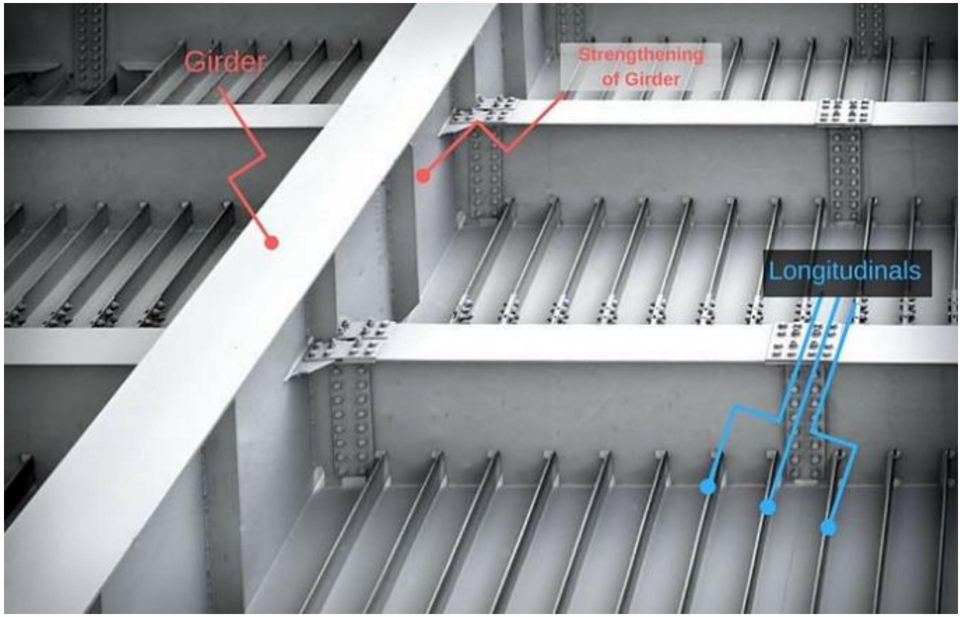
DEPARTAMENTO DE ENGEHARIA NAVAL E OCEÂNICA ESCOLA POLITÉCNICA DA USP

Análise de Vigas : $\delta\,(mm)$



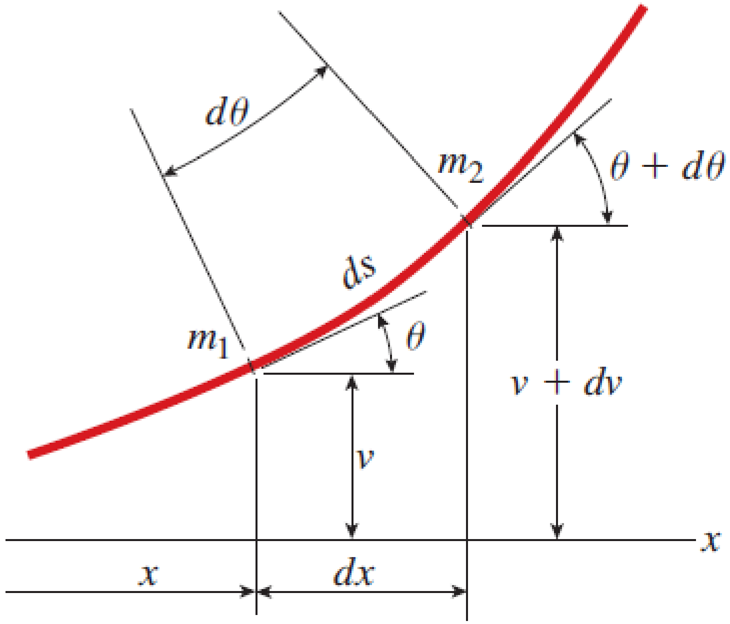
PNV 3212 – Mecânica Dos Sólidos I
2020

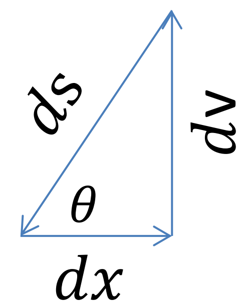
Deflexões em vigas

$$\boldsymbol{\kappa(x) = \frac{1}{\rho(x)}}$$

$$\boxed{\boldsymbol{\kappa = \frac{d\theta}{ds}}}$$

Variação da inclinação por unidade de comprimento





$$\frac{dv}{dx} = \tan\theta \qquad \frac{dv}{ds} = \sin\theta \qquad \frac{dx}{ds} = \cos\theta$$

very small angles of rotation, very small deflections, and very small curvatures.

