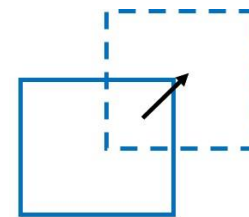
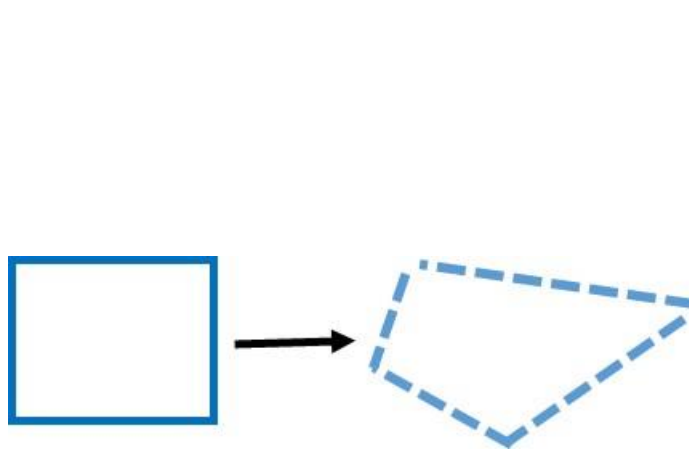
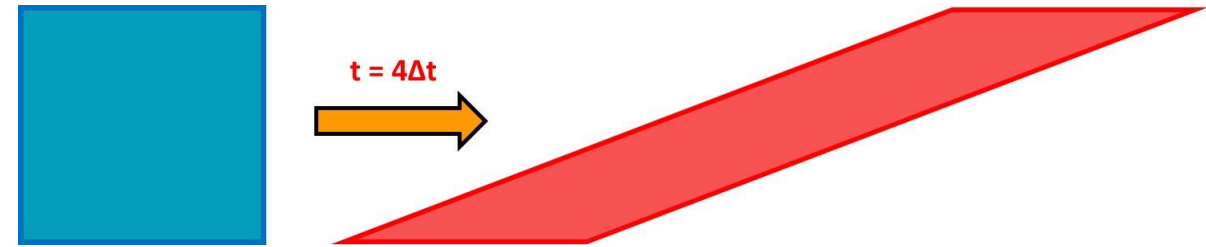
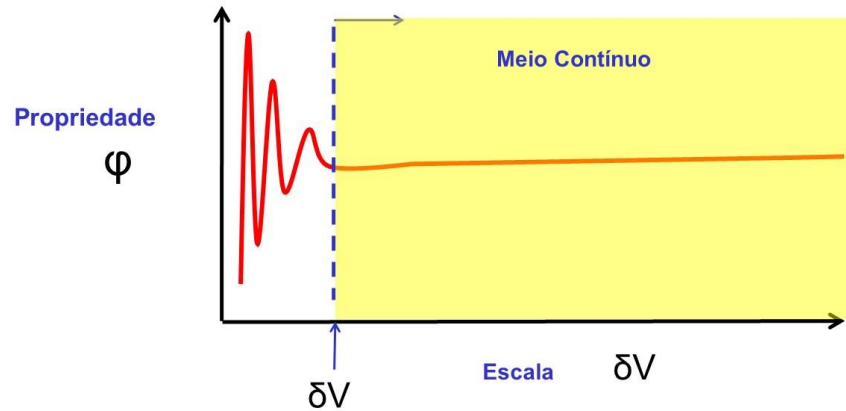
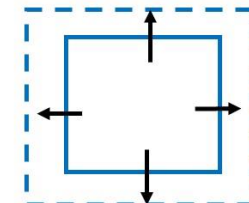


