Rotação quântica

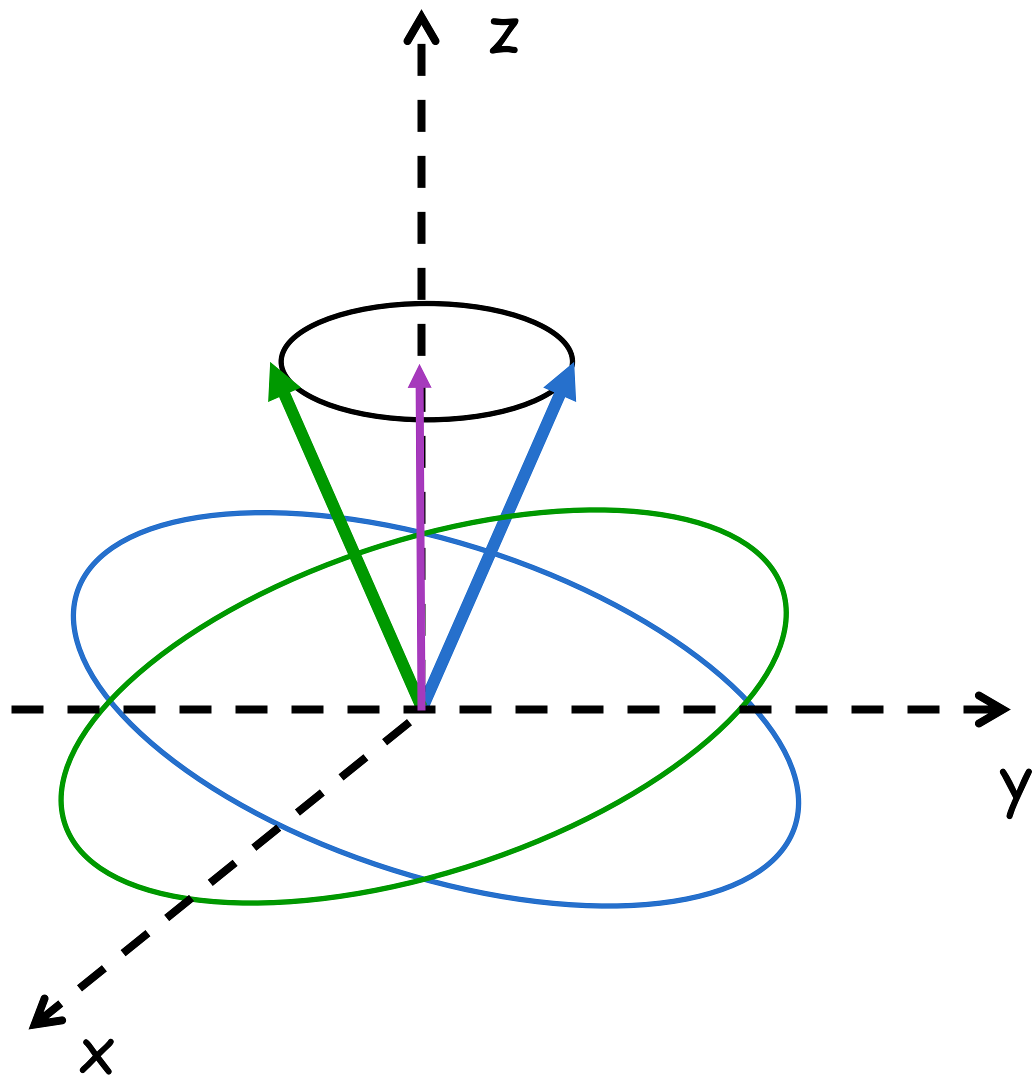
O eixo não está completamente determinado !

Começo do curso

Final do curso

Quando medimos (Stern-Gerlach) determinamos L_z e L

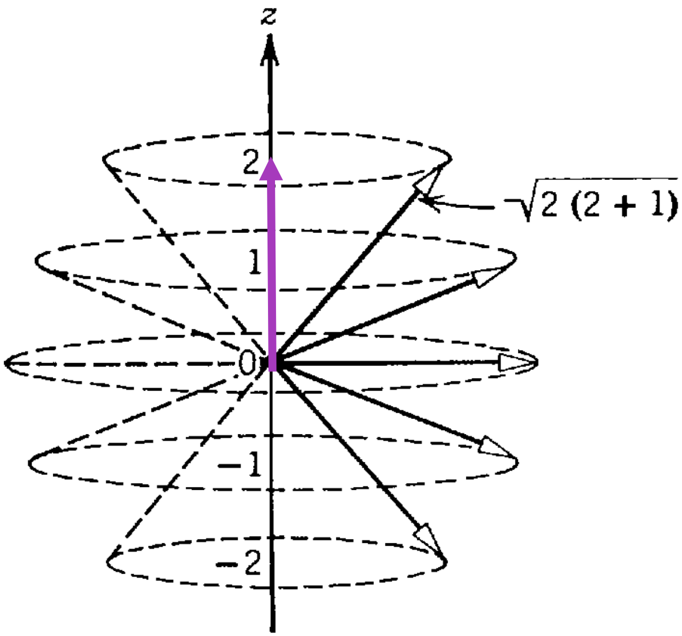
L_x e L_y não estão determinados !

