# Problemas de Valor Inicial - Parte I

Elias S. Helou Neto

# Equações Diferenciais Ordinárias
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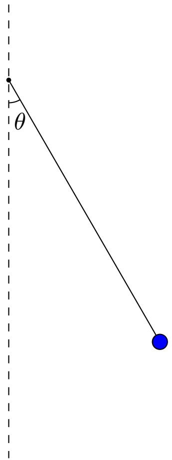
- $d$ é a ordem da EDO
- $y : \mathbb{R} \to \mathbb{R}$ é a incógnita
- $f : \mathbb{R}^{d+1} \to \mathbb{R}$ é dada

# Equações Diferenciais Ordinárias

## Exemplo



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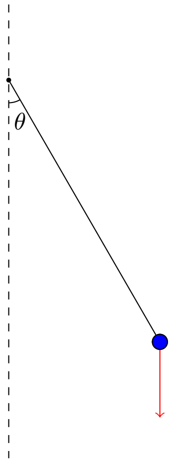

# Equações Diferenciais Ordinárias

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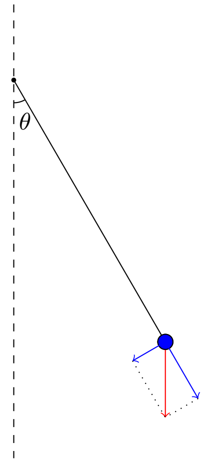

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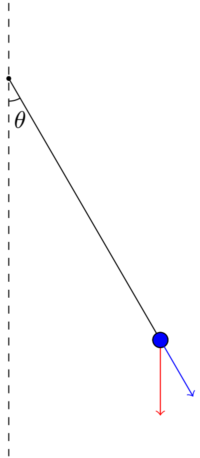

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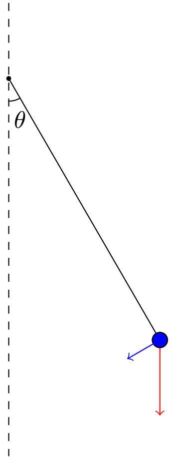
## Exemplo



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- $L$ é o comprimento do pêndulo
- $g$ é a aceleração da gravidade

# Equações Diferenciais Ordinárias

## Exemplo

$$\theta''(t) \approx -\frac{g}{L}\theta(t)$$

# Equações Diferenciais Ordinárias
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$$\theta(t) \approx A \sin\left(\sqrt{\frac{g}{L}}t + \phi\right)$$

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- $\sqrt{\frac{g}{L}}$ é a frequência
- $A$ é a amplitude
- $\phi$ é a fase

- A EDO determina uma família de funções
- Nem toda EDO pode ser resolvida exatamente

# Problemas de Valores Iniciais

## Definição

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# Problemas de Valores Iniciais

## Definição

$$y^{(d)}(t) = f\left(t, y(t), y'(t), y''(t), \ldots, y^{(d-1)}(t)\right)$$

$$\begin{aligned} y\left(t_0\right) &= v_0 \\ y'\left(t_0\right) &= v_1 \\ y''\left(t_0\right) &= v_2 \\ &\vdots \\ y^{(d-1)}\left(t_0\right) &= v_{d-1} \end{aligned}$$

$$\boldsymbol{y}'(t) = \boldsymbol{f}(t, \boldsymbol{y}(t))$$

# Problemas de Valores Iniciais

## Sistemas

$$\boldsymbol{y}'(t) = \boldsymbol{f}(t, \boldsymbol{y}(t))$$

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# Problemas de Valores Iniciais

