UNIVERSIDAI LO
FACULDADE DE ZOOTECNIA E F DE ALIMENTOS - FZEA
DEPARTAMENTO DE CI AS - ZAB0162

Yara Viviane Apª C. da Silva.

Curso: Engenharia de Alimentos / Noturno (FZEA-USP)

MONITORI CULO I

Resoluções dá Cálculo I

Relatório referente à m culo I no primeiro semestre de 2007

Docente: Prof. Dr. Andrés Vercik

Discente: Priscila Missano Florido

N° USP: 5628211

P GA

LISTA 1:

Números Reais. Desigualdades. Funções

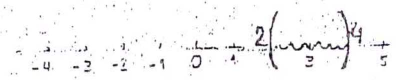
Lista 1: Números Reais. Desigualdades. Funções

1. a) $|x-3|<1$

Reescrever a desigualdade: $-1 < x-3 < 1$

Somar 3 às partes: $2 < x < 4$

Escrever na notação intervalar: $(2,4)$

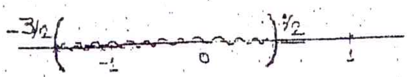
Representar em uma reta: 

b) $|2x+1|<2$

A desigualdade pode ser reescrita como: $|2| * |x+1/2| < 2 \therefore |x+1/2|<1$

Reescrever a desigualdade: $-1 < x + 1/2 < 1$

Subtrair 1/2 de cada parcela: $-3/2 < x < 1/2$

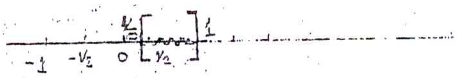
Representar em uma reta: 

c) $|2-3x| \leq 1$

A desigualdade pode ser reescrita como: $|-3| * |x - 2/3| \leq 1 \therefore |x-2/3| \leq 1/3$

Reescrever a desigualdade: $-1/3 \leq x - 2/3 \leq 1/3$

Somar 2/3 às partes: $1/3 \leq x \leq 1$

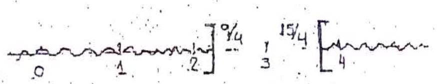
Representar em uma reta: 

d) $|12 - 4x| \geq 3$

A desigualdade pode ser reescrita como: $|-4| * |x-3| \geq 3 \therefore |x-3| \geq 3/4$

Reescrever a desigualdade: $x - 3 \leq -3/4 \quad \cup \quad x - 3 \geq 3/4$

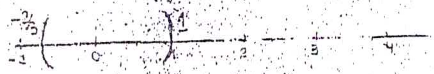
Somar 3 a todas as parcelas: $x \leq 9/4 \quad \cup \quad x \geq 15/4$

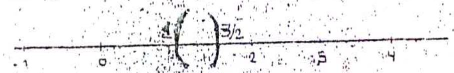
Representar em uma reta: 

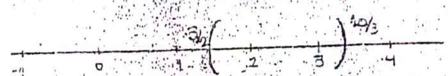
e) $|x-1|+|2x-3|<6$

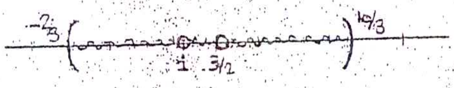
	$x<1$	$1<x<3/2$	$x>3/2$
$\lvert x-1 \rvert$	$\lvert a \rvert = -a$	$\lvert a \rvert = a$	$\lvert a \rvert = a$
$\lvert 2x-3 \rvert$	$\lvert a \rvert = -a$	$\lvert a \rvert = -a$	$\lvert a \rvert = a$

... $x > -2/3 \rightarrow$ (I) $x > -2/3 \wedge x < 1$

I. $x > -2/3 \wedge x < 1$: 

II. $x > -4 \wedge 1 < x < 3/2$: 

III. $x < 10/3 \wedge x > 3/2$: 

Solução I ∪ II ∪ III 

2. Para ser função de A em B todos os componentes do domínio de A devem possuir uma única imagem em B.

a) $f(1) = a \quad f(2) = b \quad f(3) = c$

Não caracteriza uma função pois o elemento 4 não possui imagem.

b) $f(1) = c \quad f(2) = c \quad f(3) = c \quad f(4) = c$

É uma função de A em B.

c) $f(1) = a \quad f(2) = c \quad f(3) = c \quad f(4) = d \quad f(1) = c \quad f(4) = a$

Não caracteriza função pois os elementos 1 e 4 possuem mais de uma imagem.

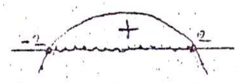
3. a) $f(x) = \sqrt{x}$

Condição de Existência: $x \geq 0$ ∴ Dom: $\{x \in \mathbb{R} / x \geq 0\}$

b) $f(x) = 1/x$

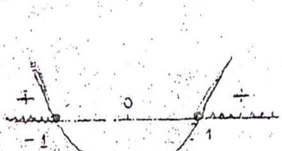
Condição de Existência: $x \neq 0$ ∴ Dom: $\{x \in \mathbb{R} / x \neq 0\}$

c) $f(x) = \dfrac{\sqrt{4 - x^2}}{2 - 3x}$

Condição de Existência: $\begin{cases} 4 - x^2 \geq 0 \\ 2 - 3x \neq 0 \rightarrow x \neq 2/3 \end{cases}$  $\rightarrow -2 \leq x \leq 2$

DOM: $\{x \in \mathbb{R} / -2 \leq x \leq 2 \wedge x \neq 2/3\}$

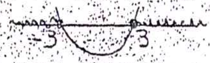
d) $f(x) = \sqrt{x^2 - 1}$

Condição de Existência: $x^2 - 1 \geq 0$  $\rightarrow x \leq -1$ ou $x \geq 1$

DOM: $\{x \in \mathbb{R} / x \leq -1 \cup x \geq 1\}$

e) $f(x) = \dfrac{2x}{x + 4}$

f) $f(x)=\dfrac{x^{3/4}-1}{(x^2-9)^{3/2}-1}$ Condição de Existência $\sqrt[4]{x^3}$ $x \geq 0$

$\sqrt[2]{(x^2-9)^3}$ $x^2-9 \geq 0$  $x \leq -3$ ou $x \geq 3$

$x^2-9 \neq 1$ $x \neq \pm\sqrt{10}$ DOM: $\{ x \in \mathbb{R} / \ x \geq 3, \ x \neq \sqrt{10} \}$

g) $f(x)=\dfrac{1}{\sqrt{x^2-4}}$ Condição de Existência $x^2-4>0$

 $x<-2$ ou $x>2$

DOM: $\{ x \in \mathbb{R} / \ x<-2 \cup x>2 \}$

4. a) $f(x)=x^4, \ -\infty < x < \infty$

Qualquer número elevado a quarta potência resulta em um valor positivo. Logo: Imagem $\mathbb{R} \geq 0$.

b) $f(x)=1+x^2, \ -\infty < x < \infty$

Como x está elevado ao quadrado sempre resultará em um valor positivo. O menor valor assumido por f(x) será 1. Logo:

Imagem $\mathbb{R} \geq 1$.

c) $f(x)=\sqrt{x-1}, \quad x \geq 1$

O mínimo valor que f(x) pode assumir é 0 (zero) dada a condição de existência. Logo: Imagem $\mathbb{R} \geq 0$.

d) $f(x)=1/x, \quad x \neq 0$

f(x) pode assumir qualquer valor desde que $x \neq 0$. logo:

Imagem $\mathbb{R}-\{0\}$

e) $f(x)=1/\sqrt{x}, \quad x>0$

Pela condição de existência f(x) pode assumir valores para os quais $x>0$. Logo: Imagem $\mathbb{R}>0$.

f) $f(x)=\dfrac{2}{3+x^2}$

Como x está elevado ao quadrado sempre resultará em um valor positivo. O menor valor que f(x) pode assumir é 2/3

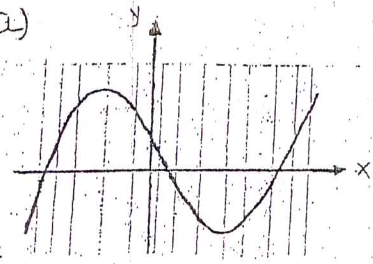
Logo: Imagem $\mathbb{R} \geq 2/3$

5. Para verificar se a curva dada é gráfico de uma função $y = f(x)$ devemos realizar o Teste da Reta Vertical.

"Uma curva no plano XY é o Gráfico de alguma função f se e somente se nenhuma reta vertical intercepta a ~~curva~~ curva mais de uma vez (Anton)."

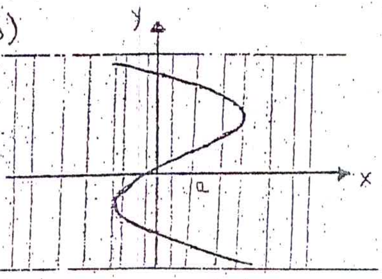
A explicação para este fato é que não existe uma função f que possa atribuir mais de um valor para um mesmo ponto do domínio.

a)



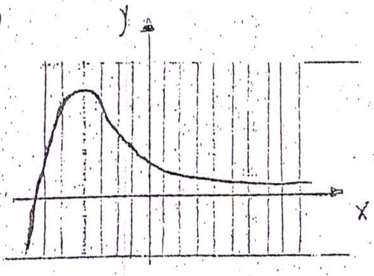
Todas as retas verticais interceptam a curva somente uma vez. É o gráfico de uma função.

b)



Não é função pois para um determinado a em x, existe mais de um correspondente em y.

c)



Todas as retas verticais interceptam a curva somente uma vez. É o gráfico de uma função.

6. Para uma função ser injetora a seguinte condição deve ser satisfeita: $f(x_1) \neq f(x_2)$ sempre que $x_1 \neq x_2$. Realizar o Teste da Reta Horizontal.

a) A função é injetora pois cada valor de x assume um único correspondente em y que não se repete para outro valor de x

b) Não é função injetora pois não se verifica $f(x_1) \neq f(x_2)$, $\forall x_1 \neq x_2$.

c) A curva apresentada não é o gráfico de uma função.

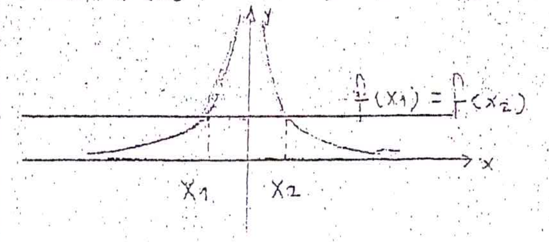
d) A função é injetora.

1. Para $f(x)$ possuir função inversa deve atender às condições:

- $f(x_1) \neq f(x_2)$, $\forall x_1 \neq x_2$
- $Dom\ f^{-1}(x) = Imagem\ f(x)$ e $Dom\ f(x) = Imagem\ f^{-1}(x)$

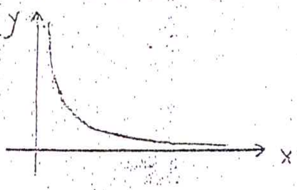
Se as condições forem satisfeitas escrevemos $y = f(x)$, resolvemos esta equação para x em termos de y. Trocar x por y e obter $y = f^{-1}(x)$

a) $f(x) = 1/x^2$, $x \neq 0$



Não possui função inversa pois não cumpre $f(x_1) \neq f(x_2)$, $\forall x_1 \neq x_2$

b) $f(x) = 1/x^2$, $x > 0$



A função possui inversa: $y = 1/x^2$

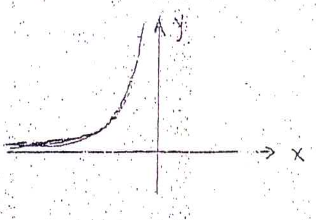
Isolar x: $x^2 = 1/y$ $\therefore$ $x = 1/\sqrt{y}$

$f^{-1}(x) = 1/\sqrt{x}$

$Dom\ f^{-1}(x) = Imagem\ f(x)$: $Dom\ f^{-1}(x) \{x \in \mathbb{R} / x > 0\}$

Contra-Domínio : $\mathbb{R} > 0$

c) $f(x) = 1/x^2$, $x < 0$

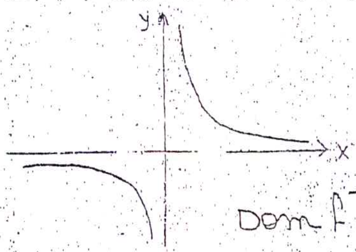


$f^{-1}(x) = 1/\sqrt{x}$ (ver letra b)

$Dom\ f^{-1}(x) : \{x \in \mathbb{R} / x > 0\}$

Contra-Domínio : $\mathbb{R} > 0$

d) $f(x) = 1/x^3$. $x \neq 0$



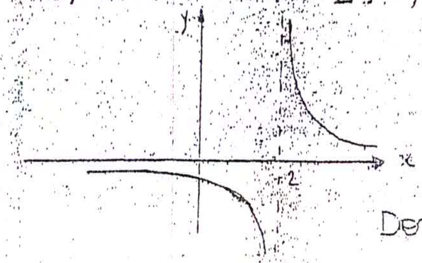
$y = 1/x^3$ $\therefore$ $x^3 = 1/y$

$f^{-1}(x) = 1/\sqrt[3]{x}$

$Dom\ f^{-1}(x)$ $\{x \in \mathbb{R} / x \neq 0\}$

Contra-Domínio : $\mathbb{R} - \{0\}$

e) $f(x) = 1/(x-2)$, $x \neq 2$



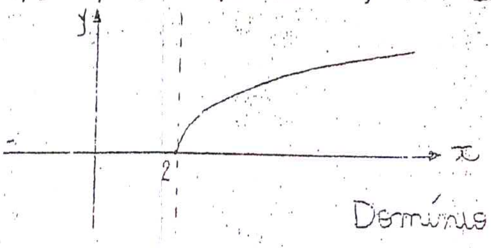
$y = \frac{1}{(x-2)} \rightarrow x - 2 = \frac{1}{y} \therefore x = \frac{1}{y} + 2$

$f^{-1}(x) = 1/x + 2$

Domínio:

Contra Domínio:

f) $f(x) = \sqrt{x-2}$; $x \geq 2$



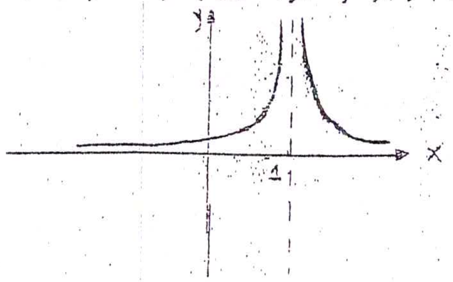
$y = \sqrt{x-2} \rightarrow y^2 = x - 2 \therefore x = y^2 + 2$

$f^{-1}(x) = x^2 + 2$

Domínio:

Contra Domínio:

8. $f(x) = 1/(x-1)^2$, $x > 1$



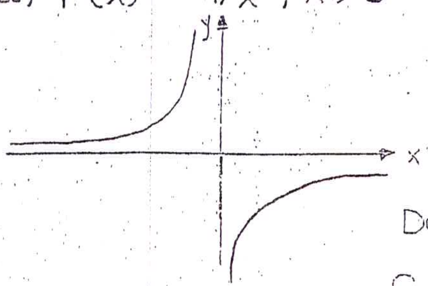
$y = 1/(x-1)^2 \rightarrow (x-1)^2 = 1/y \therefore x = \frac{1}{\sqrt{y}} + 1$

$f^{-1}(x) = 1/\sqrt{x} + 1$

Domínio $f^{-1}(x)$:

Contra Domínio:

9. a) $f(x) = -1/x$, $x > 0$



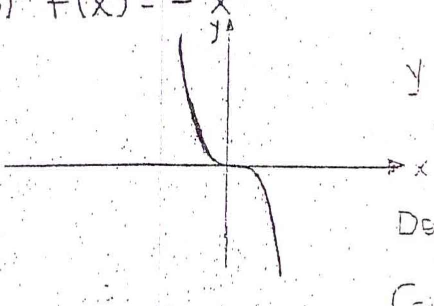
$y = -1/x \rightarrow x = -\frac{1}{y}$

$f^{-1}(x) = -\frac{1}{x}$

Domínio $f^{-1}(x)$:

Contra Domínio $f^{-1}(x)$:

b) $f(x) = -x^3$



$y = -x^3 \rightarrow x^3 = -y \therefore x = \sqrt[3]{-y}$

$f^{-1}(x) = -\sqrt[3]{x}$

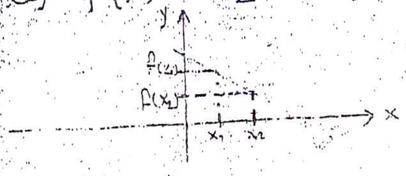
Domínio $f^{-1}(x)$:

Contra-Domínio:

10. Função Crescente: $f(x_1) < f(x_2) \Rightarrow x_1 < x_2$

Função Decrescente: $f(x_1) > f(x_2) \Rightarrow x_1 < x_2$

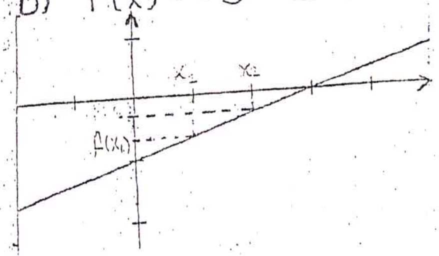
a) $f(x) = -2x + 7$



$x_2 > x_1 \rightarrow f(x_2) < f(x_1)$

A função é decrescente: $(-\infty, +\infty)$

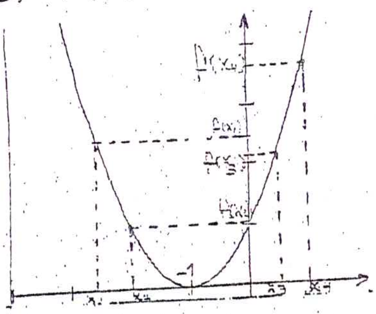
b) $f(x) = x/3 - 1$



$x_2 > x_1 \rightarrow f(x_2) > f(x_1)$

A função é crescente: $(-\infty, +\infty)$

c) $f(x) = (x+1)^2$



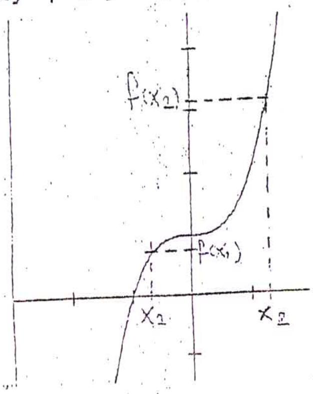
$x_2 > x_1 \rightarrow f(x_2) < f(x_1)$

A função é decrescente: $(-\infty, -1)$

$x_4 > x_3 \rightarrow f(x_4) > f(x_3)$

A função é crescente: $(-1, +\infty)$

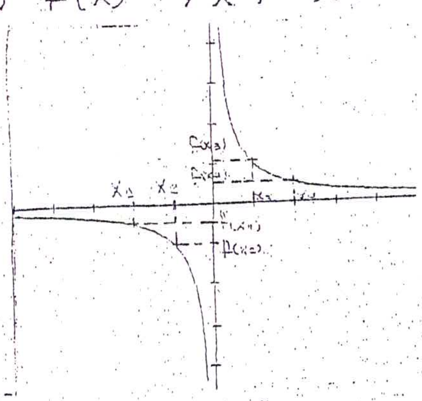
d) $f(x) = x^3 + 1$



$x_2 > x_1 \rightarrow f(x_2) > f(x_1)$

A função é crescente: $(-\infty, +\infty)$

e) $f(x) = 1/x, \quad x \neq 0$



$x_2 > x_1 \rightarrow f(x_2) < f(x_1)$

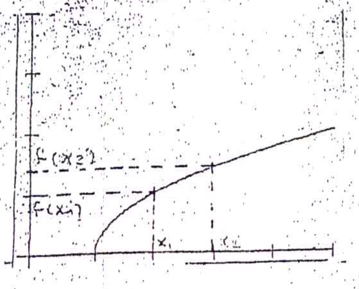
A função é decrescente: $(-\infty, 0)$

$x_4 > x_3 \rightarrow f(x_4) < f(x_3)$

A função é decrescente: $(0, +\infty)$

$\rightarrow$ Decrescente: $(-\infty, 0) \cup (0, +\infty)$

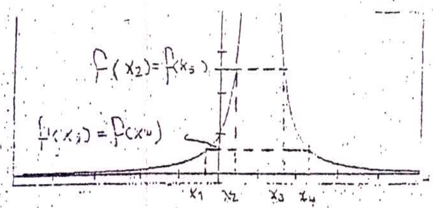
f) $f(x) = \sqrt{x-1}$, $x \geq 1$



$x_2 > x_1 \rightarrow f(x_2) > f(x_1)$

A função é crescente: $(1, +\infty)$

g) $f(x) = 1/(x-1)^2$, $x \neq 1$

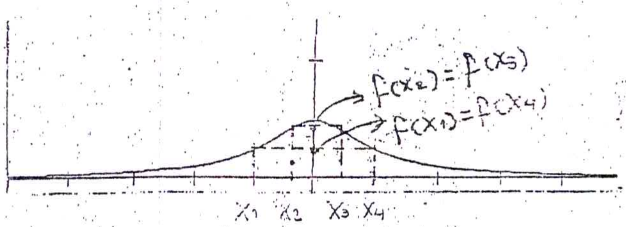


$x_2 > x_1 \longrightarrow f(x_2) > f(x_1)$

A função é crescente: $(-\infty, 1)$

$x_4 > x_3 \longrightarrow f(x_4) < f(x_3)$ ∴ Decrescente $(1, +\infty)$

h) $f(x) = 1/1+x^2$



$x_2 > x_1 \rightarrow f(x_2) > f(x_1)$

A função é crescente: $(-\infty, 0)$

$x_4 > x_3 \rightarrow f(x_4) < f(x_3)$

A função é decrescente: $(0, +\infty)$

11. Função Par: $f(x) = f(-x)$ → Simetria ao redor do eixo y

Função Ímpar: $f(x) = -f(-x)$ → Simetria em torno da origem

a) $f(x) = 2x$

$f(-x) = 2(-x) = -2x$ ∴ $f(x) = -f(-x)$

A função é ímpar com simetria ao redor da origem.

b) $f(x) = 2x - 1$

$f(-x) = 2(-x) - 1 = -2x - 1$ → A função não é par nem ímpar

c) $f(x) = x^4 - 3x^2 + 1$

$f(-x) = (-x)^4 - 3(-x)^2 + 1 = x^4 - 3x^2 + 1$ ∴ $f(x) = f(-x)$

A função é par com simetria em torno do eixo y

d) $f(x) = x^3 - 3x + 1$ $\quad -x^3 + x + 1$

$f(-x) = (-x)^3 - 3(-x) + 1 = -x^3 + 3x + 1$ → A função não é par nem ímpar

f) $f(x) = x/x^2+1$

$f(-x) = -x/(-x)^2+1 = -\frac{x}{x^2+1}$ $\therefore f(x) = -f(-x)$

A função é ímpar com simetria em torno da origem.

g) $f(x) = x^2/x^2+1$

$f(-x) = (-x)^2/(-x)^2+1 = x^2/x^2+1$ $\therefore f(x) = f(-x)$

A função é par com simetria em torno do eixo y.

h) $f(x) = 1-|x|$

$f(-x) = 1-|-x| = 1-|-1||x| = 1-|x|$ $\therefore f(x) = f(-x)$

A função é par com simetria em torno do eixo y.

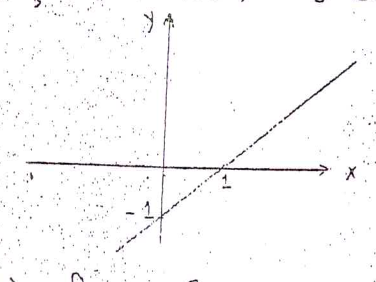
i) $f(x) = x/|x|+1$

$f(-x) = -x/|-x|+1 = -\frac{x}{|x|+1}$ $\therefore f(x) = -f(-x)$

A função é ímpar com simetria ao redor da origem.

12. a) $f(x) = x-1$

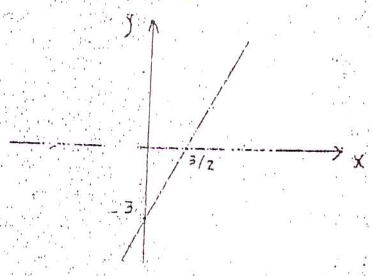
A inclinação é dada pelo coeficiente angular. Se a função estiver na forma $y = ax+b$ a corresponde ao coeficiente angular (inclinação da reta) e b corresponde ao coeficiente linear (intersecção com eixo y).



Inclinação: $m = 1$

Intersecção com o eixo y: $(0, -1)$

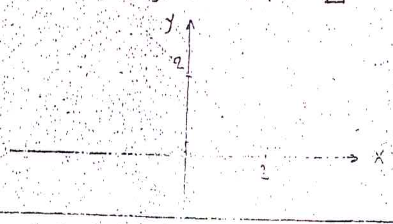
b) $f(x) = 2x-3$



Inclinação: $m = 2$

Intersecção com o eixo y: $(0, -3)$

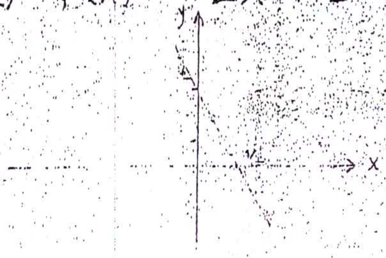
c) $f(x) = -x+2$



Inclinação: $m = -1$

Intersecção com o eixo y: $(0, 2)$

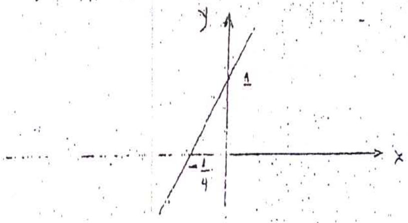
d) $f(x) = -2x + 1$



Inclinação: $m = -2$

Intersecção com eixo y: $(0, 1)$

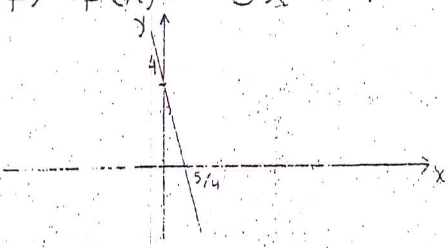
e) $f(x) = 4x + 1$



Inclinação: $m = 4$

Intersecção com eixo y: $(0, 1)$

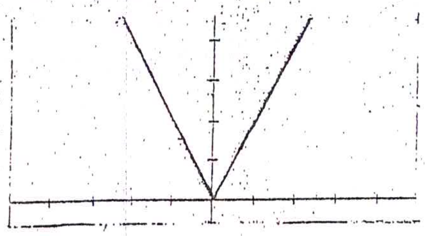
f) $f(x) = -5x + 4$



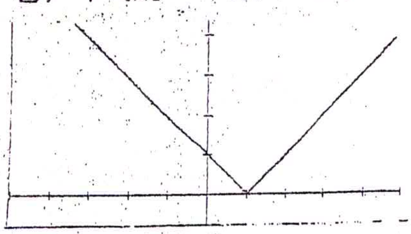
Inclinação: $m = -5$

Intersecção com eixo y: $(0, 4)$

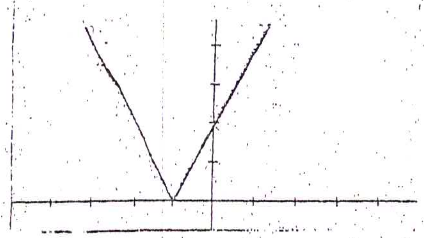
13. a) $f(x) = 2|x|$



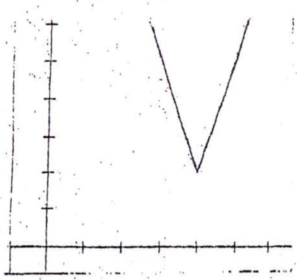
b) $f(x) = |x - 1|$



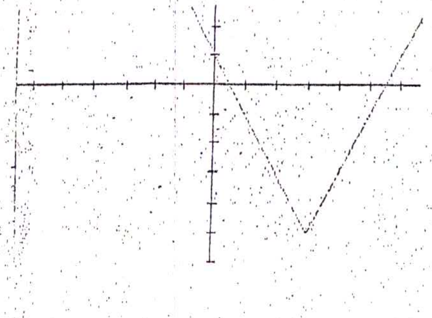
c) $f(x) = |2x + 2|$



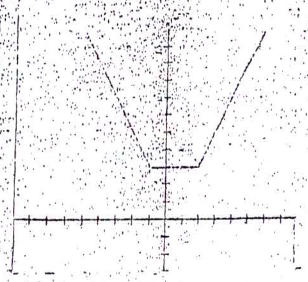
d) $f(x) = 3|x - 4| + 2$



e) $f(x) = |6 - 2x| - 5$



f) $f(x) = |x+1| + |x-2|$

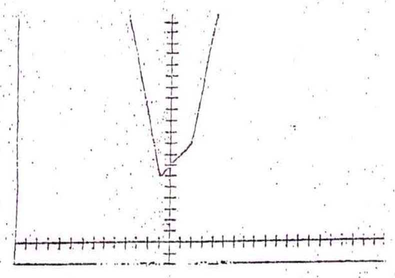


$x < -1 : f(x) = -x-1-x+2 = -2x+1$

$-1 < x < 2 : f(x) = x+1-x+2 = 3$

$x > 2 : f(x) = x+1+x-2 = 2x-1$

g) $f(x) = 2|x-2| + 3|x+1|$



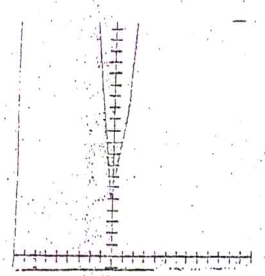
$x < -1 : 2(-x+2) + 3(-x-1) = -2x+4-3x-3$

$f(x) = -5x+1$

$-1 < x < 2 : f(x) = -2x+4+3x+3 = x+7$

$x > 2 : f(x) = 2x-4+3x+3 = 5x-1$

h) $f(x) = |3x \mp 4| + 2|4x+1|$

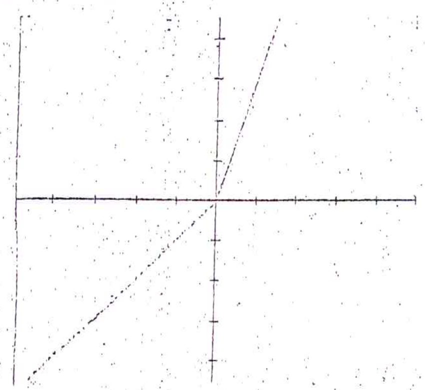


$x < -1/4 : f(x) = -3x+4-8x-2 = -11x+2$

$-1/4 < x < 4/3 : f(x) = -3x+4+8x+2 = 5x+6$

$x > 4/3 : f(x) = 3x-4+8x+2 = 11x-2$

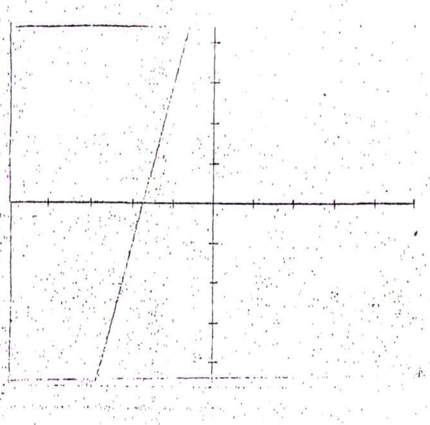
i) $f(x) = 2x + |x|$



$x < 0 : f(x) = 2x - x = x$

$x > 0 : f(x) = 2x + x = 3x$

j) $f(x) = 6x+3+|x-1|+|3-x|$



$x < 1 : f(x) = 6x+3-x+1-3+x = 6x+1$

$1 < x < 3 : f(x) = 6x+3+x-1-3+x = 8x-1$

$x > 3 : f(x) = 6x+3+x-1+3-x = 6x+5$

14. Para encontrar a função resolver

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

a) (1,2) e (2,5)

$$\begin{vmatrix} x & y & 1 \\ 1 & 2 & 1 \\ 2 & 5 & 1 \end{vmatrix} = x(2-5) - y(1-2) + 1(5-4) = 0$$

$$-3x + y + 1 = 0 \therefore y = 3x - 1$$

b) (2,4) e (-1,2)

$$\begin{vmatrix} x & y & 1 \\ 2 & 4 & 1 \\ -1 & 2 & 1 \end{vmatrix} = x(4-2) - y(2+1) + (4+4) = 0$$

$$2x - 3y + 8 = 0 \therefore y = \frac{2}{3}x + \frac{8}{3}$$

c) (0,-2) e (-1,1)

$$\begin{vmatrix} x & y & 1 \\ 0 & -2 & 1 \\ -1 & 1 & 1 \end{vmatrix} = x(-2-1) - y(1) + (0-2) = 0$$

$$-3x - y - 2 = 0 \therefore y = -3x - 2$$

15. (1,3) (2,4) (-1,2)

$$\begin{vmatrix} x & y & 1 \\ 1 & 3 & 1 \\ 2 & 4 & 1 \end{vmatrix} = x(3-4) - y(1-2) + (4-6) = 0$$

$$-x + y - 2 = 0 \therefore y = x + 2$$

Testando o ponto (-1,2) : $2 = (-1) + 2 \rightarrow \underline{2 \neq 1}$

Logo: Os pontos não estão alinhados

16. a) $m = -3$ (0,2)

$y = ax + b$ com a = inclinação e b = intersecção com eixo y

$y = -3x + 2$

b) $m = 1/2$ (0,-1) : $y = ax + b \therefore y = \frac{1}{2}x - 1$

c) $m = -1/2$ (0,0) : $y = ax + b \therefore y = -\frac{1}{2}x$

e) $m = 0$ $(0, -5/4)$ $\rightarrow y = ax+b \therefore y = -5/4$

f) Reta vertical passando por $(-2, 1/3) \therefore X = -2$

g) $m = -2$ $(0, 3)$ $\rightarrow y = ax+b \therefore y = -2X+3$

h) $(2,1)$ e $(-1,5)$

$$\begin{vmatrix} x & y & 1 \\ 2 & 1 & 1 \\ -1 & 5 & 1 \end{vmatrix} = x(1-5) - y(2+1) + (10+1) = 0$$

$$-4x - 3y + 11 = 0 \therefore y = -\frac{4}{3}x + \frac{11}{3}$$

i) $(2/3, 1)$ e $(-4/7, 1)$

$$\begin{vmatrix} x & y & 1 \\ 2/3 & 1 & 1 \\ -4/7 & 1 & 1 \end{vmatrix} = x(1-1) - y(2/3 + 4/7) + (2/3 + 4/7) = 0$$

$$y = 1$$

J) $(3/2, -1/2)$ paralela a $y = -\frac{x}{2} + \frac{3}{2}$

$m = -1/2 \therefore y = -1/2 X + b$

Substituindo $(3/2, -1/2)$: $-1/2 = -1/2(3/2) + b \therefore b = 1/4$

$y = -1/2 X + 1/4$

K) $(0,4)$ perpendicular $y = -1/2 x + 3/2$

$m = -1/_{-1/2} = 2 \therefore y = 2x + b \therefore$ Logo: $y = 2x + 4$

17. taxa variável: 0,09 reais/minutos

taxa fixa mensal: 6,50 reais

Valor mensal da conta = taxa fixa mensal + (taxa variável * tempo total)

$V(t) = 6,50 + 0,09t$

18.

19. $f(x) = x^2 - 3x + 2$

a) Para 1 ser raiz $f(1) = 0$. Verificando: $(1)^2 - 3(1) + 2 = 1 - 3 + 2 = 0$

1 é raiz.

b)

	1	-3	2
1	1	-2	0

$\longrightarrow x - 2 \quad \therefore f(x) = (x-1)(x-2)$

$g(x) = x - 2$

20. a) $-x^2 + 6x + 11$

21\. a) $y = x^2 - 5x - 14$ $\quad x_1 \cdot x_2 = 14$ e $x_1 + x_2 = 5$ $\left\langle \begin{matrix} x_1 = -2 \\ x_2 = 7 \end{matrix} \right.$

$y = (x-7)(x+2)$ $\quad$ Negativo em: $(-2, 7)$

b) $y = 2x^2 - x - 1$: $y = 2\left(x^2 - \frac{1}{2}x - \frac{1}{2}\right)$

$x_1 \cdot x_2 = -1/2$ e $x_1 + x_2 = 1/2$ $\left\langle \begin{matrix} x_1 = -1/2 \\ x_2 = 1 \end{matrix} \right.$ $\therefore y = 2(x-1)(x+1/2)$

Negativo em: $(-1/2, 1)$

c) $y = x^2 + 5/6\, x + 1/6$ $\quad x_1 \cdot x_2 = 1/6$ e $x_1 + x_2 = -5/6$ $\left\langle \begin{matrix} x_1 = -1/2 \\ x_2 = -1/3 \end{matrix} \right.$

$y = (x + 1/3)(x + 1/2)$ $\quad$ Negativo em: $(-1/2, -1/3)$

d) $y = 4x^2 - 9$ $\quad$ Para fatorar diferença de quadrados: $x^2 - y^2 = (x+y)(x-y)$

$y = (2x+3)(2x-3)$ $\quad$ Negativo em: $(-3/2, 3/2)$

e) $y = 1/4\, x^2 - 2x + 3$: $y = 1/4\,(x^2 - 8x + 12)$

$x_1 + x_2 = 8$ e $x_1 \cdot x_2 = 12$ $\left\langle \begin{matrix} x_1 = 2 \\ x_2 = 6 \end{matrix} \right.$ $\therefore y = 1/4\,(x-2)(x-6)$

Negativo em: $(2, 6)$

f) $y = 3x^2 + 5x - 2$: $y = 3\left(x^2 + 5/3\, x - 2/3\right)$

$x_1 + x_2 = -5/3$ e $x_1 \cdot x_2 = -2/3$ $\left\langle \begin{matrix} x_1 = -2 \\ x_2 = 1/3 \end{matrix} \right.$ $\therefore y = 3(x+2)(x - 1/3)$

Negativo em: $(-2, 1/3)$

22\. $f(x) = x^3 + 1$

a) Para -1 ser raiz de $f(x)$, $f(-1) = 0$.

Verificar: $(-1)^3 + 1 = -1 + 1 = 0$ $\therefore -1$ é raiz de $f(x)$

b)

$$\begin{array}{r|rrr|r} & 1 & \overset{-1}{0} & \overset{1}{0} & \overset{-1}{1} \\ \hline -1 & 1 & -1 & 1 & 0 \end{array} \longrightarrow (x^2 - x + 1)(x + 1) = y$$

$g(x) = x^2 - x + 1$

23\. a) $f(x) = x^5 - 3x^4 + 7x^3 + 6x^2 - x - 8$. Reescrever $f(x)$ como:

$f(x) = x^5\left(1 - \frac{3}{x} + \frac{7}{x^2} + \frac{6}{x^3} - \frac{1}{x^4} - \frac{8}{x^5}\right)$

Quando x aumenta sem limite $f(x) \cong x^5$ $\therefore$ f(x) aumenta sem limite

Quando x diminui sem limite $f(x) \cong x^5$ $\therefore$ f(x) diminui sem limite

b) $f(x) = -3x^4 + x^2 - 9$ Reescrever f(x) como: $f(x) = x^4\left(-3 + 1/x^2 - 9/x^4\right)$

Quando x aumenta sem limite $f(x) \cong -3x^4$ $\therefore$ f(x) diminui sem limite

Quando x diminui sem limite $f(x) \cong -3x^4$ $\therefore$ f(x) diminui sem limite

c) $f(x) = -0{,}01x^3 + x^2 + 0{,}1x - 10$ Reescrever $f(x)$ como

$f(x) = x^3(-0{,}01 + 1/x + 0{,}1/x^2 - 10/x^3)$

Quando x aumenta sem limite $f(x) \simeq -0{,}01x^3 \therefore f(x)$ diminui sem limite

Quando x diminui sem limite $f(x) \simeq -0{,}01x^3 \therefore f(x)$ aumenta sem limite

d) $f(x) = 100x^6 - 82x^4 + 31x^2 - 144$ Reescrever $f(x)$ como

$f(x) = x^6(100 - 82/x^2 + 31/x^4 - 144/x^6)$

Quando x aumenta sem limite $f(x) \simeq 100x^6 \therefore f(x)$ aumenta sem limite

Quando x diminui sem limite $f(x) \simeq 100x^6 \therefore f(x)$ aumenta sem limite

24. Para determinar se n é par ou ímpar devemos observar que:
Se o gráfico de $p(x)$ é tangente ao eixo x em $x = r$ (raiz) mas não cruza o eixo x neste ponto, a função é par. Se for tangente e cruzar o eixo x, a função é ímpar.

25. a) $2^4 = 2*2*2*2 = 4*4 = 16$ b) $(-4)^3 = (-4)(-4)(-4) = 16*(-4) = -64$

c) $(0,3)^3 = \frac{3}{10}*\frac{3}{10}*\frac{3}{10} = \frac{27}{1000} = 0{,}027$ d) $4^{1/2} = \sqrt{4} = 2$

e) $4^{-2} = 1/4^2 = 1/16$ f) $(0{,}008)^{1/3} = \sqrt[3]{\frac{8}{1000}} = \frac{2}{10} = 0{,}2$

g) $\left(\frac{1}{3}\right)^{-3} = 3^3 = 27$ h) $\left(\frac{1}{27}\right)^{1/3} = \sqrt[3]{\frac{1}{3^3}} = 1/3$

i) $8^{2/3} = \sqrt[3]{2^3 \cdot 2^3} = 4$ j) $\left(\frac{1}{8}\right)^{-1/3} = \sqrt[3]{8} = 2$

26. a) $f(9) = (9+7)^{1/2} = \sqrt[2]{16} = 4$

b) $f(9) = (9)^{1/2} + 3 = \sqrt{9} + 3 = 6$

c) $f(9) = 4(9)^{1/2} = 4*3 = 12$

d) $f(9) = (4*9)^{-1/2} = 1/\sqrt{36} = 1/6$

e) $f(9) = (1/9)^{3/2} = \sqrt{\frac{1}{9^3}} = \frac{1}{27}$

f) $f(9) = (1/9)^{-3/2} = \sqrt{9^3} = 27$

27. a) $g(f(2)) = g(1/2) = (1/2)^2 + 1 = 5/4$

b) $f(g(2)) = f(2^2+1) = 1/5$

c) $g(g(2)) = g(2^2+1) = 5^2 + 1 = 26$

d) $g(1+f(2)) = g(1 + 1/2) = \left(\frac{3}{2}\right)^2 + 1 = 13/4$

e) $1 + g(f(2)) = 1 + 5/4 = 9/4$

f) $f(g(f(2))) = f(g(1/2)) = f(5/4) = 4/5$

LISTA 2:

Limites

$\sqrt{x}$ -

$$\frac{3-\sqrt{x}}{x-9}$$

$$\frac{3-\sqrt{x}}{(\sqrt{x}-9)(\sqrt{x}+9)} = \frac{3-\sqrt{9}}{}$$

Lista 2: Limites

1. a) $\lim\limits_{x\to -2}(3x-1) = \lim\limits_{x\to -2}(3x) + \lim\limits_{x\to -2}(-1) = (3*(-2)) +* (-1) = -7;$

b) $\lim\limits_{x\to 3}(x^2+2) = \lim\limits_{x\to 3}(x^2) + \lim\limits_{x\to 3} 2 = 3^2+2 = 11;$

c) $\lim\limits_{x\to 4} x = 4;$ d) $\lim\limits_{x\to -3} -x = -(-3) = 3;$ e) $\lim\limits_{x\to 100} 7 = 7;$ f) $\lim\limits_{x\to -1} \pi = \pi;$

g) $\lim\limits_{x\to -1}\dfrac{x+4}{2x+1} = \dfrac{\lim\limits_{x\to -1}(x+4)}{\lim\limits_{x\to -1}(2x+1)} = \dfrac{\lim\limits_{x\to -1} x + \lim\limits_{x\to -1} 4}{\lim\limits_{x\to -1} 2x + \lim\limits_{x\to -1} 1} = \dfrac{-1+4}{2(-1)+1} = \dfrac{3}{-1} = -3;$

h) $\lim\limits_{x\to 5}\dfrac{x+2}{x-4} = \dfrac{\lim\limits_{x\to 5}(x+2)}{\lim\limits_{x\to 5}(x-4)} = \dfrac{\lim\limits_{x\to 5} x + \lim\limits_{x\to 5} 2}{\lim\limits_{x\to 5} x + \lim\limits_{x\to 5} -4} = \dfrac{5+2}{5-4} = \dfrac{7}{1} = 7;$

2. a) $\lim\limits_{x\to -3}\dfrac{(x+3)(x-4)}{(x+3)(x+1)} = \lim\limits_{x\to -3}\dfrac{x-4}{x+1} = \dfrac{\lim\limits_{x\to -3}(x-4)}{\lim\limits_{x\to -3}(x+1)} = \dfrac{\lim\limits_{x\to -3} x + \lim\limits_{x\to -3} -4}{\lim\limits_{x\to -3} x + \lim\limits_{x\to -3} 1} = \dfrac{-3-4}{-3+1} = \dfrac{-7}{-2} = \dfrac{7}{2};$

b) $\lim\limits_{x\to -1}\dfrac{(x+1)(x^2+3)}{(x+1)} = \lim\limits_{x\to -1} x^2+3 = \lim\limits_{x\to -1} x^2 + \lim\limits_{x\to -1} 3 = (-1)^2+3 = 1+3 = 4;$

c) $\lim\limits_{x\to 2}\dfrac{x^2-4}{x-2} = \lim\limits_{x\to 2}\dfrac{(x+2)(x-2)}{x-2} = \lim\limits_{x\to 2}(x+2) = \lim\limits_{x\to 2} x + \lim\limits_{x\to 2} 2 = 2+2 = 4;$

d) $\lim\limits_{x\to 3}\dfrac{2x^3-6x^2+x-3}{x-3} = \lim\limits_{x\to 3}\dfrac{2x^2(x-3)+(x-3)}{x-3} = \lim\limits_{x\to 3} 2x^2+1 = 2(3)^2+1 = 2*9+1 = 19;$

e) $\lim\limits_{r\to 1}\dfrac{r^2-r}{2r^2+5r-7} = \lim\limits_{r\to 1}\dfrac{r(r-1)}{2(r-1)(r+7/2)} = \lim\limits_{r\to 1}\dfrac{r}{2r+7} = \dfrac{\lim\limits_{r\to 1} r}{\lim\limits_{r\to 1}(2r+7)} = \dfrac{1}{2+7} = \dfrac{1}{9};$

$2r-2 \Rightarrow 2r^2 - 2r + 7r - 7$

f) $\lim\limits_{k\to 4}\dfrac{k^2-16}{\sqrt{k}-2} = \lim\limits_{k\to 4}\dfrac{(k+4)(k-4)}{\sqrt{k}-2} = \lim\limits_{k\to 4}\dfrac{(k+4)(\sqrt{k}+2)(\sqrt{k}-2)}{\sqrt{k}-2} = \lim\limits_{k\to 4}(k+4)(\sqrt{k}+2) = 8*4 = 32;$

g) $\lim\limits_{x\to 25}\dfrac{\sqrt{x}-5}{x-25} = \lim\limits_{x\to 25}\dfrac{\sqrt{x}-5}{(\sqrt{x}+5)(\sqrt{x}-5)} = \lim\limits_{x\to 25}\dfrac{1}{\sqrt{x}+5} = \dfrac{\lim\limits_{x\to 25} 1}{\lim\limits_{x\to 25}\sqrt{x} + \lim\limits_{x\to 25} 5} = \dfrac{1}{5+5} = \dfrac{1}{10};$

h) $\lim\limits_{h\to 0}\dfrac{(x+h)^2-x^2}{h} = \lim\limits_{h\to 0}\dfrac{x^2+2xh+h^2-x^2}{h} = \lim\limits_{h\to 0}\dfrac{2xh+h^2}{h} = \lim\limits_{h\to 0} 2x+h = 2x;$

i) $\lim\limits_{h\to 0}\dfrac{(x+h)^3-x^3}{h} = \lim\limits_{h\to 0}\dfrac{x^3+3x^2h+3xh^2+h^3-x^3}{h} = \lim\limits_{h\to 0} 3x^2+3xh+h^2 = 3x^2;$

j) $\lim\limits_{h\to -2}\dfrac{h^3+8}{h+2} = \lim\limits_{h\to -2}\dfrac{(h+2)(h^2-2h+4)}{h+2} = \lim\limits_{h\to -2} h^2-2h+4 = 4+4+4 = 12;$

k) $\lim_{z\to -2}\frac{z^2-4}{z^2-2z-8}=\lim_{z\to -2}\frac{z^2-4}{(z-4)(z+2)}=\lim_{z\to -2}\frac{1}{z+2}=\frac{1}{0}\Rightarrow$ *não existe o* lim *ite* ;

l) $\lim_{z\to 5}\frac{z-5}{z^2-10z+25}=\lim_{z\to 5}\frac{z-5}{(z-5)(z-5)}=\lim_{z\to 5}\frac{1}{z-5}=\frac{1}{0}\Rightarrow$ *não existe o* lim *ite* ;

3. a) $\begin{cases}\lim\limits_{x\to 4-}\frac{|x-4|}{x-4}=\lim\limits_{x\to 4-}\frac{-(x-4)}{x-4}=-1;\\ \lim\limits_{x\to 4+}\frac{|x-4|}{x-4}=\lim\limits_{x\to 4+}\frac{x-4}{x-4}=1;\end{cases}\Rightarrow \lim\limits_{x\to 4}\frac{|x-4|}{x-4}\Rightarrow NE$;

b) $\begin{cases}\lim\limits_{x\to -5-}\frac{x-5}{|x-5|}=\lim\limits_{x\to -5-}\frac{x-5}{-(x-5)}=-1;\\ \lim\limits_{x\to -5+}\frac{x-5}{|x-5|}=\lim\limits_{x\to -5+}\frac{x-5}{x-5}=1;\end{cases}\Rightarrow \lim\limits_{x\to -5}\frac{x-5}{|x-5|}\Rightarrow NE$;

c) $\begin{cases}\lim\limits_{x\to -6-}\sqrt{x+6}+x\Rightarrow NE;\\ \lim\limits_{x\to -6+}\sqrt{x+6}+x=\lim\limits_{x\to -6+}\sqrt{0}+(-6)=-6;\end{cases}\Rightarrow \lim\limits_{x\to -6}\sqrt{x+6}+x\Rightarrow NE$;

d) $\begin{cases}\lim\limits_{x\to 5/2-}\sqrt{5-2x}-x^2=\lim\limits_{x\to 5/2-}\sqrt{0}-\left(\frac{5}{2}\right)^2=\frac{-25}{4};\\ \lim\limits_{x\to 5/2+}\sqrt{5-2x}-x^2\Rightarrow NE;\end{cases}\Rightarrow \lim\limits_{x\to 4}\sqrt{5-2x}-x^2\Rightarrow NE$;

?

e) $\begin{cases}\lim\limits_{x\to 0-}\frac{1}{x^3}=-\infty;\\ \lim\limits_{x\to 0+}\frac{1}{x^3}=+\infty;\end{cases}\Rightarrow \lim\limits_{x\to 0}\frac{1}{x^3}\Rightarrow$ *não existe*; f) $\begin{cases}\lim\limits_{x\to 8-}\frac{1}{x-8}\Rightarrow NE;\\ \lim\limits_{x\to 8+}\frac{1}{x-8}\Rightarrow NE;\end{cases}\Rightarrow \lim\limits_{x\to 8}\frac{1}{x-8}\Rightarrow NE$;

$x\to 8^+$ $x\to 8^-$

4. a) $\begin{cases}\lim\limits_{x\to 2-}f(x)=3\\ \lim\limits_{x\to 2+}f(x)=1\\ \lim\limits_{x\to 2}f(x)\to NE\end{cases}$ $\begin{cases}\lim\limits_{x\to 0-}f(x)=2\\ \lim\limits_{x\to 0+}f(x)=2\\ \lim\limits_{x\to 0}f(x)=2\end{cases}$

b) $\begin{cases}\lim\limits_{x\to 2-}f(x)=4\\ \lim\limits_{x\to 2+}f(x)=4\\ \lim\limits_{x\to 2}f(x)=4\end{cases}$ $\begin{cases}\lim\limits_{x\to 0-}f(x)=1\\ \lim\limits_{x\to 0+}f(x)=1\\ \lim\limits_{x\to 0}f(x)=1\end{cases}$

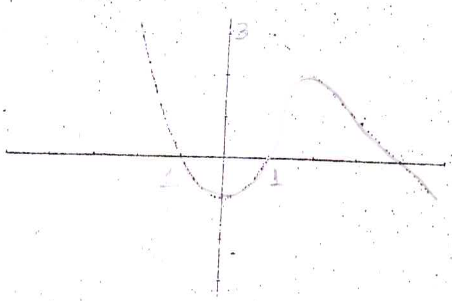
c) $\begin{cases}\lim\limits_{x\to 2-}f(x)=1\\ \lim\limits_{x\to 2+}f(x)=1\\ \lim\limits_{x\to 2}f(x)=1\end{cases}$ $\begin{cases}\lim\limits_{x\to 0-}f(x)=3\\ \lim\limits_{x\to 0+}f(x)=3\\ \lim\limits_{x\to 0}f(x)=3\end{cases}$

d) $\begin{cases}\lim\limits_{x\to 2-}f(x)=1\\ \lim\limits_{x\to 2+}f(x)=2\\ \lim\limits_{x\to 2}f(x)\to NE\end{cases}$ $\begin{cases}\lim\limits_{x\to 0-}f(x)=-1\\ \lim\limits_{x\to 0+}f(x)=-1\\ \lim\limits_{x\to 0}f(x)=-1\end{cases}$

e) $\begin{cases}\lim\limits_{x\to 2-}f(x)=-\infty\\ \lim\limits_{x\to 2+}f(x)=+\infty\\ \lim\limits_{x\to 2}f(x)\to NE\end{cases}$ $\begin{cases}\lim\limits_{x\to 0-}f(x)\to NE\\ \lim\limits_{x\to 0+}f(x)=0\\ \lim\limits_{x\to 0}f(x)\to NE\end{cases}$

f) $\begin{cases}\lim\limits_{x\to 2-}f(x)=-1\\ \lim\limits_{x\to 2+}f(x)=-1\\ \lim\limits_{x\to 2}f(x)=-1\end{cases}$ $\begin{cases}\lim\limits_{x\to 0-}f(x)\to NE\\ \lim\limits_{x\to 0+}f(x)=1\\ \lim\limits_{x\to 0}f(x)\to NE\end{cases}$

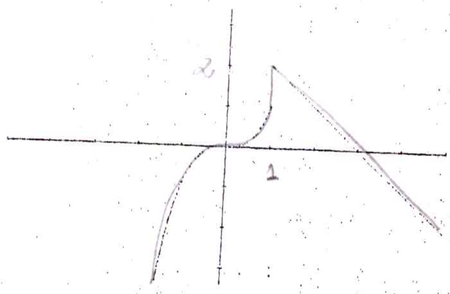
5. a)



$$\begin{cases} \lim\limits_{x \to 1-} f(x) = 0 \\ \lim\limits_{x \to 1+} f(x) = 3 \\ \lim\limits_{x \to 1} f(x) \to NE \end{cases}$$

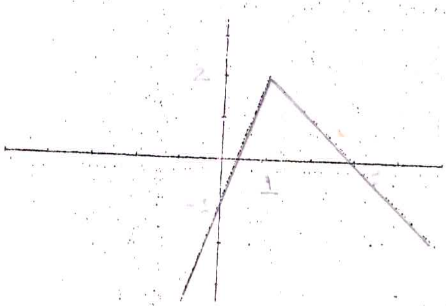
La Bella
Propuestas
de Invierno

b)



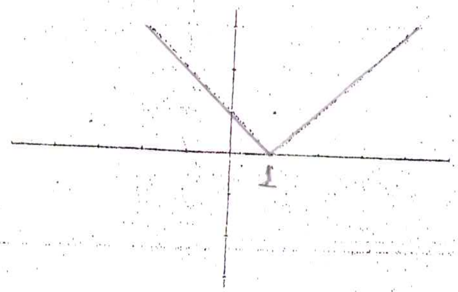
$$\begin{cases} \lim\limits_{x \to 1-} f(x) = 1 \\ \lim\limits_{x \to 1+} f(x) = 2 \\ \lim\limits_{x \to 1} f(x) \to NE \end{cases}$$

c)



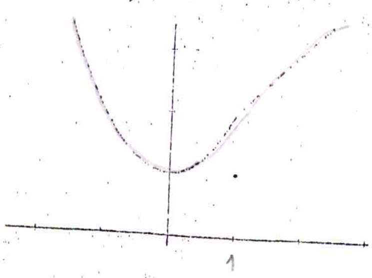
$$\begin{cases} \lim\limits_{x \to 1-} f(x) = 2 \\ \lim\limits_{x \to 1+} f(x) = 2 \\ \lim\limits_{x \to 1} f(x) = 2 \end{cases}$$

d)



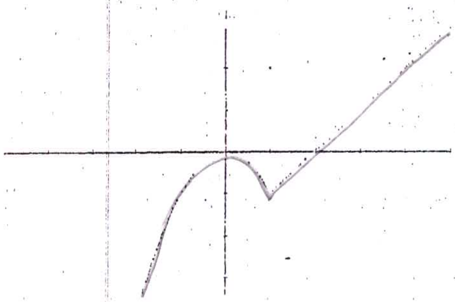
$$\begin{cases} \lim\limits_{x \to 1-} f(x) = 0 \\ \lim\limits_{x \to 1+} f(x) = 0 \\ \lim\limits_{x \to 1} f(x) = 0 \end{cases}$$

e)



$$\begin{cases} \lim\limits_{x \to 1-} f(x) = 2 \\ \lim\limits_{x \to 1+} f(x) = 2 \\ \lim\limits_{x \to 1} f(x) = 2 \end{cases}$$

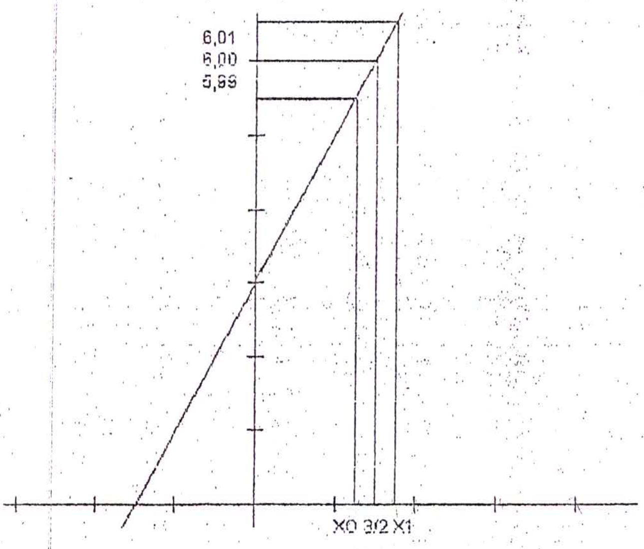
d)



$$\begin{cases} \lim_{x \to 1^-} f(x) = -1 \\ \lim_{x \to 1^+} f(x) = -1 \\ \lim_{x \to 1} f(x) = -1 \end{cases}$$

6. Gráficos fora de escala para melhor visualização

a)



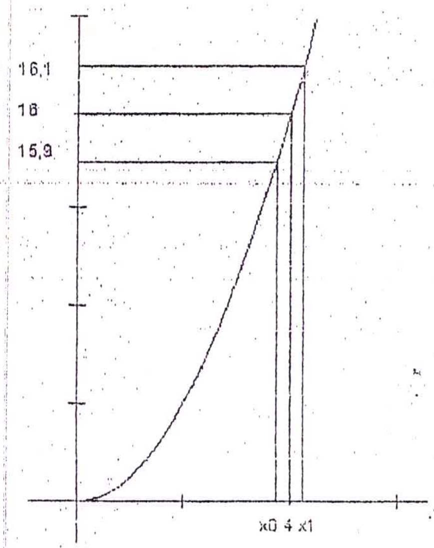
+ 0,005   - 0,05

$X_0=1,495$ e $X_1=1,505$

$\delta_1=\delta_2= 0,005$

Logo $\delta max = 0,005$

b)



$X_0=3,98748$ e $X_1=4,01248$

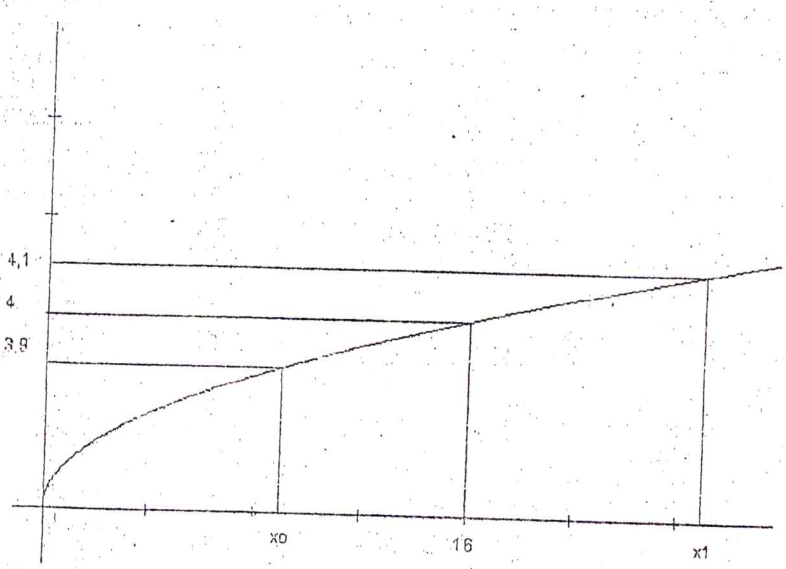
$\delta_1= 0,01252$

$\delta_2= 0,01248$

Logo $\delta max = 0,01248$

0,01248

c)



X_0=15,21 e X_1=16,81

δ_1= 0,79

δ_2= 0,81

Logo δmax = 0,79

7. a) Se $\begin{cases} |x-5|<\delta \Rightarrow |(-4x)-(-20)|<\varepsilon \\ |x-5|<\delta \Rightarrow |-4|\,|x-5|<\varepsilon \\ |x-5|<\delta \Rightarrow |x-5|<\dfrac{\varepsilon}{4} \end{cases} \Rightarrow \delta = \dfrac{\varepsilon}{4};$

b) Se $\begin{cases} |x-4|<\delta \Rightarrow |(15-8x)-(-17)|<\varepsilon \\ |x-4|<\delta \Rightarrow |-8x+32|<\varepsilon \\ |x-4|<\delta \Rightarrow |-8|\,|x-4|<\varepsilon \\ |x-4|<\delta \Rightarrow |x-4|<\dfrac{\varepsilon}{8} \end{cases} \Rightarrow \delta = \dfrac{\varepsilon}{8};$

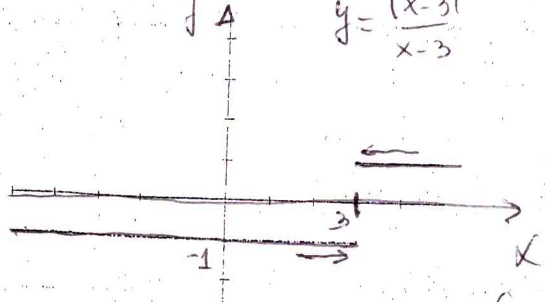
c)Se $\begin{cases} |x-3|<\delta \Rightarrow |(5)-(5)|<\varepsilon \\ |x-3|<\delta \Rightarrow |0|<\varepsilon \end{cases} \Rightarrow \varepsilon > 0$

Para qualquer $\delta \rightarrow \varepsilon > 0.$

Verifica – se a existência do lim *para qualquer valor de* ε

7 a)

$y = \dfrac{|x-3|}{x-3}$



$\begin{cases} \lim\limits_{x\to 3-} f(x) = -1 \\ \lim\limits_{x\to 3+} f(x) = 1 \\ \lim\limits_{x\to 3} f(x) \rightarrow NE \end{cases}$ $\neq$

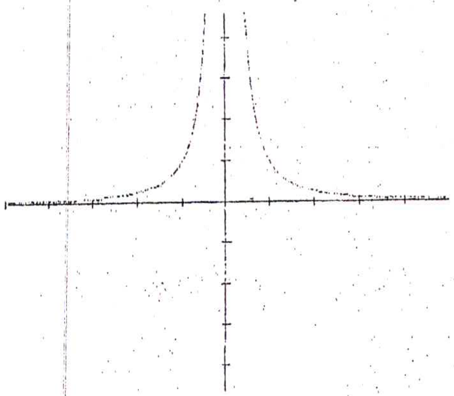
$y = \dfrac{|x-3|}{x-3} = \begin{cases} 1, & \text{se } x>3 \\ -1, & \text{se } x<3 \end{cases}$

$|x-3| \geq 0$

$|x-3| = x-3$

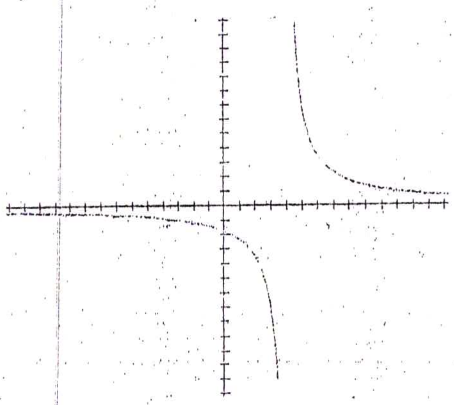
$\dfrac{|x-3|}{x-3} = \dfrac{x-3}{x-3} = 1$

b)



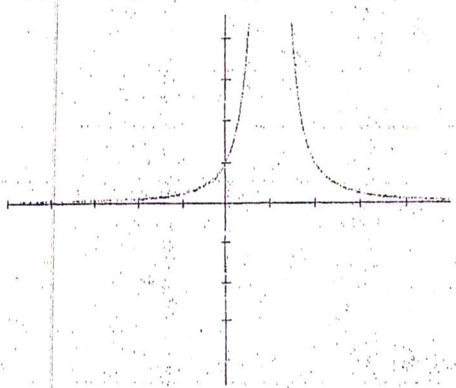
$$\begin{cases} \lim\limits_{x\to 0-} f(x) = +\infty \\ \lim\limits_{x\to 0+} f(x) = +\infty \Rightarrow f(x)\ \textit{cresce sem}\ \lim \\ \lim\limits_{x\to 0} f(x) = +\infty \end{cases}$$

c)



$$\begin{cases} \lim\limits_{x\to 4-} f(x) = -\infty \\ \lim\limits_{x\to 4+} f(x) = +\infty \\ \lim\limits_{x\to 4} f(x) \to NE \end{cases}$$

d)



$$\begin{cases} \lim\limits_{x\to 1-} f(x) = +\infty \\ \lim\limits_{x\to 1+} f(x) = +\infty \Rightarrow f(x)\ \textit{cresce sem}\ \lim \\ \lim\limits_{x\to 1} f(x) = +\infty \end{cases}$$

8. a) $\lim\limits_{x\to -2}(-3x+1) = \lim\limits_{x\to -2}(-3x) + \lim\limits_{x\to -2} 1 = (-3)(-2)+1 = 7$;

b) $\lim\limits_{x\to 4} \dfrac{2x-1}{3x+1} = \dfrac{\lim\limits_{x\to 4} 2x + \lim\limits_{x\to 4} -1}{\lim\limits_{x\to 4} 3x + \lim\limits_{x\to 4} 1} = \dfrac{8-1}{12+1} = \dfrac{7}{13}$;

c) $\lim\limits_{x\to 1}(-2x+5)^4 = \left(\lim\limits_{x\to 1} -2x + \lim\limits_{x\to 1} 5\right)^4 = (-2+5)^4 = 3^4 = 81$;

d) $\lim\limits_{x\to -2}(3x^3-2x+7) = \lim\limits_{x\to -2} 3x^3 + \lim\limits_{x\to -2} -2x + \lim\limits_{x\to -2} 7 = 3(-2)^3 - 2(-2) + 7 = -13$;

e) $\lim\limits_{s\to 4}\dfrac{6s-1}{2s-9}=\dfrac{\lim\limits_{s\to 4}6s+\lim\limits_{s\to 4}-1}{\lim\limits_{s\to 4}2s+\lim\limits_{s\to 4}-9}=\dfrac{24-1}{8-9}=-23$;

f) $\lim\limits_{x\to 1/2}\dfrac{2x^2+5x-3}{6x^2-7x+2}=\lim\limits_{x\to 1/2}\dfrac{2(x-1/2)(x+3)}{6(x-1/2)(x-2/3)}=\lim\limits_{x\to 1/2}\dfrac{2x+6}{6x-4}=\dfrac{\lim\limits_{x\to 1/2}2x+\lim\limits_{x\to 1/2}6}{\lim\limits_{x\to 1/2}6x+\lim\limits_{x\to 1/2}-4}=\dfrac{1+6}{3-4}=-7$;

g) $\lim\limits_{x\to 2}\dfrac{x-2}{x^3-8}=\lim\limits_{x\to 2}\dfrac{x-2}{(x-2)(x^2+2x+4)}=\lim\limits_{x\to 2}\dfrac{1}{x^2+2x+4}=\dfrac{\lim\limits_{x\to 2}1}{\lim\limits_{x\to 2}x^2+2x+4}=\dfrac{1}{12}$;

h) $\lim\limits_{x\to -2}\dfrac{x^2+2x-3}{x^2+5x+6}=\lim\limits_{x\to -2}\dfrac{(x-1)(x+3)}{(x+2)(x+3)}=\lim\limits_{x\to -2}\dfrac{x-1}{x+2}=\dfrac{-3}{0}\Rightarrow \text{NE}$;

i) $\lim\limits_{x\to -2}\dfrac{x^3+8}{x^4-16}=\lim\limits_{x\to -2}\dfrac{(x+2)(x^2-2x+4)}{(x^2+4)(x+2)(x-2)}=\lim\limits_{x\to -2}\dfrac{x^2-2x+4}{(x^2+4)(x-2)}=\dfrac{\lim\limits_{x\to -2}x^2-2x+4}{\lim\limits_{x\to -2}(x^2+4)\lim\limits_{x\to -2}(x-2)}=\dfrac{12}{8*(-4)}=-\dfrac{3}{8}$;

j) $\lim\limits_{x\to 16}\dfrac{x-16}{\sqrt{x}-4}=\lim\limits_{x\to 16}\dfrac{(\sqrt{x}+4)(\sqrt{x}-4)}{\sqrt{x}-4}=\lim\limits_{x\to 16}\sqrt{x}+4=\lim\limits_{x\to 16}\sqrt{x}+\lim\limits_{x\to 16}4=\sqrt{16}+4=8$;

k) $\lim\limits_{x\to 2}\dfrac{(1/x)-(1/2)}{x-2}=\lim\limits_{x\to 2}\dfrac{2-x/2x}{x-2}=\lim\limits_{x\to 2}\dfrac{-(x-2)}{2x}*\dfrac{1}{x-2}=\lim\limits_{x\to 2}\dfrac{1}{2x}=\dfrac{\lim\limits_{x\to 2}-1}{\lim\limits_{x\to 2}2x}=-\dfrac{1}{4}$;

l) $\lim\limits_{x\to -3}\dfrac{x+3}{(1/x)+(1/3)}=\lim\limits_{x\to -3}\dfrac{x+3}{3+x/3x}=\lim\limits_{x\to -3}(x+3)*\dfrac{3x}{x+3}=\lim\limits_{x\to -3}3x=3*(-3)=-9$;

m) $\lim\limits_{x\to 1}\left(\dfrac{x^2}{x-1}-\dfrac{1}{x-1}\right)=\lim\limits_{x\to 1}\dfrac{x^2-1}{x-1}=\lim\limits_{x\to 1}\dfrac{(x+1)(x-1)}{x-1}=\lim\limits_{x\to 1}x+1=1+1=2$;

n) $\lim\limits_{x\to -8}\dfrac{16x^{3/2}}{4-x^{4/3}}$;

o) $\lim\limits_{x\to 4}\sqrt[3]{x^2 \quad 5x \quad 4}=\sqrt[3]{\left(\lim\limits_{x\to 4}x^2 \quad 5x \quad 4\right)}=\sqrt[3]{16 \quad 20 \quad 4}=\sqrt[3]{-8}= \quad 2$;

p) $\lim\limits_{x\to -2}\sqrt{x^4-4x+1}=\sqrt{\left(\lim\limits_{x\to -2}x^4-4x+1\right)}=\sqrt{16+8+1}=\sqrt{25}=5$;

q) $\lim\limits_{x\to 3}\sqrt[3]{\dfrac{2+5x-3x^2}{x^2-1}}=\sqrt[3]{\dfrac{\lim\limits_{x\to 3}2+5x-3x^2}{\lim\limits_{x\to 3}x^2-1}}=\sqrt[3]{\dfrac{2+15-27}{8}}=-\dfrac{\sqrt[3]{10}}{2}$;

r) $\lim\limits_{h\to 0}\dfrac{4-\sqrt{16+h}}{h}=\lim\limits_{h\to 0}\dfrac{4-\sqrt{16+h}}{h}*\dfrac{4+\sqrt{16+h}}{4+\sqrt{16+h}}=\lim\limits_{h\to 0}\dfrac{16-16-h}{4h+h\sqrt{16+h}}=\lim\limits_{h\to 0}\dfrac{-1}{4+\sqrt{16+h}}=-\dfrac{1}{8}$;

s) $\lim\limits_{h\to 0}\left(\dfrac{1}{h}\right)\left(\dfrac{1}{\sqrt{1+h}}-1\right)=\lim\limits_{h\to 0}\dfrac{1}{h}\left(\dfrac{1-\sqrt{1+h}}{\sqrt{1+h}}\right)*\left(\dfrac{1+\sqrt{1+h}}{1+\sqrt{1+h}}\right)=\lim\limits_{h\to 0}\dfrac{1}{h}\left(\dfrac{1-1-h}{\sqrt{1+h}+1+h}\right)$

$$= \lim_{h\to 0} \frac{-1}{\sqrt{1+h}+1+h} = \frac{\lim\limits_{h\to 0} -1}{\lim\limits_{h\to 0} \sqrt{1+h}+1+h} = \frac{-1}{2};$$

t) $$\lim_{x\to 1} \frac{x^2+x-2}{x^5-1} = \lim_{x\to 1} \frac{(x-1)(x+2)}{(x-1)(x^4+x^3+x^2+x+1)} = \lim_{x\to 1} \frac{x+2}{x^4+x^3+x^2+x+1} = \frac{3}{5};$$

u) $$\lim_{x\to 2} \frac{x^2-7x+10}{x^6-64} = \lim_{x\to 2} \frac{(x-2)(x-5)}{(x-2)(x^5+2x^4+4x^3+8x^2+16x+32)} = \lim_{x\to 2} \frac{(x-5)}{(x^5+2x^4+4x^3+8x^2+16x+32)}$$

$$= \frac{\lim\limits_{x\to 2} x-5}{\lim\limits_{x\to 2}(x^5+2x^4+4x^3+8x^2+16x+32)} = \frac{2-5}{6*2^5} = \frac{-3}{6*32} = \frac{-3}{192} = -\frac{1}{64};$$

v) $$\lim_{v\to 1} v^2(3v-4)(9-v^3) = \lim_{v\to 1} 27v^3 - 3v^6 - 36v^2 + 4v^5 = 27-3-36+4 = -8;$$

w) $$\lim_{x\to 5^+} \left(\sqrt{x^2-25}\right)+3 = \sqrt{\lim_{x\to 5^+}\left(x^2-25\right)} + \lim_{x\to 5^+} 3 = 0+3 = 3;$$

x) $$\lim_{x\to 3^-} x\sqrt{9-x^2} = \lim_{x\to 3-} x * \sqrt{\lim_{x\to 3-} 9-x^2} = 3*0 = 0;$$

y) $$\lim_{x\to 3^+} \frac{\sqrt{(x-3)^2}}{x-3} = \lim_{x\to 3+} \frac{x-3}{x-3} = \lim_{x\to 3+} 1 = 1;$$

z) $$\lim_{x\to 4^+} \frac{\sqrt[4]{x^2-16}}{x+4} = \frac{\sqrt[4]{\lim\limits_{x\to 4+} x^2-16}}{\lim\limits_{x\to 4+} x+4} = \frac{0}{8} = 0.$$

