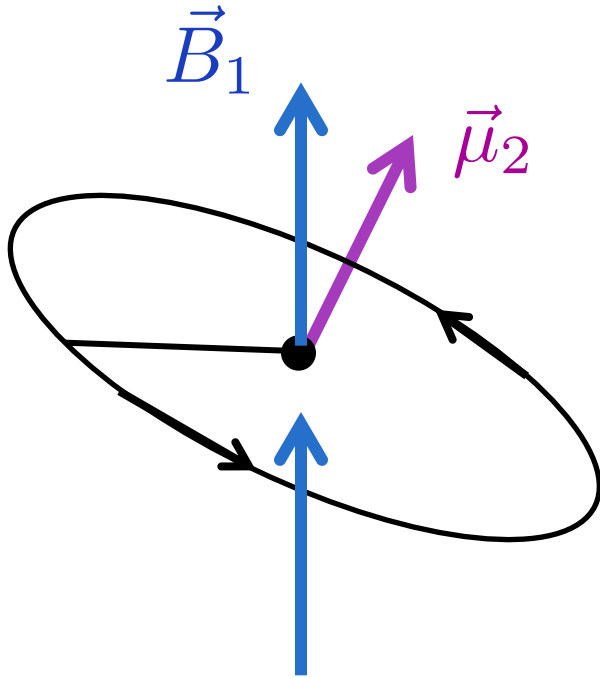
$$\vec{B} = \frac{\mu_0 I}{2a} \hat{k}$$

$$\vec{B} = \frac{\mu_0 I}{2a} \frac{\pi a^2}{\pi a^2} \hat{k}$$

$$\vec{\mu} = I A \hat{k} = I \pi a^2 \hat{k}$$

$$\vec{B} = \frac{\mu_0}{2\pi a^3} \vec{\mu}$$

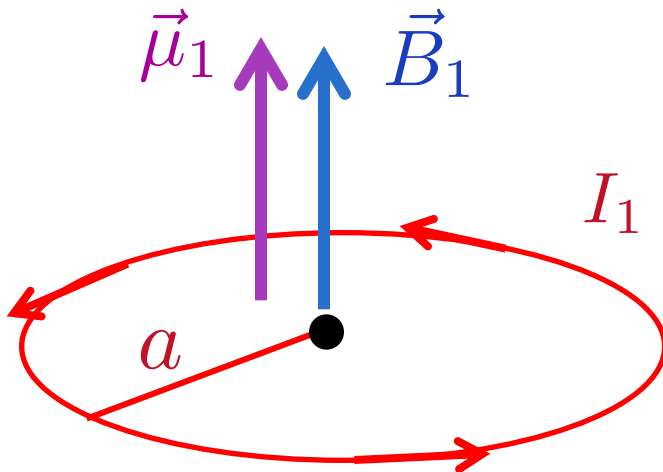
Interação entre duas espiras



$$\vec{B}_1 = \frac{\mu_0}{2\pi a^3} \vec{\mu}_1$$

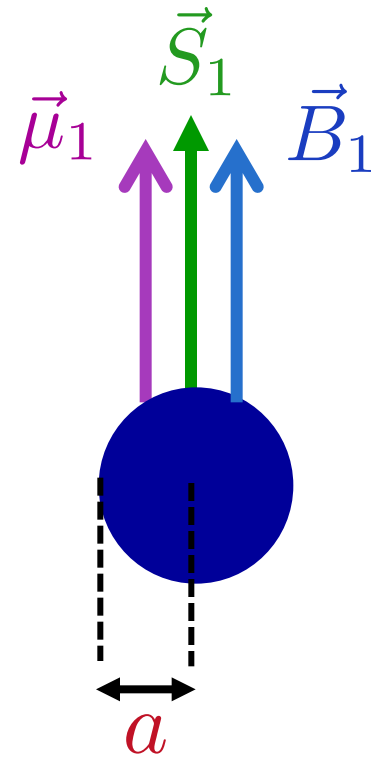
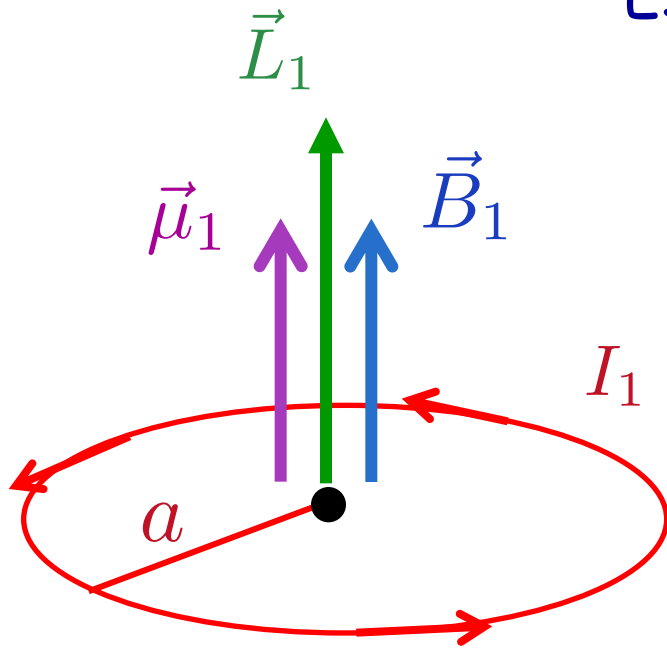
$$U = -\vec{\mu}_2 \cdot \vec{B}_1$$

$$H = U = -\frac{\mu_0}{2\pi a^3} \vec{\mu}_1 \cdot \vec{\mu}_2$$



(cargas positivas
em movimento)

Espiras -> "esferas"



$$\vec{\mu}_1 = \gamma_1 \vec{S}_1$$

$$\vec{\mu}_2 = \gamma_2 \vec{S}_2$$

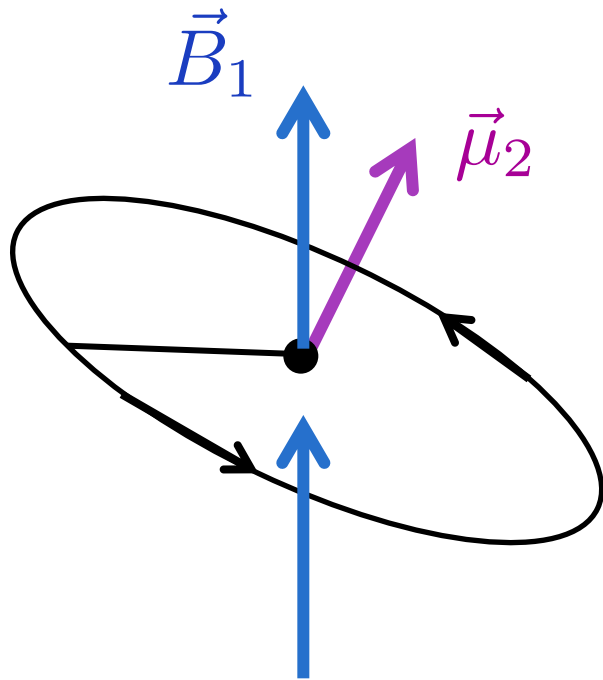
$$\gamma = \frac{g e}{2 m}$$

$$H = -\frac{\mu_0}{2 \pi a^3} \vec{\mu}_1 \cdot \vec{\mu}_2$$



$$H = -\frac{\mu_0 \gamma_1 \gamma_2}{2 \pi a^3} \vec{S}_1 \cdot \vec{S}_2$$

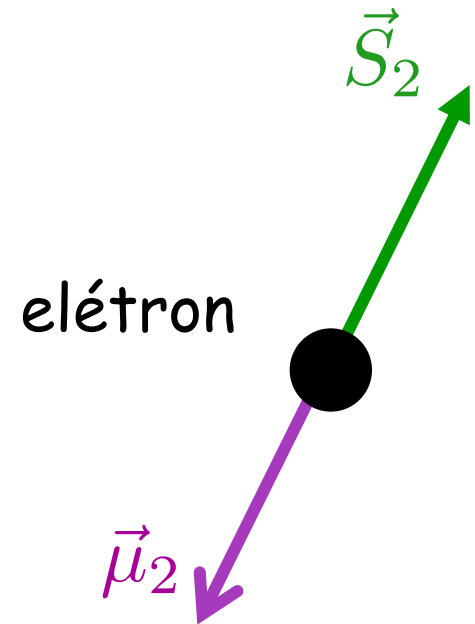
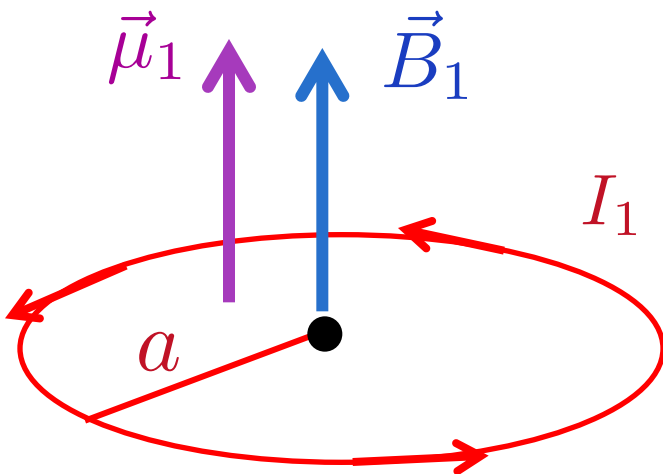
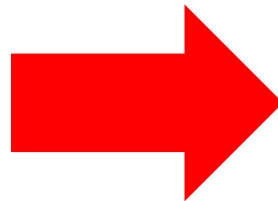
Espiras -> "esferas"