## Sistemas

$$\boldsymbol{y}'(t) = \boldsymbol{f}(t, \boldsymbol{y}(t))$$

$$\boldsymbol{y}(t_0) = \boldsymbol{y}_0$$

- $y : \mathbb{R} \to \mathbb{R}^n$ é a incógnita
- $\boldsymbol{f} : \mathbb{R} \times \mathbb{R}^n \to \mathbb{R}^n$ é dada

$$\boldsymbol{f}(t, \boldsymbol{y}) = \begin{pmatrix} f(t, y_n, y_{n-1}, \dots, y_1) \\ y_1 \\ y_2 \\ \vdots \\ y_{n-1} \end{pmatrix}$$

$$\boldsymbol{y}'(t) = \boldsymbol{f}\left(t, \boldsymbol{y}(t)\right)$$

$$\boldsymbol{y}'(t) = \begin{pmatrix} f(t, y_n(t), y_{n-1}(t), \ldots, y_1(t)) \\ y_1(t) \\ y_2(t) \\ \vdots \\ y_{n-1}(t) \end{pmatrix}$$

$$\begin{pmatrix} y_1'(t) \\ y_2'(t) \\ y_3'(t) \\ \vdots \\ y_{n-1}'(t) \\ y_n'(t) \end{pmatrix} = \begin{pmatrix} f(t, y_n(t), y_{n-1}(t), \ldots, y_1(t)) \\ y_1(t) \\ y_2(t) \\ \vdots \\ y_{n-2}(t) \\ y_{n-1}(t) \end{pmatrix}$$

$$y_{n-1}(t) = y_n'(t)$$

$$y_{n-2}(t) = y_n''(t)$$

$$y_{n-3}(t) = y_n^{(3)}(t)$$

# Problemas de Valores Iniciais

## Sistemas e EDOs Superiores

$$\cdots$$

$$y_1(t) = y_n^{(n-1)}(t)$$

$$\begin{pmatrix} y_1'(t) \\ y_2'(t) \\ y_3'(t) \\ \vdots \\ y_{n-1}'(t) \\ y_n'(t) \end{pmatrix} = \begin{pmatrix} f(t, y_n(t), y_{n-1}(t), \ldots, y_1(t)) \\ y_1(t) \\ y_2(t) \\ \vdots \\ y_{n-2}(t) \\ y_{n-1}(t) \end{pmatrix}$$

$$y_n^{(n)}(t) = f\left(t, y_n(t), y_n'(t), y_n''(t), \ldots, y_n^{(n-1)}(t)\right)$$

$$\boldsymbol{y}(t + \Delta_t) \approx \boldsymbol{y}(t) + \Delta_t \boldsymbol{y}'(t)$$

# Problemas de Valores Iniciais

## Método de Euler

$$\boldsymbol{y}(t+\Delta_t) \approx \boldsymbol{y}(t) + \Delta_t \boldsymbol{f}\big(t, \boldsymbol{y}(t)\big)$$

$$t_k = t_0 + k\Delta_t$$

$$\boldsymbol{y}(t_{k+1}) \approx \boldsymbol{y}(t_k) + \Delta_t \boldsymbol{f}\left(t_k, \boldsymbol{y}(t_k)\right)$$

$$\boldsymbol{y}_{k+1} = \boldsymbol{y}_k + \Delta_t \boldsymbol{f}(t_k, \boldsymbol{y}_k)$$

$$\boldsymbol{y}(t+\Delta_t) = \boldsymbol{y}(t) + \Delta_t \boldsymbol{f}\big(t, \boldsymbol{y}(t)\big) + O(\Delta_t^2)$$

$$\boldsymbol{y}(t+\Delta) \approx \boldsymbol{y}(t) + \Delta \boldsymbol{y}'(t) + \frac{\Delta^2}{2}\boldsymbol{y}''(t) + \cdots + \frac{\Delta^d}{d!}\boldsymbol{y}^{(d)}(t)$$

$$\boldsymbol{y}(t+\Delta) \approx \boldsymbol{y}(t) + \Delta \boldsymbol{y}'(t) + \frac{\Delta^2}{2}\boldsymbol{y}''(t)$$

$$\boldsymbol{y}(t+\Delta) \approx \boldsymbol{y}(t) + \Delta \boldsymbol{f}\left(t, \boldsymbol{y}(t)\right) + \frac{\Delta^2}{2}\boldsymbol{y}''(t)$$