10. a) $$\begin{cases} \lim\limits_{x\to 4-} f(x) = \lim\limits_{x\to 4-} \dfrac{5}{x-4} = \dfrac{5}{-0,000\ldots} = -\infty \\ \lim\limits_{x\to 4+} f(x) = \lim\limits_{x\to 4+} \dfrac{5}{x-4} = \dfrac{5}{+0,000\ldots} = +\infty \\ \lim\limits_{x\to 4} f(x) \to NE \end{cases};$$ b) $$\begin{cases} \lim\limits_{x\to -5/2-} f(x) = \lim\limits_{x\to -5/2-} \dfrac{8}{(2x+5)^3} = \dfrac{8}{-0,000\ldots} = -\infty \\ \lim\limits_{x\to -5/2+} f(x) = \lim\limits_{x\to -5/2+} \dfrac{8}{(2x+5)^3} = \dfrac{8}{+0,000\ldots} = +\infty \\ \lim\limits_{x\to -5/2} f(x) \to NE \end{cases}$$

c) $$\begin{cases} \lim\limits_{x\to -8-} f(x) = \lim\limits_{x\to -8-} \dfrac{3x}{(x+8)^2} = \dfrac{-24}{+0,000\ldots} = -\infty \\ \lim\limits_{x\to -8+} f(x) = \lim\limits_{x\to -8+} \dfrac{3x}{(x+8)^2} = \dfrac{-24}{+0,000\ldots} = -\infty \\ \lim\limits_{x\to -8} f(x) = \lim\limits_{x\to -8} \dfrac{3x}{(x+8)^2} = -\infty \end{cases};$$ d) $$\begin{cases} \lim\limits_{x\to 9/2-} \dfrac{3x^2}{(2x-9)^2} = \dfrac{243/2}{+0,000\ldots} = +\infty \\ \lim\limits_{x\to 9/2+} \dfrac{3x^2}{(2x-9)^2} = \dfrac{243/2}{+0,000\ldots} = +\infty \\ \lim\limits_{x\to 9/2} \dfrac{3x^2}{(2x-9)^2} = +\infty \end{cases}$$

e) $$\begin{cases} \lim\limits_{x\to -1-} f(x) = \lim\limits_{x\to -1-} \dfrac{2x^2}{x^2-x-2} = \dfrac{2}{+0,000\ldots} = +\infty \\ \lim\limits_{x\to -1+} f(x) = \lim\limits_{x\to -1+} \dfrac{2x^2}{x^2-x-2} = \dfrac{2}{-0,000\ldots} = -\infty \\ \lim\limits_{x\to -1} f(x) \to NE \end{cases};$$ f) $$\begin{cases} \lim\limits_{x\to 1-} \dfrac{4x}{x^2-4x+3} = \dfrac{4}{+0,000\ldots} = +\infty \\ \lim\limits_{x\to 1+} \dfrac{4x}{x^2-4x+3} = \dfrac{4}{-0,000\ldots} = -\infty \\ \lim\limits_{x\to 1} \dfrac{4x}{x^2-4x+3} \to NE \end{cases}$$

$$g)\begin{cases}\lim\limits_{x\to 3-}\dfrac{1}{x(x-3)^2}=\dfrac{1}{+0{,}000\ldots}=+\infty\\ \lim\limits_{x\to 3+}\dfrac{1}{x(x-3)^2}=\dfrac{1}{+0{,}000\ldots}=+\infty;\\ \lim\limits_{x\to 3}f(x)=+\infty\end{cases}\qquad h)\begin{cases}\lim\limits_{x\to -1-}\dfrac{1}{(x+1)^2}=\dfrac{1}{+0{,}000\ldots}=+\infty\\ \lim\limits_{x\to -1+}\dfrac{1}{(x+1)^2}=\dfrac{1}{+0{,}000\ldots}=+\infty\\ \lim\limits_{x\to -1}f(x)=+\infty\end{cases}$$

11 a) $$\lim_{x\to\infty}\frac{5x^2-3x+1}{2x^2+4x-7}=\lim_{x\to\infty}\frac{x^2\left(5-\frac{3}{x}+\frac{1}{x^2}\right)}{x^2\left(2+\frac{4}{x}-\frac{7}{x^2}\right)}=\lim_{x\to\infty}\frac{\left(5-\frac{3}{x}+\frac{1}{x^2}\right)}{\left(2+\frac{4}{x}-\frac{7}{x^2}\right)}=\frac{5-0+0}{2+0-0}=\frac{5}{2};$$

b) $$\lim_{x\to\infty}\frac{3x^3-x+1}{6x^3+2x^2-7}=\lim_{x\to\infty}\frac{x^3\left(3-\frac{1}{x^2}+\frac{1}{x^3}\right)}{x^3\left(6+\frac{2}{x}-\frac{7}{x^3}\right)}=\lim_{x\to\infty}\frac{\left(3-\frac{1}{x^2}+\frac{1}{x^3}\right)}{\left(6+\frac{2}{x}-\frac{7}{x^3}\right)}=\frac{3-0+0}{6+0-0}=\frac{3}{6}=\frac{1}{2};$$

c) $$\lim_{x\to-\infty}\frac{4-7x}{2+3x}=\lim_{x\to-\infty}\frac{x\left(\frac{4}{x}-7\right)}{x\left(\frac{2}{x}+3\right)}=\lim_{x\to-\infty}\frac{\left(\frac{4}{x}-7\right)}{\left(\frac{2}{x}+3\right)}=\frac{0-7}{0+3}=-\frac{7}{3};$$

d) $$\lim_{x\to-\infty}\frac{(3x+4)(x-1)}{(2x+7)(x+2)}=\lim_{x\to-\infty}\frac{3x^2+4x-7}{2x^2+11x+14}=\lim_{x\to-\infty}\frac{x^2\left(3+\frac{4}{x}-\frac{7}{x^2}\right)}{x^2\left(2+\frac{11}{x}+\frac{14}{x^2}\right)}=\lim_{x\to-\infty}\frac{\left(3+\frac{4}{x}-\frac{7}{x^2}\right)}{\left(2+\frac{11}{x}+\frac{14}{x^2}\right)}=\frac{3}{2};$$

e) $$\lim_{x\to-\infty}\frac{2x^2-3}{4x^3+5x}=\lim_{x\to-\infty}\frac{x^3\left(\frac{2}{x}-\frac{3}{x^3}\right)}{x^3\left(4+\frac{5}{x^2}\right)}=\lim_{x\to-\infty}\frac{\left(\frac{2}{x}-\frac{3}{x^3}\right)}{\left(4+\frac{5}{x^2}\right)}=\frac{0}{4}=0;$$

f) $$\lim_{x\to\infty}\frac{2x^2-x+3}{x^3+1}=\lim_{x\to\infty}\frac{x^3\left(\frac{2}{x}-\frac{1}{x^2}+\frac{3}{x^3}\right)}{x^3\left(1+\frac{1}{x^3}\right)}=\lim_{x\to\infty}\frac{\left(\frac{2}{x}-\frac{1}{x^2}+\frac{3}{x^3}\right)}{\left(1+\frac{1}{x^3}\right)}=\frac{0}{1}=0;$$

g) $$\lim_{x\to\infty}\frac{-x^3+2x}{2x^2-3}=\lim_{x\to\infty}\frac{x^2\left(-x+\frac{2}{x^2}\right)}{x^2\left(2-\frac{3}{x^2}\right)}=\lim_{x\to\infty}\frac{\left(-x+\frac{2}{x^2}\right)}{\left(2-\frac{3}{x^2}\right)}=\frac{-\infty}{2}=-\infty;$$

h) $$\lim_{x\to-\infty}\frac{x^2+2}{x-1}=\lim_{x\to-\infty}\frac{x\left(x+\frac{2}{x}\right)}{x\left(1-\frac{1}{x}\right)}=\lim_{x\to-\infty}\frac{\left(x+\frac{2}{x}\right)}{\left(1-\frac{1}{x}\right)}=\frac{-\infty}{1}=-\infty;$$

i) $\lim\limits_{x\to\infty}\dfrac{2-x^2}{x+3}=\lim\limits_{x\to\infty}\dfrac{x\left(\dfrac{2}{x}-x\right)}{x\left(1+\dfrac{3}{x}\right)}=\lim\limits_{x\to\infty}\dfrac{\left(\dfrac{2}{x}-x\right)}{\left(1+\dfrac{3}{x}\right)}=\dfrac{-\infty}{1}=-\infty$;

j) $\lim\limits_{x\to\infty}\dfrac{3x^4+x+1}{x^2-5}=\lim\limits_{x\to\infty}\dfrac{x^2\left(3x^2+\dfrac{1}{x}+\dfrac{1}{x^2}\right)}{x^2\left(1-\dfrac{5}{x^2}\right)}=\lim\limits_{x\to\infty}\dfrac{\left(3x^2+\dfrac{1}{x}+\dfrac{1}{x^2}\right)}{\left(1-\dfrac{5}{x^2}\right)}=+\infty$;

k) $\lim\limits_{x\to\infty}\sqrt[3]{\dfrac{8+x^2}{x(x+1)}}=\sqrt[3]{\lim\limits_{x\to\infty}\dfrac{x^2\left(\dfrac{8}{x^2}+1\right)}{x^2\left(1+\dfrac{1}{x}\right)}}=\sqrt[3]{\lim\limits_{x\to\infty}\dfrac{\left(\dfrac{8}{x^2}+1\right)}{\left(1+\dfrac{1}{x}\right)}}=\sqrt[3]{\dfrac{1}{1}}=1$;

l) $\lim\limits_{x\to-\infty}\dfrac{4x-3}{\sqrt{x^2+1}}=\lim\limits_{x\to-\infty}\dfrac{4x-3}{\sqrt{x^2+1}}*\dfrac{\sqrt{x^2+1}}{\sqrt{x^2+1}}=\lim\limits_{x\to-\infty}\dfrac{4x\sqrt{x^2+1}-3\sqrt{x^2+1}}{x^2+1}=\lim\limits_{x\to-\infty}\dfrac{x^2\left(\dfrac{4}{x}\sqrt{x^2+1}-\dfrac{3}{x^2}\sqrt{x^2+1}\right)}{x^2\left(1+\dfrac{1}{x^2}\right)}$

$\lim\limits_{x\to-\infty}\dfrac{\left(\dfrac{4}{x}\sqrt{x^2+1}-\dfrac{3}{x^2}\sqrt{x^2+1}\right)}{\left(1+\dfrac{1}{x^2}\right)}=\dfrac{0}{1}=0$;

m) $\lim\limits_{x\to\infty}\operatorname{sen} x$

n) $\lim\limits_{x\to\infty}\cos x$

Seno e cosseno são funções periódicas e, portanto, não assumem um único valor quando x tende para o infinito.

12. a) $A.H.\begin{cases}\lim\limits_{x\to\infty-}f(x)=0\\ \lim\limits_{x\to\infty+}f(x)=0\end{cases}\Rightarrow A.H\ em\ y=o\ ;\ A.V\begin{cases}\lim\limits_{x\to2-}f(x)=-\infty\\ \lim\limits_{x\to2+}f(x)=+\infty\end{cases}e\begin{cases}\lim\limits_{x\to-2-}f(x)=+\infty\\ \lim\limits_{x\to-2+}f(x)=-\infty\end{cases}\Rightarrow AV\begin{cases}x=2\\ x=-2\end{cases}$

b) $A.H.\begin{cases}\lim\limits_{x\to\infty-}f(x)=0\\ \lim\limits_{x\to\infty+}f(x)=0\end{cases}\Rightarrow A.H\ em\ y=o\ ;\ A.V\begin{cases}\lim\limits_{x\to2-}f(x)=+\infty\\ \lim\limits_{x\to2+}f(x)=-\infty\end{cases}e\begin{cases}\lim\limits_{x\to-2-}f(x)=+\infty\\ \lim\limits_{x\to-2+}f(x)=-\infty\end{cases}\Rightarrow AV\begin{cases}x=2\\ x=-2\end{cases}$

c) $A.H.\begin{cases}\lim\limits_{x\to\infty-}f(x)=2\\ \lim\limits_{x\to\infty+}f(x)=2\end{cases}\Rightarrow A.H\ em\ y=2\ ;\ A.V\to NE\ f(x)$ *é continua para* $\forall x\in\Re$

d) $A.H.\begin{cases}\lim\limits_{x\to\infty-}f(x)=0\\ \lim\limits_{x\to\infty+}f(x)=0\end{cases}\Rightarrow A.H\ em\ y=0\ ;\ A.V\to NE\ f(x)$ *é continua para* $\forall x\in\Re$

e) $A.H.\begin{cases}\lim\limits_{x\to\infty-}f(x)=0\\ \lim\limits_{x\to\infty+}f(x)=0\end{cases}\Rightarrow A.H\ y=o;\ A.V\begin{cases}\lim\limits_{x\to2-}f(x)=-\infty\\ \lim\limits_{x\to2+}f(x)=+\infty\end{cases}\begin{cases}\lim\limits_{x\to-3-}f(x)=-\infty\\ \lim\limits_{x\to-3+}f(x)=+\infty\end{cases}\begin{cases}\lim\limits_{x\to0-}f(x)=+\infty\\ \lim\limits_{x\to0+}f(x)=-\infty\end{cases}\Rightarrow AV\begin{cases}x=-3\\ x=0\\ x=2\end{cases}$

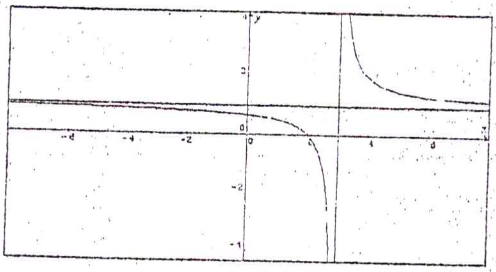
$$f)\ A.H.\begin{cases}\lim\limits_{x\to\infty-} f(x)=-1\\ \lim\limits_{x\to\infty+} f(x)=-1\end{cases}\Rightarrow A.H.em\ y=-1\,;\ A.V\begin{cases}\lim\limits_{x\to 4-} f(x)=+\infty\\ \lim\limits_{x\to 4+} f(x)=-\infty\end{cases} e\begin{cases}\lim\limits_{x\to -4-} f(x)=-\infty\\ \lim\limits_{x\to -4+} f(x)=+\infty\end{cases}\Rightarrow AV\begin{cases}x=4\\ x=-4\end{cases}$$

$$g)\ A.H.\begin{cases}\lim\limits_{x\to\infty-} f(x)=1\\ \lim\limits_{x\to\infty+} f(x)=1\end{cases}\Rightarrow A.H.em\ y=1\,;\ A.V\begin{cases}\lim\limits_{x\to 1-} f(x)=-\infty\\ \lim\limits_{x\to 1+} f(x)=+\infty\end{cases} e\begin{cases}\lim\limits_{x\to -3-} f(x)=+\infty\\ \lim\limits_{x\to -3+} f(x)=-\infty\end{cases}\Rightarrow AV\begin{cases}x=-3\\ x=1\end{cases}$$

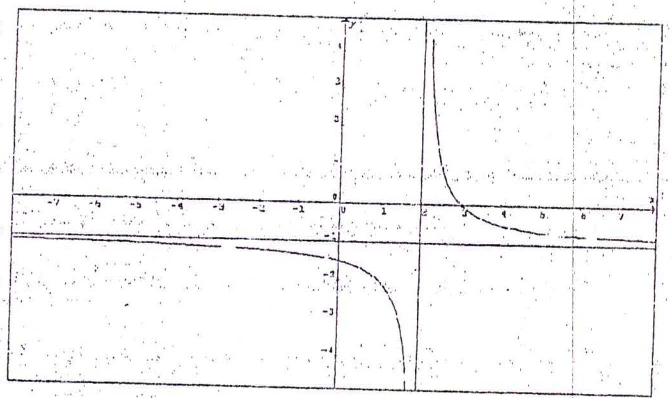
$$h)\ A.H.\begin{cases}\lim\limits_{x\to\infty-} f(x)=1\\ \lim\limits_{x\to\infty+} f(x)=1\end{cases}\Rightarrow A.H.em\ y=1\,;\ A.V\begin{cases}\lim\limits_{x\to 5-} f(x)=-\infty\\ \lim\limits_{x\to 5+} f(x)=+\infty\end{cases} e\begin{cases}\lim\limits_{x\to -5-} f(x)=+\infty\\ \lim\limits_{x\to -5+} f(x)=-\infty\end{cases}\Rightarrow AV\begin{cases}x=5\\ x=-5\end{cases}$$

$$i)\ A.H.\begin{cases}\lim\limits_{x\to\infty-} f(x)=0\\ \lim\limits_{x\to\infty+} f(x)=0\end{cases}\Rightarrow A.H.em\ y=0\,;\ A.V\begin{cases}\lim\limits_{x\to 4-} f(x)=-\infty\\ \lim\limits_{x\to 4+} f(x)=+\infty\end{cases} e\begin{cases}\lim\limits_{x\to -4-} f(x)=+\infty\\ \lim\limits_{x\to -4+} f(x)=-\infty\end{cases}\Rightarrow AV\begin{cases}x=4\\ x=-4\end{cases}$$

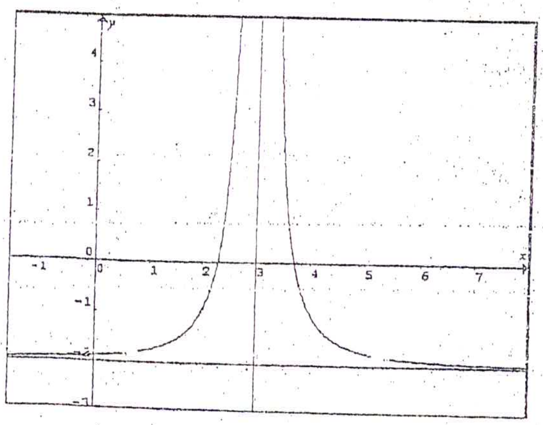
13. a)



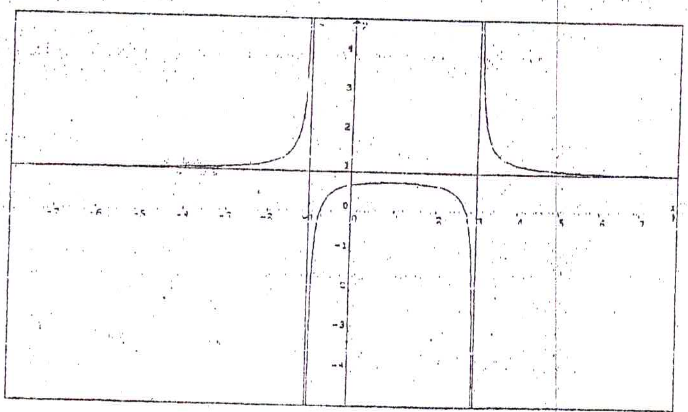
b)



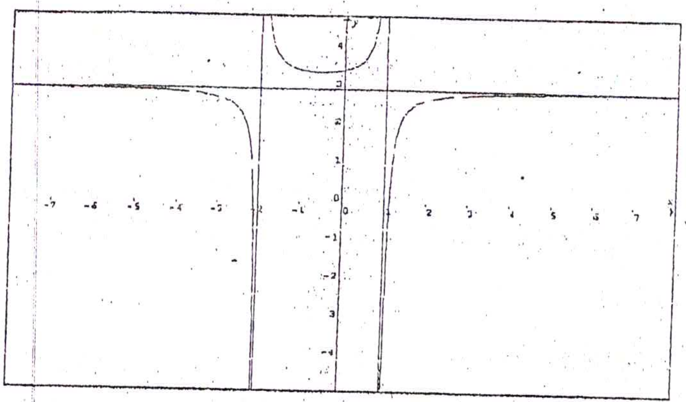
c)



d)



e)



14. (ex.4) a) salto; b) Removível; c) Removível; d) Salto; e) Infinita; f) Removível.
(ex.5) a) Salto; b) Salto; c) Não há descontinuidade; d) Removível; e) Removível; f) Removível.

15.
$a) \lim_{x \to 4} f(x) = f(4) = \sqrt{3} + 12 \qquad c) \lim_{x \to -2} f(x) = f(-2) = 19 - 1/\sqrt{2}$

$b) \lim_{x \to -5} f(x) = f(-5) = 3 \qquad d) \lim_{x \to 8} f(x) = f(8) = 2/17$

16. a) f(x) não é contínua em a, pois f(a) não esta definida.

b) f(x) não é contínua em a, pois f(a) não esta definida.

c) f(x) não é contínua em a, pois $\lim_{x \to 3} f(x) \neq f(3)$.

d) f(x) não é contínua em a, pois $\lim_{x \to -3} f(x) \neq f(-3)$.

e) f(x) não é contínua em a, pois $\lim_{x \to 3} f(x) \neq f(3)$.

f) f(x) não é contínua em a, pois $\lim_{x \to 3} f(x)$ não existe.

17. a) $f(x) = \dfrac{3}{x^2 + x - 6} \Rightarrow x^2 + x - 6 \neq 0;$ *Descontinuidade em* : x= -3 e x= 2;

b) $f(x) = \dfrac{5}{x^2 - 4x - 12} \Rightarrow x^4 - 4x - 12 \neq 0;$ *Descontinuidade em* : x= 6 e x= -2;

c) $f(x) = \dfrac{x-1}{x^2 + x - 2} \Rightarrow x^2 + x - 2 \neq 0;$ *Descontinuidade em* : x= -2 e x= 1;

d) $f(x) = \dfrac{x-4}{x^2 - x - 12} \Rightarrow x^2 - x - 12 \neq 0;$ *Descontinuidade em* : x= -3 e x= 4.

18. A função será contínua no intervalo dado se for contínua em todos os pontos do intervalo.

a) Domf(x)=R≥4 Logo o intervalo [4,8] pertence ao domínio da função sendo f contínua no intervalo dado;

b) Domf(x)=R≤16. Logo o intervalo (-∞,16) pertence ao domínio da função sendo f contínua no intervalo dado;

c) Domf(x)=R≠0. Logo o intervalo (0,∞) pertence ao domínio da função sendo f contínua no intervalo dado;

d) Domf(x)=R≠1. Logo o intervalo (1,3) pertence ao domínio da função sendo f contínua no intervalo dado;

19. a) $f(x)=\dfrac{3x-5}{2x^2-x-3} \Rightarrow 2x^2-x-3\neq 0 \therefore 2(x+1)\left(x-\frac{3}{2}\right)\neq 0$.

Então: $\{x\in\Re/x\neq 3/2 \cup x\neq -1\}$;

b) $f(x)=\dfrac{x^2-9}{x-3}=\dfrac{(x+3)(x-3)}{x-3}=x+3$. Então: $\{x\in\Re,\forall x\}$;

c) $f(x)=\sqrt{2x-3}+x^2 \Rightarrow 2x-3\geq 0$. Então: $\{x\in\Re/x\geq 3/2\}$;

d) $f(x)=\dfrac{x}{\sqrt[3]{x-4}} \Rightarrow x-4\neq 0$. Então: $\{x\in\Re/x\neq 4\}$;

e) $f(x)=\dfrac{x-1}{\sqrt{x^2-1}} \Rightarrow x^2-1>0$. Então: $\{x\in\Re/-1<x<1\}$;

$x^2>1$
$x>1$

f) $f(x)=\dfrac{x}{\sqrt{1-x^2}} \Rightarrow 1-x^2>0$. Então: $\{x\in\Re/x<-1\cup x>1\}$;

$x<-1$ $x>-1$ $x<-1 \cup x>1$

g) $f(x)=\dfrac{|x+9|}{x+9} \Rightarrow x+9\neq 0$. Então: $\{x\in\Re/x\neq -9\}$;

$-1<x<1$ $x<-1 \cup x>1$

h) $f(x)=\dfrac{x}{x^2+1} \Rightarrow \{x\in\Re,\forall x\}$;

i) $f(x)=\dfrac{5}{x^3-x^2} \Rightarrow x^2(x-1)\neq 0$. Então: $\{x\in\Re/x\neq 0\cup x\neq 1\}$;

j) $f(x)=\dfrac{4x-7}{(x+3)(x^2+2x-8)} \Rightarrow x+3\neq 0 \wedge x^2+2x-8\neq 0$. Então:

$\{x\in\Re/x\neq -4\cup x\neq -3\cup x\neq 2\}$;

k) $f(x)=\dfrac{\sqrt{x^2-9}\sqrt{25-x^2}}{x-4} \Rightarrow x-4\neq 0 \wedge x^2-9\geq 0 \wedge 25-x^2\geq 0$. Então:

$\{x\in\Re/-5\leq x\leq -3\cup 3\leq x\leq 5,\ x\neq 4\}$;

l) $f(x)=\dfrac{\sqrt{9-x}}{\sqrt{x-6}} \Rightarrow 9-x\geq 0 \wedge x-6>0$. Então: $\{x\in\Re/6<x\leq 9\}$;

m) $f(x)=\tan 2x=\dfrac{sen 2x}{\cos 2x} \Rightarrow \cos 2x\neq 0$. Então: $\{x\in\Re/x\neq \pi/4+n\pi/2,\ n\in I\}$

n) $f(x) = \cot\frac{1}{3}x = \frac{\cos\left(x/3\right)}{sen\left(x/3\right)} \Rightarrow sen\left(x/3\right) \neq 0$. Então: $\{x \in \Re \,/\, x \neq 3\pi n,\, n \in \mathrm{I}\}$;

o) $f(x) = \csc\frac{1}{2}x = \frac{1}{sen\left(x/2\right)} \Rightarrow sen\left(x/2\right) \neq 0$. Então: $\{x \in \Re \,/\, x \neq 2n\pi,\, n \in \mathrm{I}\}$;

p) $f(x) = \sec 3x = \frac{1}{\cos 3x} \Rightarrow \cos 3x \neq 0$. Então: $\{x \in \Re \,/\, x \neq \pi/6 + n\pi/3,\, n \in \mathrm{I}\}$.

LISTA 3:

Derivadas

Lista 3: Derivadas

10a+5h-4.

1.a) (i) $m=\lim\limits_{h\to 0}\frac{5(a+h)^2-4(a+h)-5a^2+4a}{h}=\lim\limits_{h\to 0}\frac{\cancel{5a^2}+10ah+5h^2\cancel{-4a}-4h\cancel{-5a^2}\cancel{+4a}}{h}=10a-4;$

$(ii)\ f(x)=f(2)+f'(2)(x-2)=12+16(x-2)=16x-20;$

b) (i) $m=\lim\limits_{h\to 0}\frac{(a+h)^3-a^3}{h}=\lim\limits_{h\to 0}\frac{a^3+3a^2h+3ah^2+h^3-a^3}{h}=3a^2;$

$(ii)\ f(x)=f(2)+f'(2)(x-2)=8+12(x-2)=12x-16;$

c) (i) $m=\lim\limits_{h\to 0}\frac{3(a+h)+2-3a-2}{h}=\lim\limits_{h\to 0}\frac{3a+3h+\cancel{2}\cancel{-3a}\cancel{-2}}{h}=3;$

$(ii)\ f(x)=f(2)+f'(2)(x-2)=8+3(x-2)=3x+2;$

d) (i) $m=\lim\limits_{h\to 0}\frac{3-2(a+h)^2-3+2a^2}{h}=\lim\limits_{h\to 0}\frac{\cancel{3}\cancel{-2a^2}-4ah-2h^2\cancel{-3}\cancel{+2a^2}}{h}=-4a;$

$(ii)\ f(x)=f(2)+f'(2)(x-2)=-5-8(x-2)=-8x+11;$

e) (i) $m=\lim\limits_{h\to 0}\frac{(a+h)^4-a^4}{h}=\lim\limits_{h\to 0}\frac{a^4+4a^3h+6a^2h^2+4ah^3+h^4-a^4}{h}=4a^3;$

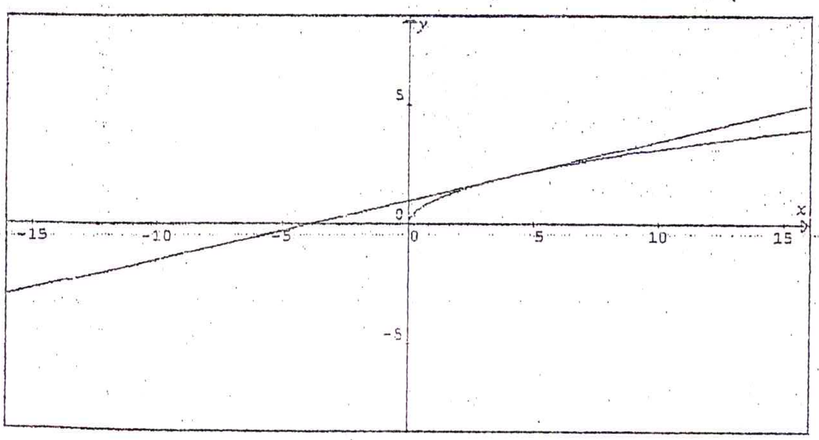
$(ii)\ f(x)=f(2)+f'(2)(x-2)=16+32(x-2)=32x-48;$

f) (i) $m=\lim\limits_{h\to 0}\frac{4-2(a+h)-4+2a}{h}=\lim\limits_{h\to 0}\frac{4-2a-2h-4+2a}{h}=-2;$

$(ii)\ f(x)=f(2)+f'(2)(x-2)=-2(x-2)=-2x+4;$

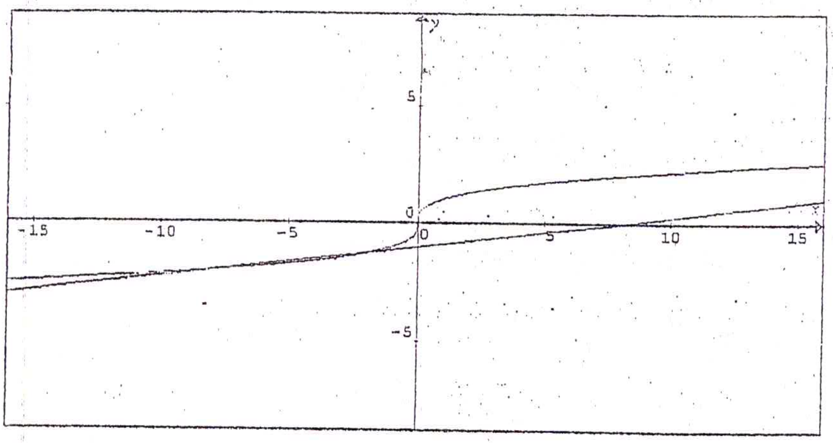
2. a) (i) $m=\lim\limits_{h\to 0}\frac{\sqrt{a+h}-\sqrt{a}}{h}=\lim\limits_{h\to 0}\frac{\sqrt{a+h}-\sqrt{a}}{h}*\frac{\sqrt{a+h}+\sqrt{a}}{\sqrt{a+h}+\sqrt{a}}=\lim\limits_{h\to 0}\frac{a+h-a}{h\left[(\sqrt{a+h})+\sqrt{a}\right]}=\lim\limits_{h\to 0}\frac{1}{(\sqrt{a+h})+\sqrt{a}}=\frac{1}{2\sqrt{a}};$

$(ii)\ f(x)=f(4)+f'(4)(x-4)=2+\frac{1}{4}(x-4)=\boxed{\frac{x}{4}+1;}$



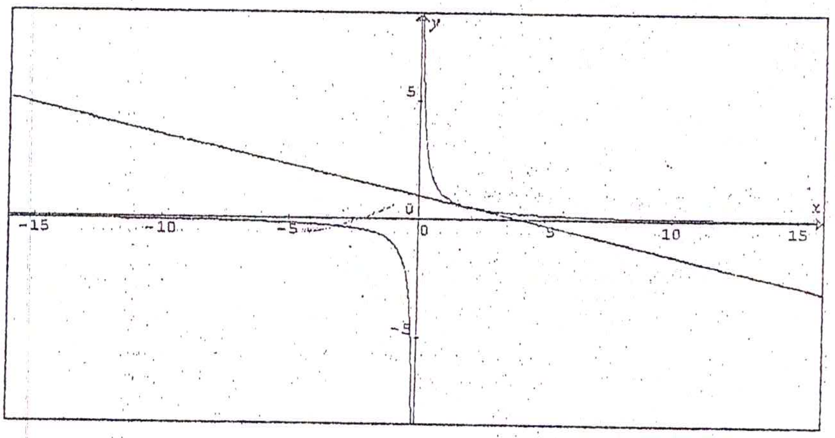
b) (i) $m=\lim\limits_{h\to 0}\frac{\sqrt[3]{a+h}-\sqrt[3]{a}}{h}=\lim\limits_{h\to 0}\frac{\sqrt[3]{a+h}-\sqrt[3]{a}}{h}*\frac{\sqrt[3]{(a+h)^2}+\sqrt[3]{a^2}}{\sqrt[3]{(a+h)^2}+\sqrt[3]{a^2}}=\lim\limits_{h\to 0}\frac{a+h-a}{h\left[\sqrt[3]{(a+h)^2}+\sqrt[3]{a^2}\right]}=\lim\limits_{h\to 0}\frac{1}{\left[\sqrt[3]{a+h}+\sqrt[3]{a}\right]}=\frac{1}{2\sqrt[3]{a^2}};$

$(ii)\ f(x)=f(-8)+f'(-8)(x+8)=-2+\frac{1}{8}(x+8)=\frac{x}{8}-1$



c) (i) $m=\lim\limits_{h\to 0}\dfrac{\dfrac{1}{a+h}-\dfrac{1}{a}}{h}=\lim\limits_{h\to 0}\dfrac{\dfrac{a-a-h}{a(a+h)}}{h}=\lim\limits_{h\to 0}\dfrac{-1}{a(a+h)}=-\dfrac{1}{a^2}$;

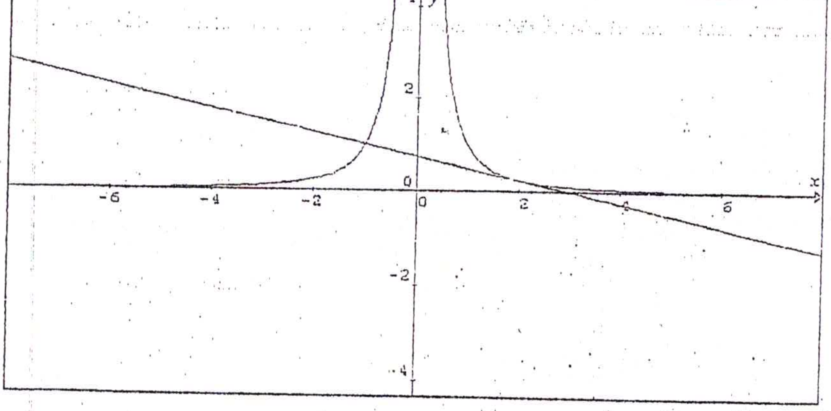
(ii) $f(x)=f(2)+f'(2)(x-2)=\dfrac{1}{2}-\dfrac{1}{4}(x-2)=-\dfrac{x}{4}+1$;



d) (i) $m=\lim\limits_{h\to 0}\dfrac{\dfrac{1}{(a+h)^2}-\dfrac{1}{a^2}}{h}=\lim\limits_{h\to 0}\dfrac{\dfrac{a^2-a^2-2ah-h^2}{a^2(a+h)^2}}{h}=\lim\limits_{h\to 0}\dfrac{-2a-h}{a^2(a+h)^2}=-\dfrac{-2}{a^3}$;

(ii) $f(x)=f(2)+f'(2)(x-2)=\dfrac{1}{4}-\dfrac{2}{8}(x-2)=-\dfrac{x}{4}+\dfrac{3}{4}$;

$\dfrac{-7+3}{4} = -\dfrac{4}{4} - 1$



3. a) (i) $m = y' = 2x$ (*regra da potência*)

Equação das Retas Tangente:

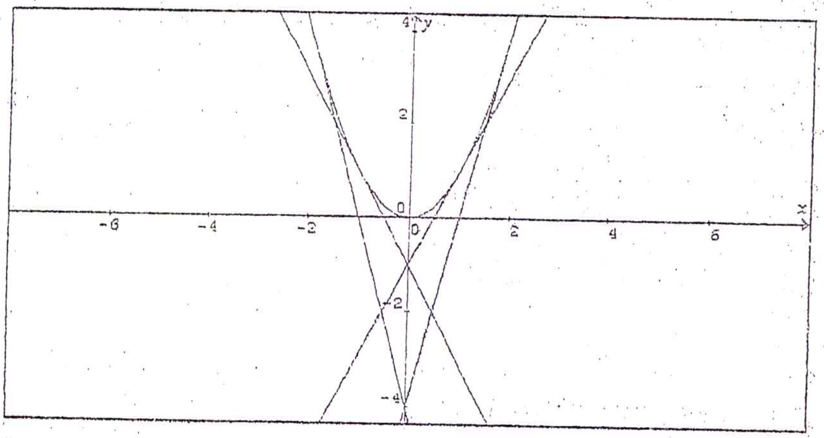
x= -2: $f(x) = f(-2) + f'(-2)(x+2) = 4 - 4(x+2) = -4x - 4$

x= -1: $f(x) = f(-1) + f'(-1)(x+1) = 1 - 2(x+1) = -2x - 1$

x=1: $f(x) = f(1) + f'(1)(x-1) = 1 + 2(x-1) = 2x - 1$

x=2: $f(x) = f(2) + f'(2)(x-2) = 4 + 4(x-2) = 4x - 4$

Esboçar o gráfico:



(ii) $m = 2x = 6 \Rightarrow x = 3 \therefore y = 9$; **Ponto: (3,9)**

b) $m = y' = 3x^2$ (*regra da potência*)

Equação das Retas Tangente:

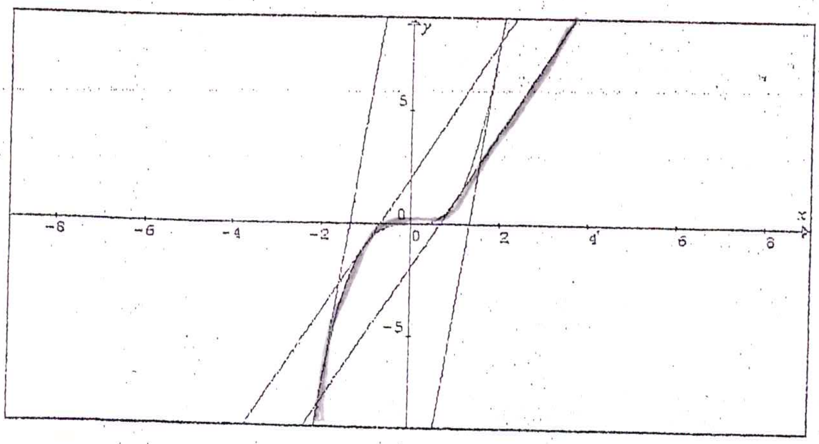
x= -2: $f(x) = f(-2) + f'(-2)(x+2) = -8 + 12(x+2) = 12x + 16$

x= -1: $f(x) = f(-1) + f'(-1)(x+1) = -1 + 3(x+1) = 3x + 2$

x=1: $f(x) = f(1) + f'(1)(x-1) = 1 + 3(x-1) = 3x - 2$

x=2: $f(x) = f(2) + f'(2)(x-2) = 8 + 12(x-2) = 12x - 16$

Esboçar o gráfico:



(ii) $m = 3x^2 = 9 \Rightarrow x = \pm\sqrt{3} \therefore y = \pm 3^{3/2}$

4. a) $(i)\ f'(x)=\lim_{h\to 0}\dfrac{-5(a+h)^2+8(a+h)+2+5a^2-8a-2}{h}=$

$$\lim_{h\to 0}\frac{-5a^2-10ah-5h^2+8a+8h+2+5a^2-8a-2}{h}=\lim_{h\to 0}-10a-5h+8=-10a+8$$

$(ii)\ Dom:\{x\in\Re\};$

$$(iii)\ P(-1,-11)\to\begin{cases}f(x)=f(-1)+f'(1)(x+1)\\ f(x)=-11+18(x+1)\\ f(x)=18x+7\to \text{equação da reta tangente em } P(-1,-11)\end{cases}$$

(iv) tangente horizontal : $m=0$

$$\begin{cases}-10a+8=0\\ 10a=8\to a=\dfrac{4}{5}\end{cases}\Rightarrow x=\frac{4}{5}\therefore y=-5\left(\frac{16}{25}\right)+8\left(\frac{4}{5}\right)+2=\frac{26}{5}$$

$m=0$ em $P\left(\dfrac{4}{5},\dfrac{26}{5}\right);$

b) $(i)\ f'(x)=\lim_{h\to 0}\dfrac{3(a+h)^2-2(a+h)-4-3a^2+2a+4}{h}=$

$$\lim_{h\to 0}\frac{3a^2+6ah+3h^2-2a-2h-4-3a^2+2a+4}{h}=\lim_{h\to 0}6a+3h-2=6a-2$$

$(ii)\ Dom:\{x\in\Re\}$

$$(iii)\ P(2,4)\to\begin{cases}f(x)=f(2)+f'(2)(x-2)\\ f(x)=4+10(x-2)\\ f(x)=10x-16\to \text{equação da reta tangente em } P(2,4)\end{cases}$$

(iv) tangente horizontal : $m=0$

$$\begin{cases}6a-2=0\\ 6a=2\to a=\dfrac{1}{3}\end{cases}\Rightarrow x=\frac{1}{3}\therefore y=3\left(\frac{1}{9}\right)-2\left(\frac{1}{3}\right)-4=\frac{-13}{3}$$

$m=0$ em $P\left(\dfrac{1}{3},\dfrac{-13}{3}\right);$

c) $(i)\ f'(x)=\lim_{h\to 0}\dfrac{(a+h)^3+(a+h)-a^3-a}{h}=$

$$\lim_{h\to 0}\frac{\cancel{a^3}+3a^2\cancel{h}+3ah^{\cancel{2}}+h^{\cancel{3}}\cancel{+a}+\cancel{h}-\cancel{a^3}-\cancel{a}}{\cancel{h}}=\lim_{h\to 0}3a^2+3ah+h^2+1=3a^2+1$$

$(ii)\ Dom:\{x\in\Re\}$

$$(iii)\ P(1,2) \to \begin{cases} f(x) = f(1) + f'(1)(x-1) \\ f(x) = 2 + 4(x-1) \\ f(x) = 4x - 2 \to \textit{equação da reta tangente em } P(1,2) \end{cases}$$

(iv) tangente horizontal : $m = 0$

$$\begin{cases} 3a^2 + 1 = 0 \\ 3a^2 = -1 \end{cases} \Rightarrow \textit{não existe pontos } (x, y) \in \Re / m = 0$$

d) $(i)\ f'(x) = \lim\limits_{h \to 0} \dfrac{(a+h)^3 - 4(a+h) - a^3 + 4a}{h} =$

$$\lim_{h \to 0} \frac{a^3 + 3a^2h + 3ah^2 + h^3 - 4a - 4h - a^3 + 4a}{h} = \lim_{h \to 0} 3a^2 + 3ah + h^2 - 4 = 3a^2 - 4$$

$(ii)\ Dom : \{x \in \Re\}$

$$(iii)\ P(2,0) \to \begin{cases} f(x) = f(2) + f'(2)(x-2) \\ f(x) = 0 + 8(x-2) \\ f(x) = 8x - 16 \to \textit{equação da reta tangente em } P(2,0) \end{cases}$$

(iv) tangente horizontal : $m = 0$

$$\begin{cases} 3a^2 - 4 = 0 \\ 3a^2 = 4 \to a = \pm\dfrac{2\sqrt{3}}{3} \end{cases} \Rightarrow x = \pm\frac{2\sqrt{3}}{3} \therefore y = \mp\frac{16\sqrt{3}}{9}$$

$$m = 0 \quad em \quad P\left(\pm\frac{2\sqrt{3}}{3}, \mp\frac{16\sqrt{3}}{9}\right)$$

5 a) f(x) = 9x-2

(i) Pela regra da potência: f'(x) = 9

$(ii)\ Dom : \{x \in \Re\}$

$$(iii)\ P(3,25) \to \begin{cases} f(x) = f(3) + f'(3)(x-3) \\ f(x) = 25 + 9(x-3) \\ f(x) = 9x - 2 \to \textit{equação da reta tangente em } P(3,25) \end{cases}$$

(iv) tangente horizontal : $m = 0 \to 9 \neq 0, \forall x \therefore$ não existe pontos $(x, y) \in \Re / m = 0$

b) f(x) = -4x+3

(i) Pela regra da potência: f'(x) = -4

$(ii)\ Dom : \{x \in \Re\}$

$$(iii)\ P(-2{,}11) \rightarrow \begin{cases} f(x) = f(-2) + f'(-2)(x+2) \\ f(x) = 11 - 4(x+2) \\ f(x) = -4x + 3 \rightarrow \textit{equação da reta tangente em } P(-2{,}11) \end{cases}$$

(iv) tangente horizontal : $m = 0 \rightarrow -4 \neq 0, \forall x \therefore$ *não existe pontos* $(x, y) \in \Re / m = 0$

c) f(x) = 37

(i) Derivada de uma constante: f'(x) = 0

$(ii)\ Dom : \{x \in \Re\}$

$$(iii)\ P(0{,}37) \rightarrow \begin{cases} f(x) = f(0) + f'(0)(x-0) \\ f(x) = 37 - 0(x-0) \\ f(x) = 37 \rightarrow \textit{equação da reta tangente em } P(0{,}37) \end{cases}$$

(iv) tangente horizontal : $m = 0 \rightarrow \forall x, f'(x) = 0 \therefore m = 0 \forall (x, y)$

d) f(x) = π^2

(i) Derivada de uma constante: f'(x) = 0

$(ii)\ Dom : \{x \in \Re\}$

$$(iii)\ P(5, \pi^2) \rightarrow \begin{cases} f(x) = f(5) + f'(5)(x-5) \\ f(x) = \pi^2 - 0(x-5) \\ f(x) = \pi^2 \rightarrow \textit{equação da reta tangente em } P(5, \pi^2) \end{cases}$$

(iv) tangente horizontal : $m = 0 \rightarrow \forall x, f'(x) = 0 \therefore m = 0 \forall (x, y)$

e) f(x) = $1/x^3 = x^{-3}$

(i) Pela regra da potência: f'(x) = $-3x^{-4}$

$(ii)\ Dom : \{x \in \Re / x \neq 0\}$

$$(iii)\ P\left(2, \frac{1}{8}\right) \rightarrow \begin{cases} f(x) = f(2) + f'(2)(x-2) \\ f(x) = \frac{1}{8} - \frac{3}{16}(x-2) \\ f(x) = -\frac{3}{16}x + \frac{1}{2} \rightarrow \textit{equação da reta tangente em } P\left(2, \frac{1}{8}\right) \end{cases}$$

(iv) tangente horizontal : $m = 0$

$$\left\{ \frac{-3}{x^4} = 0 \Rightarrow \textit{não existe pontos } (x, y) \in \Re / m = 0 \right.$$

f) f(x) = $1/x^4 = x^{-4}$

(i) Pela regra da potência: f'(x) = $-4x^{-5}$

$(ii)\ Dom : \{x \in \Re / x \neq 0\}$

$$(iii)\ P(1,1) \to \begin{cases} f(x) = f(1) + f'(1)(x-1) \\ f(x) = 1 - 4(x-1) \\ f(x) = -4x + 5 \to \textit{equação da reta tangente em } P(1,1) \end{cases}$$

(iv) tangente horizontal : $m = 0$

$$\left\{ \frac{-4}{x^5} = 0 \Rightarrow \textit{não existe pontos } (x, y) \in \Re / m = 0 \right.$$

g) $f(x) = 4x^{1/4}$

(i) Pela regra da potência: $f'(x) = x^{-3/4}$

(ii) $Dom : \{x \in \Re / x > 0\}$

$$(iii)\ P(81,12) \to \begin{cases} f(x) = f(81) + f'(81)(x-81) \\ f(x) = 12 + \dfrac{1}{27}(x-81) \\ f(x) = \dfrac{x}{27} + 9 \to \textit{equação da reta tangente em } P(81,12) \end{cases}$$

(iv) tangente horizontal : $m = 0$

$$\left\{ \frac{1}{\sqrt[4]{x^3}} = 0 \Rightarrow \textit{não existe pontos } (x, y) \in \Re / m = 0 \right.$$

h) $f(x) = 12x^{1/3}$

(i) Pela regra da potência: $f'(x) = 4x^{-2/3}$

(ii) $Dom : \{x \in \Re / x \neq 0\}$

$$(iii)\ P(-27,-36) \to \begin{cases} f(x) = f(-27) + f'(-27)(x+27) \\ f(x) = -36 + \dfrac{4}{9}(x+27) \\ f(x) = \dfrac{4}{9}x - 24 \to \textit{equação da reta tangente em } P(-27,-36) \end{cases}$$

(iv) tangente horizontal : $m = 0$

$$\left\{ \frac{4}{x^{2/3}} = 0 \Rightarrow \textit{não existe pontos } (x, y) \in \Re / m = 0 \right.$$

6 a) $f(x) = 3x^6 \to \textit{Regra da Potência} \begin{cases} f'(x) = 18x^5 \\ f''(x) = 90x^4 \\ f'''(x) = 360x^3 \end{cases}$

b) $f(x)=6x^4 \rightarrow \text{Regra da Potência} \begin{cases} f'(x)=24x^3 \\ f''(x)=72x^2 \\ f'''(x)=144x \end{cases}$

c) $f(x)=9x^{2/3} \rightarrow \text{Regra da Potência} \begin{cases} f'(x)=6x^{-1/3} \\ f''(x)=-2x^{-4/3} \\ f'''(x)=\dfrac{8}{3}x^{-7/3} \end{cases}$

d) $f(x)=3x^{7/3} \rightarrow \text{Regra da Potência} \begin{cases} f'(x)=7x^{4/3} \\ f''(x)=\dfrac{28}{3}x^{1/3} \\ f'''(x)=\dfrac{28}{9}x^{-2/3} \end{cases}$

e) $z=25t^{9/5} \rightarrow \text{Regra da Potência} \begin{cases} D_tZ=45t^{4/5} \\ D_t^2Z=36t^{-1/5} \end{cases} \Rightarrow D_t^2Z=36t^{-1/5}$

f) $y=3x+5 \rightarrow \text{Regra da Potência} \begin{cases} D_ty=3 \\ D_t^2Z=0 \Rightarrow D_t^3Z=0 \\ D_t^3Z=0 \end{cases}$

g) $y=-4x+7 \rightarrow \text{Regra da Potência} \begin{cases} \dfrac{dy}{dx}=-4 \\ \dfrac{d^2y}{dx^2}=0 \Rightarrow \dfrac{d^3y}{dx^3}=0 \\ \dfrac{d^3y}{dx^3}=0 \end{cases}$

h) $z=64\sqrt[4]{t^3}=64t^{3/4} \rightarrow \text{Regra da Potência} \begin{cases} \dfrac{dz}{dt}=48t^{-1/4} \\ \dfrac{d^2z}{dt^2}=-12t^{-5/4} \end{cases} \Rightarrow \dfrac{d^2z}{dt^2}=-12t^{-5/4}$

7 Para uma função ser diferenciável em um dado intervalo a derivada desta função deve existir em todos os pontos pertencentes a este intervalo.

a) $f(x)=\dfrac{1}{x} \rightarrow f'(x)=-\dfrac{1}{x^2}$; Condição de Existência: $x \neq 0$.

(i) [0,2]: O intervalo inclui o ponto em que x=0; neste ponto a função não é diferenciável e, portanto, a função não é diferenciável no intervalo dado.

(ii) [1,3]: Em todos os pontos pertencentes ao intervalo a função é diferenciável, portanto, a função é diferenciável no intervalo dado.

b) $f(x) = \sqrt[3]{x} = x^{1/3} \rightarrow f'(x) = \dfrac{1}{3x^{2/3}}$; Condição de Existência: $x \neq 0$.

(i) [-1,1]: O intervalo inclui o ponto em que x=0; neste ponto a função não é diferenciável e, portanto, a função não é diferenciável no intervalo dado.

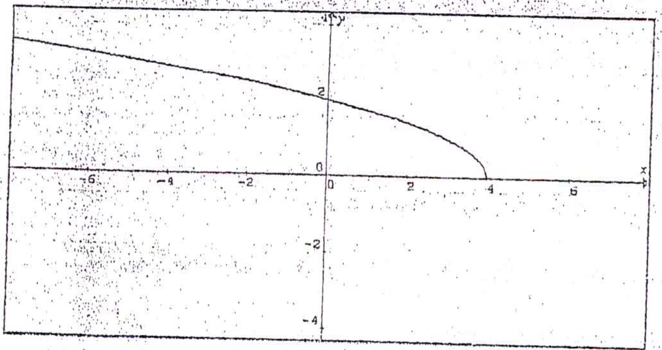
(ii) [-2,-1]: Em todos os pontos pertencentes ao intervalo a função é diferenciável, portanto, a função é diferenciável no intervalo dado.

8. Analisando o gráfico da função podemos encontrar os pontos em que esta não é diferenciável; pontos que não possuem reta tangente indicam pontos não diferenciáveis.

a) Analisando o gráfico percebemos que o ponto (4,0) é um ponto não diferenciável.

(i) [0,4]: A função não é diferenciável no intervalo dado.

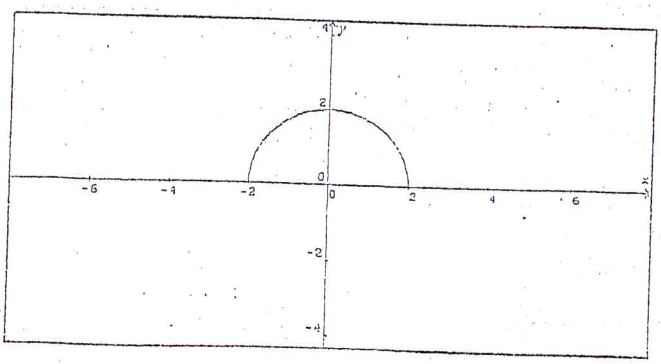
~~(ii) [-5,0]: Em todos os pontos~~ pertencentes ao intervalo a função é diferenciável, portanto, a função é diferenciável no intervalo dado.



b) Analisando o gráfico percebemos que os pontos (-2,0) e (2,0) são pontos não diferenciáveis.

(i) [-2,2]: A função não é diferenciável no intervalo dado.

(ii) [-1,1]: Em todos os pontos pertencentes ao intervalo a função é diferenciável, portanto, a função é diferenciável no intervalo dado.



9. Definição: (i) Tangente vertical em um ponto a: $\lim_{x\to a} f'(x)=\infty$; (ii) Ponto de reversão em um determinado a: $f'(x)$ *NE*.

a) $f(x)=x^{1/3} \rightarrow f'(x)=\dfrac{1}{3x^{2/3}} \begin{cases} \textit{Tangente Vertical}: \lim_{x\to 0}\dfrac{1}{3x^{2/3}}=+\infty \\ \textit{Ponto de Reversão}: f'(0)\ \textit{não está definido} \end{cases}$

(i) f(x) possui tangente vertical em (0,0);

(ii) f(x) possui ponto de reversão em (0,0);

b) $f(x)=x^{5/3} \rightarrow f'(x)=\dfrac{5x^{2/3}}{3} \begin{cases} \textit{Tangente Vertical}: \lim_{x\to 0}\dfrac{5x^{2/3}}{3}=0 \\ \textit{Ponto de Reversão}: f'(0)=0 \end{cases}$

(i) f(x) não possui tangente vertical em (0,0);

(ii) f(x) não possui ponto de reversão em (0,0);

c) $f(x)=x^{2/5} \rightarrow f'(x)=\dfrac{2}{5x^{3/5}} \begin{cases} \textit{Tangente Vertical}: \lim_{x\to 0}\dfrac{2}{5x^{3/5}}=+\infty \\ \textit{Ponto de Reversão}: f'(0)\ \textit{não está definido} \end{cases}$

(i) f(x) possui tangente vertical em (0,0);

(ii) f(x) possui ponto de reversão em (0,0);

d) $f(x)=x^{1/4} \rightarrow f'(x)=\dfrac{1}{4x^{3/4}} \begin{cases} \textit{Tangente Vertical}: \lim_{x\to 0}\dfrac{1}{4x^{3/4}}=+\infty \\ \textit{Ponto de Reversão}: f'(0)\ \textit{não está definido} \end{cases}$

(i) f(x) possui tangente vertical em (0,0);

(ii) f(x) possui ponto de reversão em (0,0);

e) $f(x)=5x^{3/2} \rightarrow f'(x)=\dfrac{15x^{1/2}}{2} \begin{cases} \textit{Tangente Vertical}: \lim_{x\to 0}\dfrac{15x^{1/2}}{2}=0 \\ \textit{Ponto de Reversão}: f'(0)=0 \end{cases}$

(i) f(x) não possui tangente vertical em (0,0);

(ii) f(x) não possui ponto de reversão em (0,0);

f) $f(x)=7x^{4/3} \rightarrow f'(x)=\dfrac{28x^{1/3}}{3} \begin{cases} \textit{Tangente Vertical}: \lim_{x\to 0}\dfrac{28x^{1/3}}{3}=0 \\ \textit{Ponto de Reversão}: f'(0)=0 \end{cases}$

(i) f(x) não possui tangente vertical em (0,0);

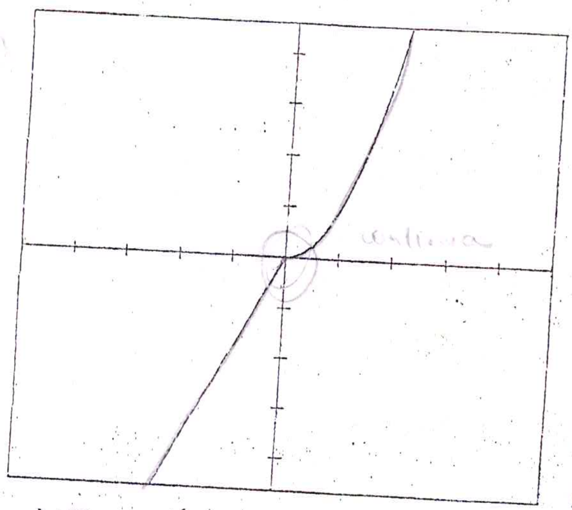
(ii) f(x) não possui ponto de reversão em (0,0);

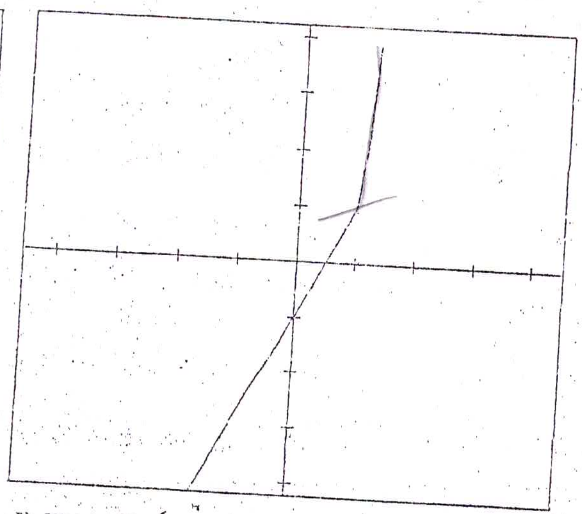
10. a) $f(x)=\begin{cases} x-5, & x>5 \\ -x+5, & x<5 \end{cases} \Rightarrow \left.\begin{cases} f'(5^+)=(x-5)'=1 \\ f'(5^-)=(-x+5)'=-1 \end{cases}\right\} f'(5^+)\neq f'(5^-)$ ∴ Não é diferenciável em a=5;

b) $f(x)=\begin{cases} x+2, & x>-2 \\ -x-2, & x<-2 \end{cases} \Rightarrow \left.\begin{cases} f'(-2^+)=(x+2)'=1 \\ f'(-2^-)=(-x-2)'=-1 \end{cases}\right\} f'(-2^+)\neq f'(-2^-)$ ∴ Não é diferenciável em a=-2;

11 a) Domf': $\{x\in\Re / x\neq 0\}$

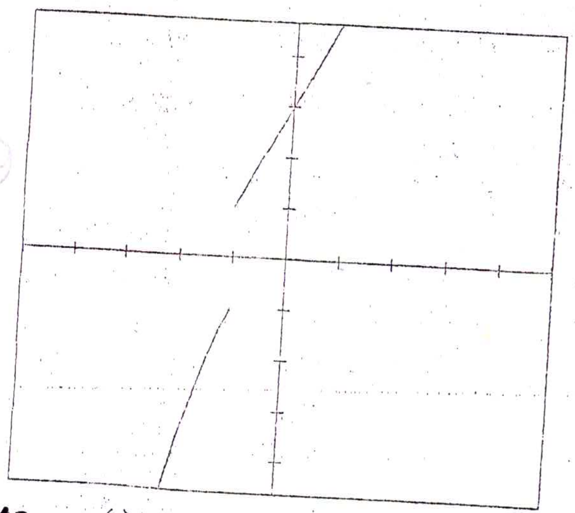
b) Domf': $\{x\in\Re / x\neq 1\}$

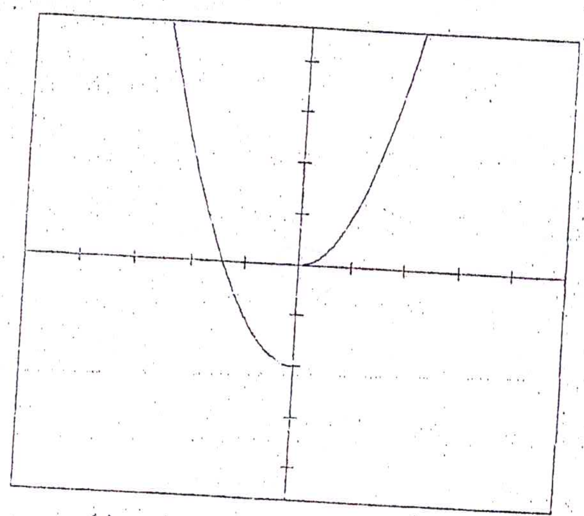




c) Domf': $\{x\in\Re / x\neq -1\}$

d) Domf': $\{x\in\Re / x\neq 0\}$





12. a) $g(t)=6t^{5/3} \rightarrow$ *Pela regra da potência* : $g'(t)=10t^{2/3}$;

b) $h(z)=8z^{3/2} \rightarrow$ *Pela regra da potência* : $h'(z)=12z^{1/2}$;

c) $f(s)=15-s+4s^2-5s^4 \rightarrow$ *Pela regra da potência* : $f'(s)=-1+8s-20s^3$;

d) $f(x)=3x^2+x^{4/3} \rightarrow$ *Pela regra da potência* : $f'(x)=6x+\dfrac{4}{3}x^{1/3}$;

e) $g(x)=(x^3-7)(2x^2+3) \rightarrow$ *Pela regra do produto* : $g'(x)=3x^2(2x^2+3)+4x(x^3-7)$

$g'(x)=6x^4+9x^2+4x^4-28x \rightarrow g'(x)=10x^4+9x^2-28x$;

f) $k(x)=(2x^2-4x+1)(6x-5) \rightarrow$ *Pela regra do produto*: $k'(x)=(4x-4)(6x-5)+6(2x^2-4x+1)$

$k'(x)=24x^2-20x-24x+20+12x^2-24x+6 \rightarrow k'(x)=36x^2-68x+26$;

g) $f(x)=x^{1/2}(x^2+x-4) \rightarrow$ *Pela regra do produto*: $f'(x)=\frac{1}{2x^{1/2}}(x^2+x-4)+x^{1/2}(2x+1)$

$f'(x)=\frac{1x^{3/2}}{2}+\frac{1}{2}x^{1/2}-\frac{2}{x^{1/2}}+2x^{3/2}+x^{1/2} \rightarrow f'(x)=\frac{5}{2}x^{3/2}+\frac{3}{2}x^{1/2}-2x^{-1/2}$;

h) $h(x)=x^{2/3}(3x^2-2x+5) \rightarrow$ *Pela regra do produto*: $f'(x)=\frac{2}{3x^{1/3}}(3x^2-2x+5)+x^{2/3}(6x-2)$

$h'(x)=2x^{5/3}-\frac{4x^{2/3}}{3}+\frac{10}{3x^{1/3}}+6x^{5/3}-2x^{2/3} \rightarrow h'(x)=8x^{2/3}-\frac{10x^{2/3}}{3}+\frac{10}{3x^{1/3}}$

i) $h(r)=r^2(3r^4-7r+2) \rightarrow$ *Pela regra do produta* $h'(r)=2r(3r^4-7r+2)+r^2(12r^3-7)$

$h'(r)=6r^5-14r^2+4r+12r^5-7r^2 \rightarrow h'(r)=18r^5-21r^2+4r$;

j) $k(v)=v^3(-2v^3+v-3) \rightarrow$ *Pela regra do produta* $k'(v)=3v^2(-2v^3+v-3)+v^3(-6v^2+1)$

$k'(v)=-6v^5+3v^3-9v^2-6v^5+v^3 \rightarrow k'(v)=-12v^5+4v^3-9v^2$;

k) $g(x)=(8x^2-5x)(13x^2+4) \rightarrow$ *Pela regra do produta* $g'(x)=(16x-5)(13x^2+4)+(8x^2-5x)(26x)$

$g'(x)=208x^3+64x-65x^2-20+208x^3-130x^2 \rightarrow g'(x)=416x^3-195x^2+64x-20$

l) $H(z)=(z^5-2z^3)(7z^2+z-8) \rightarrow$ *Pela regrado produta* $H'(z)=(5z^4-6z^2)(7z^2+z-8)+(z^5-2z^3)(14z+1)$

$H'(z)=35z^6+5z^5-40z^4-42z^4-6z^3+48z^2+14z^6+z^5-28z^4-2z^3$

$H'(z)=49z^6+6z^5-110z^4-8z^3+48z^2$;

m) $f(x)=\frac{4x-5}{3x+2} \rightarrow$ *pela regrado quociente* $f'(x)=\frac{1}{(3x+2)^2}(4*(3x+2)-3*(4x-5))$

$f'(x)=\frac{12x+8-12x+15}{(3x+2)^2}=\frac{23}{(3x+2)^2}$;

n) $h(z)=\frac{8-z+3z^2}{2-9z} \rightarrow$ *pela regrado quociente* $h'(z)=\frac{1}{(2-9z)^2}((-1+6z)(2-9z)+9(8-z+3z^2))$

$h'(z)=\frac{-2+9z+12z-54z^2+72-9z+27z^2}{(2-9z)^2}=\frac{-27z^2+12z+70}{(2-9z)^2}$;

o) $g(t)=\frac{\sqrt[3]{t^2}}{3t-5} \rightarrow$ *pela regrado quociente* $g'(t)=\frac{1}{(3t-5)^2}\left(\frac{2}{3}t^{-1/3}(3t-5)-t^{2/3}*3\right)$

$$g'(t) = \frac{2t^{2/3} - \frac{10}{3}t^{-1/3} - 3t^{2/3}}{(3t-5)^2} = \frac{-t^{2/3} - \frac{10}{3}t^{-1/3}}{(3t-5)^2};$$

p) $f(x) = \frac{1}{1+x+x^2+x^3} \rightarrow$ *pela regra do quociente* $f'(x) = \frac{1}{(1+x+x^2+x^3)^2}((0)(1+x+x^2+x^3) - 1(1+2x+3x^2))$

$$f'(x) = \frac{-1-2x-3x^2}{(1+x+x^2+x^3)^2} = \frac{-(1+2x+3x^2)}{(1+x+x^2+x^3)^2};$$

q) $f(x) = 1 + \frac{1}{x} + \frac{1}{x^2} + \frac{1}{x^3}$; Técnica de diferenciação utilizada : $\frac{d}{dx}[f(x)+g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$

$$f'(x) = -\frac{1}{x^2} - \frac{2}{x^3} - \frac{3}{x^4} = -\frac{(x^2+2x+3)}{x^4};$$

r) $f(t) = t^2 + \frac{1}{t^2}$; Técnica de diferenciação utilizada: $\frac{d}{dx}[f(x)+g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$;

$$f'(t) = 2t - \frac{2}{t^3} = \frac{2(t^4-1)}{t^3};$$

s) $g(r) = (5r-4)^{-2} \rightarrow$ Re *gra da cadeia* : $\begin{cases} f(r) = r^{-2}; \\ u(r) = 5r-4 \end{cases}; \rightarrow \frac{d}{dr}[f(u(r))] = \frac{d}{du}f * \frac{d}{dx}u$

$$g'(r) = -2(5r-4)^{-3} * 5 = -\frac{10}{(5r-4)^3};$$

t) $f(t) = \frac{\frac{3}{5t} - 1}{\frac{2}{t^2} + 7} = \frac{\frac{3-5t}{5t}}{\frac{2+7t^2}{t^2}} = \frac{3-5t}{5t} * \frac{t^2}{2+7t^2} = \frac{3t-5t^2}{10+35t^2}$ errado

Pela regra do quociente: $f'(t) = \frac{1}{(10+35t^2)^2}[(3-10t)(10+35t^2) - (3t-5t^2)70t]$

$$f'(t) = \frac{30+105t^2-100t-350t^3-210t^2+350t^3}{(10+35t^2)^2} = \frac{-105t^2-100t+30}{(10+35t^2)^2}$$

u) $N(z) = \frac{\frac{4}{z^2}}{\frac{3}{z}+2} = \frac{\frac{4}{z^2}}{\frac{3+2z}{z}} = \frac{4}{z^2} * \frac{z}{3+2z} = \frac{4}{3z+2z^2}$

Pela regra do quociente: $N'(z) = \frac{1}{(3z+2z^2)^2}[-4(3+4z)] = -\frac{12+16z}{(3z+2z^2)^2};$

$$D_x y = 6x^2 - 6x - 36 = 0 \rightarrow 6(x^2 - x - 6) = 0 \therefore x^2 - x - 6 = 0$$

13 a)
$$\Delta = (-1)^2 - 4*1*(-6) = 25 \rightarrow x = \frac{-(-1) \pm \sqrt{25}}{2*1} = \begin{cases} x_1 = -2 \\ x_2 = 3 \end{cases}$$

Solução: {x= -2 e x=3}.

$$D_x y=\frac{1}{(x-2)^2}\left[(x-2)(4x+3)-\left(2x^2+3x-6\right)\right]=\frac{4x^2+3x-8x-6-2x^2-3x+6}{(x-2)^2}=0$$

b)
$$\frac{2x^2-8x}{(x-2)^2}=0\rightarrow 2x(x-4)=0\begin{cases}x=0\\ x-4=0\rightarrow x=4\end{cases}\Rightarrow \textit{Solução}:\{x=0\quad e\quad x=4\}$$

14. a)
$$D_x^2 y=\frac{d}{dx}\left(24x^3+72x^2-1080x\right)=0\rightarrow 72x^2+144x-1080=72\left(x^2+2x-15\right)=0$$

$$x^2+2x-15=0\therefore\Delta=4-4*1*(-15)=64\quad e\quad x=\frac{-2\pm\sqrt{64}}{2*1}\begin{cases}x_1=-5\\ x_2=3\end{cases}$$

Solução: {x= -68 e x=76}.

b)
$$D_x^2 y=\frac{d}{dx}\left(5x^4-20x^3-90x^2+11\right)=0\rightarrow 20x^3-60x^2-180x=0\therefore 20x\left(x^2-3x-9\right)=0$$

$$\begin{cases}x=0\\ x^2-3x-9=0\end{cases}\Rightarrow\Delta=9+36=45\quad e\quad x=\frac{3\pm\sqrt{45}}{2}=\frac{3\pm3\sqrt{5}}{2}$$

$$\text{Solução:}\left\{x=0;\quad x=\frac{3+3\sqrt{5}}{2};\quad x=\frac{3-3\sqrt{5}}{2}\right\}.$$

15. a)
$$(i)\ y'=\frac{1}{x^4}\left[3x^2-2x(3x-1)\right]=\frac{3x^{\cancel{2}}-6x^{\cancel{2}}+2\cancel{x}}{x^4}=\frac{-3x+2}{x^3};$$

$$(ii)\ y'=3*x^{-2}+(3x-1)*\left(-2x^{-3}\right)=\frac{3}{x^2}-\frac{2(3x-1)}{x^3}=\frac{3x-6x+2}{x^3}=\frac{-3x+2}{x^3}$$

b)
$$(i)\ y'=\frac{1}{x^3}\left[2x^{3/2}-(2x+3)\left(\frac{3}{2}x^{1/2}\right)\right]=\frac{2x^{3/2}-3x^{3/2}-\frac{9}{2}x^{1/2}}{x^3}=-\left(x^{-3/2}+\frac{9}{2}x^{-5/2}\right);$$

$$(ii)\ y'=2x^{-3/2}+(2x+3)\left(-\frac{3}{2}x^{-5/2}\right)=2x^{-3/2}-3x^{-3/2}-\frac{9}{2}x^{-5/2}=-\left(x^{-3/2}+\frac{9}{2}x^{-5/2}\right);$$

16. a)
$$\begin{cases}\dfrac{dy}{dx}=\dfrac{1}{(x+1)^2}[3x+3-3x-4]=\dfrac{-1}{(x+1)^2}\\[2ex] \dfrac{d^2y}{dx^2}=\dfrac{d}{dx}\left(\dfrac{dy}{dx}\right)=\dfrac{1}{(x+1)^4}\left[0-\left(-2(x+1)\right)\right]=\dfrac{2}{(x+1)^3}\end{cases}\Rightarrow\frac{d^2y}{dx^2}=\frac{2}{(x+1)^3}$$

b)
$$\begin{cases}\dfrac{dy}{dx}=\dfrac{1}{(2x+3)^2}[2x+3-2x-6]=\dfrac{-3}{(2x+3)^2}\\[2ex] \dfrac{d^2y}{dx^2}=\dfrac{d}{dx}\left(\dfrac{dy}{dx}\right)=\dfrac{1}{(2x+3)^4}\left[0-\left(-6(2x+3)\cancel{*2}\right)\right]=\dfrac{12}{(2x+3)^3}\end{cases}\Rightarrow\frac{d^2y}{dx^2}=\frac{12}{(2x+3)^3}$$

17. Para tangente ser perpendicular a y = -x/2 + 7/2 → y'= 2. $\Rightarrow -\frac{1}{2}$

$$y=x^{5/3}+x^{1/3}\rightarrow y'=\frac{5}{3}x^{2/3}+\frac{1}{3}x^{-2/3}=2$$

$y = x^{2/3}$

$y' = 5x^{2/3} + x^{-2/3} = 6 \Rightarrow 5x^{4/3} - 6x^{2/3} + 1 = 0 \therefore 5x^{4/3} - 6x^{2/3} = -1$ $5y^2 - 6y + 1 = 0$

Essa condição é verificada para os pontos em que x=1 e x= -1, logo, a solução para o problema são os pontos: (1,2) e (-1,-2).