PQI 3203: FENÔMENOS DE TRANSPORTE I

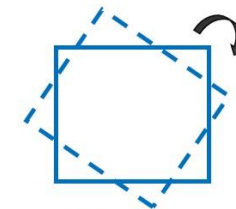
Campo de velocidades



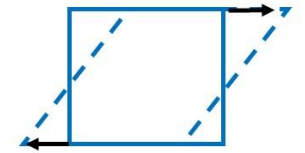
translação



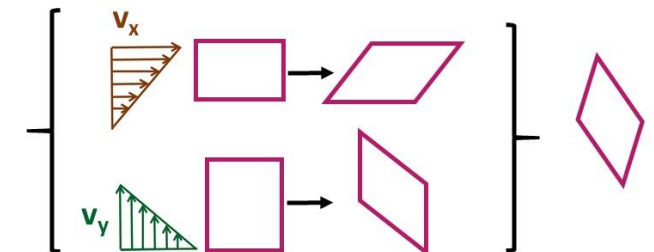
dilatação/contração



rotação



Cisalhamento simples



PQI 3203: FENÔMENOS DE TRANSPORTE I

Derivadas no tempo- Observações

- MEIO CONTÍNUO – CAMPO

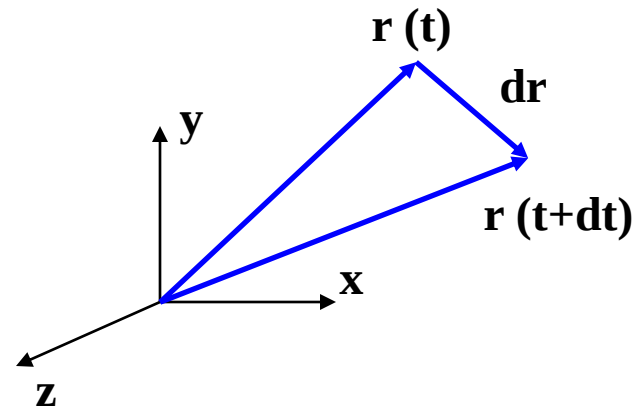
PROPRIEDADE : $\varphi(\vec{r}(t), t)$

$$d\varphi = \left(\frac{d\varphi}{dt} \right)_{\vec{r}} dt + \left(\frac{d\varphi}{d\vec{r}} \right)_t d\vec{r}$$

$$d\varphi = \left(\frac{d\varphi}{dt} \right)_{\vec{r}} dt + \text{grad} \varphi \bullet d\vec{r}$$

$$\frac{d\varphi}{dt} = \left(\frac{\partial \varphi}{\partial t} \right) + \text{grad} \varphi \bullet \vec{w}$$

instantâneo



$$\vec{w} = \frac{d\vec{r}}{dt}$$

Velocidade de observação

PQI 3203: FENÔMENOS DE TRANSPORTE I
Derivadas no tempo- Observações Euler e Lagrange

$$d\varphi = \left(\frac{d\varphi}{dt}\right)_{\vec{r}} dt + \left(\frac{d\varphi}{d\vec{r}}\right)_t d\vec{r} \longrightarrow \frac{d\varphi}{dt} = \left(\frac{d\varphi}{dt}\right)_{\vec{r}} \frac{dt}{dt} + \left(\frac{d\varphi}{d\vec{r}}\right)_t \frac{d\vec{r}}{dt}$$

$$\varphi(x, y, z, t) = \varphi[x(t), y(t), z(t), t]$$

$$\frac{d\varphi}{dt} = \left(\frac{\partial\varphi}{\partial t}\right)_{x,y,z} + \left(\frac{\partial\varphi}{\partial x}\right)_t \left(\frac{dx}{dt}\right) + \left(\frac{\partial\varphi}{\partial y}\right)_t \left(\frac{dy}{dt}\right) + \left(\frac{\partial\varphi}{\partial z}\right)_t \left(\frac{dz}{dt}\right)$$

$$\frac{d\varphi}{dt} = \left(\frac{\partial\varphi}{\partial t}\right)_{x,y,z} + \left[v_x \vec{i} + v_y \vec{j} + v_z \vec{k}\right] \cdot \left[\left(\frac{\partial\varphi}{\partial x}\right) \vec{i} + \left(\frac{\partial\varphi}{\partial y}\right) \vec{j} + \left(\frac{\partial\varphi}{\partial z}\right) \vec{k}\right]$$