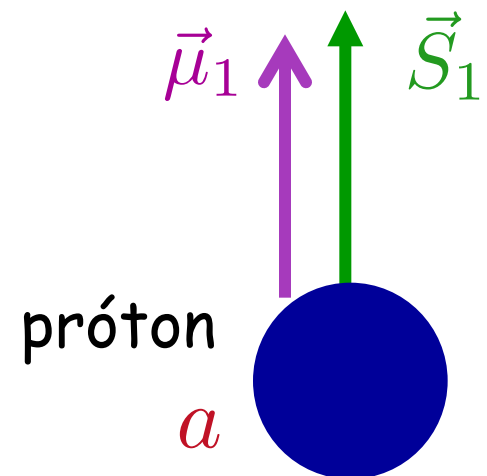
(cargas negativas em movimento)

$$\gamma_2 \rightarrow -\gamma_2$$

$$\vec{\mu}_2 = -\gamma_2 \vec{S}_2$$



elétron



próton

$$H = -\frac{\mu_0 \gamma_1 \gamma_2}{2 \pi a^3} \vec{S}_1 \cdot \vec{S}_2 \quad \xrightarrow{\gamma_2 \rightarrow -\gamma_2} \quad H = \frac{\mu_0 \gamma_1 \gamma_2}{2 \pi a^3} \vec{S}_1 \cdot \vec{S}_2$$

$$\vec{S} = \vec{S}_1 + \vec{S}_2$$

$$\vec{S}^2 = \vec{S}_1^2 + \vec{S}_2^2 + 2 \vec{S}_1 \cdot \vec{S}_2$$

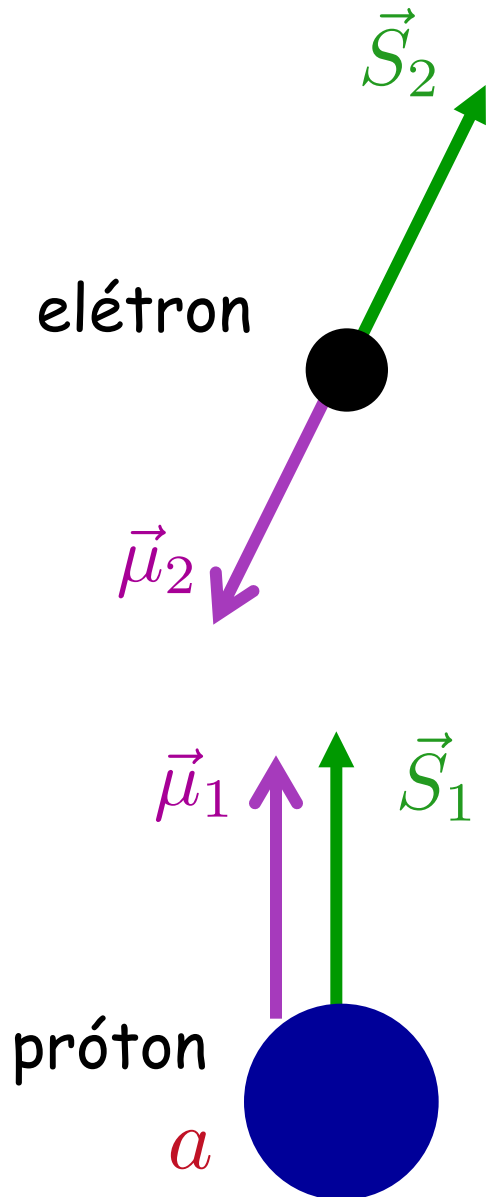
$$\vec{S}_1 \cdot \vec{S}_2 = \frac{\vec{S}^2 - \vec{S}_1^2 - \vec{S}_2^2}{2}$$

$$H = \frac{\mu_0 \gamma_1 \gamma_2}{2 \pi a^3} \frac{\vec{S}^2 - \vec{S}_1^2 - \vec{S}_2^2}{2}$$

versão quântica: funções viram operadores !

$$\left\{ \begin{array}{l} \vec{S} \rightarrow \hat{S} \\ H \rightarrow \hat{H} \end{array} \right.$$

Energia de interação hiperfina (spin-spin)



$$E = \langle \hat{H} \rangle = \langle \chi | \hat{H} | \chi \rangle$$

$$| \chi \rangle = | s, m \rangle$$

Estado de spin de duas partículas
com spin total s e projeção m

$$\text{Tripleto : } \begin{cases} |1 \ 1\rangle \\ |1 \ 0\rangle \\ |1 \ -1\rangle \end{cases}$$

$$\text{Singleto : } |0 \ 0\rangle$$

$$E = \frac{\mu_0 \gamma_1 \gamma_2}{2 \pi a^3} \left(\frac{\langle s m | \vec{S}^2 | s m \rangle - \langle s m | \vec{S}_1^2 | s m \rangle - \langle s m | \vec{S}_2^2 | s m \rangle}{2} \right)$$

Singleto : $|0\ 0\rangle = \frac{1}{\sqrt{2}} \left| \frac{1}{2} \right\rangle + \frac{1}{2} \left\rangle \left| \frac{1}{2} \right\rangle - \frac{1}{2} \left\rangle - \frac{1}{\sqrt{2}} \left| \frac{1}{2} \right\rangle - \frac{1}{2} \left\rangle \left| \frac{1}{2} \right\rangle + \frac{1}{2} \left\rangle$

$$\langle 0\ 0 | \hat{S}^2 | 0\ 0 \rangle = 0$$

$$\langle 0\ 0 | \hat{S}_1^2 | 0\ 0 \rangle = \frac{3}{4} \hbar^2$$

$$\langle 0\ 0 | \hat{S}_2^2 | 0\ 0 \rangle = \frac{3}{4} \hbar^2$$

$$E_{singleto} = \frac{\mu_0 \gamma_1 \gamma_2}{2 \pi a^3} \left(-\frac{3}{4} \hbar^2 \right)$$

$$E = \frac{\mu_0 \gamma_1 \gamma_2}{2 \pi a^3} \left(\frac{\langle s m | \vec{S}^2 | s m \rangle - \langle s m | \vec{S}_1^2 | s m \rangle - \langle s m | \vec{S}_2^2 | s m \rangle}{2} \right)$$

Tripleto : $|1 \ -1\rangle = |\frac{1}{2} \ -\frac{1}{2}\rangle |\frac{1}{2} \ -\frac{1}{2}\rangle$ (por exemplo)

$$\langle 1 \ -1 | \hat{S}^2 | 1 \ -1 \rangle = 2 \hbar^2 \quad (\text{veja aula 9})$$

$$\langle 1 \ -1 | \hat{S}_1^2 | 1 \ -1 \rangle = \frac{3}{4} \hbar^2$$

$$\langle 1 \ -1 | \hat{S}_2^2 | 1 \ -1 \rangle = \frac{3}{4} \hbar^2$$

$$E_{\text{tripleto}} = \frac{\mu_0 \gamma_1 \gamma_2}{2 \pi a^3} \left(+\frac{1}{4} \hbar^2 \right)$$

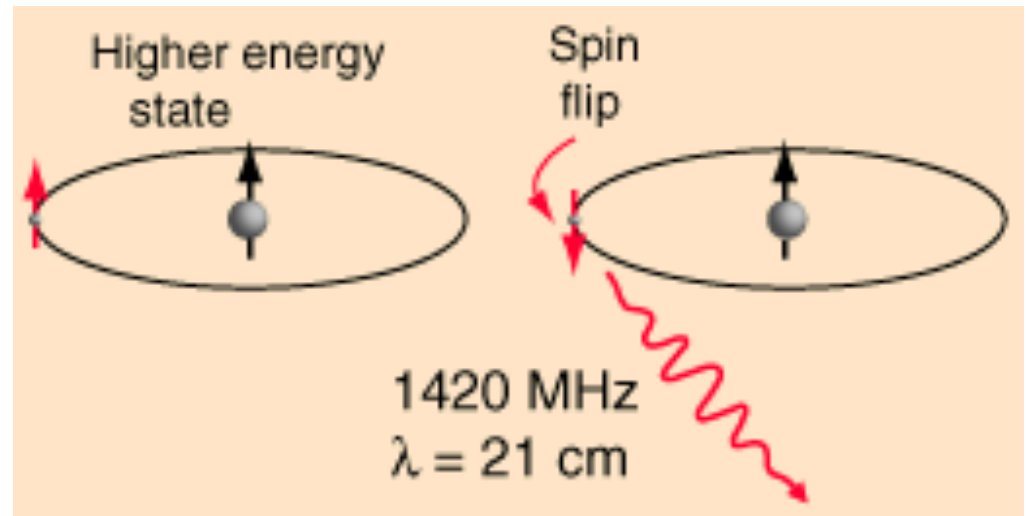
$$\Delta E_{hf} = E_{triplet} - E_{singlet} = \frac{\mu_0 \gamma_1 \gamma_2}{2 \pi a^3} \hbar^2 \quad \gamma = \frac{g e}{2 m}$$

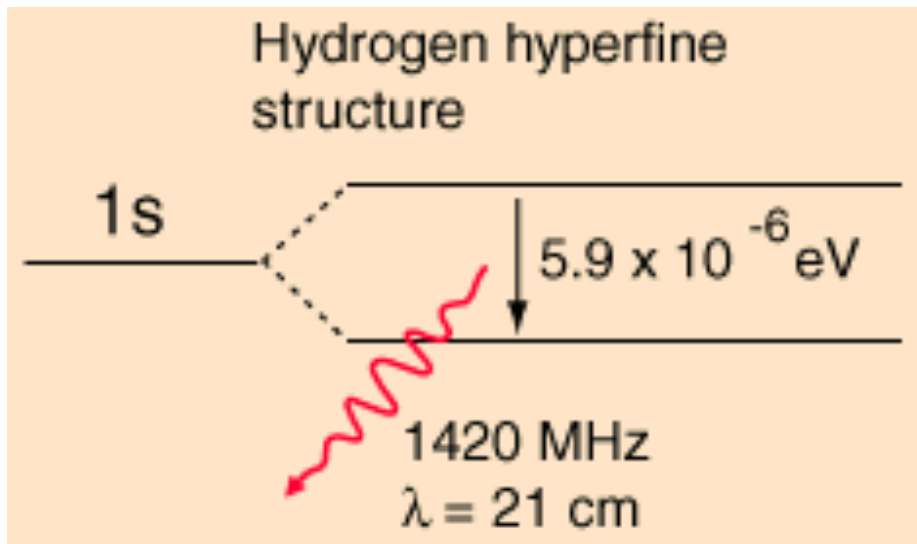
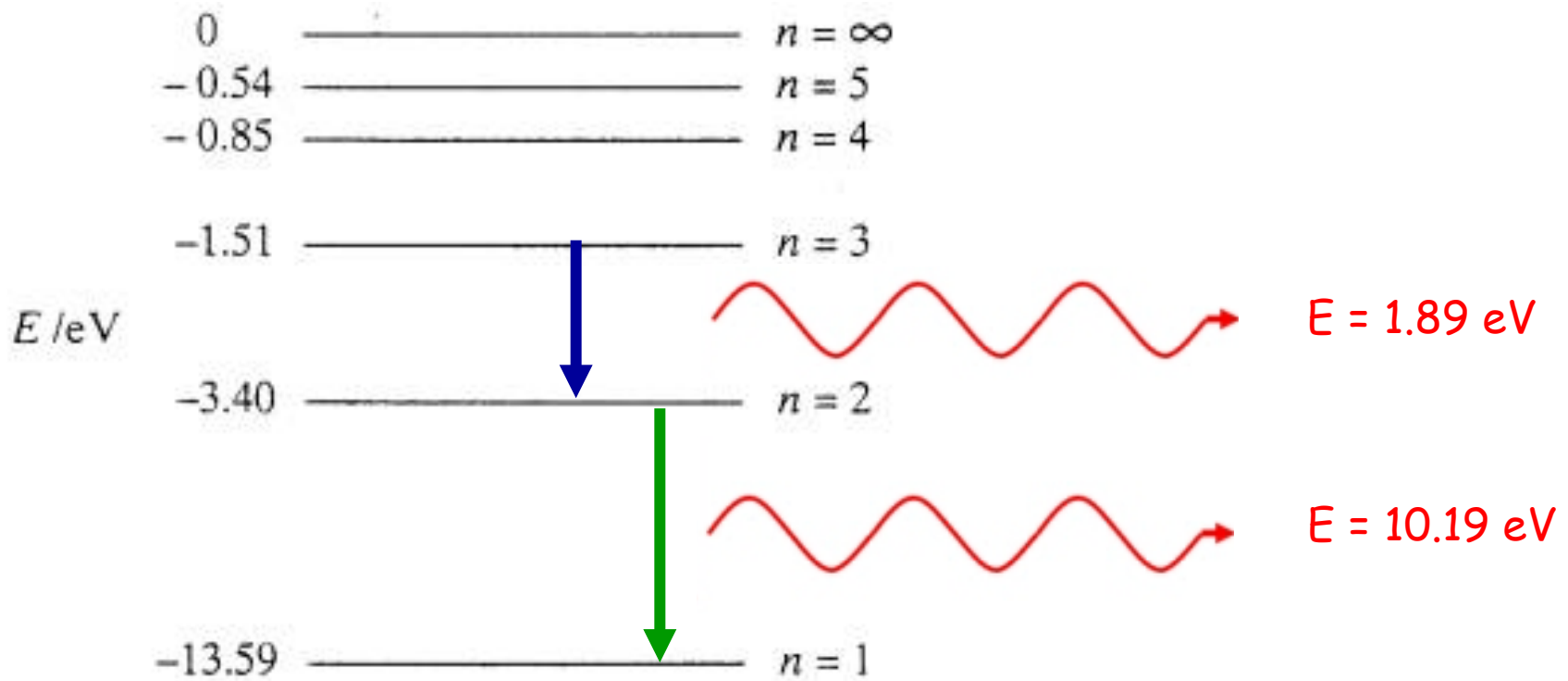
$$\Delta E_{hf} \simeq 5.884 \times 10^{-6} \text{ eV}$$