18. a) $f'(2) + g'(2) = -1 + 2 = 1$;

b) $f'(2) - g'(2) = -1 - 2 = -3$;

c) $4f'(2) = 4*(-1) = -4$;

d) $f'(2)g(2) + f(2)g'(2) = (-1)*(-5) + 3*2 = 11$;

e) $\frac{1}{g(2)^2}[f'(2)g(2) - f(2)g'(2)] = \frac{5-6}{25} = \frac{-1}{25}$;

f) $\frac{1}{f(2)^2}[0 - f'(2)] = \frac{-(-1)}{9} = \frac{1}{9}$;

g) $f'(2)f(2) + f(2)f'(2) = 2*(-3) = -6$

h) $\frac{1}{(f(2)+g(2))^2}[-f'(2) - g'(2)] = \frac{-1}{4} =$

i) $\frac{1}{(f(2)+g(2))^2}[f'(2)(f(2)+g(2)) - (f'(2)+g'(2))f(2)] = \frac{2-3}{4} = \frac{-1}{4}$

19. a) $f'(x) = 4(\cos x)' = -4 sen x$;

b) $H'(z) = 7(tg z)' = 7\sec^2 z$;

c) $G'(v) = 5(v \csc v)' = 5[\csc v + v(-\csc v * ctg v)] = 5\csc v(1 - v ctg v)$;

d) $f'(x) = 3(x sen x)' = 3[sen x + x\cos x] = 3 sen x + 3x\cos x$;

e) $k'(t) = (t)' - (t^2\cos t)' = 1 - (2t\cos t - t^2 sen t) = (sen t)t^2 - 2(\cos t)t + 1$;

f) $p'(w) = (w^2)' + (w sen w)' = 2w + (sen w + w\cos w) = (2 + \cos w)w + sen w$;

g) $f'(\theta) = \frac{1}{\theta^2}(\theta\cos\theta - sen\theta) = \frac{\cos\theta}{\theta} - \frac{sen\theta}{\theta^2}$;

h) $g'(\alpha) = \frac{1}{\alpha^2}(\alpha sen\alpha - 1 + \cos\alpha) = \frac{sen\alpha}{\alpha} + \frac{\cos\alpha - 1}{\alpha^2}$;

i) $g'(t) = 3t^2 sen t + t^3\cos t$;

j) $T'(r) = 2r\sec r + r^2(\sec r\, tg r) = \sec r[(tg r)r^2 + 2r]$;

k) $f'(x) = (2x\cot x)' + (x^2 tg x)' = 2(\cot x + x(-\csc^2 x)) + (2x tg x + x^2\sec^2 x)$

$f'(x) = 2\cot x - 2x\csc^2 x + 2x tg x + x^2\sec^2 x = 2\frac{\cos x}{sen x} - 2x\frac{1}{sen^2 x} + 2x\frac{sen x}{\cos x} + x^2\frac{1}{\cos^2 x}$

$$f'(x)=x^2\sec^2 x+2x(tgx-\csc^2 x)+2\cot x;$$

l) $h'(z)=\dfrac{1}{(1+\cos z)^2}[senz(1+\cos z)+senz(1-\cos z)]=\dfrac{senz(1+\cos z+1-\cos z)}{(1+\cos z)^2}=\dfrac{2senz}{(1+\cos z)^2};$

m)
$$g'(x)=\frac{1}{(senxtgx)^2}[-(\cos xtgx+senx\sec^2 x)]=\frac{-(senx+senx\sec^2 x)}{(senxtgx)^2}=-\frac{senx(1+\sec^2 x)}{sen^2xtg^2x}$$
$$g'(x)=-\frac{1+\sec^2 x}{senxtg^2x}=-\frac{1+\cos^2 x}{\cos^2 x}*\frac{\cos^2 x}{sen^3x}=-\frac{1+\cos^2 x}{sen^3x};$$

n) $g'(x)=(1+(-\csc xctgx))ctgx+(x+\csc x)(-\csc^2 x)=ctgx-\csc xctg^2x-x\csc^2 x-\csc^3 x;$

$$g'(x)=ctgx-\csc xctg^2x-x\csc^2 x-\csc^3 x=\frac{\cos x}{senx}-\frac{\cos^2 x}{sen^3x}-x\frac{1}{sen^2x}-\frac{1}{sen^3x}$$

$$g'(x)=\frac{sen^2x\cos x-\cos^2 x-xsenx-1}{sen^3x};$$

o)
$$k'(\theta)=2(sen\theta+\cos\theta)*(\cos\theta-sen\theta)=2(\cos^2\theta-sen^2\theta)=2(\cos^2\theta-1+\cos^2\theta)$$
$$k'(\theta)=4\cos^2\theta-2;$$

p) Reescrever $H(\phi)$ como: $H(\phi)=1-\cos\phi+\sec\phi-1=(1-\cos^2\phi)/\cos\phi$.

$$H'(\phi)=\frac{1}{\cos^2\phi}[(2sen\phi\cos\phi)\cos\phi+sen\phi(1-\cos^2\phi)]=2sen\phi+\frac{sen^3\phi}{\cos^2\phi};$$

q)
$$f'(x)=\frac{1}{(tgx+senx)^2}[(tgx+senx)(\sec xtgx)-(1+\sec x)(\sec^2 x+\cos x)]$$
$$f'(x)=\frac{\sec xtg^2x+tg^2x-\sec^2 x-\cos x-\sec^3 x-1}{(tgx+senx)^2}=\frac{tg^2x(1+\sec x)-\sec^2 x(1+\sec x)-\cos x-1}{(tgx+senx)^2}$$

$$f'(x)=\frac{(1+\sec x)(tg^2x-\sec^2 x)-(1+\cos x)}{(tgx+senx)^2}=\frac{(1+\sec x)(-1)-(1+\cos x)}{(tgx+senx)^2}=\frac{-2-\sec x-\cos x}{(tgx+senx)^2}$$

20 a) $f'(x)=\sec xtgx\rightarrow\begin{cases}f(x)=f(\pi/4)+f'(\pi/4)(x-\pi/4)\\ f(x)=\sqrt{2}+\sqrt{2}(x-\pi/4)\\ f(x)=\sqrt{2}x+\sqrt{2}\left(1-\dfrac{\pi}{4}\right)\rightarrow equação\ da\ reta\ tangente\ em\ (\pi/4,\sqrt{2})\end{cases}$

Reta normal possui $m=-1/\sqrt{2}$:

$$m=\frac{-\sqrt{2}}{2}\rightarrow\begin{cases}f(x)=f(\pi/4)+f'(\pi/4)(x-\pi/4)\\ f(x)=\sqrt{2}-\dfrac{\sqrt{2}}{2}(x-\pi/4)\\ f(x)=-\dfrac{\sqrt{2}}{2}x+\sqrt{2}\left(1+\dfrac{\pi}{8}\right)\rightarrow equação\ da\ reta\ normal\ em\ (\pi/4,\sqrt{2})\end{cases}$$

23.

$$a)\ f(x)=\cos x\begin{cases} f'(x)=-senx\\ f''(x)=-\cos x\\ f'''(x)=senx\\ f^{iv}(x)=\cos x\end{cases};$$

$b)$ *A cada quatro derivadas obtemos a função inicial* : $\frac{99}{4}=(4*24)+3 \therefore f^{99}=f^{3}=senx;$

$$24.\ a)\ f(x)=\cot x\begin{cases} f'(x)=-\csc^2 x\\ f''(x)=-(2\csc x*(-\csc xctgx))=2\csc^2 xctgx\\ f'''(x)=2[ctgx(\csc^2 x)+\csc^2 x(ctgx)]=-4\csc^2 xctg^2x-2\csc^4 x\end{cases}$$

$$b)\ f(x)=tgx\begin{cases} D_x y=\sec^2 x\\ D^2_x y=\sec x(\sec xtgx)+\sec x(\sec xtgx)=2\sec^2 xtgx\\ D^3_x y=2[tgx(\sec^2 x)+\sec^2 x(\sec^2 x)]=4\sec^2 xtg^2x+2\sec^4 x\end{cases}$$

$$c)\ f(x)=\sec x\begin{cases} \frac{dy}{dx}=\sec xtgx\\ \frac{d^2y}{dx^2}=\sec x(\sec^2 x)+tgx(\sec xtgx)=\sec^3 x+\sec xtg^2x\\ \frac{d^3y}{dx^3}=3\sec^2 x(\sec xtgx)+\sec x(2tgx\sec^2 x)+tg^2x(\sec xtgx)=5\sec^3 xtgx+tg^3x\sec x\end{cases}$$

25. **a)** $D_x \cot x=D_x\frac{\cos x}{senx}=\frac{1}{sen^2x}[(-senx)(senx)-(\cos x)(\cos x)]=\frac{-1(sen^2x+\cos^2 x)}{sen^2x}=\frac{-1}{sen^2x}=-\csc^2 x;$

b) $D_x \csc x=D_x\frac{1}{senx}=\frac{1}{sen^2x}(0-(1*\cos x))=\frac{-\cos x}{senx}*\frac{1}{senx}=-\cot x\csc x;$

c) $D_x sen2x=D_x(2senx\cos x)=2(senx(-senx)+\cos x(\cos x))=2(\cos^2 x-sen^2x)=2\cos 2x;$

d) $D_x \cos 2x=D_x(\cos^2 x-sen^2x)=2\cos x(-senx)-2senx(\cos x)=-4senx\cos x=-2sen2x;$

26. **a)** $f'(x)=2x+4+\frac{1}{x}$; **b)** $f'(t)=6t^2-\frac{3}{t}$; **c)** $f'(x)=-\frac{1}{x}$; **d)** $f'(x)=\frac{2}{x}+\frac{1}{x^2}$;

e) $f'(x)=\frac{1}{x^2}(x(\ln x+1)-1(x\ln x+1))=\frac{x-1}{x^2}$; **f)** $f'(x)=\frac{1}{4}\left(\frac{2}{x}-0\right)=\frac{1}{2x}$;

g) $f'(t)=3e^t-2t$; **h)** $f'(x)=2e^x+\frac{1}{2}x^{-3/2}$; **i)** $f'(x)=e^x-\frac{1}{x}$; **j)** $f'(t)=\frac{1}{4}(2e^t-0)=\frac{e^t}{2}$;

k) $f'(t)=e^t-\frac{1}{t^2}-\frac{2}{t^3}$; **l)** Reescrever a equação como: f(x) = x + e^x +x = 2x + e^x;

portanto: $f'(x)=2+e^x$;

27. **a)** $f'(x)=(1*e^{-x})+(-1e^{-x}*x)=e^{-x}-xe^{-x}=e^{-x}(1-x);$

b) $f'(x)=\left(\left(3x^2-2x\right)*e^{-x}\right)+\left(-1e^{-x}*\left(x^3-x^2+4\right)\right)=3x^2e^{-x}-2xe^{-x}-x^3e^{-x}+x^2e^{-x}-4e^{-x}$
$f'(x)=-x^3e^{-x}+4x^2e^{-x}-2xe^{-x}-4e^{-x}=e^{-x}\left(-x^3+4x^2-2x-4\right)$;

c) $f'(x)=\ln x\left(-x^{-2}\right)+\left(x^{-1}\right)\left(\frac{1}{x}\right)=\frac{1-\ln x}{x^2}$;

d) $f'(x)=4x^3e^{2x}+x^4 2e^{2x}=2e^{2x}\left(x^4+2x^3\right)$;

e) $f'(x)=\frac{1}{x}e^{-3x}-3\ln xe^{-3x}=e^{-3x}\left(\frac{1}{x}-3\ln x\right)$;

f) $f'(x)=\frac{1}{2\sqrt{x}}\left(x-\frac{1}{x}\right)+\sqrt{x}\left(1+\frac{1}{x^2}\right)$;

23. a) $f'(x)=\frac{1}{\left(x^2+4\right)^2}\left(2x\left(x^2+4\right)-2x\left(x^2-2\right)\right)=\frac{8x+4x}{\left(x^2+4\right)^2}=\frac{12x}{\left(x^2+4\right)^2}$;

b) $f'(x)=\frac{1}{(x+1)^2}\left(\frac{1}{x}(x+1)-1(\ln x)\right)=\frac{1+x^{-1}-\ln x}{(x+1)^2}$;

c) $f'(x)=\frac{1}{\left(e^x+1\right)^2}\left[\left(e^x+1\right)-xe^x\right]=\frac{e^x(1-x)+1}{\left(e^x+1\right)^2}$;

d) $f'(x)=\frac{1}{(2x-3)^2}\left(3(2x-3)-2(3x+1)\right)=\frac{6x-9-6x-2}{(2x-3)^2}=\frac{-11}{(2x-3)^2}$;

e) $f'(x)=\frac{1}{\left(e^x+1\right)^2}\left(e^x\left(e^x+1\right)-e^x\left(e^x-1\right)\right)=\frac{2e^x}{\left(e^x+1\right)^2}$;

f) $f'(x)=\frac{1}{(\ln x)^2}\left(1(\ln x)-x\left(1/x\right)\right)=\frac{\ln x-1}{(\ln x)^2}$;

LISTA 4:

Derivadas II

Lista 4: Derivadas II

1. a) $y = 2x^2 - 4x + 5$

(i) $\Delta y = f(x+\Delta x) - f(x) = 2(x+\Delta x)^2 - 4(x+\Delta x) + 5 - 2x^2 + 4x - 5$

$\Delta y = 2x^2 + 4x\Delta x + 2(\Delta x)^2 - 4x - 4\Delta x - 2x^2 + 4x = 4x(\Delta x) + 2(\Delta x)^2 - 4(\Delta x)$

$\Delta y = \Delta x(4x - 4) + 2(\Delta x)^2$

$dy = f'(x) \cdot dx = (4x - 4)\, dx$

(ii) $x = a = 2 \quad \Delta x = -0{,}2$ se Δx é muito pequeno: $dx \simeq \Delta x$

$\Delta y = -0{,}2(4*2-4) + (-0{,}2)^2 = -0{,}72$

$dy = (4*2-4)*(-0{,}2) = -0{,}8$

b) $y = x^3 - 4$

(i) $\Delta y = (x+\Delta x)^3 - 4 - x^3 + 4 = x^3 + 3x^2(\Delta x) + 3x(\Delta x)^2 + (\Delta x)^3 - x^3$

$\Delta y = 3x^2(\Delta x) + 3x(\Delta x)^2 + (\Delta x)^3$

$dy = 3x^2 dx$

(ii) $x = a = -1 \quad \Delta x = 0{,}1$

$\Delta y = 3(-1)^2(0{,}1) + 3(-1)(0{,}1)^2 + (0{,}1)^3 = 0{,}271$

$dy = 3(-1)^2 * 0{,}1 = 0{,}3$

c) $y = 1/x^2$

(i) $\Delta y = 1/(x+\Delta x)^2 - 1/x^2 = \dfrac{1}{x^2 + 2x\Delta x + (\Delta x)^2} - \dfrac{1}{x^2} = \dfrac{x^2 - x^2 - 2x\Delta x - (\Delta x)^2}{x^2(x^2 + 2x\Delta x + (\Delta x^2))}$

$\Delta y = \dfrac{-2x(\Delta x) - (\Delta x)^2}{x^2(x^2 + 2x\Delta x + (\Delta x)^2)}$

$dy = \dfrac{-2}{x^3} dx$

(ii) $\Delta y = \dfrac{-2*3*(0{,}3) - (0{,}3)^2}{9(9 + 2*3*0{,}3 + (0{,}3)^2)} = \dfrac{-1{,}89}{98{,}01} = -0{,}01928$

$dy = \dfrac{-2}{3^3} * 0{,}3 = -0{,}02222$

d) $y = 1/2 + x$

(i) $\Delta y = \dfrac{1}{2+(x+\Delta x)} - \dfrac{1}{2+x}$ e $dy = \dfrac{-1}{(2+x)^2} dx$

(ii) $\Delta y = \dfrac{1}{2-0{,}03} - \dfrac{1}{2} = 0{,}0076$

$dy = -1/2^2 \, dx = -1/4 * (-0{,}03) = 0{,}0075$

2. a) $y = 4 - 9x$

i) $\Delta y = f(x+\Delta x) - f(x) = 4 - 9x - 9\Delta x - 4 + 9x = -9\Delta x \quad \therefore \Delta y = -9\Delta x$

ii) $dy = -9dx$

iii) $dy - \Delta y = -9dx + 9\Delta x = 9(\Delta x - dx)$

b) $y = 7x + 12$

(i) $\Delta y = 7x + 7\Delta x + 12 - 7x - 12 = 7\Delta x \quad \therefore \Delta y = 7\Delta x$

(ii) $dy = 7dx$

(iii) $dy - \Delta y = 7dx - 7\Delta x = 7(dx - \Delta x)$

c) $y = 3x^2 + 5x - 2$

(i) $\Delta y = 3(x^2 + 2x\Delta x + \Delta x^2) + \cancel{5x} + 5\Delta x - \cancel{2} - 3x^2 - \cancel{5x} + \cancel{2} = 6x\Delta x + 3(\Delta x)^2 + 5\Delta x$

$\Delta y = 3(\Delta x)^2 + \Delta x(6x+5)$

(ii) $dy = (6x+5)dy$

(iii) $dy - \Delta y = (6x-5)dy - (6x+5)\Delta x - 3(\Delta x)^2 = (6x+5)(dy - \Delta x) - 3(\Delta x)^2$

d) $y = 4 - 7x - 2x^2$

(i) $\Delta y = \cancel{4} - \cancel{7x} - 7\Delta x - 2(x^2 + 2x\Delta x + (\Delta x)^2) - \cancel{4} + \cancel{7x} + 2x^2$

$\Delta y = -7\Delta x - 4x\Delta x - 2(\Delta x)^2 = (-7 - 4x)\Delta x - 2(\Delta x)^2$

(ii) $dy = (-7 - 4x)dx$

(iii) $dy - \Delta y = (-7 - 4x)dx - (-7 - 4x)\Delta x - 2(\Delta x)^2 = (-7 - 4x)(dx - \Delta x) - 2(\Delta x)^2$

e) $y = 1/x$

(i) $\Delta y = \frac{1}{x+\Delta x} - \frac{1}{x} = \frac{x - x - \Delta x}{x^2 + x\Delta x} = \frac{-\Delta x}{x^2 + x\Delta x}$

(ii) $dy = \frac{-1}{x^2} dx$

(iii) $dy - \Delta y = \frac{-1}{x^2} dx + \frac{\Delta x}{x^2 + x\Delta x}$

f) $y = 1/x^2$

(i) $\Delta y = 1/(x+\Delta x)^2 - 1/x^2$

(ii) $dy = -2/x^3 \, dx$

(iii) $dy - \Delta y = -\frac{2dx}{x^3} - \frac{1}{(x+\Delta x)^2} + \frac{1}{x^2}$

3. a) $f(x) = 4x^5 - 6x^4 + 3x^2 - 5$, $a = 1$ e $b = 1,03$ $\therefore \Delta x = (b-a) = 0,03$

$f(1,03) = f(1) + f'(1) . \Delta x$

$\rightarrow f(1) = -4(1)^5 - 6(1)^4 + 3(1)^2 - 5 = -4$

$\rightarrow f'(1) = 20(1)^4 - 24(1)^3 + 6(1) - 8 = 2$

$f(1,03) = -4 + 2 * (0,03) = -3,94$

b) $f(x) = -3x^3 - 8x - 7$ $a = 4$ $b = 3,96$ $\therefore \Delta x = (b-a) = -0,04$

$f(3,96) = f(4) + f'(4) . \Delta x$

$\rightarrow f(4) = -3 * (4)^3 - 8(4) - 7 = -231$

$\rightarrow f'(4) = -9(4)^2 - 8 = -44$

$f(3,96) = -231 + (-44)(-0,04) = -229,24$

c) $f(\theta) = 2 \operatorname{sen}\theta + \cos\theta$ $a = 30°$ $b = 27°$

$1° = 2\pi/360$ $\therefore \Delta x = (b-a)\, 2\pi/360 = -\frac{3 * 2\pi}{360} = -\pi/60$

$\rightarrow f(30°) = 2 \operatorname{sen} 30 + \cos 30 = 1 + \sqrt{3}/2$

$\rightarrow f'(30) = 2\cos 30 - \operatorname{sen} 30 = \sqrt{3} - 1/2$

$\therefore f'(27) = 1 + \frac{\sqrt{3}}{2} + \left(\sqrt{3} - \frac{1}{2}\right) - \frac{\pi}{60} = 1 + \frac{\sqrt{3}}{2} - \frac{\pi\sqrt{3}}{60} + \frac{\pi}{120} \cong 1,841$

d) $f(\phi) = \csc\phi + \cot\phi$ $a = 45°$ $b = 46°$ $\Delta x = (b-a)\, 2\pi/360 = \pi/180$

$f'(\phi) = -\csc\phi \cot\phi - \csc^2\phi$

$\rightarrow f(45°) = \csc 45° + \cot 45° = \sqrt{2} + 1$

$\rightarrow f'(45°) = -\csc 45° \cot 45° - \csc^2 45° = -\sqrt{2} - 2$

$f(46°) = f(45°) + f'(45°)\,\Delta x = \sqrt{2} + 1 - (\sqrt{2}+2)\, \pi/180 \cong 2,3654$

4. a) $y = 3x^4$ $x = 2$ $\Delta x = \pm 0,01$

$\Delta y = f(x + \Delta x) - f(x)$

$\rightarrow \Delta y = f(2,01) - f(2) = 3(2,01)^4 - 3(2)^4 = 0,9672$

$\rightarrow \Delta y = f(1,99) - f(2) = 3(1,99)^4 - 3(2)^4 = 0,9528$

Erro médio $= \frac{0,9672 + 0,9528}{2} = 0,96 = \Delta y$

Para $y = 48$ e $\Delta y = 0,96$

Erro percentual $= \frac{0,96}{48} * 100 = 2\%$

b) $y = x^3 + 5x \quad x = 1 \quad \Delta x = \pm 0,1 \longrightarrow \frac{dx}{x} = \pm 0,1$

$dy = (3x^2+5)\,dx \longrightarrow \frac{dy}{y} = \frac{(3x^2+5)}{x^3+5x}\,dx = \frac{(3x^2+5)}{(x^2+5)} * \frac{dx}{x}$

Logo: $\frac{dy}{y} = \frac{3x^2+5}{x^2+5}(\pm 0,1) = \pm 0,13$ para $x = 1$

Erro percentual $= 0,13 = 13\%$

Erro médio $= \frac{|\Delta y_1| + |\Delta y_2|}{2} = \frac{|f(1,1) - f(1)| + |f(0,9) - f(1)|}{2} = 0,801$

c) $y = 4\sqrt{x} + 3x \;;\; x = 4 \quad \Delta x = \pm 0,2 \quad \ldots$ para $x = 4 \quad y = 20$

Erro médio: $\Delta y_1 = 4\sqrt{4,2} + 3(4,2) - 20 = 0,7976$

$\Delta y_2 = 4\sqrt{3,8} + 3(3,8) - 20 = -0,8026$

Erro Médio $= \frac{|\Delta y_1| + |\Delta y_2|}{2} = \frac{0,7976 + 0,8026}{2} = 0,8001$

Erro Percentual: $\frac{0,8001}{20} * 100 = 4,00\%$

d) $y = 6\sqrt[3]{x} \qquad x = 8 \qquad \Delta x = \pm 0,03$

$y(8) = 12$

Erro Médio: $\Delta y_1 = 6\sqrt[3]{8,03} - 12 = 0,01498$

$\Delta y_2 = 6\sqrt[3]{7,97} - 12 = -0,01502$

Erro Médio $= \frac{0,01498 + 0,01502}{2} = 0,015$

Erro Percentual $= \frac{0,015}{12} * 100 = 0,125\%$

5. a) $A = 3x^2 - x \longrightarrow dA = (6x - 1)\,dx$

dA para $x = 2$ e $dx = 0,1$: $dA = (6 * 2 - 1) * 0,1 = 1,1$

b) $P = 6t^{2/3} + t^2$

$dP = (4t^{-1/3} + 2t)\,dt$

dP para $t = 8$ e $dt = 0,2$: $dP = \left(\frac{4}{\sqrt[3]{8}} + 2 * 8\right) 0,2 = 3,6$

6. a) $z = 40w^{-2/5} \longrightarrow \frac{dz}{z} = \frac{16w^{-3/5}}{z} dw \therefore \frac{dz}{z} = \frac{16}{w^{3/5}\,40w^{-2/5}} dw$

$\frac{dz}{z} = 16/40 \frac{dw}{w} \therefore \frac{dz}{z} = 16/40\,(\pm 0{,}08) = \pm 0{,}032$

b) $S = 10\pi x^2$

$dS = 20\pi x\, dx \longrightarrow \frac{dS}{S} = \frac{20\pi x\, dx}{S} \therefore \pm 0{,}1 = \frac{20\pi x}{10\pi x^2} dx$

$\longrightarrow \pm 0{,}1 = 2\frac{dx}{x} \therefore \frac{dx}{x} = \frac{\pm 0{,}1}{2} = \pm 0{,}05$

Erro percentual é $\pm 5\%$

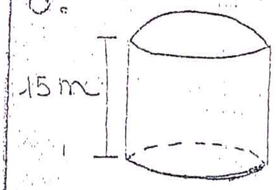
7. raio = 40 cm $\quad dr = 0{,}15\,cm \quad A = \pi r^2 = 1600\pi\, cm^2$

$dA = 2\pi r\, dr \therefore dA = 2\pi(40cm)(0{,}15cm)$

$dA = \pm 12\pi\, cm^2$

Erro Percentual $= \frac{dA}{A} = \frac{\pm 12\pi}{1600\pi} = 0{,}0075 = 0{,}75\%$

8.



$C = 2\pi r = 10m \therefore r = \frac{C}{2\pi} m \quad dC = \pm 0{,}15m$

Volume silo = Vcilindro + $\frac{\text{Vesfera}}{2} = \pi r^2 h + \frac{4\pi r^3}{3*2}$

Volume silo $= 15\pi r^2 + \frac{2}{3}\pi r^3 = \frac{15\pi}{4\pi^2}C^2 + \frac{2\pi}{24\pi^3}C^3 = \frac{15}{4\pi}C^2 + \frac{1}{12\pi^2}C^3$

$V = \frac{15}{4\pi}C^2 + \frac{C^3}{12\pi^2} = \frac{15(10)^2}{4\pi} + \frac{(10)^3}{12\pi^2} = 127{,}81\, m^3$

$dV = \left(\frac{15}{2\pi}C + \frac{C^2}{4\pi^2}\right) dC = \left(\frac{150}{2\pi} + \frac{100}{4\pi^2}\right)(\pm 0{,}15\, m) = \pm 3{,}961$

Erro percentual $= \frac{dV}{V} = \frac{\pm 3{,}961}{127{,}81} = \pm 0{,}031 = \pm 3{,}1\%$

9.

$$(48x^3 + 54x - 56x^2 - 63)^5$$

~~10~~ (7) a) $y = u^2 \qquad u = x^3 - 4$

$$\frac{dy}{dx} = \frac{dy}{du} * \frac{du}{dx} = 2u * (3x^2) = 6(x^5 - 4x^2) \quad \therefore \frac{dy}{dx} = 6(x^5 - 4x^2)$$

b) $y = u^{1/3} \qquad u = x^2 + 5x$

$$\frac{dy}{dx} = \frac{dy}{du} * \frac{du}{dx} = \frac{1}{3}u^{-2/3} * (2x+5) = \frac{(2x+5)}{3(x^2+5x)^{2/3}} \quad \therefore \frac{dy}{dx} = \frac{(2x+5)}{3(x^2+5x)^{2/3}}$$

c) $y = u^{-1} \qquad u = \sqrt{3x-2} = (3x-2)^{1/2}$

$$\frac{dy}{dx} = \frac{dy}{du} * \frac{du}{dx} = -\frac{1}{u^2} * \frac{3}{2}(3x-2)^{-1/2} = \frac{-3}{2(3x-2)(3x-2)^{1/2}} = \frac{-3}{2(3x-2)^{3/2}}$$

d) $y = 3u^2 + 2u \qquad u = 4x$

$$\frac{dy}{dx} = \frac{dy}{du} * \frac{du}{dx} = (6u+2) * 4 = 8(12x+1) \quad \therefore \frac{dy}{dx} = 8(12x+1)$$

e) $y = tg\,u \qquad u = x^2$

$$\frac{dy}{dx} = \frac{dy}{du} * \frac{du}{dx} = \sec^2 u * 2x = 2x\sec^2 x^2 \quad \therefore \frac{dy}{dx} = \frac{2x}{\cos^2 x^2}$$

f) $y = \boxed{u\,\text{sen}\,u} \qquad u = x^3$

$$\frac{dy}{dx} = \frac{dy}{du} * \frac{du}{dx} = (\text{sen}\,u + u\cos u)3x^2 = 3x^2(\text{sen}\,x^3 + x^3\cos x^3)$$

~~11~~ (8) a) $f(x) = (x^2 - 3x + 8)^3 \quad \begin{cases} u = x^2 - 3x + 8 \\ f(x) = u^3 \end{cases}$

$$\frac{dy}{dx} = \frac{dy}{du} * \frac{du}{dx} = 3u^2 * (2x-3) = 3(x^2 - 3x + 8)^2 * (2x-3)$$

b) $f(x) = (8x-7)^{-5} \quad \begin{cases} u = 8x - 7 \\ f(x) = u^{-5} \end{cases}$

$$\frac{dy}{dx} = \frac{dy}{du} * \frac{du}{dx} = -5u^{-6} * 8 = \frac{-40}{(8x-7)^6}$$

c) $f(x) = \dfrac{x}{(x^2-1)^4}$ $\quad$ Regra Quociente $\rightarrow$ $f'(x) = \dfrac{1}{(x^2-1)^8} * \left((x^2-1)^4 - x\left[(x^2-1)^4\right]'\right)$

$\rightarrow \quad u = x^2 - 1 \qquad \dfrac{dy}{dx} = \dfrac{dy}{du} * \dfrac{du}{dx} = 4u^3 * 2x = 8x(x^2-1)^3$

$f(x) = u^4$

Logo $f'(x) = \dfrac{1}{(x^2-1)^8}\left((x^2-1)^4 - 8x^2(x^2-1)^3\right) = \dfrac{x^2 - 1 - 8x^2}{(x^2-1)^5} = \dfrac{-7x^2-1}{(x^2-1)^5}$

d) $f(x) = (8x^3 - 2x^2 + x - 7)^5 \qquad u = 8x^3 - 2x^2 + x - 7$

$f(x) = u^5$

$$\frac{dy}{dx} = \frac{dy}{du} * \frac{du}{dx} = 5u^4 * (24x^2 - 4x + 1) = 5(8x^3 - 2x^2 + x - 7)^4 * (24x^2 - 4x + 1)$$

e) $f(v) = (17v-5)^{1000}$ $\begin{cases} u = 17v-5 \\ f(v) = u^{1000} \end{cases}$ $\therefore \dfrac{df}{dv} = \dfrac{df}{du} * \dfrac{du}{dv} = 1000u^{999} * 17 = 17000(17v-5)^{999}$

$f'(v) = 17.000(17v-5)^{999}$

d) $N(x) = (6x-7)^3 * (8x^2+9)^2 \xrightarrow{R.\ Produto} N'(x) = 18(6x-7)^2 * (8x^2+9)^2 + (6x-7)^3 * 32x * (8x^2+9)$

I) $g(x) = (6x-7)^3$ $\begin{cases} u = 6x-7 \\ g(u) = u^3 \end{cases}$ $\therefore \dfrac{dg}{dx} = \dfrac{dg}{du} * \dfrac{du}{dx} = 3u^2 * 6 = 18(6x-7)^2$

II) $h(x) = (8x^2+9)^2$ $\begin{cases} u = 8x^2+9 \\ h(u) = u^2 \end{cases}$ $\therefore \dfrac{dh}{dx} = \dfrac{dh}{du} * \dfrac{du}{dx} = 2u * 16x = 32x(8x^2+9)$

Logo: $N'(x) = 2(6x-7)^2 * (8x^2+9) * \left[9(8x^2+9) + 16x(6x-7)\right]$

g) $g(z) = \left(z^2 - \dfrac{1}{z^2}\right)^6$ $\begin{cases} u = z^2 - z^{-2} \\ g(u) = u^6 \end{cases}$ $\therefore \dfrac{dg}{dz} = \dfrac{dg}{du} \cdot \dfrac{du}{dz} = 6u^5 * (2z + 2z^{-3})$

$g'(z) = 12\left(z^2 - \dfrac{1}{z^2}\right)^5 * \left(z + \dfrac{1}{z^3}\right)$

h) $K(r) = (8r^3+27)^{1/3}$ $\begin{cases} u = 8r^3+27 \\ K(u) = u^{1/3} \end{cases}$ $\therefore \dfrac{dK}{dr} = \dfrac{dK}{du} * \dfrac{du}{dr} = \dfrac{1}{3u^{2/3}} * (24r^2)$

$K'(r) = \dfrac{8r^2}{(8r^3+27)^{2/3}}$

$(v^5-32)^{\frac{1}{5}}$ $\quad \frac{1}{5}(v^5-32)^{4/5}$

i) $F(v) = \dfrac{5}{\sqrt[5]{v^5-32}} \xrightarrow{Regra\ Quociente} F'(v) = \dfrac{1}{(v^5-32)^{2/5}}\left(0 - \dfrac{5 * v^4}{(v^5-32)^{4/5}}\right)$

$\rightarrow g(v) = (v^5-32)^{1/5}$ $\begin{cases} u = v^5-32 \\ g(u) = u^{1/5} \end{cases}$ $\therefore \dfrac{dg}{dv} = \dfrac{dg}{du} * \dfrac{du}{dv} = 1/5\, u^{-4/5} * 5v^4 = \dfrac{v^4}{(v^5-32)^{4/5}}$

Portanto, $F'(v) = \dfrac{-5v^4}{(v^2-32)^{6/5}}$

J) $g(w) = \dfrac{w^2 - 4w + 3}{w^{3/2}} \xrightarrow{R.\ Quociente} g'(w) = \dfrac{1}{w^3}\left((2w-4)\, w^{3/2} - (w^2-4w+3)\dfrac{3}{2}w^{1/2}\right)$

$g'(w) = \dfrac{2w^{5/2} - 4w^{3/2} - 3/2\, w^{5/2} + 6w^{3/2} - 9/2\, w^{1/2}}{w^3} = \dfrac{1/2\, w^{5/2} + 2w^{3/2} - 9/2\, w^{1/2}}{w^3}$

$g'(w) = \dfrac{w^{5/2} + 4w^{3/2} - 9w^{1/2}}{2w^3}$

K) $H(x) = \dfrac{2x+3}{(4x^2+9)^{1/2}} \xrightarrow{R.\ Quociente} \dfrac{1}{(4x^2+9)}\left(2(4x^2+9)^{1/2} - \dfrac{(2x+3)\, \cancel{8}^{4}x}{\cancel{2}(4x^2+9)^{1/2}}\right)$

$\rightarrow g(x) = (4x^2+9)^{1/2}$ $\begin{cases} u = 4x^2+9 \\ g(u) = u^{1/2} \end{cases}$ $\therefore \dfrac{dg}{dx} = \dfrac{dg}{du} * \dfrac{du}{dx} = \dfrac{1}{2u^{1/2}} * 8x = \dfrac{8x}{2(4x^2+9)^{1/2}}$

Logo: $H'(x) = \dfrac{8x^2+18 - 8x^2 - 12x}{(4x^2+9)^{3/2}} = \dfrac{18 - 12x}{(4x^2+9)^{3/2}}$

l) $K(x) = \operatorname{sen}(x^2+2)$ $\left\{\begin{array}{l} u = x^2+2 \\ K(u) = \operatorname{sen} u \end{array}\right.$ $\therefore \frac{dK}{dx} = \frac{dK}{du} * \frac{du}{dx} = \cos u * 2x = 2x\cos(x^2+2)$

$K'(x) = 2x\cos(x^2+2)$

n) $H(\theta) = (\cos 3\theta)^5$ $\left\{\begin{array}{l} u = \cos 3\theta \\ H(u) = u^5 \end{array}\right.$ $\therefore \frac{dH}{d\theta} = \frac{dH}{du} * \frac{du}{d\theta} = 5u^4 * \frac{du}{d\theta} = 5(\cos 3\theta)^4 \frac{du}{d\theta}$

mas, $u = \cos 3\theta$ $\left\{\begin{array}{l} g = 3\theta \\ u(g) = \cos g \end{array}\right.$ $\therefore \frac{du}{d\theta} = \frac{du}{dg} * \frac{dg}{d\theta} = -\operatorname{sen} g * 3 = -3\operatorname{sen} 3\theta$

logo $H'(\theta) = -15(\cos 3\theta)^4 \operatorname{sen} 3\theta$

i) $g(z) = \sec(2z+1)^2$ $\left\{\begin{array}{l} u = (2z+1)^2 \\ g(u) = \sec u \end{array}\right.$ $\therefore \frac{dg}{dz} = \frac{dg}{du} * \frac{du}{dz} = \sec u \operatorname{tg} u * \frac{du}{dz}$

mas, $u = (2z+1)^2$ $\left\{\begin{array}{l} h = 2z+1 \\ u(h) = h^2 \end{array}\right.$ $\therefore \frac{du}{dz} = \frac{du}{dh} * \frac{dh}{dz} = 2h * 2 = 4(2z+1)$

logo $g'(z) = 4(2z+1)\sec(2z+1)^2 * \tan(2z+1)^2$

) $H(s) = \cot(s^2-3s)$ $\left\{\begin{array}{l} u = s^2-3s \\ H(u) = \cot u \end{array}\right.$ $\frac{dH}{ds} = \frac{dH}{du} \cdot \frac{du}{ds} = -\csc^2 u * (2s-3)$

$H'(s) = -(2s-3)\csc^2(s^2-3s)$

) $f(x) = \cos(3x^2) + \cos^2 3x$

$f'(x) = -6x\operatorname{sen}(3x^2) - 6\cos(3x)\operatorname{sen} 3x$

q) $F(\phi) = \csc^2 2\phi = (\csc 2\phi)^2$ $\left\{\begin{array}{l} u = \csc 2\phi \\ F(u) = u^2 \end{array}\right.$ $\therefore \frac{dF}{d\phi} = \frac{dF}{du} \cdot \frac{du}{d\phi}$

$F'(\phi) = 2u * (-\csc 2\phi \cot 2\phi * 2) = -4\csc^3(2\phi)\cot(2\phi)$

) $K(z) = z^2 \cot 5z \xrightarrow{\text{R. Produto}} K'(z) = 2z\cot 5z + 5z^2(-\csc^2 5z)$

$K'(z) = 2z\cot(5z) - 5z^2\csc^2(5z)$

s) $h(\theta) = \operatorname{tg}^2\theta \sec^3\theta = \frac{\operatorname{sen}^2\theta}{\cos^5\theta} \xrightarrow{\text{R. Quocumte}} h'(\theta) = \frac{1}{\cos^{10}\theta} * \left(2\operatorname{sen}\theta\cos^6\theta + 5\cos^4\theta\operatorname{sen}^3\theta\right)$

$h'(\theta) = \frac{2\operatorname{sen}\theta\cos^2\theta + 5\operatorname{sen}^3\theta}{\cos^6\theta} = \frac{\operatorname{sen}\theta\,(2\cos^2\theta + 5\operatorname{sen}^2\theta)}{\cos^6\theta}$

$h'(\theta) = \operatorname{tg}\theta \frac{(2\cos^2\theta + 5\operatorname{sen}^2\theta)}{\cos^5\theta}$

t) $N(x) = (\text{sen}5x - \cos 5x)^5$ $\begin{cases} u = \text{sen}5x - \cos 5x \\ N(u) = u^5 \end{cases}$ $\therefore \frac{dN}{dx} = 5u^4 \cdot \frac{du}{dx} = 5(\text{sen}5x - \cos 5x)^4 \frac{du}{dx}$

$N'(x) = 5(\text{sen}5x - \cos 5x)^4 * (5\cos 5x + 5\,\text{sen}5x)$ –

u) $T(W) = \cot^3(3W+1)$ $\begin{cases} u = 3W+1 \\ T(u) = (\cot u)^3 \end{cases}$ $\therefore \frac{dT}{dW} = \frac{dT}{du} * \frac{du}{dW}$

mas, $T(u) = (\cot u)^3$ $\begin{cases} h = \cot u \\ T(h) = h^3 \end{cases}$ $\therefore \frac{dT}{du} = \frac{dT}{dh} \cdot \frac{dh}{du} = 3h^2 * (-\csc^2 u)$

$$\frac{dT}{du} = -3h^2 \csc^2(3W+1) = -3\cot^2(3W+1)\csc^2(3W+1)$$

$$T'(W) = -\frac{9\cos^2(3W+1)}{\text{sen}^2(3W+1)} * \frac{1}{\text{sen}^2(3W+1)} = -\frac{9\cos^2(3W+1)}{\text{sen}^4(3W+1)}$$

v) $h(w) = \frac{\cos 4w}{1 - \text{sen}4w} \xrightarrow{\text{R. Quociente}} h'(w) = \frac{1}{(1-\text{sen}4w)^2} * \left(-4\,\text{sen}4w(1-\text{sen}4w) + 4\cos^2 4w\right)$

$$h'(w) = \frac{-4\,\text{sen}4w + 4\,\text{sen}^2 4w + 4\cos^2 4w}{(1-\text{sen}4w)^2} = \frac{4(1-\text{sen}4w)}{(1-\text{sen}4w)^2}$$

$$h'(w) = \frac{4}{(1-\text{sen}4w)}$$

w) $f(x) = tg^3 2x - \sec^3 2x$

$$f'(x) = 3tg^2 2x * 2\sec^2 2x - 3\sec^2 2x * 2\sec 2x\, tg 2x$$

$$f'(x) = \frac{6\,\text{sen}^2 2x}{\cos^4 2x} - \frac{6\,\text{sen}2x}{\cos^4 2x} = \frac{6\,\text{sen}2x(\text{sen}2x - 1)}{\cos^4 2x}$$

x) $f(x) = \text{sen}\sqrt{x} + \sqrt{\text{sen}x}$

$\rightarrow \text{sen}\sqrt{x}$ $\begin{cases} u = \sqrt{x} \\ f(u) = \text{sen}u \end{cases}$ $\therefore \frac{df}{dx} = \frac{df}{du} \cdot \frac{du}{dx} = \cos u * \frac{1}{2\sqrt{x}} = \frac{\cos\sqrt{x}}{2\sqrt{x}}$

$\rightarrow (\text{sen}x)^{1/2}$ $\begin{cases} u = \text{sen}x \\ g(u) = u^{1/2} \end{cases}$ $\therefore \frac{dg}{dx} = \frac{dg}{du} \cdot \frac{du}{dx} = \frac{1}{2\sqrt{u}} * \cos x = \frac{\cos x}{2\sqrt{\text{sen}x}}$

$$f'(x) = \frac{\cos\sqrt{x}}{2\sqrt{x}} + \frac{\cos x}{2\sqrt{\text{sen}x}}$$

y) $K(\theta) = \cos^2\sqrt{3-8\theta}$

$$K'(\theta) = 2(\cos\sqrt{3-8\theta}) * (-\text{sen}\sqrt{3-8\theta}) * \left(\frac{-4}{\sqrt{3-8\theta}}\right) = \frac{4\,\text{sen}2\sqrt{3-8\theta}}{\sqrt{3-8\theta}}$$

z) $g(x) = (x^2+1)^{1/2}\, tg(x^2+1)^{1/2}$

$$g'(x) = \frac{2x\, tg(x^2+1)^{1/2}}{2(x^2+1)^{1/2}} + (x^2+1)^{1/2}\sec^2(x^2+1)^{1/2} * \frac{1}{2(x^2+1)^{1/2}} * 2x$$

9. $g'(x) = \dfrac{x\, tg\sqrt{x^2+1}}{(x^2+1)^{1/2}} + \dfrac{x}{\cos^2(x^2+1)^{1/2}}$

$\cancel{10}$. a) $g(z) = (3z+1)^{1/2}$

$g'(z) = \dfrac{3}{2(3z+1)^{1/2}}$ $\quad g'' = -\dfrac{9}{4(3z+1)^{3/2}}$

b) $K(s) = (s^2+4)^{2/3}$

$K'(s) = \dfrac{4s}{3(s^2+4)^{1/3}}$. $\quad K'' = \dfrac{1}{9(s^2+4)^{2/3}}\left(12(s^2+4)^{1/3} - 4s\,(s^2+4)^{-2/3} * 2s\right)$

$\therefore K'' = \dfrac{1}{9(s^2+4)^{2/3}}\left(\dfrac{12(s^2+4) - 8s^2}{(s^2+4)^{2/3}}\right) = \dfrac{4(s^2+12)}{9\,(s^2+4)^{4/3}} = \dfrac{4\,(s^2+12)}{9\,(s^2+4)^{4/3}}$

c) $K(r) = (4r+7)^5$

$u = 4r+7 \quad \therefore \dfrac{dK}{dr} = \dfrac{dK}{du} * \dfrac{du}{dr} = 5u^4 * 4 = 20(4r+7)^4$

$K(u) = u^5$

$\therefore K'(r) = 20(4r+7)^4$

$K''(r) = \left[20(4r+7)^4\right]'$ $\quad \left\{ \begin{array}{l} g = 4r+7 \\ K(g) = u^4 \end{array} \right. \quad \therefore K'' = 20\left(4u^3 * 4\right)$

$K'' = 20 * 16(4r+7)^3$

$K'' = 320(4r+7)^3$

d) $f(x) = (10x+7)^{1/5}$

$u = 10x+7$
$f(u) = u^{1/5}$ $\longrightarrow \dfrac{df}{dx} = \dfrac{df}{du}\cdot\dfrac{du}{dx} = \dfrac{1}{5} * (u)^{-4/5} * 10 = 2(10x+7)^{-4/5}$

$f'(x) = 2(10x+7)^{-4/5}$

$f''(x) = 2\left(-\dfrac{4}{5}\right)(10x+7)^{-9/5} * 10 = \dfrac{-16}{(10x+7)^{9/5}}$

e) $f(x) = (\operatorname{sen}x)^3$

$u = \operatorname{sen}x \quad \therefore \dfrac{df}{dx} = \dfrac{df}{du}\cdot\dfrac{du}{dx} = 3u^2 * \cos x = 3\operatorname{sen}^2 x \cos x$

$f(u) = u^3$

$f'(x) = 3\operatorname{sen}^2 x \cos x$

$\rightarrow g(x) = (\operatorname{sen}x)^2 \quad \left\{ \begin{array}{l} u = \operatorname{sen}x \\ g(u) = u^2 \end{array} \right. \quad \therefore \dfrac{dg}{dx} = \dfrac{dg}{du}\cdot\dfrac{du}{dx} = 2u * \cos x = 2\operatorname{sen}x\cos x$

$f''(x) \overset{\text{P. Produto}}{=} 3\left(2\operatorname{sen}x\cos^2 x + \operatorname{sen}^2 x(-\operatorname{sen}x)\right) = 3\left(2\operatorname{sen}x\cos^2 x - \operatorname{sen}^3 x\right)$

f) $G(t) = \sec^2 4t = (\sec 4t)^2$

$\longrightarrow u = \sec 4t \quad \therefore \dfrac{dG}{dt} = \dfrac{dG}{du}\cdot\dfrac{du}{dt} = 2u * 4\sec 4t * tg\,4t = 8\sec^2 4t * \tan 4t$

$G(u) = u^2$

$G'(t) = 8\operatorname{sen}4t/\cos^3 4t$

$G''(t) = 8\left(\dfrac{1}{\cos^6 4t} * \left[4\cos^4 4t - 3\operatorname{sen}4t(\cos^2 4t)(-4\operatorname{sen}4t)\right]\right) = \dfrac{8(4\cos^2 4t + 12\operatorname{sen}^2 4t)}{\cos^4 4t}$

~~13~~ 10. a) $8x^2 + y^2 = 10 \longrightarrow 16x + 2yy' = 0 \quad \therefore y' = -\frac{8x}{y}$

b) $4x^3 - 2y^3 = x \longrightarrow 12x^2 - 6y^2y' = 1$

$y' = \frac{12x^2 - 1}{6y^2}$

b) $2x^3 + x^2y + y^3 = 1$

$6x^2 + 2xy + x^2y' + 3y^2y' = 0 \longrightarrow 6x^2 + 2xy + y'(x^2 + 3y^2) = 0$

$y' = \frac{-(6x^2 + 2xy)}{x^2 + 3y^2}$

d) $5x^2 + 2x^2y + y^2 = 8$

$10x + 4xy + 2x^2y' + 2yy' = 0 \longrightarrow y'(2x^2 + 2y) = -(10x + 4xy)$

$y' = \frac{-(10x + 4xy)}{(2x^2 + 2y)} = \frac{-(5x + 2xy)}{x^2 + y}$

e) $5x^2 - xy - 4y^2 = 0$

$10x - y - xy' - 8yy' = 0 \longrightarrow y'(-x - 8y) = -10x + y$

$y' = \frac{10x - y}{x + 8y}$

f) $x^4 + 4x^2y^2 - 3xy^3 + 2x = 0$

$4x^3 + 8xy^2 + 8x^2yy' - 3y^3 - 9xy^2y' + 2 = 0$

$4x^3 + 8xy^2 - 3y^3 + 2 = y'(9xy^2 - 8x^2y)$

$y' = \frac{4x^3 + 8xy^2 - 3y^3 + 2}{(9xy^2 - 8x^2y)}$

g) $x^{1/2} + y^{1/2} = 100$

$\frac{1}{2\sqrt{x}} + \frac{y'}{2\sqrt{y}} = 0 \longrightarrow y' = -\frac{\sqrt{y}}{\sqrt{x}} = -\sqrt{\frac{y}{x}}$

h) $x^{2/3} + y^{2/3} = 4$

$\frac{2}{3x^{1/3}} + \frac{2y'}{3y^{1/3}} = 0 \longrightarrow \frac{2y'}{3y^{1/3}} = -\frac{2}{3x^{1/3}} \longrightarrow y' = -\sqrt[3]{\frac{y}{x}} \quad \therefore y' = -\sqrt[3]{\frac{y}{x}}$

i) $x^2 + (xy)^{1/2} = 7$

$2x + \frac{1}{2(xy)^{1/2}}(y + xy') = 0 \longrightarrow 2x + \frac{y}{2(xy)^{1/2}} + \frac{xy'}{2(xy)^{1/2}} = 0$

$y' = -\frac{2x(2\sqrt{xy})}{x} - \frac{2y(xy)^{1/2}}{2(xy)^{1/2}} = -4\sqrt{xy} - y$

$y' = -(4\sqrt{xy} + y)$

f) $2x - (xy)^{1/2} + y^3 = 16$

$2 - \frac{1}{2(xy)^{1/2}} \cdot (y + xy') + 3y^2y' = 0 \longrightarrow 2 - \frac{y}{2(xy)^{1/2}} - \frac{xy'}{2(xy)^{1/2}} + 3y^2y' = 0$

$y'\left(3y^2 - \frac{x}{2(xy)^{1/2}}\right) = \frac{y}{2(xy)^{1/2}} - 2 \therefore y' = \frac{y - 4(xy)^{1/2}}{6y^2(xy)^{1/2} - x}$

g) $\text{sen}^2 3y = x + y - 1$

$2\text{sen}3y \cdot (3\cos 3y)y' = 1 + y' \longrightarrow (3\,\text{sen}6y)y' = 1 + y'$

$y' = \frac{1}{3\text{sen}6y - 1}$

l) $x = \text{sen}(xy)$

$1 = y\cos(xy) + x\cos(xy)y' \longrightarrow y' = \frac{1 - y\cos(xy)}{x\cos(xy)}$

m) $y = \csc(xy)$

$y' = -y\cot(xy)\csc(xy) + x\cot(xy)\csc(xy)y'$

$y'(1 + x\cot(xy)\csc(xy)) = -y\cot(xy)\csc(xy)$

$y' = \frac{-y\cot(xy)\csc(xy)}{(1 + x\cot(xy)\csc(xy))} = \frac{-y\cos(xy)}{\text{sen}^2xy + x\cos xy}$

n) $y^2 + 1 = x^2\sec y$

$2yy' = 2x\sec y - x^2(\tan y\sec y)y'$

$y'(2y + x^2\tan y\sec y) = 2x\sec y \longrightarrow y' = \frac{2x\sec y}{2y + x^2\tan y\sec y}$

o) $y^2 = x\cos y$

$2yy' = \cos y - x(\text{sen}y)y' \longrightarrow y'(2y + x\text{sen}y) = \cos y \therefore y' = \frac{\cos y}{2y + x\text{sen}y}$

p) $xy = \text{tg}y$

$y + xy' = (\sec^2 y)y' \longrightarrow y = y'(\sec^2 y - x) \therefore y' = \frac{y}{\sec^2 y - x}$

q) $x^2 + (\text{sen}y)^{1/2} - y^2 = 1$

$2x + \frac{\cos y}{2\sqrt{\text{sen}y}}y' - 2yy' = 0 \longrightarrow y'\left(2y - \frac{\cos y}{2\sqrt{\text{sen}y}}\right) = 2x$

$y' = \frac{4x\sqrt{\text{sen}y}}{4y\sqrt{\text{sen}y} - \cos y}$

r) $\text{sen}\sqrt{y} - 3x = 2$

$\frac{\cos\sqrt{y}}{2\sqrt{y}} \cdot y' - 3 = 0 \longrightarrow y' = \frac{6\sqrt{y}}{\cos\sqrt{y}}$

14. a) $3x^2+4y^2=4 \longrightarrow 6x+8y\cdot y'=0 \rightarrow y'=-\frac{3x}{4y}$

$$y''=\frac{1}{(4y)^2}\left(-12y+12x\,y'\right)=\frac{-12y+12x\left(\frac{-3x}{4y}\right)}{16y^2}=\frac{-3\left(\frac{4y^2+3x^2}{4y}\right)}{4\;y^2}$$

$$y''=\frac{-3}{16}\left(\frac{4y^2+3x^2}{y^3}\right)$$

b) $5x^2-2y^2=4$

$10x-4yy'=0 \rightarrow y'=\frac{5x}{2y} \;\therefore\; y''=\frac{1}{4y^2}\left(10y-10xy'\right)=\frac{5}{2y^2}\left(y-x\left(\frac{5x}{2y}\right)\right)$

$$y''=\frac{5}{2y^2}\left(y-\frac{5x^2}{2y}\right)=\frac{5}{2y^2}\left(\frac{2y^2-5x^2}{2y}\right)$$

c) $x^3-y^3=1$

$3x^2-3y^2y'=0 \;\therefore\; y'=\frac{x^2}{y^2} \;\therefore\; y''=\frac{1}{y^4}\left(2xy^2-2yx^2y'\right)$

$$y''=\frac{1}{y^4}\left(2xy^2-2yx^2\left(\frac{x^2}{y^2}\right)\right)=\frac{2x}{y^2}-\frac{2x^4}{y^5}$$

d) $x^2y^3=1$

$2xy^3+3x^2y^2y'=0 \longrightarrow y'=\frac{-2xy^3}{3x^2y^2}=-\frac{2y}{3x}$

$$y''=\frac{1}{9x^2}\left(-6xy'+6y\right)=\frac{2}{3x^2}\left(-x\left(\frac{-2y}{3x}\right)+y\right)$$

$$y''=\frac{2}{3x^2}\left(\frac{2y}{3}+y\right)=\frac{2}{3x^2}\left(\frac{5y}{3}\right)=\frac{10y}{9x^2}$$

e) $\operatorname{sen}y+y=x$

$y'(\cos y+1)=1 \rightarrow y'=\frac{1}{\cos y+1}$

$$y''=\frac{1}{(\cos y+1)^2}\left(+\operatorname{sen}y\cdot y'\right)=\frac{\operatorname{sen}y}{(\cos y+1)^3}$$

f) $\cos y=x$

$-\operatorname{sen}y\cdot y'=1 \;\therefore\; y'=-1/\operatorname{sen}y$

$$y''=\frac{1}{\operatorname{sen}^2y}\left(\cos y\cdot y'\right)=\frac{-\cos y}{\operatorname{sen}^3y}$$

LISTA 5:

Extremos de Funções

Lista 5: Extremos de Funções

1. a) (i) $[-3,3]$ Extremos em $x=-3$; $x=-1$; $x=1$

(ii) $(-3,\sqrt{3})$ Extremos em $x=-1$; $x=1$.

(iii) $[-\sqrt{3},1)$ Extremos em $x=-\sqrt{3}$; $x=-1$

(iv) $[0,3]$ Extremos em $x=0$; $x=3$; $x=1$

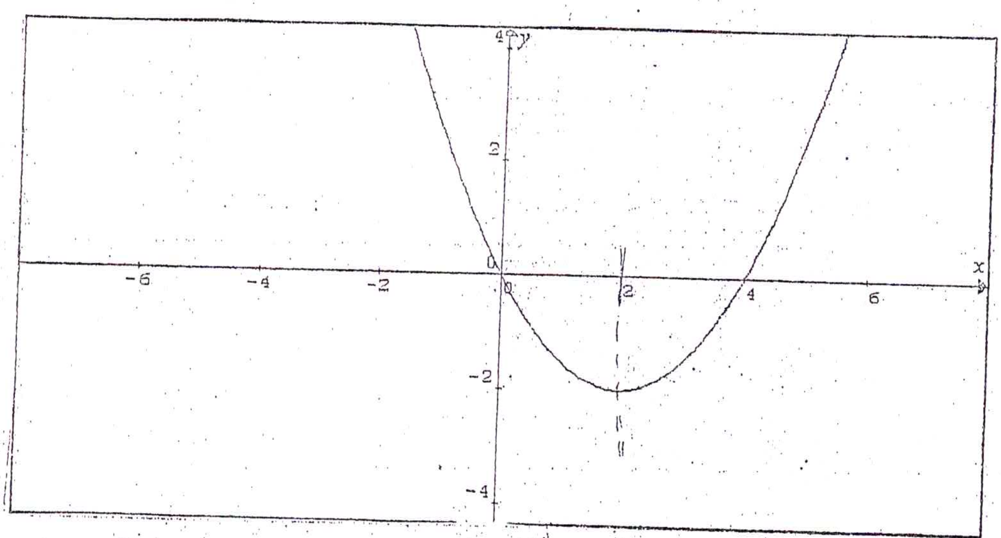
b) (i) $[-2,2]$ Extremos em $x=-2$; $x=2$; $x=-1$; $x=0$; $x=1$.

(ii) $(0,2)$ Extremos em $x=1$

(iii) $(-1,1)$ Extremos em $x=0$

(iv) $[-2,-1)$ Extremos em $x=-2$,

2. a) $f(x)=1/2\,x^2-2x$



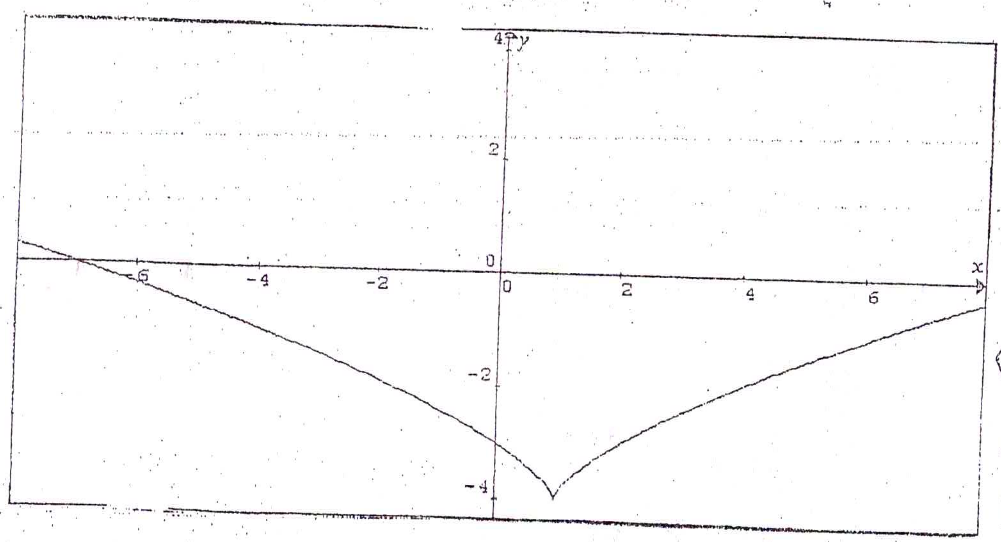
(i) $[0,5]$ Extremos em $x=0$; $x=5$; $x=\cancel{+2}\ 2$

(ii) $(0,2)$. Não possui

(iii) $(0,4)$. Extremo em $x=2$

(iv) $[2,5]$ Extremo em $x=2$; $x=5$

b) $f(x)=(x-1)^{2/3}-4$



(i) $[0,9]$. Extremos em $x=0$; $x=9$; $x=1$

(ii) $(1,2]$ Extremos em $x=2$.

(iii) ~~$(-7,2)$ Extremos~~ em $x=1$

(iv) $[0,1)$ Extremo em $x=0$.

a) $f(x) = 5 + 6x^2 - 2x^3$; $[0,9]$

$f'(x) = -12x - 6x^2 = -6x(2+x)$

→ Pontos críticos : $f'(x) = 0 \xrightarrow{x\neq 0} x = 0$ ou $x = -2$

Extremos : $x = 0$; $x = 9$

b) $f(x) = 3x^2 - 10x + 7$; $[-1,3]$

$f'(x) = 6x - 10$ ∴ Ponto crítico ; $f'(x) = 0 \longrightarrow x = 5/3$

Extremos : $x = -1$; $x = 3$; $x = 5/3$

c) $f(x) = 1 - x^{2/3}$; $[1,8]$

$f'(x) = -\frac{2}{3}x^{-1/3}$ ∴ Pontos Críticos ; $f'(x) = 0$ $\nexists x / f'(x) = 0$

e $\nexists f'(x)$ em $x = 0$.