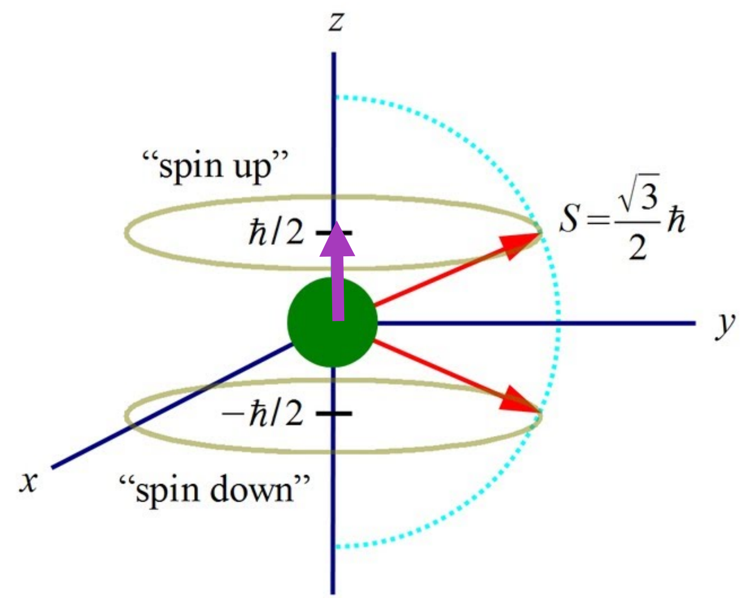


Momento angular orbital

Momento angular intrínseco

Spin





$$L = \sqrt{l\,(l+1)}\,\hbar$$

$$S = \sqrt{s\,(s+1)}\,\hbar$$

$$L_z = m_l\,\hbar$$

$$S_z = m_s\,\hbar$$

$$-l \leq m_l \leq l$$

$$l = 0\,,\,1\,,\,2\,,\,\ldots$$

$$s = \frac{1}{2} \qquad m_s = -\frac{1}{2}\,,\,+\frac{1}{2}$$

As grandezas físicas satisfazem equações de autovalores

$$\hat{O}\,|\;\rangle = o\,|\;\rangle$$

operador autoestado autovalor autoestado

Exemplo mãe:

$$\hat{H}\,|\,\psi\,\rangle = E\,|\,\psi\,\rangle$$

Equação de Schroedinger independente do tempo

Equação de autovalores para o spin

$$\left\{ \begin{array}{l} \hat{S}^2\,\psi = s\,(s+1)\,\hbar^2\,\psi \\[2em] \hat{S}_z\,\psi = m_s\,\hbar\,\psi \end{array} \right.$$

operadores
$$\left\{ \begin{array}{l} \hat{S}_x = \dfrac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\[2em] \hat{S}_y = \dfrac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \\[2em] \hat{S}_z = \dfrac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \end{array} \right.$$

Pauli

Comutadores e operadores compatíveis

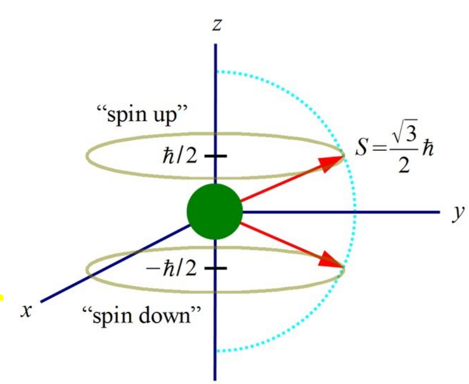
$$[\,S_x\,,\,S_y\,] = i\,\hbar\,S_z \qquad\qquad [\,S^2\,,\,S_x\,] = 0$$

$$[\,S_y\,,\,S_z\,] = i\,\hbar\,S_x \qquad\qquad [\,S^2\,,\,S_y\,] = 0$$

$$[\,S_z\,,\,S_x\,] = i\,\hbar\,S_y \qquad\qquad [\,S^2\,,\,S_z\,] = 0$$

Não têm autoestados comuns !

Não podem ser conhecidos
ao mesmo tempo !



Autoestados

$$|\,x_+\,\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} \qquad |\,x_-\,\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$$|\,y_+\,\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ i \end{pmatrix} \qquad |\,y_-\,\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -i \end{pmatrix}$$

$$|z_+\,\rangle = |\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |z_-\,\rangle = |\downarrow\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

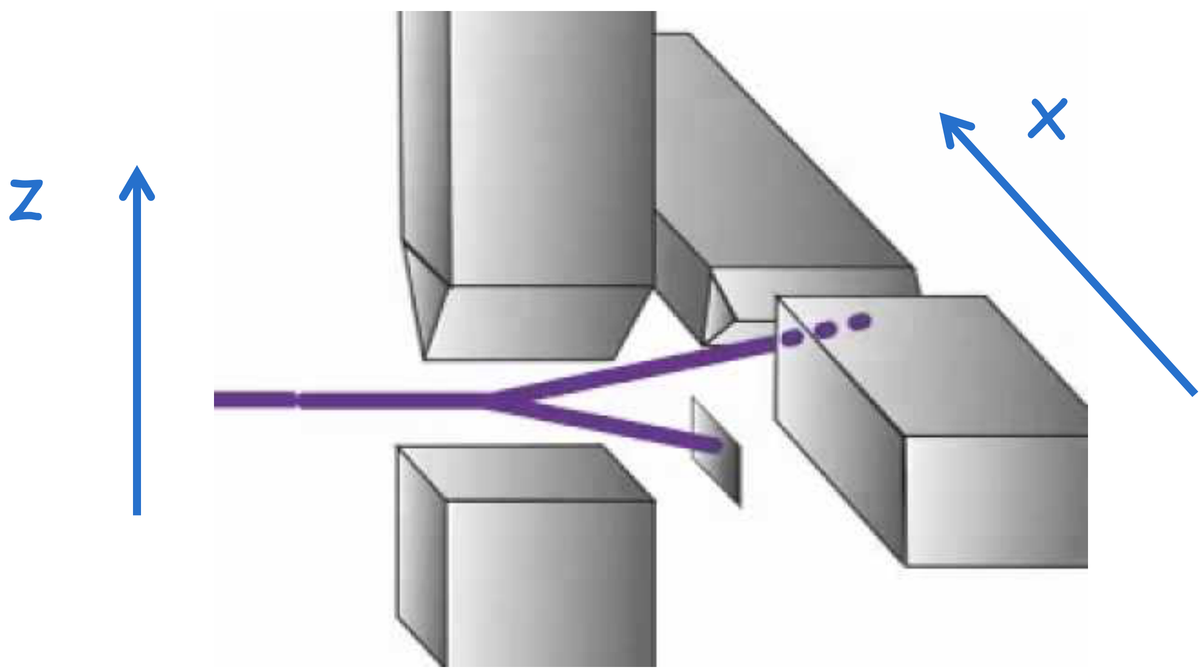
São três bases !   Cada uma com dois estados ortonormais !