OPERADOR

$$\frac{D\varphi}{Dt} = \frac{\partial\varphi}{\partial t} + \vec{v} \cdot \text{grad}\varphi$$

$$\frac{D\varphi}{Dt} = \frac{\partial\varphi}{\partial t} + \vec{v} \cdot \vec{\nabla}\varphi$$

instantâneo

PQI 3203: FENÔMENOS DE TRANSPORTE I

Derivadas no tempo- Observações Euler e Lagrange

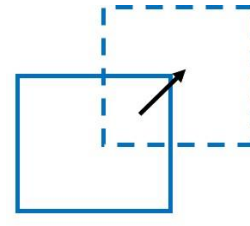
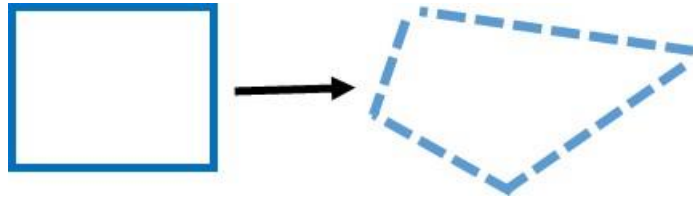
Observação	Observação	$\left(\frac{d \varphi}{d t} \right)$
$\vec{w} = \mathbf{0}$	EULER VARIAÇÃO LOCAL	$\left(\frac{\partial \varphi}{\partial t} \right)_{\vec{r}}$
$\vec{w} = \vec{v}$	LAGRANGE MATERIAL OU SUBSTANTIVA	$\left(\frac{D \varphi}{D t} \right) = \frac{\partial \varphi}{\partial t} + \vec{v} \cdot \text{grad} \varphi$
$\vec{w}$	TOTAL	$\left(\frac{d \varphi}{d t} \right) = \frac{\partial \varphi}{\partial t} + \vec{w} \cdot \text{grad} \varphi$

OPERADOR

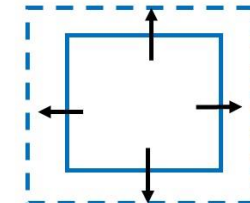
$$\frac{D}{D t} = \frac{\partial}{\partial t} + \vec{v} \cdot \text{grad}$$

PQI 3203: FENÔMENOS DE TRANSPORTE I

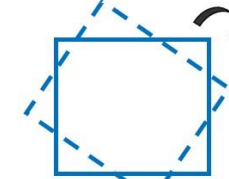
Campo de velocidades



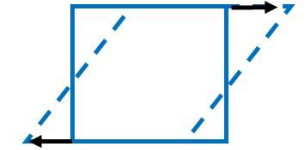
translação



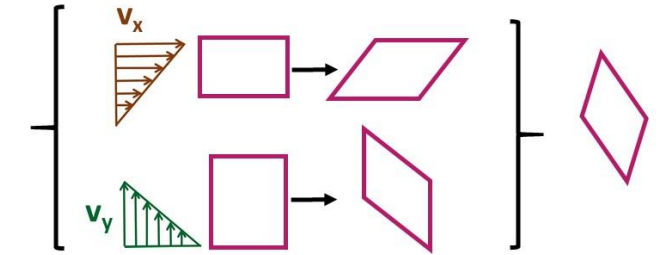
dilação/contração



rotação



Cisalhamento simples



$$\vec{a} = \frac{D\vec{v}}{Dt} = \frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \text{grad} \vec{v}$$

$$\frac{dv_x}{dt} = \left(\frac{\partial v_x}{\partial t} \right)_{x,y,z} + \left(\frac{\partial v_x}{\partial x} \right)_t v_x + \left(\frac{\partial v_x}{\partial y} \right)_t v_y + \left(\frac{\partial v_x}{\partial z} \right)_t v_z$$