Extremos: $x = 1$; $x = 8$.

d) $f(x) = x^4 - 5x^2 + 4$; $[0,2]$

$f'(x) = 4x^3 - 10x \longrightarrow f'(x) = 2x(2x^2 - 5)$

Ponto Crítico: $f'(x) = 0 \longrightarrow x = 0$ $x = \pm\sqrt{5/2}$

Extremos em $x = 0$; $x = +\sqrt{5/2}$; $x = 2$

a) $f(x) = 4x^2 - 3x + 2$

$f'(x) = 8x - 3$. Pontos críticos: $f'(x) = 0$ ou $\nexists f'(x)$

$f'(x) = 8x - 3 = 0$ ∴ $x = 3/8$ → PONTO CRÍTICO : $x = 3/8$

b) $F(w) = w^4 - 32w$

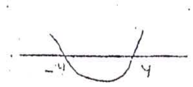
$F'(w) = 4w^3 - 32$. Pontos críticos: $F'(w) = 0$ ou $\nexists F'(w)$

$F'(w) = 4w^3 - 32 = 0$ ∴ $w = \sqrt[3]{8} = 2$ → Ponto crítico: $x = 2$.

c) $f(z) = \sqrt{z^2 - 16} = (z^2 - 16)^{1/2}$

$f'(z) = \frac{1}{2}(z^2 - 16)^{-1/2} * 2z = \frac{z}{\sqrt{z^2 - 16}}$. Pontos Críticos $f'(z) = 0$ e $\nexists f'(z)$

$f'(z) = \frac{z}{\sqrt{z^2 - 16}} = 0$ ∴ $z = 0$ ou $\nexists f'(z)$ ∴ $z^2 - 16 = 0$:



PONTOS críticos : $x = 0$, $x = -4$ e $x = 4$

d) $h(x) = (2x-5)(x^2-4)^{1/2}$ Regra do Produto e regra da cadeia

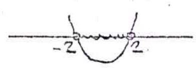
$h'(x) = 2(x^2-4)^{1/2} + (2x-5)\,\frac{1}{2}(x^2-4)^{-1/2} * 2x$

$h'(x) = 2(x^2-4)^{1/2} + \frac{x(2x-5)}{(x^2-4)^{1/2}} = \frac{2(x^2-4)+(2x^2-5x)}{(x^2-4)^{1/2}}$

$h'(x) = \frac{2x^2-8+2x^2-5x}{\sqrt{x^2-4}} = \frac{4x^2-5x-8}{\sqrt{x^2-4}}$

Pontos Críticos: $h'(x)=0 \therefore 4x^2-5x-8=0 \rightarrow \Delta = 25-4(4)(-8)$

$\Delta = 25+128 = 153$; $x = \frac{-(-5)\pm\sqrt{153}}{2*(4)} = \frac{5\pm3\sqrt{17}}{8}$

Pontos Críticos: $\nexists h'(x) \rightarrow x^2-4 \leq 0$  $\therefore -2 \leq x \leq 2$

Pontos Críticos: $x = \frac{5+3\sqrt{17}}{8}$; $x = \frac{5-3\sqrt{17}}{8}$; $-2 \leq x \leq 2$

e) $g(t) = t^2(2t-5)^{1/3}$ Regra do Produto e Regra da cadeia

$g'(t) = 2t(2t-5)^{1/3} + t^2 * \frac{1}{3}(2t-5)^{-2/3} * 2$

$g'(t) = 2t\sqrt[3]{(2t-5)} + \frac{2t^2}{3\sqrt[3]{(2t-5)^2}} = \frac{6t(2t-5)+2t^2}{3(2t-5)^{2/3}}$

$g'(t) = \frac{12t^2-30t+2t^2}{3(2t-5)^{2/3}} = \frac{14t^2-30t}{3(2t-5)^{2/3}} = \frac{2t(7t-15)}{3(2t-5)^{2/3}}$

Pontos Críticos: $g'(x)=0 \therefore 2t(7t-15)=0$ se $t=0$ ou $t=15/7$

$\nexists g'(x) \therefore 2t-5=0 \rightarrow t=5/2$

Pontos Críticos: $t=0$; $t=15/7$; $t=5/2$

f) $G(x) = \frac{2x-3}{x^2-9} \xrightarrow{\text{Regra Quociente}} G'(x) = \frac{1}{(x^2-9)^2} * \left[2(x^2-9) - 2x(2x-3)\right]$

$G'(x) = \frac{2x^2-9-4x^2+6x}{(x^2-9)^2} = \frac{-2x^2+6x-9}{(x^2-9)^2}$

Pontos Críticos: $G'(x)=0 \therefore -2x^2+6x-9=0 \rightarrow \Delta = 36-4(-2)(-9) < 0$

$\Delta = -25 = 25i^2 \therefore x = \frac{-6\pm5i}{-4}$ $\begin{cases} x_1 = \frac{-6+5i}{-4} = \frac{6-5i}{4} \\ x_2 = \frac{-6-5i}{-4} = \frac{6+5i}{4} \end{cases}$

$\nexists G'(x) \rightarrow x^2-9=0 \therefore x = \pm 3$

Pontos Críticos: $x=-3$; $x=+3$; $x_1 = \frac{6-5i}{4}$; $x_2 = \frac{6+5i}{4}$, x_1 e $x_2 \in \mathbb{C}$

g) $f(s) = \dfrac{s^2}{5s+4} \xrightarrow{\text{Regra Quociente}} f'(s) = \dfrac{1}{(5s+4)^2} * \left[2s(5s+4) - 5s^2\right]$

$f'(s) = \dfrac{5s^2 + 8s}{(5s+4)^2} = \dfrac{s(5s+8)}{(5s+4)^2}$

Ponto Crítico: $f'(s) = 0 \therefore s = 0$ ou $s = -8/5$

e $\nexists f'(s) \Rightarrow s = -4/5$

PONTOS CRÍTICOS: $s = -8/5$; $s = -4/5$; $s = 0$

h) $f(t) = (\operatorname{sen} t)^2 - \cos t \longrightarrow f'(t) = 2\operatorname{sen} t \cos t + \operatorname{sen} t = (2\cos t + 1)\operatorname{sen} t$

Pontos Críticos: $f'(t) = 0 \therefore \operatorname{sen} t = 0$ ou $2\cos t + 1 = 0$

$\rightarrow \operatorname{sen} t = 0 \therefore t = 0 + n\pi,\ n \in \mathbb{Z}$

$\rightarrow 2\cos t + 1 = 0 \rightarrow \cos t = -1/2 \therefore t_1 = 2\pi/3 + 2n\pi,\ n \in \mathbb{Z}$

$t_2 = 4\pi/3 + 2n\pi,\ n \in \mathbb{Z}$

Pontos críticos: $t = n\pi$, $t = 2\pi/3 + 2n\pi$, $t = 4\pi/3 + 2n\pi$, $n \in \mathbb{Z}$

i) $f(x) = 8\cos^3 x - 3\operatorname{sen} 2x - 6x$

$f'(x) = [24\cos^2 x * (-\operatorname{sen} x)] - [6\cos 2x] - 6$

$f'(x) = -24\cos^2 x \operatorname{sen} x - 6\cos 2x - 6$ se $\operatorname{sen} x = 0$, $\cos 2x = -1 \rightarrow$

$f'(x) = 0 \longrightarrow x = \pi + 2n\pi,\ n \in \mathbb{Z}$

J) $f(x) = \dfrac{1 + \operatorname{sen} x}{1 - \operatorname{sen} x} \xrightarrow{\text{Regra Quociente}} f'(x) = \dfrac{1}{(1-\operatorname{sen} x)^2} * \left[\cos x(1 - \operatorname{sen} x) + \cos x(1 + \operatorname{sen} x)\right]$

$f'(x) = \dfrac{\cos x(1 - \operatorname{sen} x + 1 + \operatorname{sen} x)}{(1 - \operatorname{sen} x)^2} = \dfrac{2\cos x}{(1 - \operatorname{sen} x)^2} \rightarrow$

$f'(x) = 0 \longrightarrow 2\cos x = 0 \therefore \cos x = 0 \rightarrow x_1 = \pi/2 + 2n\pi,\ n \in \mathbb{Z}$

$x_2 = 3\pi/2 + 2n\pi,\ n \in \mathbb{Z}$

$\nexists f'(x) \rightarrow \operatorname{sen} x = 1 \therefore x = \pi/2 + 2n\pi,\ n \in \mathbb{Z}$

Pontos Críticos: $x = \dfrac{\pi}{2} + n\pi,\ n \in \mathbb{Z}$.

k) $K(u) = u - \operatorname{tg} u \longrightarrow K'(u) = 1 - \sec^2 u = 1 - 1/\cos^2 u$

$K'(u) = 0 \rightarrow \cos^2 u = 1 \therefore u = 0 + n\pi,\ n \in \mathbb{Z}$

$\nexists K'(u) \rightarrow \cos u = 0 \therefore u = \pi/2 + n\pi,\ n \in \mathbb{Z}$

Pontos críticos: $u = n\pi$; $u = \pi/2 + n\pi$, $n \in \mathbb{Z}$

g) $\frac{du}{dt} = 2 + 2u + t + tu = 2(1+u) + t(1+u) = (1+u)(2+t)$

$\frac{1}{1+u}\,du = (2+t)\,dt \longrightarrow \int \frac{1}{1+u}\,du = \int (2+t)\,dt$

$\ln|1+u| = 2t + t^2/2 + C \longrightarrow |1+u| = e^{t^2/2 + 2t + C} = K e^{t^2/2 + 2t}$ $(K = e^C)$

$u = \pm K e^{t^2/2 + 2t} - 1$ ou $u = -1 \pm K e^{t^2/2 + 2t}$

h) $\frac{dz}{dt} + e^{t+z} = 0 \longrightarrow \frac{dz}{dt} = -e^t . e^z \quad \therefore \frac{1}{e^z}\,dz = -e^t\,dt$

$\int 1/e^z\,dz = -\int e^t\,dt \Rightarrow -e^{-z} = -e^t + C \quad \therefore e^{-z} = e^t - C$

$e^z = \frac{1}{e^t - C} \longrightarrow z = -\ln(e^t - C)$

7. a) $\frac{dy}{dx} = y^2 + 1$, $y(1) = 0$

$\frac{1}{y^2+1}\,dy = 1\,dx \longrightarrow \int \frac{1}{y^2+1}\,dy = \int 1\,dx \quad \therefore \tan^{-1} y = x + C$

$\tan^{-1} 0 = 1 + C \quad \therefore 0 = 1 + C \longrightarrow C = -1$

$y = \tan(x - 1)$

b) $\frac{dy}{dx} = \frac{1+x}{xy}$, $x > 0$, $y(1) = -4$

$y\,dy = \left(\frac{1}{x} + 1\right)dx \longrightarrow \int y\,dy = \int \frac{1}{x} + 1\,dx \quad \therefore y^2/2 = \ln|x| + x + C$

$16/2 = \ln|1| + 1 + C \quad \therefore C = 7$

$y^2 = 2\ln|x| + 2x + 14$

c) $xe^{-t}\frac{dx}{dt} = t$, $x(0) = 1$

$x\,dx = te^t\,dt \longrightarrow \int x\,dx = \int te^t\,dt \quad \therefore \frac{x^2}{2} = te^t - e^t + C$

$\rightarrow \int te^t\,dt \left\langle \begin{array}{l} u = t \rightarrow du = dt \\ v = e^t \leftarrow dv = e^t\,dt \end{array} \right. \quad \therefore \int te^t\,dt = te^t - \int e^t\,dt = \underline{te^t - e^t + C}$

Se $x(0) = 1 \rightarrow 1/2 = 0 - e^0 + C \rightarrow C = 3/2 \quad \therefore x^2 = 2e^t(t-1) + 3$

$x = \sqrt{2e^t(t-1) + 3}$

d) $x + 2y\sqrt{x^2+1}\,\frac{dy}{dx} = 0$, $y(0) = 1$

$2y\,dy = \frac{-x}{\sqrt{x^2+1}}\,dx \longrightarrow \int 2y\,dy = \int \frac{-x}{\sqrt{x^2+1}}\,dx \quad \therefore y^2 = -\sqrt{x^2+1} + C$

$\rightarrow -\int \frac{x}{\sqrt{x^2+1}}\,dx \left\langle \begin{array}{l} u = x^2+1 \\ du = 2x\,dx \end{array} \right. \quad \therefore -\int \frac{x}{\sqrt{x^2+1}}\,dx = -\int \frac{1}{2\sqrt{u}}\,du = -\sqrt{u} + C = -\sqrt{x^2+1} + C$

Logo, $1^2 = -\sqrt{1} + C \rightarrow C = 2$

$y^2 = 2 - \sqrt{x^2+1}$

e) $\frac{du}{dt} = \frac{2t + \sec^2 t}{2u}$, $u(0) = -5$

$2u\,du = 2t + \sec^2 t\,dt \longrightarrow \int 2u\,du = \int 2t + \sec^2 t\,dt \therefore u^2 = t^2 + \tan t + C$

Se $u(0) = -5$: $25 = 0 + \tan 0 + C \therefore C = 25$

Se $u(0) = -5$, devemos ter: $u = -\sqrt{t^2 + \tan t + 25}$

f) $\frac{dy}{dt} = te^y$, $y(1) = 0$

$e^{-y}dy = t\,dt \longrightarrow \int e^{-y}dy = \int t\,dt \therefore -e^{-y} = \frac{t^2}{2} + C$

Se $y(1) = 0$: $-e^0 = 1/2 + C \therefore C = -3/2$

$e^{-y} = 3/2 - t^2/2 \longrightarrow e^y = \frac{2}{3-t^2} \longrightarrow y = \ln 2 - \ln(3 - t^2)$ p/ $|t| < \sqrt{3}$

3. $\frac{dy}{dx} = 4x^3 y$, $y(0) = 7$

$\frac{1}{y}dy = 4x^3 dx \longrightarrow \int \frac{1}{y}dy = \int 4x^3 dx \therefore \ln|y| = x^4 + C$

$|y| = e^{x^4 + C} \longrightarrow |y| = e^c e^{x^4} \longrightarrow y = Ae^{x^4}$

$y(0) = 7$: $7 = Ae^0 \therefore A = 7$. Solução: $y = 7e^{x^4}$

1. $\frac{dy}{dx} = \frac{y^2}{x^3}$, $y(1) = 1$

$\frac{1}{y^2}dy = \frac{1}{x^3}dx \longrightarrow \int \frac{1}{y^2}dy = \int \frac{1}{x^3}dx \longrightarrow -\frac{1}{y} = -\frac{1}{2x^2} + C$

Se $y(1) = 1$: $-1 = -1/2 + C \therefore C = -1/2$.

$-1/y = -1/2x^2 - 1/2 \longrightarrow y = \frac{2x^2}{x^2 + 1}$

). $P(t) = Ae^{kT}$ $k = 0{,}7944$

$P(t) = Ae^{0{,}7944T}$. $P(0) = 2 \longrightarrow P(0) = Ae^0 = 2 \therefore A = 2$

$P(t) = 2e^{0{,}7944T}$. Portanto, $P(6) = 2e^{0{,}7944 * 6} \cong 235$ membros.

. a) $P(t) = Ae^{kT}$

$P(\frac{1}{3}) = Ae^{k/3}$ e $P(0) = Ae^{kT} = 60 \therefore A = 60$

$P(\frac{1}{3}) = 60e^{k/3} = 120 \therefore e^{k/3} = 2 \longrightarrow k/3 = \ln 2$

$k = 3\ln 2 \therefore k = \ln 2^3 = \ln 8$.

b) $P(t) = Ae^{kT} \therefore P(t) = 60e^{(\ln 8)t} = 60 * 8^t$

c) $P(8) = 60 * 8^8 = 60 * 2^{24} \cong 1{,}01 * 10^9$ células

d) $\frac{dP}{dt} = kP \longrightarrow P'(8) = kP(8) = \ln 8 * P(8) \simeq 2{,}093$ bilhões células/h

e) $P(t) = 20.000$ células $= 60 * 8^t \longrightarrow 8^t = \frac{2000}{6}$

$t \ln 8 = \ln\left(\frac{1000}{3}\right) \longrightarrow t = \frac{\ln 1000 - \ln 3}{\ln 8} \simeq 2{,}79$ horas

12. a) $P(t) = Ae^{kT} \longrightarrow P(0) = Ae^0 = 500 \therefore A = 500$

$P(t) = 500e^{kT} \longrightarrow P(3) = 500e^{3k} = 8000 \therefore e^{3k} = \frac{80}{5} = 16$

$3k = \ln 16 \therefore k = \frac{\ln 16}{3} = \ln 16^{1/3}$

$P(t) = 500e^{(\ln 16^{1/3})t} = 500 * 16^{t/3} \longrightarrow P(t) = 500 * 16^{t/3}$

b) $P(4) = 500 * 16^{4/3} = 20.159$ células

c) $\frac{dP}{dt} = kP \longrightarrow P'(4) = \ln 16^{1/3} * P(4) = 18.631$ células/h

d) $P(t) = 30.000 = 500 * 16^{t/3} \longrightarrow 16^{t/3} = 60 \therefore t/3 \ln 16 = \ln 60$

$t/3 = \frac{\ln 60}{\ln 16} \longrightarrow t = 3 * \frac{\ln 60}{\ln 16} \simeq 4{,}4$ horas.

13. a) $P(t) = Ae^{kT}$

$P(2) = Ae^{2k} = 400$

$P(6) = Ae^{6k} = 25.600$ $\Longrightarrow \frac{P(6)}{P(2)} = \frac{Ae^{6k}}{Ae^{2k}} = \frac{25600}{400}$

$\therefore e^{4k} = 64 \longrightarrow 4k = \ln 64 \therefore k = \ln 64^{1/4}$

$Ae^{2 * \frac{\ln 64}{4}} = 400 \longrightarrow 64^{1/2} A = 400 \therefore A = \frac{400}{\sqrt{64}} = 50$

$P(t) = 50e^{\ln 64^{t/4}} = 50 * 64^{t/4}$

$P(0) = 50$

b) $P(t) = 50 * 64^{t/4}$ (ver item a)

$P(t) = 50 * 8^{t/2}$

c) $P(t) = 2P(0) = 100$

$P(t) = 50 * 8^{t/2} = 100 \longrightarrow 8^{t/2} = 2 \therefore t/2 \ln 8 = \ln 2 \rightarrow 3/2\, t = 1$

$t = 2/3\, h = 40$ min

d) $P(t) = 100.000 \longrightarrow 50 * 8^{t/2} = 100000 \rightarrow 8^{t/2} = 2000 \therefore t/2 \ln 8 = \ln 2000$

$t = 2\left(\frac{\ln 2000}{\ln 8}\right) \simeq 7{,}3h$

14. a) $P(t) = P(1750)e^{k(t-1750)}$, $P(1800) = 728e^{k(50)} = 906$

$50k = \ln\left(\frac{906}{728}\right) \longrightarrow k = \frac{\ln\left(\frac{906}{728}\right)}{50} \simeq 0{,}0043748$

$P(t) = 728\, e^{0,0043748(t-1750)} \rightarrow P(1900) = 728\, e^{150(0,0043748)} \cong 1403$ milhões

$P(1500) = 728\, e^{200(0,0043748)} \cong 1746$ milhões

b) $P(t) = P(1850)\, e^{k(t-1850)} \longrightarrow P(1900) = 1171 e^{k50} = 1608$

$50k = \ln\left(\frac{1608}{1171}\right) \therefore k = \frac{1}{50} \cdot \ln\left(\frac{1608}{1171}\right) \simeq 0,006343$

Logo : $P(t) = 1171\, e^{0,006343(t-1850)}$

Para $t = 1950$: $P(1950) = 1171\, e^{0,006343(1950-1850)} = 2208$ milhões

15. $P(t) = P(1900)\, e^{k(t-1900)} \longrightarrow P(1950) = 1608\, e^{k(50)} = 2517$

$50k = \ln\left(\frac{2517}{1608}\right) \therefore k = \frac{1}{50} \ln\left(\frac{2517}{1608}\right) = 0,008962$

Logo : $P(t) = 1608\, e^{0,008962(t-1900)}$

Para $t = 1992$: $P(1992) = 1608\, e^{0,008962(1992-1900)} = 3667$ milhões

6. (a) $y = \text{sen}kt \longrightarrow y' = k\cos kt \longrightarrow y'' = -k^2 \text{sen}kt$

Dada a equação diferencial $y'' + 9y = 0$, substituir y'' e y :

$-k^2 \text{sen}kt + 9\,\text{sen}kt = 0 \therefore \text{sen}kt\,(9 - k^2) = 0 \therefore k = \pm 3$

(b) $y = A\,\text{sen}kt + B\cos kt \longrightarrow y' = Ak\cos kt - Bk\,\text{sen}kt$

$y'' = -Ak^2 \text{sen}kt - Bk^2 \cos kt$. Substituir em $y'' + 9y = 0$

$-Ak^2 \text{sen}kt - Bk^2 \cos kt + 9(A\,\text{sen}kt + B\cos kt) = 0 \therefore -k^2(A\,\text{sen}kt + B\cos kt) + 9(A\,\text{sen}kt + B\cos kt) = 0$

$(A\,\text{sen}kt + B\cos kt)(9 - k^2) = 0 \longrightarrow (9-k^2)(A\,\text{sen}kt) + (9-k^2)(B\cos kt) = 0$

Os valores não-nulos de k serão $k = \pm 3$

7. $y = e^{rt} \longrightarrow y' = r e^{rt} \longrightarrow y'' = r^2 e^{rt}$

$y'' + y' - 6y = 0 \therefore r^2 e^{rt} + r e^{rt} - 6 e^{rt} = 0$

$e^{rt}(r^2 + r - 6) = 0 \therefore (r+3)(r-2) = 0 \Rightarrow r = -3$ ou $r = 2$.

Nari-favarina@hot

Lista 6: Regra de L'Hôpital - continuação pág.

1

a) $\lim\limits_{x\to -1} \frac{x^2-1}{x+1}$: Indeterminação $\frac{0}{0}$. Aplicando L'Hopital: $\lim\limits_{x\to -1} \frac{2x}{1} = 2(-1) = -2$

b) $\lim\limits_{x\to -2} \frac{x+2}{x^2+3x+2}$: Indeterminação $\frac{0}{0}$. Aplicar L'Hopital: $\lim\limits_{x\to -2} \frac{d/dx\,(x+2)}{d/dx\,(x^2+3x+2)} = \lim\limits_{x\to -2} \frac{1}{2x+3} = -1$

c) $\lim\limits_{x\to 1} \frac{x^9-1}{x^5-1}$: Indeterminação $\frac{0}{0}$. Aplicando L'Hopital:

$\lim\limits_{x\to 1} \frac{d/dx\,(x^9-1)}{d/dx\,(x^5-1)} = \lim\limits_{x\to 1} \frac{9x^8}{5x^4} = \frac{9(1)^8}{5(1)^4} = \frac{9}{5}$

d) $\lim\limits_{x\to 1} \frac{x^a-1}{x^b-1}$: Indeterminação $\frac{0}{0}$. Aplicar L'Hopital:

$\lim\limits_{x\to 1} \frac{d/dx\,(x^a-1)}{d/dx\,(x^b-1)} = \lim\limits_{x\to 1} \frac{a.x^{a-1}}{b\,x^{b-1}} = \frac{a}{b}$

e) $\lim\limits_{x\to 0} \frac{e^x-1}{\operatorname{sen} x}$: Indeterminação $\frac{0}{0}$. Aplicar L'Hopital: $\lim\limits_{x\to 0} \frac{d/dx\,(e^x-1)}{d/dx\,(\operatorname{sen} x)} = \lim\limits_{x\to 0} \frac{e^x}{\cos x} = \frac{1}{1} = 1$

f) $\lim\limits_{x\to 0} \frac{x+\operatorname{tg} x}{x}$: Indeterminação $\frac{0}{0}$. Aplicar L'Hopital:

$\lim\limits_{x\to 0} \frac{d/dx\,(x+\operatorname{tg}x)}{d/dx\,(x)} = \lim\limits_{x\to 0} \frac{1+\sec^2 x}{1} = \lim\limits_{x\to 0} \frac{1+\frac{1}{\cos^2 x}}{1} = \frac{1+1}{1} = 2$

g) $\lim\limits_{x\to 0} \frac{\operatorname{sen} x}{x^3}$: Indeterminação $\frac{0}{0}$. Aplicar L'Hopital:

$\lim\limits_{x\to 0} \frac{d/dx\,(\operatorname{sen}x)}{d/dx\,(x^3)} = \lim\limits_{x\to 0} \frac{\cos x}{3x^2} = \frac{\cos 0}{3\times 0} = \frac{1}{0} = +\infty$

h) $\lim\limits_{x\to \pi} \frac{\operatorname{tg} x}{x} = \frac{\operatorname{tg}\pi}{\pi} = \frac{0}{\pi} = 0 \to$ Não se aplica L'Hopital.

i) $\lim\limits_{x\to 0} \frac{\operatorname{tg} px}{\operatorname{tg} qx}$: Indeterminação $\frac{0}{0}$. Aplicar L'Hopital:

$\lim\limits_{x\to 0} \frac{d/dx\,(\operatorname{tg} px)}{d/dx\,(\operatorname{tg} qx)} = \lim\limits_{x\to 0} \frac{p\sec^2 px}{q\sec^2 qx} = \lim\limits_{x\to 0} \frac{p\cos^2 qx}{q\cos^2 px} = \frac{p}{q}$

j) $\lim\limits_{x\to \frac{3\pi}{2}} \frac{\cos x}{x-(\frac{3\pi}{2})}$: Indeterminação $\frac{0}{0}$. Aplicar L'Hopital:

$\lim\limits_{x\to 3\pi/2} \frac{d/dx\,(\cos x)}{d/dx\,(x-\frac{3\pi}{2})} = \lim\limits_{x\to 3\pi/2} \frac{-\operatorname{sen} x}{1} = -\operatorname{sen}(\frac{3\pi}{2}) = -(-1) = 1$

k) $\lim\limits_{x\to \infty} \frac{\ln x}{x}$: Indeterminação $\frac{\infty}{\infty}$. Aplicar L'Hopital:

$\lim\limits_{x\to \infty} \frac{d/dx\,(\ln x)}{d/dx\,(x)} = \lim\limits_{x\to\infty} \frac{1/x}{1} = \frac{1}{\infty} = 0$

$\frac{d a^x}{dx} = a^x \ln a$

l) $\lim\limits_{x\to \infty} \frac{e^x}{x}$: Indeterminação $\frac{\infty}{\infty}$. Aplicar L'Hopital:

$\lim\limits_{x\to \infty} \frac{d/dx\,(e^x)}{d/dx\,(x)} = \lim\limits_{x\to\infty} \frac{e^x}{1} = e^\infty = +\infty$

m) $\lim\limits_{x\to 0^+} \frac{\ln x}{x} = \frac{\ln 0^+}{0^+} = -\infty$ Não se aplica L'Hopital

n) $\lim\limits_{x\to \infty} \frac{\ln \ln x}{x}$: Indeterminação $\frac{\infty}{\infty}$. Aplicar L'Hopital: $\frac{1}{x}$

$\lim\limits_{x\to \infty} \frac{d/dx\,(\ln\ln x)}{d/dx\,(x)} = \lim\limits_{x\to\infty} \frac{1}{x\ln x} = \frac{1}{\infty} = 0$

o) $\lim\limits_{t\to 0} \frac{5^t-3^t}{t}$: Indeterminação $\frac{0}{0}$. Aplicar L'Hopital:

$\lim\limits_{t\to 0} \frac{d/dt\,(5^t-3^t)}{d/dt\,(t)} = \lim\limits_{t\to 0} \frac{5^t\ln 5 - 3^t\ln 3}{1} = \ln\frac{5}{3}$

p) $\lim\limits_{t\to 16} \frac{\sqrt[4]{t}-2}{t-16}$: Indeterminação $\frac{0}{0}$. Aplicar L'Hopital:

$$\lim_{t\to 16} \frac{d/dt\,(\sqrt[4]{t}-2)}{d/dt\,(t-16)} = \lim_{t\to 16} \frac{1}{4} t^{-3/4} = \frac{1}{32}$$

q) $\lim_{x\to 1} \frac{e^x-1-x}{x^2} = \frac{e-2}{1}$ não se aplica L'Hopital

r) $\lim_{x\to 1} \frac{e^x-1-x-(x^2/2)}{x^3} = e-\frac{5}{2}$ Não se aplica L'Hopital.

s) $\lim_{x\to\infty} \frac{e^x}{x^3}$: Indeterminação $\frac{\infty}{\infty}$. Aplicar L'Hopital:

$\lim_{x\to\infty} \frac{d/dx(e^x)}{d/dx(x^3)} = \lim_{x\to\infty} \frac{e^x}{3x^2} \longrightarrow$ Indeterminação $\frac{\infty}{\infty}$. Aplicar L'Hopital:

$\lim_{x\to\infty} \frac{d/dx(e^x)}{d/dx(3x^2)} = \lim_{x\to\infty} \frac{e^x}{6x} \longrightarrow$ Indeterminação $\frac{\infty}{\infty}$. Aplicar L'Hopital:

$\lim_{x\to\infty} \frac{d/dx(e^x)}{d/dx(6x)} = \lim_{x\to\infty} \frac{e^x}{6} = +\infty$

t) $\lim_{x\to\infty} \frac{(\ln x)^3}{x^2}$: Indeterminação $\frac{\infty}{\infty}$. Aplicar L'Hopital:

$\lim_{x\to\infty} \frac{d/dx[(\ln x)^3]}{d/dx\,(x^2)} = \lim_{x\to\infty} \frac{3(\ln x)^2 * 1/x}{2x} = \lim_{x\to\infty} \frac{3(\ln x)^2}{2x^2} \longrightarrow$ Indeterminação $\frac{\infty}{\infty}$. Aplicar L'Hopital:

$\lim_{x\to\infty} \frac{d/dx\, 3(\ln x)^2}{d/dx\, 2x^2} = \lim_{x\to\infty} \frac{6\ln x * 1/x}{4x} = \lim_{x\to\infty} \frac{6\ln x}{4x^2} \longrightarrow$ Indeterminação $\frac{\infty}{\infty}$. Aplicar L'Hopital.

$\lim_{x\to\infty} \frac{d/dx\, 6\ln x}{d/dx\, 4x^2} = \lim_{x\to\infty} \frac{6}{8x^2} = \frac{6}{\infty} = 0$

u) $\lim_{x\to 0} \frac{\operatorname{sen}^{-1}x}{x}$: Indeterminação $\frac{0}{0}$. Para aplicar L'Hopital encontrar a derivada de $\operatorname{sen}^{-1}x$.

1, x, y, $\sqrt{1-x^2}$

$\operatorname{sen} y = x \therefore$ derivando $\to \cos y \cdot y' = 1$

$y' = \frac{1}{\cos y} = \frac{1}{\sqrt{1-x^2}}$

Portanto, $\lim_{x\to 0} \frac{\operatorname{sen}^{-1}x}{x} = \lim_{x\to 0} \frac{1/\sqrt{1-x^2}}{1} = 1$.

v) $\lim_{x\to 0} \frac{1-\cos x}{x^2}$: Indeterminação $\frac{0}{0}$. Aplicar L'Hopital:

$\lim_{x\to 0} \frac{d/dx\,(1-\cos x)}{d/dx\,(x^2)} = \lim_{x\to 0} \frac{\operatorname{sen} x}{2x} \longrightarrow$ Indeterminação $\frac{0}{0}$. Aplicar L'Hopital:

$\lim_{x\to 0} \frac{d/dx\,(\operatorname{sen} x)}{d/dx\,(2x)} = \lim_{x\to 0} \frac{\cos x}{2} = \frac{1}{2}$

w) $\lim_{x\to 0} \frac{\operatorname{sen} x}{e^x} = \frac{0}{1} = 0$. Não se aplica L'Hopital.

x) $\lim_{x\to 0} \frac{\operatorname{tg} ax}{x}$: Indeterminação $\frac{0}{0}$. Aplicar L'Hopital:

$\lim_{x\to 0} \frac{d/dx\,(\operatorname{tg} ax)}{d/dx\,(x)} = \lim_{x\to 0} \frac{a\sec^2 ax}{1} = \frac{a}{\cos^2 0} = a$

y) $\lim_{x\to 0} \frac{x}{\operatorname{tg}^{-1}(4x)}$: Indeterminação $\frac{0}{0}$. Para aplicar L'Hopital encontrar a derivada de $\operatorname{tg}^{-1}(4x)$

$\sqrt{16x^2+1}$, $4x$, y, 1

$\operatorname{tg} y = 4x \therefore$ derivando $\to y' \cdot \sec^2 y = 4$

$y' = 4\cdot\cos^2 y = 4\left(\frac{1}{16x^2+1}\right) = \frac{4}{16x^2+1}$

Portanto, $\lim_{x\to 0} \frac{x}{\operatorname{tg}^{-1}(4x)} = \lim_{x\to 0} \frac{1}{4/16x^2+1} = \lim_{x\to 0} \frac{16x^2+1}{4} = \frac{1}{4}$.

z) $\lim_{x\to\infty} \frac{x}{\ln(1+2e^x)}$: Indeterminação $\frac{\infty}{\infty}$. Aplicar L'Hopital:

$\lim_{x\to\infty} \frac{1}{2e^x/1+2e^x} = \lim_{x\to\infty} \frac{1+2e^x}{2e^x} \longrightarrow$ Indeterminação $\frac{\infty}{\infty}$. Aplicar L'Hopital:

$\lim_{x\to\infty} \frac{2e^x}{2e^x} = \lim_{x\to\infty} 1 = 1$

LISTA 6:

Regra de L'Hopital

1) $f(x) = \dfrac{4-x^2}{x+3}$ (1) Dom: $\{x \in \mathbb{R} \mid x \neq -3\}$

(2) Continuidade: $(-\infty, -3) \cup (-3, +\infty)$

(3) $f(x) = 0$ para $4 - x^2 = 0$ $\therefore$ $x = -2$ ou $x = 2$

$f(0) = 4/3$ $\therefore$ Interceptos: $(-2,0), (2,0), (0, 4/3)$

(4) $f(x) = \dfrac{4-x^2}{x+3}$ e $f(-x) = \dfrac{4-x^2}{-x+3}$ A função não é par nem ímpar.

(5) $f'(x) = \dfrac{1}{(x+3)^2}\left(-2x^2 - 6x - 4 + x^2\right) = \dfrac{-x^2-6x-4}{(x+3)^2}$

$f'(x) = 0$ para $-x^2 - 6x - 4 = 0$ Soma $= -6$ Produto $= 4$ $x = -3 \pm \sqrt{5}$

$\nexists f'(x)$ quando $x = -3$

Pontos Críticos: $x = -3$, $x = -3 \pm \sqrt{5}$

(6) p/ $x < -3-\sqrt{5}$ Decrescente $(f'(x) < 0)$

p/ x: $-3-\sqrt{5} < x < -3$ Crescente $(f'(x) > 0)$

p/ x: $\cancel{-3} < x < -3+\sqrt{5}$ Crescente $(f'(x) > 0)$

p/ x: $x > -3+\sqrt{5}$ Decrescente $(f'(x) < 0)$

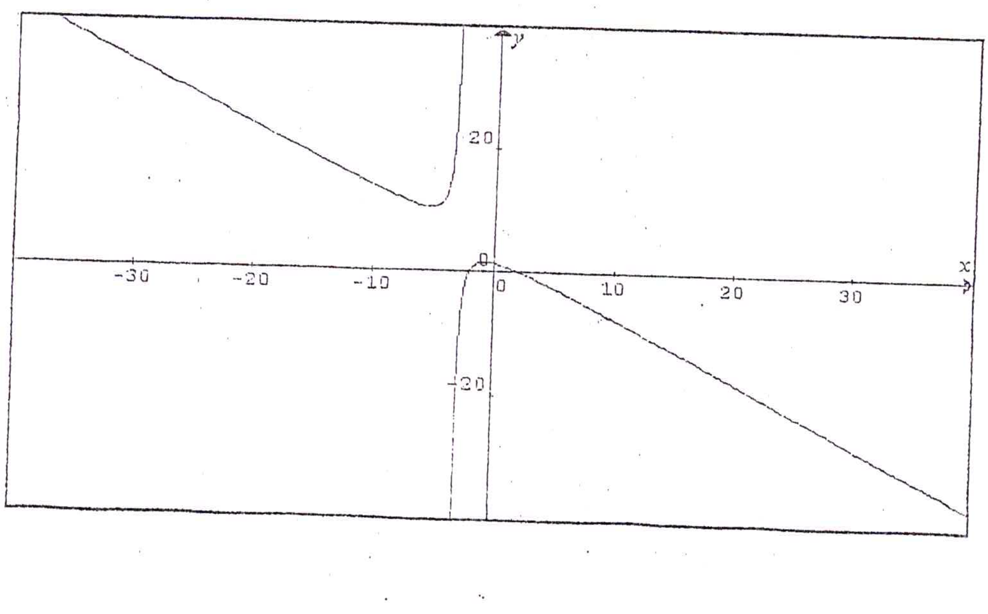
(7) $f''(x) = \dfrac{1}{(x+3)^4}\left((-2x-6)(x+3)^2 - 2(-x^2-6x-4)(x+3)\right)$

$f''(x) = \dfrac{-2x^2 - 12x - 18 + 2x^2 + 12x + 8}{(x+3)^3} = \dfrac{-10}{(x+3)^3}$

Se $x < -3$ $f''(x) > 0$ Concavidade p/ cima

$x > -3$ $f''(x) < 0$ Concavidade p/ baixo

8) Assíntota vertical: $\lim\limits_{x \to -3^-} f(x) = +\infty$ $\lim\limits_{x \to -3^+} f(x) = -\infty$



h) $f(x) = \dfrac{x^2}{x+1}$ (1): Dom $\{x \in \mathbb{R} / x \neq -1\}$

(2): Continuidade : $(-\infty,-1) \cup (-1,+\infty)$

(3) : $f(x)=0 \rightarrow x=0$ e $f(0)=0$ ∴ Intercepto : $(0,0)$

(4) : $f(x) = x^2/x+1$ $f(-x) = x^2/-x+1$ Não é par nem ímpar

(5) : $f'(x) = \dfrac{1}{(x+1)^2}\left(2x^2+2x - x^2\right) = \dfrac{x^2+2x}{(x+1)^2} = \dfrac{x(x+2)}{(x+1)^2}$

$$\begin{cases} f'(x)=0 & \text{para } x=0 \text{ ou } x=-2 \\ \nexists f'(x) & \text{para } x=-1 \end{cases}$$

(6) $x < -2$ ou $x>0$ Crescente

$-2 < x < 0$ Decrescente

(7) $f''(x) = \dfrac{1}{(x+1)^4}\left((2x+2)(x+1)^2 - 2(\cancel{2}x+1)(x^2+2x)\right)$

$f''(x) = \dfrac{1}{(x+1)^3}\left((2x+2)(x+1) - 2(x^2+2x)\right) = \dfrac{2}{(x+1)^3}\left((x+1)^2 - 2x^2-4x\right)$

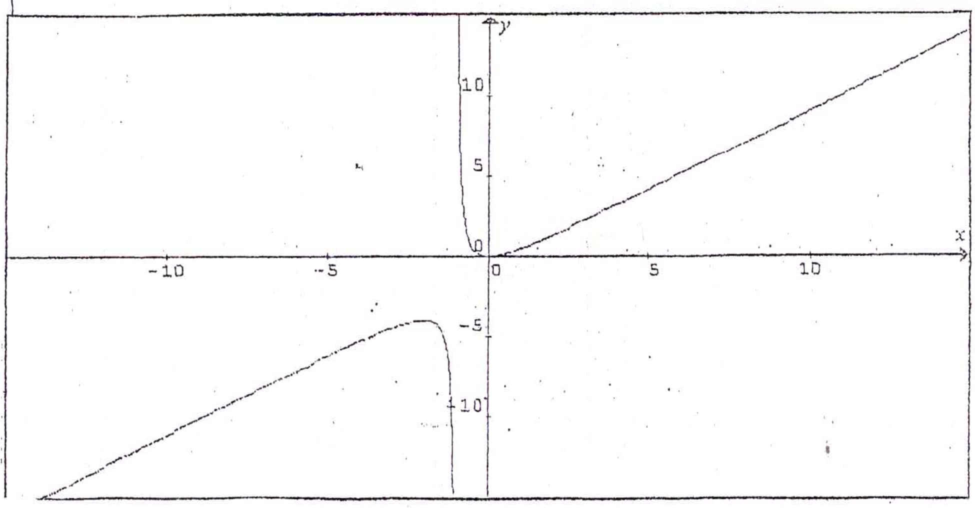
$f''(x) = \dfrac{2}{(x+1)^3}\left(-x^2-2x+1\right)$

P/cima : $x > -1$
P/baixo : $x < -1$

(8) Assíntotas verticais : $\lim\limits_{x\to -1^-} f(x) = -\infty$ $\lim\limits_{x\to -1^+} f(x) = +\infty$

Não possui assíntotas horizontais.

(9) Gráfico:



8) $f(x) = \frac{x^2 - x - 6}{x+1}$ (1): Dom $\{x \in \mathbb{R} / x \neq -1\}$

(2) Continuidade: $(-\infty, -1) \cup (-1, +\infty)$

(3) $f(x) = 0 \therefore x^2 - \cancel{x} - 6 = 0 \Rightarrow (x+2)(x-3) = 0 \therefore x = -2$ e $x = 3$

$f(0) = -6$. Interceptos: $(-2,0)$; $(3,0)$; $(0,-6)$

(4) $f(x) = \frac{x^2 - x - 6}{x+1}$ $f(-x) = \frac{x^2 + x - 6}{-x+1}$ não é par nem ímpar

(5) $f'(x) = \frac{1}{(x+1)^2} * \left((2x-1)(x+1) - x^2 + x + 6\right) = \frac{x^2 + 2x + 5}{(x+1)^2}$

- $f'(x) = 0 \Rightarrow \nexists x \in \mathbb{R} / f'(x) = 0$
- $\nexists f'(x)$ para $x = -1$.

6) Crescente: $(-\infty, -1) \cup (-1, +\infty)$

7) $f''(x) = \frac{1}{(x+1)^4} \left(2(x+1)(x+1)^2 - 2(x+1)(x^2+2x+5)\right)$

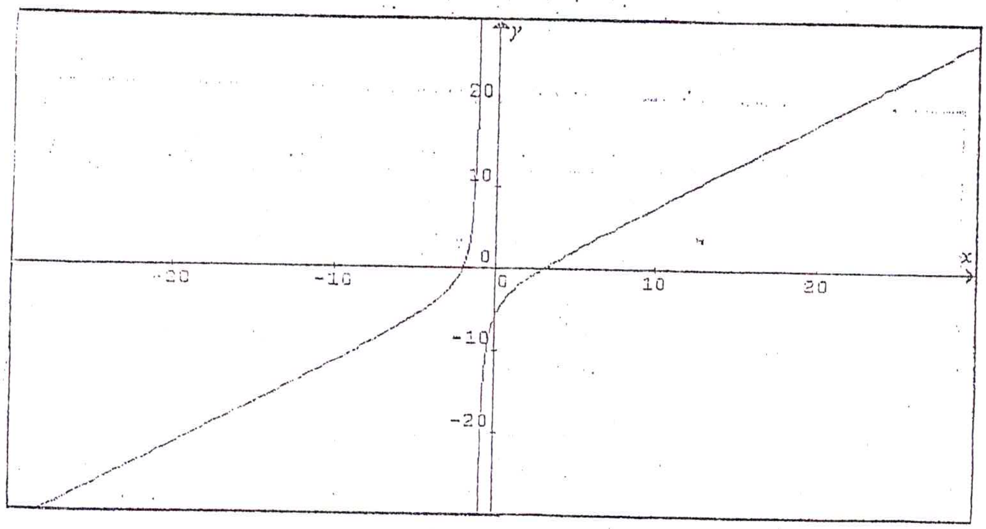
$f''(x) = \frac{1}{(x+1)^3} \left(\cancel{2x^2} + \cancel{4x} + 2 - 2x^3 - 4x^2 - 20x - \cancel{2x^2} - \cancel{4x} - 20 \right)$

$f''(x) = \frac{-2x^3 - 4x^2 - 20x - 18}{(x+1)^3} = \frac{-2(x^3 + 2x^2 + 10x + 9)}{(x+1)^3}$

Concavidade para cima: $(-1, +\infty)$
concavidade para baixo: $(-\infty, 1)$

8) Assintotas verticais: $\lim_{x \to -1^-} f(x) = +\infty$ $\lim_{x \to -1^+} f(x) = -\infty$

9) Gráfico:



f) $f(x) = \dfrac{-3x}{\sqrt{x^2+4}}$ (1) Dom: $x \in \mathbb{R}$ ou $\mathbb{R}$

(2). Continuidade: $(-\infty, +\infty)$

(3) $f(0) = 0$ $f(x) = 0$ em $x = 0$, Intercepto: $(0,0)$

(4) $f(x) = \dfrac{-3x}{\sqrt{x^2+4}}$ e $f(-x) = \dfrac{3}{\sqrt{x^2+4}}$ $\therefore f(x) = -f(-x)$ Simetria ímpar ao redor da origem

(5) $f'(x) = \dfrac{1}{x^2+4}\left(-3\sqrt{x^2+4} + 3x\dfrac{(\frac{1}{2}2x)}{\sqrt{x^2+4}}\right)$

$f'(x) = \dfrac{1}{x^2+4}\left(\dfrac{-3(x^2+4)+3x^2}{\sqrt{x^2+4}}\right) = \dfrac{-12}{(x^2+4)^{3/2}}$

- $f'(x) = 0 \rightarrow \nexists x / f'(x) = 0$
- $\nexists f'(x) \rightarrow \nexists x$ que satisfaça a condição

(6) $f'(x) < 0$ Qualquer que seja x $\therefore$ Função Decrescente

(7) $f''(x) = \dfrac{1}{(x^2+4)^3}\left(12 * \dfrac{3}{\cancel{2}}(x^2+4)^{1/2} * \cancel{2}x\right) = \dfrac{36x}{(x^2+4)^{5/2}}$ $\dfrac{36x(x^2+4)^{1/2}}{(x^2+4)^3}$?

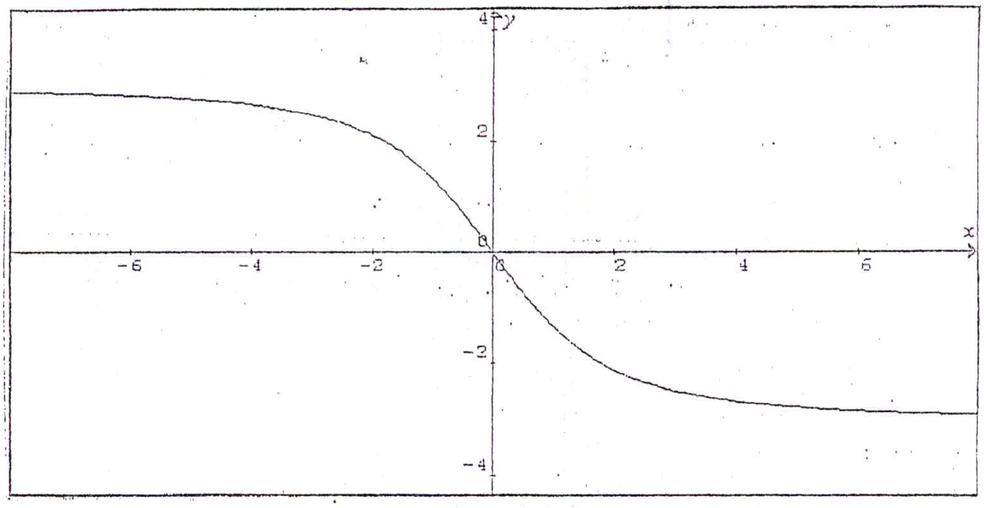
Concavidade p/ cima: $f''(x) > 0 \rightarrow x > 0$

Concavidade p/ baixo: $f''(x) < 0 \rightarrow x < 0$

(8) Assíntotas Verticais $\lim\limits_{x \to a} f(x) = \infty$ (não possui assíntotas verticais)

Assíntotas Horizontais $\lim\limits_{x \to \infty} f(x) = -3$. $\lim\limits_{x \to -\infty} = +3$

(9) Gráficos:



20. a) $f(x) = \dfrac{2x-5}{x+3}$. (1) Restrição $x+3 \neq 0 \therefore x \neq -3$

Dom: $\{ x \in \mathbb{R} \mid x \neq -3 \}$ ou $\mathbb{R} - \{-3\}$

(2) Continuidade : $(-\infty, -3) \cup (-3, +\infty)$

3) $f(x) = 0 \longrightarrow 2x - 5 = 0 \therefore x = 5/2$ e $f(0) = -5/3$

$\therefore$ Interceptos : $\begin{cases} x = 5/2 \text{ quando } f(x) = 0 & (5/2, 0) \\ f(0) = -5/3 \text{ quando } x = 0 & (0, -5/3) \end{cases}$

(4) $f(x) = \dfrac{2x-5}{x+3}$, $f(-x) = \dfrac{-2x-5}{-x+3}$ A função não é par nem ímpar.

5) $f'(x) = \dfrac{1}{(x+3)^2}\left(2x+6-2x+5\right) = \dfrac{11}{(x+3)^2}$

- $f'(x) = 0 \longrightarrow \nexists x \mid f'(x) = 0$
- $\nexists f'(x)$ para $x+3=0 \therefore x = -3$

Ponto Crítico : $x = -3$

6) Como $f'(x) > 0, \forall x$ a função é sempre crescente.

7) $f''(x) = \dfrac{1}{(x+3)^4}\left(0 - 11 * 2(x+3)\right) = \dfrac{-22(x+3)}{(x+3)^4} = \dfrac{-22}{(x+3)^3}$

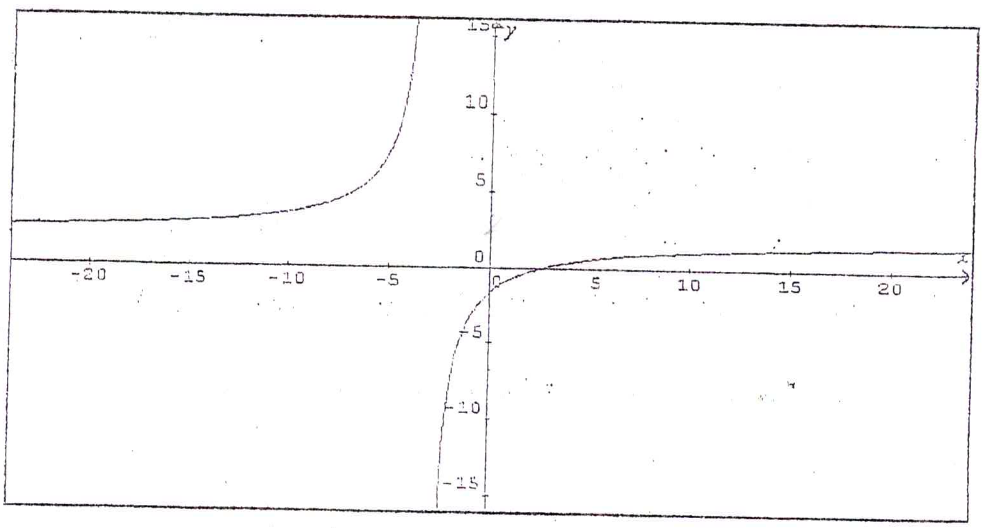
P/ $x < -3$: $f''(x) < 0 \therefore$ Concavidade para cima

P/ $x > -3$: $f''(x) > 0 \therefore$ Concavidade para baixo

) Assíntotas Verticais: $\lim\limits_{x \to -3^+} f(x) = -\infty$ $\lim\limits_{x \to -3^-} = +\infty$

Assíntotas Horizontais: $\lim\limits_{x \to \infty} f(x) = \lim\limits_{x \to \infty} \dfrac{2 - 5/x}{1 + 3/x} = 2$

) Gráfico :



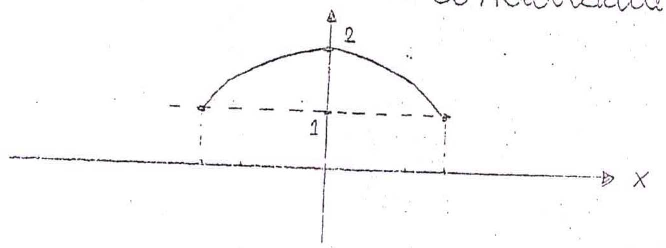
b) Dados: $f(0) = 2$; PONTOS CRÍTICOS: $f'(0) = 0$

$f(2) = f(-2) = 1$

$f'(x) > 0$ se $x < 0$: Função Crescente

$f'(x) < 0$ se $x > 0$: Função Decrescente

Concavidade: $\begin{cases} f''(x) > 0 \text{ se } |x| > 2 \rightarrow \text{Concavidade p/ cima} \\ f''(x) < 0 \text{ se } |x| < 2 \rightarrow \text{Concavidade p/ baixo} \end{cases}$



c) Dados: $f(-2) = f(6) = -2$

$f(0) = f(4) = 0$; PONTOS CRÍTICOS: $\nexists f'(2)$, $\nexists f'(6)$ e $f'(0) = 1$

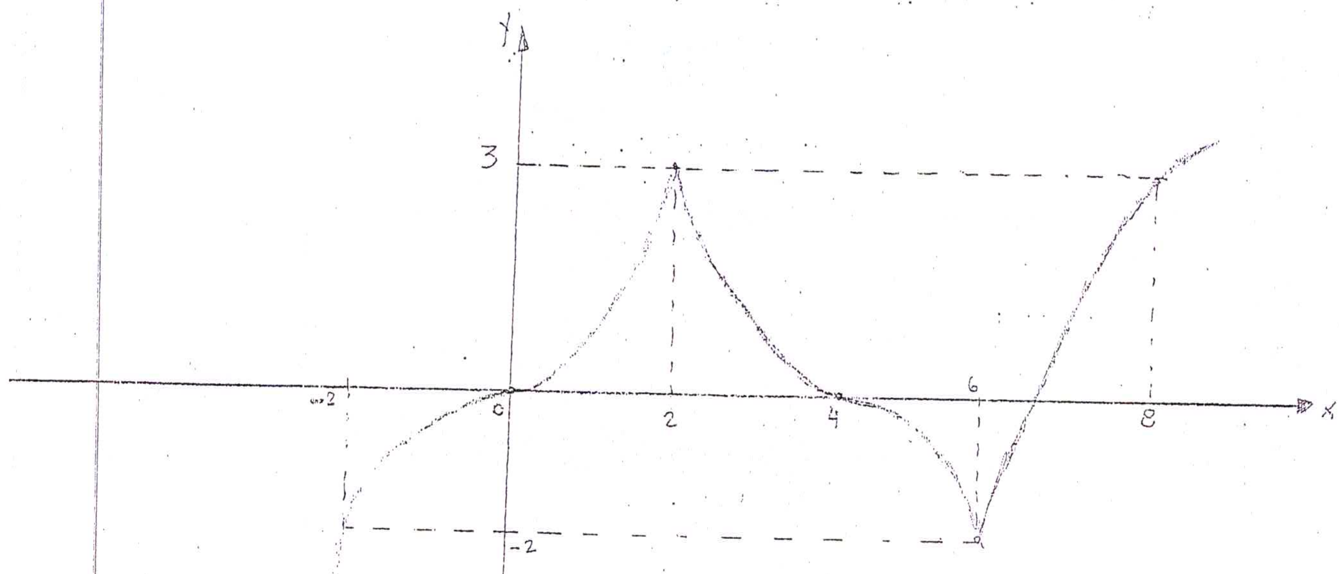
$f(2) = f(8) = 3$

$f'(x) > 0$: $(-\infty, 2)$ e $(6, \infty) \rightarrow$ Função Crescente

$f'(x) < 0$: $|x-4| < 2 \rightarrow$ Função Decrescente

$f''(x) < 0$: $(-\infty, 0)$, $(2,6)$ e $(6,\infty) \rightarrow$ Concav. p/ baixo

$f''(x) > 0$: $(0,2)$, $(2,4) \rightarrow$ Conc. p/ cima.



d) $f(x) = \dfrac{2x^2}{x^2+1}$ (1) Dom: $\mathbb{R}$

(2) Continuidade: $(-\infty, +\infty)$

(3) $f(x) = 0$ para $x = 0$ Intercepto: $(0,0)$

(4) $f(x) = \dfrac{2x^2}{x^2+1}$ e $f(-x) = \dfrac{2x^2}{x^2+1}$ $\therefore f(x) = f(-x)$ Simetria Par ao redor do eixo y

(5) $f'(x) = \dfrac{1}{(x^2+1)^2}\left(4x(x^2+1) - 2x^2(2x)\right) = \dfrac{4x^3 + 4x - 4x^3}{(x^2+1)^2} = \dfrac{4x}{(x^2+1)^2}$

$f'(x) = 0$ para $x = 0$ Ponto Crítico $\therefore x = 0$

(6) p/ $x < 0 \to f'(x) < 0$ Função Decrescente

p/ $x > 0 \to f'(x) > 0$ Função Crescente

(7) $f''(x) = \dfrac{1}{(x^2+1)^4}\left(4(x^2+1)^2 - 8x(x^2+1)\cdot 2x\right) = \dfrac{4x^2+4-16x^2}{(x^2+1)^3}$

$f''(x) = \dfrac{-12x^2+4}{(x^2+1)^3}$

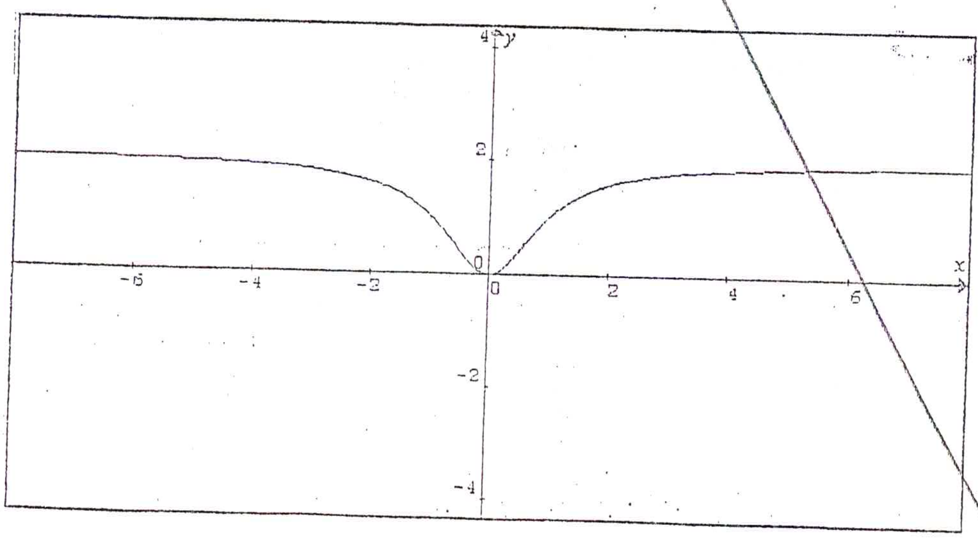
p/ $-12x^2+4 > 0 \therefore -3/4 < x < 3/4$ Concavidade p/cima

$-12x^2+4 < 0 \therefore x < -3/4$ ou $x > 3/4$ Concavidade p/baixo

(8) Assíntotas Verticais: $\lim\limits_{x\to 0^+} f(x) = +\infty$ $\lim\limits_{x\to 0^-} f(x) = +\infty$

Assíntotas Horizontais: $\lim\limits_{x\to\infty} \dfrac{2}{1+1/x^2} = 2$

(9) Gráfico:



e) $f(x) = \dfrac{x+4}{\sqrt{x}}$ (1) Dom: $\{x \in \mathbb{R} \,/\, x > 0\}$ ou $\mathbb{R} > 0$

(2) Continuidade: $(+0, +\infty)$

(3) $f(x) = 0$ para $x = -4$. Intercepto: $(-4, 0)$

(4) $f(x) = \dfrac{x+4}{\sqrt{x}}$ e $f(-x) \Rightarrow \nexists f(-x)$

(5) $f'(x) = \dfrac{1}{x}\left(\sqrt{x} - \dfrac{(x+4)}{2\sqrt{x}}\right) = \dfrac{2x-x-4}{2x\sqrt{x}} = \dfrac{x-4}{2x\sqrt{x}} = \dfrac{x-4}{2x^{3/2}}$

$2\sqrt[2]{x^3}$

- $f'(x) = 0$ para $x = +4$
- $\nexists f'(x)$ para $x = 0$

(6) Pl $0 < x < 4 \rightarrow f'(x) < 0$ decrescente

$x > 4 \rightarrow f'(x) > 0$ Crescente

(7) $f''(x) = \dfrac{1}{4x^3}\left(2x^{3/2} - 3x^{1/2}(x-4)\right) = \dfrac{-x^{3/2} + 12x^{1/2}}{4x^3}$

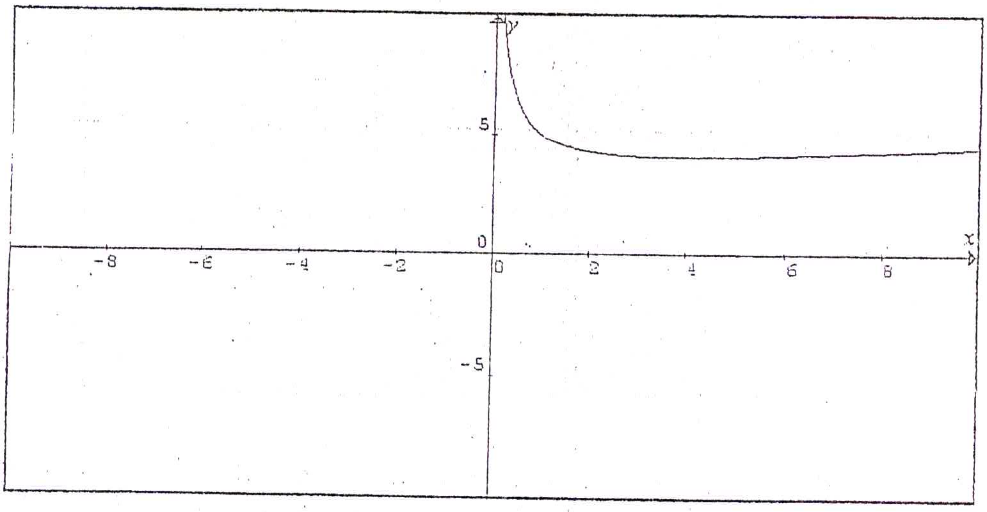
$f''(x) = \dfrac{x^{1/2}(-x+12)}{4x^3} = \dfrac{-x+12}{4x^{5/2}} = \dfrac{-x+12}{4\sqrt{x^5}}$

- $f''(x) > 0$ para $x < 12$ Concavidade para cima
- $f''(x) < 0$ para $x > 12$ Concavidade para baixo

(8) Assíntota Horizontal: $\lim\limits_{x \to \infty} f(x)$

Assíntota Vertical: $\lim\limits_{x \to 0^-} f(x) \Rightarrow \nexists$ $\lim\limits_{x \to 0^+} f(x) = +\infty$

(9) Gráfico:



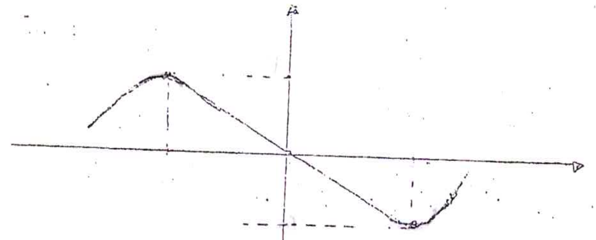
d) Dados: $f(-5) = 4$; PONTOS CRÍTICOS : $f'(-5) = f'(0) = f'(5) = 0$

$f(0) = 0$

$f(5) = -4$

$f'(x) > 0$ se $|x| > 5$: Crescente

$f'(x) < 0$ se $0 < |x| < 5$: Decrescente



17. Resolvido no exercício 13.

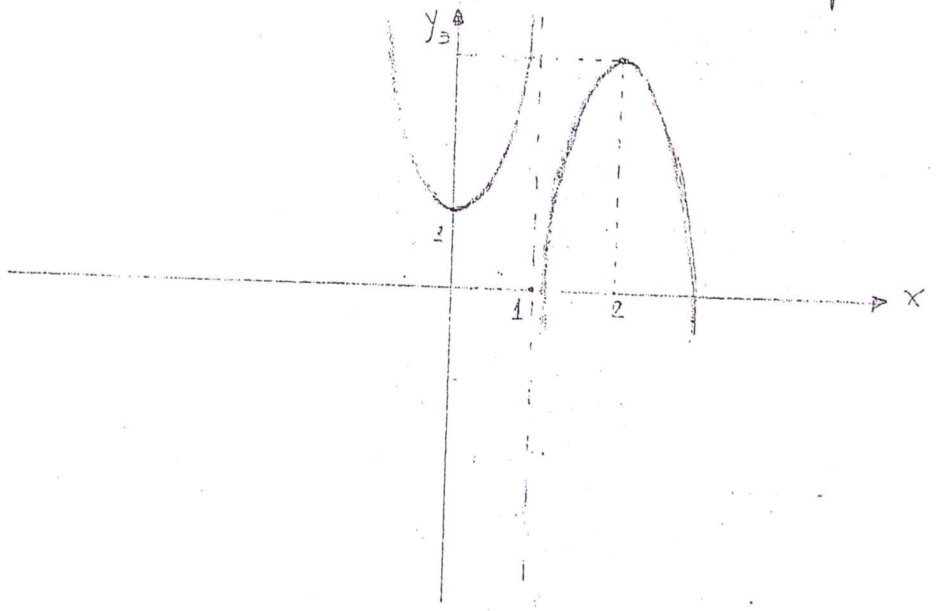
18. Enunciado incompleto.

19. a) Dados: $f(0) = 1$, $f(2) = 3$; Pontos Críticos : $f'(0) = f'(2) = 0$

$f'(x) < 0$: $|x-1| > 1$: Função Decrescente

$f'(x) > 0$: $|x-1| < 1$: Função Crescente

Concavidade $\begin{cases} f''(x) > 0 \text{ se } x < 1 \therefore \text{Concavidade p/ cima} \\ f''(x) < 0 \text{ se } x > 1 \therefore \text{Concavidade p/ baixo} \end{cases}$

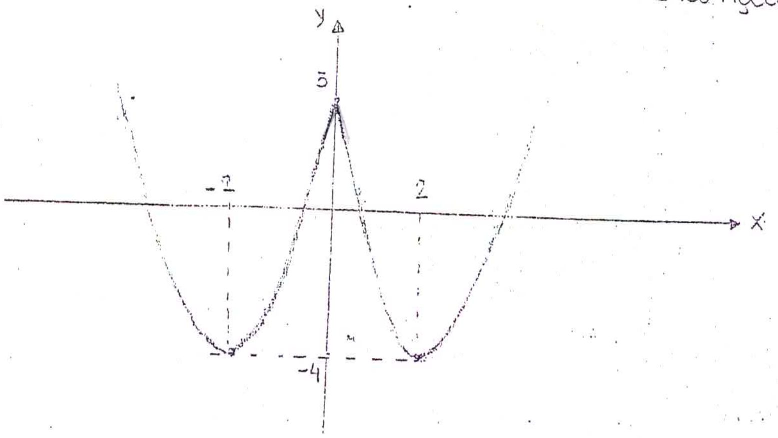


16. a) Dados: $f(0) = 3$; PONTOS CRÍTICOS: $\nexists f'(0)$

$f(-2) = f(2) = -4$; $f'(-2) = f'(2) = 0$

$f'(x) > 0$: $-2 < x < 0$ ou $x > 2$: Função Crescente

$f'(x) < 0$: $x < -2$ ou $0 < x < 2$: Função Decrescente

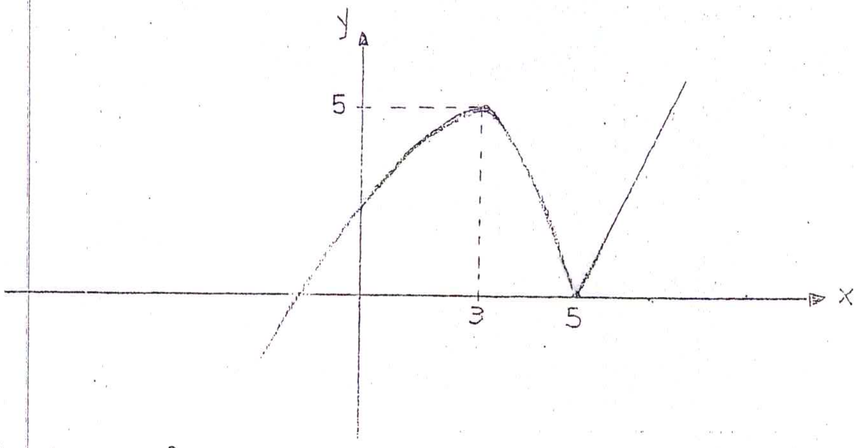


b) Dados: $f(3) = 5$; PONTOS CRÍTICOS: $\nexists f'(5)$

$f(5) = 0$; $f'(3) = 0$

$f'(x) > 0$: $x < 3$ ou $x > 5$: Função Crescente

$f'(x) < 0$: $3 < x < 5$: Função Decrescente

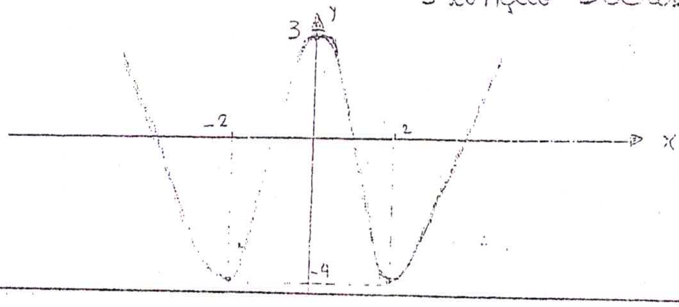


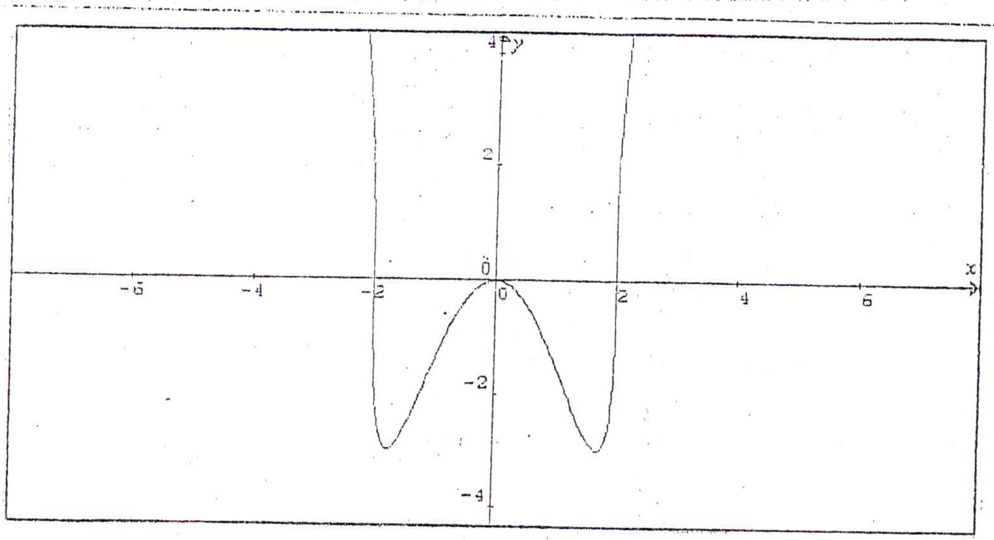
c) Dados: $f(0) = 3$; PONTOS CRÍTICOS: $f'(-2) = f'(0) = f'(2) = 0$

$f(-2) = f(2) = -4$

$f'(x) > 0$: $-2 < x < 0$ ou $x > 2$: Função Crescente

$f'(x) < 0$: $x < -2$ ou $0 < x < 2$: Função Decrescente



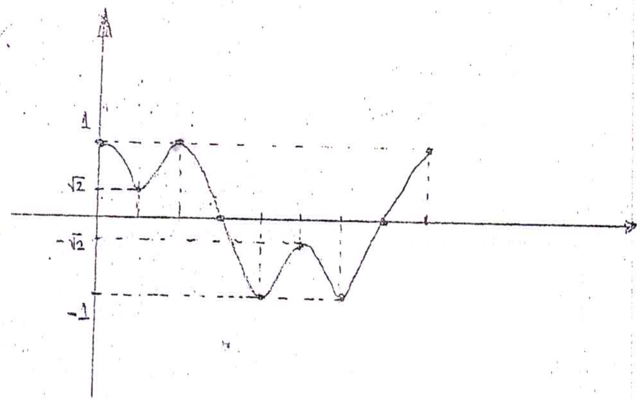


14. a) $f(x) = \cos x + \operatorname{sen} x$

Considerar o intervalo $[0, 2\pi]$

x	$f(x)$
0	1
$\pi/4$	$\sqrt{2}$
$\pi/2$	1
$3\pi/4$	0
π	-1

x	$f(x)$
$5\pi/4$	$-\sqrt{2}$
$3\pi/2$	-1
$7\pi/4$	0
2π	1



Máximo local: $x = \pi/2$ e $x = 5\pi/4$

mínimo local: $x = \pi/4$, π e $3\pi/2$

e) $f(x) = x^{2/3}(8-x)$

$f'(x) = 2/3\, x^{-1/3}(8-x) + x^{2/3}(-1) = \dfrac{2(8-x)}{3x^{1/3}} - x^{2/3} = \dfrac{16-2x-3x}{3x^{1/3}}$

$f'(x) = \dfrac{16-5x}{3x^{1/3}} \longrightarrow$ Pontos Críticos: $f'(x)=0$

$16-5x=0 \therefore x = 16/5$ ou $\nexists f'(x) \to x=0$

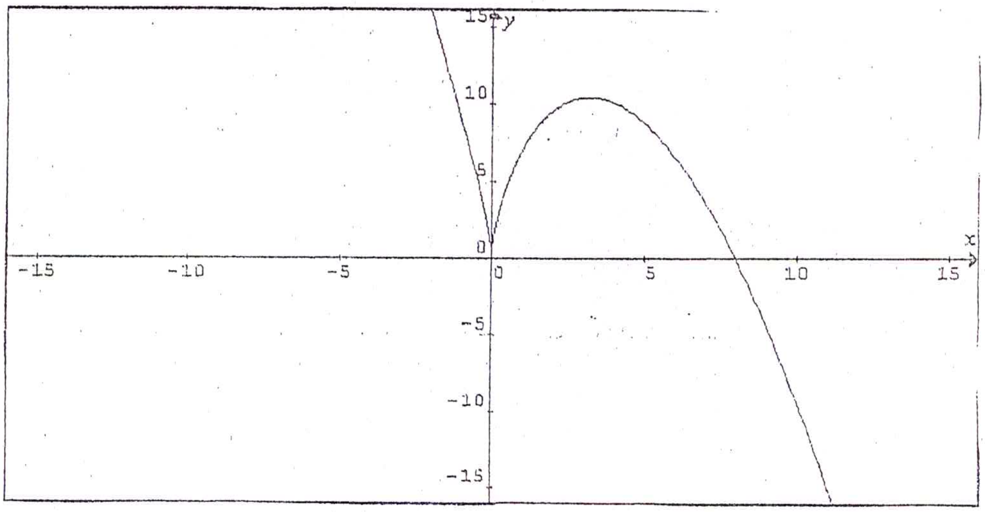
$f''(x) = \dfrac{1}{3x^{2/3}} * \left[-15x^{1/3} - \dfrac{(16-5x)}{x^{2/3}}\right] = \dfrac{1}{3x^{2/3}} * \left[\dfrac{-15x-16+5x}{x^{2/3}}\right]$

$f''(x) = \dfrac{-10x-16}{3x^{4/9}}$

$\to f'(16/5) = 0 \quad f''(16/5) < 0 \to$ Máximo local

P/ $x=0 \to \begin{cases} \text{se } x<0 \therefore f'(x)<0 \\ \text{se } 0<x<16/5 \therefore f'(x)>0 \end{cases}$ } mínimo local

Crescimento: $(0, 16/5)$ e Decrescente: $(-\infty,0) \cup (16/5,+\infty)$



f) $f(x) = x^2(x^2-4)^{1/3}$

$f'(x) = 2x(x^2-4)^{1/3} + x^2 * 1/3\,(x^2-4)^{-2/3} * 2x$

$f'(x) = 2x(x^2-4)^{1/3} + \dfrac{2x^3}{3(x^2-4)^{2/3}} = \dfrac{6x(x^2-4)+2x^3}{3(x^2-4)^{2/3}} = \dfrac{8x^3-24x}{3(x^2-4)^{2/3}}$

PONTO CRÍTICO: $f'(x)=0 \to 8x(x^2-3)=0$

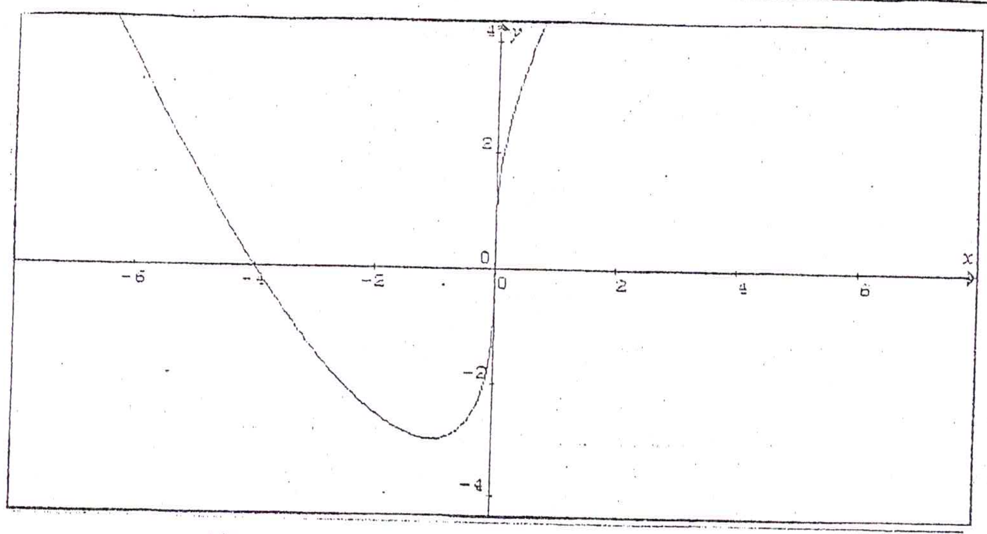
$x=0$; $x=-\sqrt{3}$; $x=\sqrt{3}$

ou $\nexists f'(x) \to x=\pm 2$

Crescimento $(-\sqrt{3},0) \cup (\sqrt{3},+\infty)$

Decrescente $(-\infty,-\sqrt{3}) \cup (0,\sqrt{3})$

$\therefore$ máximo local em $x=0$
mínimo local em $x=\pm\sqrt{3}$



d) $f(x) = x(x-5)^{1/3}$

$f'(x) = (x-5)^{1/3} + x/3\,(x-5)^{-2/3} = \dfrac{3x-15+x}{3(x-5)^{2/3}} = \dfrac{4x-15}{3(x-5)^{2/3}}$

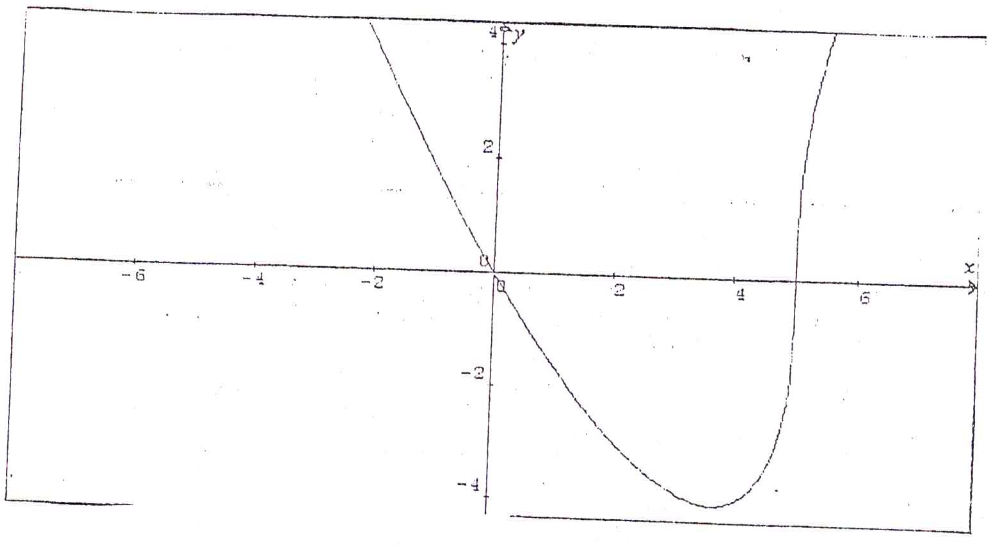
Pontos Críticos: $\begin{cases} f'(x)=0 \rightarrow 4x-15=0 \therefore x=15/4 \\ \nexists f'(x) \rightarrow x=5 \end{cases}$

$x < 15/4 \rightarrow f'(x) < 0$

$x > 15/4 \rightarrow f'(x) > 0$

$\therefore x = \frac{15}{4}$ é mínimo local.

