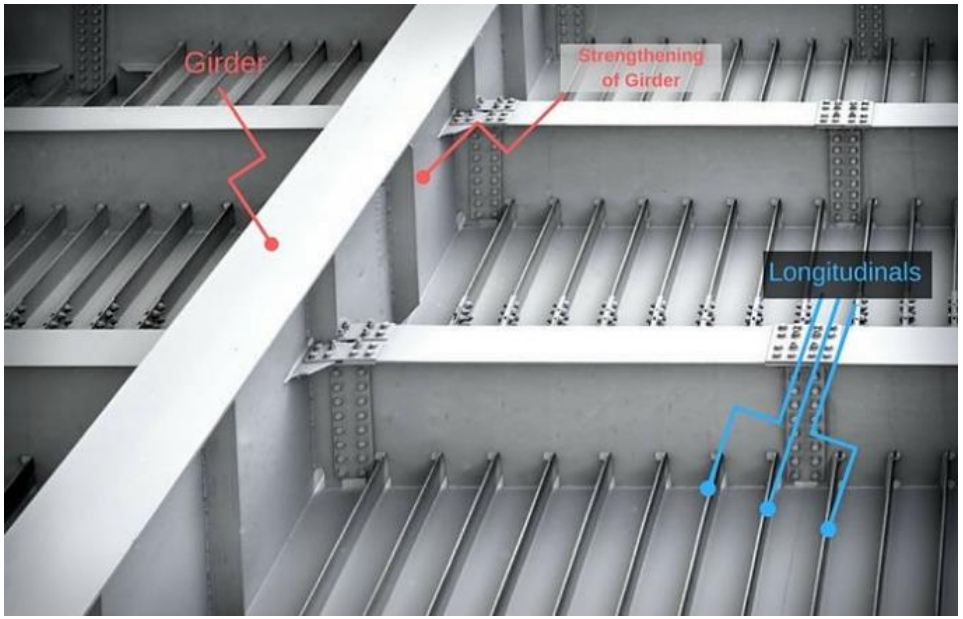
DEPARTAMENTO DE ENGEHARIA NAVAL E OCEÂNICA
ESCOLA POLITÉCNICA DA USP

Análise de Vigas : $\delta\,(mm)$

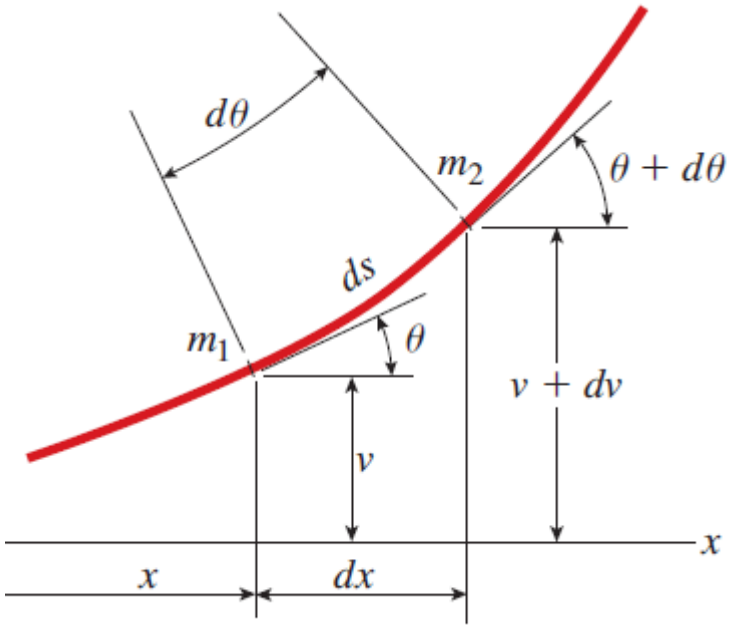


PNV 3212 – Mecânica Dos Sólidos I

2020

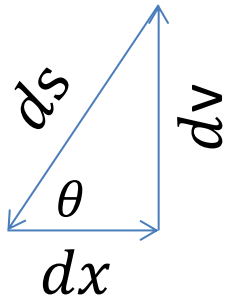
Deflexões em vigas

$$\boldsymbol{\kappa(x) = \frac{1}{\rho(x)}}$$



$$\boldsymbol{\kappa = \frac{d\theta}{ds}}$$

Variação da inclinação por unidade de comprimento



$$\frac{dv}{dx} = \tan\theta \qquad \frac{dv}{ds} = \sin\theta \qquad \frac{dx}{ds} = \cos\theta$$

very small angles of rotation, very small deflections, and very small curvatures.