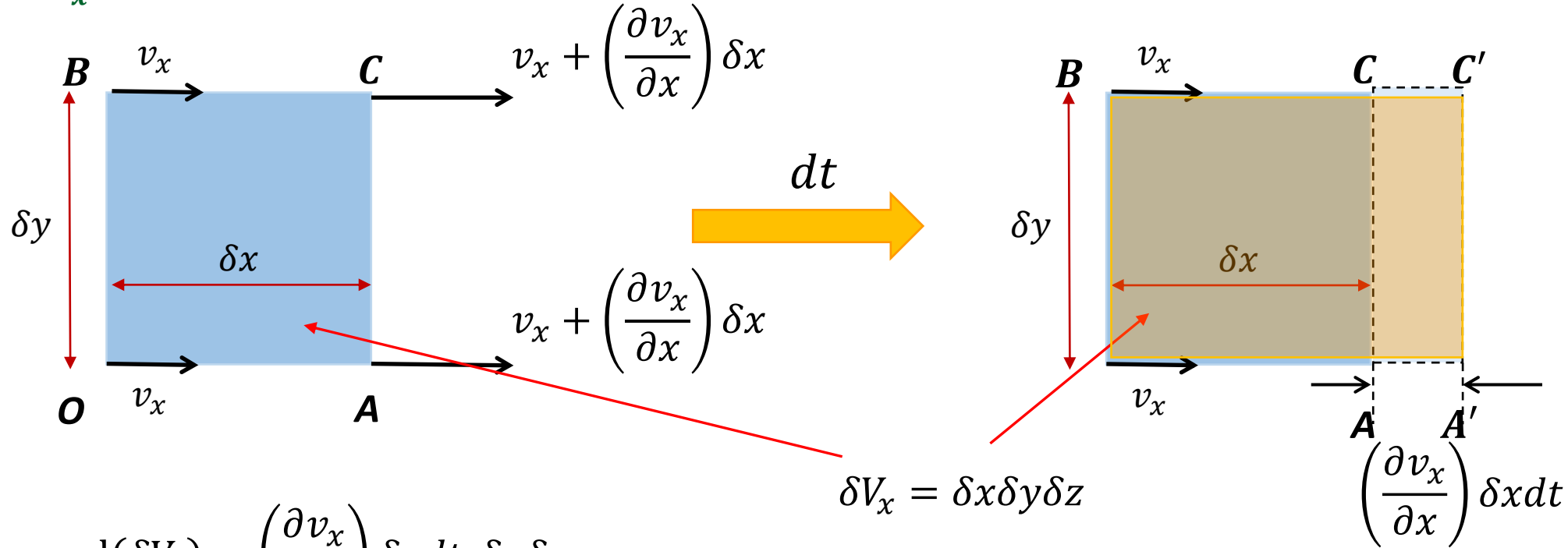
$$\frac{dv_y}{dt} = \left(\frac{\partial v_y}{\partial t} \right)_{x,y,z} + \left(\frac{\partial v_y}{\partial x} \right)_t v_x + \left(\frac{\partial v_y}{\partial y} \right)_t v_y + \left(\frac{\partial v_y}{\partial z} \right)_t v_z$$

$$\frac{dv_z}{dt} = \left(\frac{\partial v_z}{\partial t} \right)_{x,y,z} + \left(\frac{\partial v_z}{\partial x} \right)_t v_x + \left(\frac{\partial v_z}{\partial y} \right)_t v_y + \left(\frac{\partial v_z}{\partial z} \right)_t v_z$$

PQI 3203: FENÔMENOS DE TRANSPORTE I

Campo de velocidades

v_x varia com x



$$d(\delta V_x) = \left(\frac{\partial v_x}{\partial x} \right) \delta x dt \cdot \delta y \delta z$$