Valor médio de um operador

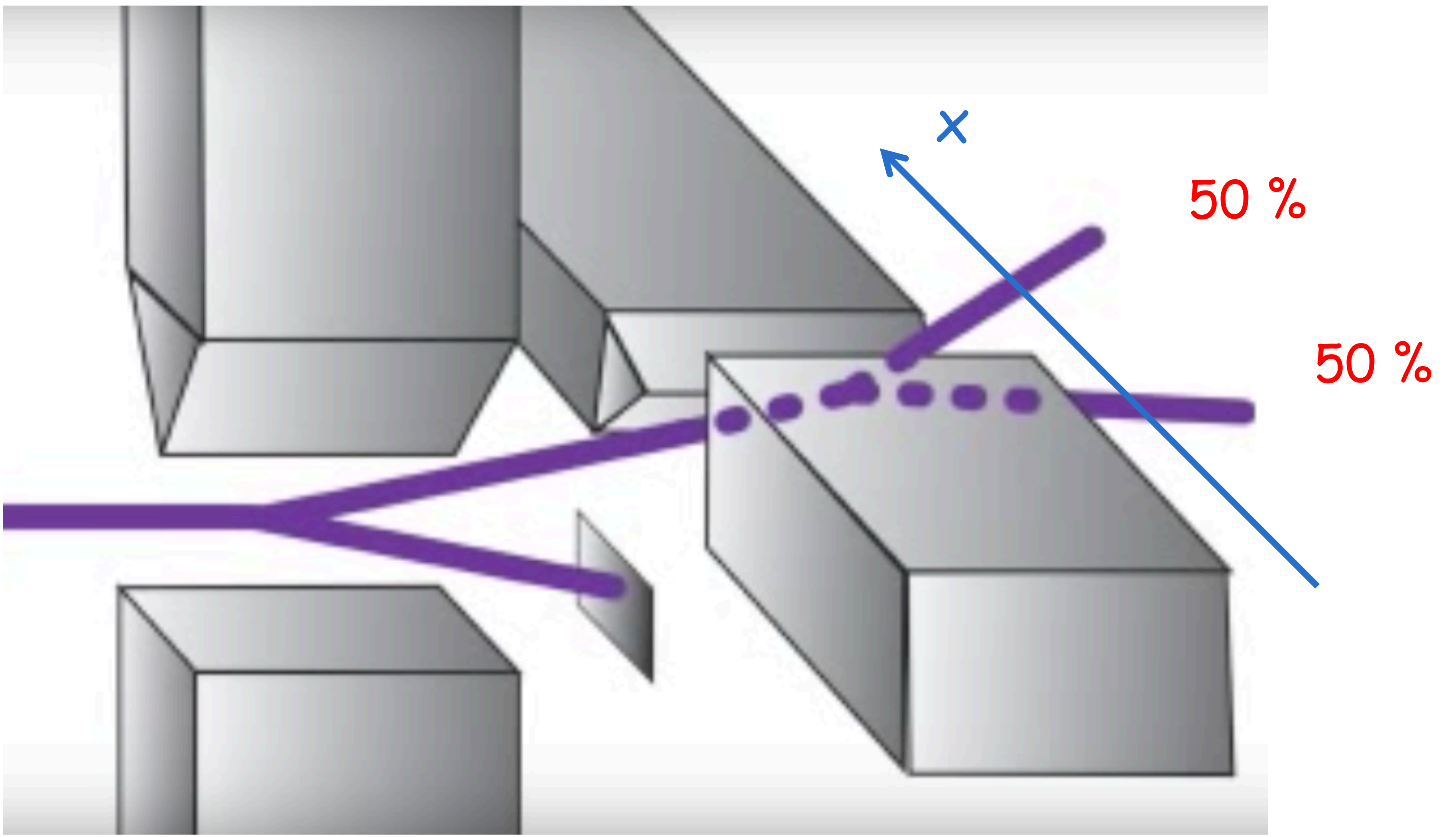
$$\langle A \rangle = \langle \chi | \hat{A} | \chi \rangle$$

Média dos valores medidos da grandeza A no estado $|\chi\rangle$
(o número de medidas deve ser infinito)

Exemplo: medidas da componente do spin na direção x quando o sistema está no estado $|\uparrow\rangle$



