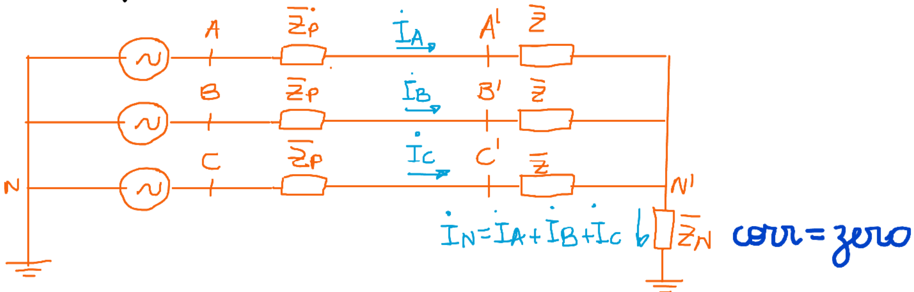
Resolução de circuitos trifásicos pelo equivalente monofásico:

- gerador simétrico
- rede equilibrada
- carga equilibrada



$$\dot{V}_{AN} = \dot{I}_A \bar{Z}_P + \dot{I}_A \bar{Z} + (\dot{I}_A + \dot{I}_B + \dot{I}_C)\bar{Z}_N$$
$$\dot{V}_{BN} = \dot{I}_B \bar{Z}_P + \dot{I}_B \bar{Z} + (\dot{I}_A + \dot{I}_B + \dot{I}_C)\bar{Z}_N +$$
$$\dot{V}_{CN} = \dot{I}_C \bar{Z}_P + \dot{I}_C \bar{Z} + (\dot{I}_A + \dot{I}_B + \dot{I}_C)\bar{Z}_N$$

$\bar{Z}$ e $\bar{Z}_N$ fazem parte da carga, e não apresentam acoplamento magnético, como acontece na rede/linha.

$$\dot{V}_{AN} + \dot{V}_{BN} + \dot{V}_{CN} = \dot{I}_A(\bar{Z}_P + \bar{Z}) + \dot{I}_B(\bar{Z}_P + \bar{Z}) + \dot{I}_C(\bar{Z}_P + \bar{Z}) + 3(\dot{I}_A + \dot{I}_B + \dot{I}_C)\bar{Z}_N$$
$$\dot{V}_{AN} + \dot{V}_{BN} + \dot{V}_{CN} = (\dot{I}_A + \dot{I}_B + \dot{I}_C)(\bar{Z}_P + \bar{Z}) + 3(\dot{I}_A + \dot{I}_B + \dot{I}_C)\bar{Z}_N$$

$$\underbrace{\dot{V}_{AN} + \dot{V}_{BN} + \dot{V}_{CN}}_{\text{sist. simetrico}} = (\dot{I}_A + \dot{I}_B + \dot{I}_C)\underbrace{(\bar{Z}_P + \bar{Z} + 3\bar{Z}_N)}_{\neq 0}$$

$\dot{V}_{AN}(1 + \alpha^2 + \alpha)$
ou
$\dot{V}_{AN}(1 + \alpha + \alpha^2)$
$\shortparallel$
0 (zero)

soma das correntes = 0

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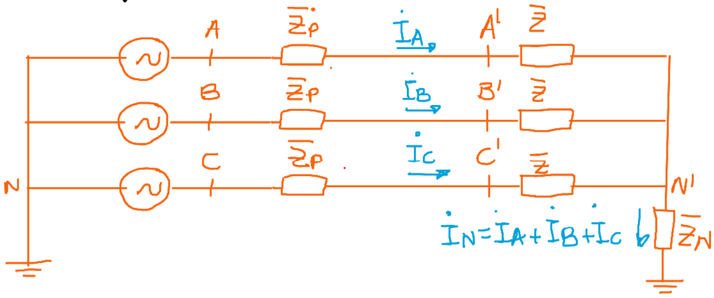


