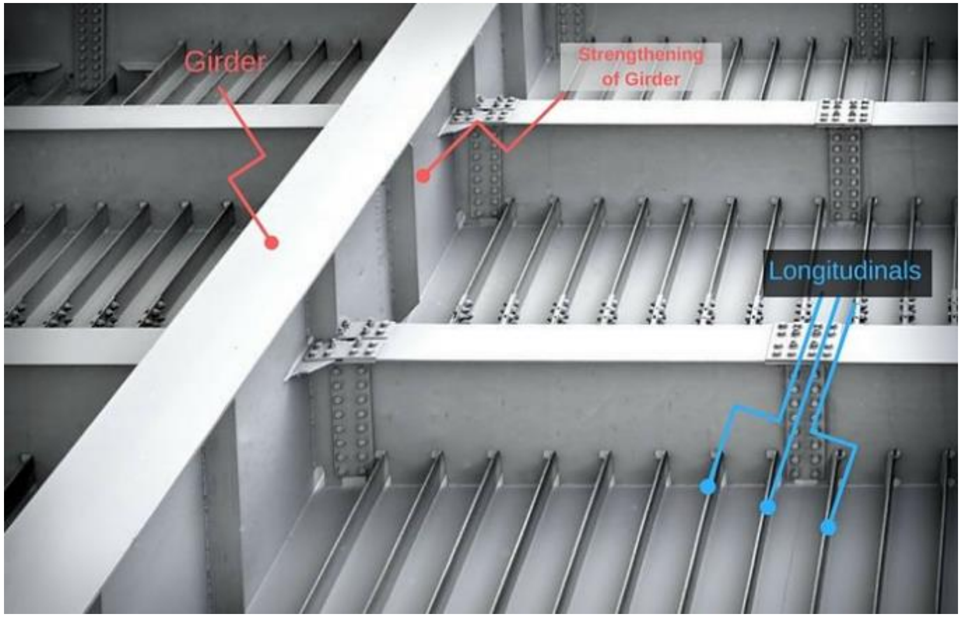
DEPARTAMENTO DE ENGEHARIA NAVAL E OCEÂNICA
ESCOLA POLITÉCNICA DA USP

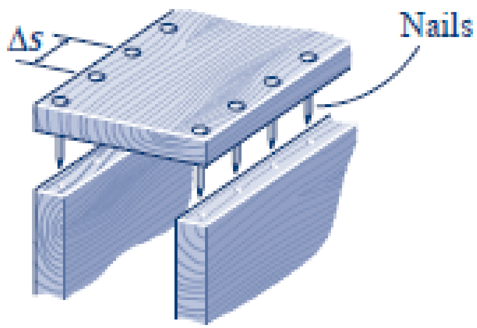
Análise de Vigas : σ_x e τ_{xy}

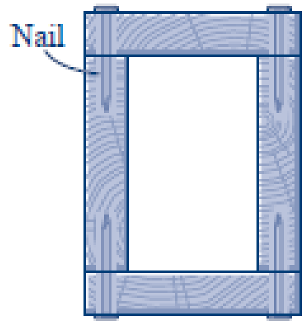


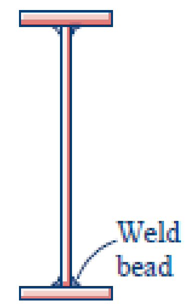
PNV 3212 – Mecânica Dos Sólidos I

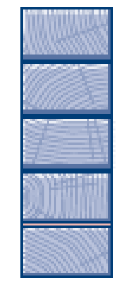
2020

Shear Flow

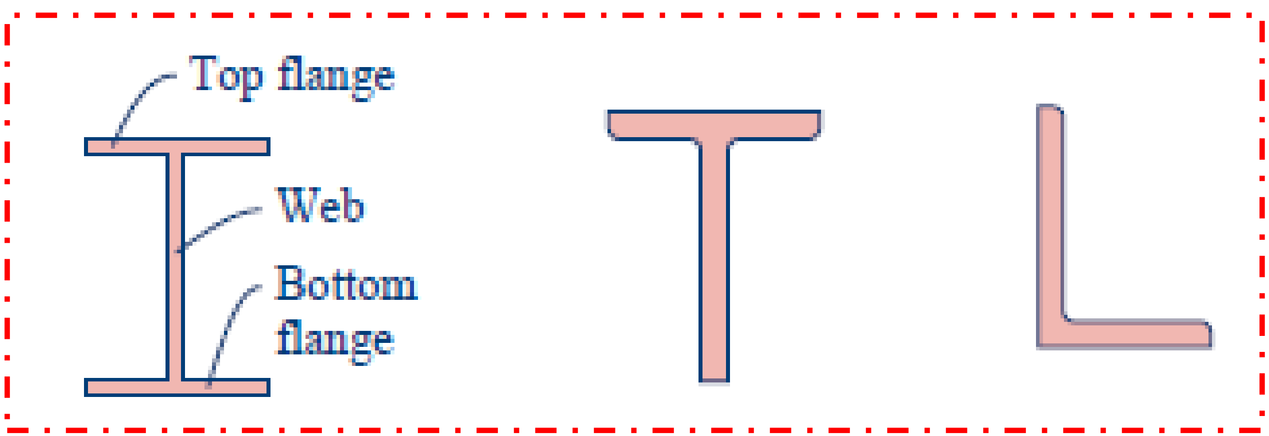








SHEAR STRESS IN THIN-WALL BEAMS



$t \ll b, h$

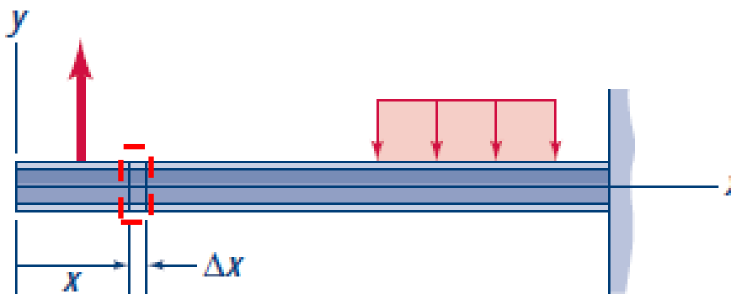
Open section

Tensões de Cisalhamento

- **Hipóteses**
  - Problema é independente do tempo.
  - O formato da viga é um prisma reto, cujo comprimento é muito maior que as outras dimensões (**Esbelta**).
  - A viga é constituída de um material **linearmente elástico**.
  - O efeito Poisson é negligenciável.
  - A seção transversal é simétrica em relação ao plano vertical.
  - Planos perpendiculares à linha neutra permanecem quase planos e perpendiculares ao eixo deformado depois da deformação (**Navier**).
  - O ângulo de rotação da seção transversal é muito pequeno.
  - O efeitos de momento de inércia da rotação é desprezado.
  - ~~**Flexão Pura**~~.
  - A viga é constituída de material homogêneo .
  - The distribution of flexural stress on a given cross section is not affected by the deformation due to shear.
  - Distorção da seção transversal é pequena o suficiente para ser desprezada!