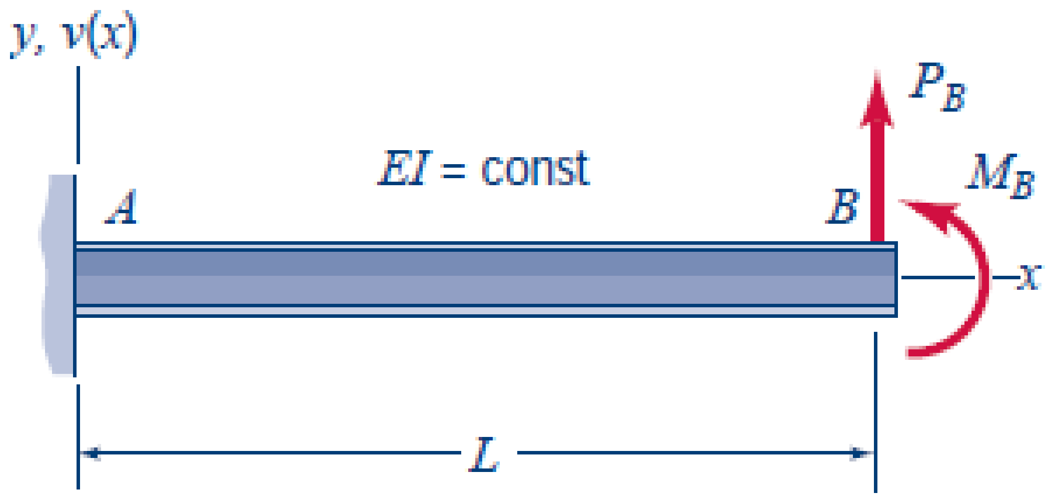
$$\frac{M(x)}{\mathrm{EI}} = \frac{d\boldsymbol{\theta}}{d\boldsymbol{x}}$$

$$\boxed{\boldsymbol{\frac{M(x)}{\mathrm{EI}} = \frac{d^2v(x)}{dx^2}}}$$

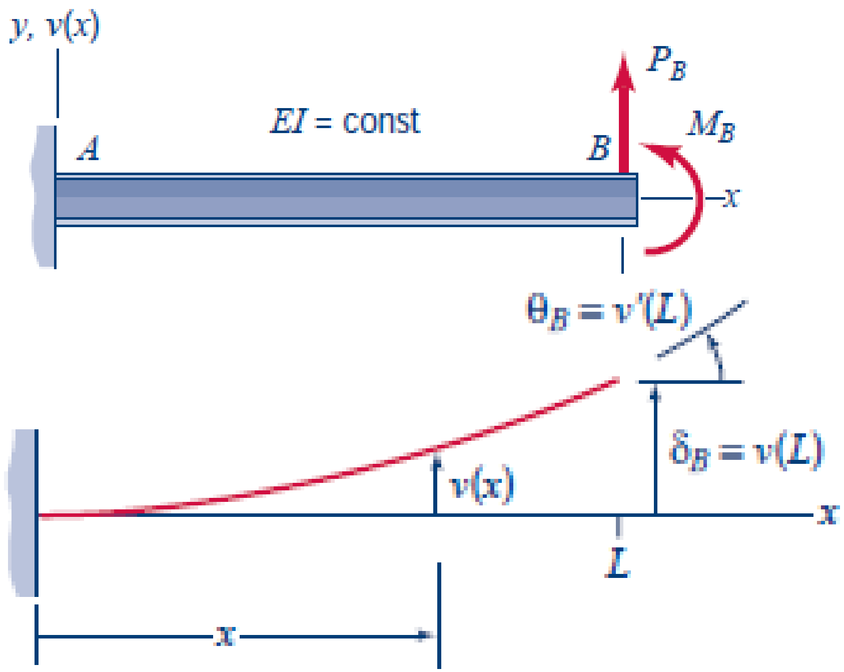
Flexão de uma viga → deflexão vertical + **rotação**