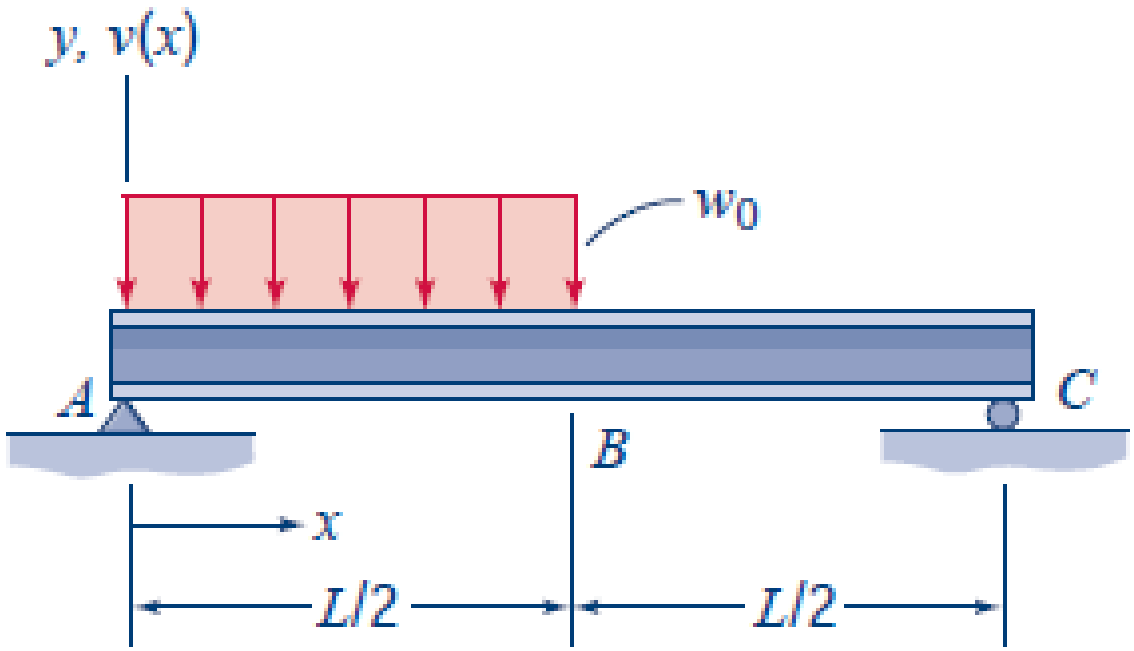
$$\frac{M(x)}{\mathrm{EI}} = \boldsymbol{\frac{d\theta}{dx}}$$

$$\boldsymbol{\frac{M(x)}{\mathrm{EI}} = \frac{d^2v(x)}{dx^2}}$$

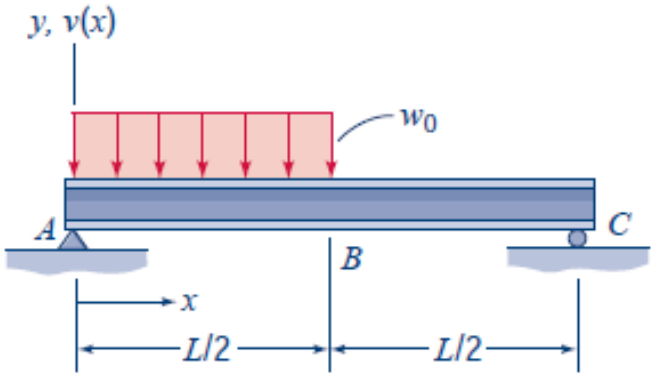
Flexão de uma viga → deflexão vertical + **rotação**