corr = zero

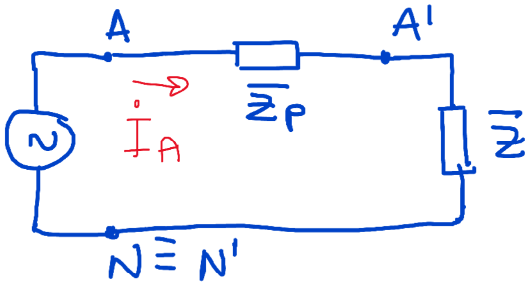
$$\boxed{\begin{aligned} \dot{V}_{AN} &= \dot{I}_A \bar{Z}_P + \dot{I}_A \bar{Z} \\ \dot{V}_{BN} &= \dot{I}_B \bar{Z}_P + \dot{I}_B \bar{Z} \\ \dot{V}_{CN} &= \dot{I}_C \bar{Z}_P + \dot{I}_C \bar{Z} \end{aligned}} \begin{aligned} &+ \cancel{(\dot{I}_A + \dot{I}_B + \dot{I}_C)\bar{Z}_N} \\ &+ \cancel{(\dot{I}_A + \dot{I}_B + \dot{I}_C)\bar{Z}_N} + \\ &+ \cancel{(\dot{I}_A + \dot{I}_B + \dot{I}_C)\bar{Z}_N} \end{aligned}$$

$$(\dot{I}_A + \dot{I}_B + \dot{I}_C) = \text{zero}$$

$$\dot{V}_{AN} = \dot{I}_A(\bar{Z}_P + \bar{Z})$$

$$\dot{V}_{BN} = \dot{I}_B(\bar{Z}_P + \bar{Z})$$

$$\dot{V}_{CN} = \dot{I}_C(\bar{Z}_P + \bar{Z})$$



$$\dot{I}_A = \frac{\dot{V}_{AN}}{\bar{Z}_P + \bar{Z}} \; ; \; \dot{I}_B = \frac{\dot{V}_{BN}}{\bar{Z}_P + \bar{Z}} \; ; \; \dot{I}_C = \frac{\dot{V}_{CN}}{\bar{Z}_P + \bar{Z}}$$

mesma defasagem em relação a $\dot{I}_A$ que entre $\dot{V}_{BN}$ e $\dot{V}_{AN}$

PEA339x - Solução delta -1

segunda-feira, 4 de maio de 2020 08:58

$$\begin{cases} -\dot V_{AB} + \cancel{I_{AA'}\bar Z_P} + \dot I_{A'B'}\bar Z - \cancel{I_{BB'}\bar Z_P} = 0 \\ -\dot V_{BC} + \cancel{I_{BB'}\bar Z_P} + \dot I_{B'C'}\bar Z - \cancel{I_{CC'}\bar Z_P} = 0 \\ -\dot V_{CA} + \cancel{I_{CC'}\bar Z_P} + I_{C'A'}\bar Z - \cancel{I_{AA'}\bar Z_P} = 0 \end{cases} +$$

6 incógnitas!

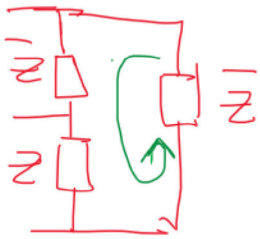
$\dot V_{AB}, \dot V_{BC}, \dot V_{CA}$ conhecidos

$\bar Z_P, \bar Z$ conhecidas

$$-\underbrace{(\dot V_{AB} + \dot V_{BC} + \dot V_{CA})}_{\dot V_{AB}(1+\alpha+\alpha^2) = \emptyset} + (I_{A'B'} + I_{B'C'} + I_{C'A'})\bar Z = 0$$

soma $= \emptyset$

$\bar Z \neq 0$



$$\underbrace{\dot V_{AB} + \dot V_{BC} + \dot V_{CA}}_{=0} = \underbrace{\overbrace{(I_{A'B'} + I_{B'C'} + I_{C'A'})}^{=0}\bar Z}_{=0}$$