Deflexões em vigas

Exemplo 1: Determine a curva elástica para a viga engastada mostrada na figura. Quais são os valores da deflexão e rotação na extremidade ?.

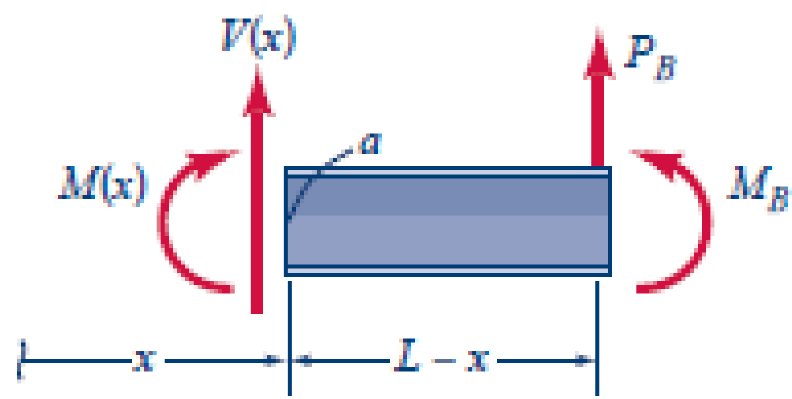


Deflexões em vigas



$$\frac{M(x)}{EI} = \frac{d^2v(x)}{dx^2}$$

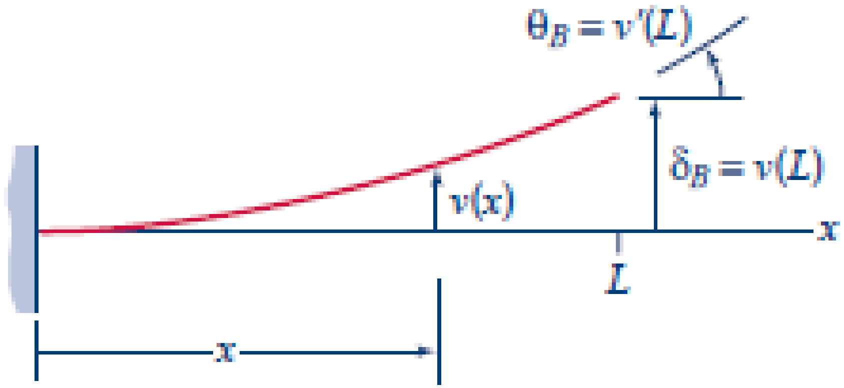
D.C.L



$$\sum M_z = 0 \quad +$$

$$M(x) = P_B(L - x) + M_B$$

Deflexões em vigas



$$\frac{M(x)}{EI} = \frac{d^2 v(x)}{dx^2}$$

$$M(x) = P_B(L - x) + M_B$$

$$\Downarrow$$

$$\frac{d^2 v(x)}{dx^2} = \frac{P_B(L - x) + M_B)}{EI}$$

$$\Downarrow$$

$$\frac{d}{dx}\left(\frac{dv(x)}{dx}\right) = \frac{P_B(L - x) + M_B)}{EI}$$

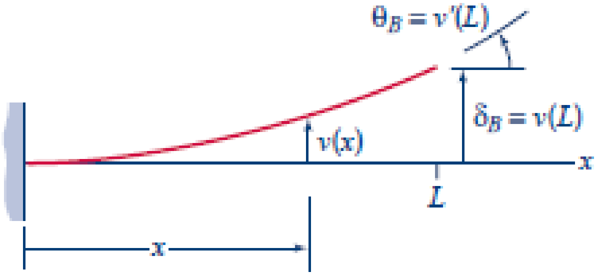
$$\Downarrow$$

$$\int d\left(\frac{dv(x)}{dx}\right) = \int \frac{P_B(L - x) + M_B)}{EI}\,dx$$

$$\Leftarrow$$

$$\frac{dv(x)}{dx} = \int \frac{P_B(L - x) + M_B)}{EI}\,dx$$

Deflexões em vigas



$$\frac{dv(x)}{dx} = \int \left[\frac{P_B(L-x) + M_B)}{EI}\right] dx + C_1$$