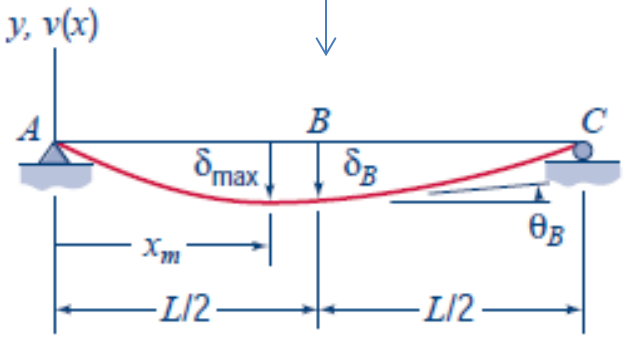
Deflexões em vigas

Exemplo 2: Determine a curva elástica para a viga biapoiada mostrada na figura. Quais são os valores da deflexão e rotação em x=L/2 ?.

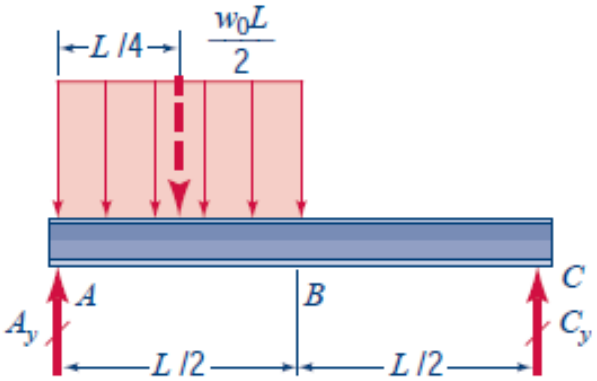


Deflexões em vigas





1) Cálculo das Reações



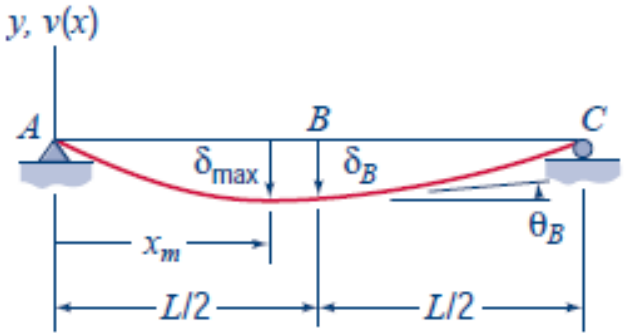
D.C.L

$$\left(\sum M\right)_A = 0: \longrightarrow C_Y L = \frac{w_0 L}{2}\frac{L}{4}$$

$$\left(\sum M\right)_C = 0: \longrightarrow A_Y = \frac{3w_0 L}{8}$$

$$C_Y = \frac{w_0 L}{8}$$

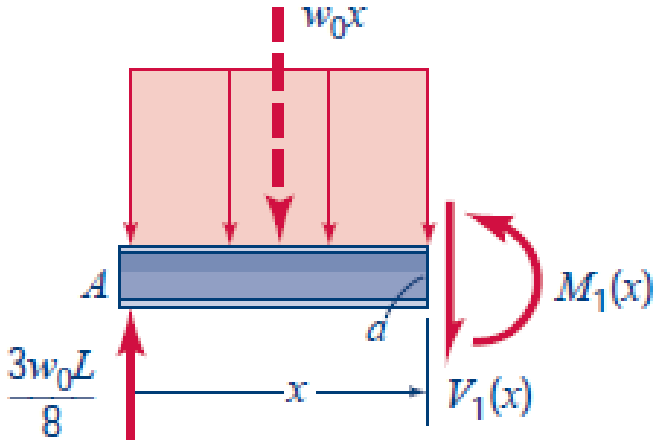
Deflexões em vigas



2) Diagramas de corpo livre

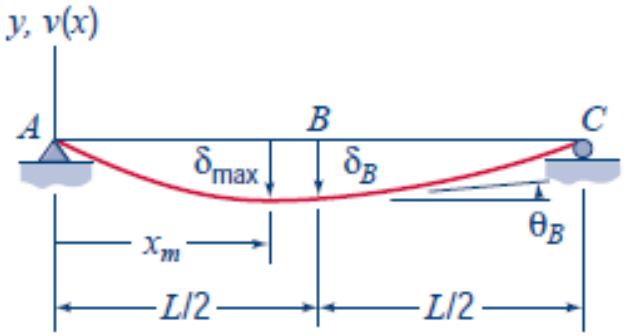
0 <= x < L/2

.C.L



$$\left(\sum M\right)_a = 0: \qquad M_1(x) = \frac{3w_0Lx}{8} - \frac{w_0x^2}{2}$$