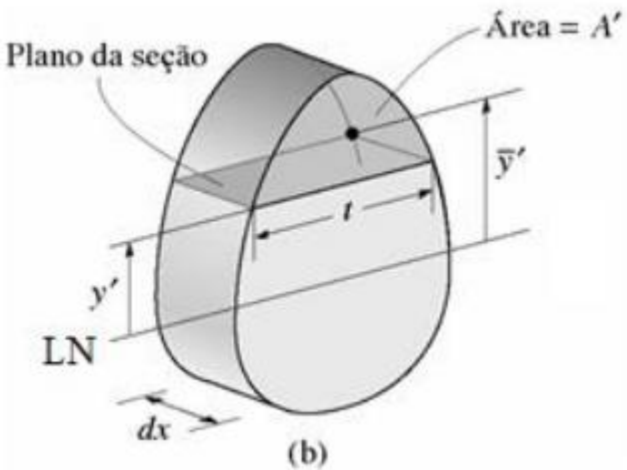
Tensões de Cisalhamento

- Fórmula



$$\tau(x) = \frac{V(x) \times Q}{I(x) \times t}$$

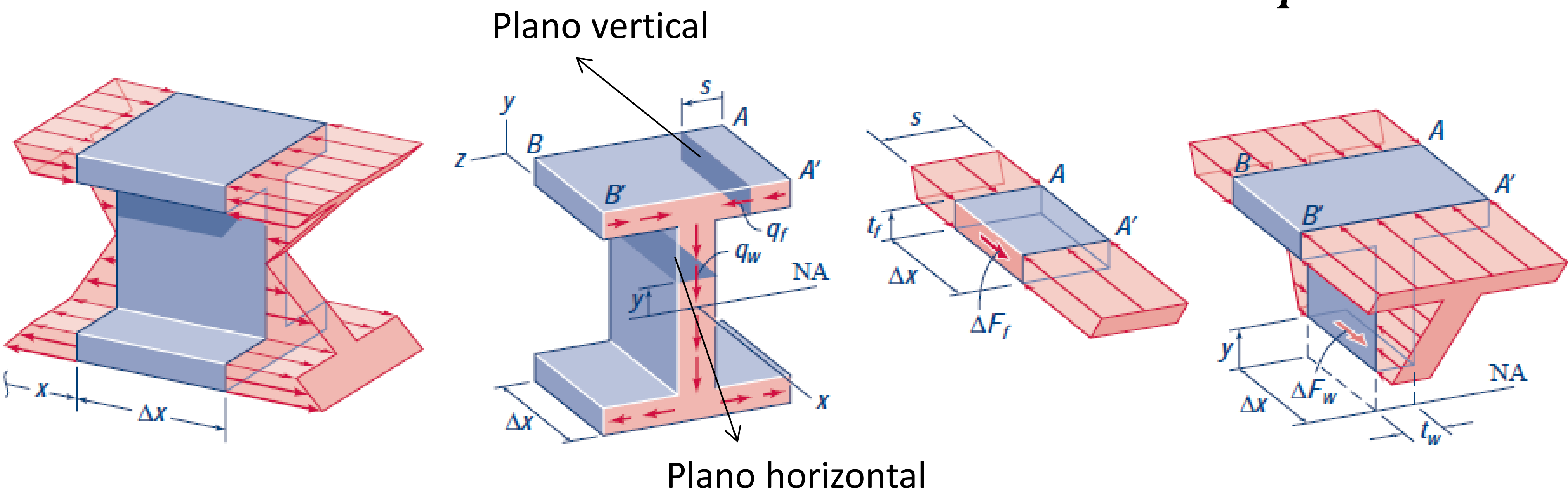
$$Q = \int_{A'} y\,dA'$$



Tensões de Cisalhamento

$$q \equiv \lim_{\Delta x \to 0} \frac{\Delta H}{\Delta x}$$

$$q = \frac{VQ}{I} \quad \text{[ton/m]}$$



Momentos fletores diferentes agindo em seções adjacentes

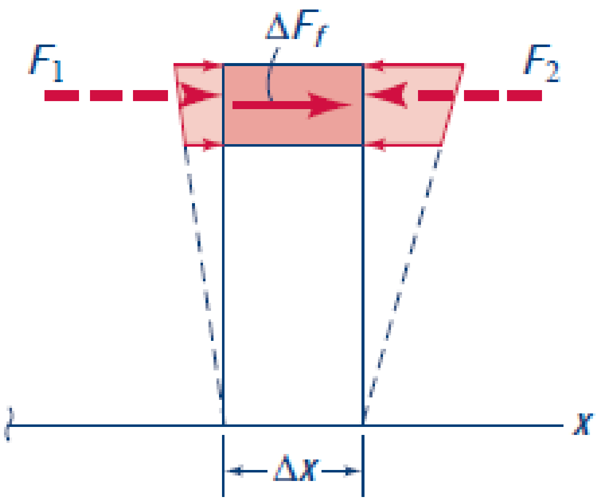
Planos de corte utilizados para:
a) Flange
b) Alma

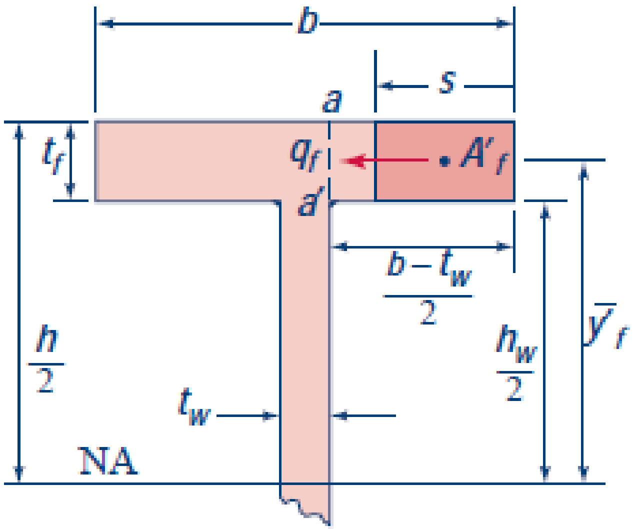
Diagrama de Corpo Livre para determinar ***q*** no flange