$$\frac{dv(x)}{dx} = \frac{P_B L}{EI}x - \frac{P_B}{2EI}x^2 + \frac{M_B}{EI}x + C_1$$

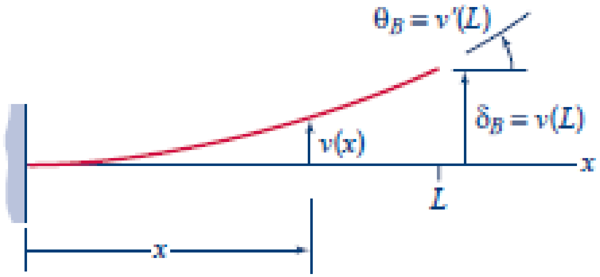
$$\Downarrow$$

$$dv(x) = \left(\frac{P_B L}{EI}x - \frac{P_B}{2EI}x^2 + \frac{M_B}{EI}x + C_1\right) dx$$

$$\Downarrow$$

$$\int dv(x) = \int \left(\frac{P_B L}{EI}x - \frac{P_B}{2EI}x^2 + \frac{M_B}{EI}x + C_1\right) dx + C_2$$

Deflexões em vigas



$$\int dv(x) = \int \left(\frac{P_B L}{EI} x - \frac{P_B}{2EI} x^2 + \frac{M_B}{EI} x + C_1\right) dx + C_2$$

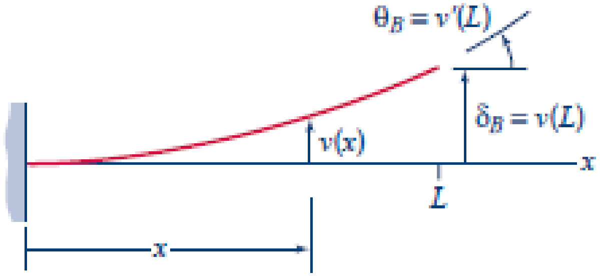
$$\downarrow$$

$$v(x) = \int \left(\frac{P_B L}{EI} x - \frac{P_B}{2EI} x^2 + \frac{M_B}{EI} x + C_1\right) dx + C_2$$

$$\downarrow$$

$$\boxed{v(x) = \frac{P_B L}{2EI} x^2 - \frac{P_B}{6EI} x^3 + \frac{M_B}{2EI} x^2 + C_1 x + C_2}$$

Deflexões em vigas



$$v(x) = \frac{P_B L}{2EI}x^2 - \frac{P_B}{6EI}x^3 + \frac{M_B}{2EI}x^2 + C_1 x + C_2$$

Condição de Contorno

$$\frac{d}{dx}v(0) = 0$$

$$\frac{dv(x)}{dx} = \frac{P_B L}{EI}x - \frac{P_B}{2EI}x^2 + \frac{M_B}{EI}x + C_1$$