Decrescente: $(-\infty, 15/4)$ e crescente $(15/4, 5) \cup (5, +\infty)$



PONTOS CRÍTICOS: $x = 0$; $x = 1$; $x = 3/5$

$f''(x) = 200x^3 - 240x^2 + 60x$

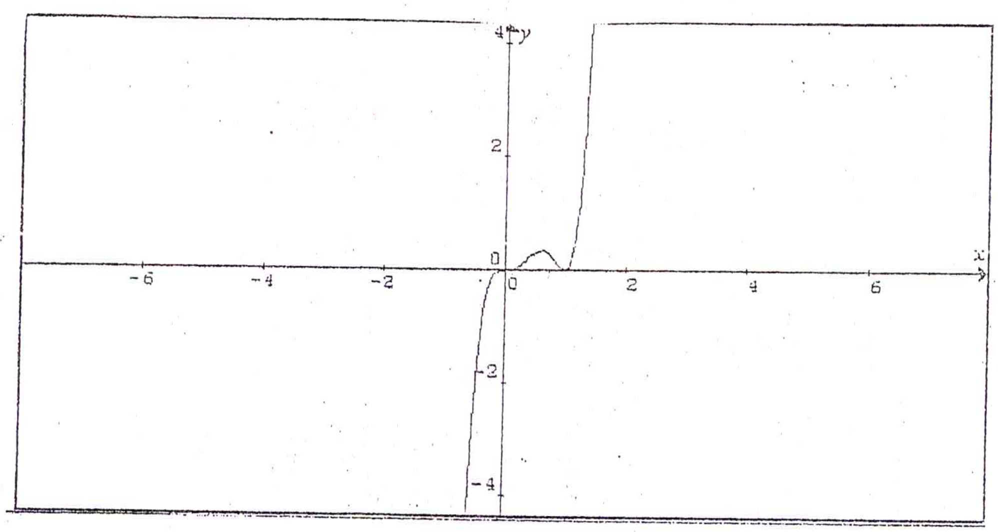
$\rightarrow f'(0) = 0$ e $f''(0) = 0$ $\rightarrow$ não é máximo nem mínimo

$\rightarrow f'(1) = 0$ e $f''(1) > 0$ mínimo local

$\rightarrow f'(3/5) = 0$ e $f''(3/5) < 0$ máximo local

Função Crescente: $f'(x) > 0 \rightarrow x < 3/5$ ou $x > 1$

Função Decrescente: $f'(x) < 0 \rightarrow 3/5 < x < 1$



c) $f(x) = x^{4/3} + 4x^{1/3}$

$f'(x) = \frac{4}{3}x^{1/3} + \frac{4}{3}x^{-2/3} = \frac{4}{3}\left(x^{1/3} + x^{-2/3}\right) \longrightarrow x^{1/3} = -\frac{1}{x^{2/3}} \therefore x = -1$

PONTOS CRÍTICOS: $f'(x) = 0 \rightarrow x = -1$ ou $\nexists f'(x) \therefore x = 0$

PONTOS CRÍTICOS: $x = 0$; $x = -1$

$f''(x) = 4/9\, x^{-2/3} - 8/9\, x^{-5/3} = 4/9x^{2/3} - 8/9x^{5/3}$

$\rightarrow f'(0) = 0$ e $\nexists f''(0) \Rightarrow$ não é máximo nem mínimo

$\rightarrow f'(-1) = 0$ e $f''(-1) > 0$ é mínimo local.

P/ $x = -1$ $\begin{cases} \text{se } x < -1 \rightarrow f'(x) < 0 \therefore (-\infty, -1) \text{ decrescente} \\ \text{se } x > -1 \rightarrow f'(x) > 0 \quad (-1, 0) \cup (0, \infty) \text{ crescente} \end{cases}$

12. $f(x) = \operatorname{sen} x$ $\quad f'(x) = \cos x$

Considerando valores arbitrários u e v: $f(u) = \operatorname{sen} u$ e $f(v) = \operatorname{sen} v$

$$f'(c) = \frac{|f(u) - f(v)|}{|u-v|} = \cos c$$

Sabendo que $\cos x$ terá valor máximo 1: $\frac{|\operatorname{sen} u - \operatorname{sen} v|}{|u-v|} \leq 1$

$|u-v| \geq |\operatorname{sen} u - \operatorname{sen} v|$

13. a) $f(x) = 2x^3 + x^2 - 20x + 1$

$f'(x) = 6x^2 + 2x - 20 \longrightarrow$ Pontos Críticos: $6x^2 + 2x - 20 = 0$

$6\left(x^2 + \frac{1}{3}x - \frac{10}{3}\right) = 0 \left\{ \begin{array}{l} \text{Soma} = -1/3 \\ \text{Produto} = -10/3 \end{array} \right. \longrightarrow x_1 = -2$ e $x_2 = \frac{5}{3}$

Pontos Críticos: $x_1 = -2$ e $x_2 = 5/3$

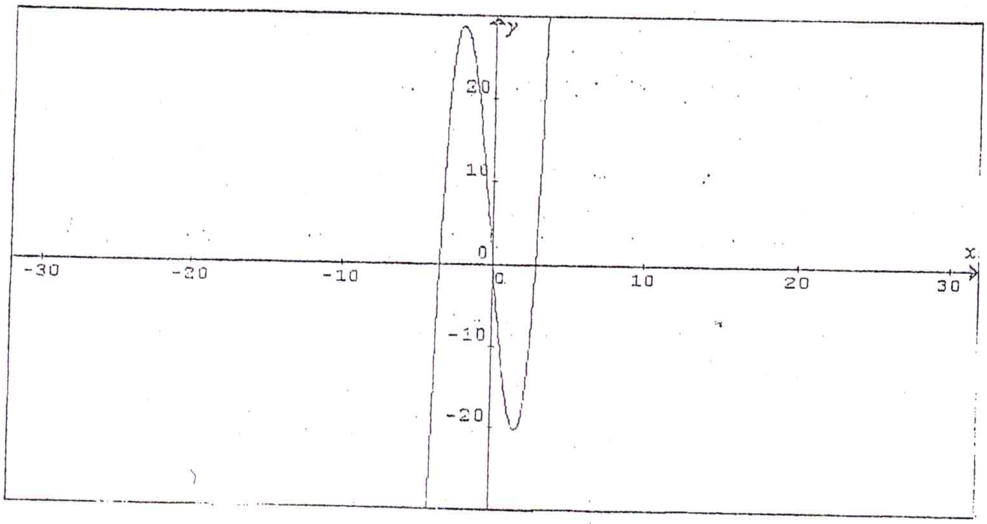
$f''(x) = 12x + 2$

$f'(-2) = 0$ e $f''(-2) = -24 + 2 < 0 \therefore x = -2$ é máximo local.

$f'(5/3) = 0$ e $f''(5/3) = 12 \cdot \frac{5}{3} + 2 > 0 \therefore x = 5/3$ é mínimo local.

Função crescente quando $f'(x) > 0$: $x < -2$ ou $x > 5/3$

Função decrescente quando $f'(x) < 0$: $-2 < x < 5/3$



b) $f(x) = 10x^3(x-1)^2$

$f'(x) = 30x^2(x-1)^2 + 20x^3(x-1) = 30x^2(x^2 - 2x + 1) + 20x^4 - 20x^3$

$f'(x) = 50x^4 - 80x^3 + 30x^2 = x^2(50x^2 - 80x + 30)$

Pontos Críticos: $10x^2(5x^2 - 8x + 3) = 0 \quad x = 0$ ou $5x^2 - 8x + 3 = 0$

$f'(c) = 1 - \frac{4}{c^2} \quad \therefore \quad 1 - \frac{4}{c^2} = \frac{f(4) - f(1)}{3} = \frac{5-5}{3} = 0$

$\rightarrow \frac{4}{c^2} = 1 \rightarrow c = \pm 2.$

d) $f(x) = (x+2)^{2/3} \quad , \quad [-1,6].$

$f'(x) = \frac{2}{3(x+2)^{1/3}}$ Restrição p/ $f'(x)$: $x \neq -2$

1) $f(x)$ é contínua em $[-1,6]$

2) $f(x)$ é diferenciável em $(-1,6)$

$f'(c) = \frac{2}{3(c+2)^{1/3}} = \frac{f(6) - f(-1)}{7} \longrightarrow \frac{2}{3(c+2)^{1/3}} = \frac{3}{7}$

$(c+2)^{1/3} = \frac{14}{9} \rightarrow c+2 = \left(\frac{14}{9}\right)^3 \Rightarrow c = \frac{1286}{729}.$

e) $f(x) = tg\,x \;;\; [0, \pi/4]$ Restrição $x \neq \pi/2 + n\pi,\ n \in \mathbb{R}$

$f'(x) = \sec^2 x = \frac{1}{\cos^2 x}$ Restrição $f'(x) \rightarrow x \neq \pi/2 + n\pi,\ n \in \mathbb{R}$

1) $f(x)$ é contínua em $[0, \pi/4]$

2) $f(x)$ é diferenciável em $(0, \pi/4)$

$f'(c) = \frac{1}{\cos^2 c} = \frac{f(\pi/4) - f(0)}{\pi/4} = \frac{4}{\pi} \longrightarrow \cos^2 c = \frac{\pi}{4} \quad \therefore \quad c = \cos^{-1}\left(\pm\sqrt{\frac{\pi}{4}}\right)$

9. $f(x) = 5 + 3(x-1)^{2/3}$

$f(0) = 5 + 3(-1)^{2/3} = 8$ e $f(2) = 5 + 3(2-1)^{2/3} = 8 \quad \therefore \quad f(0) = f(2)$

$f'(x) = \frac{2}{(x-1)^{1/3}}$: Restrição $x \neq 1 \longrightarrow f'(x) \neq 0$

$\rightarrow f(x)$ não é diferenciável em $(-1,1)$

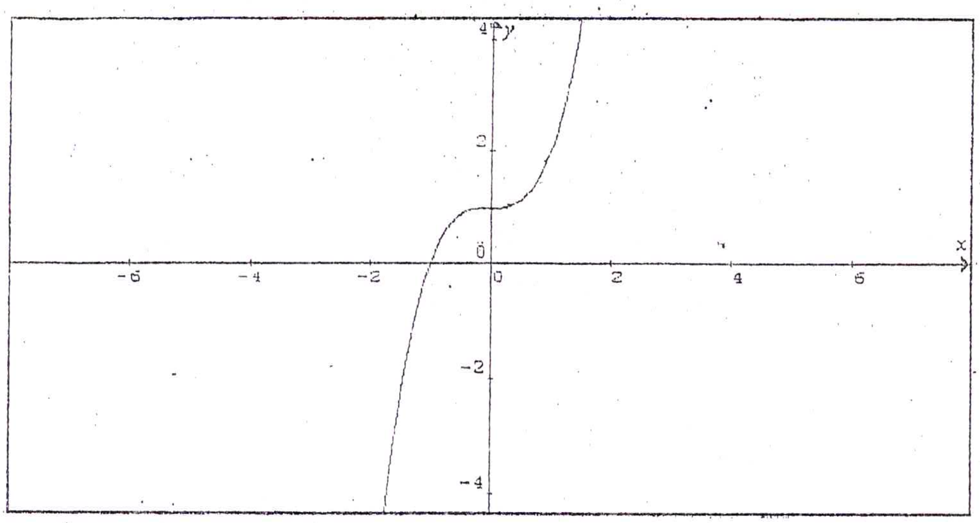
10. $f(x) = 4/x$ Restrição $x \neq 0.$

$f'(x) = -4/x^2$

No intervalo $[-1,4]$ $f(x)$ não é contínua e não é diferenciável.
Não é possível aplicar o Teorema do Valor médio.

11.

b)



c) $f(x) = x^3 + 1$. Dom: $\{x \in \mathbb{R}\}$ $\longrightarrow$ O ponto $(0,1)$ está contido no domínio de $f(x)$. Logo, a função é contínua em $(0,1)$.

Em $x=0$ não há extremo. (ver letra a).

1. a) $f(x) = 3x^2 + x - 4$, $[1,5]$

1) $f(x)$ é contínua em $[a,b]$; f é contínua em $[1,5]$

2) $f(x)$ é diferenciável em $(1,5)$

Se 1 e 2 se verificam : $\exists c \in (a,b) \,/\, f'(c) = \dfrac{f(b) - f(a)}{b-a}$

$f'(x) = 6x + 1$

$f'(c) = 6c + 1 = \dfrac{f(5) - f(1)}{4} = \dfrac{76 - 0}{4} \therefore 6c = \dfrac{76}{4} - 1$

$c = \dfrac{18}{6} = 3 \longrightarrow c = 3$

b) $f(x) = \dfrac{x+3}{x-2}$, $[-2, 3]$ Restrição : $x - 2 \neq 0 \therefore x \neq 2$

1) $f(x)$ não é contínua no intervalo $[-2, 3]$ pois $\nexists f(-2)$.

não satisfaz as hipóteses do teorema do valor médio.

c) $f(x) = x + 4/x$, $[1, 4]$ Restrição $x \neq 0$

$f'(x) = 1 - \dfrac{4}{x^2}$

l) $H(\phi) = \cot\phi + \csc\phi \longrightarrow H'(\phi) = -\csc^2 x - \csc x \cot x$

$H'(\phi) = \frac{-1}{\text{sen}^2 x} - \frac{1}{\text{sen}x} \cdot \frac{\cos x}{\text{sen} x} = \frac{-1-\cos x}{\text{sen}^2 x}$

$H'(\phi) = 0 \longrightarrow \cos x = -1 \;\therefore\; x = \pi + 2n\pi,\ n \in \mathbb{Z}$

$\nexists H'(\phi) \Rightarrow \text{sen} x = 0 \;\therefore\; x_1 = 0 + 2n\pi,\ n \in \mathbb{Z}$

$x_2 = \pi + 2n\pi,\ n \in \mathbb{Z}$

Pontos Críticos: $x = 2\pi n,\ n \in \mathbb{Z}$

5. a) $f(x) = x^{1/3}$

$f'(x) = \frac{1}{3} x^{-2/3} = \frac{1}{3x^{2/3}} \longrightarrow$ Ponto Crítico: $f'(x) = 0$ ou $\nexists f'(x)$

$\begin{cases} f'(x) = 0 \rightarrow \nexists x \,/\, f'(x) = 0 \\ \nexists f(x) \rightarrow x = 0 \end{cases} \longrightarrow$ Ponto Crítico $\therefore x = 0$

Para ser extremo deve marcar mudança de crescimento e decrescimento.

$f'(x) = \frac{1}{3\sqrt[3]{x^2}} \rightarrow$ p/ $x<0$ e $x>0$; $f'(x) > 0$ sempre.

Não é extremo local.

b) $f(x) = x^{2/3} \longrightarrow f'(x) = \frac{2}{3} x^{-1/3} = \frac{2}{3x^{1/3}}$

$\begin{cases} \text{Ponto Crítico: } f'(x) = 0 \rightarrow \nexists x \,/\, f'(x) = 0 \\ \nexists f'(x) \rightarrow x^{1/3} = 0 \;\therefore\; x = 0 \end{cases} \longrightarrow$ Ponto Crítico: $x = 0$

$\left.\begin{array}{l} \text{p/ } x<0 : f'(x) < 0 \\ \text{p/ } x>0 : f'(x) > 0 \end{array}\right\}$ $x=0$ é mínimo local.

6. $f(x) = |x| \longrightarrow f(x) = \begin{cases} x, & x>0 \\ -x, & x<0 \end{cases} \longrightarrow f'(0^+) = 1$ e $f'(0^-) = -1$

Como $f'(0^+) \neq f'(0^-) \rightarrow x = 0$

é ponto crítico $\rightarrow$ único ponto em que $\nexists f'(x)$.

$\left.\begin{array}{l} \text{p/ } x<0 \rightarrow f'(x) < 0 \\ \text{p/ } x>0 \rightarrow f'(x) > 0 \end{array}\right\}$ $x=0$ é mínimo local.

7. $f(x) = x^3 + 1 \longrightarrow$ a) $f'(x) = 3x^2$

p/ ter extremos: Encontrar pontos críticos.

$\rightarrow f'(x) = 0 \rightarrow \therefore x = 0$ ∴ p/ ser extremo deve marcar mudança de cresc/// e decrescimento. Porém $f'(x) = 3x^2 > 0\ \forall x$. Não é extremo

$\rightarrow \nexists f(x)$: não há um valor de x que satisfaça a condição.

LISTA 7:

Integrais I

$\triangleright\ tg\,2x$

$u = 2x \qquad u' = 2$

$f(u) = tg\,u \longrightarrow sec^2 u \triangleright sec^2 2x$

$tg'\,2x \qquad 2 \cdot sec^2 2x$

aa) $\lim\limits_{x\to 0} \dfrac{x + tg\,2x}{x - tg\,2x}$: Indeterminação $\frac{0}{0}$. Aplicar L'Hopital:

$\lim\limits_{x\to 0} \dfrac{1+2\sec^2 2x}{1-2\sec^2 2x} = \dfrac{1+2}{1-2} = -3$.

bb) $\lim\limits_{x\to 0} \dfrac{tg\,2x}{tgh\,3x}$: Indeterminação $\frac{0}{0}$. Aplicar L'Hopital:

$\lim\limits_{x\to 0} \dfrac{2\sec^2 2x}{3\,\text{sech}^2 3x} = \lim\limits_{x\to 0} \dfrac{2}{3} * \dfrac{\cosh^2 3x}{\cos^2 2x} = \lim\limits_{x\to 0} \dfrac{2}{3} * \dfrac{\left(\frac{e^{3x}+e^{-3x}}{2}\right)^2}{\cos^2 2x} = \dfrac{2}{3}$

cc) $\lim\limits_{x\to 0} \dfrac{1-e^{-2x}}{\sec x} = \frac{0}{1} = 0$ não se aplica L'Hopital.

2. a) $\lim\limits_{x\to 0^+} \sqrt{x}\,.\ln x$: Indeterminação $0 * \infty$

Reescrever : $\lim\limits_{x\to 0^+} \dfrac{\ln x}{1/\sqrt{x}} \longrightarrow$ Indeterminação $\frac{\infty}{\infty}$. Aplicar L'Hopital:

$\lim\limits_{x\to 0^+} \dfrac{1/x}{-\frac{1}{2}x^{-3/2}} = \lim\limits_{x\to 0^+} -2x^{1/2} = 0$

b) $\lim\limits_{x\to\infty} \dfrac{\ln x}{e^x}$: Indeterminação $\frac{\infty}{\infty}$. Aplicar L'Hopital:

$\lim\limits_{x\to\infty} \dfrac{1}{x e^x} = \dfrac{1}{\infty} = 0$

c) $\lim\limits_{x\to\infty} \dfrac{x^3}{e^{x^2}}$: Indeterminação $\frac{\infty}{\infty}$. Aplicar L'Hopital:

$\lim\limits_{x\to\infty} \dfrac{3x^2}{2x\,e^{x^2}} = \lim\limits_{x\to\infty} \dfrac{3x}{2e^{x^2}} \longrightarrow$ Indeterminação $\frac{\infty}{\infty}$. Aplicar L'Hopital

$\lim\limits_{x\to\infty} \dfrac{3}{4x e^{x^2}} = \dfrac{3}{\infty} = 0$.

d) $\lim\limits_{x\to\pi} (x-\pi)\cot x = \lim\limits_{x\to\pi} \dfrac{(x-\pi)}{tg\,x}$: Indeterminação $\frac{0}{0}$. Aplicar L'Hopital:

$\lim\limits_{x\to\pi} \dfrac{1}{\sec^2 x} = \dfrac{1}{(-1)^2} = 1$

e) $\lim\limits_{x\to 1^+} (x-1)\,tg\left(\frac{\pi x}{2}\right)$: Indeterminação $0 * \infty$

Reescrever : $\lim\limits_{x\to 1^+} \dfrac{(x-1)}{\frac{1}{tg(\pi x/2)}} \longrightarrow$ Indeterminação $\frac{0}{0}$. Aplicar L'Hopital:

$\lim\limits_{x\to 1^+} \dfrac{1}{-tg^{-2}(\frac{\pi x}{2}) * \sec^2(\frac{\pi x}{2}) * \frac{\pi}{2}} = \lim\limits_{x\to 1^+} \dfrac{-2\,tg^2(\pi x/2)}{\pi\,\sec^2(\pi x/2)} = \lim\limits_{x\to 1^+} -\dfrac{2}{\pi} * \text{sen}^2(\frac{\pi x}{2}) = -\dfrac{2}{\pi}$

f) $\lim\limits_{x\to -\infty} x^2 e^x = \lim\limits_{x\to -\infty} \dfrac{x^2}{e^{-x}}$: Indeterminação $\frac{\infty}{\infty}$. Aplicar L'Hopital:

$\lim\limits_{x\to -\infty} \dfrac{2x}{-e^{-x}} \longrightarrow$ Indeterminação $\frac{\infty}{\infty}$. Aplicar L'Hopital:

$\lim\limits_{x\to -\infty} \dfrac{2}{e^{-x}} = 0$

g) $\lim\limits_{x\to(\pi/2)^-} \sec 7x \cos 3x = \lim\limits_{x\to(\pi/2)^-} \dfrac{\cos 3x}{\cos 7x}$: Indeterminação $\frac{0}{0}$. Aplicar L'Hopital:

$\lim\limits_{x\to(\pi/2)^-} \dfrac{-3\,\text{sen}\,3x}{-7\,\text{sen}\,7x} = \dfrac{3}{7}$

h) $\lim\limits_{x\to 0^+} \sqrt{x}\sec x = \lim\limits_{x\to 0^+} \dfrac{\sqrt{x}}{\cos x} = \dfrac{\sqrt{0}}{1} = 0$.

i) $\lim\limits_{x\to 0^+} x^{\text{sen}\,x}$: Indeterminação 0^0

Calcular $\lim\limits_{x\to 0^+} \text{sen}\,x\,\ln x$: Indeterminação $0 . \infty$. Aplicar L'Hopital:

Reescrever : $\lim\limits_{x\to 0^+} \dfrac{\ln x}{(\text{sen}\,x)^{-1}} = \lim\limits_{x\to 0^+} -\dfrac{\text{sen}^2 x}{x\cos x} \longrightarrow$ Indeterminação $\frac{0}{0}$. Aplicar L'Hopital:

$\lim\limits_{x\to 0^+} \dfrac{-2\,\text{sen}\,x\cos x}{\cos x - x\,\text{sen}\,x} = 0$ $\therefore$ $\lim\limits_{x\to 0^+} x^{\text{sen}\,x} = e^0 = 1$.

j) $\lim\limits_{x\to 0} (1-2x)^{1/x}$: Indeterminação 1^{∞}

Calcular $\lim\limits_{x\to 0} \frac{1}{x}\ln(1-2x)$: Indeterminação $\frac{0}{0}$. Aplicar L'Hopital:

$\lim\limits_{x\to 0} \frac{\frac{-2}{1-2x}}{1} = \frac{-2}{1} = -2 \therefore \lim\limits_{x\to 0} (1-2x)^{1/x} = e^{-2}$

k) $\lim\limits_{x\to 0^+} (\operatorname{sen} x)^{tg x}$; Indeterminação 0^0

Calcular $\lim\limits_{x\to 0^+} tg x \ln(\operatorname{sen} x)$: Indeterminação $0.\infty$. Reescrever:

$\lim\limits_{x\to 0^+} \frac{\ln(\operatorname{sen} x)}{1/tg x}$: Indeterminação $\frac{\infty}{\infty}$ Aplicar L'Hopital:

$$\lim_{x\to 0^+} \frac{\frac{1}{\operatorname{sen} x} * \cos x}{\frac{1}{tg^2 x} * \left[0 - 1 * \sec^2 x\right]} = \lim_{x\to 0^+} \frac{\frac{\cos x}{\operatorname{sen} x}}{\frac{-\cos^2 x}{\operatorname{sen}^2 x * \cos^2 x}} = \lim_{x\to 0^+} \frac{-\cos x * \operatorname{sen}^2 x}{\operatorname{sen} x} = 0 \therefore$$

$\lim\limits_{x\to 0^+} (\operatorname{sen} x)^{tg x} = e^0 = 1$

l) $\lim\limits_{x\to\infty} \left(1+\frac{a}{x}\right)^{bx}$: Indeterminação 1^{∞}

Calcular $\lim\limits_{x\to\infty} bx \ln\left(1+\frac{a}{x}\right)$: Indeterminação $\infty.0$. Reescrever:

$\lim\limits_{x\to\infty} \frac{\ln\left(1+\frac{a}{x}\right)}{1/bx}$: Indeterminação $\frac{0}{0}$ Aplicar L'Hopital:

$$\lim_{x\to\infty} \frac{\frac{1}{1+\frac{a}{x}} * \frac{-a}{x^2}}{\frac{-1}{bx^2}} = \lim_{x\to\infty} \frac{ab}{1+\frac{a}{x}} = ab \therefore \lim_{x\to\infty} \left(1+\frac{a}{x}\right)^{bx} = e^{ab}$$

$a\,bx^2$

m) $\lim\limits_{x\to\infty} x^{1/x}$: Indeterminação ∞^0

Calcular $\lim\limits_{x\to\infty} \frac{1}{x} * \ln x$: Indeterminação $\frac{\infty}{\infty}$. Aplicar L'Hopital:

$\lim\limits_{x\to\infty} \frac{1/x}{1} = 0 \therefore \lim\limits_{x\to\infty} x^{1/x} = e^0 = 1.$

n) $\lim\limits_{x\to\infty} \left(\frac{x}{x+1}\right)^x$ Quando $x\to\infty$, ou seja, x é suficientemente grande $\frac{x}{x+1} \approx \frac{x}{x} \approx 1$

Logo, forma de indeterminação 1^{∞}.

Calcular: $\lim\limits_{x\to\infty} x \ln\left(\frac{x}{x+1}\right)$: Indeterminação $\infty.0$

$\lim\limits_{x\to\infty} \frac{\ln\left(\frac{x}{x+1}\right)}{1/x}$: Indeterminação $\frac{0}{0}$. Aplicar L'Hopital:

$$\lim_{x\to\infty} \frac{\left(\frac{1}{\frac{x}{x+1}}\right) * \left(\frac{1}{(x+1)^2}\left[x+1-x\right]\right)}{-\frac{1}{x^2}} = \lim_{x\to\infty} \frac{\frac{x+1}{x} * \frac{1}{(x+1)^2}}{-\frac{1}{x^2}} = \lim_{x\to\infty} \frac{-x}{x+1}$$: Indeterminação $\frac{\infty}{\infty}$

Aplicando L'Hopital: $\lim\limits_{x\to\infty} \frac{-1}{1} = -1 \therefore \lim\limits_{x\to\infty} \left(\frac{x}{x+1}\right)^x = e^{-1}$

O) $\lim_{x\to 0^+} (-\ln x)^x$: Indeterminação ∞^0

Calcular: $\lim_{x\to 0^+} x\ln(-\ln x)$: Indeterminação $0.\infty$. Reescrever:

$\lim_{x\to 0^+} \frac{\ln|-\ln x|}{1/x}$: Indeterminação $\frac{\infty}{\infty}$. Aplicar L'Hopital:

$\lim_{x\to 0^+} \frac{\frac{1}{-\ln x} * \frac{-1}{x}}{-1/x^2} = \lim_{x\to 0^+} \frac{-x}{\ln x} = 0 \therefore \lim_{x\to 0^+} (-\ln x)^x = e^0 = 1$

P) $\lim_{x\to\infty} x^{(\ln 2)/(1+\ln x)}$: Indeterminação ∞^0.

Calcular: $\lim_{x\to\infty} \frac{\ln 2}{1+\ln x} * \ln x$: Indeterminação $\frac{\infty}{\infty}$ Aplicar L'Hopital:

$\lim_{x\to\infty} \frac{\ln 2/x}{1/x} = \ln 2 \therefore \lim_{x\to\infty} x^{(\ln 2)/(1+\ln x)} = e^{\ln 2} = 2$.

q) $\lim_{x\to\infty} (e^x + x)^{1/x}$: Indeterminação ∞^0.

Calcular: $\lim_{x\to\infty} \frac{1}{x} * \ln(e^x+x)$ Indeterminação $\frac{\infty}{\infty}$. Aplicar L'Hopital:

$\lim_{x\to\infty} \frac{e^x+1}{e^x+x}$: Indeterminação $\frac{\infty}{\infty}$. Aplicar L'Hopital:

$\lim_{x\to\infty} \frac{e^x}{e^x+1}$: Indeterminação $\frac{\infty}{\infty}$. Aplicar L'Hopital: $\lim_{x\to\infty} \frac{e^x}{e^x} = 1$.

$\lim_{x\to\infty} (e^x + x)^{1/x} = e^1 = e$

R) $\lim_{x\to 0} (\cos 3x)^{5/x}$: Indeterminação 1^∞.

Calcular: $\lim_{x\to 0} \frac{5}{x} \cdot \ln(\cos 3x)$ Indeterminação $\frac{0}{0}$. Aplicar L'Hopital:

$\lim_{x\to 0} \frac{-5/(\cos 3x) * 3\operatorname{sen} 3x}{1} = \lim_{x\to 0} \frac{-15\operatorname{sen} 3x}{\cos 3x} = 0 \therefore \lim_{x\to 0} (\cos 3x)^{5/x} = e^0 = 1$.

s) $\lim_{x\to\infty} \left(\frac{2x-3}{2x+5}\right)^{2x+1}$ Quando $x\to\infty$, $\frac{2x-3}{2x+5} \to 1 \therefore$ Indeterminação 1^∞.

Calcular: $\lim_{x\to\infty} (2x+1)\ln\left(\frac{2x-3}{2x+5}\right)$ Indeterminação $\infty.0$. Reescrever:

$\lim_{x\to\infty} \frac{\ln\left|\frac{2x-3}{2x+5}\right|}{1/(2x+1)}$ Indeterminação $\frac{0}{0}$. Aplicar L'Hopital:

$\lim_{x\to\infty} \frac{\frac{1}{\left(\frac{2x-3}{2x+5}\right)} * \left[\frac{1}{(2x+5)^2}\left(2(2x+5) - 2(2x-3)\right)\right]}{-2/(2x+1)^2} = \lim_{x\to\infty} \frac{\frac{4x+10-4x+6}{(2x-3)(2x+5)}}{-2/(2x+1)^2}$

$\lim_{x\to\infty} \frac{-16(2x+1)^2}{2(2x-3)(2x+5)}$ Indeterminação $\frac{\infty}{\infty}$. Aplicar L'Hopital:

$\lim_{x\to\infty} \frac{-8(2x+1)^2}{4x^2+4x-15} = \lim_{x\to\infty} \frac{-16(2x+1)*2}{8x+4} = \lim_{x\to\infty} \frac{-8(8x+4)}{(8x+4)} = -8$

$\therefore \lim_{x\to\infty} \left(\frac{2x-3}{2x+5}\right)^{2x+1} = e^{-8}$

t) $\lim\limits_{x \to 0} \left[\frac{1}{x^4} - \frac{1}{x^2} \right] = \lim\limits_{x \to 0} \frac{1 - x^2}{x^4} = \frac{1}{0} = +\infty$

u) $\lim\limits_{x \to 0} (\text{cossec } x - \cot x) = \lim\limits_{x \to 0} \left(\frac{1}{\text{sen}x} - \frac{\cos x}{\text{sen}x} \right) = \lim\limits_{x \to 0} \left(\frac{1 - \cos x}{\text{sen}x} \right)$

Indeterminação $\frac{0}{0}$: $\lim\limits_{x \to 0} \frac{\text{sen}x}{\cos x} = \frac{0}{1} = 0$

v) $\lim\limits_{x \to 1} \left[\frac{1}{\ln x} - \frac{1}{x-1} \right] = \lim\limits_{x \to 1} \frac{(x-1) - \ln x}{(x-1)\ln x}$: Indeterminação $\frac{0}{0}$.

$\lim\limits_{x \to 1} \frac{1 - \frac{1}{x}}{\ln x + \frac{1}{x}(x-1)} = \lim\limits_{x \to 1} \frac{1 - \frac{1}{x}}{\ln x + 1 - \frac{1}{x}} = \lim\limits_{x \to 1} \frac{x-1}{x\ln x + x - 1}$ Indeterminação $\frac{0}{0}$

Aplicar L'Hopital: $\lim\limits_{x \to 1} \frac{1}{\ln x + 1 + 1} = \frac{1}{2}$

w) $\lim\limits_{x \to \infty} (xe^{1/x} - x) = \lim\limits_{x \to \infty} (e^{1/x} - 1)x$ Indeterminação $0 \cdot \infty$.

Reescrever: $\lim\limits_{x \to \infty} \frac{(e^{1/x} - 1)}{1/x}$ Indeterminação $\frac{0}{0}$ Aplicar L'Hopital:

$\lim\limits_{x \to \infty} \frac{-\frac{1}{x^2}e^{1/x}}{-\frac{1}{x^2}} = e^0 = 1$

x) $\lim\limits_{x \to 0} \left(\frac{1}{x} - \text{cossec}x \right) = \lim\limits_{x \to 0} \left(\frac{1}{x} - \frac{1}{\text{sen}x} \right) = \lim\limits_{x \to 0} \left(\frac{\text{sen}x - x}{x\,\text{sen}x} \right)$ Indeterminação $\frac{0}{0}$

Aplicar L'Hopital: $\lim\limits_{x \to 0} \frac{\cos x - 1}{\text{sen}x + x\cos x}$ Indeterminação $\frac{0}{0}$

Aplicar L'Hopital: $\lim\limits_{x \to 0} \frac{-\text{sen}x}{\cos x + \cos x - x\,\text{sen}x} = \frac{0}{2} = 0$

y) $\lim\limits_{x \to \infty} \left(x - \sqrt{x^2 - 1} \right) * \frac{(x + \sqrt{x^2-1})}{(x + \sqrt{x^2-1})} = \lim\limits_{x \to \infty} \frac{\cancel{x^2} - \cancel{x^2} + 1}{(x + \sqrt{x^2-1})}$

$\lim\limits_{x \to \infty} \frac{1}{x + \sqrt{x^2-1}} = \frac{1}{\infty} = 0$

z) $\lim\limits_{x \to \infty} \left(\frac{1}{x} - \frac{1}{e^x - 1} \right) = \lim\limits_{x \to \infty} \frac{1}{x} - \lim\limits_{x \to \infty} \frac{1}{e^x - 1} = 0 - 0 = 0$

$\lim\limits_{x \to \infty} \left(x - \sqrt{x^2-1} \right) * \frac{(x + \sqrt{x^2-1})}{(x + \sqrt{x^2-1})} = x^2 + x\sqrt{x^2-1} - x\sqrt{x^2-1}$

Lista 7: Integrais I

1. a) $f(x) = \sqrt{x}$

$F(x) = \int f(x)dx = \int x^{1/2}dx = \frac{x^{1/2+1}}{1/2+1} + C = \frac{2x^{3/2}}{3} + C$

b) $h(s) = s - 1/s$

$H(s) = \int h(s)\,ds = \int (s - 1/s).ds = \frac{s^{1+1}}{1+1} - \ln|s| + C = \frac{s^2}{2} - \ln|s| + C$

c) $f(t) = e^{-2}$

$F(t) = \int f(t)dt = \int e^{-2}dt = te^{-2} + C$

d) $g(x) = 1/\sqrt{x} = x^{-1/2}$

$G(x) = \int g(x)dx = \int x^{-1/2}dx = \frac{x^{-1/2+1}}{-1/2+1} + C = 2x^{1/2} + C$

e) $f(x) = x^5 - \sqrt[3]{x} + x^{-4} = x^5 - x^{1/3} + x^{-4}$

$F(x) = \int f(x)dx = \int (x^5 - x^{1/3} + x^{-4})\,dx = \frac{x^{5+1}}{5+1} - \frac{x^{1/3+1}}{1/3+1} + \frac{x^{-4+1}}{-4+1} + C$

$F(x) = \frac{x^6}{6} - \frac{3}{4}x^{4/3} - \frac{x^{-3}}{3} + C$

f) $p(t) = 100e^{-0,02t} - \frac{50}{t}$

$P(t) = \int p(t)dt = \int \left[100e^{-0,02t} - \frac{50}{t}\right] dt = 50\left(\frac{2e^{-0,02t}}{-0,02} - \ln|t|\right) + C$

$P(t) = -50\left(100e^{-0,02t} + \ln|t|\right) + C$

g) $A(r) = \pi r^2 - 2\pi r$

$= \int (\pi r^2 - 2\pi r)\,dr = \frac{\pi r^{2+1}}{2+1} - \frac{2\pi r^{1+1}}{1+1} + C = \pi\left(\frac{r^3}{3} - r^2\right) + C$

h) $q(s) = As^{0,7}$

$Q(s) = \int q(s)\,ds = \int As^{0,7}ds = A\left(\frac{s^{0,7+1}}{0,7+1}\right) + C = \frac{As^{1,7}}{1,7} + C$

2. a) $f(x) = 2xe^{x^2}$, $F(x) = 1 + e^{x^2}$

$F'(x) = f(x) \therefore f(x) = \frac{d}{dx}\left(1 + e^{x^2}\right) = 0 + e^{x^2}(2x)$

$\rightarrow f(x) = 2xe^{x^2}$

b) $f(x) = xe^{x/2}$, $F(x) = 2e^{x/2}(x-2)$

$F'(x) = f(x) \therefore f(x) = \frac{d}{dx}\left(2e^{x/2}(x-2)\right) = 2e^{x/2}\left(\tfrac{1}{2}\right)(x-2) + 2e^{x/2} \cdot 1$

$f(x) = e^{x/2}(x-2) + 2e^{x/2} = xe^{x/2} - 2e^{x/2} + 2e^{x/2} = xe^{x/2}$

$\rightarrow f(x) = xe^{x/2}$

c) $f(x) = \dfrac{2}{x^2-1}$, $F(x) = \ln\left|\dfrac{x-1}{x+1}\right|$

$F'(x) = f(x)$ ∴ $f(x) = \dfrac{d}{dx}\left(\ln\left|\dfrac{x-1}{x+1}\right|\right) = \dfrac{1}{\left(\frac{x-1}{x+1}\right)} * \left[\dfrac{1}{(x+1)^2}\Big((x+1)-(x-1)\Big)\right]$

$f(x) = \dfrac{x+1}{x-1} * \left[\dfrac{x+1-x+1}{(x+1)^2}\right] = \dfrac{2}{(x-1)(x+1)} = \dfrac{2}{x^2+x-x-1} = \dfrac{2}{x^2-1}$

d) $f(x) = \ln x$, $F(x) = x\ln x - x$

$F'(x) = f(x)$ ∴ $f(x) = \dfrac{d}{dx}\left[x\ln x - x\right] = (1*\ln x + x*1/x) - 1 = \ln x + 1 - 1 = \ln x$

$\rightarrow f(x) = \ln x$

3. a) $f(x) = (2x-3)^5$, $F'(x) = A(2x-3)^6$ $\frac{d}{dx}(2x-3)$?

$f(x) = F'(x) \longrightarrow (2x-3)^5 = 6A(2x-3)^5 * 2$ ∴ $A = \dfrac{(2x-3)^5}{12(2x-3)^5} = \dfrac{1}{12}$

b) $f(x) = \dfrac{x}{x^2+1}$, $F(x) = A\ln(x^2+1)$

$f(x) = F'(x) \longrightarrow \dfrac{x}{x^2+1} = \dfrac{2Ax}{x^2+1}$ ∴ $A = \dfrac{1}{2}$

c) $f(x) = xe^{-x}$, $F(x) = Ae^{-x}(x+1)$

$f(x) = F'(x) \longrightarrow xe^{-x} = A\left[e^{-x} + \dfrac{(x+1)e^{-x}}{-1}\right] = A\left[e^{-x} - xe^{-x} - e^{-x}\right]$

logo $xe^{-x} = -Axe^{-x}$ ∴ $A = -1$

4. a) $\int (2t-1)^5\,dt = A(2t-1)^6 + C$

$\dfrac{(2t-1)^{5+1}}{(5+1)*2} + C = A(2t-1)^6 + C \longrightarrow \dfrac{(2t-1)^6}{12} + C = A(2t-1)^6 + C$

∴ $A = 1/12$

b) $\int (3x+1)^{1/2}\,dx = A(3x+1)^{3/2} + C$

$\dfrac{(3x+1)^{1/2+1}}{(1/2+1)*3} + C = A(3x+1)^{3/2} + C \longrightarrow \dfrac{(3x+1)^{3/2}}{3/2*3} + C = A(3x+1)^{3/2} + C$

$A = 2/9$

c) $\int \dfrac{dt}{t^2-t} = A\ln\left|\dfrac{t}{t-1}\right| + C$

$\int \dfrac{1}{t(t-1)}\,dt = \int\left(\dfrac{-1}{t} + \dfrac{1}{t-1}\right)dt = -\ln|t| + \ln|t-1| + C$

* $\dfrac{A}{t} + \dfrac{B}{t-1} = \dfrac{At - A + Bt}{t^2-t}$ ∴ $(A+B)t - A = 1 \rightarrow A = -B$ e $-A = 1$ ∴ $A = -1$, $B = 1$

P! $\int \dfrac{dt}{\ldots} = \ln\left|\dfrac{t}{t-1}\right| \rightarrow A = -1$

c) $\int te^{-t^2} dt = Ae^{-t^2} + C$ $\qquad u = t^2 \;\; du = 2t\,dt$

$$\int \frac{e^{-u}}{2} du = Ae^{-t^2} + C \longrightarrow \frac{e^{-u}}{-2} + C = Ae^{-t^2} + C \;\therefore\; \frac{e^{-t^2}}{-2} = Ae^{-t^2} \;\therefore\; A = -1/2$$

5. a) $\int \frac{dx}{x^2+x-6} = A \ln|x-2| + B \ln|x+3| + C$

$$\Rightarrow \frac{1}{x^2+x-6} = \frac{1}{(x+3)(x-2)} = \frac{a}{(x+3)} + \frac{b}{(x-2)} = \frac{ax-2a+bx+3b}{x^2+x-6}$$

Logo $(a+b)x + (3b-2a) = 1 \longrightarrow a = -b$

$3b - 2a = 1 \therefore 3b + 2b = 1 \longrightarrow \begin{cases} b = 1/5 \\ a = -1/5 \end{cases}$

$$\int \frac{dx}{x^2+x-6} = \int \frac{-1}{5(x+3)} + \frac{1}{5(x-2)} dx = -\frac{1}{5} \ln|x+3| + \frac{1}{5} \ln|x-2| + C$$

Comparando: $\frac{1}{5}\ln|x-2| - \frac{1}{5}\ln|x+3| + C = A \ln|x-2| + B \ln|x+3| + C$

$A = 1/5$ e $B = -1/5$

b) $\int te^{0,1t} dt = Ate^{0,1t} + Be^{0,1t} + C$

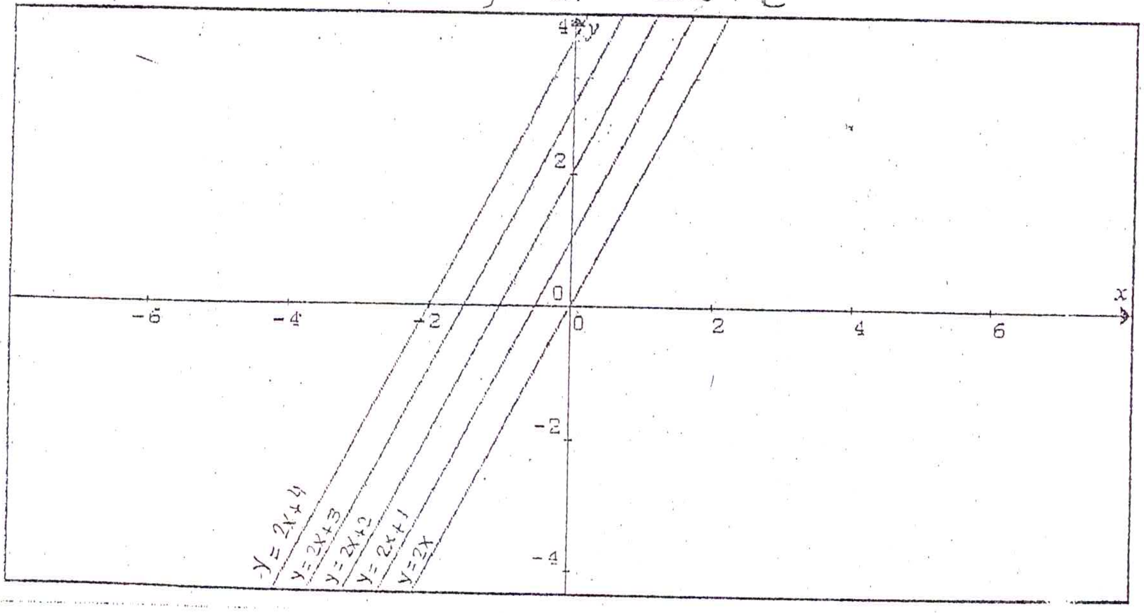
$$\left.\begin{matrix} u = t \rightarrow du = dt \\ v = \frac{e^{0,1t}}{0,1} \leftarrow dv = e^{0,1t} dt \end{matrix}\right\} \int te^{0,1t} dt = 10te^{0,1t} - \int 10e^{0,1t} dt$$

Logo: $\int te^{0,1t} dt = 10te^{0,1t} - \frac{10e^{0,1t}}{0,1} = 10te^{0,1t} - 100e^{0,1t} + C$

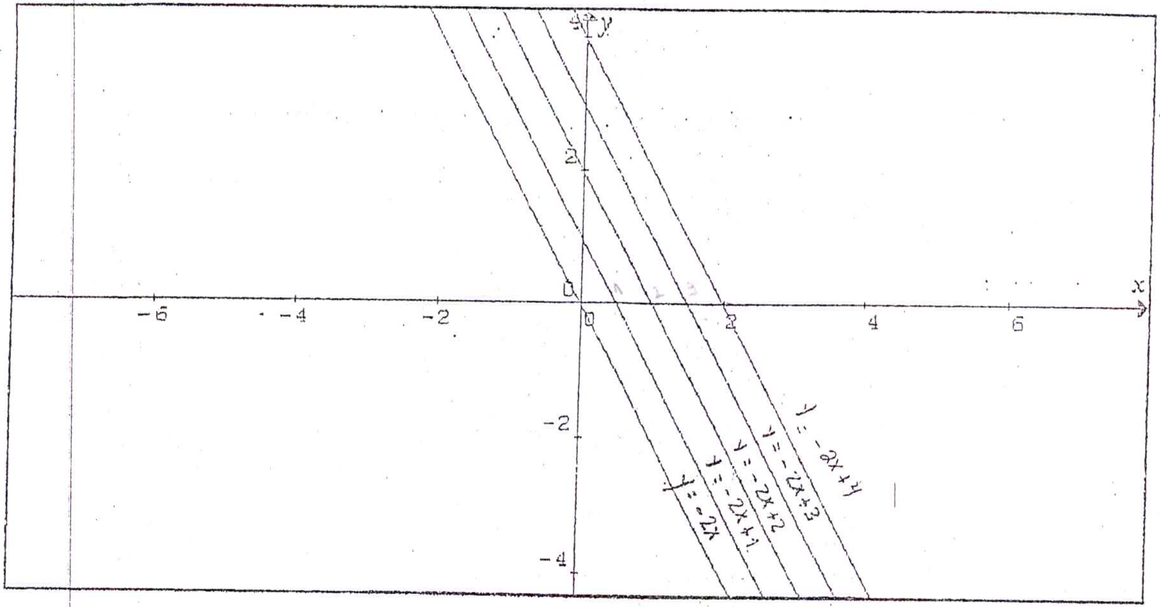
Comparando: $10te^{0,1t} - 100e^{0,1t} = Ate^{0,1t} + Be^{0,1t}$

$A = 10$ e $B = -100$

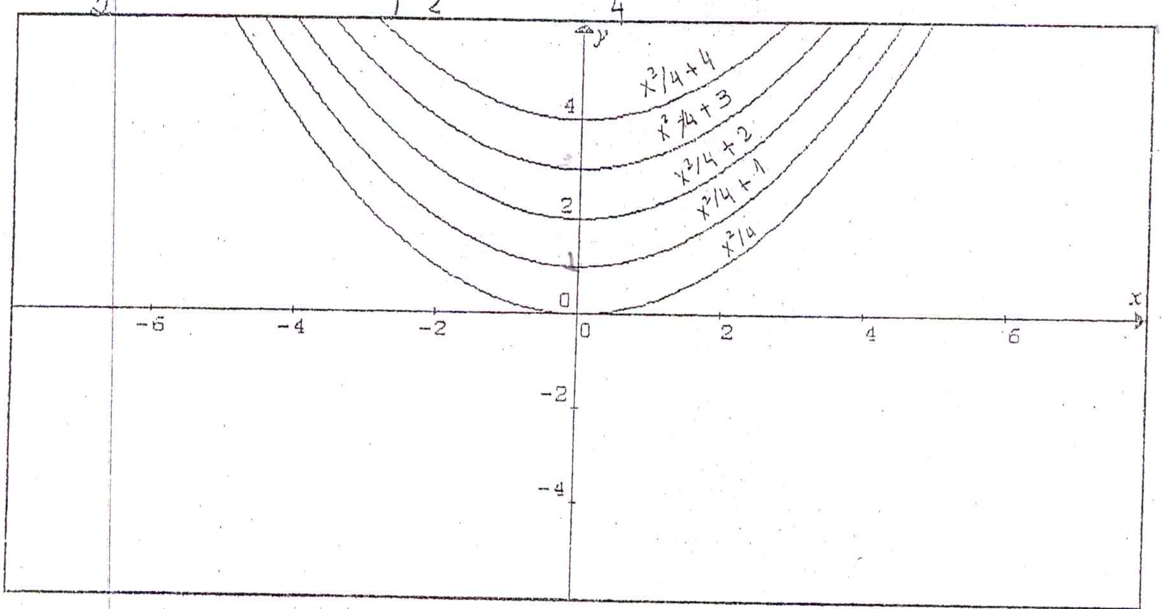
6. a) $f(x) = 2 \longrightarrow F(x) = \int 2dx = 2x + C$



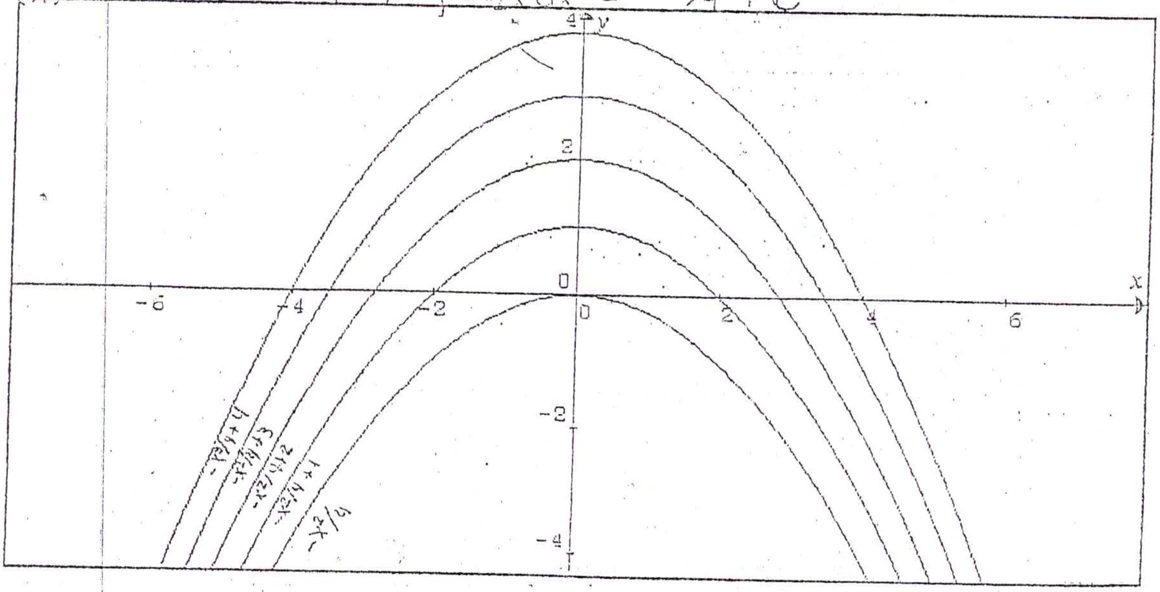
b) $f(x) = -2 \rightarrow F(x) = \int -2\,dx = -2x + C$



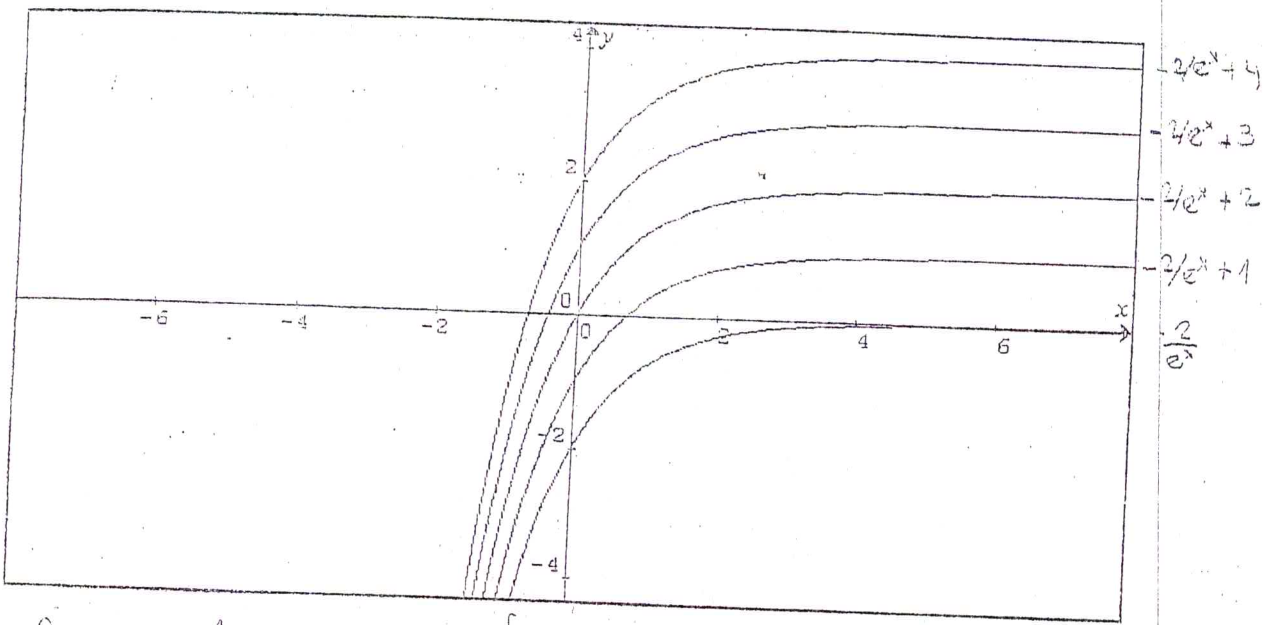
c) $f(x) = \frac{1}{2}x \rightarrow F(x) = \int \frac{1}{2}x\,dx = \frac{x^2}{4} + C$



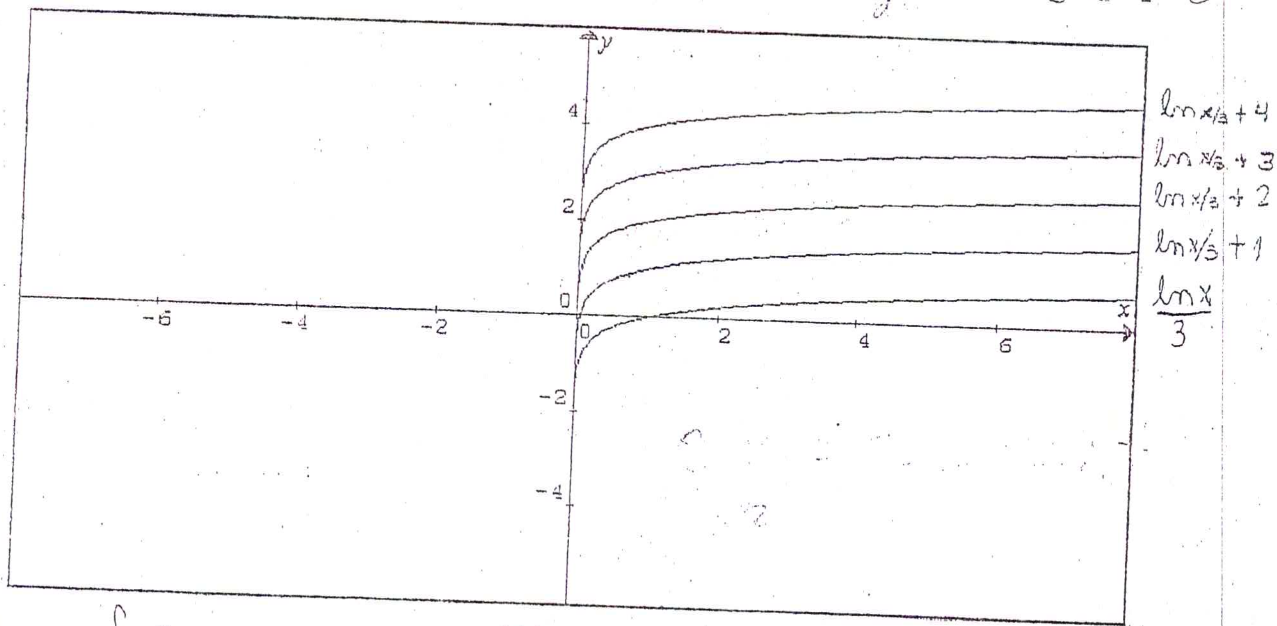
d) $f(x) = -\frac{1}{2}x \rightarrow F(x) = \int -\frac{1}{2}x\,dx = -x^2/4 + C$



e) $f(x) = 2e^{-x} \longrightarrow F(x) = \int 2e^{-x}dx = -2e^{-x} + C$



f) $f(x) = 1/3x \longrightarrow F(x) = \int \frac{1}{3x}dx = \frac{\ln|x|}{3} + C$



7. a) $\int (x^2 - 3x + 1)\,dx = \frac{x^{2+1}}{2+1} - \frac{3x^{1+1}}{1+1} + x + C = \frac{x^3}{3} - \frac{3x^2}{2} + x + C$

Verificando a resposta: $\frac{d}{dx}\left[\frac{x^3}{3} - \frac{3x^2}{2} + x + C\right] = \frac{3x^2}{3} - \frac{6x}{2} + 1 = x^2 - 3x + 1$

b) $\int \frac{2}{x^3}dx = 2\int x^{-3}dx = \frac{2x^{-3+1}}{-3+1} + C = \frac{\cancel{2}x^{-2}}{\cancel{-2}} + C = -x^{-2} + C$

Verificando a resposta: $\frac{d}{dx}\left[-\frac{1}{x^2} + C\right] = -(-2)x^{-2-1} = 2/x^3$

c) $\int e^{3t}dt = \frac{e^{3t}}{3} + C$

Verificando a resposta: $\frac{d}{dt}\left[\frac{e^{3t}}{3} + C\right] = \frac{e^{3t}}{3} * 3 = e^{3t}$

d) $\int \frac{1}{2}(e^x + e^{-x})\,dx = \frac{1}{2}\left(e^x + \frac{e^{-x}}{-1}\right) + C = \frac{1}{2}(e^x - e^{-x}) + C$

Verificando a resposta: $\frac{d}{dx}\left[\frac{e^x}{2} - \frac{e^{-x}}{2} + C\right] = \frac{1}{2}\left[e^x + e^{-x}\right]$

e) $\int y^2(5y^3 + 1/y)\,dy = \int (5y^5 + y)\,dy = \frac{5y^{5+1}}{5+1} + \frac{y^{1+1}}{1+1} + C = \frac{5y^6}{6} + \frac{y^2}{2} + C$

Verificando a resposta: $\frac{d}{dy}\left[\frac{5y^6}{6} + \frac{y^2}{2} + C\right] = \frac{30}{6}y^5 + \frac{2y}{2} = 5y^5 + y = y^2(5y^3 + 1/y)$

f) $\int \sqrt{t}\,dt = \int t^{1/2}\,dt = \frac{t^{1/2+1}}{1/2+1} + C = \frac{2t^{3/2}}{3} + C$

Verificando a resposta:

$d/dt\left[\frac{2t^{3/2}}{3} + C\right] = 2/3 * 3/2\, t^{1/2} = t^{1/2} = \sqrt{t}$

g) $\int e^{-0,5t}\,dt = \frac{e^{-0,5t}}{-0,5} + C = -2e^{-0,5t} + C$

Verificando a resposta: $\frac{d}{dt}\left[-2e^{-0,5t}\right] = -2(-0,5)e^{-0,5t} = e^{-0,5t}$

h) $\int (1-t)^2\,dt = \int (1 - 2t + t^2)\,dt = t - \frac{2t^2}{2} + \frac{t^3}{3} + C = t - t^2 + \frac{t^3}{3} + C$

Verificando a resposta: $\frac{d}{dt}\left[t - t^2 + \frac{t^3}{3} + C\right] = 1 - 2t + \frac{3t^2}{3} = (1-t)^2$

i) $\int (e^{2x} + e^{3/2})\,dx$

$= \frac{e^{2x}}{2} + \boxed{x}e^{3/2} + C$

Verificando a resposta: $\frac{d}{dx}\left[\frac{e^{2x}}{2} + xe^{3/2} + C\right] = \frac{2e^{2x}}{2} + e^{3/2} = e^{2x} + e^{3/2}$

8. a) $\int \frac{2x}{x^2+1}\,dx \qquad u = x^2 + 1, \quad du = \underline{2x\,dx}$

$= \int \frac{du}{u} = \ln|u| + C = \ln|x^2+1| + C$

b) $\int \frac{\ln x}{x}\,dx \qquad u = \ln x, \quad du = \frac{1}{x}\,dx$

$= \int u\,du = \frac{u^2}{2} + C = \frac{\ln^2|x|}{2} + C$

c) $\int \sqrt{3x+4}\,dx \qquad u = 3x + 4, \quad du = 3dx$

$= \int \frac{u^{1/2}}{3}\,du = \frac{u^{1/2+1}}{3(1/2+1)} + C = \frac{2(3x+4)^{3/2}}{9} + C$

d) $\int \frac{x\ln(x^2+1)}{x^2+1}\,dx \qquad u = \ln(x^2+1), \quad du = \frac{1}{x^2+1} * 2x\,dx$

$= \int \frac{u}{2}\,du = \frac{u^2}{4} + C = \frac{\ln^2(x^2+1)}{4} + C$

e) $\int \frac{e^{2x}}{e^x+1}\,dx \qquad u = 1 + e^x, \quad du = e^x\,dx$

$= \int \frac{(u-1)}{u}\,du = \int\left(1 - \frac{1}{u}\right)du = u - \ln|u| + C$

$= 1 + e^x - \ln|1+e^x| + C$

f) $\int e^{-t}\,dt \quad u=-t \;,\; du=-dt$

$= \int -e^{u}\,du = -e^{u}+C = -e^{-t}+C$

g) $\int \frac{e^t}{(2+e^t)^2}\,dt \qquad u = 2+e^t \;,\; du = e^t\,dt$

$= \int \frac{1}{u^2}\,du = \frac{u^{-2+1}}{-2+1} = -\frac{1}{u}+C = -\frac{1}{2+e^t}+C$

$x^{-1/2} \quad \frac{1}{2}x^{-1/2} = \frac{1}{2}\frac{1}{\sqrt{x}}$

h) $\int \frac{1-\sqrt{x}}{1+\sqrt{x}}\,dx \qquad y = 1+\sqrt{x} \qquad dy = \frac{1}{2\sqrt{x}}\,dx$

$1 - \cancel{1} - \sqrt{x} + \cancel{1} \quad \boxed{1-\sqrt{x}}$

$= \int \frac{1-y+1}{y}\,2\sqrt{x}\,dy = 2\int \frac{(2-y)}{y} * (y-1)\,dy$

$= 2\int \frac{2y-2-y^2+y}{y}\,dy = 2\int 2-\frac{2}{y}-y+1\,dy = 2\int 3-\frac{2}{y}-y\,dy$

$= 2\left(3y-2\ln|y|-\frac{y^2}{2}\right)+C = 6(1+\sqrt{x}) - 4\ln|1+\sqrt{x}| - (1+\sqrt{x})^2 + C$

i) $\int \frac{1}{e^x+e^{-x}+2}\,dx = \int \frac{1}{e^x+\frac{1}{e^x}+2}\,dx = \int \frac{e^x}{e^{2x}+1+2e^x}\,dx$