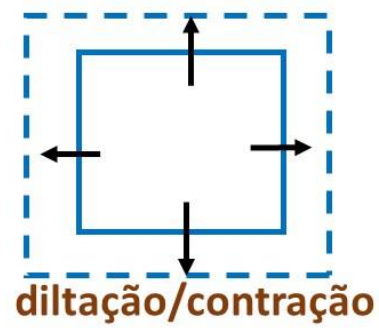
$$\frac{1}{\delta V_x} \frac{d(\delta V_x)}{dt} = \left(\frac{\partial v_x}{\partial x} \right)$$

$$\frac{1}{\delta V_y} \frac{d(\delta V_y)}{dt} = \left(\frac{\partial v_y}{\partial y} \right)$$

$$\frac{1}{\delta V_z} \frac{d(\delta V_z)}{dt} = \left(\frac{\partial v_z}{\partial z} \right)$$

$$\frac{1}{\delta V} \frac{d(\delta V)}{dt} = \left(\frac{\partial v_x}{\partial x} \right) + \left(\frac{\partial v_y}{\partial y} \right) + \left(\frac{\partial v_z}{\partial z} \right) = \text{div } \vec{v}$$

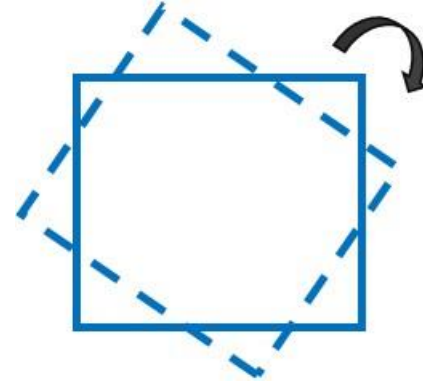
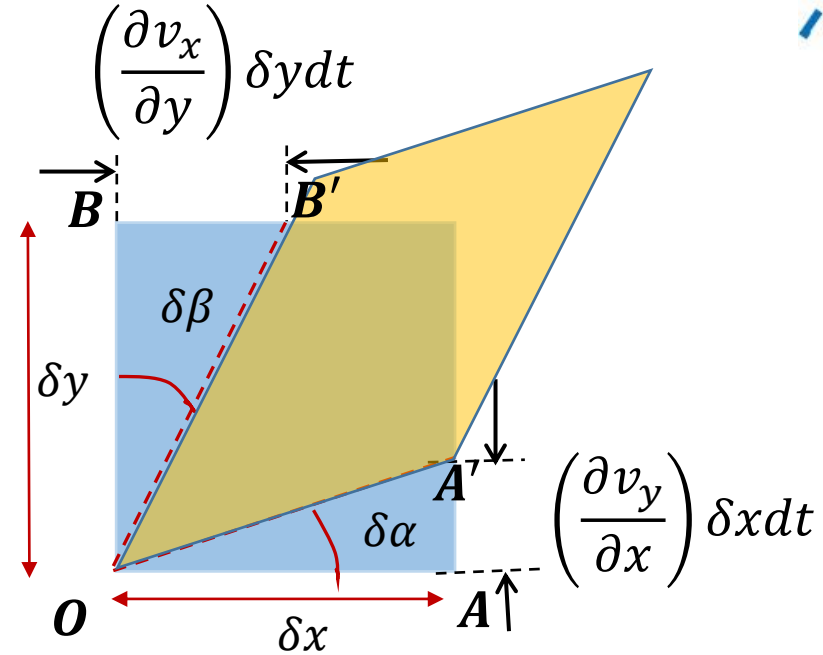
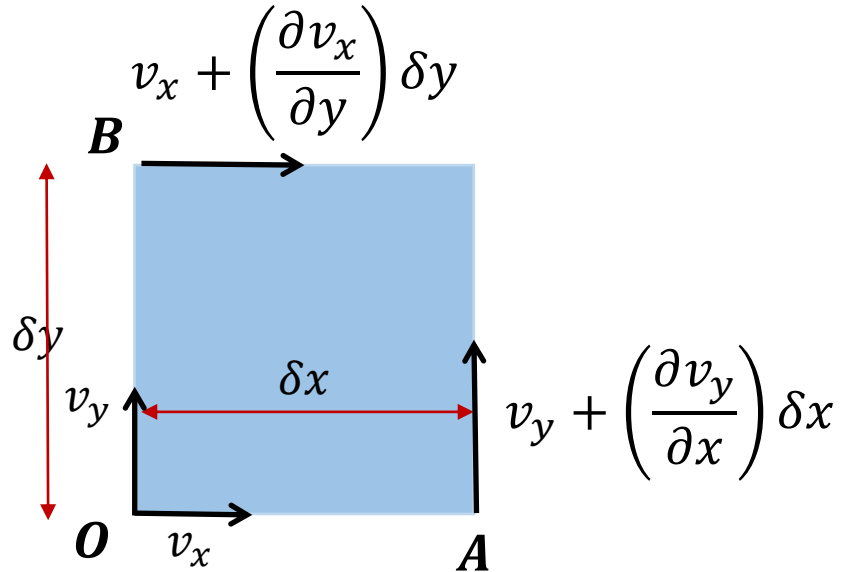
Escoamento incompressível $\Rightarrow \text{div } \vec{v} = 0$