$$0 = \frac{P_B L}{EI}0 - \frac{P_B}{2EI}0^2 + \frac{M_B}{EI}0 + C_1$$

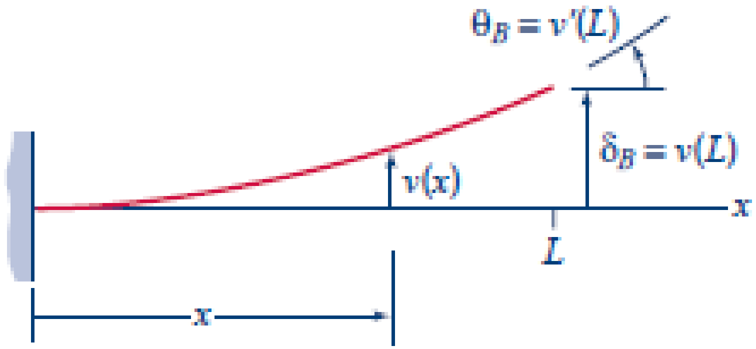
$$0 = C_1$$

$$v(0) = 0$$

$$0 = \frac{P_B L}{2EI}0^2 - \frac{P_B}{6EI}0^3 + \frac{M_B}{2EI}0^2 + C_1 0 + C_2$$

$$0 = C_2$$

Deflexões em vigas

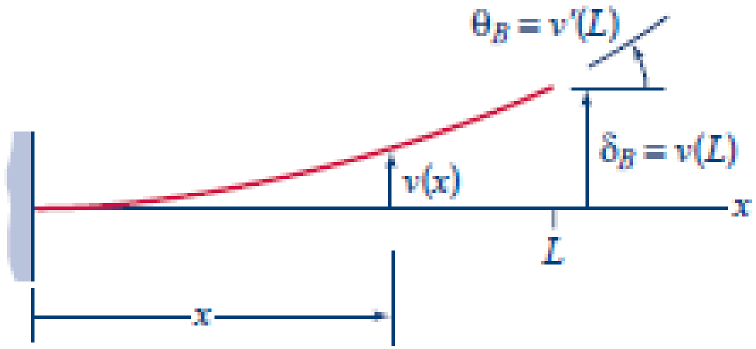


$$v(x) = \frac{P_B L}{2EI}x^2 - \frac{P_B}{6EI}x^3 + \frac{M_B}{2EI}x^2$$

$$v(x) = \left(\frac{M_B}{2EI} + \frac{P_B L}{2EI}\right)x^2 - \frac{P_B}{6EI}x^3$$

$$v(x) = \frac{1}{2EI}\left[(M_B + P_B L)x^2 - \frac{P_B}{3}x^3\right]$$

Deflexões em vigas



$$v(x) = \frac{1}{2EI}\left[(M_B + P_B L)x^2 - \frac{P_B}{3}x^3\right]$$

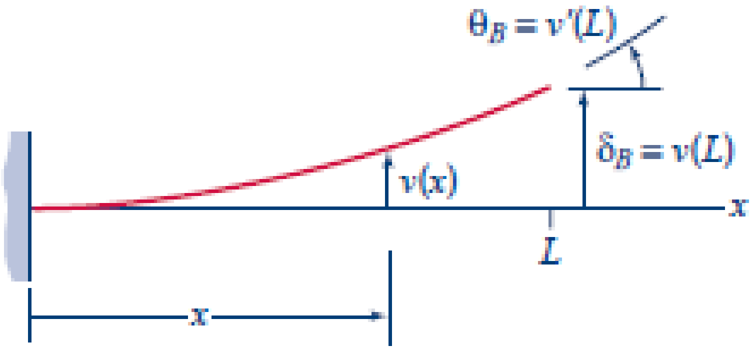
x=L

$$v(L) = \delta_B = \frac{1}{2EI}\left[M_B L^2 + \frac{2P_B}{3}L^3\right]$$

$$\frac{dv(L)}{dx} = \theta_B = \frac{P_B L}{EI}L - \frac{P_B}{2EI}L^2 + \frac{M_B}{EI}L$$

$$\boxed{\frac{dv(L)}{dx} = \theta_B = \frac{P_B}{2EI}L^2 + \frac{M_B}{EI}L}$$

Deflexões em vigas



$$v(L) = \delta_B = \frac{1}{2EI}\left[M_B L^2 + \frac{2P_B}{3} L^3\right]$$

M_B=0

$$\delta_B = \frac{P_B L^3}{3EI}$$

$$\frac{Nmm^3}{\frac{N}{mm^2} mm^4}$$

Superposição de Efeitos!!!

Check Unidades!!!