$= \int \frac{e^x}{(e^x+1)^2}\,dx \qquad u = e^x+1 \;,\; du = e^x\,dx$

$= \int \frac{1}{u^2}\,du = \int u^{-2}\,du = \frac{u^{-2+1}}{-2+1} = -\frac{1}{\boxed{u}}+C = -\frac{1}{e^x+1}+C$

j) $\int (x^2-1)\,e^{x^3-3x+1}\,dx \qquad u = x^3-3x+1 \;,\; du = (3x^2-3)\,dx$

$\int \frac{e^u}{3}\,du = \frac{e^u}{3}+C = \frac{e^{x^3-3x+1}}{3}+C$

Verificação das respostas obtidas no exercício 8

a) $\frac{d}{dx}\left[\ln|x^2+1|+C\right] = \frac{2x}{x^2+1}$

b) $\frac{d}{dx}\left[\frac{\ln^2|x|}{2}+C\right] = \frac{2\ln x}{2x} = \frac{\ln x}{x}$

c) $\frac{d}{dx}\left[\frac{2(3x+4)^{3/2}}{9}+C\right] = \frac{2}{9}*\frac{3}{2}(3x+4)^{1/2} * 3 = \sqrt{3x+4}$

d) $\frac{d}{dx}\left[\frac{\ln^2(x^2+1)}{4}+C\right] = \frac{2\ln(x^2+1)}{4} * 2x * \frac{1}{x^2+1} = \frac{x\ln(x^2+1)}{x^2+1}$

e) $\frac{d}{dx}\left[1+e^x-\ln|1+e^x|+C\right] = e^x - \frac{e^x}{1+e^x} = \frac{e^x+e^{2x}-e^x}{1+e^x} = \frac{e^{2x}}{e^x+1}$

f) $\frac{d}{dt}\left[-e^{-t}+C\right] = -e^{-t} * (-1) = e^{-t}$

g) $\frac{d}{dt}\left[\frac{-1}{2+e^{t}}\right] = \frac{1}{(2+e^{t})^2}\left[0-(-1)e^{t}\right] = \frac{e^{t}}{(2+e^{t})^2}$

h) $\frac{d}{dx}\left[6(1+\sqrt{x}) - 4\ln|1+\sqrt{x}| - (1+\sqrt{x})^2 + C\right]$

$= \frac{d}{dx}\left[6+6\sqrt{x} - 4\ln|1+\sqrt{x}| - 1 - 2\sqrt{x} - \cancel{x} + C\right]$

$= \frac{6}{2\sqrt{x}} - \left(\frac{4}{1+\sqrt{x}} * \frac{1}{2\sqrt{x}}\right) - \frac{2}{2\sqrt{x}} - 1 = \frac{3}{\sqrt{x}} - \frac{2}{\sqrt{x}(1+\sqrt{x})} - \frac{1}{\sqrt{x}} - 1$

$= \frac{3(1+\sqrt{x}) - 2 - (1+\sqrt{x}) - \sqrt{x}(1+\sqrt{x})}{\sqrt{x}(1+\sqrt{x})} = \frac{3+3\sqrt{x}-3-\sqrt{x}-\sqrt{x}-x}{\sqrt{x}(1+\sqrt{x})}$

$= \frac{\sqrt{x}-x}{\sqrt{x}(1+\sqrt{x})} = \frac{\sqrt{x}(1-\sqrt{x})}{\sqrt{x}(1+\sqrt{x})} = \frac{(1-\sqrt{x})}{(1+\sqrt{x})}$

i) $\frac{d}{dx}\left[\frac{-1}{e^{x}+1}+C\right] = \frac{1}{(e^{x}+1)^2}\left[-(-1)e^{x}\right] = \frac{e^{x}}{(e^{x}+1)^2} = \frac{e^{x}}{e^{2x}+2e^{x}+1}$

$= \frac{1}{e^{x}+2+e^{-x}} = \frac{1}{e^{x}+e^{-x}+2}$

j) $\frac{d}{dx}\left[\frac{e^{x^3-3x+1}}{3}+C\right] = \frac{(3x^2-3)\,e^{x^3-3x+1}}{3} = (x^2-1)\,e^{x^3-3x+1}$

9. a) $\int \frac{2x}{\sqrt{x^2+4}}\,dx$ Resolver por substituição: $u = x^2+4$
$du = 2x\,dx$

$= \int \frac{1}{\sqrt{u}}\,du = \int u^{-1/2}du = \frac{u^{-1/2+1}}{-1/2+1} = 2u^{1/2} = 2\sqrt{x^2+4} + C$

Derivando para verificação: $\frac{d}{dx}\left[2(x^2+4)^{1/2}+C\right]$

Usar regra da cadeia $v = x^2+4$

$\frac{df}{dx} = \frac{df}{dv}\cdot\frac{dv}{dx} = \left(2 * \frac{1}{2} v^{-1/2}\right) * 2x = \frac{2x}{\sqrt{x^2+4}}$

b) $\int (x^2-3)^5\,x\,dx$ Resolver por substituição: $u = x^2-3$
$du = 2x\,dx$

$= \int \frac{u^5}{2}\,du = \frac{1}{2}\left(\frac{u^{5+1}}{5+1}\right) = \frac{u^6}{12} = \frac{(x^2-3)^6}{12} + C$

Derivando para verificação: $\frac{d}{dx}\left[\frac{(x^2-3)^6}{12}+C\right] = \frac{6(x^2-3)^5\,x\,2x}{12} = (x^2-3)^5 x$

c) $\int \frac{2x+1}{(x^2+x+1)^3}\,dx$ Resolver por substituição $u = x^2+x+1$

$du = (2x+1)\,dx$

$$= \int \frac{1}{u^3}\,du = \int u^{-3}\,du = \frac{u^{-3+1}}{-3+1} = \frac{(x^2+x+1)^{-2}}{-2} + C$$

Derivando para verificação: $\frac{d}{dx}\left[\frac{(x^2+x+1)^{-2}}{-2} + C\right] = \frac{-2(x^2+x+1)^{-3} * (2x+1)}{-2}$

$$= \frac{2x+1}{(x^2+x+1)^3}$$

d) $\int t^3 e^{-0,1t^4}\,dt$ Resolver por substituição: $u = -0,1t^4$

$du = -0,4t^3\,dt$

$$= \int \frac{e^u}{-0,4}\,du = \frac{e^u}{-0,4} + C = \frac{e^{-0,1t^4}}{-0,4} + C$$

Derivando para verificação: $\frac{d}{dt}\left[\frac{-e^{-0,1t^4}}{0,4} + C\right] = -\frac{(-0,4t^3)\,e^{-0,1t^4}}{0,4}$

$$= t^3 e^{-0,1t^4}$$

e) $\int \frac{3}{2x+1}\,dx$ Resolver por substituição: $u = 2x+1$

$du = 2dx$

$$= \int \frac{3}{2}\frac{du}{u} = \frac{3}{2}\ln u + C = \frac{3}{2}\ln|2x+1| + C$$

Derivando para verificação: $\frac{d}{dx}\left[\frac{3}{2}\ln|2x+1| + C\right] = \frac{3 * 2}{2(2x+1)} = \frac{3}{2x+1}$

f) $\int \frac{\ln(t^2)}{t}\,dt$

$u = \ln(t^2)$

$du = \frac{2t}{t^2}\,dt \rightarrow du = \frac{2}{t}\,dt$

$$= \int \frac{u}{2}\,du = \frac{u^{1+1}}{2(1+1)} + C = \frac{u^2}{4} + C = \frac{\ln^2(t^2)}{4} + C$$

Derivando para verificação: $\frac{d}{dt}\left[\frac{\ln^2(t^2)}{4} + C\right] = \frac{2\ln(t^2)}{4} * \frac{1}{t^2} * 2t$

$\frac{u^2}{2} \cdot \frac{1}{2} \cdot \frac{u^2}{4}$

$= \boxed{\frac{\ln^4(t^2)}{t}}$

g) $\int (2x+5)^7\,dx$ Resolver por substituição: $u = 2x+5$

$du = 2dx$

$$= \int \frac{u^7}{2}\,du = \frac{u^{7+1}}{2(7+1)} = \frac{u^8}{16} + C = \frac{(2x+5)^8}{16} + C$$

Derivando para verificação: $\frac{d}{dx}\left[\frac{(2x+5)^8}{16} + C\right] = \frac{8(2x+5)^7 * 2}{16} = (2x+5)^7$

h) $\int t^3 \sqrt{t^4+1}\, dt$ Resolver por substituição: $u = t^4+1$
$du = 4t^3 dt$

$= \int \frac{\sqrt{u}}{4} du = \frac{u^{1/2+1}}{4(1/2+1)} = \frac{u^{3/2}}{6} = \frac{(t^4+1)^{3/2}}{6} + C$

Derivando para verificação: $\frac{d}{dt}\left[\frac{(t^4+1)^{3/2}}{6} + C\right] = \frac{3}{2} \frac{(t^4+1)^{1/2}}{6} * 4t^3$
$= t^3 \sqrt{t^4+1}$

i) $\int \frac{x+1}{x^2+2x+3} dx$ Resolver por substituição: $u = x^2+2x+3$
$du = 2(x+1)dx$

$= \int \frac{1}{2u} du = \frac{\ln|u|}{2} + C = \frac{\ln|x^2+2x+3|}{2} + C$

Derivando para verificação: $\frac{d}{dt}\left[\frac{\ln|x^2+2x+3|}{2}\right] = \frac{2x+2}{2(x^2+2x+3)} = \frac{x+1}{(x^2+2x+3)}$

j) $\int e^{-4t} dt$ Resolver por substituição: $u = -4t$
$du = -4dt$

$= \int \frac{e^u}{-4} du = -\frac{e^u}{4} + C = \frac{-e^{-4t}}{4} + C$

Derivando para verificação: $\frac{d}{dt}\left[\frac{-e^{-4t}}{4} + C\right] = \frac{-(-4)e^{-4t}}{4} = e^{-4t}$

k) $\int \frac{(\ln t)^2}{t} dt$ Resolver por substituição: $u = \ln t$
$du = \frac{1}{t} dt$

$= \int u^2 du = \frac{u^{2+1}}{2+1} + C = \frac{\ln^3 t}{3} + C$

Derivando para verificação: $\frac{d}{dt}\left[\frac{\ln^3 t}{3} + C\right] = \frac{3\ln^2 t * (1/t)}{3} = \frac{\ln^2 t}{t}$

l) $\int \frac{e^t}{1+e^t} dt$ Resolver por substituição: $u = 1+e^t$
$du = e^t dt$

$= \int \frac{1}{u} du = \ln|u| + C = \ln|1+e^t| + C$

Derivando para verificação: $\frac{d}{dt}\left[\ln|1+e^t| + C\right] = \frac{e^t}{1+e^t}$

m) $\int \frac{dt}{t\ln t}$ Resolver por substituição: $u = \ln t$
$du = \frac{1}{t} dt$

$= \int \frac{1}{u} du = \ln|u| + C = \ln(\ln t) + C$

Derivando para verificação: $\frac{d}{dt}\left[\ln(\ln t) + C\right] = \frac{1}{\ln t} * \frac{1}{t} = \frac{1}{t\ln t}$

n) $\int \frac{\sqrt{x}}{1+\sqrt{x}}\,dx$ Resolver por substituição $u = 1+\sqrt{x}$ $du = \frac{1}{2\sqrt{x}}\,dx$

$$= \int \frac{(u-1)}{u}\,2\sqrt{x}\,du = 2\int \frac{(u-1)}{u}(u-1)\,du = 2\int \frac{(u-1)^2}{u}\,du$$

$$= 2\int \frac{u^2-2u+1}{u}\,du = 2\int \left(u-2+\frac{1}{u}\right)du = 2\left(\frac{u^2}{2} - 2u + \ln u\right) + C$$

$$= 2\left(\frac{(1+\sqrt{x})^2}{2} - 2(1+\sqrt{x}) + \ln|1+\sqrt{x}|\right) = (1+\sqrt{x})^2 - 4(1+\sqrt{x}) + 2\ln(1+\sqrt{x}) + C$$

Derivando para verificar:

$$= \frac{2(1+\sqrt{x})}{2\sqrt{x}} - \frac{4}{2\sqrt{x}} + \frac{2}{1+\sqrt{x}} * \frac{1}{2\sqrt{x}} = \frac{(1+\sqrt{x})}{\sqrt{x}} - \frac{2}{\sqrt{x}} + \frac{1}{\sqrt{x}(1+\sqrt{x})}$$

$$= \frac{(1+\sqrt{x})^2 - 2(1+\sqrt{x}) + 1}{\sqrt{x}(\sqrt{x}+1)} = \frac{1+2\sqrt{x}+x-2-2\sqrt{x}+1}{\sqrt{x}(\sqrt{x}+1)} = \frac{x}{\sqrt{x}(\sqrt{x}+1)}$$

$$= \frac{x}{x+\sqrt{x}} = \frac{x}{x(1+x^{-1/2})} = \frac{1}{1+\frac{1}{x^{1/2}}} = \frac{1}{\frac{x^{1/2}+1}{x^{1/2}}} = \frac{\sqrt{x}}{1+\sqrt{x}}$$

o) $\int e^{2x}\sqrt{1+e^{2x}}\,dx$ Resolver por substituição $u = 1+e^{2x}$ $du = 2e^{2x}\,dx$

$$= \int \frac{\sqrt{u}}{2}\,du = \frac{u^{1/2+1}}{2(1/2+1)} = \frac{u^{3/2}}{2(3/2)} + C = \frac{(1+e^{2x})^{3/2}}{3} + C$$

Derivando para verificar:

$$= \frac{3/2\,(1+e^{2x})^{1/2} * 2e^{2x}}{3} = e^{2x}(1+e^{2x})^{1/2}$$

p) $\int \frac{1}{1+x^{1/3}}\,dx = \int \frac{x^{1/3}\cdot x^{1/3}}{x^{2/3}(1+x^{1/3})}\,dx$ Resolver por substituição: $u = x^{1/3}+1$ $du = \frac{dx}{3x^{2/3}}$

$$= \int 3\frac{(u-1)^2}{u}\,du = 3\int \frac{u^2-2u+1}{u}\,du = 3\int \left[u-2+\frac{1}{u}\right]du$$

$$= 3\left(\frac{u^2}{2} - 2u + \ln u\right) + C = 3\left(\frac{(x^{1/3}+1)^2}{2} - 2(x^{1/3}+1) + \ln|x^{1/3}+1|\right) + C$$

Derivando para verificar:

$$= 3\left[\frac{2(x^{1/3}+1)}{2} * \left(\frac{1}{3x^{2/3}}\right) - \frac{2}{3x^{2/3}} + \frac{1}{x^{1/3}+1} * \left(\frac{1}{3x^{2/3}}\right)\right]$$

$$= \frac{x^{1/3}+1}{x^{2/3}} - \frac{2}{x^{2/3}} + \frac{1}{x^{2/3}(x^{1/3}+1)} = \frac{(x^{1/3}+1)^2 - 2(x^{1/3}+1) + 1}{x^{2/3}(x^{1/3}+1)} = \frac{x^{2/3} + 2x^{1/3} + 1 - 2x^{1/3} - 2 + 1}{x^{2/3}(x^{1/3}+1)}$$

$$= \frac{x^{2/3}}{x^{2/3}(x^{1/3}+1)} = \frac{1}{1+x^{1/3}}$$

10. a) $\frac{dy}{dt} = \sqrt{4t+1}$, $y(0)=3$

$Y = \int \sqrt{4t+1}\, dt$ POR substituição: $u = 4t+1$
$du = 4dt$

$Y = \int \frac{u^{1/2}}{4} du = \frac{u^{1/2+1}}{4(1/2+1)} = \frac{u^{3/2}}{4(3/2)} + C = \frac{(4t+1)^{3/2}}{6} + C$

$\rightarrow Y(0) = 1/6 + C = 3 \therefore C = 3 - \frac{1}{6} = \frac{18-1}{6} = \frac{17}{6}$

Solução: $Y = \frac{\sqrt{(4t+1)^3} + 17}{6}$

b) $\frac{dy}{dt} = \sqrt{4t+1}$, $y(2) = 3$

$Y = \int \sqrt{4t+1}\, dt$ POR substituição: $u = 4t+1$, $du = 4dt$

$Y = \int \frac{u^{1/2}}{4} du = \frac{u^{1/2+1}}{4(1/2+1)} = \frac{u^{3/2}}{4(3/2)} + C = \frac{(4t+1)^{3/2}}{6} + C$

$\rightarrow Y(2) = \frac{9^{3/2}}{6} + C = 3 \rightarrow C = 3 - \frac{27}{6} = \frac{18-27}{6} = -\frac{9}{6} = -\frac{3}{2}$

$Y = \frac{(4t+1)^{3/2} - 9}{6}$

c) $\frac{dr}{dx} = \frac{e^x - e^{-x}}{e^x + e^{-x}}$, $r(0) = 4$

$r = \int \frac{e^x - e^{-x}}{e^x + e^{-x}} dx$ POR substituição: $u = e^x + e^{-x}$
$du = (e^x - e^{-x})dx$

$r = \int \frac{du}{u} = \ln u + C = \ln(e^x + e^{-x}) + C$

$\rightarrow r(0) = \ln(e^0 + e^0) + C = 4 \therefore C = 4 - \ln 2$

Solução: $r = \ln(e^x + e^{-x}) + 4 - \ln 2$

d) $\frac{dy}{dx} = \frac{2x}{(x-2)^2}$, $y(1) = 4$

$Y = \int \frac{2x}{(x-2)^2} dx$

11. a) $\int x e^{2x} dx$ $\qquad u = x \longrightarrow du = dx$

$v = \frac{e^{2x}}{2} \longleftarrow dv = e^{2x} dx$

$$\int x e^{2x} dx = \frac{x e^{2x}}{2} - \int \frac{e^{2x}}{2} dx = \frac{x e^{2x}}{2} - \frac{e^{2x}}{4} + C$$

b) $\int \ln x \, dx$ $\qquad u = \ln x \longrightarrow du = \frac{1}{x} dx$

$v = x \longleftarrow dv = 1 dx$

$$\int \ln x \, dx = x \ln x - \int x \cdot \frac{1}{x} dx = x \ln x - x + C$$

12. a) $\int t e^{-t/2} dt$ $\qquad u = t \longrightarrow du = dt$

$v = \frac{e^{-t/2}}{-1/2} \longleftarrow dv = e^{-t/2} dt$

$$\int t e^{-t/2} dt = -2t e^{-t/2} - \int \frac{e^{-t/2}}{-1/2} dt = -2t e^{-t/2} + 2\int e^{-t/2} dt = -2te^{-t/2} - 4e^{-t/2} + C$$

b) $\int t^2 e^{-t} dt$ $\qquad u = t^2 \rightarrow du = 2t\,dt$

$v = -e^{-t} \longleftarrow dv = e^{-t} dt$

$$\int t^2 e^{-t} dt = -t^2 e^{-t} + 2\int e^{-t}\, t\, dt \Rightarrow$$ $\qquad u = t \rightarrow du = dt$

$v = -e^{-t} \longleftarrow dv = e^{-t} dt$

$$\boxed{\int t^2 e^{-t} dt =} -t^2 e^{-t} + 2\left[-te^{-t} + \int 1 e^{-t} dt\right] = -t^2 e^{-t} - 2te^{-t} - 2e^{-t} + C$$

u.v

$e^{-t} = \frac{e^{-t}}{-1} = -e^{-t}$ //

c) $\int x^2 \ln x \, dx$ $\qquad u = \ln x \longrightarrow du = \frac{1}{x} dx$

$v = x^3/3 \longleftarrow dv = x^2 dx$

$$\int x^2 \ln x \, dx = \frac{x^3 \ln x}{3} - \int \frac{x^3}{3} \cdot \frac{1}{x} dx = \frac{x^3 \ln x}{3} - \frac{1}{3}\left(\frac{x^3}{3}\right) = \frac{x^3 \ln x}{3} - \frac{x^3}{9} + C$$

d) $\int x \sqrt{3x+2}\, dx$ $\qquad u = x \longrightarrow du = dx$

$v = \frac{2}{3}\frac{(3x+2)^{3/2}}{3} \leftarrow dv = (3x+2)^{1/2} dx$

$$\int x\sqrt{3x+2}\, dx = \frac{2x(3x+2)^{3/2}}{9} - \frac{2}{9}\int (3x+2)^{3/2} dx = \frac{2x(3x+2)^{3/2}}{9} - \frac{2}{9}\left(\frac{2}{15}(3x+2)^{5/2}\right) + C$$

$$= \frac{2x(3x+2)^{3/2}}{9} - \frac{4}{135}(3x+2)^{5/2} + C$$

e) $\int x^3 \sqrt{x^2+1}\, dx = \int x^2 x \sqrt{x^2+1}\, dx$ $\qquad u = x^2 \rightarrow du = 2x\,dx$

$v = \frac{(x^2+1)^{3/2}}{3} \leftarrow dv = (x(x^2+1))^{1/2} dx$

$$\int x^3 \sqrt{x^2+1}\, dx = \frac{x^2 (x^2+1)^{3/2}}{3} - \int \frac{2x(x^2+1)^{3/2}}{3} dx$$ Por substituição: $u = x^2+1$, $du = 2x\,dx$

$$= \frac{x^2(x^2+1)^{3/2}}{3} - \frac{1}{3}\int u^{3/2} du = \frac{x^2(x^2+1)^{3/2}}{3} - \frac{2}{15}(x^2+1)^{5/2} + C$$

f) $\int t \ln t \, dt$ $\qquad u = \ln t \longrightarrow du = \frac{1}{t}\,dt$
$\qquad v = t^2/2 \longleftarrow dv = t\,dt$

$\int t \ln t \, dt = \frac{t^2 \ln t}{2} - \int \frac{t}{2}\,dt = \frac{t^2 \ln t}{2} - \frac{t^2}{4} + C$

$z = t^2 \to dz = 2t\,dt$ $\quad \frac{1}{2}\int z \cdot t\,dt\,dz \quad dz\left(\frac{1}{2}\right)$ $\quad \frac{1}{2}\left[t^2\,dz\right]$

g) $\int x (\ln x)^2 dx$ $\qquad u = (\ln x)^2 \rightarrow du = \frac{2\ln x}{x}\,dx$
$\qquad v = \frac{x^2}{2} \longleftarrow dv = x\,dx$

$\int x(\ln x)^2 dx = \frac{x^2}{2}(\ln x)^2 - \int x \ln x \, dx \Rightarrow$ $u = \ln x \quad du = 1/x\,dx$; $v = x^2/2 \quad dv = x\,dx$

$\int x(\ln x)^2 dx = \frac{x^2}{2}(\ln x)^2 - \frac{x^2}{2}\ln x + \int \frac{x}{2}\,dx = \frac{x^2}{2}(\ln x)^2 - \frac{x^2}{2}\ln x + \frac{x^2}{4} + C$

h) $\int t^3 e^t \, dt$ $\qquad u = t^3 \rightarrow du = 3t^2 dt$
$\qquad v = e^t \leftarrow dv = e^t dt$

$\int t^3 e^t dt = t^3 e^t - \int 3t^2 e^t dt \Rightarrow u = t^2 \rightarrow du = 2t\,dt$
$\qquad v = e^t \leftarrow dv = e^t dt$

$\int t^3 e^t dt = t^3 e^t - 3\left[t^2 e^t - \int 2te^t dt\right] = t^3 e^t - 3t^2 e^t + 6\int te^t dt$

** $\int te^t dt$ $\quad u = t \quad du = dt$; $v = e^t \quad dv = e^t dt$ $\quad \therefore = te^t - \int e^t dt = te^t - e^t$

Logo, $\int t^3 e^t dt = t^3 e^t - 3t^2 e^t + 6te^t - 6e^t$

i) $\int \frac{x^2}{\sqrt{x+1}}\,dx$ $\qquad u = x^2 \quad du = 2x\,dx$
$\qquad v = 2\sqrt{x+1} \quad dv = \boxed{\frac{1}{\sqrt{x+1}}}\,dx$

$\frac{(x+1)^{1/2+1}}{\frac{1}{2}} - 2(x+1)^{1/2}$

$\int \frac{x^2}{\sqrt{x+1}}\,dx = 2x^2\sqrt{x+1} - 4\int x\sqrt{x+1}\,dx \Rightarrow u = x \quad du = dx$; $v = \frac{2}{3}(x+1)^{3/2} \quad dv = \sqrt{x+1}\,dx$

$\int \frac{x^2}{\sqrt{x+1}}\,dx = 2x^2\sqrt{x+1} - 4\left(\frac{2x(x+1)^{3/2}}{3}\right) + \frac{4}{5}\int \frac{2}{3}(x+1)^{3/2}dx$

$= 2x^2\sqrt{x+1} - \frac{8}{3}x(x+1)^{3/2} + \frac{16}{15}(x+1)^{5/2} + C$

j) $\int x^3 e^{x^2} dx = \int x^2 \cdot x e^{x^2} dx$ $\qquad u = x^2 \quad du = 2x\,dx$
$\qquad v = \frac{e^{x^2}}{2} \quad dv = x e^{x^2} dx$

$\int x^3 e^{x^2} dx = \frac{x^2 e^{x^2}}{2} - \int x e^{x^2} dx = \boxed{\frac{x^2 e^{x^2}}{2} - \frac{e^{x^2}}{2} + C}$

3. Repetido
4. Repetido

15. a) $\frac{1}{(x-1)(x+5)} = \frac{A}{(x-1)} + \frac{B}{(x+5)} = \frac{Ax+5A+Bx-B}{(x-1)(x+5)} \therefore (A+B)x+(5A-B)=1$

$\to A=-B$ e $5A+A=1$

$A=1/6$

$= \frac{1}{6(x-1)} - \frac{1}{6(x+5)}$

b) $\frac{1}{x^2-9} = \frac{1}{(x+3)(x-3)} = \frac{A}{(x+3)} + \frac{B}{(x-3)} = \frac{Ax-3A+Bx+3B}{x^2-9} \therefore (A+B)x+(3B-3A)$

$A=-B$ e $3B-3A=1$

Logo $-3A-3A=1 \therefore A=-1/6$ e $B=1/6$

$\frac{1}{x^2-9} = \frac{-1}{6(x+3)} + \frac{1}{6(x-3)}$

16. a) $\int \frac{dx}{x^2-9x+20} \to \frac{1}{x^2-9x+20} = \frac{1}{(x-5)(x-4)} = \frac{A}{(x-5)} + \frac{B}{(x-4)}$

$\frac{Ax-4A+Bx-5B}{(x-5)(x-4)} \to (A+B)x - 4A-5B = 1$

$\to A=-B$

$\to -4A+5A=1 \therefore A=1$, $B=-1$

$\int \frac{dx}{x^2-9x+20} = \int \frac{1}{(x-5)} - \frac{1}{(x-4)}\,dx = \ln|x-5| - \ln|x-4| + C$

$= \ln\left|\frac{x-5}{x-4}\right| + C$

b) $\int \frac{dt}{t^2-t} = \int \frac{1}{t(t-1)}dt \Rightarrow \frac{1}{t(t-1)} = \frac{A}{t} + \frac{B}{t-1} = \frac{At-A+Bt}{t^2-t}$

Logo $(A+B)t - A = 1$

$\to A=-B$

$\to A=-1$

$A=-1$ e $B=1$

$\int \frac{dt}{t^2-t} = \int -\frac{1}{t} + \frac{1}{t-1}\,dt = \ln|t-1| - \ln|t| + C$

$= \ln\left|\frac{t-1}{t}\right| + C$

c) $\int \frac{dt}{t^2+t} = \int \frac{1}{t(t+1)}dt \Rightarrow \frac{1}{t(t+1)} = \frac{A}{t} + \frac{B}{t+1} = \frac{At+A+Bt}{t^2+t}$

Logo $(A+B)t + A = 1$

$\to A=-B$

$\to A=1$

$A=1$ e $B=-1$

$\int \frac{dt}{t^2+t} = \int \frac{1}{t} - \frac{1}{t+1}\,dt = \ln|t| - \ln|t+1| + C = \ln\left|\frac{t}{t+1}\right| + C$

d) $\int \frac{1}{t^2+t-6}dt = \int \frac{1}{(t-2)(t+3)}dt \Rightarrow \frac{1}{t^2+t-6} = \frac{A}{t-2} + \frac{B}{t+3} = \frac{At+3A+Bt-2B}{t^2+t-6}$

Logo $(A+B)t + (3A-2B)$

$\to A=-B$

$\to 3A-2B=1 \to A=1/5$ e $B=-1/5$

$\int \frac{1}{t^2+t-6}dt = \int \frac{1}{5(t-2)} - \frac{1}{5(t+3)}\,dt = \frac{1}{5}\left(\ln|t-2| - \ln|t+3|\right) + C = \frac{1}{5}\ln\left|\frac{t-2}{t+3}\right| + C$

e) $\int \frac{dx}{(x-a)(x-b)}$, $(a \neq b)$ $\qquad \frac{1}{(x-a)(x-b)} = \frac{A}{x-a} + \frac{B}{x-b} = \frac{Ax - bA + Bx - aB}{(x-a)(x-b)}$

$(A+B)x - bA - aB = 1 \quad \rightarrow A = -B$

$-bA + aA = 1 \quad \therefore A = 1/(a-b)$

$$\int \frac{dx}{(x-a)(x-b)} = \int \frac{1}{(a-b)(x-a)} - \frac{1}{(a-b)(x-b)}\, dx = \frac{1}{a-b}\left(\ln|x-a| - \ln|x-b|\right) + C$$

$$= \frac{\ln\left|\frac{x-a}{x-b}\right|}{(a-b)} + C$$

f) $\int \frac{dp}{p(1-(p/K))} \Rightarrow \frac{A}{p} + \frac{B}{1-p/K} = \frac{A - \frac{A}{K}p + Bp}{p(1-(p/K))}$

$\left(B - \frac{A}{K}\right)p + A = 1 \quad \therefore A = 1$

$B = \frac{1}{K}$

$$\int \frac{dp}{p(1-(p/K))} = \int \frac{1}{p} + \frac{1}{K-p}\, dp = \ln|p| + \ln|K-p| + C$$

$$= \ln\left|\frac{p}{K-p}\right| + C$$

17. a) $\int (3x+5)^9 dx = \frac{(3x+5)^{9+1}}{(9+1)*3} = \frac{(3x+5)^{10}}{30} + C$

b) $\int \frac{dx}{4x+1} = \frac{\ln|4x+1|}{4} + C \qquad \ln|4+1| - 4 \quad \frac{1}{4x+1}$

c) $\int \frac{2}{x+4} - \frac{3}{x-1}\, dx = 2\ln|x+4| - 3\ln|x-1| + C$

d) $\int \frac{t^3}{t^4+1} dt \qquad u = t^4+1, \quad du = 4t^3 dt \qquad \therefore \int \frac{t^3}{t^4+1} dt = \int \frac{1}{4u} du = \frac{\ln|u|}{4} + C$

$= \frac{\ln|t^4+1|}{4} + C$

e) $\int t\sqrt{t^2+9}\, dt \qquad u = t^2+9, \quad du = 2t\,dt \qquad \therefore \int \frac{\sqrt{u}}{2} du = \frac{2}{3}\frac{u^{3/2}}{2} = \frac{(t^2+9)^{3/2}}{3} + C$

f) $\int x^6 \ln x\, dx \qquad u = \ln x \quad du = \frac{1}{x}dx$

$v = x^7/7 \quad dv = x^6 dx$

$$\int x^6 \ln x\, dx = \frac{x^7 \ln x}{7} - \int \frac{x^6}{7} dx = \frac{x^7 \ln x}{7} - \frac{x^7}{49} + C$$

$\int \frac{1}{t^2-1}\,dt \Rightarrow \frac{1}{(t+1)(t-1)} = \frac{A}{t+1} + \frac{B}{t-1} = \frac{At - A + Bt + B}{t^2-1} = \frac{(A+B)t - A + B}{t^2-1}$

$\therefore A = -B \quad \& \quad -A + B = 1 \quad \therefore \quad A = -\frac{1}{2} \quad \& \quad B = \frac{1}{2}$

$$\int \frac{1}{t^2-1}\,dt = \int \frac{-1}{2(t+1)} + \frac{1}{2(t-1)}\,dt = \frac{1}{2}\left(\ln|t-1| - \ln|t+1|\right) + C$$

$$= \frac{\ln\left|\frac{t-1}{t+1}\right|}{2} + C$$

h) $\int \frac{t}{t^2-1}\,dt \qquad u = t^2 - 1 \quad du = 2t\,dt$

$$= \int \frac{1}{2u}\,du = \frac{\ln|u|}{2} + C = \frac{\ln|t^2-1|}{2}$$

i) $\int \frac{1}{(t-1)^2}\,dt \qquad u = (t-1) \quad du = dt \quad \therefore = \int \frac{1}{u^2}\,du = \int u^{-2}\,du = \frac{u^{-1}}{-1} = \frac{-1}{(t-1)} + C$

j) $\int \frac{t}{(t-1)^2}\,dt \qquad u = t-1 \quad du = dt \quad = \int \frac{u-1}{u^2}\,du$

$\rightarrow \frac{u-1}{u^2} = \frac{A}{u^2} + \frac{B}{u} = \frac{A + Bu}{u^2} \qquad \rightarrow A = -1 \quad \rightarrow B = 1$

$$= \int \frac{-1}{u^2} + \frac{1}{u}\,du = u^{-1} + \ln|u| + C = \ln|t-1| + \frac{1}{t-1} + C$$

K) $\int x^3 (x^4 - 4)^5\,dx \qquad u = x^4 - 4 \quad du = 4x^3\,dx \quad \therefore = \int \frac{u^5}{4}\,du = \frac{u^6}{24} + C$

$$= \frac{(x^4-4)^6}{24} + C$$

l) $\int (2t+1)(t^2+t-1)^8\,dt \qquad u = t^2 + t - 1 \quad du = (2t+1)\,dt$

$$= \int u^8\,du = \frac{u^9}{9} + C = \frac{(t^2+t-1)^9}{9} + C$$

m) $\int (t + e^{4t})\,dt = \frac{t^2}{2} + \frac{e^{4t}}{4} + C$

n) $\int t e^{4t}\,dt \qquad u = t \quad du = dt \qquad v = \frac{e^{4t}}{4} \quad dv = e^{4t}\,dt$

$$= \frac{te^{4t}}{4} - \int \frac{e^{4t}}{4}\,dt = \frac{te^{4t}}{4} - \frac{e^{4t}}{16} + C$$

o) $\int x(1+\ln x)\,dx$ $\quad u = 1+\ln x \quad du = \frac{1}{x}dx$
$\quad v = x^2/2 \quad dv = x\,dx$

$= \frac{x^2}{2}(1+\ln x) - \int \frac{x}{2}dx = \frac{x^2}{2}(1+\ln x) - \frac{x^2}{4} + C$

p) $\int \frac{1+\ln x}{x}\,dx$ $\quad u = (1+\ln x) \quad du = \frac{1}{x}dx$

$= \int u\,du = \frac{u^2}{2} + C = \frac{(1+\ln x)^2}{2} + C$

q) $\int t e^{0,5t^2}dt$ $\quad u = 0,5t^2 \quad du = t\,dt$

$= \int e^u du = e^{0,5t^2} + C$

r) $\int t^2 e^{0,5t}dt$ $\quad u = t^2 \quad du = 2t\,dt$
$\quad v = \frac{e^{0,5t}}{0,5} \quad dv = e^{0,5t}dt$

$= 2t^2 e^{0,5t} - 4\int t e^{0,5t}dt$ $\quad u = t \quad du = dt$
$\quad v = \frac{e^{0,5t}}{0,5} \quad dv = e^{0,5t}dt$

$= 2t^2 e^{0,5t} - 4\left(\frac{te^{0,5t}}{0,5} - \int \frac{e^{0,5t}}{0,5}dt\right)$

$= 2t^2 e^{0,5t} - 8te^{0,5t} + 16e^{0,5t} + C$

s) $\int \frac{dx}{x^2-3x+2}$ $\quad \frac{1}{x^2-3x+2} = \frac{1}{(x-2)(x-1)} \rightarrow \frac{A}{(x-2)} + \frac{B}{(x-1)}$

$Ax - A + Bx - 2B \therefore (A+B)x - A - 2B = 1 \rightarrow A = -B$
$-A-2B = 1 \therefore A = 1$
$B = -1$

$= \int \frac{1}{x-2} - \frac{1}{(x-1)}dx = \ln|x-2| - \ln|x-1| + C = \ln\left|\frac{x-2}{x-1}\right| + C$

t) $\int (x-1)e^{2x}dx$ $\quad u = x-1 \quad du = dx$
$\quad v = e^{2x}/2 \quad dv = e^{2x}dx$

$= \frac{e^{2x}}{2}(x-1) - \int \frac{e^{2x}}{2}dx = \frac{e^{2x}}{2}(x-1) - \frac{e^{2x}}{4} + C$

u) $\int (\ln x)^3 dx$ $\quad u = (\ln x)^3 \quad du = \frac{3\ln^2 x}{x}dx$
$\quad v = x\,dx \quad dv = 1dx$

$= x(\ln x)^3 - 3\int \ln^2 x\,dx$

** $\int (\ln x)^2 dx$ $\quad u = \ln^2 x \quad du = \frac{2\ln x}{x}dx$
$\quad v = x \quad dv = 1dx$

$\int \ln^2 x\,dx = x\ln^2 x - 2\int \ln x\,dx$

Logo: $\int (\ln x)^3 dx = x(\ln x)^3 - 3x(\ln x)^2 + 6\int \ln x\,dx$

Resolvir $\int \ln x\,dx$ $\quad u = \ln x \quad du = \frac{1}{x}dx \quad v = x \quad dv = 1dx \quad \therefore = x\ln x - \int dx = x\ln x - x$

Resposta. $\int (\ln x)^3 dx = x(\ln x)^3 - 3x(\ln x)^2 + 6x\ln x - 6x + C$

v) $\int \ln(x^3)\,dx \qquad u = \ln(x^3) \quad du = \frac{3x^2}{x^3}dx = \frac{3}{x}dx \quad v = x \quad dv = dx$

$= x\ln(x^3) - \int 3dx = x\ln(x^3) - 3x + C$

w) $\int \frac{2t}{t^2-3t}dt = \int \frac{2}{t-3}dt = 2\ln|t-3| + C$

x) $\int \frac{te^{-t}}{(t-1)^2}dt$

$e^{-x/2} \quad \frac{1}{e^{x/2}}$

y) $\int \frac{1-e^x}{1+e^x}dx = \int \frac{e^{-x/2}(1-e^x)}{e^{-x/2}(1+e^x)}dx = \int \frac{e^{-x/2}-e^{x/2}}{e^{-x/2}+e^{x/2}}dx$

$u = e^{-x/2} + e^{x/2}$

$du = \left[-\frac{1}{2}e^{-x/2} + \frac{1}{2}e^{x/2}\right]dx = -\frac{1}{2}\left(e^{-x/2} - e^{x/2}\right)dx$

Logo: $\int \frac{1-e^x}{1+e^x}dx = \int \frac{e^{-x/2}-e^{x/2}}{e^{-x/2}+e^{x/2}}dx = \int -\frac{2}{u}du = -2\ln|u| + C$

$= -2\ln|e^{-x/2} + e^{x/2}| + C$

z) $\int \frac{1}{1+e^x}dx \qquad u = 1+e^x \quad du = e^x dx \quad \therefore \int \frac{1}{u}\frac{du}{e^x} = \int \frac{1}{u(u-1)}du$

$\rightarrow \frac{A}{u} + \frac{B}{u-1} = \frac{Au - A + Bu}{u^2-u} \quad \therefore (A+B)u - A = 1 \quad \begin{cases} A = -B \\ A = -1 \end{cases} \quad \therefore B = 1$

$\int \frac{1}{1+e^x}dx = \int \frac{1}{u(u-1)}du = \int \frac{1}{u-1} - \frac{1}{u}du = \ln\left|\frac{u-1}{u}\right| + C$

$= \ln\left|\frac{e^x}{1+e^x}\right| + C$

aa) $\int \frac{1}{e^x - e^{-x}}\,dx = \int \frac{1}{e^x - \frac{1}{e^x}}\,dx = \int \frac{e^x}{e^{2x}-1}\,dx \qquad u = e^x \quad du = e^x dx$

$= \int \frac{1}{u^2-1}\,du = \int \frac{1}{(u+1)(u-1)}\,du \longrightarrow \frac{A}{u+1} + \frac{B}{u-1} = \frac{Au - A + Bu + B}{u^2-1}$

Então $(A+B)u + B - A = 1 \rightarrow A = -B,\ B - A = 1 \quad \therefore B = 1/2 \quad A = -1/2$

$= \int \frac{-1}{2(u+1)} + \frac{1}{2(u-1)}\,du = \frac{\ln\left|\frac{u-1}{u+1}\right|}{2} + C = \frac{\ln\left|\frac{e^x-1}{e^x+1}\right|}{2} + C$

bb) $\int \frac{e^x}{e^{2x} - 4e^x + 3}\,dx \qquad u = e^x,\ du = e^x dx$

$= \int \frac{1}{u^2-4u+3}\,du = \int \frac{1}{(u-3)(u-1)}\,du \Rightarrow \frac{A}{u-3} + \frac{B}{u-1} = \frac{Au - A + Bu - 3B}{(u-3)(u-1)}$

$(A+B)u - A - 3B = 1 \rightarrow A = -B,\ A = 1/2 \quad \therefore A = 1/2 \text{ e } B = -1/2$

$= \int \frac{1}{2(u-3)} - \frac{1}{2(u-1)}\,dx = \frac{\ln|u-3| - \ln|u-1|}{2} + C = \frac{\ln\left|\frac{u-3}{u-1}\right|}{2} + C$

$= \frac{1}{2}\ln\left|\frac{e^x-3}{e^x-1}\right| + C$

cc) $\int \frac{x}{\sqrt{1+\sqrt{x}}}\,dx \ : \quad u = 1+\sqrt{x},\ du = \frac{1}{2\sqrt{x}}\,dx \quad \therefore = 2\int \frac{(u-1)^2}{u}(u-1)\,du$

$= 2\int \frac{u^3 - 3u^2 + 3u - 1}{u}\,du = 2\int u^2 - 3u + 3 - \frac{1}{u}\,du$

$= 2\left(\frac{(1+\sqrt{x})^3}{3} - \frac{3(1+\sqrt{x})^2}{2} + 3(1+\sqrt{x}) - \ln|1+\sqrt{x}|\right) + C$

dd) $\int \frac{x}{\sqrt{1+x}}\,dx \qquad u = \sqrt{1+x} \qquad du = \frac{1}{2\sqrt{1+x}}\,dx$

$= \int 2(u^2-1)\,du = 2\left(\frac{u^3}{3} - u\right) + C = \frac{2}{3}(1+x)^{3/2} - 2(1+x)^{1/2} + C$

ee) $\int \frac{1}{\sqrt{x}(\sqrt{x}-1)(\sqrt{x}+2)}\,dx \qquad u = \sqrt{x},\ du = \frac{1}{2\sqrt{x}}\,dx$

$= \int \frac{2}{(u-1)(u+2)}\,du \Rightarrow \frac{A}{u-1} + \frac{B}{u+2} = \frac{Au + 2A + Bu - B}{(u-1)(u+2)} \rightarrow A + B = 0,\ 2A - B = 2$

$= \int \frac{2}{3(u-1)} - \frac{2}{3(u+2)}\,du = \frac{2}{3}\ln\left|\frac{u-1}{u+2}\right| + C = \frac{2}{3}\ln\left|\frac{\sqrt{x}-1}{\sqrt{x}+2}\right| + C$

ff) $\int \frac{1}{(\sqrt{x}-2)(\sqrt{x}+3)}\,dx \qquad u=\sqrt{x} \qquad du = \frac{1}{2\sqrt{x}}\,dx$

$= \int \frac{2u}{(u-2)(u+3)}\,du \Rightarrow \frac{A}{u-2} + \frac{B}{u+3} = \frac{Au+3A+Bu-2B}{(u-2)(u+3)}$

$\rightarrow (A+B)u + 3A-2B \Longrightarrow 3A = 2B \;\; e \;\; A+B = 2$

$A = \frac{2}{3}B \longrightarrow \frac{2}{3}B + B = 2 \;\therefore\; \frac{5B}{3} = 2$

$B = \frac{6}{5} \;\; e \;\; A = \frac{2}{3} * \frac{6}{5} = \frac{4}{5}$

$= \int \frac{4}{5(u-2)} + \frac{6}{5(u+3)}\,du = \frac{4}{5}\ln|u-2| + \frac{6}{5}\ln|u+3| + C$

$= \frac{4}{5}\ln|\sqrt{x}-2| + \frac{6}{5}\ln|\sqrt{x}+3| + C$.

18. a) $\frac{dy}{dt} = \sqrt{2t+1}$, $y(0)=1$

$y = \int (2t+1)^{1/2}\,dt = \frac{\frac{2}{3}(2t+1)^{3/2}}{2} = \frac{(2t+1)^{3/2}}{3} + C$

$y(0) = \frac{1}{3} + C = 1 \;\therefore\; C = 1 - \frac{1}{3} = \frac{3-1}{3} = 2/3$

$y = \frac{(2t+1)^{3/2} + 2}{3}$

b) $\frac{dy}{dt} = t - te^t$, $y(0) = -1$

$y = \int t - te^t\,dt = \frac{t^2}{2} - e^t t + \int e^t\,dt \qquad u=t \;\; du=dt \quad v=e^t \;\; dv=e^t dt$

$y = \frac{t^2}{2} - e^t t + e^t + C$

$y(0) = e^0 + C = -1 \;\therefore\; C = -2 \rightarrow y = \frac{t^2}{2} - e^t t + e^t - 2$

c) $\frac{dy}{dt} = (\ln t)^2 - t \qquad y(1) = 2$

$\int \ln^2 t\,dt - \int t\,dt = t\ln^2 t - 2t\ln t + 2t - \frac{t^2}{2} + C$

(Resolvido no exercício 17 u)

$y(1) = 2 - \frac{1}{2} + C = 2 \;\therefore\; C = 1/2$

$y = t\ln^2 t - 2t\ln t + 2t - t^2/2 + 1/2$

d) $\frac{dy}{dt} = \frac{1}{t^2+3t+2} \rightarrow y = \int \frac{1}{t^2+3t+2}\,dt = \int \frac{1}{(t+1)(t+2)}\,dt$

* $\frac{A}{t+1} + \frac{B}{t+2} = \frac{At+2A+Bt+B}{t^2+3t+2}$ $\therefore A=-B$, $2A+B=1$ $\therefore A=1$ e $B=-1$

$y = \int \frac{1}{t+1} - \frac{1}{t+2}\,dt = \ln\left|\frac{t+1}{t+2}\right| + C$

$y(0) = \ln 1/2 + C = 0 \quad \therefore C = -\ln 1/2 = \ln 2$

$y = \ln\left|\frac{t+1}{t+2}\right| + \ln 2$

e) $\frac{dy}{dt} = t\sqrt{3t+1}$, $y(0)=0$

$y = \int t\sqrt{3t+1}\,dt$ $\quad u = t \quad du = dt$; $\quad v = \frac{2}{9}(3t+1)^{3/2} \quad dv = \sqrt{3t+1}\,dt$

$y = \frac{2}{9}t(3t+1)^{3/2} - \frac{2}{9}\int (3t+1)^{3/2}dt = \frac{2}{9}t(3t+1)^{3/2} - \frac{2}{9} * \frac{2}{5} * \frac{1}{3}(3t+1)^{5/2} + C$

$y = \frac{2}{9}t(3t+1)^{3/2} - \frac{4}{135}(3t+1)^{5/2} + C$

$y(0) = -\frac{4}{135} + C = 0 \quad \therefore C = +4/135$

$y = \frac{2}{9}t(3t+1)^{3/2} - \frac{4}{135}(3t-1)^{5/2} + \frac{4}{135}$

f) $\frac{dy}{dt} = e^{2t}\sqrt{e^t+2}\,dt$, $y(0) = 1$

$y = \int e^{2t}\sqrt{e^t+2}\,dt$ $\quad u = e^t+2 \quad du = e^t\,dt$

$y = \int (u-2)\sqrt{u}\,du = \int (u-2)u^{1/2}du = \int u^{3/2} - 2u^{1/2}\,du$

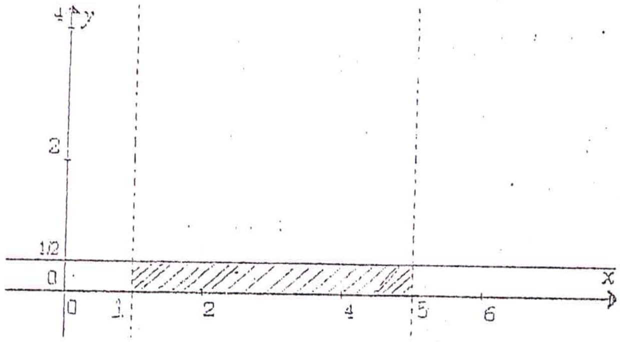
$y = \frac{2}{5}u^{5/2} - 2 * \frac{2}{3}u^{3/2} + C = \frac{2}{5}(e^t+2)^{5/2} - \frac{4}{3}(e^t+2)^{3/2} + C$

$y(0) = 2/5\,(3)^{5/2} - 4/3\,(3)^{3/2} + C = 1$

$\therefore C = 1 - \frac{2}{5}(9\sqrt{3}) + \frac{4}{3}(3\sqrt{3}) = 1 - \frac{18\sqrt{3}}{5} + 4\sqrt{3}$

13. a) $f(x) = 1/2$; $a = 1$ e $b = 5$

$A(x) = \int_1^5 f(x)dx = \int_1^5 1/2\, dx = \left(\frac{x}{2}\right)\Big|_1^5 = \frac{5}{2} - \frac{1}{2} = \frac{4}{2} = 2 u.a$



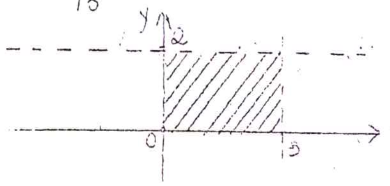
Por geometria elementar:

A = área retângulo

$A = b \times h = (5-1) * 1/2 = \frac{4}{2} = 2 u.a.$

b) $f(x) = 2$, $a = 0$ e $b = 3$

$A(x) = \int_0^3 2\, dx = (2x)\Big|_0^3 = 2 * 3 = 6 u.a$

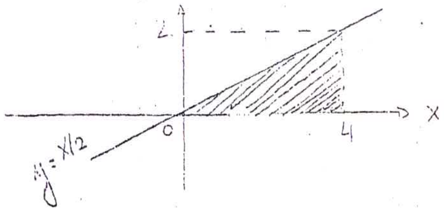


Por geometria elementar

$A = b \times h = 3 * 2 = 6 u.a$

c) $f(x) = x/2$, $a = 0$ e $b = 4$

$A(x) = \int_0^4 \frac{x}{2}\, dx = \left(\frac{x^2}{4}\right)\Big|_0^4 = \frac{16}{4} = 4 u.a$

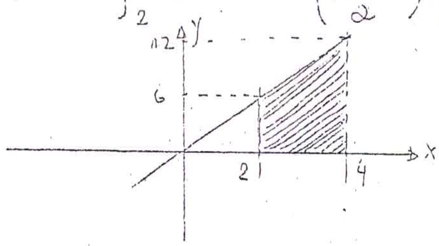


Por geometria elementar:

$A = \frac{b \times h}{2} = \frac{4 \times 2}{2} = 4 u.a.$

d) $f(x) = 3x$, $a = 2$ e $b = 4$

$A(x) = \int_2^4 3x\, dx = \left(\frac{3x^2}{2}\right)\Big|_2^4$ $\quad \frac{27}{2} - \frac{12}{2} = \frac{15}{2}$

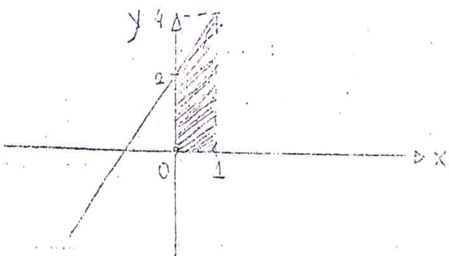


Por geometria elementar:

$A = \frac{(b+B)h}{2} = \frac{(6+12)2}{2} = 18 u.a$

e) $f(x) = 2x + 2$, $a = 0$ e $b = 1$

$A(x) = \int_0^1 2x+2\, dx = (x^2+2x)\Big|_0^1 = 1+2 = 3 u.a$

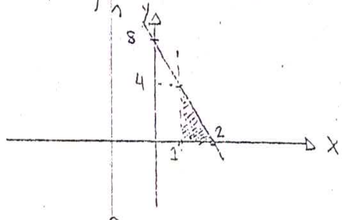


Por geometria elementar

$A = \frac{(B+b)h}{2} = \frac{(2+4)\,1}{2} = \frac{6}{2} = 3 u.a$

f) $f(x) = -4x+8$, $a=1$ e $b=2$

$A(x) = \int_1^2 -4x+8\,dx = (-2x^2+8x)\Big|_1^2 = -8+16+2-8 = 2u.a$

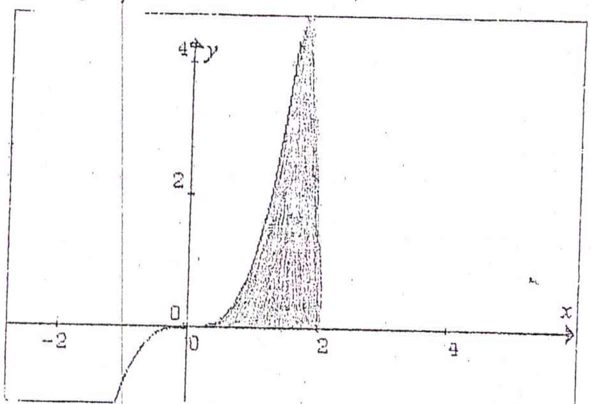


Por geometria elementar

$A = \frac{b \times h}{2} = \frac{1 * 4}{2} = 2u.a$

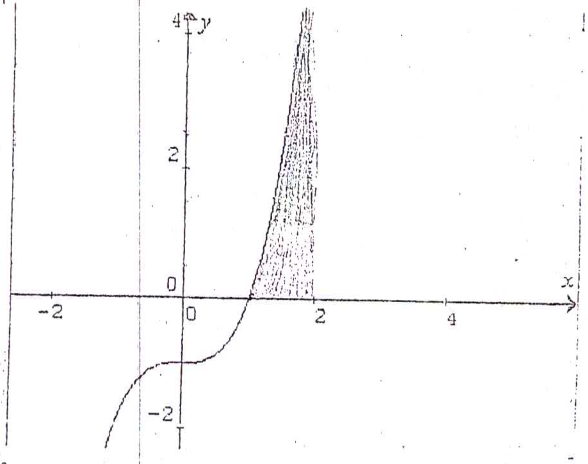
20. a) $f(x) = x^3$, $a=0$ e $b=2$

$A(x) = \int f(x) \cdot dx = f(b) - f(a)$



$A(x) = \int_0^2 x^3\,dx = \left(\frac{x^4}{4}\right)\Big|_0^2 = \frac{16}{4} = 4$

b) $f(x) = x^3 - 1$, $a=1$ e $b=2$



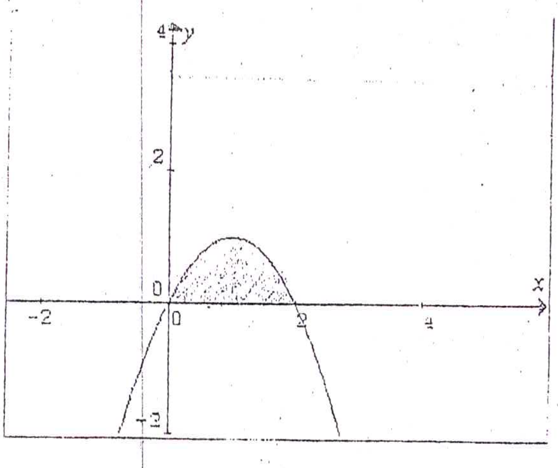
$A(x) = \int_1^2 (x^3-1)\,dx$, $F(x)$

$F(x_0) + y_0$

$A(x) = \left(\frac{x^4}{4} - x\right)\Big|_1^2 = \frac{16}{4} - 2 - \frac{1}{4} + 1$

$A(x) = 4 - 1 - \frac{1}{4} = 3 - \frac{1}{4} = \frac{12-1}{4} = \frac{11}{4}$

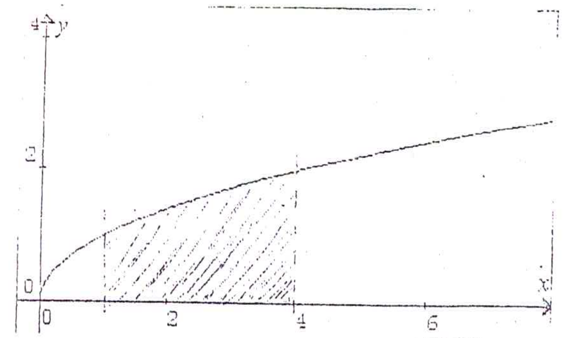
c) $f(x) = 2x - x^2$, $a=0$ e $b=2$



$A(x) = \int_0^2 (2x - x^2)\,dx$

$A(x) = \left(x^2 - \frac{x^3}{3}\right)\Big|_0^2 = 4 - \frac{8}{3} = \frac{12-8}{3} = \frac{4}{3}$

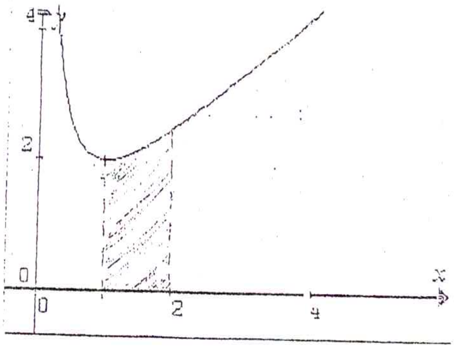
d) $f(x) = \sqrt{x}$, $a = 1$ e $b = 4$



$$A(x) = \int_1^4 \sqrt{x}\, dx = \left(\frac{2}{3} x^{3/2}\right)\Big|_1^4 = \frac{2}{3}\left(4^{3/2} - 1\right)$$

$$A(x) = \frac{2}{3}\sqrt{4^3} - \frac{2}{3} = \frac{16}{3} - \frac{2}{3} = \frac{14}{3}$$

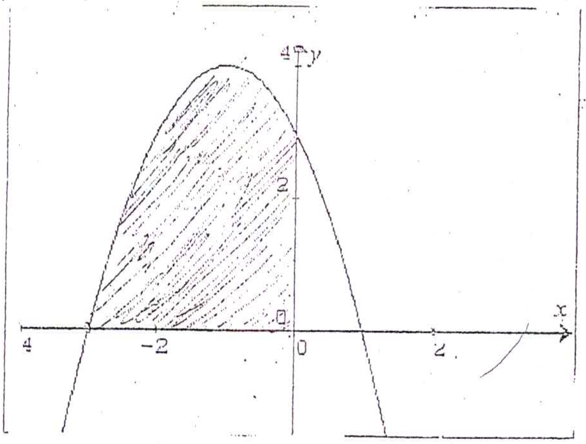
e) $f(x) = x + \frac{1}{x}$, $a = 1$ e $b = 2$



$$A(x) = \int_1^2 x + \frac{1}{x}\, dx = \left(\frac{x^2}{2} + \ln x\right)\Big|_1^2$$

$$A(x) = \frac{4}{2} + \ln 2 - \frac{1}{2} = \ln 2 + \frac{3}{2}$$

f) $f(x) = 3 - 2x - x^2$, $a = -3$ e $b = 0$

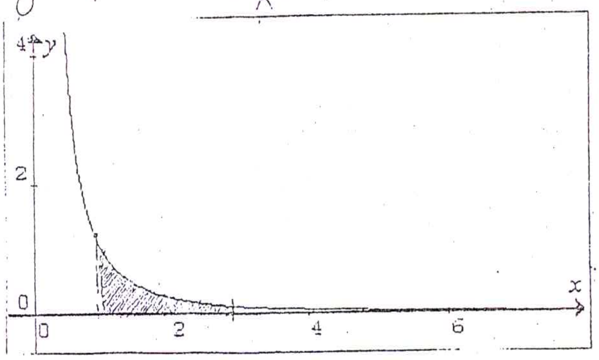


$$A(x) = \int_{-3}^{0} (3 - 2x - x^2)\, dx$$

$$A(x) = \left(3x - x^2 - \frac{x^3}{3}\right)\Big|_{-3}^{0} = 9 + 9 - 9 = 9$$

$- 9 - 9 + 9 . = - 9.$

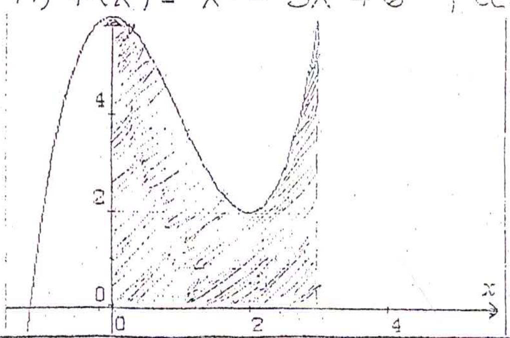
g) $f(x) = 1/x^2$, $a = 1$ e $b = 3$



$$A(x) = \int_1^3 \frac{1}{x^2}\, dx = \int_1^3 x^{-2}\, dx$$

$$A(x) = \left(\frac{x^{-2+1}}{-2+1}\right)\Big|_1^3 = \left(-\frac{1}{x}\right)\Big|_1^3 = -\frac{1}{3} + 1 = \frac{2}{3}$$

h) $f(x) = x^3 - 3x^2 + 6$, $a = 0$ e $b = 3$



$$A(x) = \int_0^3 (x^3 - 3x^2 + 6)\, dx$$

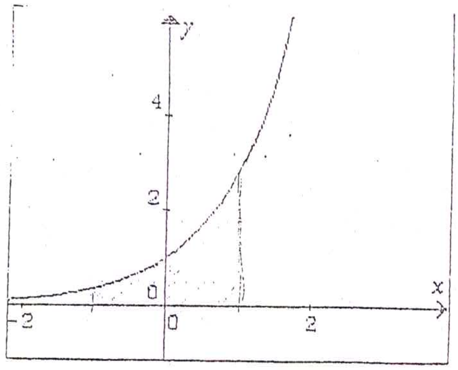
$$A(x) = \left(\frac{x^4}{4} - x^3 + 6x\right)\Big|_0^3 = \frac{81}{4} - 27 + 18$$

$$A(x) = \frac{153 - 108}{4} = \frac{45}{4}$$

i) $f(x) = e^x$, $a = -1$ e $b = 1$

$A(x) = \int_{-1}^{1} e^x dx = (e^x) \Big|_{-1}^{1}$

$A(x) = e^x - \frac{1}{e^x} = \frac{e^{2x} - 1}{e^x}$



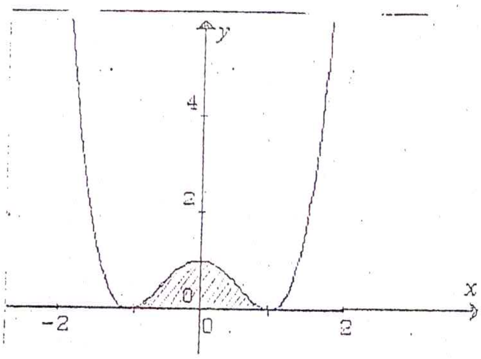
j) $f(x) = x^4 - 2x^2 + 1$, $a = -1$ e $b = 1$

$A(x) = \int_{-1}^{1} (x^4 - 2x^2 + 1)\, dx$

$A(x) = \left(\frac{x^5}{5} - \frac{2x^3}{3} + x\right) \Big|_{-1}^{1}$

$A(x) = \frac{1}{5} - \frac{2}{3} + 1 + \frac{1}{5} - \frac{2}{3} + 1 = \frac{2}{5} + 2 - \frac{4}{3}$

$A(x) = \frac{6 + 30 - 20}{15} = \frac{16}{15}$.



21. a) $\int_0^1 (x - x^3)\, dx = \left(\frac{x^2}{2} - \frac{x^4}{4}\right) \Big|_0^1 = \frac{1}{2} - \frac{1}{4} = \frac{2 - 1}{4} = \frac{1}{4}$

b) $\int_{-1}^{1} (x - x^3) dx = \left(\frac{x^2}{2} - \frac{x^4}{4}\right) \Big|_{-1}^{1} = \frac{1}{2} - \frac{1}{4} - \frac{1}{2} + \frac{1}{4} = 0$

c) $\int$

d) $\int_{-3}^{1} x dx = \left(\frac{x^2}{2}\right) \Big|_{-3}^{1} = \frac{1}{2} - \frac{9}{2} = \frac{1-9}{2} = -\frac{8}{2} = -4$

e) $\int_0^2 (x^3 - 4x^2 + 3x) dx = \left(\frac{x^4}{4} - \frac{4x^3}{3} + \frac{3x^2}{2}\right) \Big|_0^2 = \frac{16}{4} - \frac{32}{3} + \frac{12}{2} = -\frac{2}{3}$

f) $\int_{-1}^{1} (e^x - 1) dx = (e^x - x) \Big|_{-1}^{1} = e^x - 1 - e^{-x} - 1 = e^x - 2 - e^{-x}$

g) $\int_0^1 x^4 - 4x^3 dx = \left(\frac{x^5}{5} - x^4\right) \Big|_0^1 = \frac{1}{5} - 1 = -4/5$

h) $\int_{-2}^{-1} \frac{1}{x} dx = (\ln|x|) \Big|_{-2}^{-1} = \ln|-1| - \ln|-2| = -\ln 2$

i) $\int_{-1}^{1} x^{1/3} dx = \left(\frac{3x^{4/3}}{4}\right) \Big|_{-1}^{1} = \frac{3}{4} - \frac{3}{4} = 0$

j) $\int_0^1 (3x - 2e^x)\, dx = \left(\frac{3x^2}{2} - 2e^x\right) \Big|_0^1 = \frac{3}{2} - 2e + 2 = \frac{7}{2} - 2e$

LISTA 8:

Integrais II

Lista 8 · Integrais II

1. a) $\int_0^x \sqrt{1+2t}\, dt = \int_0^x (1+2t)^{1/2} dt = \left(\frac{(1+2t)^{1/2+1}}{(1/2+1)\,2} \right)\Big|_0^x = \left[\frac{1}{3}(1+2t)^{3/2} \right]_0^x$

$= 1/3\,(1+2x)^{3/2} - 1/3\,,$

$\frac{d}{dx}\left[\int_0^x \sqrt{1+2t}\, dt \right] = \frac{d}{dx}\left[\frac{4}{3}(1+2x)^{3/2} - \frac{1}{3} \right] = \frac{1}{3} * \frac{3}{2}(1+2x)^{1/2} * 2$

Logo, $\frac{d}{dx}\left[\int_0^x \sqrt{1+2t}\, dt \right] = \sqrt{(1+2x)}$

b) $g(x) = \int_1^x \ln t\, dt = (t\ln t - t)\Big|_1^x = x\ln x - x - 1\ln 1 + 1 = x\ln x - x + 1$

$\frac{d}{dx}\left[\int_1^x \ln t\, dt \right] = \frac{d}{dx}\left[x\ln x - x + 1 \right] = \cancel{x} * \frac{1}{\cancel{x}} + 1.\ln x - 1 = \ln x$

Logo, $\frac{d}{dx}\left[\int_1^x \ln t\, dt \right] = \ln x$

c) $g(y) = \int_2^y t^2 \operatorname{sen} t\, dt = \left[-t^2\cos t + 2\int t\cos t\, dt \right]_2^y$

$u = t^2 \rightarrow du = 2t\,dt$
$v = -\cos t \leftarrow dv = \operatorname{sen} t\, dt$

$g(y) = \left[-t^2\cos t + 2\left(t\operatorname{sen} t - \int \operatorname{sen} t\, dt \right) \right]_2^y$

$u = t \qquad du = dt$
$v = \operatorname{sen} t \qquad dv = \cos t\, dt$

$g(y) = \left[-t^2\cos t + 2t\operatorname{sen} t + 2\cos t \right]_2^y$

$g(y) = \int_2^y t^2 \operatorname{sen} t\, dt = -y^2\cos y + 2y\operatorname{sen} y + 2\cos y + 4\cos 2 - 4\operatorname{sen} 2 - 2\cos 2$

Logo, $\frac{d}{dy}\left[\int_2^y t^2 \operatorname{sen} t\, dt \right] = -2y\cos y + y^2\operatorname{sen} y + 2\operatorname{sen} y + 2y\cos y - 2\operatorname{sen} y$

$\frac{d}{dy}\left[\int_2^y t^2 \operatorname{sen} t\, dt \right] = y^2 \operatorname{sen} y$

d) Não é possível integrar.

e) $g(x) = \int_x^2 \cos(t^2)\,dt = -\int_2^x \cos(t^2)\,dt$

$\frac{dg}{dx}\left[\int_2^x \cos t^2\,dt\right] = -\cos x^2$

f) $g(x) = \int_x^{10} tg\theta\, d\theta = -\int_{10}^x tg\theta\, d\theta = \int_{10}^x \frac{-sen\theta d\theta}{\cos\theta} = \int_{10}^x \frac{1}{u}\,du$

$u = \cos\theta \quad du = -sen\theta\, d\theta$

$g(x) = (\ln|u|)\Big|_{10}^x = (\ln|\cos\theta|)\Big|_{10}^x = \ln|\cos x| - \ln(\cos 10)$

$\frac{dg}{dx}\left[\ln|\cos x| - \ln\cos 10\right] = \frac{1}{\cos x} * -senx = -tgx.$

g) $\frac{d}{dx}\left[\int_2^{1/x} arctgt\,dt\right] = arctg\left(\frac{1}{x}\right) * \left(\frac{1}{x}\right)'$

$\frac{d}{dx}\left[\int_2^{1/x} arctg\,t\,dt\right] = -\frac{arctg(1/x)}{x^2}$

h) $g(x) = \int_0^{x^2} \sqrt{1+r^3}\,dr$

$\frac{dg}{dx}\left[\int_0^{x^2} \sqrt{1+r^3}\,dr\right] = \sqrt{1+x^6} * 2X = 2X\sqrt{1+x^6}$

i) $y = \int_3^{\sqrt{x}} \frac{\cos t}{t}\,dt$

$\frac{dy}{dx}\left[\int_3^{\sqrt{x}} \frac{\cos t}{t}\,dt\right] = \frac{\cos\sqrt{x}}{\sqrt{x}} * \frac{1}{2\sqrt{x}} = \frac{\cos\sqrt{x}}{2x}$

J) $\frac{dy}{dx}\left[\int_1^{\cos x} (t + sent)\,dt\right] = (\cos x + sen(\cos x)) * -senx$

$= -senx\ (\cos x + sen(\cos x))$

K) $\frac{dy}{dx}\left[-\int_1^{1-3x} \frac{u^3}{1+u^2}\,du\right] = -\left(\frac{(1-3x)^3}{1+(1-3x)^2}\right) * -3$

$\frac{dy}{dx}\left[-\int_1^{1-3x} \frac{u^3}{1+u^2}\,du\right] = \frac{3(1-3x)^3}{1+(1-3x)^2}$

l) $\frac{dy}{dx}\left[-\int_0^{e^x} sen^3 t\,dt\right] = -(sen^3 e^x)\,e^x$

$= -e^x\,(sen^3 e^x)$

2. a) $\int_{-1}^{3} x^5 dx = \left(\frac{x^{5+1}}{5+1}\right)\Big|_{-1}^{3} = \left(\frac{x^6}{6}\right)\Big|_{-1}^{3} = \frac{3^6}{6} - \frac{(-1)^6}{6} = \frac{3^6-1}{6} = \frac{728}{6} = \frac{364}{3}$

b) $\int_2^8 (4x+3)\,dx = \int_2^8 4x\,dx + \int_2^8 3\,dx = \left(\frac{4x^{1+1}}{1+1} + \frac{3x}{1}\right)\Big|_2^8 = \frac{4.8^2}{2} + \frac{3.8}{1} - \frac{4(2)^2}{2} - \frac{3(2)}{1}$

$= 128 + 24 - 8 - 6 = 138$

c) $\int_0^4 \sqrt{x}\,dx = \int_0^4 (x)^{1/2} dx = \left(\frac{x^{1/2+1}}{1/2+1}\right)\Big|_0^4 = \left(\frac{2x^{3/2}}{3}\right)\Big|_0^4 = \frac{2\,(4)^{3/2}}{3} = \frac{16}{3}$

d) $\int_1^2 x^{-2} dx = \left(\frac{x^{-2+1}}{-2+1}\right)\Big|_1^2 = \left(\frac{x^{-1}}{-1}\right)\Big|_1^2 = -\frac{1}{2} + 1 = \frac{1}{2}$

e) $\int_0^4 (1+3y-y^2)\,dy = \int_0^4 1\,dy + \int_0^4 3y\,dy - \int_0^4 y^2 dy$

$= \left(y + \frac{3y^{1+1}}{1+1} - \frac{y^{2+1}}{2+1}\right)\Big|_0^4 = \left(y + \frac{3}{2}y^2 - \frac{y^3}{3}\right)\Big|_0^4 = 4 + \frac{3}{2}(4)^2 - \frac{(4)^3}{3} = 28 - \frac{64}{3}$

$= \frac{20}{3}$

f) $\int_0^1 x^{3/7} dx = \left(\frac{x^{\frac{3}{7}+1}}{\frac{3}{7}+1}\right)\Big|_0^1 = \left(\frac{7x^{10/7}}{10}\right)\Big|_0^1 = \frac{7}{10}$

g) $\int_1^2 3t^{-4} dt = \left(\frac{3t^{-4+1}}{-4+1}\right)\Big|_1^2 = \left(\frac{3t^{-3}}{-3}\right)\Big|_1^2 = \left(-\frac{1}{t^3}\right)\Big|_1^2 = -\frac{1}{8} + 1$

$= 7/8$

h) $\int_{-1}^{1} 3/t^4\, dt$ não se aplica o Teorema Fundamental do Cálculo pois no intervalo $[-1;1]$ há descontinuidade

i) $\int_3^3 \sqrt{x^2+2}\,dx = 0$

j) $\int_{\pi}^{2\pi} \cos\theta\, d\theta = (\mathrm{sen}\,\theta)\Big|_{\pi}^{2\pi} = \mathrm{sen}\,2\pi - \mathrm{sen}\,\pi = 0$

k) $\int_{-4}^{2} 2/x^6\, dx$ Não se aplica o Teorema Fundamental do Cálculo pois no intervalo $[2,-4]$ há descontinuidade.

l) $\int_1^4 x^{-1/2} dx = \left(\frac{x^{-1/2+1}}{-1/2+1}\right)\Big|_1^4 = \left(\frac{x^{1/2}}{1/2}\right)\Big|_1^4 = (2x^{1/2})\Big|_1^4 = 2(\sqrt{4} - \sqrt{1}) = 2$

m) $\int_{\pi/4}^{\pi/3} \mathrm{sen}\,t\,dt = (-\cos t)\Big|_{\pi/4}^{\pi/3} = -\cos\pi/3 + \cos\pi/4 = -\frac{1}{2} + \frac{\sqrt{2}}{2} = \frac{\sqrt{2}-1}{2}$

n) $\int_0^1 (3 + x^{3/2})\,dx = \left(3x + \frac{x^{3/2+1}}{3/2+1}\right)\Big|_0^1 = \left(3x + \frac{x^{5/2}}{5/2}\right)\Big|_0^1 = 3 + \frac{2}{5} = \frac{17}{5}$

o) $\int_{\pi/4}^{\pi} \sec^2\theta\, d\theta$ Não se aplica o Teorema Fundamental do Cálculo pois há descontinuidade no intervalo $[\pi/4, \pi]$

p) $\int_1^9 \frac{1}{2x}\,dx = \frac{1}{2}\int_1^9 1/x\,dx = \left(\frac{\ln|x|}{2}\right)\Big|_1^9 = \frac{\ln 9}{2} - \frac{\ln 1}{2} = \ln 3$

q) $\int_{\ln 3}^{\ln 6} 8e^x\,dx = (8e^x)\Big|_{\ln 3}^{\ln 6} = 8(e^{\ln 6} - e^{\ln 3}) = 8(6-3) = 24$

r) $\int_8^9 2^t\,dt = \left(\frac{2^t}{\ln 2}\right)\Big|_8^9 = \frac{2^9 - 2^8}{\ln 2} = \frac{256}{\ln 2}$

s) $\int_{-e^2}^{-e} \frac{3}{x}\,dx = (3\ln|x|)\Big|_{-e^2}^{-e} = 3\ln|e| - 3\ln|-e^2|$

$= 3\ln e - 6\ln e = 3 - 6 = -3$

t) $\int_1^{\sqrt{3}} \frac{6}{1+x^2}\,dx = 6(\tan^{-1}x)\Big|_1^{\sqrt{3}} = 6(\tan^{-1}\sqrt{3} - \tan^{-1}1) = 6\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$

$= \frac{6\pi}{12} = \frac{\pi}{2}$

u) $\int_0^{0,5}$ Não se aplica o Teorema Fundamental do Cálculo pois há descontinuidade no intervalo $[0, 0,5]$.