PQI 3203: FENÔMENOS DE TRANSPORTE I

Rotação

v_x varia com y e v_y varia com x



$$\tan(\delta\alpha) \cong \delta\alpha \cong \left(\frac{\partial v_y}{\partial x}\right) \frac{\delta x dt}{\delta x} = \left(\frac{\partial v_y}{\partial x}\right) dt$$

$$\omega_{OA} = \lim_{dt \rightarrow 0} \frac{\delta\alpha}{dt} = \left(\frac{\partial v_y}{\partial x}\right)$$

Se $\left(\frac{\partial v_y}{\partial x}\right) > 0$ anti horário

$$\omega_{OB} = \lim_{dt \rightarrow 0} \frac{\delta\beta}{dt} = \left(\frac{\partial v_x}{\partial y}\right)$$

Se $\left(\frac{\partial v_x}{\partial y}\right) > 0$ horário

PQI 3203: FENÔMENOS DE TRANSPORTE I

Rotação

$$\omega_{OA} = \lim_{dt \rightarrow 0} \frac{\delta\alpha}{dt} = \left(\frac{\partial v_y}{\partial x} \right)$$

Se $\left(\frac{\partial v_y}{\partial x} \right) > 0$ anti horário

$$\omega_{OB} = \lim_{dt \rightarrow 0} \frac{\delta\beta}{dt} = \left(\frac{\partial v_x}{\partial y} \right)$$

Se $\left(\frac{\partial v_x}{\partial y} \right) > 0$ horário

Velocidade angular média, ω_z

$$\omega_z = \frac{\omega_{OA} - \omega_{OB}}{2} = \frac{1}{2} \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \right)$$

Velocidade angular média, ω

$$\omega_x = \frac{1}{2} \left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z} \right)$$

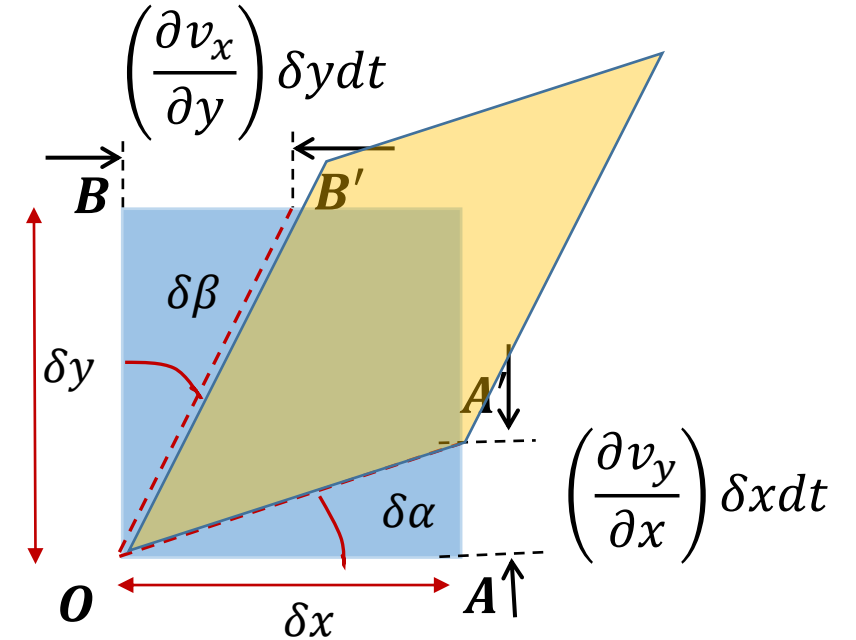
$$\omega_y = \frac{1}{2} \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} \right)$$