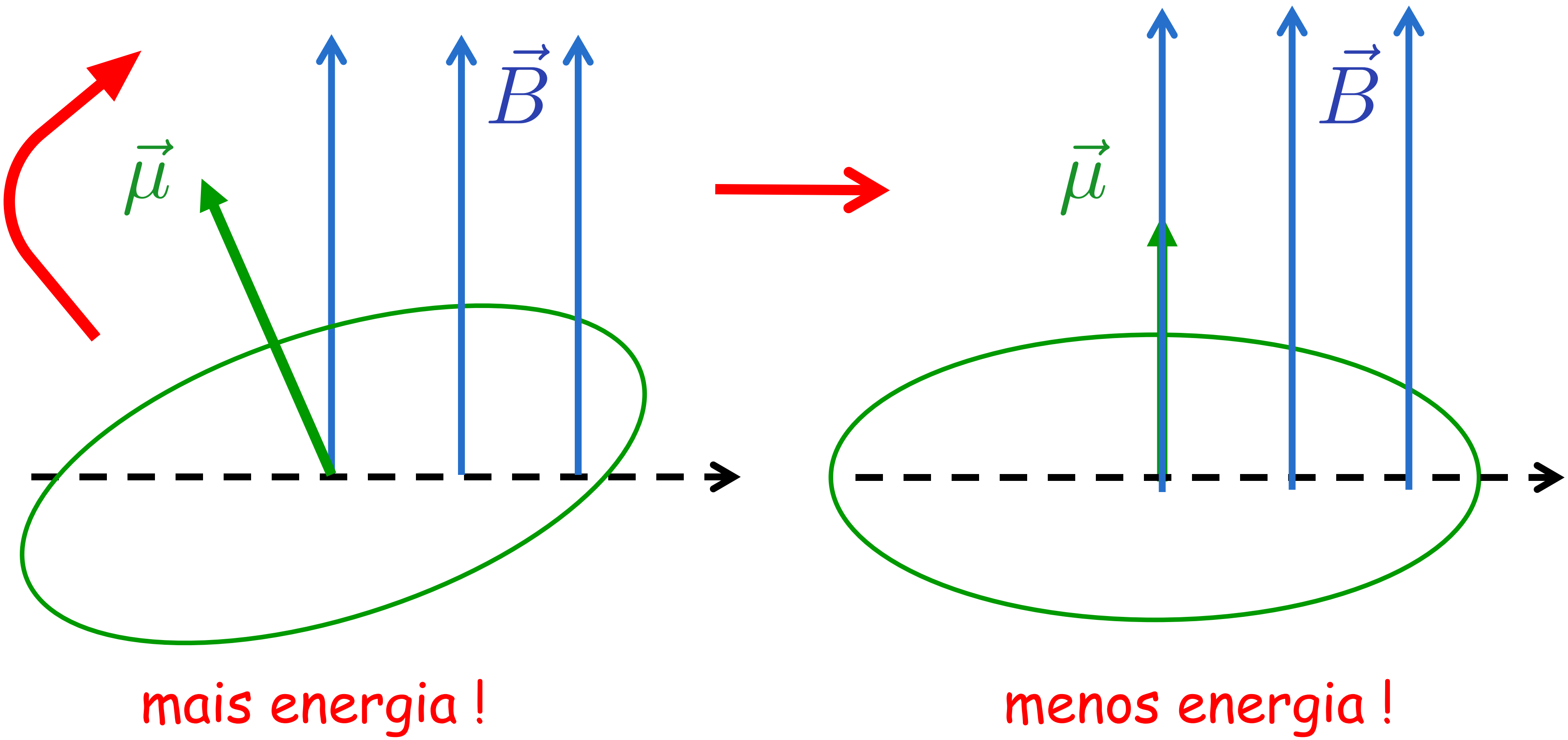
Na experiência vemos que :



Aula 8

Precessão de Larmor

Espira tenta se alinhar como uma bússola !