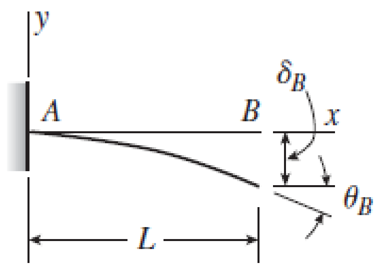
$$\frac{dv(L)}{dx} = \theta_B = \frac{P_B}{2EI} L^2 + \frac{M_B}{EI} L$$

$$\theta_B = \frac{P_B L^2}{2EI}$$

$$\frac{Nmm^2}{\frac{N}{mm^2} mm^4}$$

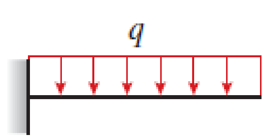
Deflections and Slopes of Beams

TABLE G-1 DEFLECTIONS AND SLOPES OF CANTILEVER BEAMS



v = deflection in the y direction (positive upward)
$v' = dv/dx$ = slope of the deflection curve
$\delta_B = -v(L)$ = deflection at end B of the beam (positive downward)
$\theta_B = -v'(L)$ = angle of rotation at end B of the beam (positive clockwise)
EI = constant

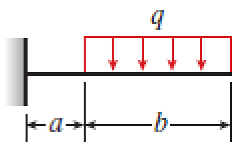
1



$$v = -\frac{qx^2}{24EI}(6L^2 - 4Lx + x^2) \qquad v' = -\frac{qx}{6EI}(3L^2 - 3Lx + x^2)$$

$$\delta_B = \frac{qL^4}{8EI} \qquad \theta_B = \frac{qL^3}{6EI}$$

3



$$v = -\frac{qbx^2}{12EI}(3L + 3a - 2x) \qquad (0 \le x \le a)$$

$$v' = -\frac{qbx}{2EI}(L + a - x) \qquad (0 \le x \le a)$$

$$v = -\frac{q}{24EI}(x^4 - 4Lx^3 + 6L^2x^2 - 4a^3x + a^4) \qquad (a \le x \le L)$$

$$v' = -\frac{q}{6EI}(x^3 - 3Lx^2 + 3L^2x - a^3) \qquad (a \le x \le L)$$

$$\text{At } x = a: \quad v = -\frac{qa^2b}{12EI}(3L + a) \qquad v' = -\frac{qabL}{2EI}$$

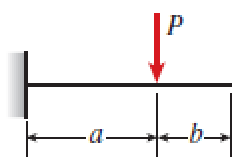
$$\delta_B = \frac{q}{24EI}(3L^4 - 4a^3L + a^4) \qquad \theta_B = \frac{q}{6EI}(L^3 - a^3)$$

4

$$v = -\frac{Px^2}{6EI}(3L - x) \qquad v' = -\frac{Px}{2EI}(2L - x)$$

$$\delta_B = \frac{PL^3}{3EI} \qquad \theta_B = \frac{PL^2}{2EI}$$

5



$$v = -\frac{Px^2}{6EI}(3a - x) \qquad v' = -\frac{Px}{2EI}(2a - x) \qquad (0 \le x \le a)$$

$$v = -\frac{Pa^2}{6EI}(3x - a) \qquad v' = -\frac{Pa^2}{2EI} \qquad (a \le x \le L)$$

$$\text{At } x = a: \quad v = -\frac{Pa^3}{3EI} \qquad v' = -\frac{Pa^2}{2EI}$$

$$\delta_B = \frac{Pa^2}{6EI}(3L - a) \qquad \theta_B = \frac{Pa^2}{2EI}$$

6

$$v = -\frac{M_0x^2}{2EI} \qquad v' = -\frac{M_0x}{EI}$$

$$\delta_B = \frac{M_0L^2}{2EI} \qquad \theta_B = \frac{M_0L}{EI}$$

Deflexões em vigas

Exemplo 2: Determine a curva elástica para a viga biapoiada mostrada na figura. Quais são os valores da deflexão e rotação em x=L/2 ?.