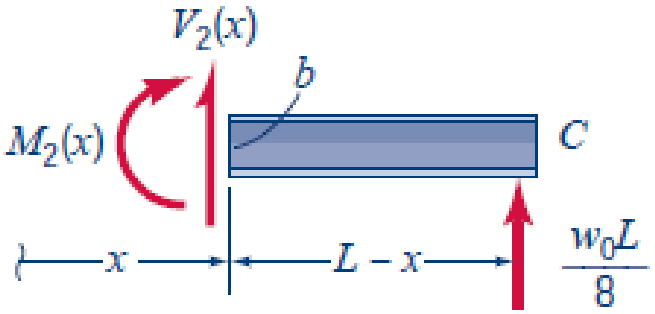
Deflexões em vigas



2) Diagramas de corpo livre

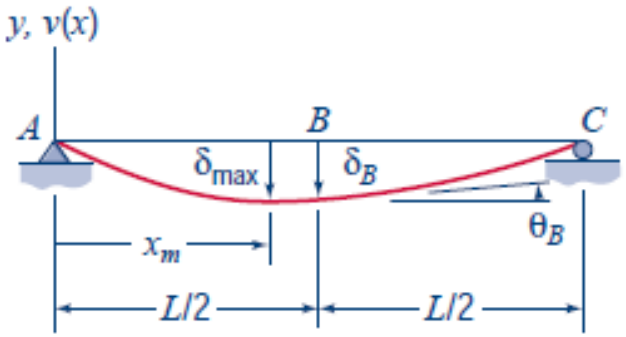
L/2 < x <= L

D.C.L



$$\left(\sum M\right)_b = 0: \qquad M_2(x) = \frac{w_0 L}{8}(L - x)$$

Deflexões em vigas



3) Equações Momento-curvatura

0 <= x < L/2

$$EI\,\frac{d^2v(x)}{dx^2} = \frac{3w_0L}{8}x - \frac{w_0}{2}x^2$$

$$EI\,\frac{d}{dx}v(x) = \frac{3w_0L}{16}x^2 - \frac{w_0}{6}x^3 + C_1$$

$$EIv_1(x) = \frac{w_0L}{16}x^3 - \frac{w_0}{24}x^4 + C_1x + D_1$$

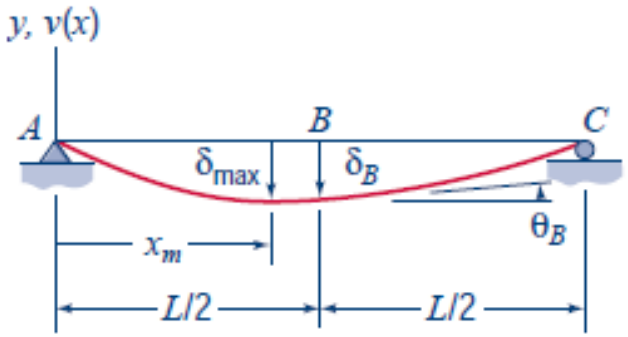
L/2 < x <= L

$$EI\,\frac{d^2v(x)}{dx^2} = \frac{w_0L}{8}(L-x)$$

$$EI\,\frac{dv(x)}{dx} = -\frac{w_0L}{16}(L-x)^2 + C_2$$

$$EIv_2(x) = \frac{w_0L}{48}(L-x)^3 + C_2x + D_2$$

Deflexões em vigas



3) Condições de contorno

0 <= x < L/2

$$v_1(0) = 0$$

$$\downarrow$$

$$EIv_1(0) = \frac{w_0 L}{16}0^2 - \frac{w_0}{24}0^4 + C_1 0 + D_1$$

$$\downarrow$$

$$0 = D_1$$

L/2 < x <= L

$$v_2(L) = 0$$

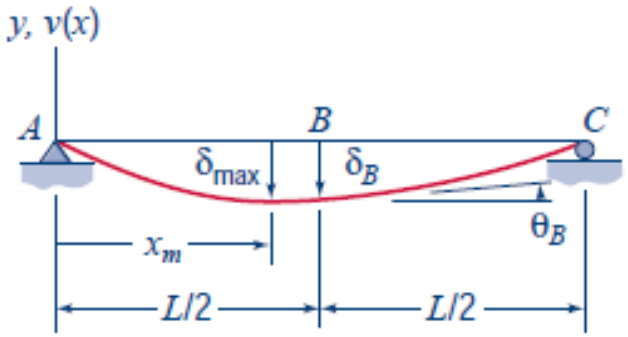
$$\downarrow$$

$$EIv_2(0) = \frac{w_0 L}{48}(L - L)^3 + C_2 L + D_2$$

$$\downarrow$$

$$0 = C_2 L + D_2$$