$$\boldsymbol{y}(t+\Delta) \approx \boldsymbol{y}(t) + \Delta \boldsymbol{f}\left(t, \boldsymbol{y}(t)\right) + \frac{\Delta^2}{2}\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{f}\left(t, \boldsymbol{y}(t)\right)$$

$$\boldsymbol{f}\left(t, \boldsymbol{y}(t)\right) = \boldsymbol{g}\left(\boldsymbol{h}(t)\right)$$

$$\boldsymbol{g}(\boldsymbol{z}) = \boldsymbol{f}\Big(z_0, (z_1, z_2, \ldots, z_n)\Big)$$

$$J\boldsymbol{g}(\boldsymbol{z})^T = \begin{pmatrix} \boldsymbol{f}_x(z_0, (z_1, z_2, \ldots, z_n))^T \\ \boldsymbol{f}_{\boldsymbol{y}}(z_0, (z_1, z_2, \ldots, z_n))^T \end{pmatrix}$$

$$\boldsymbol{h}(t) = \begin{pmatrix} t \\ \boldsymbol{y}(t) \end{pmatrix}$$

$$J\boldsymbol{h}(t) = \begin{pmatrix} 1 \\ \boldsymbol{y}'(t) \end{pmatrix}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{f}\big(t,\boldsymbol{y}(t)\big) = J\boldsymbol{g}(\boldsymbol{h}(t))J\boldsymbol{h}(t)$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{f}\left(t,\boldsymbol{y}(t)\right)=\boldsymbol{f}_x\left(t,\boldsymbol{y}(t)\right)+\boldsymbol{f}_{\boldsymbol{y}}\left(t,\boldsymbol{y}(t)\right)\boldsymbol{y}'(t)$$

$$\boldsymbol{y}(t+\Delta) \approx \boldsymbol{y}(t) + \Delta \boldsymbol{f}\left(t, \boldsymbol{y}(t)\right) + \frac{\Delta^2}{2}\left[\boldsymbol{f}_x\left(t, \boldsymbol{y}(t)\right) + \boldsymbol{f}_{\boldsymbol{y}}\left(t, \boldsymbol{y}(t)\right)\boldsymbol{y}'(t)\right]$$

$$\boldsymbol{y}(t+\Delta) \approx \boldsymbol{y}(t) + \Delta \boldsymbol{f}\left(t, \boldsymbol{y}(t)\right) + \frac{\Delta^2}{2}\left[\boldsymbol{f}_x\left(t, \boldsymbol{y}(t)\right) + \boldsymbol{f}_{\boldsymbol{y}}\left(t, \boldsymbol{y}(t)\right)\boldsymbol{f}\left(t, \boldsymbol{y}(t)\right)\right]$$

$$\boldsymbol{y}(t_{k+1}) \approx \boldsymbol{y}(t_k) + \Delta \boldsymbol{f}\left(t_k, \boldsymbol{y}(t_k)\right) + \frac{\Delta^2}{2}\left[\boldsymbol{f}_x\left(t_k, \boldsymbol{y}(t_k)\right) + \boldsymbol{f}_{\boldsymbol{y}}\left(t_k, \boldsymbol{y}(t_k)\right)\boldsymbol{f}\left(t_k, \boldsymbol{y}(t_k)\right)\right]$$

# Problemas de Valores Iniciais

## Métodos de Taylor

$$\boldsymbol{y}_{k+1} = \boldsymbol{y}_k + \Delta \boldsymbol{f}\left(t_k, \boldsymbol{y}_k\right) + \frac{\Delta^2}{2}\left[\boldsymbol{f}_x\left(t_k, \boldsymbol{y}_k\right) + \boldsymbol{f}_{\boldsymbol{y}}\left(t_k, \boldsymbol{y}_k\right)\boldsymbol{f}\left(t_k, \boldsymbol{y}_k\right)\right]$$