Diagrama de Corpo Livre para determinar ***q*** na alma

Unbalanced flexural stresses lead to shear stresses

Tensões de Cisalhamento no Flange





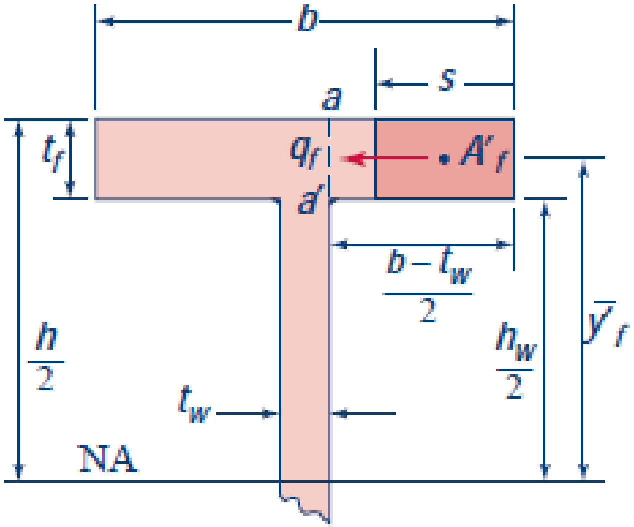
$$Q = A'_f \times y'_f$$

$$q_f = \frac{VQ}{I}$$

$$Q = st_f \times \left(\frac{h - t_f}{2}\right)$$

Unbalanced flexural stresses lead to shear stresses

Tensões de Cisalhamento no Flange



$$q_f = \frac{V}{I} s t_f \times \left(\frac{h-t_f}{2}\right)$$

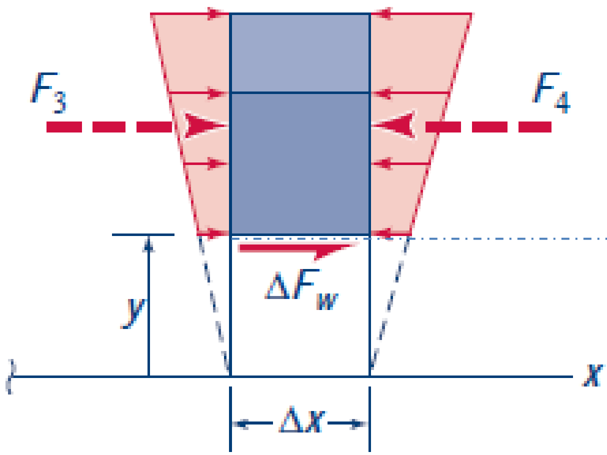
$$t_f \ll b \qquad t_f \leq \frac{b}{5}$$