Deflexões em vigas



3) Condições de contorno (continuidade em x =L/2)

$$\frac{d}{dx}v_1(L/2) = \frac{d}{dx}v_2(L/2)$$

$$\downarrow$$

$$\frac{3w_0L}{16}\left(\frac{L}{2}\right)^2 - \frac{w_0}{6}\left(\frac{L}{2}\right)^3 + C_1 = -\frac{w_0L}{16}\left(L - \frac{L}{2}\right)^2 + C_2$$

$$\downarrow$$

$$C_1 - C_2 = -\frac{w_0L^3}{64} - \frac{3w_0L^3}{64} + \frac{w_0L^3}{48}$$

$$\downarrow$$

$$C_1 - C_2 = -\frac{w_0L^3}{16} + \frac{w_0L^3}{48} = -\frac{w_0L^3}{24}$$

$$v_1(L/2) = v_2(L/2)$$

$$\downarrow$$

$$\frac{w_0L}{16}\left(\frac{L}{2}\right)^3 - \frac{w_0}{24}\left(\frac{L}{2}\right)^4 + C_1\frac{L}{2} = \frac{w_0L}{48}\left(L - \frac{L}{2}\right)^3 + C_2\frac{L}{2} + D_2$$

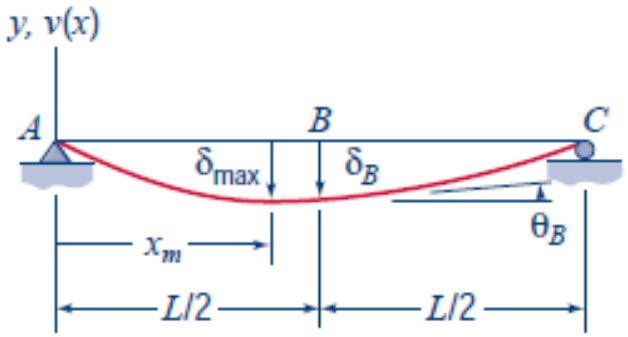
$$\downarrow$$

$$C_1\frac{L}{2} - C_2\frac{L}{2} = \frac{2w_0L^4}{384} - \frac{w_0L^4}{128} + D_2$$

$$\downarrow$$

$$C_1\frac{L}{2} - C_2\frac{L}{2} = -\frac{w_0L^4}{384} + D_2$$

Deflexões em vigas



3) Condições de contorno (continuidade em x =L/2)

$$\frac{d}{dx}v_1(L/2) = \frac{d}{dx}v_2(L/2) \qquad v_1(L/2) = v_2(L/2) \qquad v_1(0) = 0 \qquad v_2(L) = 0$$