3. a) $\int_0^2 f(x)\,dx = \int_0^1 x^4\,dx + \int_1^2 x^5\,dx$

$\int_0^2 f(x)\,dx = \left(\frac{x^{4+1}}{4+1}\right)\Big|_0^1 + \left(\frac{x^{5+1}}{5+1}\right)\Big|_1^2 = \left(\frac{x^5}{5}\right)\Big|_0^1 + \left(\frac{x^6}{6}\right)\Big|_1^2$

$\int_0^2 f(x)\,dx = 1/5 + 2^6/6 - 1/6 = 1/5 + 63/6 = 1/5 + 21/2 = \frac{2+105}{10}$

$= \frac{107}{10}$

b) $\int_{-\pi}^{\pi} f(x)\,dx = \int_{-\pi}^{0} x\,dx + \int_0^{\pi} \operatorname{sen}x\,dx = +\int_0^{\pi}\operatorname{sen}x\,dx + \int_{-\pi}^{0} x\,dx$

$\int_{-\pi}^{\pi} f(x)\,dx = (-\cos x)\Big|_0^{\pi} + \left(\frac{x^2}{2}\right)\Big|_{-\pi}^{0} = -\cos\pi + \cos 0 - \frac{\pi^2}{2} = 2 - \frac{\pi^2}{2}$

4. a) $g(x) = \int_{2x}^{3x} \frac{u^2-1}{u^2+1}\, du$

$$g(x) = \int_{2x}^{a} \frac{u^2-1}{u^2+1}\, du + \int_{a}^{3x} \frac{u^2-1}{u^2+1}\, du = -\int_{a}^{2x} \frac{u^2-1}{u^2+1}\, du + \int_{a}^{3x} \frac{u^2-1}{u^2+1}\, du$$

$$\frac{dg}{dx}\left[\int_{a}^{2x} \frac{u^2-1}{u^2+1}\, du + \int_{a}^{3x} \frac{u^2-1}{u^2+1}\, du\right] = \frac{d}{dx}\left[\int_{2x}^{3x} \frac{u^2-1}{u^2+1}\, du\right]$$

$$\frac{d}{dx}\left[\int_{2x}^{3x} \frac{u^2-1}{u^2+1}\, du\right] = -2\left(\frac{4x^2-1}{4x^2+1}\right) + 3\left(\frac{9x^2-1}{9x^2+1}\right)$$

b) $\int_{tgx}^{x^2} \frac{1}{\sqrt{2+t^4}}\, dt$

$$\frac{d}{dt}\left[\int_{tgx}^{x^2} \frac{1}{\sqrt{2+t^4}}\, dt\right] = \frac{d}{dt}\left[-\int_{a}^{tgx} \frac{1}{\sqrt{2+t^4}}\, dt\right] + \frac{d}{dt}\left[\int_{a}^{x^2} \frac{1}{\sqrt{2+t^4}}\, dt\right]$$

$$\frac{d}{dt}\left[\int_{tgx}^{x^2} \frac{1}{\sqrt{2+t^4}}\, dt\right] = \frac{-\sec^2 x}{\sqrt{2+tg^4 x}} + \frac{2x}{\sqrt{2+x^8}}$$

c) $y = \int_{\sqrt{x}}^{x^3} \sqrt{t}\, \text{sen}\, t\, dt$

$$\frac{dy}{dx} = \frac{d}{dx}\left[\int_{a}^{x^3} \sqrt{t}\, \text{sen}\, t\, dt\right] - \frac{d}{dx}\left[\int_{a}^{\sqrt{x}} \sqrt{t}\, \text{sen}\, t\, dt\right]$$

$$\frac{dy}{dx} = 3x^2\left(x^{3/2}\, \text{sen}\, \cancel{t}^3\right) - \frac{1}{2\sqrt{x}} * \left(x^{1/4}\, \text{sen}\, \sqrt{x}\right)$$

$$\frac{dy}{dx} = 3x^{7/2}\, \text{sen}(x^3) - \frac{\text{sen}(\sqrt{x})}{2(x)^{1/4}}$$

d) $y = \int_{\cos x}^{5x} \cos(u^2)\, du$

$$\frac{dy}{dx} = \frac{dy}{dx}\left[\int_{a}^{5x} \cos(u^2)\, du\right] - \frac{dy}{dx}\left[\int_{a}^{\cos x} \cos(u^2)\, du\right]$$

$$\frac{dy}{dx} = 5\cos(25x^2) + \text{sen}\, x \cos(\cos^2 x)$$

5. a)

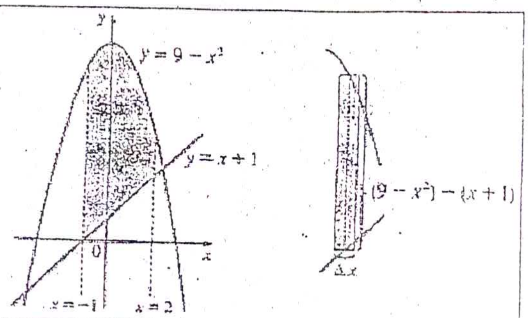
$A = \int_{-1}^{2} [(9 - x^2) - (x + 1)]\, dx$

$= \int_{-1}^{2} (8 - x - x^2)\, dx$

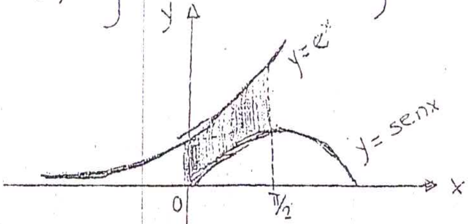
$= \left[8x - \frac{x^2}{2} - \frac{x^3}{3}\right]_{-1}^{2}$

$= \left(16 - 2 - \frac{8}{3}\right) - \left(-8 - \frac{1}{2} + \frac{1}{3}\right)$

$= 22 - 3 + \frac{1}{2} = \frac{39}{2}$ $\left(\frac{39}{2}\right)$



b) $y = \text{sen}\, x \qquad y = e^x \qquad x = 0 \qquad x = \pi/2$



$A = \int_0^{\pi/2} (e^x - \text{sen}\, x)\, dx = (e^x + \cos x)\Big|_0^{\pi/2}$

$A = e^{\pi/2} + \cos \pi/2 - e^0 - \cos 0 = e^{\pi/2} - 2$

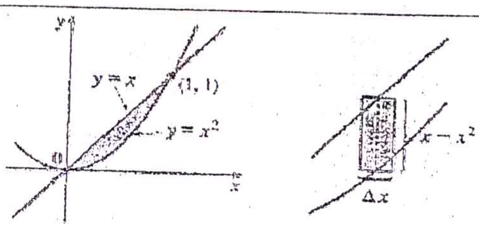
c)

$A = \int_0^1 (x - x^2)\, dx$

$= \left[\frac{1}{2}x^2 - \frac{1}{3}x^3\right]_0^1$

$= \frac{1}{2} - \frac{1}{3}$

$= \frac{1}{6}$



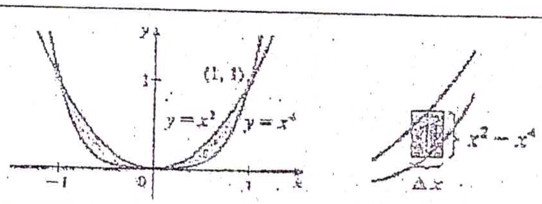
d)

$A = \int_{-1}^{1} (x^2 - x^4)\, dx$

$= 2\int_0^1 (x^2 - x^4)\, dx$

$= 2\left[\frac{1}{3}x^3 - \frac{1}{5}x^5\right]_0^1$

$= 2\left(\frac{1}{3} - \frac{1}{5}\right) = \frac{4}{15}$



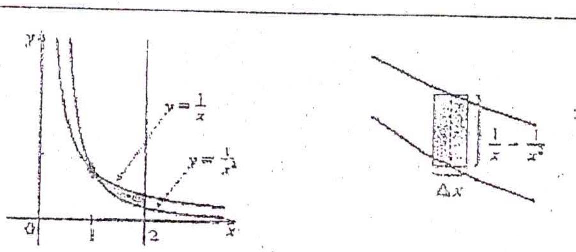
e)

$A = \int_1^2 \left(\frac{1}{x} - \frac{1}{x^2}\right) dx = \left[\ln x + \frac{1}{x}\right]_1^2$

$= \left(\ln 2 + \frac{1}{2}\right) - (\ln 1 + 1)$

$= \ln 2 - \frac{1}{2} \approx 0{,}19$

$x^{-2} =$

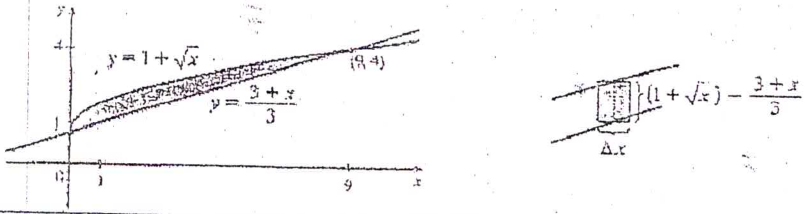


f)

$1 + \sqrt{x} = \frac{3 + x}{3} = 1 + \frac{x}{3} \Rightarrow \sqrt{x} = \frac{x}{3} \Rightarrow x = \frac{x^2}{9} \Rightarrow 9x - x^2 = 0 \Rightarrow x(9 - x) = 0 \Rightarrow x = 0$ or 9, so

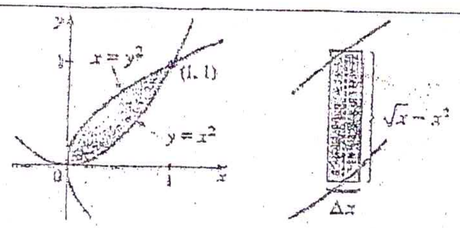
$$A = \int_0^9 \left[(1 + \sqrt{x}) - \left(\frac{3 + x}{3}\right)\right] dx = \int_0^9 \left[(1 + \sqrt{x}) - \left(1 + \frac{x}{3}\right)\right] dx$$

$$= \int_0^9 \left(\sqrt{x} - \frac{1}{3}x\right) dx = \left[\frac{2}{3}x^{3/2} - \frac{1}{6}x^2\right]_0^9 = 18 - \frac{27}{2} = \frac{9}{2}$$



g)

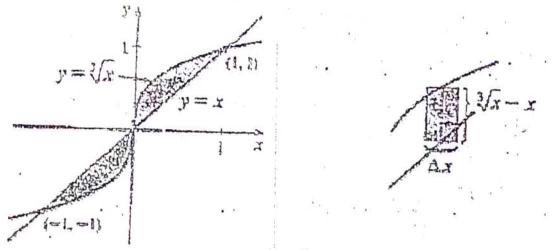
$$A = \int_0^1 (\sqrt{x} - x^2)\,dx = \left[\tfrac{2}{3}x^{3/2} - \tfrac{1}{3}x^3\right]_0^1 = \tfrac{2}{3} - \tfrac{1}{3} = \tfrac{1}{3}$$



h)

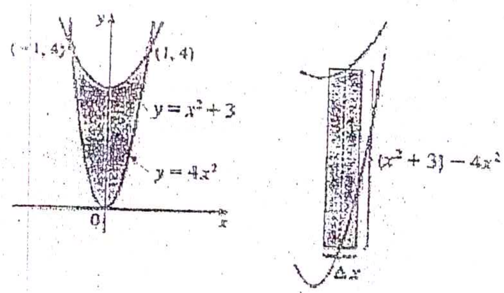
$$A = \int_{-1}^{1} |\sqrt[3]{x} - x|\,dx = \int_{-1}^{0} (x - \sqrt[3]{x})\,dx + \int_0^1 (\sqrt[3]{x} - x)\,dx = 2\int_0^1 (x^{1/3} - x)\,dx \quad \text{[by symmetry]}$$

$$= 2\left[\tfrac{3}{4}x^{4/3} - \tfrac{1}{2}x^2\right]_0^1 = 2\left(\tfrac{3}{4} - \tfrac{1}{2}\right) = \tfrac{1}{2}$$



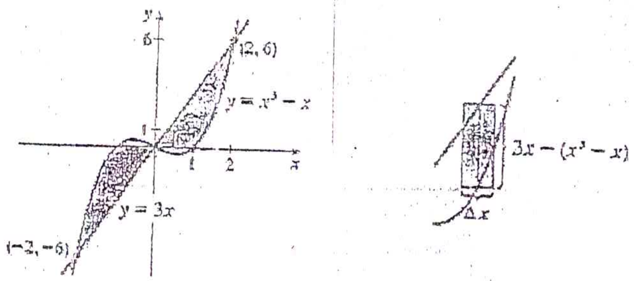
i)

$$A = \int_{-1}^{1} \left[(x^2 + 3) - 4x^2\right] dx = 2\int_0^1 (3 - 3x^2)\,dx = 2\left[3x - x^3\right]_0^1 = 2(3 - 1) = 4$$

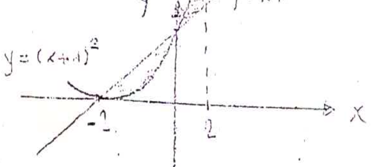


j)

$$A = \int_{-2}^{2} |3x - (x^3 - x)|\,dx = 2\int_0^2 |3x - (x^3 - x)|\,dx = 2\int_0^2 (4x - x^3)\,dx = 2\left[2x^2 - \tfrac{1}{4}x^4\right]_0^2 = 2(8 - 4) = 8$$



K) $y = x + 1 \qquad y = (x+1)^2 \qquad x = -1 \quad x = 2$



$$A = \int_{-1}^{0} \left[(x+1) - (x+1)^2\right] dx + \int_0^2 \left[(x+1)^2 - (x+1)\right] dx$$

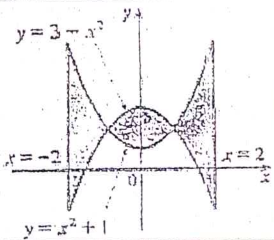
$$A = \left[\frac{x^2}{2} + x - \frac{(x+1)^3}{3}\right]_{-1}^{0} + \left[\frac{(x+1)^3}{3} - \frac{x^2}{2} - x\right]_0^2$$

$$A = -1/3 - 1/2 + 1 + 3^3/3 - 4/2 - 2 - 1/3$$

$$A = -2/3 + 1/2 + 9 - 4 = 5 - 2/3 + 1/2 = \frac{13}{3} + \frac{1}{2} = \frac{26 + 3}{6} = \frac{29}{6}$$

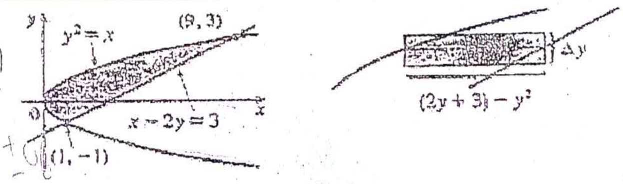
l)

$$A = \int_{-2}^{-1}\left[(x^2+1)-(3-x^2)\right]dx + \int_{-1}^{1}\left[(3-x^2)-(x^2+1)\right]dx + \int_{1}^{2}\left[(x^2+1)-(3-x^2)\right]dx$$
$$= \int_{-2}^{-1}(2x^2-2)\,dx + \int_{-1}^{1}(2-2x^2)\,dx + \int_{1}^{2}(2x^2-2)\,dx$$
$$= 2\int_0^1 (2-2x^2)\,dx + 2\int_1^2 (2x^2-2)\,dx \quad \text{[by symmetry]}$$
$$= 2\left[2x-\tfrac{2}{3}x^3\right]_0^1 + 2\left[\tfrac{2}{3}x^3-2x\right]_1^2 = 2\left(2-\tfrac{2}{3}\right) + 2\left(\tfrac{16}{3}-4\right)\boxed{-2\left(\tfrac{2}{3}-2\right)} = 8$$



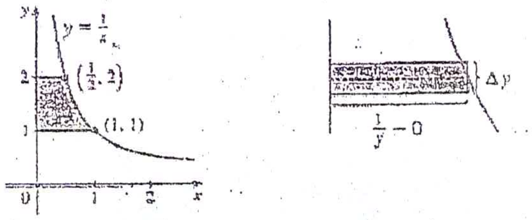
m)

$$A = \int_{-1}^{3}\left[(2y+3)-y^2\right]dy = \left[y^2+3y-\tfrac{1}{3}y^3\right]_{-1}^{3} = (9+9-9)-\left(1-3+\tfrac{1}{3}\right) = \tfrac{32}{3}$$



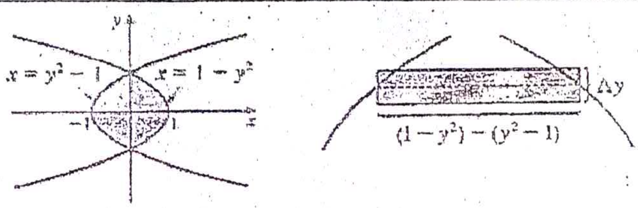
n)

$$A = \int_1^2 (1/y)\,dy = [\ln y]_1^2 = \ln 2 - \ln 1 = \ln 2 \approx 0.69$$



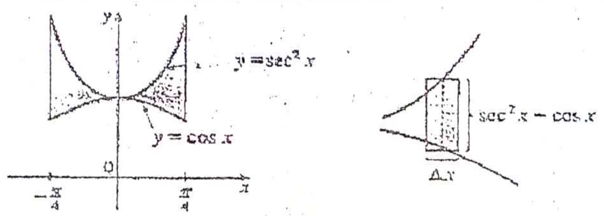
o)

$$A = \int_{-1}^{1}\left[(1-y^2)-(y^2-1)\right]dy = \int_{-1}^{1} 2(1-y^2)\,dy = 4\int_0^1 (1-y^2)\,dy = 4\left[y-\tfrac{1}{3}y^3\right]_0^1 = 4\left(1-\tfrac{1}{3}\right) = \tfrac{8}{3}$$



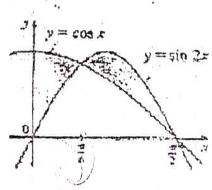
p)

$$A = \int_{-\pi/4}^{\pi/4}(\sec^2 x - \cos x)\,dx = 2\int_0^{\pi/4}(\sec^2 x - \cos x)\,dx = 2[\tan x - \sin x]_0^{\pi/4}$$
$$= 2\left(1-\tfrac{1}{\sqrt{2}}\right) = 2-\sqrt{2}$$
$$= 2\left(1-\tfrac{1}{\sqrt{2}}\right) = 2-\sqrt{2} \approx 0.59$$



q)

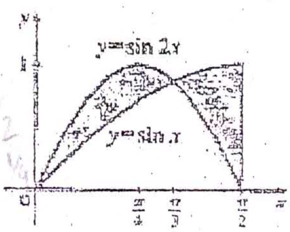
$$A = \int_0^{\pi/6}(\cos x - \sin 2x)\,dx + \int_{\pi/6}^{\pi/2}(\sin 2x - \cos x)\,dx$$
$$= \left[\sin x + \tfrac{1}{2}\cos 2x\right]_0^{\pi/6} + \left[-\tfrac{1}{2}\cos 2x - \sin x\right]_{\pi/6}^{\pi/2}$$
$$= \tfrac{1}{2} + \tfrac{1}{2}\cdot\tfrac{1}{2} - \left(0+\tfrac{1}{2}\cdot 1\right) + \left(\tfrac{1}{2}-1\right) - \left(-\tfrac{1}{2}\cdot\tfrac{1}{2}-\tfrac{1}{2}\right) = \tfrac{1}{2}$$



r)

$$A = \int_0^{\pi/3}(\sin 2x - \sin x)\,dx + \int_{\pi/3}^{\pi/2}(\sin x - \sin 2x)\,dx$$
$$= \underbrace{\left[-\tfrac{1}{2}\cos 2x + \cos x\right]_0^{\pi/3}}_{3/4} + \underbrace{\left[\tfrac{1}{2}\cos 2x - \cos x\right]_{\pi/3}^{\pi/2}}_{-1/2}$$
$$= \left[-\tfrac{1}{2}\left(-\tfrac{1}{2}\right)+\tfrac{1}{2}\right] - \left(-\tfrac{1}{2}+1\right)$$
$$+ \underbrace{\left(-\tfrac{1}{2}-0\right)}_{-1/2} - \underbrace{\left[\tfrac{1}{2}\left(-\tfrac{1}{2}\right)-\tfrac{1}{2}\right]}_{+\frac{3}{4}} = \tfrac{1}{2}$$

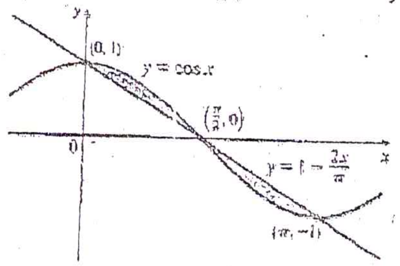
$\frac{6}{4} - 1 = \frac{2}{4} = \frac{1}{2}$

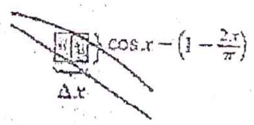


s)

$$A=\int_0^{\pi}\left|\cos x-\left(1-\frac{2x}{\pi}\right)\right|dx=2\int_0^{\pi/2}\left[\cos x-\left(1-\frac{2x}{\pi}\right)\right]dx=2\int_0^{\pi/2}\left(\cos x-1+\frac{2x}{\pi}\right)dx$$

$$=2\left[\sin x-x+\tfrac{1}{\pi}x^2\right]_0^{\pi/2}=2\left[\left(1-\tfrac{\pi}{2}+\tfrac{1}{\pi}\cdot\tfrac{\pi^2}{4}\right)-0\right]=2\left(1-\tfrac{\pi}{2}+\tfrac{\pi}{4}\right)=2-\tfrac{\pi}{2}$$

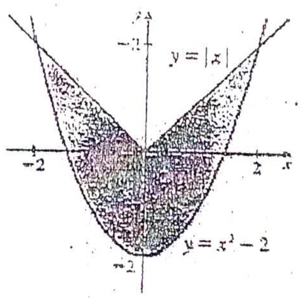


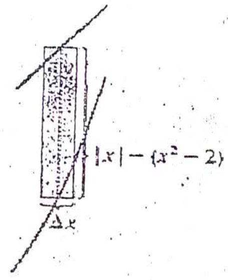


t)

$$\int_{-2}^{2}\left[|x|-(x^2-2)\right]dx=2\int_0^2\left[x-(x^2-2)\right]dx=2\int_0^2(x-x^2+2)\,dx=2\left[\tfrac12x^2-\tfrac13x^3+2x\right]_0^2$$

$$=2\left(2-\tfrac83+4\right)=\tfrac{20}{3}$$





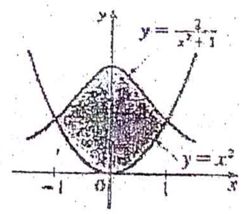
u)

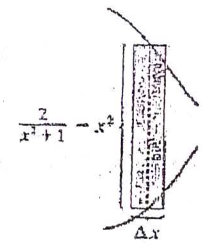
$$A=\int_{-1}^{1}\left(\frac{2}{x^2+1}-x^2\right)dx$$

$$=2\int_0^1\left(\frac{2}{x^2+1}-x^2\right)dx$$

$$=2\left[2\tan^{-1}x-\tfrac13x^3\right]_0^1=2\left(2\cdot\tfrac{\pi}{4}-\tfrac13\right)$$

$$=\pi-\tfrac23\approx 2.47$$





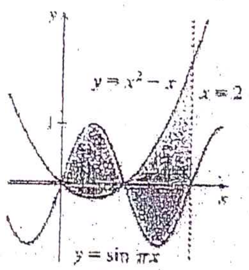
v)

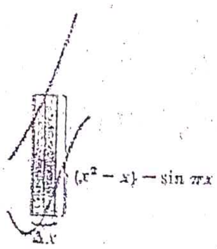
$$A=\int_0^1\left[\sin\pi x-(x^2-x)\right]dx+\int_1^2\left[(x^2-x)-\sin\pi x\right]dx$$

$$=\left[-\tfrac{1}{\pi}\cos\pi x-\tfrac13x^3+\tfrac12x^2\right]_0^1+\left[\tfrac13x^3-\tfrac12x^2+\tfrac1\pi\cos\pi x\right]_1^2$$

$$=\left(\tfrac1\pi-\tfrac13+\tfrac12\right)-\left(-\tfrac1\pi\right)+\left(\tfrac83-2+\tfrac1\pi\right)-\left(\tfrac13-\tfrac12-\tfrac1\pi\right)$$

$$=\tfrac4\pi+1$$

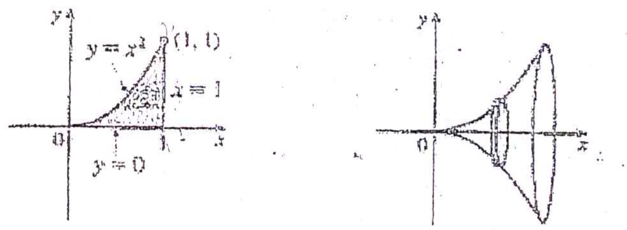


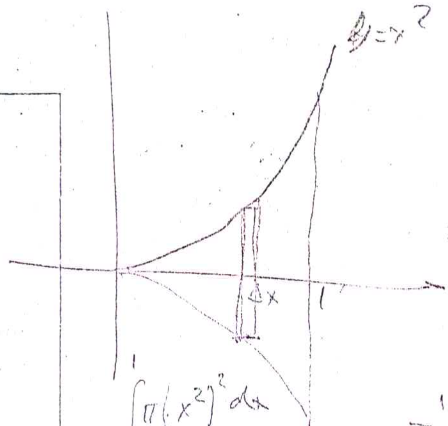


$(z^2)^2 = z^4$

6.a)

$$V = \int_0^1 A(x)\,dx = \int_0^1 \pi\,(x^2)^2\,dx = \pi\int_0^1 x^4\,dx = \pi\left[\tfrac{1}{5}x^5\right]_0^1 = \tfrac{\pi}{5}$$

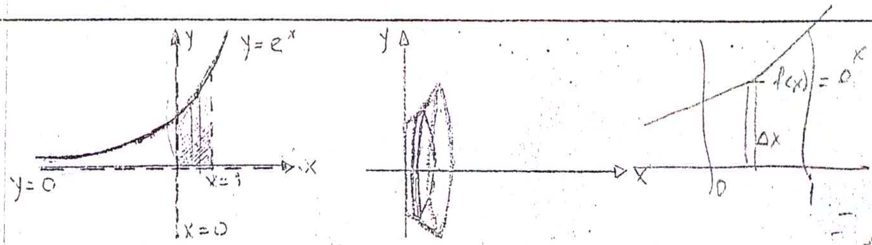




$$\int_0^1 \pi (x^2)^2\,dx$$

$$\int_0^1 x^4 \pi\,dx = \left.\frac{\pi x^5}{5}\right]_0^1 = \frac{\pi}{5}$$

b)

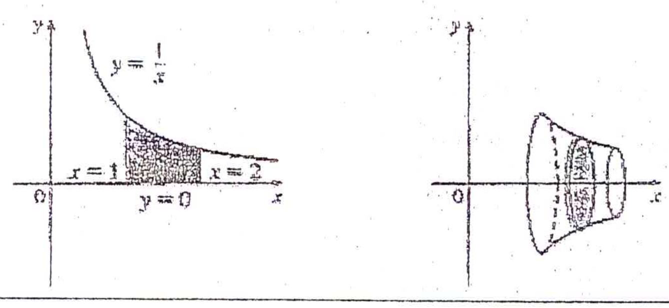


$$\int_0^1 \pi (e^x)^2\,dx = \int_0^1 \pi \cdot e^{2x}\,dx$$

$$V = \int_0^1 \pi\,(e^x)^2\,dx = \left(\frac{e^{2x}}{2}\cdot\pi\right)\Big|_0^1 = \frac{e^2\pi}{2} - \frac{\pi}{2} = \frac{\pi}{2}\left(e^2 - 1\right)$$

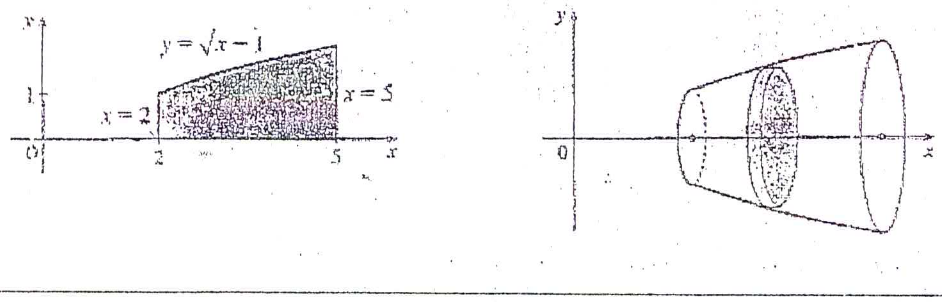
c)

$$V = \int_1^2 A(x)\,dx = \int_1^2 \pi\left(\frac{1}{x}\right)^2 dx = \pi\int_1^2 \frac{1}{x^2}\,dx = \pi\left[-\frac{1}{x}\right]_1^2 = \pi\left[-\tfrac{1}{2} - (-1)\right] = \tfrac{\pi}{2}$$



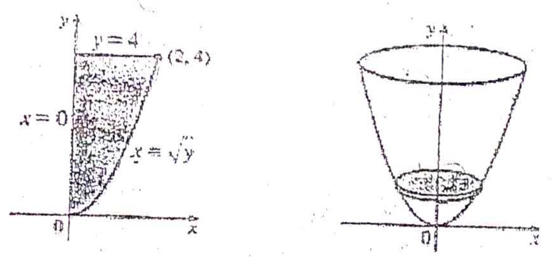
d)

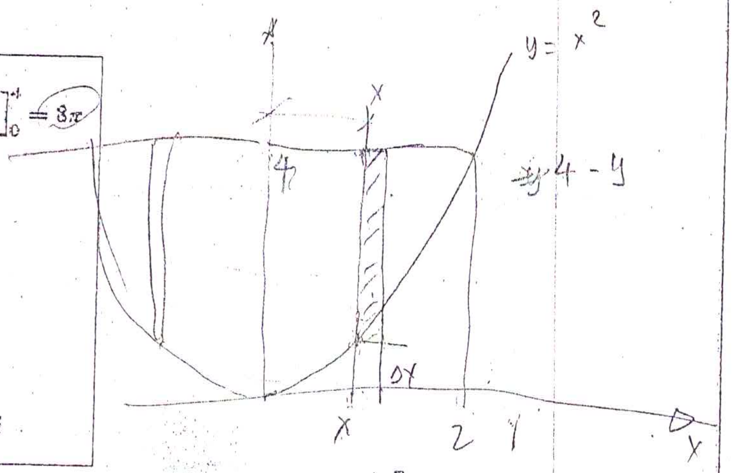
$$V = \int_2^5 A(x)\,dx = \int_2^5 \pi\,(x-1)\,dx = \pi\left[\tfrac{1}{2}x^2 - x\right]_2^5 = \pi\left(\tfrac{25}{2} - 5 - \tfrac{4}{2} + 2\right) = \tfrac{15}{2}\pi$$



e)

$$V = \int_0^4 A(y)\,dy = \int_0^4 \pi\left(\sqrt{y}\right)^2 dy = \pi\int_0^4 y\,dy = \pi\left[\tfrac{1}{2}y^2\right]_0^4 = 8\pi$$





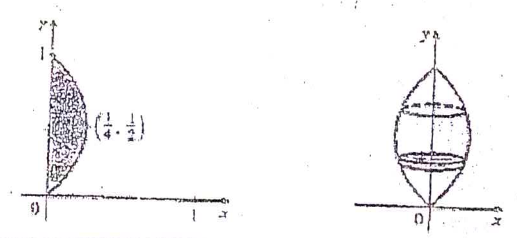
$$= \sum_0^2 (4-x^2)\,2\pi x\,\Delta x$$

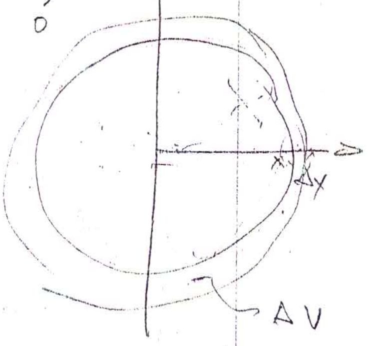
$$\int_0^2 8\pi x - 2\pi x^3\,dx$$

f)

$$V = \int_0^1 A(y)\,dy = \int_0^1 \pi\left(y-y^2\right)^2 dy = \pi\int_0^1 \left(y^4-2y^3+y^2\right)dy = \pi\left[\tfrac{1}{5}y^5-\tfrac{1}{2}y^4+\tfrac{1}{3}y^3\right]_0^1$$

$$= \pi\left(\tfrac{1}{5}-\tfrac{1}{2}+\tfrac{1}{3}\right) = \tfrac{\pi}{30}$$





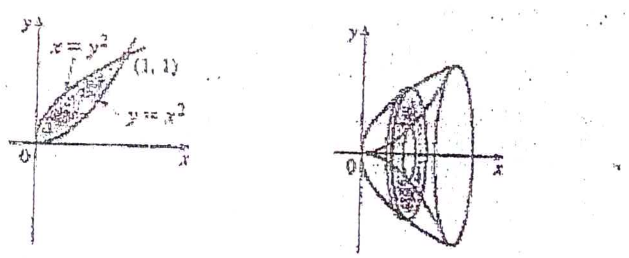
$$\Delta V = 2\pi r\cdot(4-y)\cdot\Delta x$$

$$\frac{8\pi x^2}{2} - \frac{2\pi x^4}{4}\Big]_0^2$$

$$16\pi - 8\pi = 8\pi$$

g)

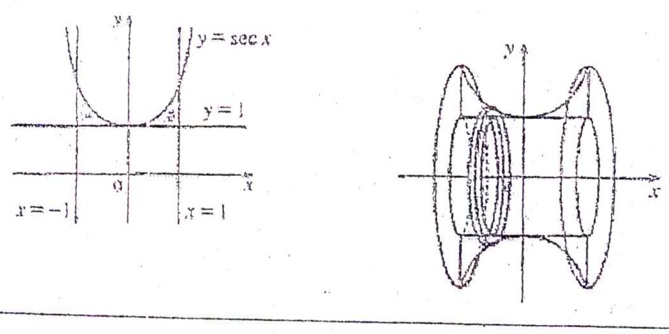
$$V = \int_0^1 A(x)\,dx = \pi\int_0^1 \left(x-x^4\right)dx = \pi\left[\tfrac{1}{2}x^2-\tfrac{1}{5}x^5\right]_0^1 = \pi\left(\tfrac{1}{2}-\tfrac{1}{5}\right) = \tfrac{3\pi}{10}$$



h)

$$V = \int_{-1}^1 A(x)\,dx = \int_{-1}^1 \pi\left(\sec^2 x - 1\right)dx = 2\pi\int_0^1\left(\sec^2 x - 1\right)dx = 2\pi\left[\tan x - x\right]_0^1 = 2\pi(\tan 1 - 1)$$

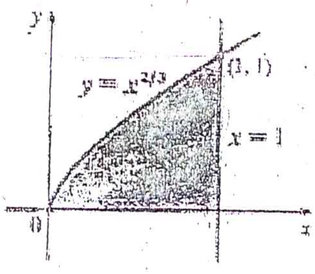
$$\approx 3.5023$$

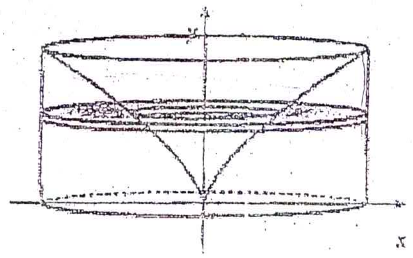


i)

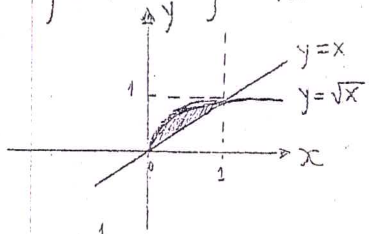
$$A(y) = \pi(1)^2 - \pi\left(y^{3/2}\right)^2 = \pi\left(1 - y^3\right)$$

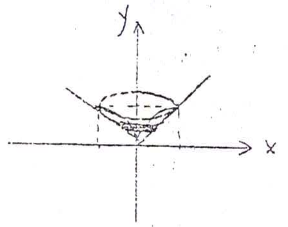
$$V = \int_0^1 A(y)\,dy = \pi \int_0^1 \left(1 - y^3\right) dy = \pi\left[y - \tfrac{1}{4}y^4\right]_0^1 = \tfrac{3}{4}\pi$$





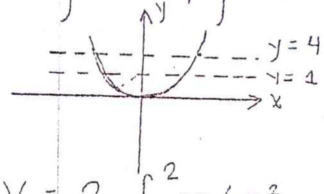
j) $y = x$, $y = \sqrt{x}$ ao redor do eixo y

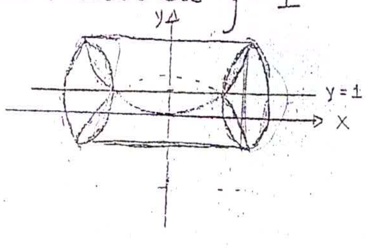




$$V = \int_0^1 \pi(y^2) - \pi(y^4)\,dy = \pi\left(\frac{y^3}{3} - \frac{y^5}{5}\right)\Big|_0^1 = \pi\left(\frac{1}{3} - \frac{1}{5}\right) = \frac{2\pi}{15}$$

k) $y = x^2$, $y = 4$ ao redor de $y = 1$

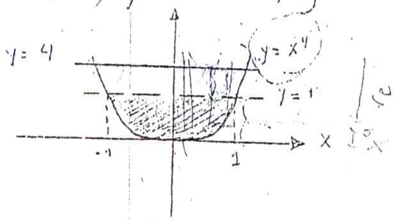




raio externo:

$$V = 2\int_0^2 \pi\,(3^2 -$$

l) $y = x^4$, $y = 1$ ao redor de $y = 4$



raio externo: $4 - x^4$

raio interno: $4 - 1 = 3$

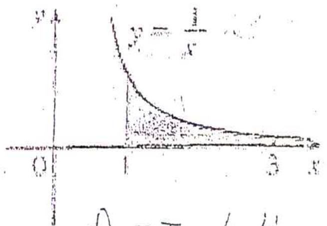
$$A = \pi\left[(4 - x^4)^2 - 9\right]$$

$$V = 2\int_0^1 \pi\,(4 - x^4)^2 - 9\,dx = 2\pi\int_0^1 (16 - 8x^4 + x^8 - 9)\,dx$$

$$V = 2\pi\int_0^1 (7 - 8x^4 + x^8)\,dx = 2\pi\left(7x - \frac{8x^5}{5} + \frac{x^9}{9}\right)\Big|_0^1 = 2\pi\left(7 - \frac{8}{5} + \frac{1}{9}\right)$$

$$\begin{array}{r} \overset{3}{1}44 \\ -\ 27 \\ \hline 117 \end{array}$$

X ... ao redor de $y = 2$



raio externo : $2 - 0$

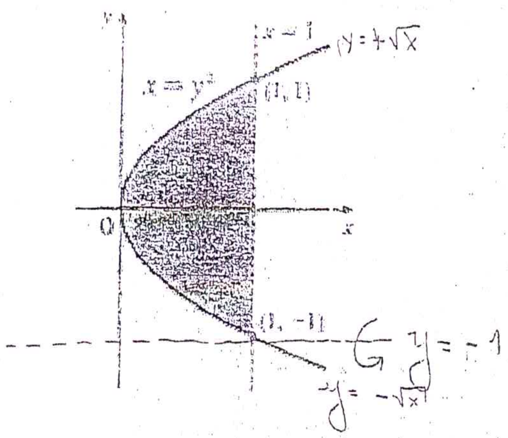
raio interno : $2 - 1/x$

$A = \pi\left(4 - (2 - 1/x)^2\right) = \pi\left(4 - 4 + 4/x - 1/x^2\right)$

$A = \pi\left(\frac{4}{x} - \frac{1}{x^2}\right) \therefore V = \int_1^3 \pi\left(\frac{4}{x} - \frac{1}{x^2}\right)dx = \pi\left(4\ln x + \frac{1}{3x^3}\right)\Big|_1^3$

$V = \pi\left(4\ln 3 + \frac{1}{81} - \ln 1 - \frac{1}{3}\right) = \pi\left(4\ln 3 + 26/3\right)$

n) $x = y^2 \quad x = 1$ ao redor de $y = -1$



$A = \pi($

o) $y = x \;,\; x = 1$ ao redor de $x = 1$

Raio : $1 - y$

$A(x) = \pi(1-y)^2 = \pi(1 - 2y + y^2)$

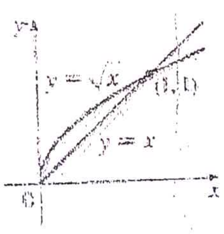
$V = \int_0^1 \pi(1 - 2y + y^2)\,dy$

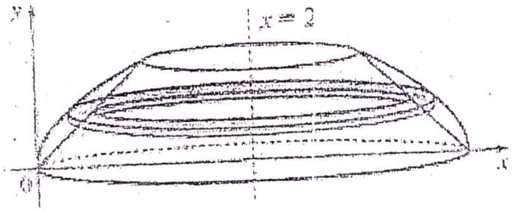
$V = \pi\left(y - y^2 + y^3/3\right)\Big|_0^1 = \pi\left(1 - 1 + 1/3\right) = \pi/3$

p)

5. $V=\int_0^1 \pi\left[(2-y^2)^2-(2-y)^2\right]dy=\pi\int_0^1\left(4-4y^2+y^4-4+4y-y^2\right)dy$

$=\pi\int_0^1\left(y^4-5y^2+4y\right)dy=\pi\left[\frac{1}{5}y^5-\frac{5}{3}y^3+2y^2\right]_0^1=\pi\left(\frac{1}{5}-\frac{5}{3}+2\right)=\frac{8}{15}\pi$



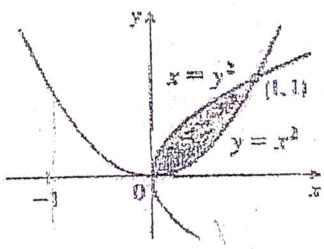


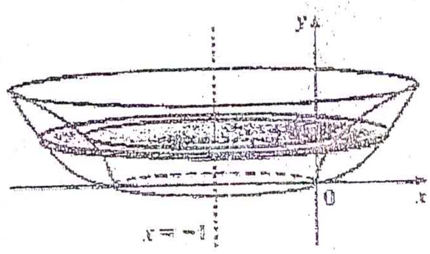
q)

$V=\int_0^1 \pi\left\{\left[\sqrt{y}-(-1)\right]^2-\left[y^2-(-1)\right]^2\right\}dy=\int_0^1 \pi\left[\left(\sqrt{y}+1\right)^2-\left(y^2+1\right)^2\right]dy$

$=\pi\int_0^1\left(y+2\sqrt{y}+1-y^4-2y^2-1\right)dy=\pi\int_0^1\left(y+2\sqrt{y}-y^4-2y^2\right)dy$

$=\pi\left[\frac{1}{2}y^2+\frac{4}{3}y^{3/2}-\frac{1}{5}y^5-\frac{2}{3}y^3\right]_0^1=\pi\left(\frac{1}{2}+\frac{4}{3}-\frac{1}{5}-\frac{2}{3}\right)=\frac{29}{30}\pi$





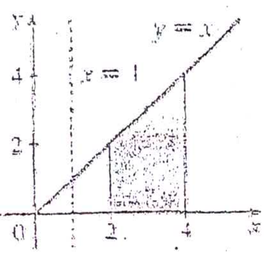
r)

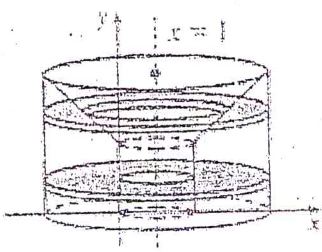
$16-8y+y^2-4+4y+y^2$

$V=\int_0^2 A(y)\,dy=\pi\int_0^2\left[(4-1)^2-(2-1)^2\right]dy+\pi\int_2^4\left[(4-1)^2-(y-1)^2\right]dy$

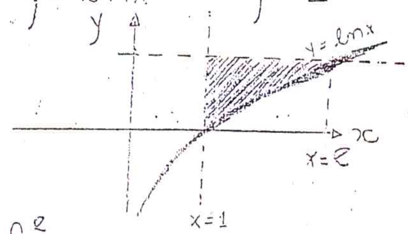
$=\pi\left[8y\right]_0^2+\pi\int_2^4\left(8+2y-y^2\right)dy=16\pi+\pi\left[8y+y^2-\frac{1}{3}y^3\right]_2^4$

$=16\pi+\pi\left[\left(32+16-\frac{64}{3}\right)-\left(16+4-\frac{8}{3}\right)\right]=\frac{76}{3}\pi$



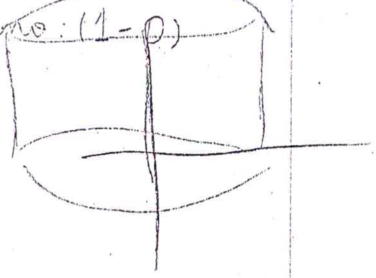


7. a) $y = \ln x \qquad y = 1 \qquad x = 1$ ao redor do eixo x.



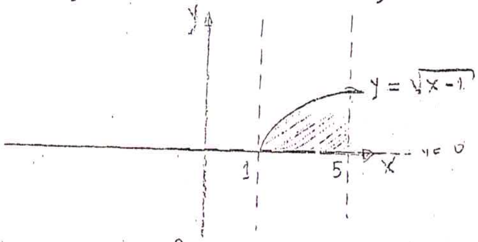
Raio interno : $\ln x$

Raio externo : $(1-0)$



$$V = \int_1^e \pi(1^2 - \ln^2 x)\,dx = \pi \int_1^e (1 - \ln^2 x)\,dx$$

b) $y = \sqrt{x-1}$, $y=0$, $x=5$ ao redor do eixo y



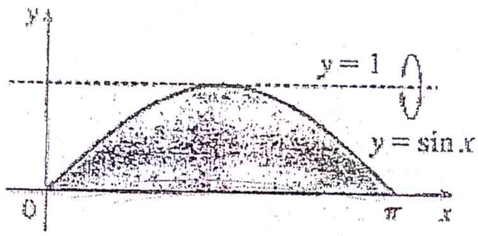
Raio externo = $5 - 0 = 5$

Raio interno = $y^2 + 1 - 0 = y^2 + 1$

$$V = \int_0^2 (25 - (y^2+1)^2)\,dy$$

c)

$$V = \pi \int_0^\pi \left[(1-0)^2 - (1-\sin x)^2\right] dx = \pi \int_0^\pi \left[1^2 - (1-\sin x)^2\right] dx$$

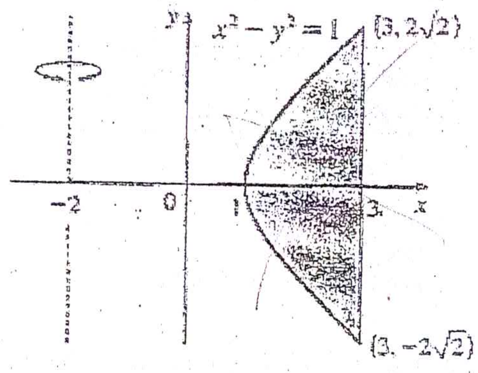


d)

$$V = \pi \int_0^\pi \left[(\sin x + 2)^2 - 2^2\right] dx$$

e)

$$V = \pi \int_{-\sqrt{8}}^{\sqrt{8}} \left\{[3-(-2)]^2 - \left[\sqrt{y^2+1} - (-2)\right]^2\right\} dy = \pi \int_{-2\sqrt{2}}^{2\sqrt{2}} \left[5^2 - \left(\sqrt{+y^2} + 2\right)^2\right] dy$$



f)

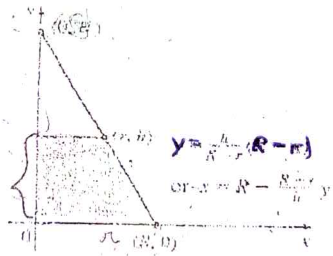
$$V = \pi \int_0^{7/2} \left[\left[4-(y-1)^2+5\right]^2 - \left(3 - \tfrac{3}{2}y + 5\right)^2\right] dy = \pi \int_0^{7/2} \left[\left[9-(y-1)\;\right]^2 - \left(8 - \tfrac{3}{2}y\right)^2\right] dy$$

$$\left(\frac{Rh}{R-h} - \frac{hx}{R-r}\right)^2$$

$$y = \frac{h}{R-r}(R-x)$$

8. a)

$$V = \pi\int_0^h \left(R - \frac{R-r}{h}y\right)^2 dy$$
$$= \pi\int_0^h \left[R^2 - \frac{2R(R-r)}{h}y + \left(\frac{R-r}{h}\right)^2 y^2\right]dy$$
$$= \pi\left[R^2 y - \frac{R(R-r)}{h}y^2 + \frac{1}{3}\left(\frac{R-r}{h}\right)^2 y^3\right]_0^h$$
$$= \pi\left[R^2 h - R(R-r)h + \tfrac{1}{3}(R-r)^2 h\right]$$
$$= \tfrac{1}{3}\pi h\left[3Rr + (R^2 - 2Rr + r^2)\right] = \tfrac{1}{3}\pi h\left(R^2 + Rr + r^2\right)$$



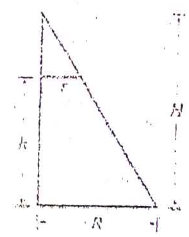
$y =$

Another Solution: $\frac{H}{R} = \frac{H-h}{r}$ by similar triangles. Therefore, $Hr = HR - hR$

$\Rightarrow hR = H(R-r) \Rightarrow H = \frac{hR}{R-r}$. Now

$V = \frac{1}{3}\pi R^2 H - \frac{1}{3}\pi r^2 (H-h)$ (by Exercise 45)

$$= \tfrac{1}{3}\pi R^2 \frac{hR}{R-r} - \tfrac{1}{3}\pi r^2 \frac{rh}{R-r} = \frac{\pi h}{3}\frac{R^3 - r^3}{R-r} = \tfrac{1}{3}\pi h\left(R^2 + Rr + r^2\right)$$
$$= \tfrac{1}{3}\left[\pi R^2 + \pi r^2 + \sqrt{(\pi R^2)(\pi r^2)}\right]h = \tfrac{1}{3}\left(A_1 + A_2 + \sqrt{A_1 A_2}\right)h$$



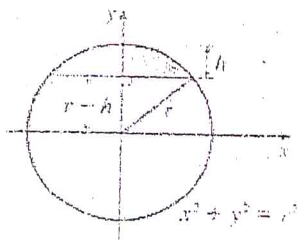
where A_1 and A_2 are the areas of the bases of the frustum. (See Exercise 48 for a related result.)

b)

$x^2 + y^2 = r^2 \Rightarrow x^2 = r^2 - y^2 \Rightarrow$

$$V = \pi\int_{r-h}^{r}\left(r^2 - y^2\right)dy = \pi\left[r^2 y - \frac{y^3}{3}\right]_{r-h}^{r}$$
$$= \pi\left(\left[r^3 - \frac{r^3}{3}\right] - \left[r^2(r-h) - \frac{(r-h)^3}{3}\right]\right)$$
$$= \pi\left(\tfrac{2}{3}r^3 - \tfrac{1}{3}(r-h)\left[3r^2 - (r-h)^2\right]\right)$$
$$= \tfrac{1}{3}\pi\left(2r^3 - (r-h)\left[3r^2 - (r^2 - 2rh + h^2)\right]\right)$$
$$= \tfrac{1}{3}\pi h^2(3r - h),\text{ or, equivalently, } \pi h^2\left(r - \frac{h}{3}\right)$$

$\pi \cdot 1\left(2 - \frac{1}{3}\right)$



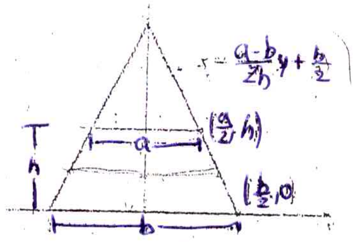
$2\pi - \frac{\pi}{3} = \frac{5}{3}\pi$

$\pi\left(2 - \frac{1}{3}\right)\frac{6}{3}\pi - \frac{1}{3}\pi$

c)

$$V = \int_0^h A(y)\,dy = \int_0^h \left[2\left(\frac{a-b}{2h}y + \frac{b}{2}\right)\right]\left[2\left(\frac{a-b}{2h}y + \frac{b}{2}\right)\right]dy$$
$$= \int_0^h \left[\frac{a-b}{h}y + b\right]^2 dy = \int_0^h \left[\frac{(a-b)^2}{h^2}y^2 + \frac{2b(a-b)}{h}y + b^2\right]dy$$
$$= \left[\frac{(a-b)^2}{3h^2}y^3 + \frac{b(a-b)}{h}y^2 + b^2 y\right]_0^h$$
$$= \tfrac{1}{3}(a-b)^2 h + b(a-b)h + b^2 h = \tfrac{1}{3}\left(a^2 - 2ab + b^2 + 3ab\right)h$$
$$= \tfrac{1}{3}\left(a^2 + ab + b^2\right)h$$

[Note that this can be written as $\frac{1}{3}\left(A_1 + A_2 + \sqrt{A_1 A_2}\right)h$, as in Exercise 46.]



$\frac{b/2}{H}$ $\frac{a/2}{h}$

$H = \frac{b}{2} \cdot \frac{h}{a/2}$

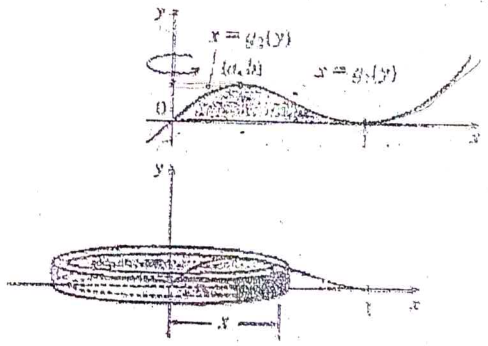
$h = 1$

$2 \cdot a$
$a \cdot b$

$\frac{1}{3}\left(9 + 27 + 81\right)1 =$

$\frac{1}{3} \cdot 117 = \frac{117}{3} = 39$

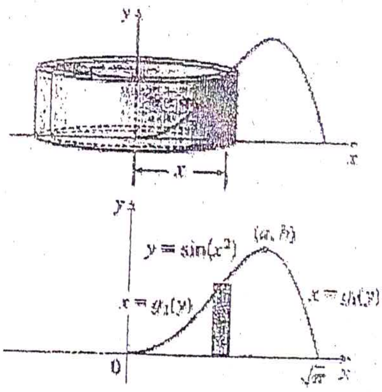
9.a)É inconveniente fatiar o sólido para obter o volume pois primeiro deveríamos encontrar o ponto de máximo local tendo de resolver equações de x em termos de y, o que envolveria a resolução de formas cúbicas. É mais simples encontrar o volume utilizando cascas cilíndricas. Circunferência: $2\pi x$; Altura: $x(x-1)^2$;



$$V = \int_0^1 2\pi x\,[x(x-1)^2]\,dx = 2\pi \int_0^1 (x^4 - 2x^3 + x^2)\,dx = 2\pi\left[\frac{x^5}{5} - 2\frac{x^4}{4} + \frac{x^3}{3}\right]_0^1 = \frac{\pi}{15}$$

10. Circunferência: $2\pi x$; Altura: $\mathrm{sen}(x^2)$;

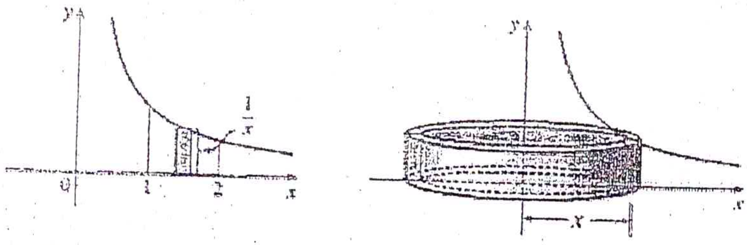
$x^3 - 2x^2 + 1$



$$V = \pi \int_0^{\pi} \sin u\,du = \pi\,[-\cos u]_0^{\pi}$$
$$= \pi\,[1 - (-1)] = 2\pi$$

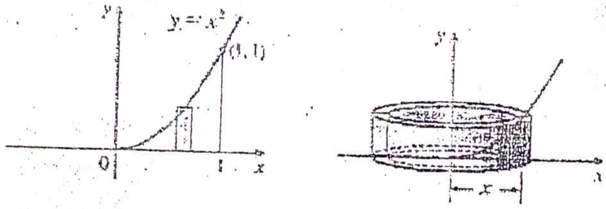
11.a)

$$V = \int_1^2 2\pi x \cdot \frac{1}{x}\,dx = 2\pi \int_1^2 1\,dx$$
$$= 2\pi\,[x]_1^2 = 2\pi\,(2-1) = 2\pi$$



b)

$$V = \int_0^1 2\pi x \cdot x^2\,dx = 2\pi \int_0^1 x^3\,dx$$
$$= 2\pi\left[\tfrac{1}{4}x^4\right]_0^1 = 2\pi \cdot \tfrac{1}{4} = \tfrac{\pi}{2}$$



$y' + 2y = 3x^2$

$p(x) = 2 \to 2x$ $\mu(x) = e^{2x}$ $y(x) = \frac{1}{e^{2x}}\left[\int 3x^2 \cdot e^{2x} + c\right]dy$
$g(x) = 3x^2$

c)

$y = e^{-x^2}$ $y = 0$ $x = 0$ $x = 1$

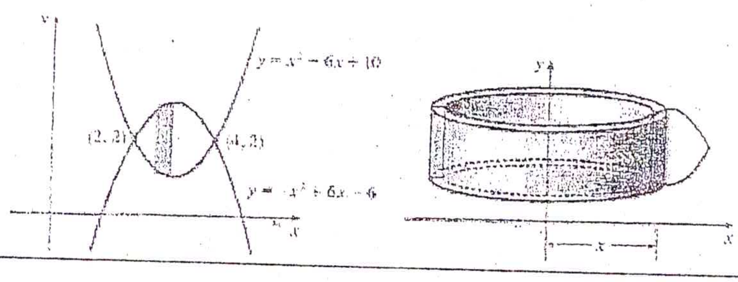
$V = \int_0^1 2\pi x\, e^{-x^2} dx = 2\pi \int_0^1 x e^{-x^2} dx$ $\begin{cases} u = x^2 \\ du = 2x\,dx \end{cases}$

$V = 2\pi \int_0^1 e^{-u} \frac{du}{2} = \pi \left(\frac{e^{-u}}{-1}\right)\Big|_0^1 = -\pi\left(e^{-1} - e^{0}\right)$

$V = \pi - \pi/e$

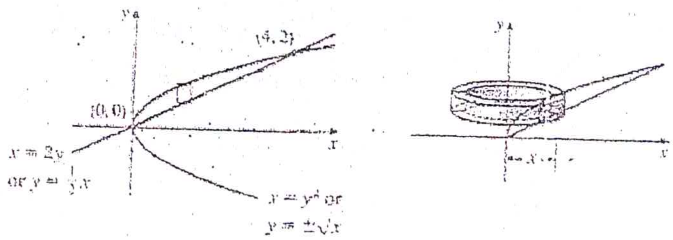
d)

$$V = \int_2^4 2\pi x\left[(-x^2+6x-6) - (x^2-6x+10)\right] dx = 2\pi \int_2^4 x(-2x^2+12x-16)\, dx$$
$$= 4\pi \int_2^4 (-x^3+6x^2-8x)\, dx = 4\pi\left[-\tfrac{1}{4}x^4 + 2x^3 - 4x^2\right]_2^4$$
$$= 4\pi\left[(-64+128-64) - (-4+16-16)\right] = 16\pi$$



e)

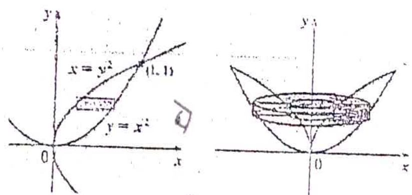
$$V = \int_0^4 2\pi x\left(\sqrt{x} - \tfrac{1}{2}x\right) dx$$
$$= 2\pi \int_0^4 x^{3/2}\, dx - \pi \int_0^4 x^2\, dx$$
$$= 2\pi\left[\tfrac{2}{5}x^{5/2}\right]_0^4 - \pi\left[\tfrac{1}{3}x^3\right]_0^4$$
$$= \tfrac{4}{5}\pi(32) - \tfrac{64}{3}\pi = \tfrac{64}{15}\pi$$



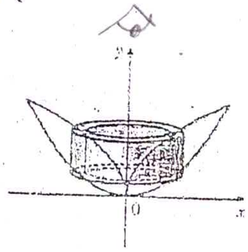
12.