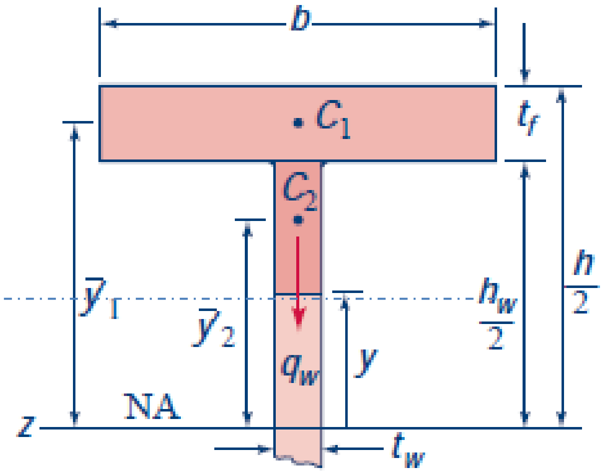
$$\tau_f = \frac{q_f}{t_f}$$

$$\tau_f = \frac{V}{I} s \times \left(\frac{h-t_f}{2}\right)$$

Unbalanced flexural stresses lead to shear stresses

Tensões de Cisalhamento na Alma





$$q_w = \frac{VQ}{I}$$

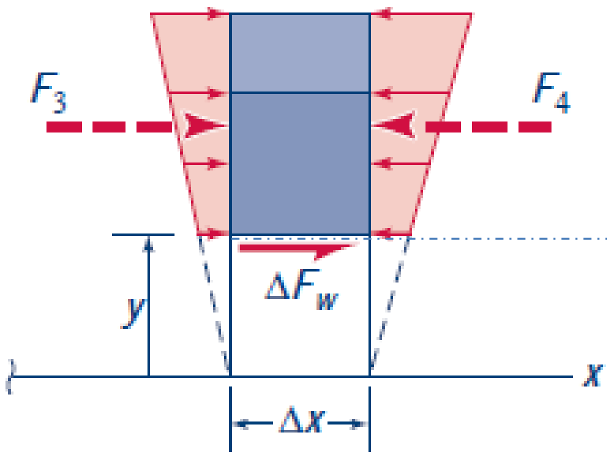
$$Q = A'_w \times y'_w$$

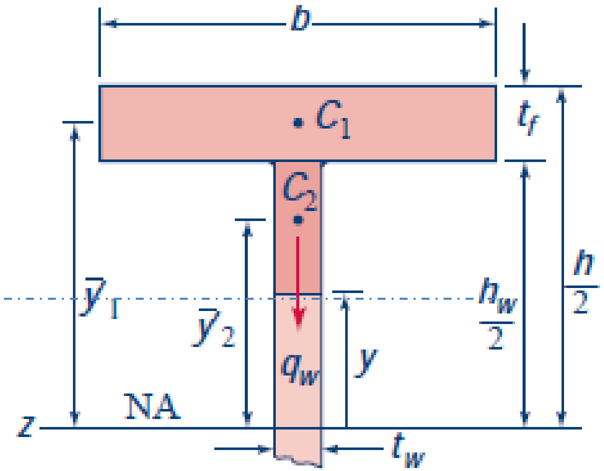
$$Q_1 = bt_f \times \left(\frac{h - t_f}{2}\right)$$

$$Q_2 = t_w \left(\frac{h_w}{2} - y\right)\left[\frac{\left(\frac{h_w}{2} - y\right)}{2} + y\right]$$

Unbalanced flexural stresses lead to shear stresses

Tensões de Cisalhamento na Alma





$$q_w = \frac{VQ}{I}$$

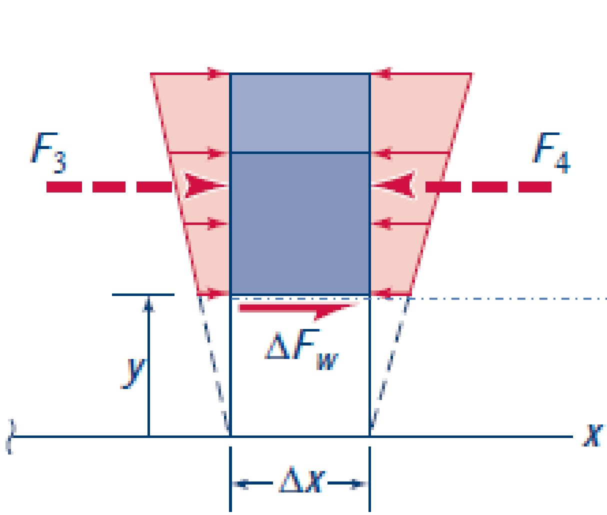
$$Q_1 = bt_f \times \left(\frac{h - t_f}{2}\right)$$

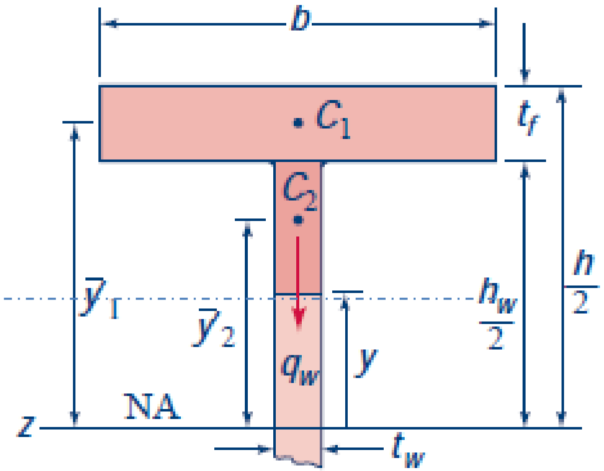
$$Q = A'_w \times y'_w$$

$$Q_2 = t_w \left(\frac{h_w}{2} - y\right)\left[\frac{h_w}{2} + y\right]\frac{1}{2}$$

Unbalanced flexural stresses lead to shear stresses

Tensões de Cisalhamento na Alma





$$q_w = \frac{VQ}{I}$$

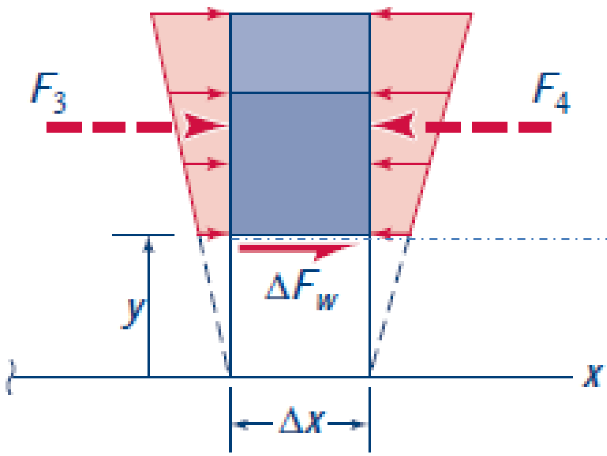
$$Q_1 = bt_f \times \left(\frac{h - t_f}{2}\right)$$

