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THE EXISTENTIAL GRAPHS

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Abstract—This introduction to Charles S. Peirce's elegant logic diagrams explains the elements and operations of the Alpha and Beta parts of the system. These parts, together, constitute a complete and consistent treatment of elementary logic. Statements and inferences involving quantifiers and relations are presented and explained, typical methods of proof are illustrated, and a proof of the consistency of the system is given. A brief sketch is presented of Peirce's attempts to extend the system and of current work on a computational model of Alpha and Beta.

The system of Existential Graphs (EG) is a diagrammatic system of logic by means of which we can express, and then examine and experiment with, statements and inferences. It has a remarkably small number of special symbols, and it shows in an especially clear way that deductive reasoning can be analyzed into certain insertions and omissions of statements, or declarative sentences. The system was invented by Charles S. Peirce in 1896 (see [1]) and as developed by him it soon became a complete and consistent treatment of elementary logic.[1] Peirce is better known for his logical algebras and his pioneering work in the logic of relations, but by 1897 [4], he preferred graphical to algebraic notations as a means of investigating these fields of inquiry. Recently EG has been used as the logical basis for a system of conceptual graphs by John Sowa [8]. Sowa connected his graphs to such topics as semantic networks and artificial intelligence, and the relevance of EG to these topics brought out by Sowa's work has been further stressed by Fritz Lehmann (see their articles in this volume).

Peirce sometimes used the terms "Alpha" and "Beta" for the parts of EG that correspond to the two major components of elementary logic, namely, the logic of statements and the logic of predicates and quantifiers. Further developments, including but not restricted to modal logic, belong to a "Gamma" part of EG which is mentioned briefly in this paper.

1. ALPHA

1.1. *The Sheet and Juxtaposition*

Because EG is a two-dimensional system, the surface on which the diagrams or graphs are to be placed becomes important. In practice, it may be a blackboard, or a sheet of paper, or something else; but whatever surface is used will be called the *sheet of assertion* (SA). SA is one of the primitive symbols of EG; it is a graph, even if it is blank, in which case it represents what philosophers and others call "the universe of discourse"—the sum total of what the reasoner understands himself to be reasoning about. Further, whatever is written on SA is asserted to be true of the (perhaps fictitious) universe represented by SA.

A pear is ripe
The lazy dog stumbles over the quick brown fox

Figure 1.

[1] A detailed treatment of EG is given in [1]. This monograph takes account of the unpublished manuscripts at Harvard [2] as well as the published material [3-6]. Deductive completeness for EG was first proved by J. Jay Zeman [7]; a different proof is given in [1]. That the system is consistent is proved in this paper (with a small improvement over the proof given in [1]).

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If we write "A pear is ripe" on **SA**, we assert that there is a pear in our universe, and it is ripe. Such a sentence, written on **SA**, constitutes a graph. If we write two or more sentences onto **SA**, we assert them all. Thus, Figure 1 means: There is a ripe pear *and* the lazy dog stumbles over the quick brown fox. Thus, writing two or more sentences (graphs) on **SA** is to join them by the conjunction "and." Graphs written together on **SA** are said to be *juxtaposed* on **SA**. Their order or arrangement, however, is of no significance in **EG**.

1.2. The Cut

To deny a statement, we enclose it in a finely drawn self-returning line, called a cut because it cuts off or separates the enclosed statement from the rest of **SA**.

Figure 2.

Thus, Figure 2 asserts "It is false that you are a cautious reasoner." The shape of the cut has no significance. Each of the following is a graph of the sentence "It does not rain."

Figure 3.

The cut separates (cuts off) one part or *area* of **SA** from another. The interior of a cut is called *the area of the cut*; the cut, together with its area and whatever graphs are on its area, is called an *enclosure*. It is important to note that the cut *per se*—the finely drawn line itself—is not a graph, but every enclosure is. Writing or drawing or otherwise placing a graph on any area will be called *scribing* a graph, and the area on which a graph is scribed is called the *place* of that graph. We abbreviate the phrase "the place of the graph P" by the use of braces: {P}. Two or more graphs scribed on the same area, that is, two or more graphs not separated by cuts, are said to be *juxtaposed* on that area, and the order or arrangement of juxtaposed graphs has no logical significance.