$$\nu = \frac{\Delta E_{hf}}{h} \simeq 1422.8 \text{ MHz}$$

$$\lambda = \frac{c}{\nu} = 21.07 \text{ cm}$$

(micro-ondas : 12 cm)





O fóton hiperfino
tem energia
MUITO MENOR !

É um só !

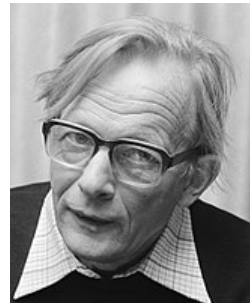
Astronomia de 21 cm !

A "linha de 21 cm"

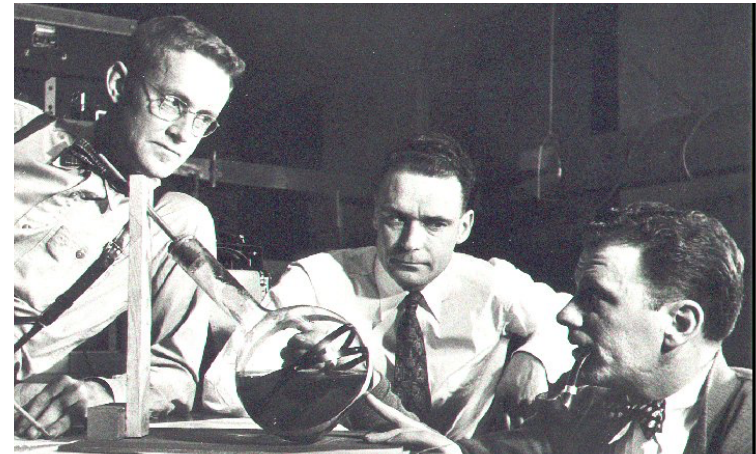
1944 Jan Oort : o Universo pode estar cheio de hidrogênio !



1945 H. Van de Hulst : H emite os ftons de 21 cm
Eles podem atravessar a poeira interestelar



1951 H. Ewen e E. Purcell: medida
da linha de 21 centímetros

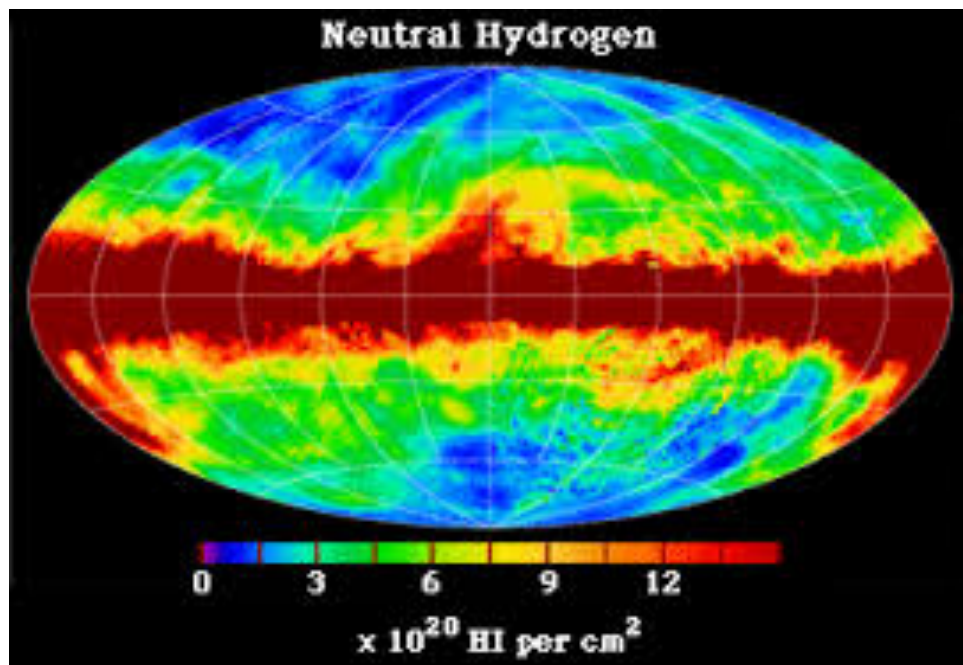


O Universo está cheio de hidrogênio !

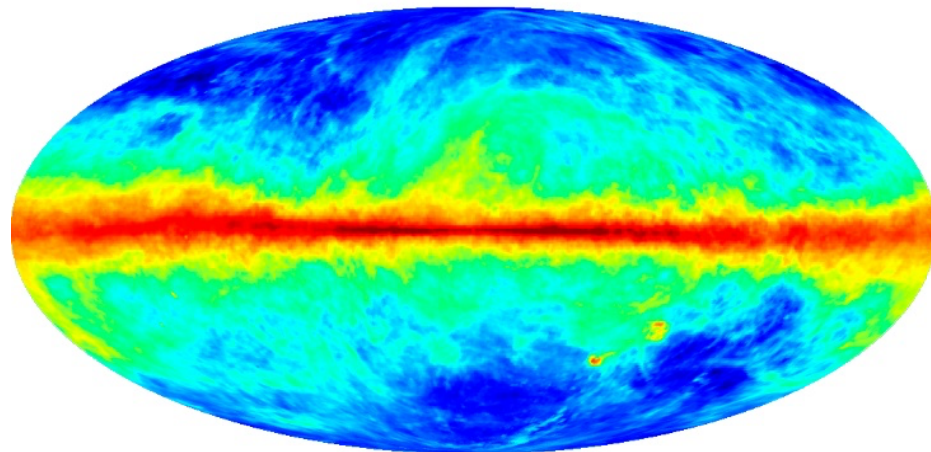
(1 átomo por cm^3)

Estes átomos emitem ftons, emitem a "linha de 21 cm" !

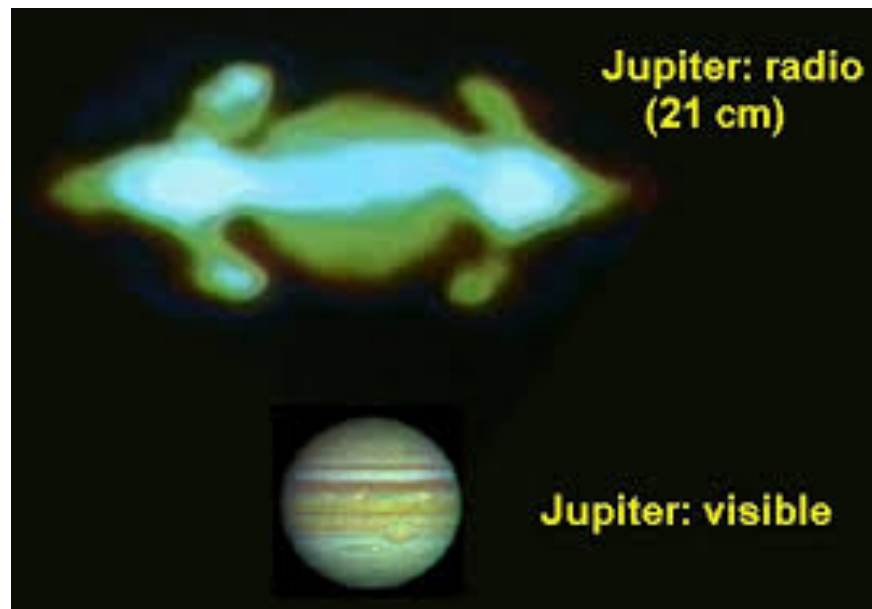
Podemos ajustar o telescópio e tirar "fotos" da "luz" de 21 cm :



21cm sky at $z=0$



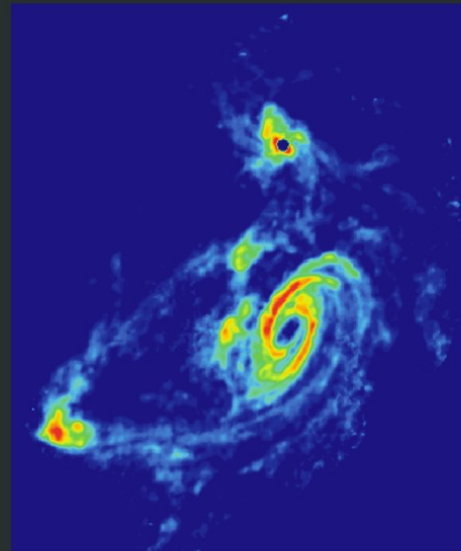
Kalberla et al 2005, from WMAP LAMBDA



Stellar Light Distribution



21 cm HI Distribution



Os ftons de 21 cm são muito penetrantes !

Atravessam o meio intergalático ! (IGM)

Trazem informações de partes remotas da galáxia !