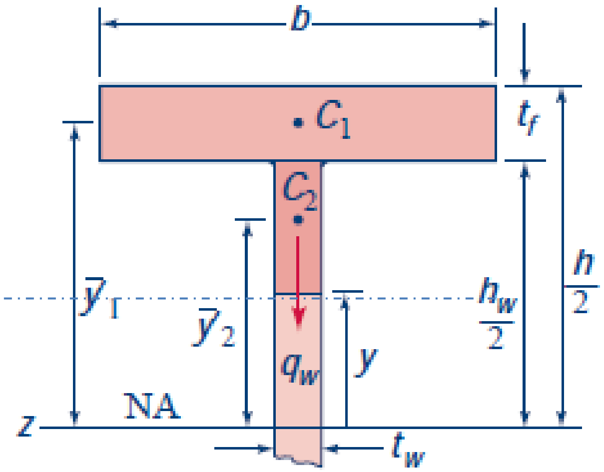
$$Q = A'_w \times y'_w$$

$$Q_2 = \frac{t_w}{2}\left[\left(\frac{h_w}{2}\right)^2 - \boldsymbol{y}^2\right]$$

Unbalanced flexural stresses lead to shear stresses

Tensões de Cisalhamento na Alma



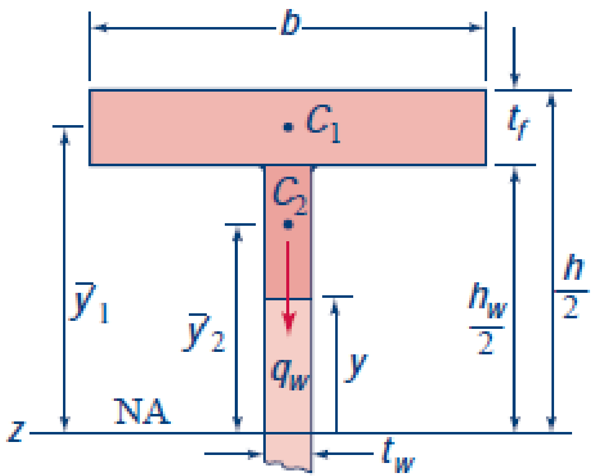


$$t_w \ll h_w$$

$$q_w = \frac{V}{I}\left\{bt_f \times \left(\frac{h - t_f}{2}\right) + \frac{t_w}{2}\left[\left(\frac{h_w}{2}\right)^2 - y^2\right]\right\}$$

Unbalanced flexural stresses lead to shear stresses

Tensões de Cisalhamento na Alma



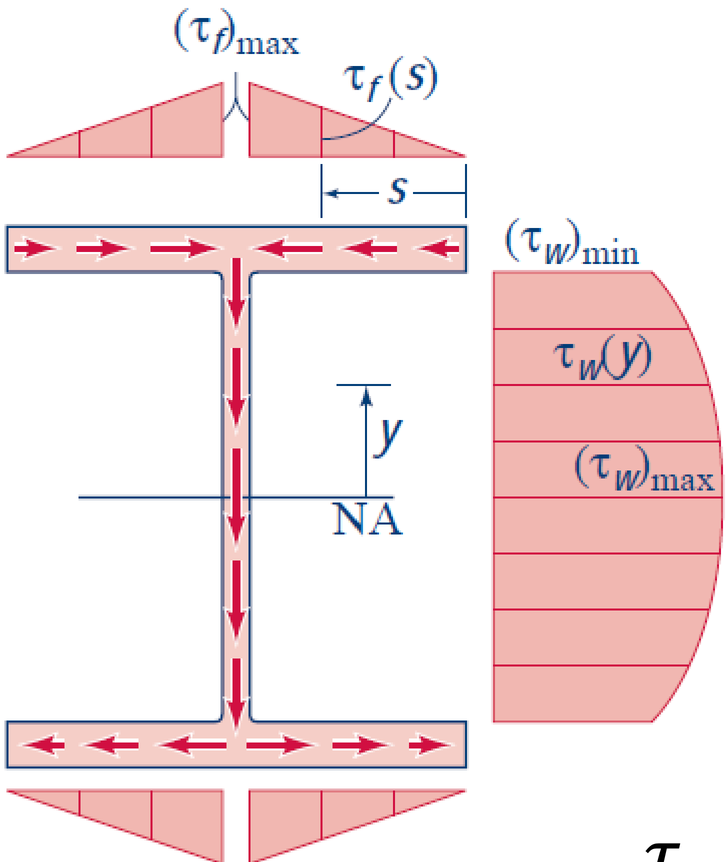
$$\tau_w = \frac{q_w}{t_w}$$

$$\downarrow$$

$$\tau_w = \frac{V}{It_w}\left\{bt_f \times \left(\frac{h - t_f}{2}\right) + \frac{t_w}{2}\left[\left(\frac{h_w}{2}\right)^2 - \boldsymbol{y}^2\right]\right\}$$