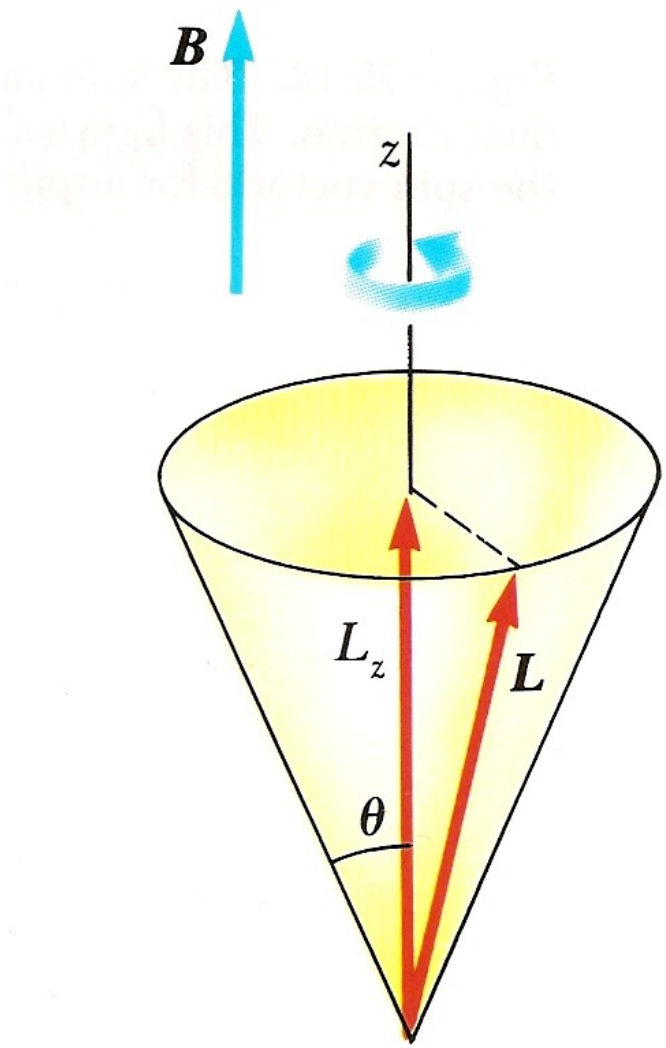


Energia potencial de orientação

$$\boxed{U = -\vec{\mu}\,.\,\vec{B}}$$

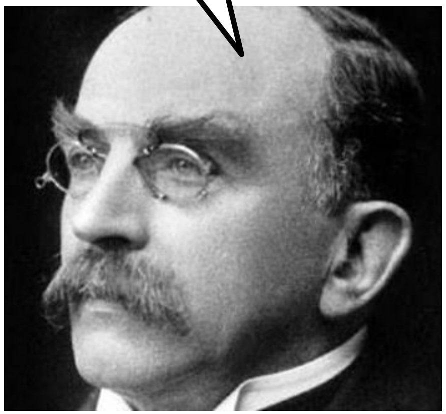
Mas para isso precisaria perder energia !

Se não tem como, NUNCA SE ALINHA ! Precessiona !



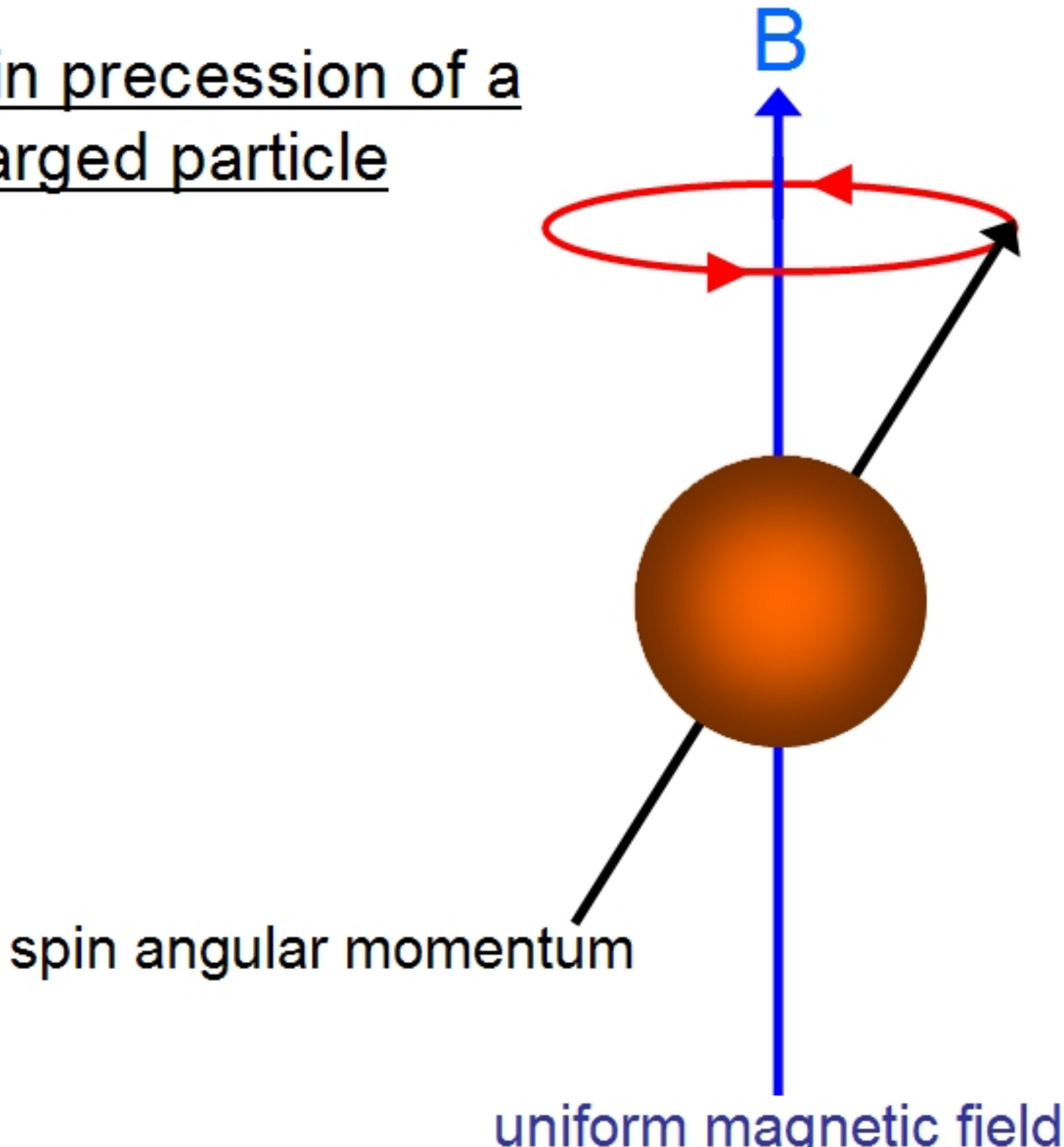
$$\vec{\mu} = \frac{g_l\,\mu_b}{\hbar}\,\vec{L}$$