Since **SA** is a graph representing all the things we take for granted, it follows that a cut made[2] on an otherwise empty area of **SA** is the denial of something taken to be true. The value of such an empty cut is always false, and it cannot, therefore, represent any possible state of the universe; since Peirce intended that all graphs scribed on **SA** should represent such possible states, he sometimes called the empty cut the *pseudograph*.

1.3. The Scroll, and Interpreting Graphs

So far, we have simple assertion, conjunction, and negation which, together, are sufficient to build any structure of the statement calculus. For example, Figure 4 expresses the conditional sentence "If P then Q," since the graph, read from the outside inward, states "It is false that both P and not-Q are true," which is equivalent to the conditional. And Figure 5, which reads "It is false that both not-P and not-Q," expresses the inclusive alternation "Either P or Q." This method of reading graphs from the outside (or least enclosed area) inward was given the name "endoporeutic" by Peirce. We will elaborate on it as we proceed.

The two-cut graph of Figure 4 is called a *scroll*, and Figure 4 can be read "P scrolls Q." {P}, the place of P, is called the *outer area* or *first area* of the scroll; {Q} is called the *inner* or *second*

[2] Because the cut is not a graph, we do not say that we "scribe" a cut, but that we "make" it.

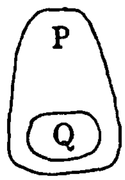


Figure 4.

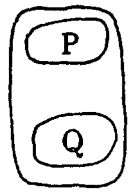


Figure 5.

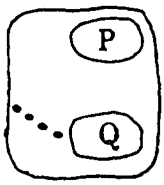


Figure 6.

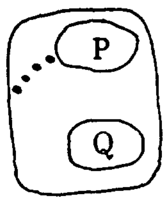


Figure 7.

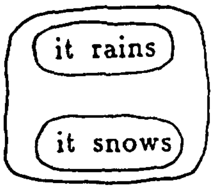


Figure 8.

area. The graph on the outer area is called the *antecedent*, that on the inner area, the *consequent*, of the scroll.

Even the graph of Figure 5 can be read as a scroll, and in two different ways, depending upon which set of two cuts is taken to constitute the scroll—the two cuts joined by the dotted line in Figure 6, or the two cuts joined in Figure 7. If we identify the scroll as in Figure 6, then cut-P is the antecedent and Q is the consequent, giving "If not-P then Q"; if we identify it as in Figure 7, then cut-Q is the antecedent and P the consequent, which gives "If not-Q then P."

One of the strengths of EG is that a graph of even slight complexity can be read in several ways, eliminating the need for additional special symbols. Facility in reading graphs becomes second nature with only a little practice. Thus, Figure 8 can readily be seen to express four equivalent statements, depending upon whether we read it endoporeutically (as a negated conjunction), or as an alternation, or as two different conditionals: (1) It is false that: it does not rain and it does not snow; (2) Either it rains or it snows (or both); (3) If it does not rain, then it snows; and (4) If it does not snow, then it rains.

1.4. Nests of Cuts and Containment

Enclosures which result from placing cuts within other cuts consist of one or more "nests" of cuts, where by *nest* we mean a collection or series of cuts each enclosing the next one. Figure 9 is a nest of three cuts with its four distinct areas "labeled" by the letters (or graphs) P, Q, R, and S. An area is said to be *oddly enclosed* if it is enclosed by any odd number of cuts (begin the count from the unenclosed SA); and it is called *evenly enclosed* if it is enclosed by an even number of cuts, or by no cuts at all. Hence, the unenclosed SA is said to be evenly enclosed.

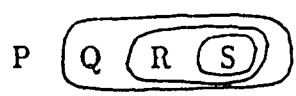


Figure 9.

This terminology applies also to graphs scribed on various areas. In Figure 9 the letters P and R are evenly enclosed, while Q and S are oddly enclosed. Cut-S, the enclosure consisting of the innermost cut with its contents, is evenly enclosed in Figure 9; and the two-cut graph, which by itself could be read "R scrolls S," is oddly enclosed in Figure 9. When a precise reference is necessary, we will use the following terminology: a graph on **SA** will be called an *unenclosed* or *level-0* graph; a graph separated from **SA** by a single cut will be called a *once-enclosed* or *level-1* graph; one separated from **SA** by two cuts, a *twice-enclosed* or *level-2* graph; and so on. This terminology will also be applied to areas.