Unbalanced flexural stresses lead to shear stresses

Tensões de Cisalhamento Alma/Flange



$$\tau_f = \frac{q_f}{t_f} \qquad \tau_w = \frac{q_w}{t_w}$$

$$\tau_f = \frac{V}{I}\, \boldsymbol{s} \times \left(\frac{h - t_f}{2}\right)$$

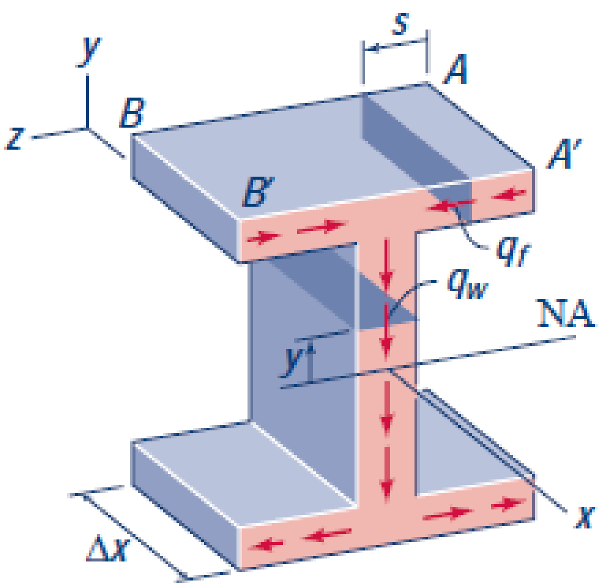
Linear

$$\tau_w = \frac{V}{I t_w}\left\{Q1 + \frac{t_w}{2}\left[\left(\frac{h_w}{2}\right)^2 - \boldsymbol{y}^2\right]\right\}$$

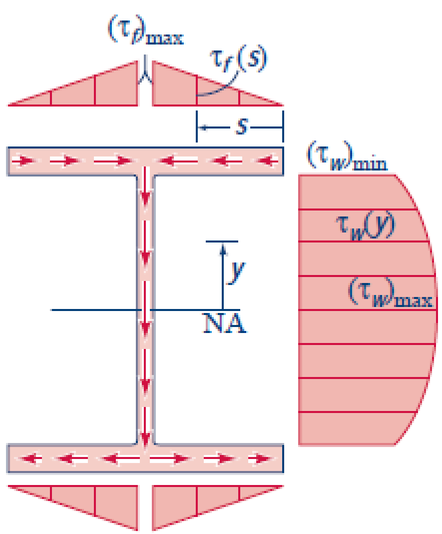
Quadrático

[*Unbalanced flexural stresses lead to shear stresses*]

Tensões de Cisalhamento

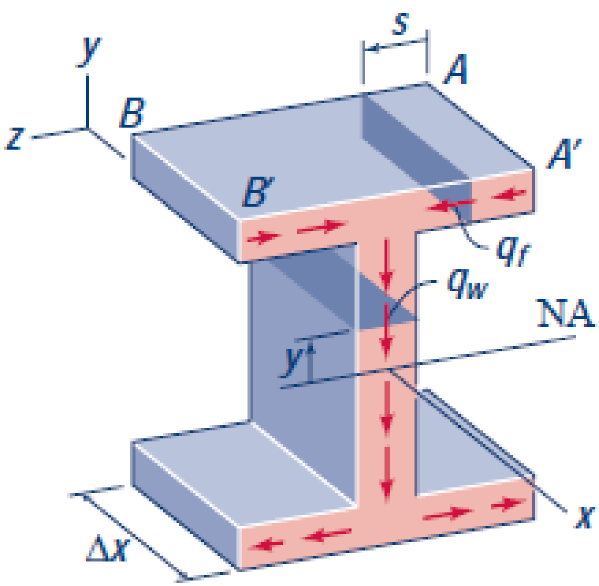


$$\sigma_{ij} = \begin{pmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{pmatrix}$$