Fatiamento: $y = \sqrt{x}$ $y = x^2$
$x = y^2$ $x = \sqrt{y}$

$$V = \int_0^1 \pi\left[(\sqrt{y})^2 - (y^2)^2\right] dy = \pi \int_0^1 (y - y^4)\, dy$$
$$= \pi\left[\tfrac{1}{2}y^2 - \tfrac{1}{5}y^5\right]_0^1 = \pi\left(\tfrac{1}{2} - \tfrac{1}{5}\right) = \tfrac{3\pi}{10}$$



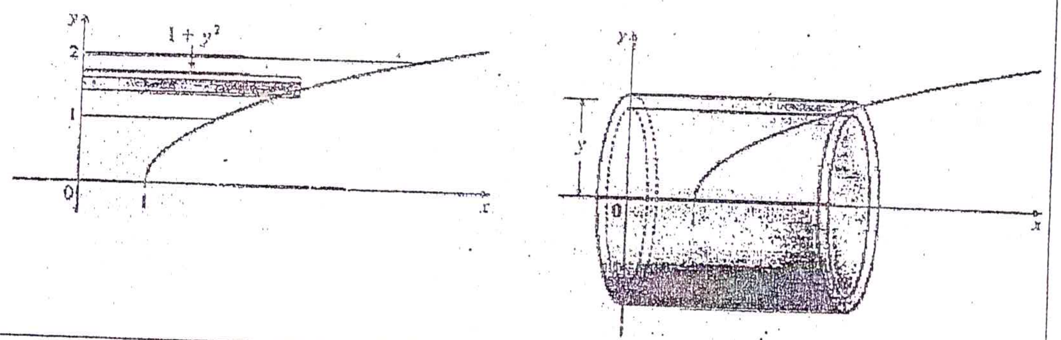
Cascas Cilíndricas:



$$V = \int_0^1 2\pi x(\sqrt{x} - x^2)\, dx = 2\pi \int_0^1 (x^{3/2} - x^3)\, dx$$
$$= 2\pi\left[\tfrac{2}{5}x^{5/2} - \tfrac{1}{4}x^4\right]_0^1 = 2\pi\left(\tfrac{2}{5} - \tfrac{1}{4}\right) = 2\pi\left(\tfrac{3}{20}\right) = \tfrac{3\pi}{10}$$

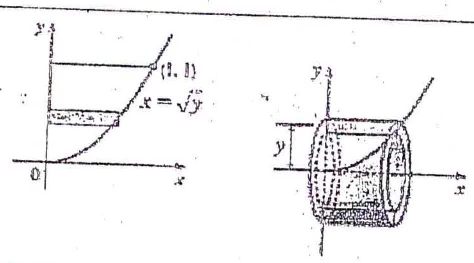
13.a)

$$V = \int_1^2 2\pi y\,(1+y^2)\,dy = 2\pi \int_1^2 (y+y^3)\,dy = 2\pi\left[\tfrac{1}{2}y^2 + \tfrac{1}{4}y^4\right]_1^2$$
$$= 2\pi\left[(2+4) - \left(\tfrac{1}{2}+\tfrac{1}{4}\right)\right] = 2\pi\left(\tfrac{21}{4}\right) = \tfrac{21\pi}{2}$$



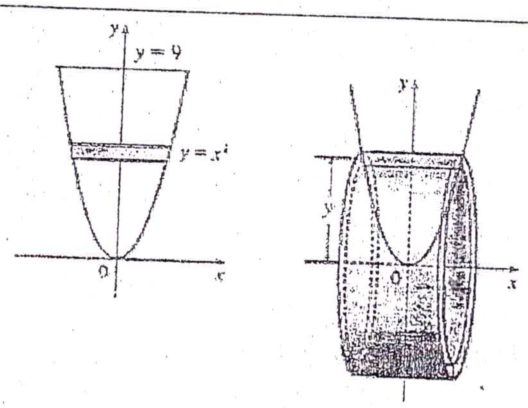
b)

$$V = \int_0^1 2\pi y\sqrt{y}\,dy = 2\pi\int_0^1 y^{3/2}\,dy$$
$$= 2\pi\left[\tfrac{2}{5}y^{5/2}\right]_0^1 = \tfrac{4\pi}{5}$$



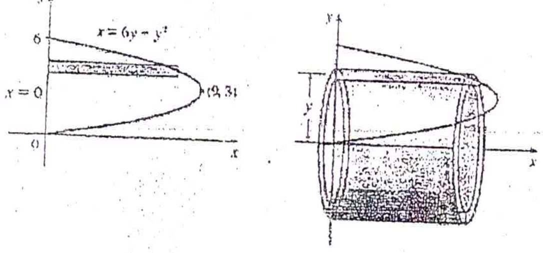
c)

$$V = \int_0^9 2\pi y\cdot 2\sqrt{y}\,dy$$
$$= 4\pi\int_0^9 y^{3/2}\,dy$$
$$= 4\pi\left[\tfrac{2}{5}y^{5/2}\right]_0^9$$
$$= \tfrac{8}{5}\pi\,(243-0)$$
$$= \tfrac{1944}{5}\pi$$



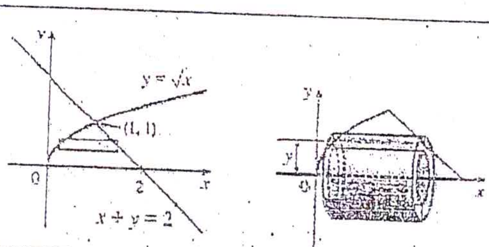
d)

$$V = \int_0^6 2\pi y\,(-y^2+6y)\,dy$$
$$= 2\pi\left[-\tfrac{1}{4}y^4 + 2y^3\right]_0^6$$
$$= 2\pi\,(-324+432) = 216\pi$$



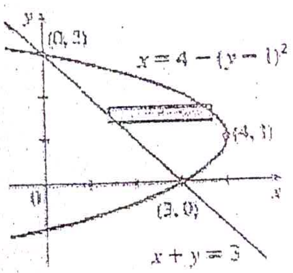
e)

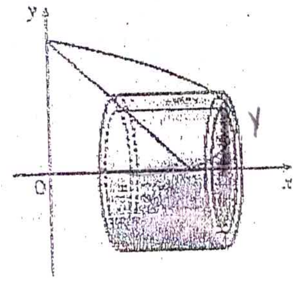
$$V = \int_0^1 2\pi y\left[(2-y) - y^2\right]dy$$
$$= 2\pi\left[y^2 - \tfrac{1}{3}y^3 - \tfrac{1}{4}y^4\right]_0^1$$
$$= 2\pi\left(1 - \tfrac{1}{3} - \tfrac{1}{4}\right) = \tfrac{5}{6}\pi$$



f)

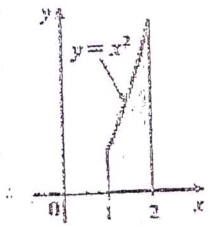
$$V = \int_0^3 2\pi y\left[4-(y-1)^2-(3-y)\right]dy$$
$$= 2\pi\int_0^3 y\left(-y^2+3y\right)dy$$
$$= 2\pi\int_0^3\left(-y^3+3y^2\right)dy = 2\pi\left[-\tfrac{1}{4}y^4+y^3\right]_0^3$$
$$= 2\pi\left(-\tfrac{81}{4}+27\right) = 2\pi\left(\tfrac{27}{4}\right) = \tfrac{27\pi}{2}$$

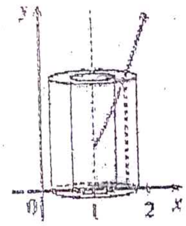




14.a)

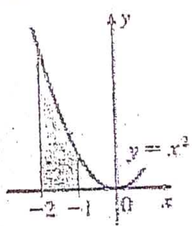
$$V = \int_1^2 2\pi(x-1)x^2\,dx = 2\pi\left[\tfrac{1}{4}x^4-\tfrac{1}{3}x^3\right]_1^2$$
$$= 2\pi\left[\left(4-\tfrac{8}{3}\right)-\left(\tfrac{1}{4}-\tfrac{1}{3}\right)\right] = \tfrac{17}{6}\pi$$

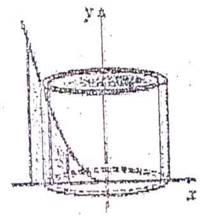




b)

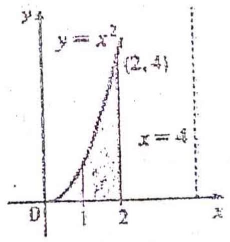
$$V = \int_{-2}^{-1} 2\pi(-x)\cdot x^2\,dx = 2\pi\left[-\tfrac{1}{4}x^4\right]_{-2}^{-1}$$
$$= 2\pi\left[\left(-\tfrac{1}{4}\right)-(-4)\right] = \tfrac{15}{2}\pi$$

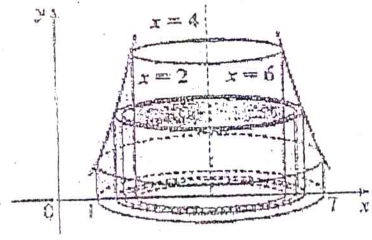




c)

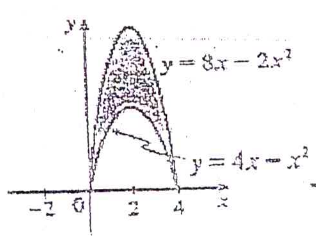
$$V = \int_1^2 2\pi(4-x)x^2\,dx = 2\pi\left[\tfrac{4}{3}x^3-\tfrac{1}{4}x^4\right]_1^2$$
$$= 2\pi\left[\left(\tfrac{32}{3}-4\right)-\left(\tfrac{4}{3}-\tfrac{1}{4}\right)\right] = \tfrac{67}{6}\pi$$

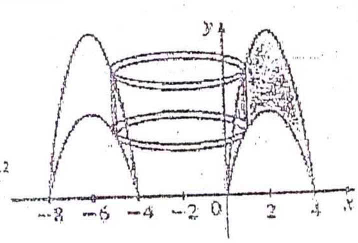




d)

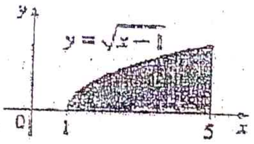
$$V = \int_0^4 2\pi(2+x)\left[\left(8x-2x^2\right)-\left(4x-x^2\right)\right]dx$$
$$= \int_0^4 2\pi(2+x)\left(4x-x^2\right)dx$$
$$= 2\pi\int_0^4\left(8x+2x^2-x^3\right)dx$$
$$= 2\pi\left[4x^2+\tfrac{2}{3}x^3-\tfrac{1}{4}x^4\right]_0^4$$
$$= 2\pi\left(64+\tfrac{128}{3}-64\right) = \tfrac{256}{3}\pi$$

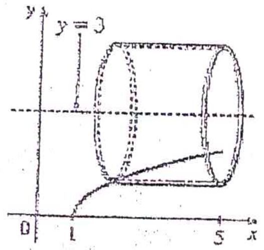




e) Não tem

$$V = \int_0^2 2\pi\,(3-y)\,(5-x)\,dy$$
$$= \int_0^2 2\pi\,(3-y)\,(5-y^2-1)\,dy$$
$$= \int_0^2 2\pi\,(12-4y-3y^2+y^3)\,dy$$
$$= 2\pi\left[12y-2y^2-y^3+\tfrac{1}{4}y^4\right]_0^2$$
$$= 2\pi\,(24-8-8+4) = 24\pi$$

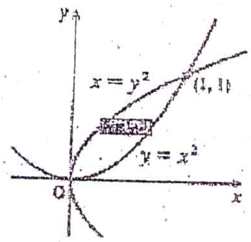


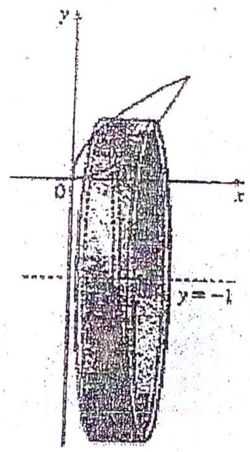


f)

$$V = \int_0^1 2\pi\,(y+1)\,(\sqrt{y}-y^2)\,dy$$
$$= 2\pi\int_0^1 (y^{3/2}+y^{1/2}-y^3-y^2)\,dy$$
$$= 2\pi\left[\tfrac{2}{5}y^{5/2}+\tfrac{2}{3}y^{3/2}-\tfrac{1}{4}y^4-\tfrac{1}{3}y^3\right]_0^1$$
$$= 2\pi\left(\tfrac{2}{5}+\tfrac{2}{3}-\tfrac{1}{4}-\tfrac{1}{3}\right) = 2\pi\left(\tfrac{29}{60}\right) = \tfrac{29\pi}{30}$$

$$\frac{6+10}{15} = \frac{16}{15} - \frac{7}{12} = \frac{192-105}{180} = \frac{87}{180}$$



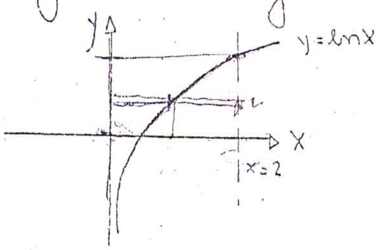


$y = \ln x$

$x = 2$

15.a)

$y = \ln x \qquad y = 0 \qquad x = 2$ ao redor do eixo y

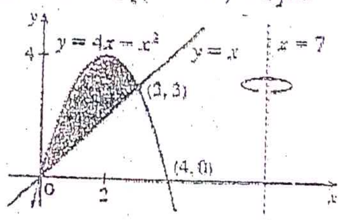


$$V = \int_0^{\ln 2} \pi\,(2^2 - \ln x)\,dx$$

$$V = \int_0^{\ln 2} \pi\,(4 - \ln x)\,dx$$

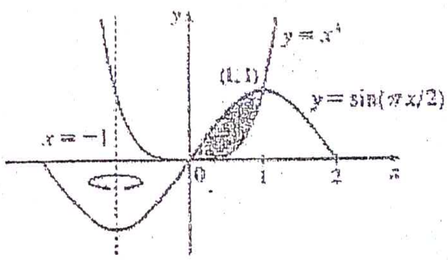
b)

$$V = \int_0^3 2\pi\,(7-x)\,[(4x-x^2)-x]\,dx$$



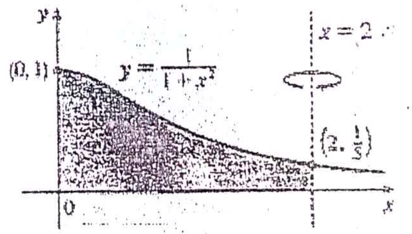
c)

$V = \int_0^1 2\pi\,[x - (-1)]\,\left(\sin \frac{\pi}{2}x - x^4\right) dx$



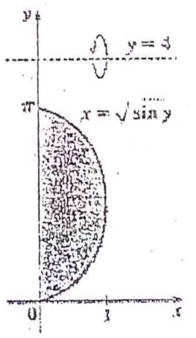
d)

$V = \int_0^2 2\pi\,(2 - x)\left(\frac{1}{1+x^2}\right) dx$



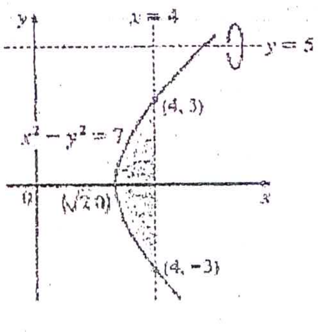
e)

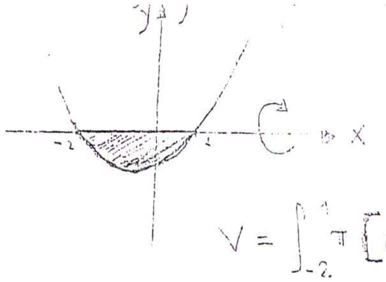
$V = \int_0^\pi 2\pi\,(4 - y)\,\sqrt{\sin y}\,dy$



f)

$V = \int_{-3}^3 2\pi\,(5 - y)\left(4 - \sqrt{y^2 + 7}\right) dy$





$$A = \pi r^2 = \pi (x^2 + x - 2)^2$$

$$V = \int_{-2}^{1} A(x)\,dx = \int_{-2}^{1} \pi (x^2+x-2)^2\,dx$$

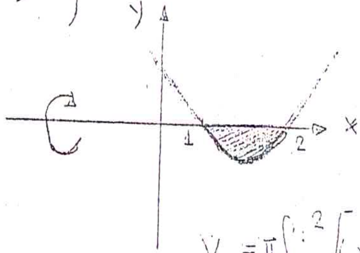
$$V = \int_{-2}^{1} \pi \left[(x^2+x)^2 - 2(x^2+x).2+4\right] dx = \pi \int_{-2}^{1} (x^4+2x^3+x^2-4x^2-4x+4)\,dx$$

$$V = \pi\left(\frac{x^5}{5} + \frac{2x^4}{4} - \frac{3x^3}{3} - \frac{4x^2}{2} + 4x\right)\Big|_{-2}^{1} = \left(\frac{1}{5}+\frac{1}{2}-1-2+4\right) - \left(-\frac{32}{5}+\frac{16}{2}+8-8-8\right)$$

$$V = \pi\left(\frac{33}{5}+\frac{3}{2}\right) = \frac{\pi(66+15)}{10} = \frac{81\pi}{10}$$

$3\frac{2}{5} + \frac{2+5-10+20+40}{10}$

b) $y = x^2 - 3x + 2 \qquad y = 0$ ao redor do eixo x



$$A = \pi (x^2 - 3x + 2)^2$$

$$V = \int_1^2 \pi (x^2-3x+2)^2\,dx$$

$$V = \pi \int_1^2 \left[(x^2-3x)^2 + 2(x^2-3x)(2) + 4\right] dx$$

$$V = \pi \int_1^2 (x^4 - 6x^3 + 9x^2 + 4x^2 - 12x + 4)\,dx = \pi \int_1^2 (x^4 - 6x^3 + 13x^2 - 12x + 4)\,dx$$

$$V = \pi\left(\frac{x^5}{5} - \frac{6x^4}{4} + \frac{13x^3}{3} - \frac{12x^2}{2} + 4x\right)\Big|_1^2 = \left(\frac{32}{5} - \frac{3}{2}(16) + \frac{13}{3}(8) - 6(4) + 8\right) - \left(\frac{1}{5} - \frac{3}{2} + \frac{13}{3} - 6 + 4\right)$$

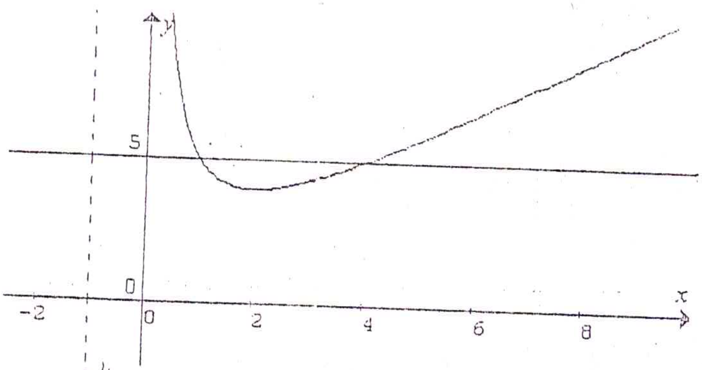
$$V = \pi\left(\frac{31}{5} - \frac{45}{2} + \frac{91}{3} - 6\right) = \frac{501\pi}{30} = \cancel{\frac{16.7\pi}{10}} \qquad \pi/30$$

c) $y = 5 \qquad y = x + (4/x)$ ao redor de $x = -1$

Interceptos: $5 = x + 4/x$

Igualando: $5 = \dfrac{x^2+4}{x} \rightarrow x^2 - 5x + 4 = 0$

$\begin{cases} \text{Soma} = 5 \\ \text{Produto} = 4 \end{cases} \therefore x_1 = 1 \text{ e } x_2 = 4$



$$V = \int_1^4 2\pi\,(x-(-1))\left[5 - \left(x + \frac{4}{x}\right)\right] dx$$

$$V = \int_1^4 2\pi (x+1)\left(5 - x - \frac{4}{x}\right) dx = 2\pi \int_1^4 \left(5x - x^2 - 4 + 5 - x - \frac{4}{x}\right) dx$$

$$V = \int_1^4 2\pi\left(-x^2 + 4x + 1 - \frac{4}{x}\right) dx = 2\pi\left(\frac{-x^3}{3} + \frac{4x^2}{2} + x - 4\ln x\right)\Big|_1^4$$

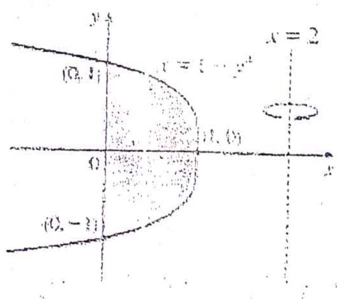
$$V = -\frac{4^3}{3} + 2(4)^2 + 4 - 4\ln 4 + \frac{1}{3} - 2 - 1 + 4\ln 1 \ldots$$

$$V = 2\pi\left(-\frac{64}{3} + 32 + 4 - 4\ln 4 + \frac{1}{3} - 3\right) = 2\pi\left(\frac{32}{3} + \frac{1}{3} + 1 - 4\ln 4\right) = (12 - 4\ln 4)\,2\pi$$

$$V = 8\pi\left(3 - \ln 4\right)$$

d)

$$V = \int_{-1}^{1} \pi \left\{[2-0]^2 - [2-(1-y^4)]^2\right\} dy$$
$$= 2\pi \int_0^1 \left[4 - (1+y^4)^2\right] dy \text{ [by symmetry]}$$
$$= 2\pi \int_0^1 [4 - (1 + 2y^4 + y^8)]\, dy = 2\pi \int_0^1 (3 - 2y^4 - y^8)\, dy$$
$$= 2\pi \left[3y - \tfrac{2}{5}y^5 - \tfrac{1}{9}y^9\right]_0^1 = 2\pi\left(3 - \tfrac{2}{5} - \tfrac{1}{9}\right) = 2\pi\left(\tfrac{112}{45}\right) = \tfrac{224\pi}{45}$$



e)

$$V = \pi \int_0^2 \left[\sqrt{1-(y-1)^2}\right]^2 dy = \pi \int_0^2 \left(2y - y^2\right) dy = \pi\left[y^2 - \tfrac{1}{3}y^3\right]_0^2 = \pi\left(4 - \tfrac{8}{3}\right) = \tfrac{4}{3}\pi$$

f) $x^2 + (y-1)^2 = 1$ ao redor do eixo x $\quad r = \sqrt{1-x^2} + 1$

17. a) $f(x) = x^2$, $[-1,1]$:

$$\bar{f} = \frac{1}{1-(-1)} \int_{-1}^{1} x^2\,dx = \frac{1}{2}\left(\frac{x^3}{3}\right)\Big|_{-1}^{1} = \frac{1}{2}\left[\frac{1}{3} - \left(-\frac{1}{3}\right)\right] = \frac{1}{3}$$

b) $f(x) = 1/x$, $[1,4]$

$$\bar{f} = \frac{1}{4-1}\int_1^4 \frac{1}{x}\,dx = \frac{1}{3}(\ln x)\Big|_1^4 = \frac{1}{3}(\ln 4 - \ln 1) = \frac{2\ln 2}{3}$$

c) $g(x) = \cos x$, $[0, \pi/2]$

$$\bar{g} = \frac{1}{\pi/2 - 0}\int_0^{\pi/2} \cos x\,dx = \frac{2}{\pi}(\text{sen}\,x)\Big|_0^{\pi/2} = \frac{2}{\pi}\left(\text{sen}\,\pi/2 - \text{sen}\,0\right) = 2/\pi$$

d) $g(x) = \sqrt{x}$, $[1,4]$

$$\bar{g} = \frac{1}{4-1}\int_1^4 \sqrt{x}\,dx = \frac{1}{3}\left(\frac{2}{3}x^{3/2}\right)\Big|_1^4 = \frac{1}{3}\left(\frac{2}{3}\sqrt{4^3} - \frac{2}{3}\sqrt{1^3}\right) = \frac{2}{9}(8-1) = \frac{14}{9}$$

e) $f(t) = te^{-t^2}$, $[0,5]$

$$\bar{f}(t) = \frac{1}{5-0}\int_0^5 te^{-t^2}dt \quad \begin{cases} t^2 = u \\ du = 2t\,dt \end{cases} \therefore = \frac{1}{5}\int_0^{25} \frac{e^{-u}}{2}\,du = \frac{1}{5}\left(\frac{-e^{-u}}{2}\right)\Big|_0^{25} = \frac{1-e^{-25}}{10}$$

f) $f(\theta) = \sec\theta\,\text{tg}\,\theta$ $[0, \pi/4]$.

$$\bar{f}(\theta) = \frac{1}{\pi/4 - 0}\int_0^{\pi/4} \sec\theta\,\text{tg}\,\theta\,d\theta = \frac{4}{\pi}(\sec\theta)\Big|_0^{\pi/4} = \frac{4}{\pi}\left(\frac{1}{\cos \pi/4} - \frac{1}{\cos 0}\right) = \frac{4}{\pi}\left(\sqrt{2} - 1\right)$$

g) $h(x) = \cos^4 x\,\text{sen}\,x$ $[0,\pi]$:

$$\bar{h} = \frac{1}{\pi - 0}\int_0^{\pi} \cos^4 x\,\text{sen}\,x\,dx \quad \begin{cases} u = \cos x \\ du = -\text{sen}\,x\,dx \end{cases} \therefore = \frac{1}{\pi}\int_1^{-1} -u^4\,du = -\frac{1}{\pi}\left(\frac{u^5}{5}\right)\Big|_1^{-1} = -\frac{1}{\pi}\left(-\frac{1}{5} - \frac{1}{5}\right)$$

$$\bar{h}(r) = \frac{1}{6-1}\int_1^6 \frac{3}{(1+r)^2} \quad \begin{cases} u = 1+r \\ du = dr \end{cases} \therefore = \frac{1}{5}\int_2^7 3u^{-2}du = -\frac{3}{5}\left(\frac{1}{u}\right)\Big|_2^7 = -\frac{3}{5}\left(\frac{1}{7}-\frac{1}{2}\right)$$

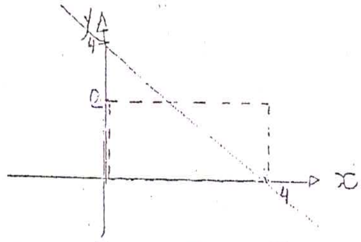
$$\bar{h}(r) = \frac{3}{5}\left(\frac{1}{2}-\frac{1}{7}\right) = \frac{3}{5}\left(\frac{5}{14}\right) = \frac{3}{14}$$

18. a) $f(x) = 4-x \quad , [0,2]$

(i) $\bar{f}(x) = \frac{1}{2-0}\int_0^2 (4-x)\,dx = \frac{1}{2}\left(4x - x^2/2\right)\Big|_0^2 = \frac{1}{2}(8-2) = \frac{6}{2} = 3$

(ii) $f(C) = 3 = 4 - C \therefore C = 4-3 = 1$

(iii)

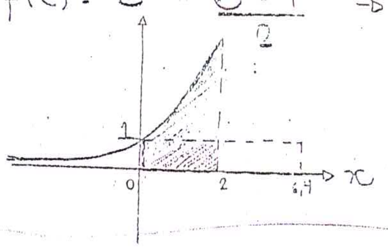


b) $f(x) = e^x \quad , [0,2]$

(i) $\bar{f}(x) = \frac{1}{2-0}\int_0^2 e^x dx = \frac{1}{2}(e^x)\Big|_0^2 = \frac{1}{2}(e^2 - e^0) = \frac{(e^2-1)}{2}$

(ii) $f(c) = e^c = \frac{e^2-1}{2} \rightarrow \ln e^c = \ln \frac{e^2-1}{2} \therefore c = \ln(e^2-1) - \ln 2$

(iii)



$$A = \int_0^2 e^x dx = (e^x)\Big|_0^2 = e^2 - 1 \simeq 6{,}4$$

19. $\boxed{L = \int_a^b \sqrt{1+(f'(x))^2}\,dx}$ $\quad -2 \leq x \leq 1 \quad$ e $f(x) = 2-3x$

$f'(x) = -3 \therefore (f'(x))^2 = (-3)^2 = 9$

$$L = \int_{-2}^1 \sqrt{1+9}\,dx = \sqrt{10}\,(x)\Big|_{-2}^1 = \sqrt{10}\,(1-(-2)) = 3\sqrt{10}$$

Para verificar: $d = \sqrt{\Delta y^2 + \Delta x^2}$ $\quad \rightarrow y = 2-3x \begin{cases} \rightarrow x = -2 \\ \rightarrow x = 1 \end{cases}$

p/ $x = -2$: $y = 2-(-2)3 = 8$

p/ $x = 1$: $y = 2-3(1) = -1$ $\quad$ Logo: $\Delta x = 1-(-2) = 3$, $\Delta y = 8-(-1) = 9$

$$d = \sqrt{9^2+3^2} = \sqrt{81+9} = \sqrt{90} = \sqrt{9*10} = 3\sqrt{10}$$

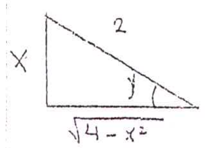
20. $L = \int_a^b \sqrt{1+(f'(x))^2}\,dx \qquad 0 \leq x \leq 2$ e $f(x) = \sqrt{4-x^2} = (4-x^2)^{1/2}$

$$f'(x) = 1/2\,(4-x^2)^{-1/2} * (-2x) = \frac{-x}{(4-x^2)^{1/2}}$$

$$L = \int_0^2 \sqrt{1+\frac{x^2}{4-x^2}}\,dx = \int_0^2 \sqrt{\frac{4-x^2+x^2}{4-x^2}}\,dx = \int_0^2 \frac{2}{\sqrt{4-x^2}}\,dx$$

$$L = 2\left(\operatorname{sen}^{-1}\frac{x}{2}\right)\Big|_0^2 = 2\left(\operatorname{sen}^{-1}1 - \operatorname{sen}^{-1}0\right) = 2(\pi/2 - 0) = \pi$$

obs:



$$\operatorname{sen} y = \frac{x}{2} \rightarrow y'\cos y = 1/2 \quad \therefore y' = \frac{1}{2\cos y} = \frac{1}{2\left(\frac{\sqrt{4-x^2}}{2}\right)}$$

$$y' = \frac{1}{\sqrt{4-x^2}}$$

Para verificar: $L_{circulo} = 2\pi r$ e $y^2 = 4-x^2 \therefore x^2+y^2 = 4 \rightarrow r = 2$

$$L = \frac{L_{circulo}}{4} = \frac{2\pi(2)}{4} = \pi$$

21. a) $y^2 = (x-1)^3$; $A(1,0)$, $B(2,1)$

$$y = (x-1)^{3/2} \qquad y' = \frac{3}{2}(x-1)^{1/2} \qquad \therefore (y')^2 = \frac{9}{4}(x-1)$$

$$L = \int_1^2 \sqrt{1+\frac{9(x-1)}{4}}\,dx = L = \int_1^2 \sqrt{\frac{4+9x-9}{4}}\,dx = \int_1^2 \frac{\sqrt{9x-5}}{2}\,dx$$

$$L = \frac{1}{2}\int_1^2 \sqrt{9x-5}\,dx \qquad \begin{cases} u = 9x-5 \\ du = 9dx \end{cases} \qquad \begin{matrix} u(2) = 13 \\ u(1) = 4 \end{matrix}$$

$$L = \frac{1}{2}\int_4^{13} \sqrt{u}\,\frac{du}{9} = \frac{1}{18}\left(\frac{2}{3}u^{3/2}\right)\Big|_4^{13} = \frac{1}{27}\left(13^{3/2} - 4^{3/2}\right) = \frac{13\sqrt{13}}{27} - \frac{8}{27}$$

$$L = \frac{13\sqrt{13} - 8}{27}$$

b) $12xy = 4y^4 + 3$; $A(7/12, 1)$, $B(67/24, 2)$

1,73600617

$x^4 + 2^x$

22. a)

$y = \frac{1}{3}(x^2+2)^{3/2} \;\Rightarrow\; dy/dx = \frac{1}{2}(x^2+2)^{1/2}(2x) = x\sqrt{x^2+2}$

$\Rightarrow\; 1 + (dy/dx)^2 = 1 + x^2(x^2+2) = (x^2+1)^2.$

$L = \int_0^1 (x^2+1)\,dx = \left[\frac{1}{3}x^3 + x\right]_0^1 = \frac{4}{3}.$

$x^4 + 2^x$

b)

$y = \frac{x^2}{2} - \frac{\ln x}{4} \;\Rightarrow\; \frac{dy}{dx} = x - \frac{1}{4x} \;\Rightarrow\; 1 + \left(\frac{dy}{dx}\right)^2 = x^2 + \frac{1}{2} + \frac{1}{16x^2}$

$L = \int_2^4 \left(x + \frac{1}{4x}\right)dx = \left[\frac{x^2}{2} + \frac{\ln x}{4}\right]_2^4 = \left(8 + \frac{2\ln 2}{4}\right) - \left(2 + \frac{\ln 2}{4}\right) = 6 + \frac{\ln 2}{4}$

c)

$y = \frac{x^4}{4} + \frac{1}{8x^2} \;\Rightarrow\; \frac{dy}{dx} = x^3 - \frac{1}{4x^3} \;\Rightarrow\; 1 + \left(\frac{dy}{dx}\right)^2 = 1 + x^6 - \frac{1}{2} + \frac{1}{16x^6} = x^6 + \frac{1}{2} + \frac{1}{16x^6}$

$L = \int_1^3 \left(x^3 + \frac{1}{4}x^{-3}\right)dx = \left[\frac{1}{4}x^4 - \frac{1}{8}x^{-2}\right]_1^3 = \left(\frac{81}{4} - \frac{1}{72}\right) - \left(\frac{1}{4} - \frac{1}{8}\right) = \frac{181}{9}.$

d)

$x = \frac{1}{3}\sqrt{y}\,(y-3) = \frac{1}{3}y^{3/2} - y^{1/2} \;\Rightarrow\; dx/dy = \frac{1}{2}y^{1/2} - \frac{1}{2}y^{-1/2} \;\Rightarrow$

$1 + (dx/dy)^2 = 1 + \frac{1}{4}(y - 2 + y^{-1}) = \frac{1}{4}(y + 2 + y^{-1}) = \left[\frac{1}{2}(y^{1/2} + y^{-1/2})\right]^2.$

$L = \int_0^9 \sqrt{1 + (dx/dy)^2}\,dy = \int_0^9 \frac{1}{2}(y^{1/2} + y^{-1/2})\,dy = \lim_{t\to 0^+} \int_t^9 \frac{1}{2}(y^{1/2} + y^{-1/2})\,dy$

$= \lim_{t\to 0^+} \left[\frac{1}{3}y^{3/2} + y^{1/2}\right]_t^9 = \lim_{t\to 0^+}\left[(9+3) - \left(\frac{1}{3}t^{3/2} + t^{1/2}\right)\right] = 12 - 0 = 12$

e)

$y = \ln(\sec x) \;\Rightarrow\; \frac{dy}{dx} = \frac{\sec x \tan x}{\sec x} = \tan x \;\Rightarrow\; 1 + \left(\frac{dy}{dx}\right)^2 = 1 + \tan^2 x = \sec^2 x.$

$L = \int_0^{\pi/4} \sqrt{\sec^2 x}\,dx = \int_0^{\pi/4} \sec x\,dx = [\ln(\sec x + \tan x)]_0^{\pi/4} = \ln\left(\sqrt{2}+1\right) - \ln(1+0) = \ln\left(\sqrt{2}+1\right)$

f)

$y = \ln(\sin x) \;\Rightarrow\; \frac{dy}{dx} = \frac{\cos x}{\sin x} = \cot x \;\Rightarrow\; 1 + \left(\frac{dy}{dx}\right)^2 = 1 + \cot^2 x = \csc^2 x.$

$L = \int_{\pi/6}^{\pi/3} \csc x\,dx = [\ln(\csc x - \cot x)]_{\pi/6}^{\pi/3} = \ln\left(\frac{2}{\sqrt{3}} - \frac{1}{\sqrt{3}}\right) - \ln\left(2 - \sqrt{3}\right) = \ln\frac{1}{\sqrt{3}(2-\sqrt{3})}$

$= \ln\frac{2+\sqrt{3}}{\sqrt{3}} = \ln\left(1 + \frac{2}{\sqrt{3}}\right)$

g)

$y = \ln(1 - x^2) \;\Rightarrow\; \frac{dy}{dx} = \frac{-2x}{1-x^2} \;\Rightarrow\; 1 + \left(\frac{dy}{dx}\right)^2 = 1 + \frac{4x^2}{(1-x^2)^2} = \frac{(1+x^2)^2}{(1-x^2)^2}$

$L = \int_0^{1/2} \frac{1+x^2}{1-x^2}\,dx = \int_0^{1/2}\left[-1 + \frac{2}{(1-x)(1+x)}\right]dx = \int_0^{1/2}\left[-1 + \frac{1}{1+x} + \frac{1}{1-x}\right]dx$

$= [-x + \ln(1+x) - \ln(1-x)]_0^{1/2} = -\frac{1}{2} + \ln\frac{3}{2} - \ln\frac{1}{2} - 0 = \ln 3 - \frac{1}{2}$

h)

$$y = \ln x \quad \Rightarrow \quad \frac{dy}{dx} = \frac{1}{x} \quad \Rightarrow \quad \sqrt{1+\left(\frac{dy}{dx}\right)^2} = \sqrt{1+\left(\frac{1}{x}\right)^2} = \frac{\sqrt{1+x^2}}{x}$$

$$L = \int_1^{\sqrt{3}} \frac{\sqrt{1+x^2}}{x}\,dx \qquad u = \sqrt{1+x^2}, \quad u^2 = 1+x^2 \text{ e } u\,du = x\,dx$$

$$L = \int_{\sqrt{2}}^{2} \frac{u}{u^2-1}\,u\,du = \int_{\sqrt{2}}^{2}\left(1+\frac{1/2}{u-1}-\frac{1/2}{u+1}\right)du = \left[u+\tfrac{1}{2}\ln|u-1|-\tfrac{1}{2}\ln|u+1|\right]_{\sqrt{2}}^{2}$$

$$= \left[u-\frac{1}{2}\ln\left|\frac{u+1}{u-1}\right|\right]_{\sqrt{2}}^{2} = 2-\sqrt{2}-\tfrac{1}{2}\ln 3+\frac{1}{2}\ln\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right) = 2-\sqrt{2}+\ln\left(\sqrt{2}+1\right)-\tfrac{1}{2}\ln 3$$

i)

$$y = \cosh x \quad \Rightarrow \quad y' = \sinh x \quad \Rightarrow \quad 1+(y')^2 = 1+\sinh^2 x = \cosh^2 x.$$

$$L = \int_0^1 \cosh x\,dx = [\sinh x]_0^1 = \sinh 1 = \tfrac{1}{2}\,(e-1/e)$$

j)

$$y^2 = 4x,\ x = \tfrac{1}{4}y^2 \quad \Rightarrow \quad \tfrac{dx}{dy} = \tfrac{1}{2}y \quad \Rightarrow \quad 1+\left(\tfrac{dx}{dy}\right)^2 = 1+\tfrac{1}{4}y^2.$$

$$L = \int_0^2 \sqrt{1+\tfrac{1}{4}y^2}\,dy = \int_0^1 \sqrt{1+u^2}\cdot 2\,du \quad (u = \tfrac{1}{2}y,\ dy = 2\,du)$$

$$= \left[u\sqrt{1+u^2}+\ln\left|u+\sqrt{1+u^2}\right|\right]_0^1 = \sqrt{2}+\ln\left(1+\sqrt{2}\right)$$

k)

$$y = e^x \quad \Rightarrow \quad y' = e^x \quad \Rightarrow \quad 1+(y')^2 = 1+e^{2x}.$$

$$L = \int_0^1 \sqrt{1+e^{2x}}\,dx = \int_1^e \sqrt{1+u^2}\,\frac{du}{u} \quad [u = e^x, \quad x = \ln u,\ dx = du/u]$$

$$= \int_1^e \frac{\sqrt{1+u^2}}{u^2}\,u\,du = \int_{\sqrt{2}}^{\sqrt{1+e^2}} \frac{v}{v^2-1}\,v\,dv \quad \left[v = \sqrt{1+u^2}, \quad v^2 = 1+u^2,\ v\,dv = u\,du\right]$$

$$= \int_{\sqrt{2}}^{\sqrt{1+e^2}}\left(1+\frac{1/2}{v-1}-\frac{1/2}{v+1}\right)dv = \left[v+\frac{1}{2}\ln\frac{v-1}{v+1}\right]_{\sqrt{2}}^{\sqrt{1+e^2}}$$

$$= \sqrt{1+e^2}-\sqrt{2}+\frac{1}{2}\ln\frac{\sqrt{1+e^2}-1}{\sqrt{1+e^2}+1}-\frac{1}{2}\ln\frac{\sqrt{2}-1}{\sqrt{2}+1}$$

$$= \sqrt{1+e^2}-\sqrt{2}+\ln\left(\sqrt{1+e^2}-1\right)-1-\ln\left(\sqrt{2}-1\right)$$

l)

$$y = \ln\left(\frac{e^x+1}{e^x-1}\right) = \ln(e^x+1)-\ln(e^x-1) \quad \Rightarrow \quad y' = \frac{e^x}{e^x+1}-\frac{e^x}{e^x-1} = \frac{-2e^x}{e^{2x}-1} \quad \Rightarrow$$

$$1+(y')^2 = 1+\frac{4e^{2x}}{(e^{2x}-1)^2} = \frac{(e^{2x}+1)^2}{(e^{2x}-1)^2} \quad \Rightarrow \quad \sqrt{1+(y')^2} = \frac{e^{2x}+1}{e^{2x}-1} = \frac{e^x+e^{-x}}{e^x-e^{-x}} = \frac{\cosh x}{\sinh x}$$

$$L = \int_a^b \frac{\cosh x}{\sinh x}\,dx = \ln\sinh x\,\big|_a^b = \ln\left(\frac{\sinh b}{\sinh a}\right) = \ln\left(\frac{e^b-e^{-b}}{e^a-e^{-a}}\right)$$

23.a)

$y = \ln x \;\Rightarrow\; ds = \sqrt{1+(dy/dx)^2}\,dx = \sqrt{1+(1/x)^2}\,dx \;\Rightarrow\; S = \int_1^3 2\pi\,(\ln x)\sqrt{1+(1/x)^2}\,dx$

b)

$y = \sin^2 x \;\Rightarrow\; ds = \sqrt{1+(dy/dx)^2}\,dx = \sqrt{1+(2\sin x\cos x)^2}\,dx \;\Rightarrow$

$S = \int_0^{\pi/2} 2\pi \sin^2 x\sqrt{1+(2\sin x\cos x)^2}\,dx$

c)

$y = \sec x \;\Rightarrow\; ds = \sqrt{1+(dy/dx)^2}\,dx = \sqrt{1+(\sec x\tan x)^2}\,dx \;\Rightarrow$

$S = \int_0^{\pi/4} 2\pi x\sqrt{1+(\sec x\tan x)^2}\,dx$

d)

$y = e^x \;\Rightarrow\; ds = \sqrt{1+(dy/dx)^2}\,dx = \sqrt{1+e^{2x}}\,dx \;\Rightarrow$

$S = \int_0^{\ln 2} 2\pi x\sqrt{1+e^{2x}}\,dx$ ou $\int_1^2 2\pi\,(\ln y)\sqrt{1+(1/y)^2}\,dy$

24. a)

$y = x^3 \;\Rightarrow\; y' = 3x^2.$

$S = \int_0^2 2\pi y\sqrt{1+(y')^2}\,dx = 2\pi\int_0^2 x^3\sqrt{1+9x^4}\,dx \quad (u = 1+9x^4,\ du = 36x^3\,dx)$

$= \frac{2\pi}{36}\int_1^{145}\sqrt{u}\,du = \frac{\pi}{18}\left[\frac{2}{3}u^{3/2}\right]_1^{145} = \frac{\pi}{27}\left(145\sqrt{145}-1\right)$

b)

A curva $y^2=4x+4$ é simétrica em relação ao eixo x, que é o eixo de rotação. Logo devemos considerar apenas a parte superior da curva dada por $y = \sqrt{4x+4} = 2\sqrt{x+1}.$

$\frac{dy}{dx} = \frac{1}{\sqrt{x+1}} \;\Rightarrow\; \sqrt{1+\left(\frac{dy}{dx}\right)^2} = \sqrt{1+\frac{1}{x+1}}$

$S = 2\pi\int_0^8 2\sqrt{x+1}\sqrt{1+\frac{1}{x+1}}\,dx = 4\pi\int_0^8\sqrt{x+2}\,dx = 4\pi\left[\frac{2}{3}(x+2)^{3/2}\right]_0^8 = \frac{8\pi}{3}\left(10\sqrt{10}-2\sqrt{2}\right)$

c)

$y = \sqrt{x} \;\Rightarrow\; 1+(dy/dx)^2 = 1+\left[1/(2\sqrt{x})\right]^2 = 1+1/(4x).$

$S = \int_4^9 2\pi y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = \int_4^9 2\pi\sqrt{x}\sqrt{1+\frac{1}{4x}}\,dx = 2\pi\int_4^9\sqrt{x+\frac{1}{4}}\,dx$

$= 2\pi\left[\frac{2}{3}\left(x+\frac{1}{4}\right)^{3/2}\right]_4^9 = \frac{4\pi}{3}\left[\frac{1}{8}(4x+1)^{3/2}\right]_4^9 = \frac{\pi}{6}\left(37\sqrt{37}-17\sqrt{17}\right)$

d)

$y = \frac{x^2}{4}-\frac{\ln x}{2} \;\Rightarrow\; \frac{dy}{dx} = \frac{x}{2}-\frac{1}{2x} \;\Rightarrow\; 1+\left(\frac{dy}{dx}\right)^2 = \frac{x^2}{4}+\frac{1}{2}+\frac{1}{4x^2}.$

$S = 2\pi\int_1^4\left(\frac{x^2}{4}-\frac{\ln x}{2}\right)\left(\frac{x}{2}+\frac{1}{2x}\right)dx = \frac{\pi}{2}\int_1^4\left(\frac{x^2}{2}-\ln x\right)\left(x+\frac{1}{x}\right)dx$

$= \frac{\pi}{2}\int_1^4\left(\frac{x^3}{2}+\frac{x}{2}-x\ln x-\frac{\ln x}{x}\right)dx = \frac{\pi}{2}\left[\frac{x^4}{8}+\frac{x^2}{4}-\frac{x^2}{2}\ln x+\frac{x^2}{4}-\frac{1}{2}(\ln x)^2\right]_1^4$

$= \frac{\pi}{2}\left[\left(32+4-8\ln 4+4-\frac{1}{2}(\ln 4)^2\right)-\left(\frac{1}{8}+\frac{1}{4}-0+\frac{1}{4}-0\right)\right] = \pi\left[\frac{315}{16}-8\ln 2-(\ln 2)^2\right]$

e)

$y = \sin x \quad \Rightarrow \quad 1 + (dy/dx)^2 = 1 + \cos^2 x.$

$S = 2\pi \int_0^{\pi} \sin x \sqrt{1 + \cos^2 x}\, dx = 2\pi \int_{-1}^{1} \sqrt{1 + u^2}\, du \quad (u = -\cos x,\ du = \sin x\, dx)$

$= 4\pi \int_0^1 \sqrt{1+u^2}\, du = 4\pi \int_0^{\pi/4} \sec^3 \theta\, d\theta \quad (u = \tan\theta,\ du = \sec^2\theta\, d\theta)$

$= 2\pi \left[\sec\theta \tan\theta + \ln|\sec\theta + \tan\theta|\right]_0^{\pi/4} = 2\pi \left[\sqrt{2} + \ln\left(\sqrt{2} + 1\right)\right]$

f)

$y = \cos 2x \quad \Rightarrow \quad ds = \sqrt{1 + (dy/dx)^2}\, dx = \sqrt{1 + (-2\sin 2x)^2}\, dx \quad \Rightarrow$

$S = \int_0^{\pi/6} 2\pi \cos 2x \sqrt{1 + 4\sin^2 2x}\, dx = 2\pi \int_0^{\sqrt{3}} \sqrt{1+u^2}\left(\frac{1}{4}\, du\right) \quad [u = 2\sin 2x,\ du = 4\cos 2x\, dx]$

$= \frac{\pi}{2}\left[\frac{1}{2}u\sqrt{1+u^2} + \frac{1}{2}\ln\left(u + \sqrt{1+u^2}\right)\right]_0^{\sqrt{3}} = \frac{\pi}{4}\left[\frac{\sqrt{3}}{2}\cdot 2 + \frac{1}{2}\ln\left(\sqrt{3}+2\right)\right] = \frac{\sqrt{3}\pi}{4} + \frac{\pi}{8}\ln\left(2+\sqrt{3}\right)$

g)

$y = \cosh x \quad \Rightarrow \quad 1 + (dy/dx)^2 = 1 + \sinh^2 x = \cosh^2 x.$

$S = 2\pi \int_0^1 \cosh x \cosh x\, dx = 2\pi \int_0^1 \frac{1}{2}(1 + \cosh 2x)\, dx = \pi \left[x + \frac{1}{2}\sinh 2x\right]_0^1$

$= \pi\left(1 + \frac{1}{2}\sinh 2\right)$ ou $\quad \pi\left[1 + \frac{1}{4}\left(e^2 - e^{-2}\right)\right]$

h)

$2y = 3x^{2/3},\ y = \frac{3}{2}x^{2/3} \quad \Rightarrow \quad dy/dx = x^{-1/3} \quad \Rightarrow \quad 1 + (dy/dx)^2 = 1 + x^{-2/3}.$

$S = 2\pi \int_1^8 \frac{3}{2}x^{2/3}\sqrt{1 + x^{-2/3}}\, dx = 3\pi \int_1^2 u^2 \sqrt{1 + 1/u^2}\, 3u^2\, du \quad (u = x^{1/3},\ x = u^3,\ dx = 3u^2\, du)$

$= 9\pi \int_1^2 u^3 \sqrt{u^2+1}\, du = \frac{9\pi}{2}\int_1^2 u^2\sqrt{u^2+1}\, 2u\, du = \frac{9\pi}{2}\int_2^5 (v-1)\sqrt{v}\, dv \quad (v = u^2+1,\ dv = 2u\, du)$

$= \frac{9\pi}{2}\int_2^5 (v^{3/2} - v^{1/2})\, dv = \frac{9\pi}{2}\left[\frac{2}{5}v^{5/2} - \frac{2}{3}v^{3/2}\right]_2^5 = 9\pi\left[\left(\frac{1}{5}\cdot 5^{5/2} - \frac{1}{3}\cdot 5^{3/2}\right) - \left(\frac{1}{5}\cdot 2^{5/2} - \frac{1}{3}\cdot 2^{3/2}\right)\right]$

$= 9\pi\left[5\sqrt{5} - \frac{5\sqrt{5}}{3} - \frac{4\sqrt{2}}{5} + \frac{2\sqrt{2}}{3}\right] = \frac{3\pi}{5}\left(50\sqrt{5} - 2\sqrt{2}\right)$

i)

$x = \frac{1}{3}(y^2+2)^{3/2} \quad \Rightarrow \quad dx/dy = \frac{1}{2}(y^2+2)^{1/2}(2y) = y\sqrt{y^2+2} \quad \Rightarrow$

$1 + (dx/dy)^2 = 1 + y^2(y^2+2) = (y^2+1)^2.$

$S = 2\pi \int_1^2 y(y^2+1)\, dy = 2\pi\left[\frac{1}{4}y^4 + \frac{1}{2}y^2\right]_1^2 = 2\pi\left(4 + 2 - \frac{1}{4} - \frac{1}{2}\right) = \frac{21\pi}{2}$

j)

$x = 1 + 2y^2 \quad \Rightarrow \quad 1 + (dx/dy)^2 = 1 + (4y)^2 = 1 + 16y^2.$

$S = 2\pi \int_1^2 y\sqrt{1 + 16y^2}\, dy = \frac{\pi}{16}\int_1^2 (16y^2+1)^{1/2}\, 32y\, dy = \frac{\pi}{16}\left[\frac{2}{3}(16y^2+1)^{3/2}\right]_1^2$

$= \frac{\pi}{24}\left(65\sqrt{65} - 17\sqrt{17}\right)$

25. a)

$y = \sqrt[3]{x} \quad \Rightarrow \quad x = y^3 \quad \Rightarrow \quad 1 + (dx/dy)^2 = 1 + 9y^4.$

$S = 2\pi \int_1^2 x\sqrt{1 + (dx/dy)^2}\, dy = 2\pi \int_1^2 y^3\sqrt{1 + 9y^4}\, dy = \frac{2\pi}{36}\int_1^2 \sqrt{1+9y^4}\, 36y^3\, dy$

$= \frac{\pi}{18}\left[\frac{2}{3}\left(1 + 9y^4\right)^{3/2}\right]_1^2 = \frac{\pi}{27}\left(145\sqrt{145} - 10\sqrt{10}\right)$

b)

$y = 1 - x^2 \quad \Rightarrow \quad 1 + (dy/dx)^2 = 1 + 4x^2 \quad \Rightarrow$

$S = 2\pi \int_0^1 x\sqrt{1+4x^2}\,dx = \frac{\pi}{4}\int_0^1 8x\sqrt{4x^2+1}\,dx = \frac{\pi}{4}\left[\frac{2}{3}(4x^2+1)^{3/2}\right]_0^1 = \frac{\pi}{6}\left(5\sqrt{5}-1\right)$

c)

$x = e^{2y} \quad \Rightarrow \quad 1 + (dx/dy)^2 = 1 + 4e^{4y}$

$S = 2\pi \int_0^{1/2} e^{2y}\sqrt{1+(2e^{2y})^2}\,dy = 2\pi \int_2^{2e} \sqrt{1+u^2}\,\frac{1}{4}\,du \quad (u = 2e^{2y},\ du = 4e^{2y}\,dy)$

$= \frac{\pi}{2}\int_2^{2e} \sqrt{1+u^2}\,du = \frac{\pi}{2}\left[\frac{1}{2}u\sqrt{1+u^2} + \frac{1}{2}\ln\left|u+\sqrt{1+u^2}\right|\right]_2^{2e} \quad (u = \tan\theta)$

$= \frac{\pi}{2}\left[e\sqrt{1+4e^2} + \frac{1}{2}\ln\left(2e+\sqrt{1+4e^2}\right) - \sqrt{5} - \frac{1}{2}\ln\left(2+\sqrt{5}\right)\right]$

$= \frac{\pi}{4}\left[2e\sqrt{1+4e^2} - 2\sqrt{5} + \ln\left(\frac{2e+\sqrt{1+4e^2}}{2+\sqrt{5}}\right)\right]$

d)

$x = \sqrt{2y-y^2} \quad \Rightarrow \quad \frac{dx}{dy} = \frac{1-y}{\sqrt{2y-y^2}} \quad \Rightarrow \quad 1 + \left(\frac{dx}{dy}\right)^2 = 1 + \frac{1-2y+y^2}{2y-y^2} = \frac{1}{2y-y^2}$

$S = 2\pi \int_0^1 \sqrt{2y-y^2}\left(\frac{1}{\sqrt{2y-y^2}}\right)dy = 2\pi \int_0^1 dy = 2\pi$

e)

$x = \frac{1}{2\sqrt{2}}(y^2 - \ln y) \quad \Rightarrow \quad \frac{dx}{dy} = \frac{1}{2\sqrt{2}}\left(2y - \frac{1}{y}\right) \quad \Rightarrow$

$1 + \left(\frac{dx}{dy}\right)^2 = 1 + \frac{1}{8}\left(2y - \frac{1}{y}\right)^2 = 1 + \frac{1}{8}\left(4y^2 - 4 + \frac{1}{y^2}\right) = \frac{1}{8}\left(4y^2 + 4 + \frac{1}{y^2}\right) = \left[\frac{1}{2\sqrt{2}}\left(2y + \frac{1}{y}\right)\right]^2$

$S = 2\pi \int_1^2 \frac{1}{2\sqrt{2}}(y^2 - \ln y)\frac{1}{2\sqrt{2}}\left(2y + \frac{1}{y}\right)dy = \frac{\pi}{4}\int_1^2 \left(2y^3 + y - 2y\ln y - \frac{\ln y}{y}\right)dy$

$= \frac{\pi}{4}\left[\frac{1}{2}y^4 + \frac{1}{2}y^2 - y^2\ln y + \frac{1}{2}y^2 - \frac{1}{2}(\ln y)^2\right]_1^2 = \frac{\pi}{8}\left[y^4 + 2y^2 - 2y^2\ln y - (\ln y)^2\right]_1^2$

$= \frac{\pi}{8}\left[16 + 8 - 8\ln 2 - (\ln 2)^2 - 1 - 2\right] = \frac{\pi}{8}\left[21 - 8\ln 2 - (\ln 2)^2\right]$

f)

$x = a\cosh(y/a) \quad \Rightarrow \quad 1 + (dx/dy)^2 = 1 + \sinh^2(y/a) = \cosh^2(y/a).$

$S = 2\pi \int_{-a}^{a} a\cosh\left(\frac{y}{a}\right)\cosh\left(\frac{y}{a}\right)dy = 4\pi a\int_0^a \cosh^2\left(\frac{y}{a}\right)dy = 2\pi a\int_0^a \left[1 + \cosh\left(\frac{2y}{a}\right)\right]dy$

$= 2\pi a\left[y + \frac{a}{2}\sinh\left(\frac{2y}{a}\right)\right]_0^a = 2\pi a\left[a + \frac{a}{2}\sinh 2\right] = 2\pi a^2\left[1 + \frac{1}{2}\sinh 2\right] \text{ or } \frac{\pi a^2\left(e^2 + 4 - e^{-2}\right)}{2}$

LISTA 9:

Equações Diferenciais

$[g(x)] = x+1$

$f(x) = [g(x)]^{3/2}$

$$\int (x+1)^{1/2} = \int = \frac{1}{2}[g(x)]^{-1/2} \cdot 1$$

$$= \frac{(x+1)^{-1/2}}{2}$$

$$(3x+2)^{1/2} \rightarrow \int \frac{[g(x)]^{-1/2}}{2} \cdot 3 \rightarrow 3\int \frac{(3x+2)^{-1/2}}{2}$$

$[g(x)] = 3x+2$

$f(x) = [g(x)]^{1/2}$

$(x+1)^{-1/2}$

$u = x+1 \quad du = 1$

Lista 9: Equações Diferenciais

1. $y = 2 + e^{-x^3} \Rightarrow y' = -3x^2 e^{-x^3}$

Dada a equação diferencial $y' + 3x^2 y = 6x^2$ substituir $y' = -3x^2 e^{-x^3}$

$-3x^2 e^{-x^3} + 3x^2 y = 6x^2$, conhecemos $y = 2 + e^{-x^3}$

$-3x^2 e^{-x^3} + 3x^2(2 + e^{-x^3}) = 6x^2$

$-3x^2 e^{-x^3} + 6x^2 + 3x^2 e^{-x^3} = 6x^2 \quad \therefore \quad 6x^2 = 6x^2$

A igualdade é verdadeira, logo $y = 2 + e^{-x^3}$ é solução da equação diferencial $y' = -3x^2 e^{-x^3}$

2. $y = \dfrac{2 + \ln x}{x}$

$x^2 y' + xy = 1$ Resolver a equação diferencial: $y' + \frac{1}{x} y = \frac{1}{x^2}$

• $\mu(x) = e^{\int \frac{1}{x} dx} = e^{\ln x} = x$

• $\int \frac{1}{x^2} \cdot x \, dx = \int \frac{1}{x} dx = \ln x$

A solução da equação diferencial será: $y = \dfrac{\ln x + C}{x}$

Se $y(1) = 2$: $y(1) = \dfrac{\ln 1 + C}{1} = 2 \quad \therefore \quad C = 2$

$y = \dfrac{\ln x + 2}{x}$ é solução da equação diferencial $x^2 y' + xy = 1$ com $y(1) = 2$.

3. Para a equação diferencial ser linear deve estar na forma $y' + p(t)y = g(t)$

a) $y' + e^x y = x^2 y^2$ Não é linear, pois não pode ser convertida na forma $y' + p(t)y = g(t)$.

b) $y + \operatorname{sen} x = x^3 y' \longrightarrow y' - \frac{1}{x^3} y = \frac{\operatorname{sen} x}{x^3} \quad \therefore \begin{cases} p(x) = -1/x^3 \\ g(x) = \operatorname{sen} x / x^3 \end{cases}$

É linear pois está na forma $y' + p(x)y = g(x)$

c) $xy' + \ln x - x^2 y = 0 \longrightarrow y' - \frac{x^2}{x} y = \frac{-\ln x}{x} \quad \therefore \quad \begin{matrix} p(x) = -x \\ g(x) = \frac{-\ln x}{x} \end{matrix}$

É linear pois está na forma $y' + p(x)y = g(x)$.

d) $yy' = \operatorname{sen} x$ Não é linear pois não pode ser convertida na forma $y' + p(t)y = g(t)$

$y(0)=0$

4\. a) $y' + 2y = 2e^x$ $\begin{cases} p(x) = 2 \\ g(x) = 2e^x \end{cases}$ $y = \dfrac{e^{x^2}+C}{2x^2}$ $y = \dfrac{e^{x^2}+e}{2x^2}$

- $\mu(x) = e^{\int 2dx} = e^{2x}$
- $\int g(x)\mu(x)dx = \int 2e^x . e^{2x} dx = \int 2e^{3x} dx = \frac{2}{3}e^{3x}$ $0 = \dfrac{e^1+C}{2}$

Solução : $y = \dfrac{2/3\, e^{3x} + C}{e^{2x}} = \dfrac{2}{3}e^x + Ce^{-2x}$

b) $y' = x + 5y \longrightarrow y' - 5y = x$ $\begin{cases} p(x) = -5 \\ g(x) = x \end{cases}$ $0 = \dfrac{e^1+C}{2}$

- $\mu(x) = e^{\int -5dx} = e^{-5x}$ $0 = e + C$ $C = -e$
- $\int g(x)\mu(x)dx = \int xe^{-5x} dx$ Resolvendo por partes $u = x \rightarrow du = dx$
$v = \dfrac{e^{-5x}}{-5} \leftarrow dv = e^{-5x} dx$

$\therefore \int xe^{-5x}dx = \dfrac{xe^{-5x}}{-5} - \int -\dfrac{e^{-5x}}{5}dx = \dfrac{-xe^{-5x}}{5} - \dfrac{e^{-5x}}{25}$

Solução : $y = \dfrac{\dfrac{-xe^{-5x}}{5} - \dfrac{e^{-5x}}{25} + C}{e^{-5x}} = \dfrac{-x}{5} - \dfrac{1}{25} + Ce^{5x}$

c) $y' - xy = x$ $\begin{cases} \boxed{p(x) = -x} \\ g(x) = x \end{cases}$

- $\mu(x) = e^{\int -x dx} = e^{-x^2/2}$
- $\int g(x)\mu(x)dx = \int xe^{-x^2/2}dx$ Resolvendo por substituição $u = -\frac{x^2}{2}$ $du = -\frac{2x}{2}dx$

$\therefore \int xe^{-x^2/2}dx = -\int e^u du = -e^u = -e^{-x^2/2}$

Solução : $y = \dfrac{-e^{-x^2/2} + C}{e^{-x^2/2}} = -1 + Ce^{x^2/2}$

$2^{x} = 2^{x}$ $\frac{2x}{2}$

d) $xy' + 2y = e^{x^2} \longrightarrow y' + \frac{2}{x}y = \frac{e^{x^2}}{x}$ $\begin{cases} p(x) = \frac{2}{x} \\ g(x) = e^{x^2}/x \end{cases}$

- $\mu(x) = e^{\int \frac{2}{x}dx} = e^{2\ln x} = x^2$
- $\int g(x)\mu(x)dx = \int \frac{e^{x^2}}{x} . x^2dx = \int xe^{x^2}dx$ Resolvendo por substituição $u = x^2$ $du = 2xdx$

$\int xe^{x^2}dx = \int \frac{e^u}{2}du = \frac{e^u}{2} = \frac{e^{x^2}}{2}$

Solução : $y = \dfrac{e^{x^2}/2 + C}{x^2} = \dfrac{e^{x^2}+C}{2x^2}$

e) $y'\cos x = y\,\text{sen}x + \text{sen}2x \longrightarrow y' - \text{tg}x\, y = \dfrac{\text{sen}2x}{\cos x}$ $\begin{cases} p(x) = -\text{tg}x \\ g(x) = \text{sen}2x/\cos x \end{cases}$

- $\mu(x) = e^{\int -\text{tg}x\, dx} = e^{\ln|\cos x|} = \cos x$
- $\int g(x)\mu(x)dx = \int \frac{\text{sen}2x}{\cos x} . \cos x\, dx = \int \text{sen}2x\, dx = -\frac{1}{2}\cos 2x$

Solução : $y = \dfrac{-\frac{1}{2}\cos 2x + C}{\cos x} = \dfrac{-\frac{1}{2}(2\cos^2 x - 1) + C}{\cos x} = \dfrac{-\cos^2 x + \frac{1}{2} + C}{\cos x}$

f) $1 + xy = xy' \longrightarrow y' - y = \frac{1}{x}$ $\begin{cases} P(x) = -1 \\ g(x) = 1/x \end{cases}$

- $\mu(x) = e^{\int -1\,dx} = e^{-x}$
- $\int g(x)\mu(x)\,dx = \int \frac{1}{x} e^{-x}\,dx$

Solução : $y = \dfrac{\int \frac{e^{-x}}{x}\,dx + C}{e^{-x}}$

g) $\frac{dy}{dx} + 2xy = x^2 \longrightarrow y' + 2xy = x^2$ $\begin{cases} P(x) = 2x \\ g(x) = x^2 \end{cases}$

- $\mu(x) = e^{\int 2x\,dx} = e^{x^2}$
- $\int g(x)\mu(x)\,dx = \int x^2 e^{x^2} = \int x \cdot x e^{x^2}\,dx$ Integrar por partes

$u = x \longrightarrow du = dx$
$v = \frac{e^{x^2}}{2} \longleftarrow dv = xe^{x^2}\,dx$ $\therefore \int x \cdot x e^{x^2}\,dx = \frac{xe^{x^2}}{2} - \int \frac{e^{x^2}}{2}\,dx$

Solução : $y = \dfrac{\frac{xe^{x^2}}{2} - \int \frac{e^{x^2}}{2}\,dx + C}{e^{x^2}} = \frac{x}{2} + e^{-x^2}C - \frac{e^{-x^2}}{2}\int e^{x^2}\,dx$

h) $\frac{dy}{dx} = x\,\text{sen}\,2x + y\tan x \,, \; -\frac{\pi}{2} < x < \frac{\pi}{2}$

$y' - \tan x\, y = x\,\text{sen}\,2x$ $\begin{cases} P(x) = -\tan x \\ g(x) = x\,\text{sen}\,2x \end{cases}$

- $\mu(x) = e^{\int -\tan x\,dx} = e^{\ln|\cos x|} = \cos x$
- $\int g(x)\mu(x)\,dx = \int x\,\text{sen}\,2x \cos x\,dx = \int 2x\cos^2 x\,\text{sen}\,x\,dx$ Integrar por partes:

$u = 2x \to du = 2dx$
$v = \text{I} \leftarrow dv = \cos^2 x\,\text{sen}\,x\,dx$

I $v = \int \cos^2 x\,\text{sen}\,x\,dx$ Resolver por substituição:
$v = -\int u^2\,du = -\frac{\cos^3 x}{3}$ $\left(\begin{matrix} u = \cos x \\ du = -\text{sen}\,x\,dx \end{matrix}\right)$

$\therefore \int 2x\cos^2 x\,\text{sen}\,x\,dx = -\frac{2x\cos^3 x}{3} + \int \frac{2\cos^3 x}{3}\,dx = \frac{-2x\cos^3 x}{3} + \frac{2}{3}\int \cos x(1-\text{sen}^2 x)\,dx$

$\int 2x\cos^2 x\,\text{sen}\,x\,dx = \frac{-2x\cos^3 x}{3} + \frac{2}{3}\left[\int \cos x\,dx - \int \text{sen}^2 x\cos x\,dx\right]$

$= \frac{-2x\cos^3 x}{3} + \frac{2}{3}\left[\text{sen}\,x - \frac{\text{sen}^3 x}{3}\right]$, logo, $\int g(x)\mu(x)\,dx = \frac{-2x\cos^3 x}{3} + \frac{2}{3}\left[\text{sen}\,x - \frac{\text{sen}^3 x}{3}\right]$

Solução: $y = \dfrac{-\frac{2x\cos^3 x}{3} + \frac{2}{3}\left[\text{sen}\,x - \frac{\text{sen}^3 x}{3}\right] + C}{\cos x}$

$\therefore y = \frac{-2x\cos^2 x}{3} + \frac{2\tan x}{3}\left[1 - \frac{\text{sen}^2 x}{3}\right] + \frac{C}{\cos x}$

i) $(1+t)\dfrac{du}{dt} + u = 1+t \quad, t>0 \longrightarrow u' + \dfrac{1}{1+t} \cdot u = 1 \quad \begin{cases} P(t) = 1/(1+t) \\ g(t) = 1 \end{cases}$

• $\mu(t) = e^{\int \frac{1}{1+t}dt} = e^{\ln|1+t|} = 1+t$

• $\int g(t)\mu(t)\,dt = \int (1+t)\,dt = t + \dfrac{t^2}{2}$

Solução : $y = \dfrac{t + t^2/2 + C}{1+t} = \dfrac{t^2+2t+C}{2(t+1)}$

J) $xy' + xy + y = e^{-x} \longrightarrow xy' + (x+1)y = e^{-x}$

$y' + \left(\dfrac{x+1}{x}\right)y = \dfrac{e^{-x}}{x} \quad \begin{cases} P(x) = \left(\frac{x+1}{x}\right) \\ g(x) = e^{-x}/x \end{cases}$

• $\mu(x) = e^{\int \frac{x+1}{x}dx} = e^{\int 1 + \frac{1}{x}dx} = e^{x+\ln x}$

• $\int g(x)\mu(x)\,dx = \int \dfrac{e^{-x}}{x} \cdot e^{x+\ln x}\,dx = \int \dfrac{e^{-x+x} \cdot e^{-x+\ln x}}{x}\,dx = \int \dfrac{e^{-x} \cdot e^{\ln x}}{x}\,dx$

$\therefore \int g(x)\mu(x)\,dx = -e^{-x}$

Solução : $y = \dfrac{-e^{-x} + C}{e^x \cdot e^{\ln x}} = \dfrac{-e^{-x} + C}{xe^x}$

5. a) $y' + y = x + e^x \quad , \quad y(0) = 0 \quad \begin{cases} P(x) = 1 \\ g(x) = x + e^x \end{cases}$

• $\mu(x) = e^{\int 1dx} = e^x$

• $\int g(x)\mu(x)\,dx = \int (xe^x + e^{2x})\,dx = \int xe^x dx + \int e^{2x}dx$

$\rightarrow \int xe^x dx$ Integrar por partes : $u = x \quad du = dx$
$v = e^x \quad dv = e^x dx$

$\int xe^x dx = xe^x - \int e^x dx$

$\rightarrow \int e^{2x}dx = e^{2x}/2$

Portanto, $\int (xe^x + e^{2x})\,dx = xe^x - e^x + e^{2x}/2$

Solução : $y = \dfrac{xe^x - e^x + e^{2x}/2 + C}{e^x} = x - 1 + \dfrac{e^x}{2} + Ce^{-x}$

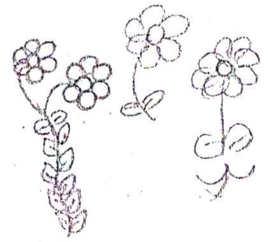
$y(0) = 0 - 1 + \dfrac{e^0}{2} + \dfrac{C}{e^0} = 0 \longrightarrow C - \dfrac{1}{2} = 0 \therefore C = 1/2$

• $y = x - 1 + \dfrac{e^x}{2} + \dfrac{e^{-x}}{2} = x - 1 + \cosh x$

b) $t\,dy/dt + 2y = t^3$, $t>0$, $y(1) = 0$

$y' + \frac{2}{t} y = t^2$ $\quad \begin{cases} p(t) = 2/t \\ g(t) = t^2 \end{cases}$

$\cdot\ \mu(t) = e^{\int 2/t\,dt} = e^{2\ln t} = t^2$



$\cdot\ \int g(t)\mu(t)\,dt = \int t^4\,dt = t^5/5$

Solução: $y = \dfrac{t^5/5 + C}{t^2} = \dfrac{t^3}{5} + C t^{-2}$

$y(1) = \frac{1}{5} + C = 0 \therefore C = -1/5$

$y = t^3/5 - t^{-2}/5$

c) $dv/dt - 2tv = 3t^2 e^{t^2}$, $v(0) = 5$

$v' - 2tv = 3t^2 e^{t^2}$ $\quad \begin{cases} p(t) = -2t \\ g(t) = 3t^2 e^{t^2} \end{cases}$

$\cdot\ \mu(t) = e^{\int -2t\,dt} = e^{-t^2}$

$\cdot\ \int g(t)\mu(t)\,dt = \int 3t^2 e^{t^2} e^{-t^2}\,dt = \int 3t^2\,dt = t^3$

Solução: $v = \dfrac{t^3 + C}{e^{-t^2}} = t^3 e^{t^2} + C e^{t^2}$

$v(0) = 0e^0 + Ce^0 = 5 \therefore C = 5$

$y = t^3 e^{t^2} + 5e^{t^2}$

d) $(1+x^2)y' + 2xy = 3\sqrt{x}$, $y(0) = 2$

$y' + (2x/1+x^2)\,y = (3\sqrt{x}/1+x^2)$ $\quad \begin{cases} p(x) = 2x/1+x^2 \\ g(x) = 3\sqrt{x}/1+x^2 \end{cases}$

$\cdot\ \mu(x) = e^{\int \frac{2x}{1+x^2}dx} = e^{\int \frac{1}{u}du}$ com $u = 1+x^2$ e $du = 2x\,dx$

$\therefore\ \mu(x) = e^{\ln u} = u = 1+x^2$

$\cdot\ \int g(x)\mu(x)\,dx = \int \frac{3\sqrt{x}}{1+x^2} * (1+x^2)\,dx = \int 3\sqrt{x} = 2x^{3/2}$

Solução: $y = \dfrac{2x^{3/2} + C}{1+x^2}$; $y(0) = \dfrac{2(0)^{3/2} + C}{1+0} = 2 \therefore C = 2$

$y = \dfrac{2x^{3/2} + 2}{1+x^2}$

e) $x^2\,dy/dx + 2xy = \cos x$, $y(\pi) = 0$

$y' + \frac{2}{x} y = \frac{\cos x}{x^2}$ $\quad \begin{cases} p(x) = 2/x \\ g(x) = \cos x/x^2 \end{cases}$

$\cdot\ \mu(x) = e^{\int +2/x\,dx} = e^{+2\ln x} = x^{+2}$

$\cdot\ \int g(x)\mu(x)\,dx = \int \cos x\,dx = \operatorname{sen} x$

Solução: $y = \dfrac{\operatorname{sen} x + C}{x^2}$; $y(\pi) = \dfrac{\operatorname{sen}(\pi) + C}{\pi^2} = 0 \therefore C = 0$

$y = \dfrac{\operatorname{sen} x}{x^2}$

f) $x \, dy/dx - y/x+1 = x$, $y(1) = 0$, $x > 0$

$y' - \dfrac{y}{x(x+1)} = 1$ $\quad P(x) = -1/x(x+1)$

$g(x) = 1$

$\cdot \mu(x) = e^{\int -\frac{1}{x(x+1)}} = e^{-(\ln|x| - \ln|x+1|)} = \dfrac{x+1}{x}$

$\cdot \int g(x)\mu(x)dx = \int \dfrac{x+1}{x} dx = \int 1 + \dfrac{1}{x} dx = x + \ln|x|$

Solução: $y = \dfrac{x + \ln|x| + C}{\frac{x+1}{x}} = \dfrac{x}{x+1}\left(x + \ln|x| + C\right)$

$y(1) = \frac{1}{2}(1 + \ln|1| + C) = 0 \therefore C = -1$

$y = x/x+1 \,(x + \ln|x| - 1)$

6. a) $dy/dx = \cancel{e^{2x}} + 1 \longrightarrow \dfrac{1}{y^2+1} dy = dx \therefore \int \dfrac{1}{y^2+1} dy = \int dx$

$tg^{-1} y = x \therefore y = tg x + C$

b) $dy/dx = e^{2x}/4y^3 \longrightarrow 4y^3 dy = e^{2x} dx \therefore \int 4y^3 dy = \int e^{2x} dx$

$y^4 = e^{2x}/2 \longrightarrow y = \pm \sqrt[4]{\frac{1}{2}e^{2x} + C}$

c) $yy' = x \longrightarrow y\,dy = x\,dx \therefore \int y\,dy = \int x\,dx \longrightarrow \dfrac{y^2}{2} = \dfrac{x^2}{2} + C$

$y = \pm \sqrt[2]{x^2 + 2C}$

d) $y' = xy \longrightarrow 1/y \, dy = x\,dx \therefore \int \frac{1}{y} dy = \int x\,dx \longrightarrow \ln|y| = \frac{x^2}{2} + C$

$y = e^c e^{x^2/2} = K e^{x^2/2}$

e) $dy/dt = te^t / y\sqrt{1+y^2} \longrightarrow y\sqrt{1+y^2}\,dy = te^t\,dt$

$\int y\sqrt{1+y^2}\,dy = \int te^t\,dt \rightarrow \frac{1}{3}(1+y^2)^{3/2} = te^t - e^t + C$

$y = \pm\sqrt{3(te^t - e^t + C)^{2/3} - 1}$

f) $y' = xy/2\ln y \quad \dfrac{2\ln y}{y} dy = x\,dx \longrightarrow 2\int \dfrac{\ln y}{y} dy = \int x\,dx$

$\longrightarrow \int \dfrac{2\ln y}{y} dy \left\langle \begin{matrix} u = \ln y \\ du = 1/y\,dy \end{matrix} \right. \therefore \int 2u\,du = u^2 = \ln^2 y$

Logo: $\ln^2 y = \dfrac{x^2}{2} + C \therefore \ln y = \pm\sqrt{x^2/2 + C} \longrightarrow y = e^{\pm\sqrt{x^2/2 + C}}$