Os átomos de H se movem junto com as estrelas

Eles emitem luz com frequência bem definida ν_0

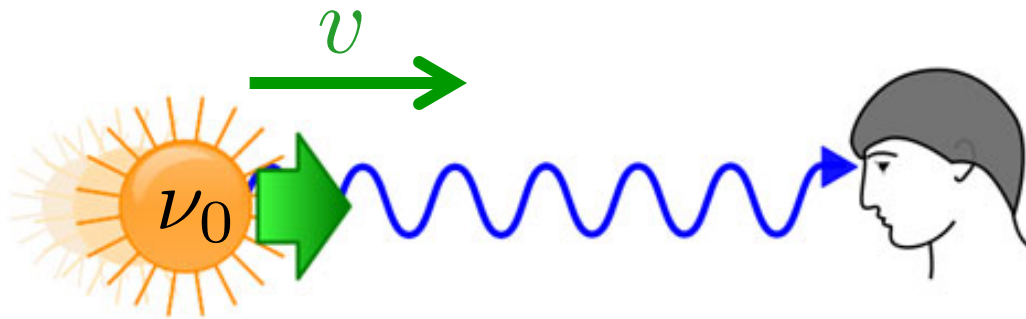
A luz emitida sofre efeito Doppler !



$$\nu = \nu_0 \sqrt{\frac{1 + v/c}{1 - v/c}}$$

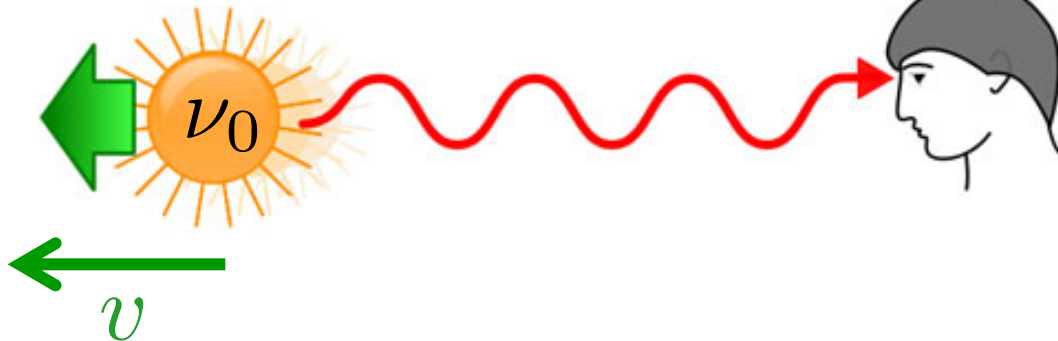
frequência
aumenta

a fonte
se aproxima



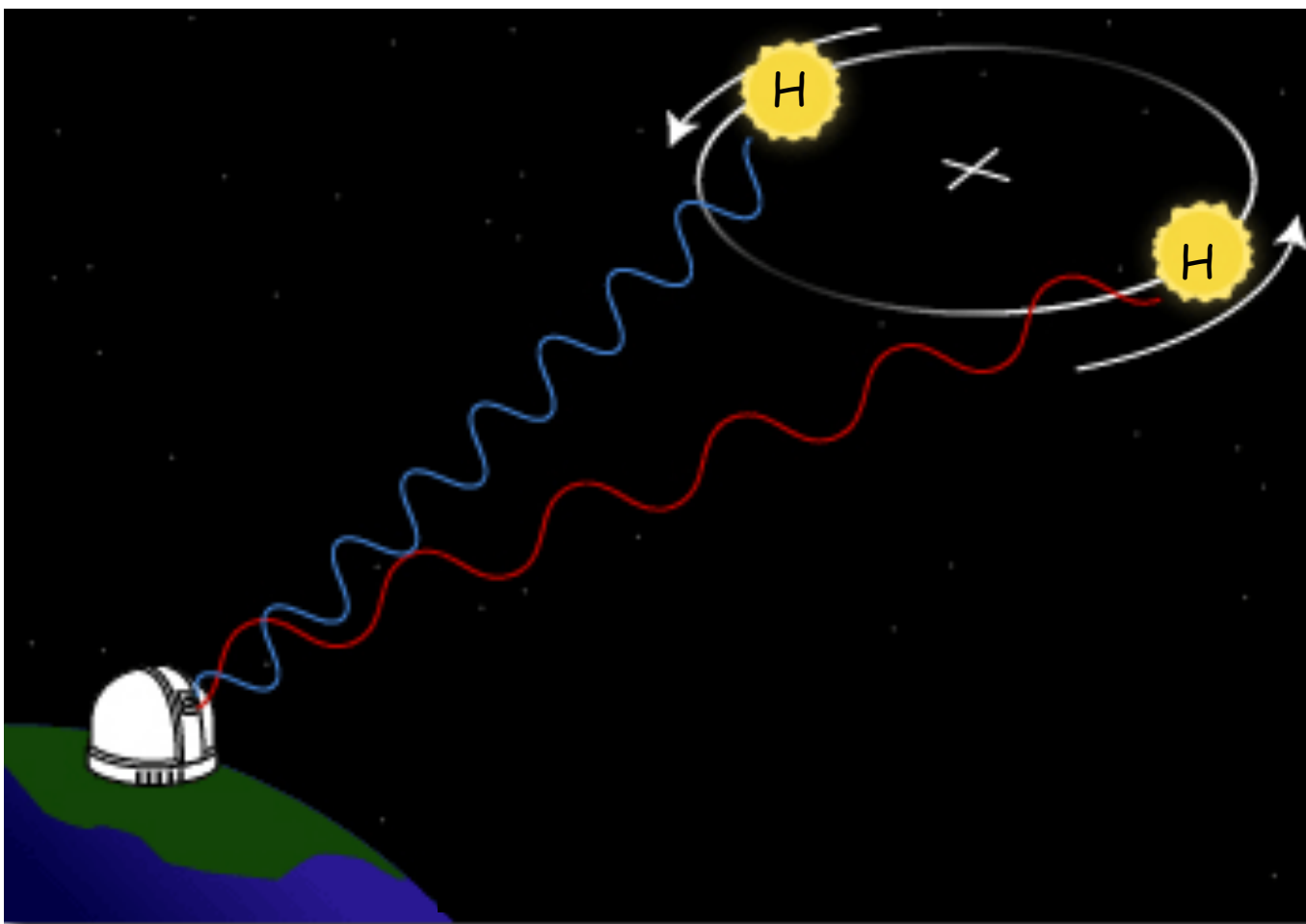
observador
parado

a fonte
se afasta



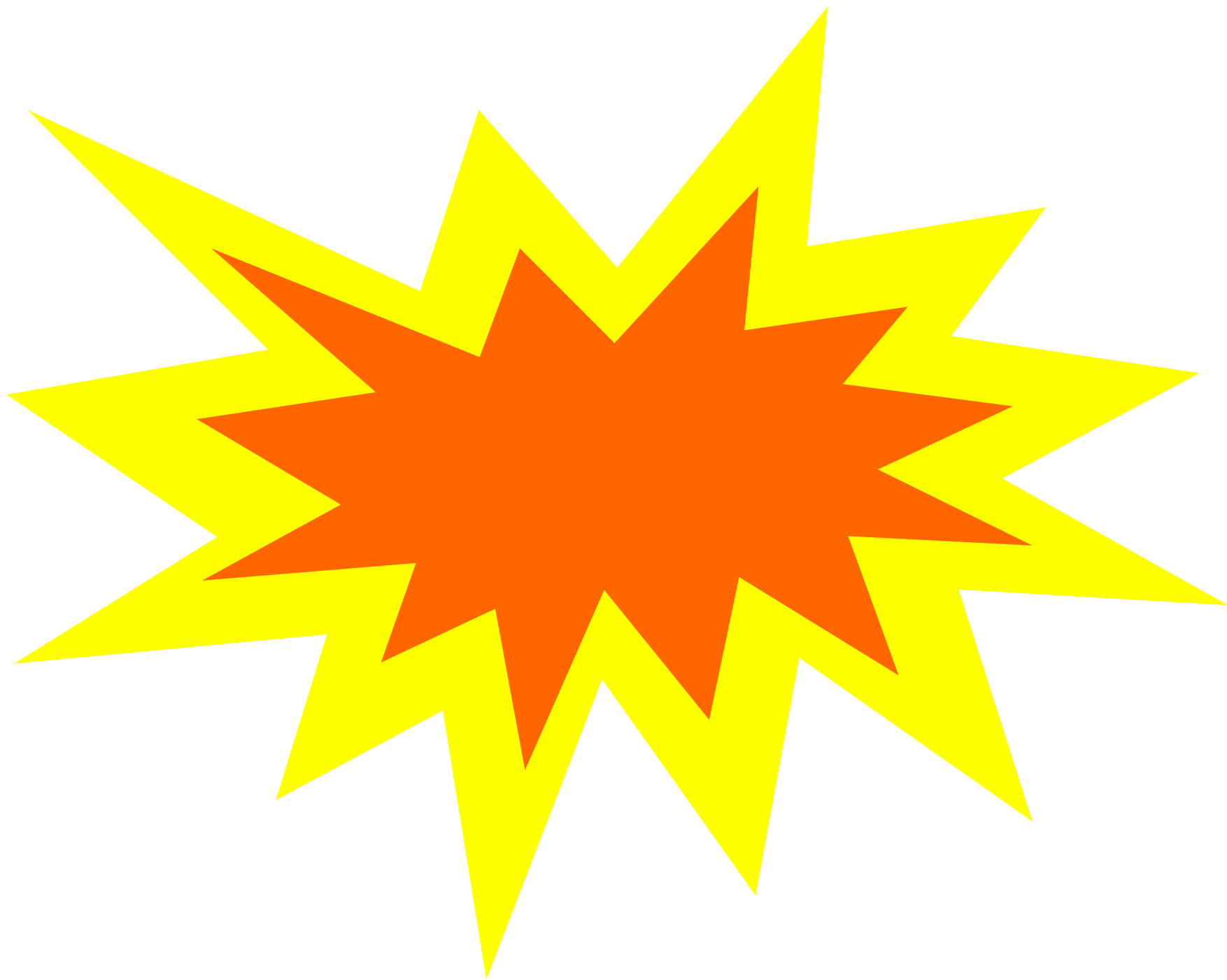
$$\nu = \nu_0 \sqrt{\frac{1 - v/c}{1 + v/c}}$$

frequência
diminui



Medimos as duas frequências e descobrimos as velocidades !