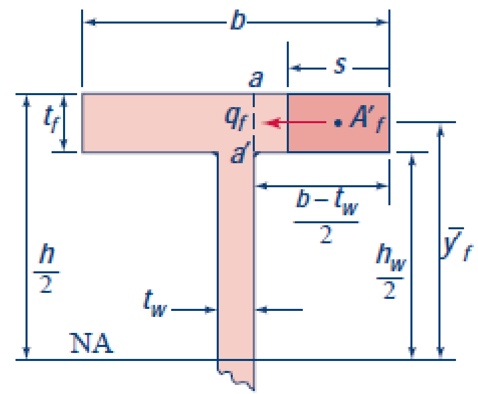


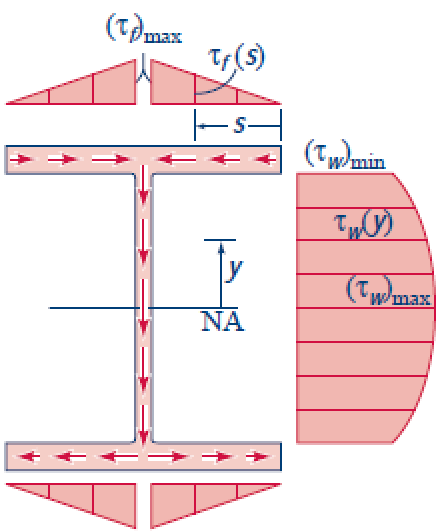
$$\sigma_{ij} = \begin{pmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & 0 & 0 \\ \tau_{zx} & 0 & 0 \end{pmatrix}$$

Tensões de Cisalhamento

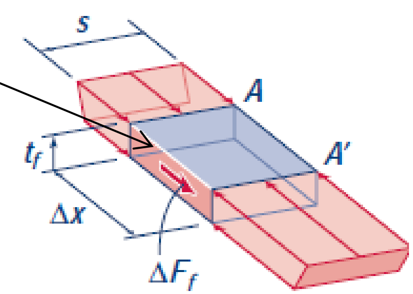


$$\sigma_{ij} = \begin{pmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{pmatrix}$$

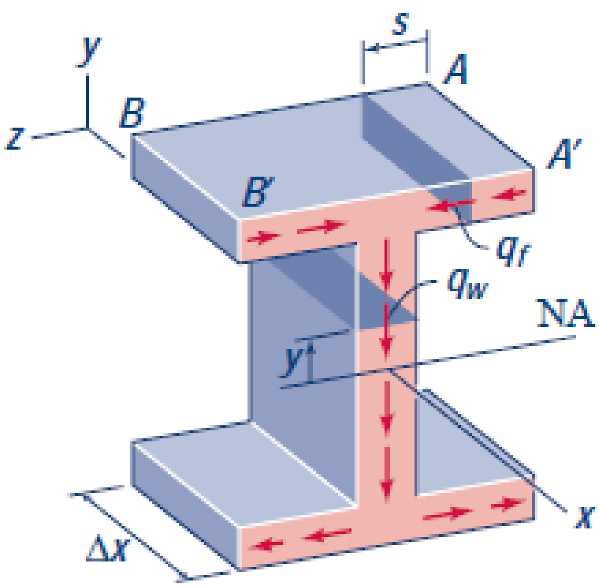




$$\sigma_{ij} = \begin{pmatrix} \sigma_{xx} & & \tau_{xz} \\ & 0 & 0 \\ \tau_{zx} & 0 & 0 \end{pmatrix}$$

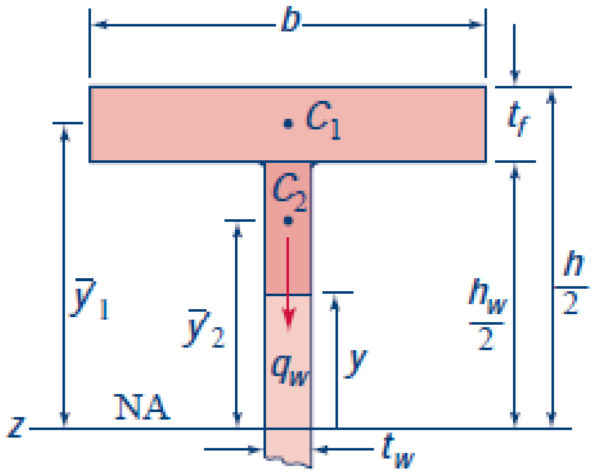


Tensões de Cisalhamento



$$\sigma_{ij} = \begin{pmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_{yy} & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{pmatrix}$$

0



$$\sigma_{ij} = \begin{pmatrix} \sigma_{xx} & \tau_{xy} & \\ \tau_{yx} & 0 & 0 \\ & 0 & 0 \end{pmatrix}$$