$$C_1 - C_2 = -\frac{w_0 L^3}{24}$$

$$C_1\frac{L}{2} - C_2\frac{L}{2} = -\frac{w_0 L^4}{384} + D_2$$

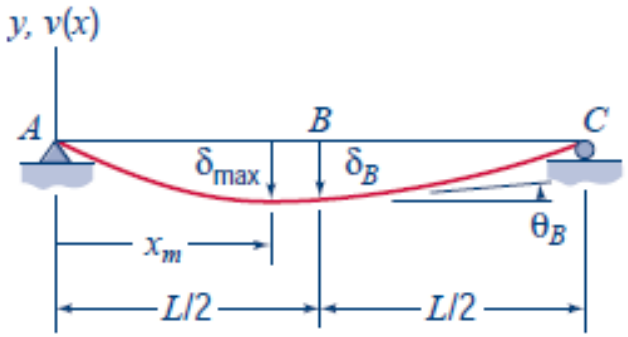
$$0 = C_2 L + D_2$$

$$C_1 = -\frac{9}{384}(w_0 L^3)$$

$$C_2 = \frac{7}{384}(w_0 L^3)$$

$$D_2 = -\frac{7}{384}(w_0 L^4)$$

Deflexões em vigas



3) SOLUÇÃO

L/2 < x <= L

$$EIv_2(x) = \frac{w_0 L}{48}(L-x)^3 + \frac{7w_0 L^3}{384}x - \frac{7w_0 L^4}{384}$$

$$v_2(x) = \frac{w_0 L^4}{384EI}\left[1 - 8\left(\frac{x}{L}\right)^3 + 24\left(\frac{x}{L}\right)^2 - 17\frac{x}{L}\right]$$

ADIMENSIONAL!!!

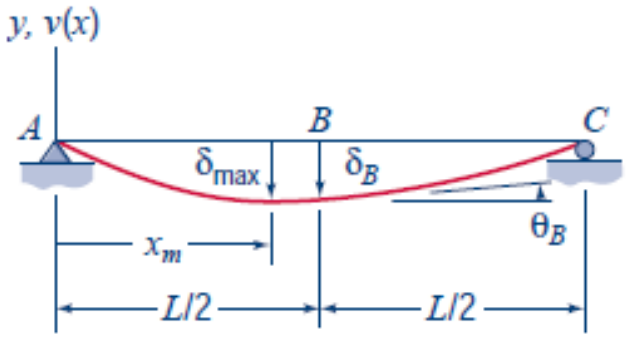
0 <= x < L/2

$$EIv_1(x) = \frac{w_0 L}{16}x^3 - \frac{w_0}{24}x^4 - \frac{9w_0 L^3}{384}x$$

$$v_1(x) = \frac{w_0 L^4}{384EI}\left[24\left(\frac{x}{L}\right)^3 - 16\left(\frac{x}{L}\right)^4 - 9\frac{x}{L}\right]$$

ADIMENSIONAL!!!

Deflexões em vigas



3) MÁXIMA DEFLEXÃO

$0 <= x < L/2$

$$v_1(x) = \frac{w_0 L^4}{384EI}\left[24\left(\frac{x}{L}\right)^3 - 16\left(\frac{x}{L}\right)^4 - 9\frac{x}{L}\right]$$

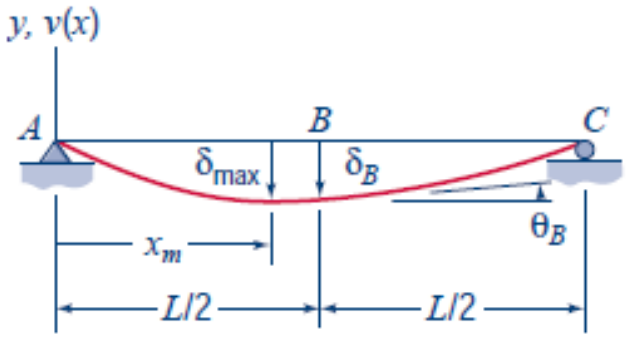
$$\frac{d}{dx}v_1(x) = 0$$

$$\frac{d}{dx}v_1(x) = \frac{w_0 L^4}{384EI}\left[24 \times 3\left(\frac{x}{L}\right)^2 - 16 \times 4\left(\frac{x}{L}\right)^3 - \frac{9}{L}\right] = 0$$

$$\left[24 \times 3\left(\frac{x}{L}\right)^2 - 16 \times 4\left(\frac{x}{L}\right)^3 - \frac{9}{L}\right] = 0$$

$$x = 0.46L$$

Deflexões em vigas



3) MÁXIMA DEFLEXÃO

$0 <= x < L/2$

$$v_1(x) = \frac{w_0 L^4}{384EI}\left[24\left(\frac{x}{L}\right)^3 - 16\left(\frac{x}{L}\right)^4 - 9\frac{x}{L}\right]$$

$$x = 0.46L$$

$$v_1(0.46L\ ) = 6.56 \times 10^{-6}\frac{w_0 L^4}{EI}$$