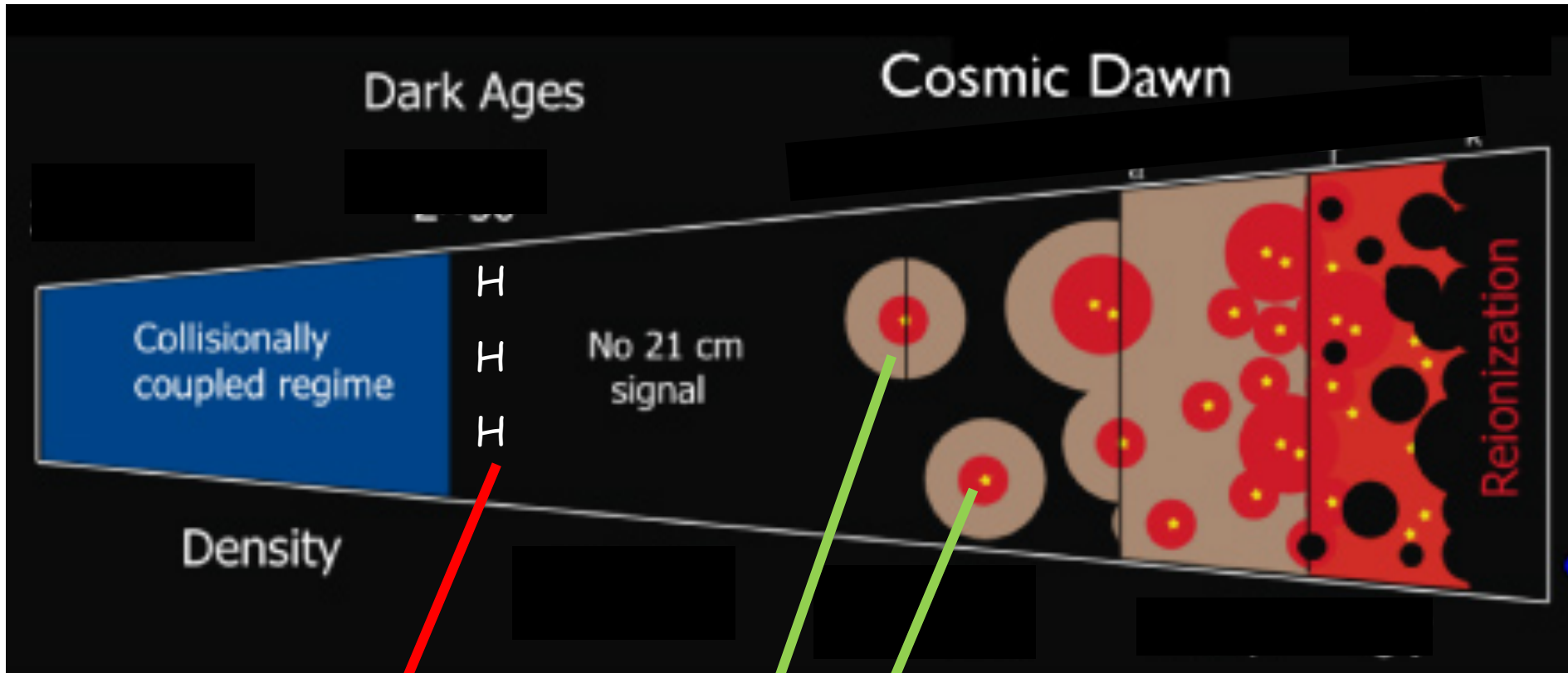
Medida precisa da rotação das galáxias !



Cosmologia de 21 cm !

Cosmologia de 21 cm !

H foi produzido no Universo primordial



Formação de H

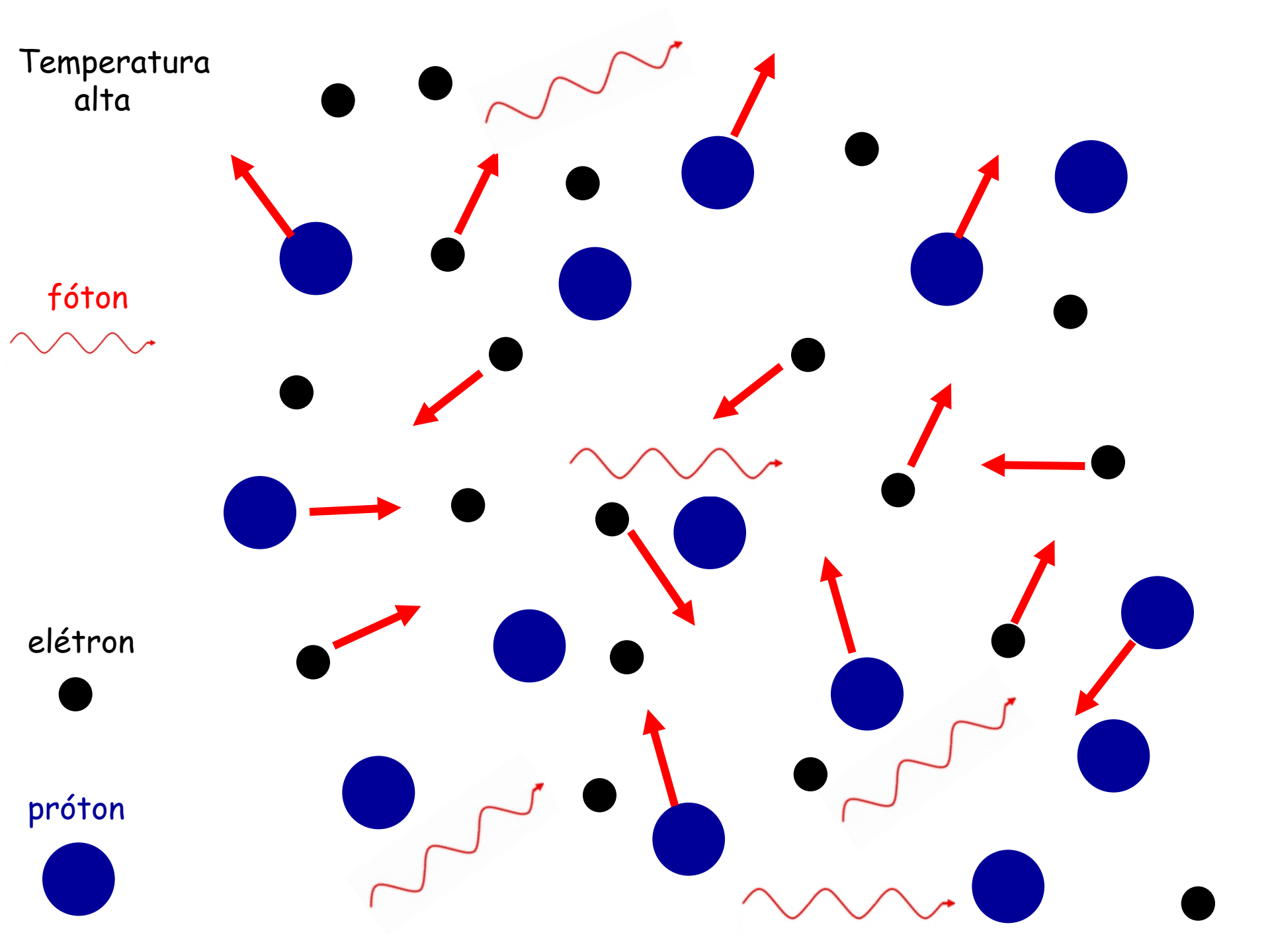
Formação das primeiras estrelas !