$\bar Z \neq 0$

soma das correntes

$$\begin{cases} -\dot V_{AB} + \dot I_{AA'}\bar Z_p + \dot I_{A'B'}\bar Z_\Delta - \dot I_{BB'}\bar Z_p = 0 & (I)\\ -\dot V_{BC} + \dot I_{BB'}\bar Z_p + \dot I_{B'C'}\bar Z_\Delta - \dot I_{CC'}\bar Z_p = 0 & (II)\\ -\dot V_{CA} + \dot I_{CC'}\bar Z_p + \dot I_{C'A'}\bar Z_\Delta - \dot I_{AA'}\bar Z_p = 0 & (III)\end{cases} +$$

$$\boxed{\dot I_{A'B'} + \dot I_{B'C'} + \dot I_{C'A'} = 0}$$

$\dot V_{AB}, \dot V_{BC}, \dot V_{CA}$ conhecidos
$\bar Z_p, \bar Z$ conhecidas

$$-\dot V_{AB} + \bar Z_p(\dot I_{AA'} - \dot I_{BB'}) + \dot I_{A'B'}\bar Z_\Delta = 0$$

$$\dot I_{AA'} = \dot I_{A'B'} - \dot I_{C'A'} \quad ; \quad \dot I_{BB'} = \dot I_{B'C'} - \dot I_{A'B'}$$

$$-\dot V_{AB} + \bar Z_p[\dot I_{A'B'} - \dot I_{C'A} - (\dot I_{B'C'} - \dot I_{A'B'})] + \dot I_{A'B'}\bar Z_\Delta = 0$$

$$-\dot V_{AB} + \bar Z_p[2\dot I_{A'B'} - \dot I_{B'C'} - \dot I_{C'A'}] + \dot I_{A'B'}\bar Z_\Delta = 0$$

$$-\dot V_{AB} + \bar Z_p[3\dot I_{A'B'} \underbrace{- \dot I_{A'B'} - \dot I_{B'C'} - \dot I_{C'A'}}_{=0}] + \dot I_{A'B'}\bar Z_\Delta = 0$$

$$\dot V_{AB} = 3\bar Z_p \dot I_{A'B'} + \bar Z \dot I_{A'B'} \rightarrow \boxed{\dot I_{A'B'} = \frac{\dot V_{AB}}{3\bar Z_p + \bar Z_\Delta}}$$

$$-\dot V_{BC} + \dot I_{BB'}\bar Z_p + \dot I_{B'C'}\bar Z_\Delta - \dot I_{CC'}\bar Z_p = 0$$

$$-\dot V_{BC} + \bar Z_p(\dot I_{BB'} - \dot I_{CC'}) + \dot I_{B'C'}\bar Z_\Delta = 0$$

$$-\dot V_{BC} + \bar Z_p[\dot I_{B'C} - \dot I_{A'B'} - (\dot I_{C'A'} - \dot I_{B'C'})] + \dot I_{B'C'}\bar Z_\Delta = 0$$

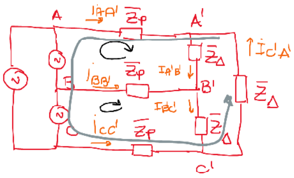
$$-\dot V_{BC} + \bar Z_p[2\dot I_{B'C'} - \dot I_{A'B'} - \dot I_{C'A'}] + \dot I_{B'C'}\bar Z_\Delta = 0$$

$$-\dot V_{BC} + \bar Z_p[3\dot I_{B'C} \underbrace{- \dot I_{B'C'} - \dot I_{A'B'} - \dot I_{C'A'}}_{=0}] + \dot I_{B'C'}\bar Z_\Delta = 0$$

$$\boxed{\dot I_{B'C'} = \frac{\dot V_{BC}}{3\bar Z_p + \bar Z_\Delta}} \quad \text{e ...} \quad \boxed{\dot I_{C'A'} = \frac{\dot V_{CA}}{3\bar Z_p + \bar Z_\Delta}}$$

Lembrando que (seq. direta ABC)

$$\dot I_{AA'} = (\sqrt3\angle -30^\circ)\dot I_{A'B'}$$
$$\dot I_{BB'} = (\sqrt3\angle -30^\circ)\dot I_{B'C'} \rightarrow$$
$$\dot I_{CC'} = (\sqrt3\angle -30^\circ)\dot I_{C'A'}$$



$$\dot I_{AA'} = \frac{(\sqrt3\angle-30^\circ)\dot V_{AB}}{3\bar Z_p + \bar Z_\Delta} = \frac{\frac{(\sqrt3\angle-30^\circ)}{3}\dot V_{AB}}{\frac{3\bar Z_p + \bar Z_\Delta}{3}}$$