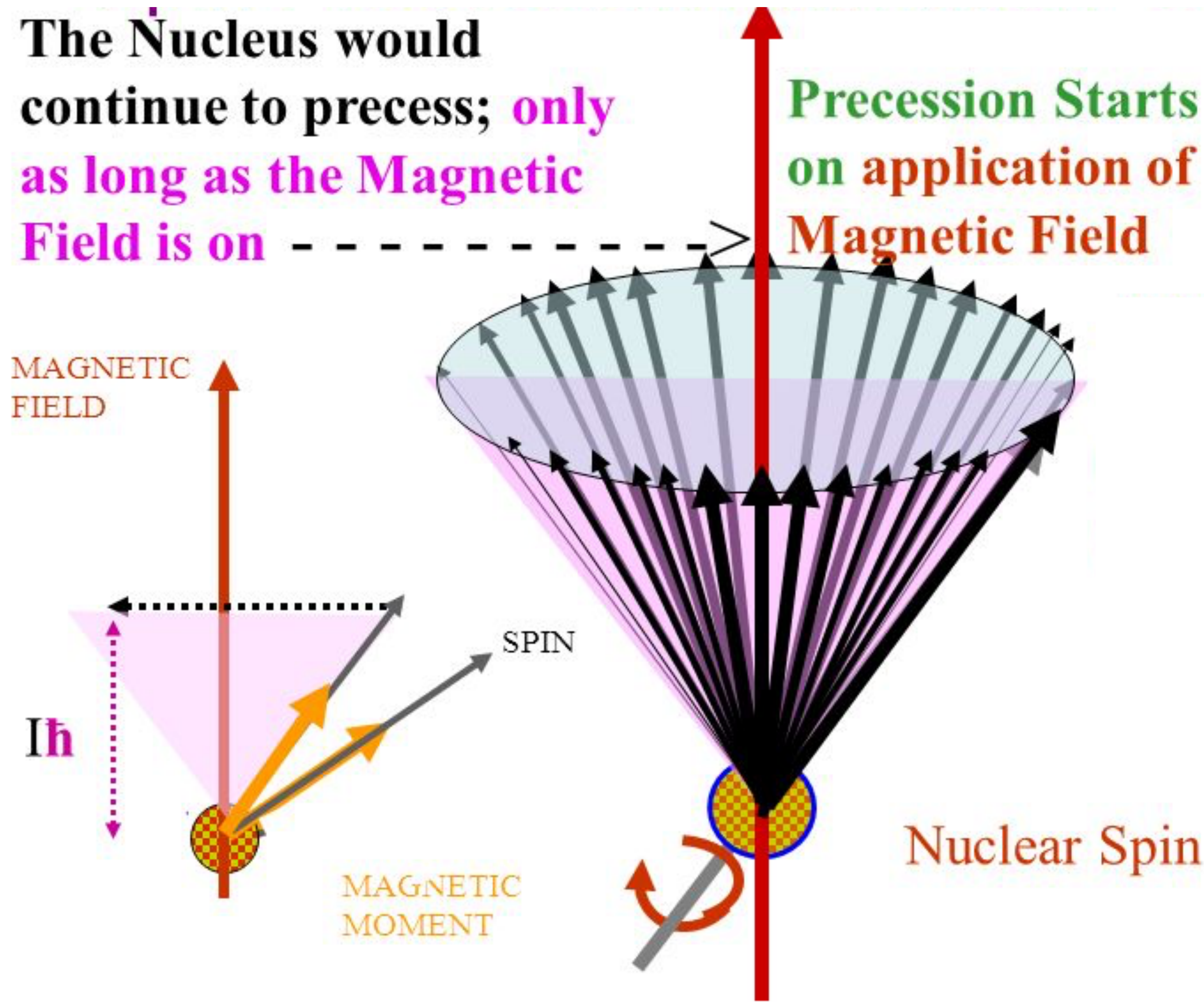
Precessão de Larmor



Joseph Larmor

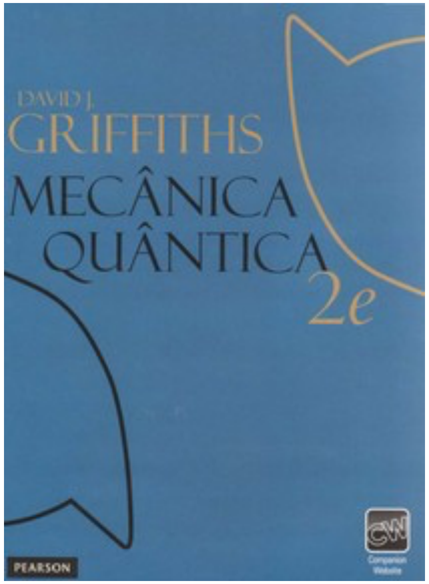
Spin precession of a charged particle




The Nucleus would continue to precess; only as long as the Magnetic Field is on
Precession Starts on application of Magnetic Field
MAGNETIC FIELD
SPIN
Iħ
MAGNETIC MOMENT
Nuclear Spin

Isto é uma descrição clássica !
...poderia ser um pião...