Temperatura
alta

fóton

elétron

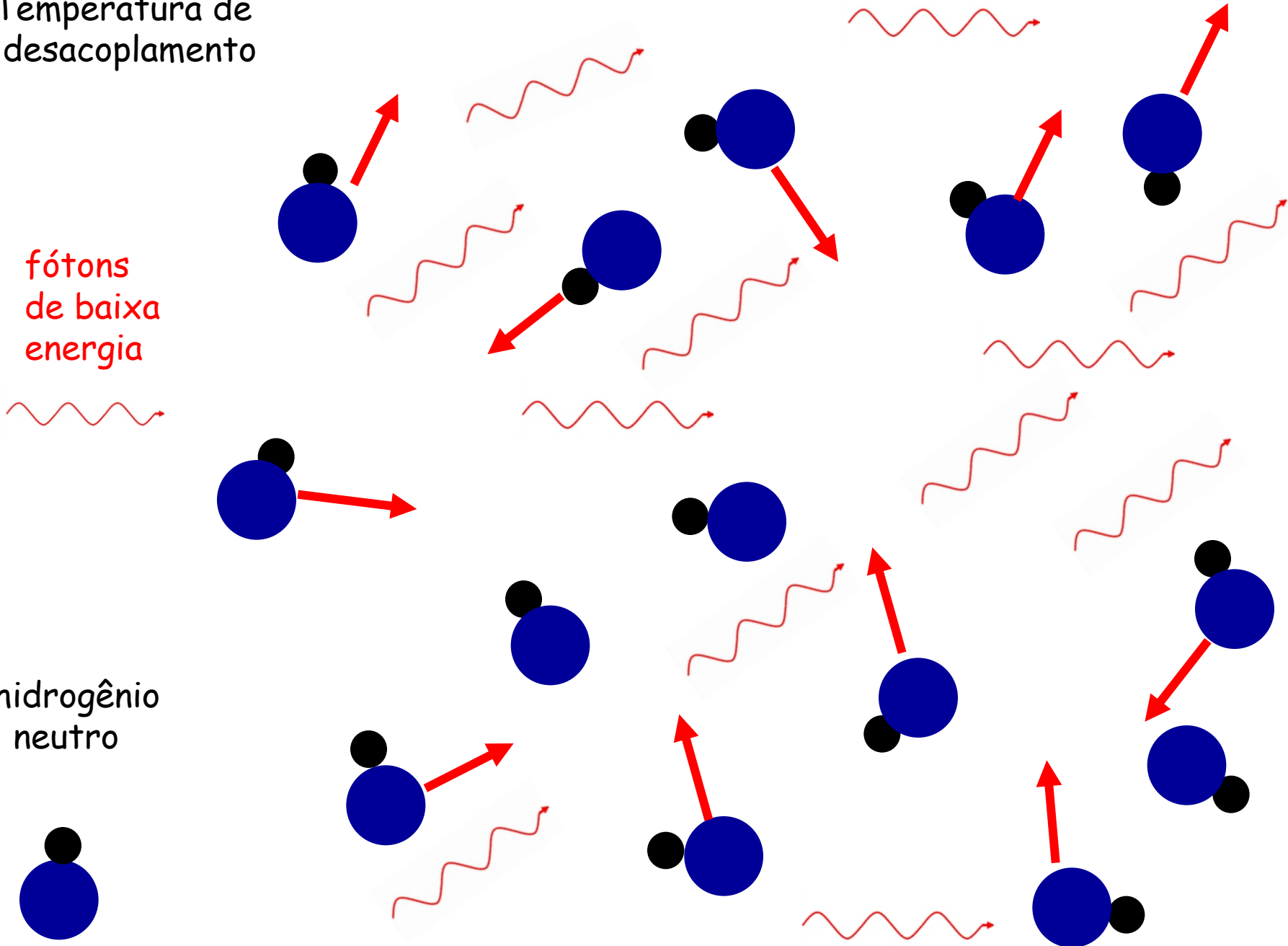
próton



Temperatura de
desacoplamento

fótons
de baixa
energia

hidrogênio
neutro

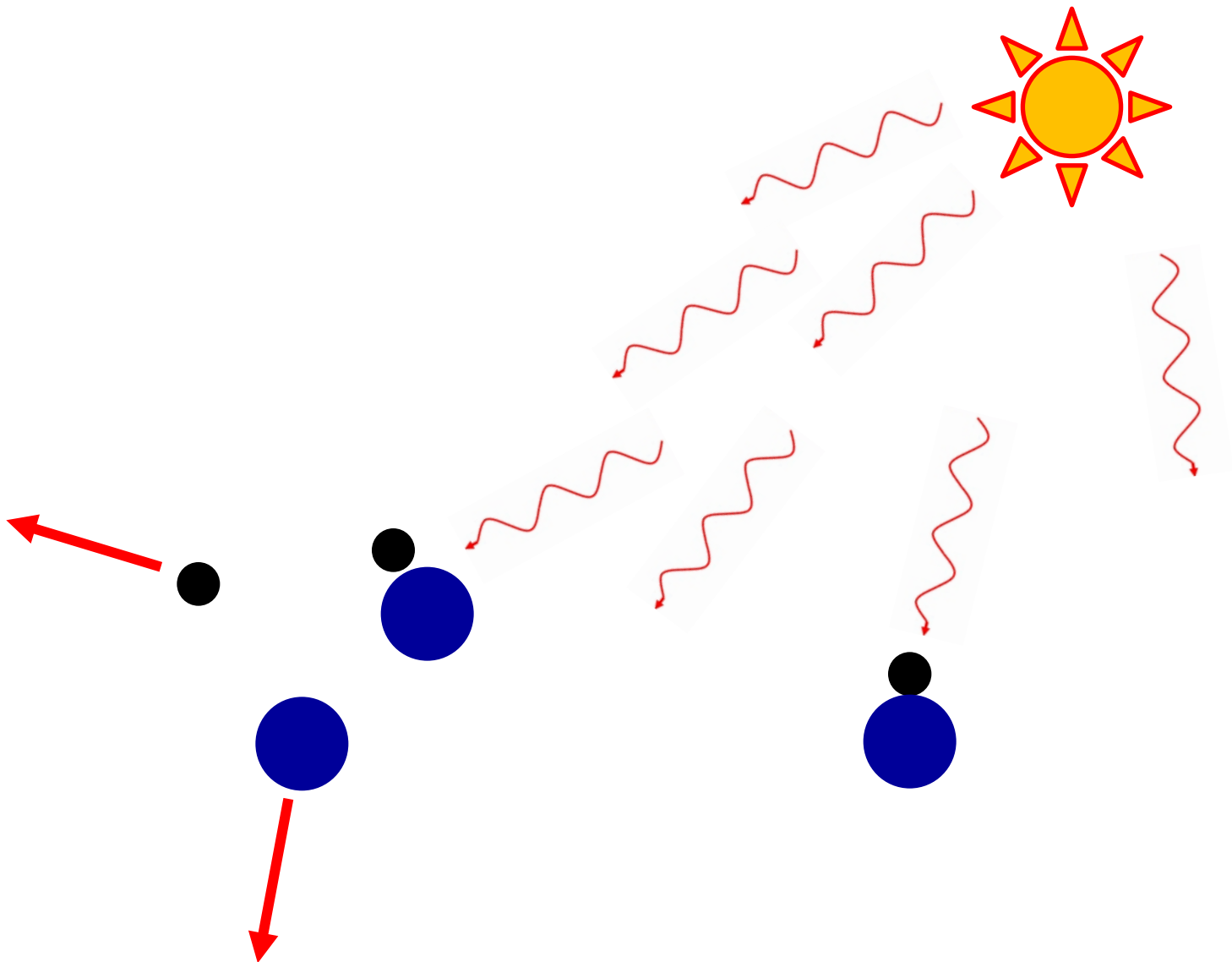
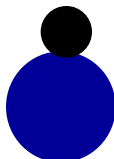


Formação das estrelas

fótons de baixa energia



hidrogênio neutro

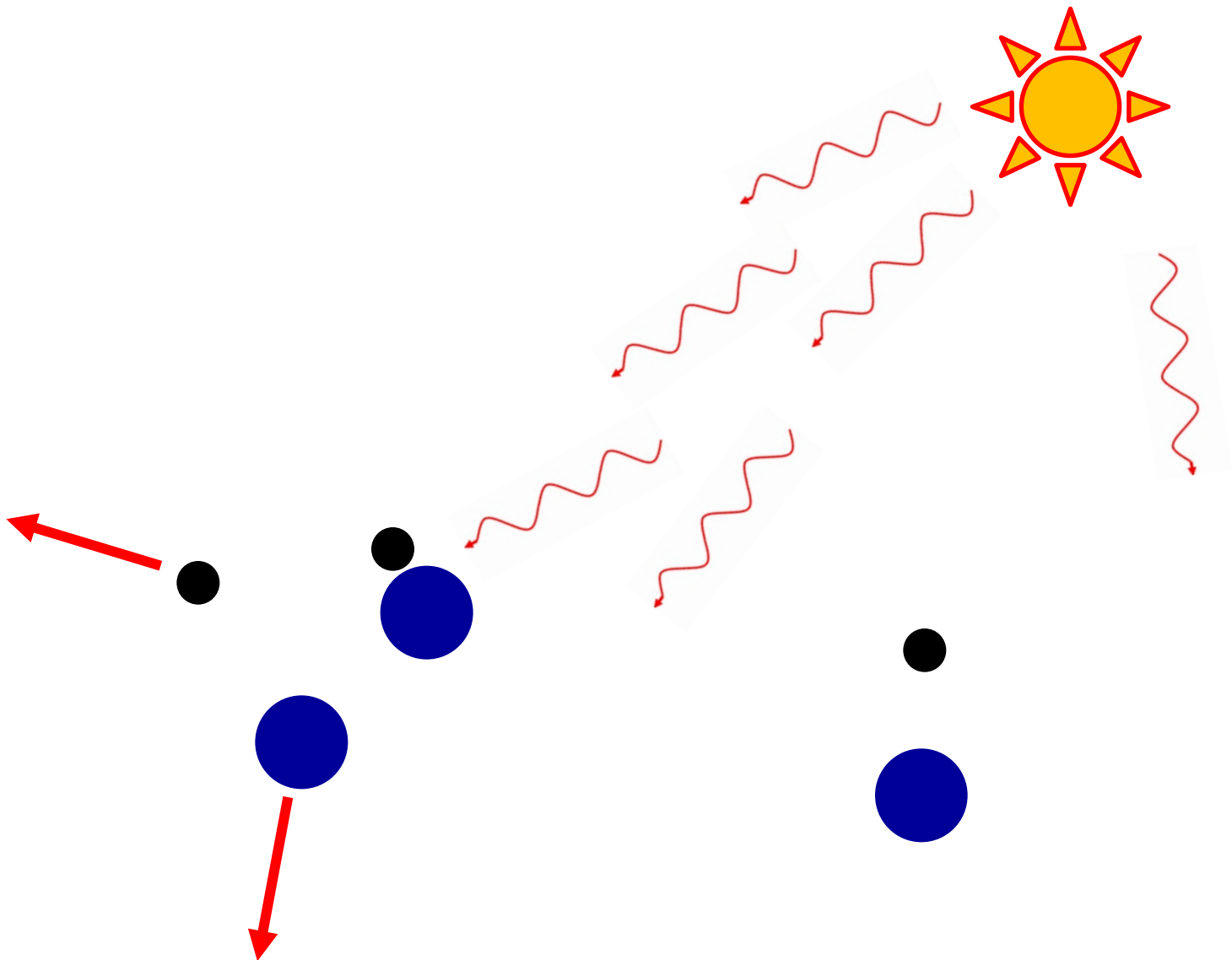
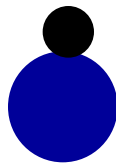


Formação
das estrelas

fótons
de baixa
energia



hidrogênio
neutro

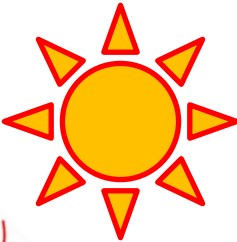
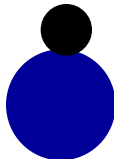


Formação
das estrelas

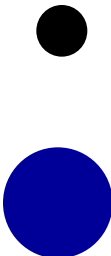
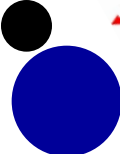
fótons
de baixa
energia



hidrogênio
neutro

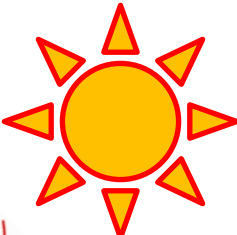
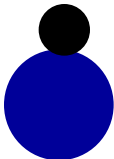
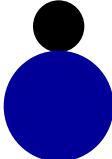
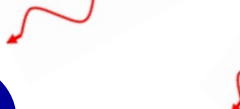
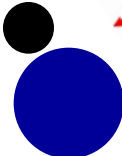


fóton de
21 cent.



Formação
das estrelas

fótons
de baixa
energia



hidrogênio
neutro

$$\vec{B} = \frac{\mu_0}{2 \pi a^3} \vec{\mu}$$

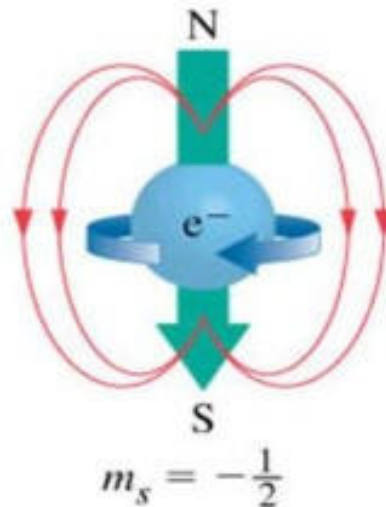
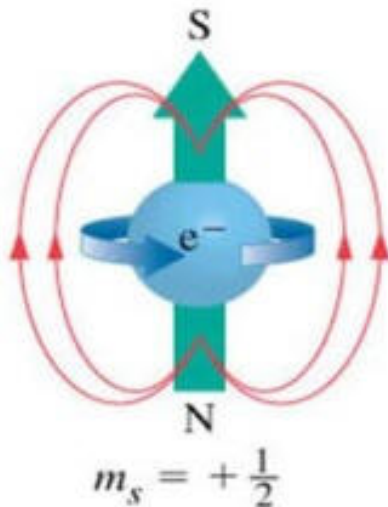
$$\vec{\mu} = \gamma \vec{S} \quad \gamma = \frac{g e}{2 m}$$

$$\vec{B} = \frac{\mu_0 I}{2 \pi a^3} \gamma \vec{S}$$

momento magnético gera
campo magnético !

spin gera momento magnético !

spin gera
campo magnético !