$$\omega_z = \frac{1}{2} \left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \right)$$

$$\vec{\omega} = \omega_x \vec{i} + \omega_y \vec{j} + \omega_z \vec{k} = \frac{1}{2} \text{rot } \vec{v} = \frac{1}{2} \nabla \times \vec{v}$$

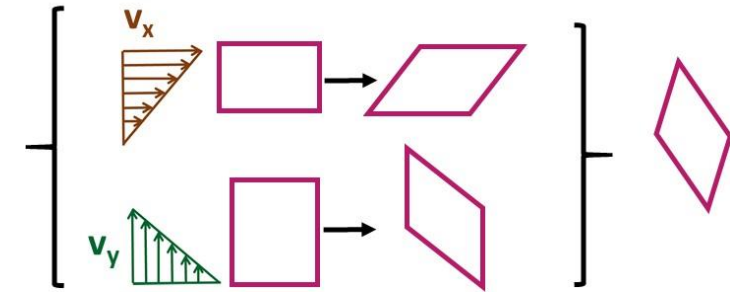
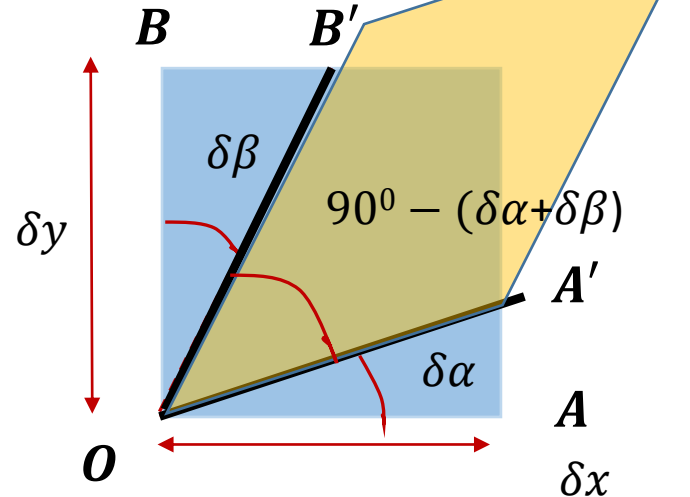
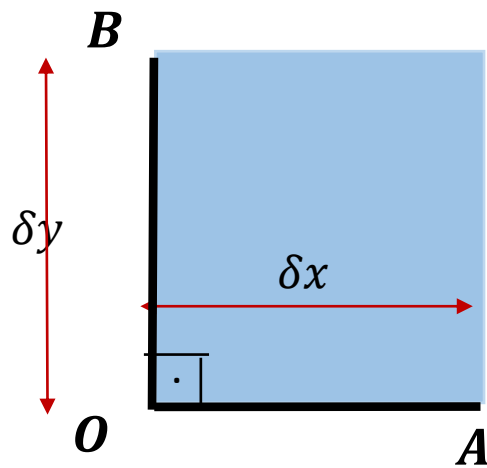
Vorticidade: $\vec{\zeta} = 2 \vec{\omega} = \text{rot } \vec{v}$

Escoamento irrotacional $\Rightarrow \text{rot } \vec{v} = 0$



PQI 3203: FENÔMENOS DE TRANSPORTE I

Deformação Angular



$$\delta\gamma = \delta\alpha + \delta\beta \quad ; \quad \delta\alpha \cong \left(\frac{\partial v_y}{\partial x} \right) dt \quad ; \quad \delta\beta \cong \left(\frac{\partial v_x}{\partial y} \right) dt$$