Δ-Δ

$$\dot I_{AA'} = \frac{\left(\frac{1}{\sqrt3}\angle-30^\circ\right)\dot V_{AB}}{\bar Z_p + \frac{\bar Z_\Delta}{3}} = \frac{\frac{\dot V_{AB}}{\sqrt3\angle+30^\circ}}{\bar Z_p + \frac{\bar Z_\Delta}{3}}$$

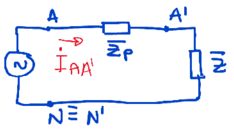
$\dot V_{AN}$ se o circuito fosse Y

impedância da carga Δ se equivalente a Y

Y-Y

$$\dot I_{AA'} = \frac{\dot V_{AN}}{\bar Z_p + \bar Z}$$

Circ. equiv. monofásico circuito Y-Y

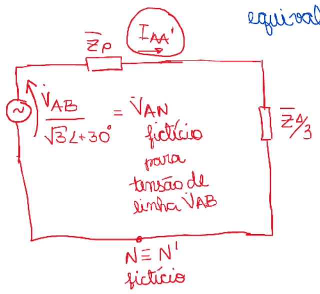


seq. inversa (CBA)

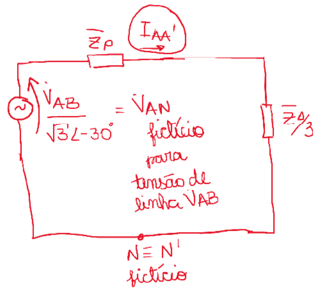
$$\dot I_{AA'} = (\sqrt3\angle30^\circ)\dot I_{A'B'}$$

$$= \frac{(\sqrt3\angle30^\circ)\dot V_{AB}}{3\bar Z_p + \bar Z_\Delta}\frac{(1/3)}{(1/3)} = \frac{\left(\frac{1}{\sqrt3}\angle30^\circ\right)\dot V_{AB}}{\bar Z_p + \bar Z_\Delta/3} = \frac{\frac{\dot V_{AB}}{\sqrt3\angle-30^\circ}}{\bar Z_p + \bar Z_\Delta/3}$$

$\dot V_{AN}$ se o circuito fosse Y



seq. dir



seq. inversa

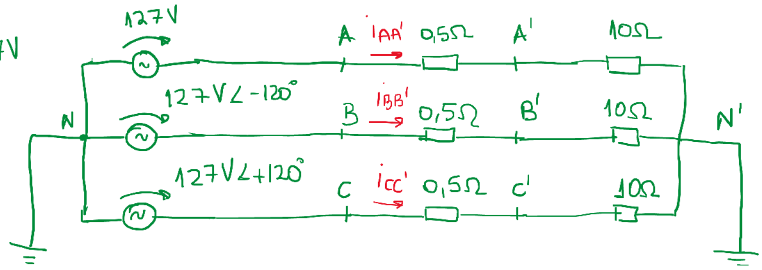
PEA339x - Solução delta -3

segunda-feira, 11 de maio de 2020 08:24

Fonte Y, Tensão de fase 127V

Carga Y, $\bar{Z} = 10\Omega$

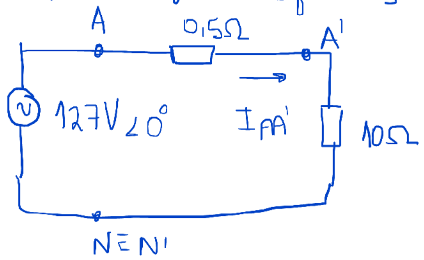
linha, $\bar{Z}_p = 0{,}5\Omega$



Tensão na carga → fase → linha

queda de tensão na linha: $\dot{V}_{AA'}$, $\dot{V}_{BB'}$, $\dot{V}_{CC'}$

Circ. equiv. monofásico: [fase A]



$$\dot{I}_{AA'} = \frac{\overset{\dot{V}_{AN}}{127}}{10+0{,}5} = 12{,}095A\angle 0°$$