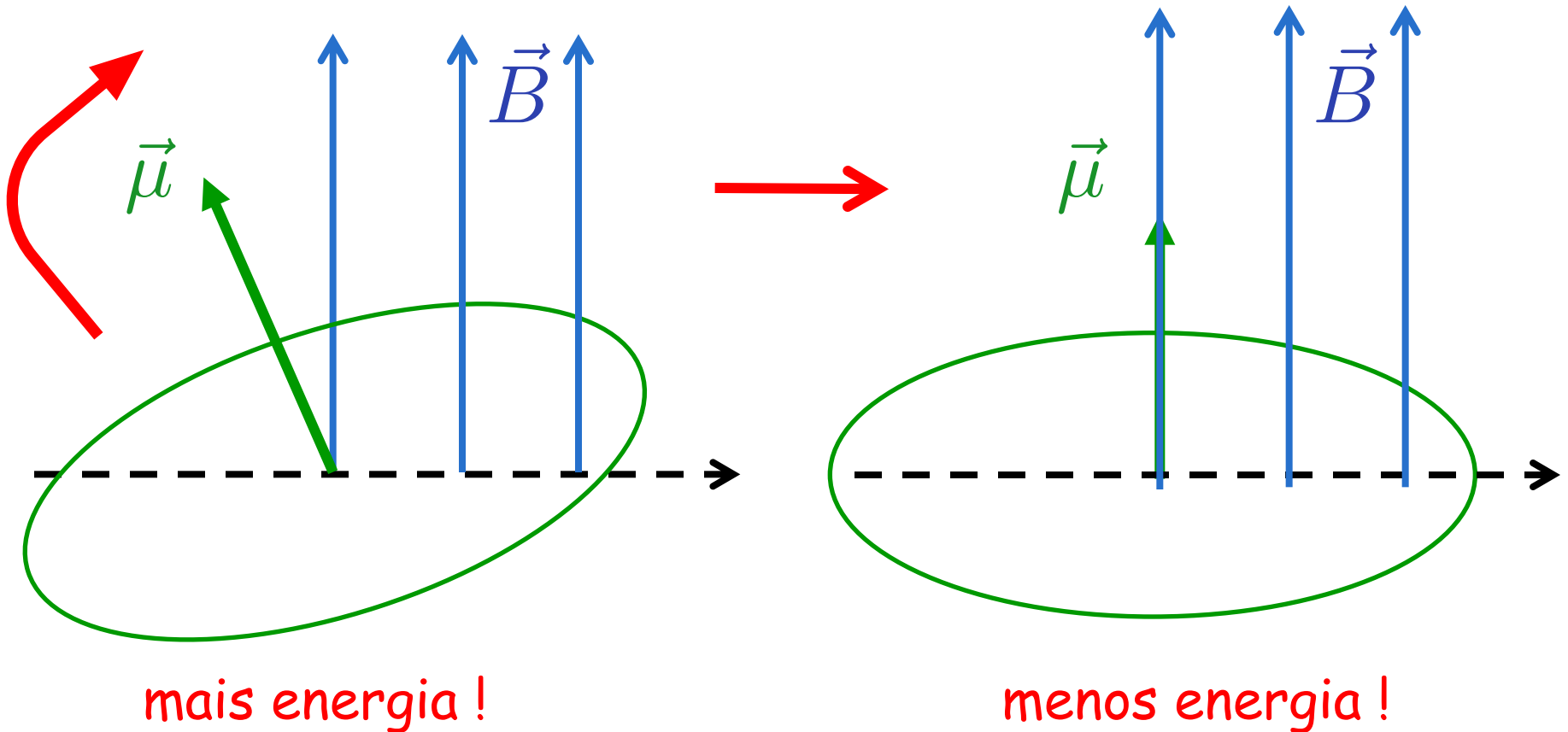


Conclusão

Spin útil em astronomia : rotação das galáxias

Spin útil em cosmologia : nascimento das estrelas

Espira tenta se alinhar como uma bússola !



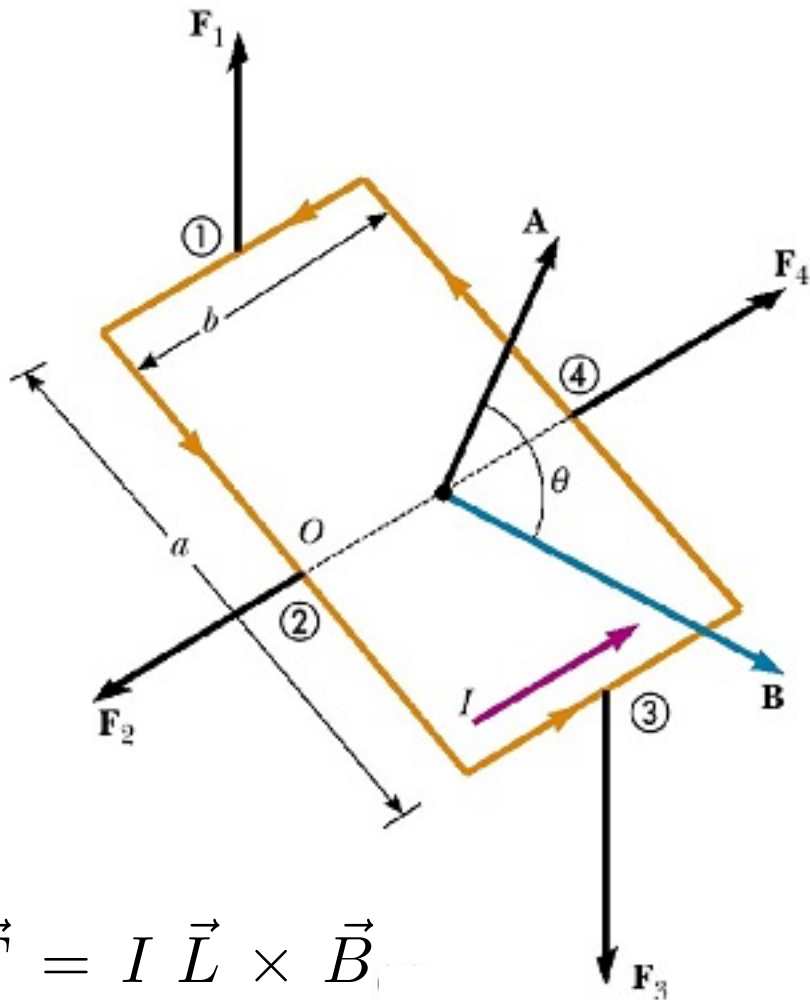
Energia potencial de orientação

$$U = -\vec{\mu} \cdot \vec{B}$$

Espira sofre um torque !

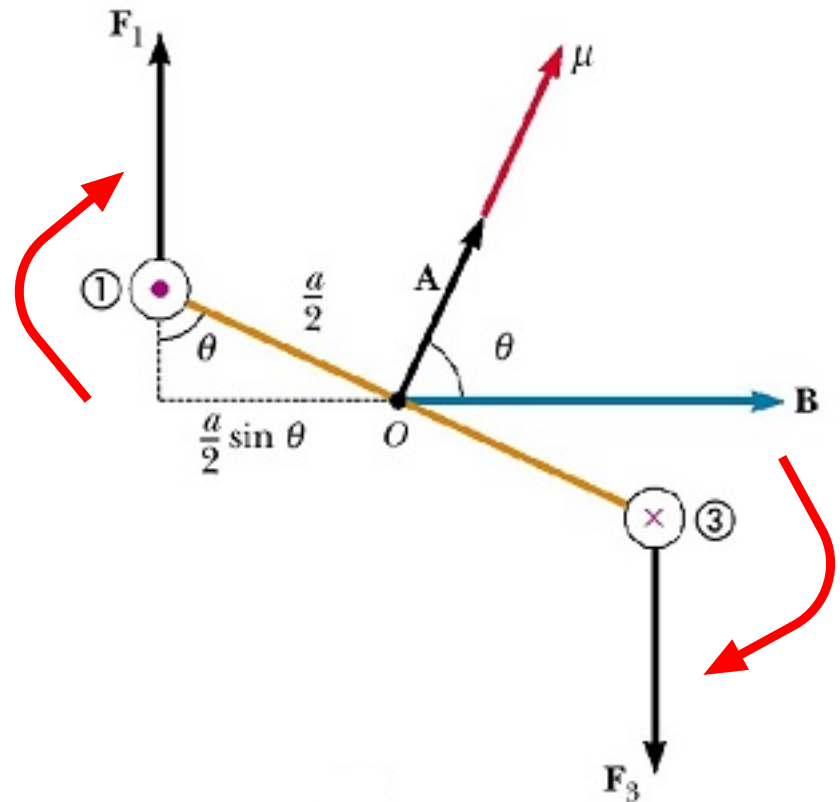
$$\vec{\mu} = I \vec{A}$$

$$\mu = I a b$$



$$\vec{F} = I \vec{L} \times \vec{B}$$

$$F = I b B$$



Trabalho da força F quando a espira vai de 1 a 2.:

$$dW = \vec{F} \cdot d\vec{l}$$

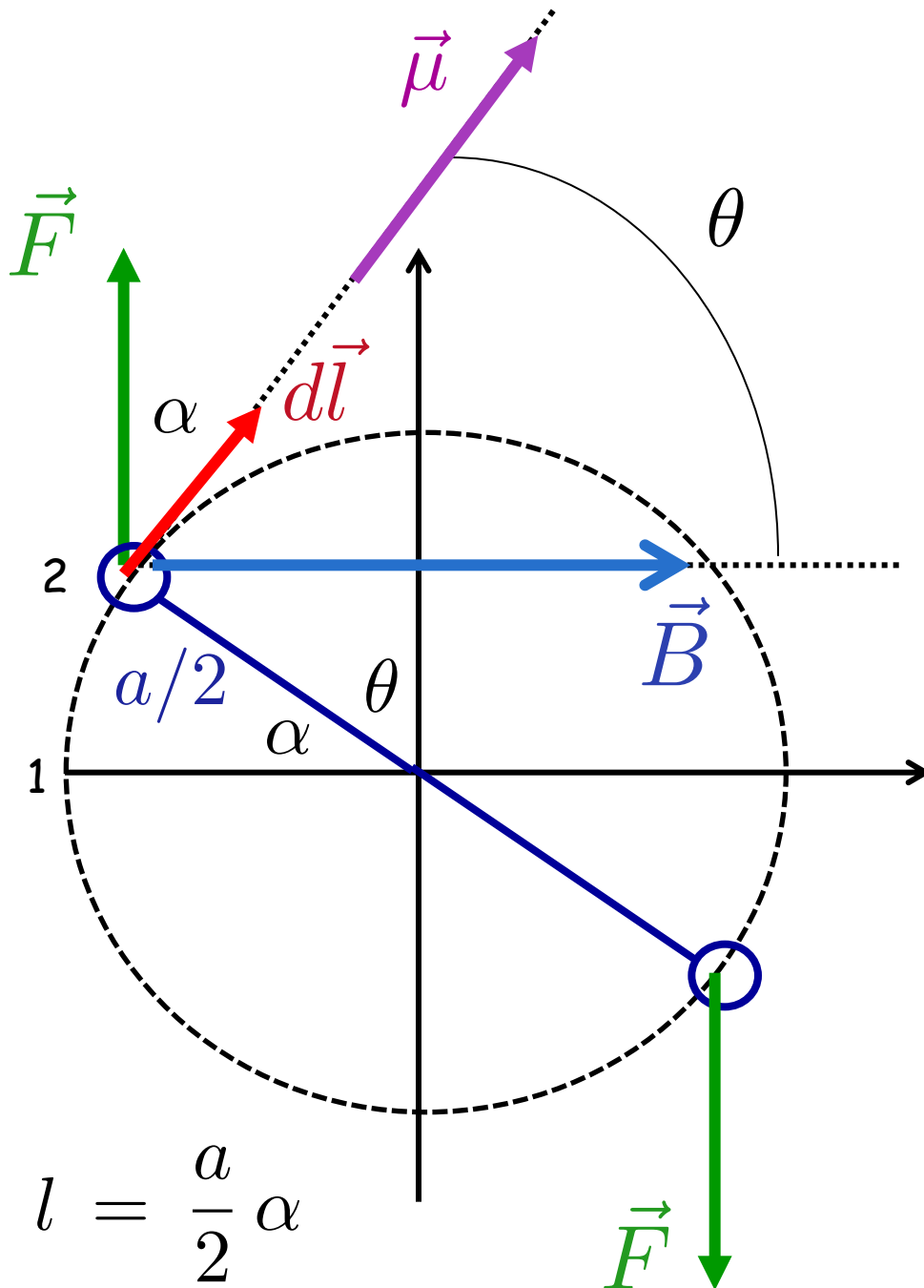
$$= F dl \cos \alpha$$

$$= F \frac{a}{2} d\alpha \cos \alpha$$

$$W = \int_0^\alpha F \frac{a}{2} \cos \alpha d\alpha$$

$$W = F \frac{a}{2} \sin \alpha$$

$$W = F \frac{a}{2} \cos \theta$$



$$W = F \frac{a}{2} \cos \theta$$

Somando a contribuição da barra inferior (3) :

$$W = F a \cos \theta$$

$$F = I b B$$

$$W = I a b B \cos \theta$$

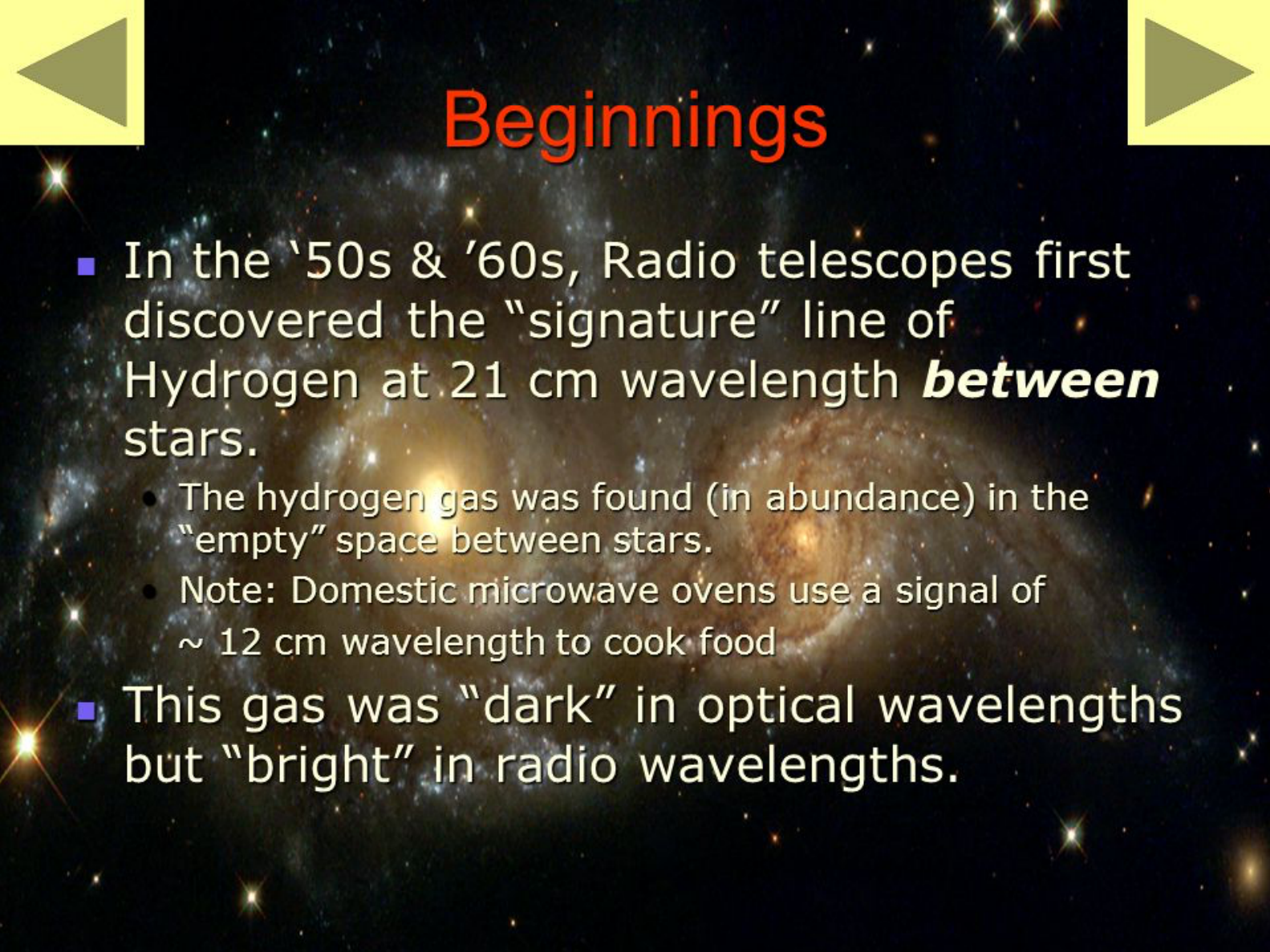
$$\mu = I a b$$

$$W = \mu B \cos \theta$$

$$W = \vec{\mu} \cdot \vec{B}$$

$$W = -U$$

$$U = -\vec{\mu} \cdot \vec{B}$$



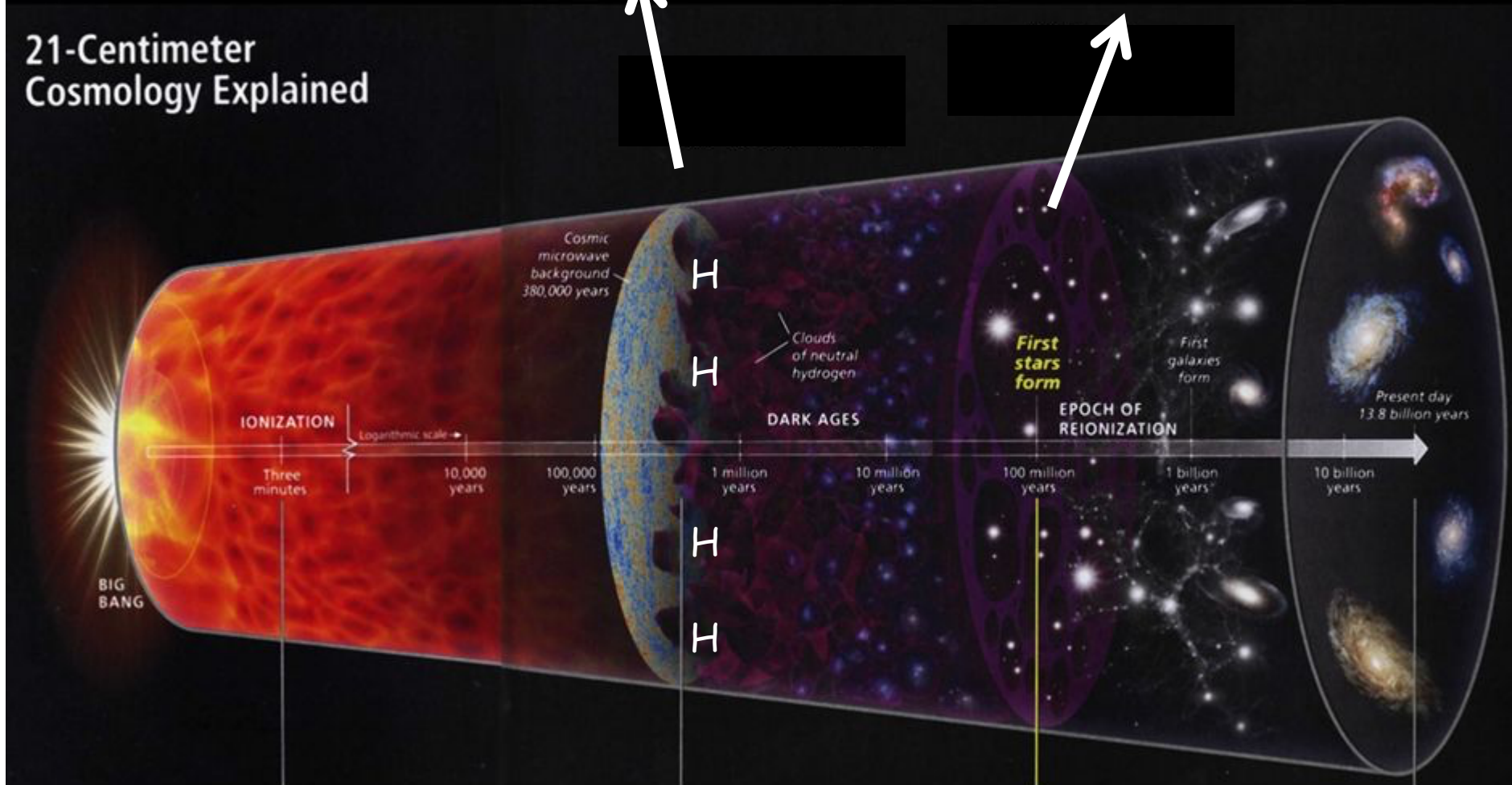
Beginnings

- In the '50s & '60s, Radio telescopes first discovered the “signature” line of Hydrogen at 21 cm wavelength ***between*** stars.
 - The hydrogen gas was found (in abundance) in the “empty” space between stars.
 - Note: Domestic microwave ovens use a signal of ~ 12 cm wavelength to cook food
- This gas was “dark” in optical wavelengths but “bright” in radio wavelengths.

21-Centimeter Cosmology Explained

Formação de H

Formação das primeiras estrelas !



What is the Reionization Era?

A Schematic Outline of the Cosmic History

Time since the
Big Bang (years)

~ 300 thousand

H H H

← The Big Bang

The Universe filled
with ionized gas

← The Universe becomes
neutral and opaque

The Dark Ages start

~ 500 million

Galaxies and Quasars
begin to form
The Reionization starts

~ 1 billion

The Cosmic Renaissance
The Dark Ages end

Reionization complete,
the Universe becomes
transparent again

~ 9 billion

Galaxies evolve

The Solar System forms

~ 13 billion

Today: Astronomers
figure it all out!

Formação de H

Formação das
primeiras estrelas !