Como fazer um estudo quântico
da precessão do spin ?



$$U = -\vec{\mu}_s \, . \, \vec{B}$$

$$\begin{cases} \vec{B} = B_0 \, \hat{k} \\ \vec{\mu}_s = \dfrac{g_s \, \mu_b}{\hbar} \, \vec{S} = \gamma \, \vec{S} \end{cases}$$

γ = razão giromagnética

$$U = -\gamma \, \vec{S} \, . \, \vec{B}$$

Função Hamiltoniana :

$H = T + U =$ energia cinética + energia potencial

$$H = -\gamma \, \vec{S} \, . \, \vec{B} = -\gamma \, B_0 \, S_z$$

$$H = -\gamma\,\vec{S}\,.\,\vec{B} = -\gamma\,B_0\,S_z$$

Quantização : funções viram operadores !

$$S_z \rightarrow \hat{S}_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

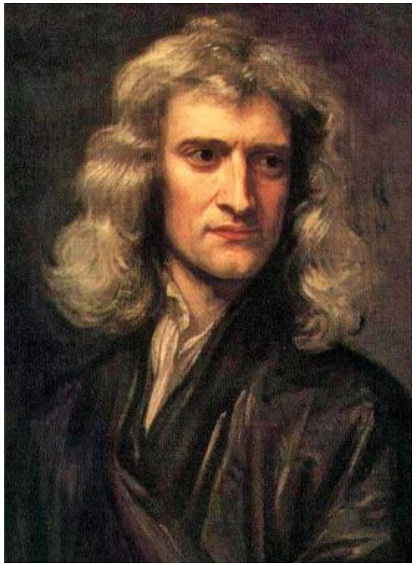
$H \rightarrow \hat{H}$ Operador Hamiltoniano

$$\hat{H} = -\gamma\,B_0\,\frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Mecânica clássica

2ª lei de Newton

$$\frac{d^2x}{dt^2} = -\frac{1}{m}\frac{\partial V}{\partial x}$$



X

Mecânica quântica

Equação de Schrödinger

$$i\hbar\frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} + V\Psi$$

$$\hat{H} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V$$

(Hamiltoniano)

$$\boxed{i\hbar\frac{\partial \Psi}{\partial t} = \hat{H}\Psi}$$

Ansatz :

$$\Psi = \psi(x,t)\ \psi_s$$

Parte do espaço e tempo

$$\psi(x,t) = N\, e^{-iE.t\,/\,\hbar}$$

$$N = 1$$

Parte do spin : matriz

$$\psi_s = |\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\Psi(t) = \psi_s\, e^{-i\,(E\,t)/\hbar} = |\chi\rangle\, e^{-i\,(E\,t)/\hbar}$$

$$\Psi(t) = \psi_s \, e^{-i\,(E\,t)/\hbar} = |\chi\rangle \, e^{-i\,(E\,t)/\hbar}$$

$$i\,\hbar \frac{\partial \Psi}{\partial t} = \hat{H}\Psi \qquad \frac{\partial \Psi}{\partial t} = (-\frac{iE}{\hbar})\,|\chi\rangle\, e^{-iE.t\,/\,\hbar}$$

$$i\cancel{\hbar}(-\frac{iE}{\cancel{\hbar}})\,|\chi\rangle\, \cancel{e^{-iE.t\,/\,\hbar}} = \hat{H}\,|\chi\rangle\, \cancel{e^{-iE.t\,/\,\hbar}}$$

$$\hat{H}\,|\chi\rangle = E\,|\chi\rangle$$

Já sabemos que só existem dois autoestados de spin:

$$|\chi\rangle = |\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad |\chi\rangle = |\downarrow\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Aplicando ao estado com spin pracima

$$\hat{H} = -\gamma B_0 \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad |\chi\rangle = |\uparrow\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\hat{H}\,|\uparrow\rangle = -\gamma B_0 \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = -\gamma B_0 \frac{\hbar}{2} \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\hat{H}\,|\uparrow\rangle = -\gamma B_0 \frac{\hbar}{2}\,|\uparrow\rangle$$

$$\hat{H}\,|\uparrow\rangle = E_+\,|\uparrow\rangle$$

$$\boxed{E_+ = -\gamma B_0 \frac{\hbar}{2}}$$

Aplicando ao estado com spin prabaixo

$$\hat{H} = -\gamma B_0 \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad |\chi\rangle = |\downarrow\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\hat{H}\,|\downarrow\rangle = -\gamma B_0 \frac{\hbar}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = +\gamma B_0 \frac{\hbar}{2} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\hat{H}\,|\downarrow\rangle = +\gamma B_0 \frac{\hbar}{2}\,|\downarrow\rangle$$

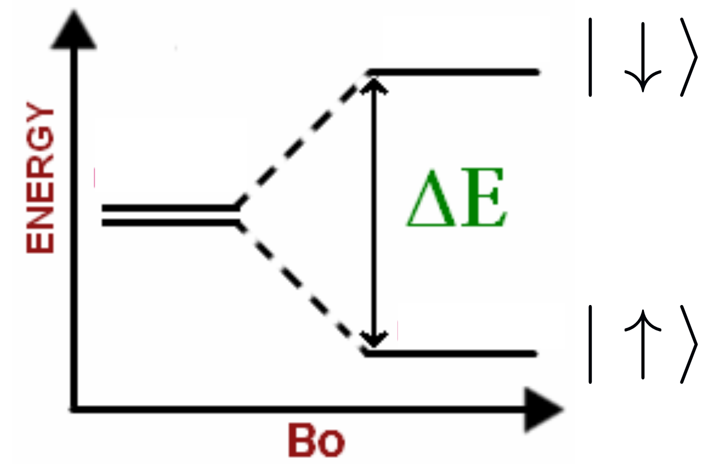
$$\hat{H}\,|\downarrow\rangle = E_-\,|\downarrow\rangle$$

$$\boxed{E_- = +\gamma B_0 \frac{\hbar}{2}}$$

$$\Psi(t) = |\chi\rangle \, e^{-i\,(E\,t)/\hbar}$$

Encontramos dois estados :

$$\begin{cases} \Psi_+(t) = |\uparrow\rangle \, e^{-i\,(E_+\,t)/\hbar} \\ \Psi_-(t) = |\downarrow\rangle \, e^{-i\,(E_-\,t)/\hbar} \end{cases}$$