$\dot{V}_{A'N'} = \dot{I}_{AA'} \cdot 10 = 120{,}95V\angle 0°$

$\dot{V}_{AA'} = \dot{I}_{AA'}\, 0{,}5 = 6{,}05V\angle 0°$

$\dot{I}_{BB} = 12{,}095 A\angle -120°$

$\dot{V}_{B'N'} = 120{,}95A\angle -120°$

$\dot{V}_{BB'} = 6{,}05V\angle -120°$

$\dot{I}_{CC'} = 12{,}095\angle 120°$

$\dot{V}_{C'N'} = 120{,}95\angle 120°$

$\dot{V}_{CC'} = 6{,}05\angle 120°$

$\dot{V}_{A'B'} = \dot{V}_{A'N'}(\sqrt{3}\angle 30°)$

$= 209{,}50V\angle 30°$

$\dot{V}_{B'C'} = 209{,}50V\angle -90°$

$\dot{V}_{C'A'} = 209{,}50V\angle 150°$

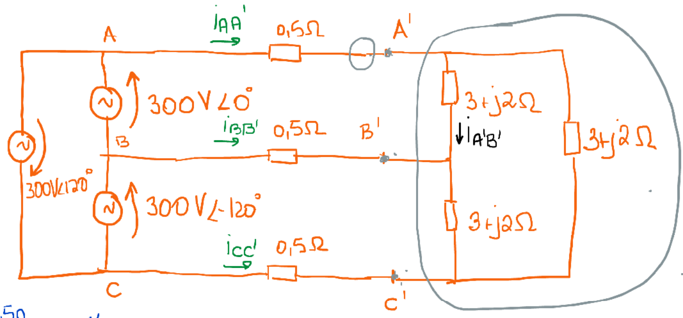
PEA339x - Solução delta -4

segunda-feira, 11 de maio de 2020 08:38

Fonte Δ, tensão fase 300 V

carga Δ, $\bar{Z}_\Delta = 3+j2\,\Omega$

linha = $0{,}5\,\Omega = \bar{Z}_P$



Δ $\bar{Z}_\Delta = (3+j2)\,\Omega$

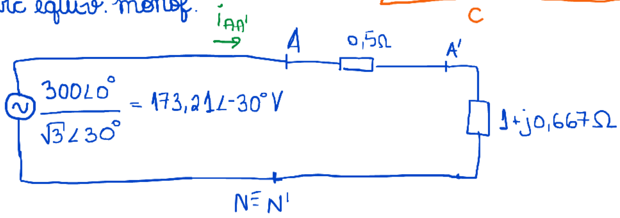
Y $\bar{Z}_Y = (1+j0{,}667)\,\Omega$

$-\dot V_{AB} + \dot I_{AA'}0{,}5 + \dot V_{A'B'} - \dot I_{BB'}0{,}5 = 0$

$\dot V_{A'B'} = \dot V_{AB} - \dot I_{AA'}0{,}5 + \dot I_{BB'}0{,}5$

$= 300 + 0{,}5\left(-105{,}52\angle{-54{,}0^\circ} + 105{,}52\angle{-174{,}0^\circ}\right)$

Circ equiv. monof.



$\dot V_{A'N'} = (105{,}52\angle{-54{,}0^\circ})(1+j0{,}667)$

$\dot I_{AA'} = 105{,}52\angle{-54{,}0^\circ}$ A

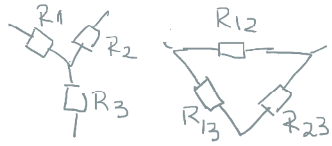
$\dot V_{AA'} = \dot I_{AA'}\cdot 0{,}5 = 52{,}76\angle{-54{,}0^\circ}$ A

$\dot I_{A'B'} = \dfrac{\dot I_{AA'}}{\sqrt{3}\angle{-30^\circ}} = 60{,}92\angle{-24{,}0^\circ}$ A

$\dot V_{A'B'} = \dot I_{A'B'}\cdot(3+j2) = 219{,}65\angle 9{,}7^\circ$ V

$\dot I_{BB'} = 105{,}52\angle{-174{,}0^\circ}$ A

$\dot V_{BB'} = 52{,}76\angle{-174{,}0^\circ}$ A



$R_1 = \dfrac{R_{12}\cdot R_{13}}{R_{12}+R_{12}+R_{23}} = \dfrac{R^2}{3R} = \dfrac{R}{3}$