Taxa de deformação angular

$$\dot{\gamma} = \lim_{dt \rightarrow 0} \frac{\delta\gamma}{dt} = \lim_{dt \rightarrow 0} \left[\frac{\left(\frac{\partial v_y}{\partial x} \right) dt + \left(\frac{\partial v_x}{\partial y} \right) dt}{dt} \right]$$

$$\dot{\gamma} = \lim_{dt \rightarrow 0} \frac{\delta\gamma}{dt} = \frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y}$$

$$\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x}$$

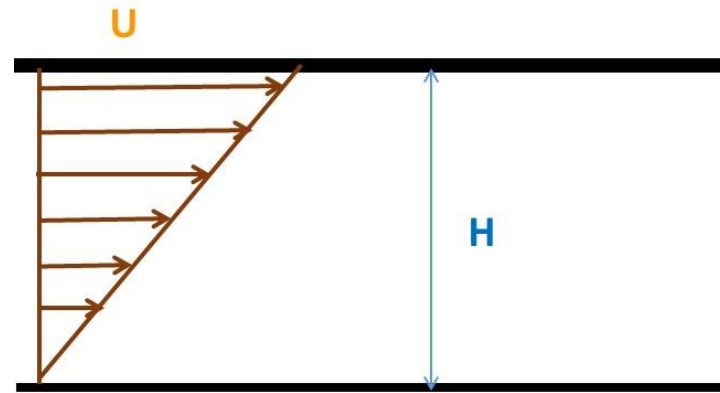
$$\frac{\partial v_z}{\partial y} + \frac{\partial v_y}{\partial z}$$

Relacionado à
Tensão de
Cisalhamento

Exercício

Considere o escoamento de um fluido entre duas placas paralelas , no qual

$$v_x = U(y/H)$$



Calcule:

- a) Divergente da velocidade. O escoamento é compressível?
- b) A vorticidade
- c) A taxa de deformação

Exercício

Divergente da velocidade

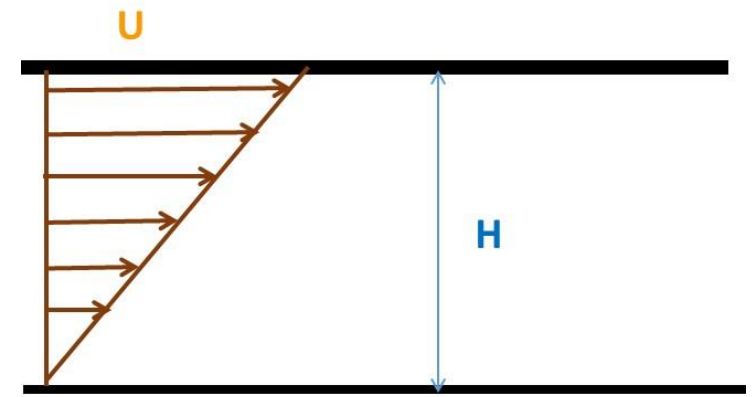
$$v_x = \frac{y}{H} U \quad v_y = 0 \quad v_z = 0$$

$$\operatorname{div} \vec{v} = \left(\frac{\partial v_x}{\partial x} \right) + \left(\frac{\partial v_y}{\partial y} \right) + \left(\frac{\partial v_z}{\partial z} \right) = 0$$

Vorticidade

$$\left(\frac{\partial v_z}{\partial y} - \frac{\partial v_y}{\partial z} \right) = 0 \quad \left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x} \right) = 0$$

$$\vec{\zeta} = \operatorname{rot} \vec{v} = -\frac{\partial v_x}{\partial y} \vec{k} = -\frac{U}{H} \vec{k}$$



$$\left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \right) = 0 - \frac{U}{H} = -\frac{U}{H}$$

Exercício

Taxa de deformação angular

$$\frac{\partial v_x}{\partial z} + \frac{\partial v_z}{\partial x} = 0$$

$$\frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y} = 0 + \frac{U}{H} = \frac{U}{H}$$

$$\frac{\partial v_z}{\partial y} + \frac{\partial v_y}{\partial z} = 0$$