Solução geral :

$$\Psi(t) = a\,\Psi_+ + b\,\Psi_- = a\,|\uparrow\rangle \, e^{-i\,(E_+\,t)/\hbar} + b\,|\downarrow\rangle \, e^{-i\,(E_-\,t)/\hbar}$$

$$|a|^2 + |b|^2 = 1 \longrightarrow \quad a = cos\,(\frac{\alpha}{2}) \qquad b = sen\,(\frac{\alpha}{2})$$

(escolha conveniente)

$$\Psi(t) \;=\; cos\,(\frac{\alpha}{2})\;\begin{pmatrix} 1 \\ 0 \end{pmatrix}\; e^{-i\,(E_+\;t)/\hbar} \;+\; sen\,(\frac{\alpha}{2})\;\begin{pmatrix} 0 \\ 1 \end{pmatrix}\; e^{-i\,(E_-\;t)/\hbar}$$

$$\Psi(t) \;=\; \begin{pmatrix} cos\,(\frac{\alpha}{2})\; e^{-i\,(E_+\;t)/\hbar} \\ \\ sen\,(\frac{\alpha}{2})\, e^{-i\,(E_-\;t)/\hbar} \end{pmatrix}$$

$$E_+ \;=\; -\,\gamma\,B_0\,\frac{\hbar}{2}$$

$$E_- \;=\; +\,\gamma\,B_0\,\frac{\hbar}{2}$$

$$\Psi(t) \;=\; \begin{pmatrix} cos\,(\frac{\alpha}{2})\; e^{+i\,(\gamma\,B_0\,t)/2} \\ \\ sen\,(\frac{\alpha}{2})\, e^{-i\,(\gamma\,B_0\,t)/2} \end{pmatrix}$$

$$\Psi^\dagger(t) \;=\; \begin{pmatrix} cos\,(\frac{\alpha}{2})\; e^{-i\,(\gamma\,B_0\,t)/2} & \quad sen\,(\frac{\alpha}{2})\, e^{+i\,(\gamma\,B_0\,t)/2} \end{pmatrix}$$

Valor esperado de S_x

$$\langle \hat{S}_x \rangle \;=\; \langle \Psi \,|\, \hat{S}_x \,|\, \Psi \rangle \;=\; \Psi^\dagger\, \hat{S}_x\, \Psi$$

$$= \left(cos\,(\frac{\alpha}{2})\; e^{-i\,(\gamma\, B_0\, t)/2} \qquad sen\,(\frac{\alpha}{2})\, e^{+i\,(\gamma\, B_0\, t)/2} \right)\, \frac{\hbar}{2} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} cos\,(\frac{\alpha}{2})\; e^{+i\,(\gamma\, B_0\, t)/2} \\ \\ sen\,(\frac{\alpha}{2})\, e^{-i\,(\gamma\, B_0\, t)/2} \end{pmatrix}$$

$$= \frac{\hbar}{2} \left(cos\,(\frac{\alpha}{2})\; e^{-i\,(\gamma\, B_0\, t)/2} \qquad sen\,(\frac{\alpha}{2})\, e^{+i\,(\gamma\, B_0\, t)/2} \right) \begin{pmatrix} sen\,(\frac{\alpha}{2})\, e^{-i\,(\gamma\, B_0\, t)/2} \\ \\ cos\,(\frac{\alpha}{2})\; e^{+i\,(\gamma\, B_0\, t)/2} \end{pmatrix}$$

$$= \frac{\hbar}{2} \left(cos\,\frac{\alpha}{2}\, sen\frac{\alpha}{2}\;\; e^{-i\,\gamma\, B_0\, t} \,+\, sen\,\frac{\alpha}{2}\, cos\,\frac{\alpha}{2}\;\; e^{+i\,\gamma\, B_0\, t} \right)$$

$$= \frac{\hbar}{2}\, cos\,\frac{\alpha}{2}\, sen\frac{\alpha}{2}\; \left(e^{-i\,\gamma\, B_0\, t} \,+\, e^{+i\,\gamma\, B_0\, t} \right)$$

$$= \frac{\hbar}{2} \cos\frac{\alpha}{2}\, sen\frac{\alpha}{2}\; \left(e^{-i\gamma B_0 t} + e^{+i\gamma B_0 t} \right)$$

$$(sen\, 2\theta = 2\, sen\,\theta\; \cos\theta)$$

$$(2\cos\theta = e^{i\theta} + e^{-i\theta})$$

$$= \frac{\hbar}{2}\;\frac{1}{2}\, sen\,\alpha \quad 2\cos(\gamma B_0 t)$$

$$\begin{cases} \langle \hat{S}_x \rangle = \dfrac{\hbar}{2}\;\dfrac{1}{2}\, sen\,\alpha \quad 2\cos(\gamma B_0 t) \\ \\ \langle \hat{S}_y \rangle = -\dfrac{\hbar}{2}\;\dfrac{1}{2}\, sen\,\alpha \quad 2\, sen(\gamma B_0 t) \end{cases}$$

(Exercício)

$$\boxed{\omega = \gamma B_0 = \frac{g_s\, \mu_b B_0}{\hbar}}$$

frequência de Larmor

$$\left\{\begin{aligned} \langle \hat{S}_x \rangle &= \frac{\hbar}{2}\;\frac{1}{2}\, sen\,\alpha \quad 2\, cos\,(\gamma\, B_0\, t) \\ \langle \hat{S}_y \rangle &= -\frac{\hbar}{2}\;\frac{1}{2}\, sen\,\alpha \quad 2\, sen\,(\gamma\, B_0\, t) \end{aligned}\right.$$