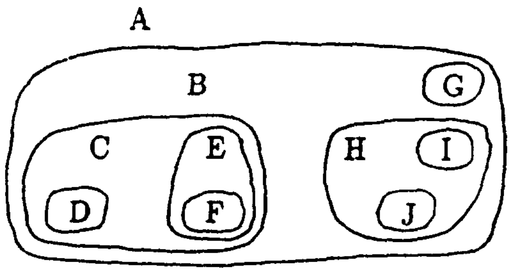


Figure 10.

An enclosure may contain more than one nest. In Figure 10, for example, there are five nests of cuts: one with 4 cuts (whose 5 areas are A-B-C-E-F); three with 3 cuts (whose corresponding areas are A-B-C-D, A-B-H-I, and A-B-H-J); and one with 2 cuts (with areas A-B-G).

Earlier, the notation $\{P\}$ was introduced to denote the place of P. Now let the relation symbol $\supseteq$, which may be read *contains*, be defined as follows: $\{A\} \supseteq \{B\}$, if and only if (iff) $\{B\}$ is enclosed by every cut that encloses $\{A\}$. Examination of Figure 10, in the light of this definition, shows, for example, that $\{A\}$ (which is **SA**) contains every area in the graph, including itself; and $\{B\}$ contains every area except $\{A\}$.

By means of the relation of containment, we can state precisely what it means for two areas to be in the same nest of cuts: Two areas belong to the *same nest of cuts* iff either area contains the other. This terminology can be applied to graphs also: Two graphs belong to the same nest of cuts iff the place of one contains the place of the other. This kind of analysis will greatly facilitate the application of **EG** rules of inference (introduced below) to graphs of any complexity. It verifies that Figure 10 does indeed contain five distinct nests of cuts. It shows that some areas in Figure 10 are not related to each other by containment—for example, there is no nest to which both E and G belong, since neither $\{E\} \supseteq \{G\}$ nor $\{G\} \supseteq \{E\}$ is true.

2. BETA

2.1. Predicates and Subjects

Suppose we take a statement and erase certain parts of it, so that it is no longer a statement but will become one, as soon as each blank is filled by a proper name (or noun or noun clause). This partly blank form of statement will be called a *predicate.* (It is the graphical analogue of what some logicians call an "open sentence," or what Russell and Whitehead called a "propositional function.") Consider, for example, the statement "Zeno was a pupil of Crates." One erasure of the sort described might produce "___ was a pupil of Crates", or it might produce the different predicate "Zeno was a pupil of ___." Two such erasures will produce "___ was a pupil of ___." We see from this that different analyses of a statement can produce different predicates. Figure 11 is an example of a predicate with three blanks; it becomes a true statement if we fill in the blanks, left to right, with "twelve," "five," and "seven."

___ is the sum of ___ and ___.

Figure 11.

According to **EG**, any statement, if it has no other statement as a proper part of itself, can be analyzed into two parts: the predicate, of which we have just seen several examples; and the *subject*, or *subjects*, defined as any part of the statement which might be replaced by a proper name (or noun or noun clause) and still leave the statement a statement. In our Figure 11 example, "twelve," "five," and "seven" are subjects. It is the subjects which identify the things represented by the predicates. Predicates *per se* do not count as graphs because, containing blanks, they cannot be said to be either true or false.

2.2. The Line of Identity

We now introduce the third special symbol of EG which, although it is not a proper name or a noun, will enable us to convert predicates into statements. That symbol is a heavily drawn dot, as in Figure 12; when it occurs by itself (not attached to a predicate) on SA, it denotes the existence of a single, individual (but otherwise undesignated) object in the universe of discourse ("something exists"). EG, thus, combines in one symbol the sign of individuality and the sign of quantification.

Figure 12.

Figure 13.

Now imagine stretching such a heavy dot into a heavy line, as scribed in Figure 13, and take this line to be an assertion of the identity of the individuals denoted by its two extremities. (In fact, any of the points on the heavy line can be taken to denote individuals, and the continuity of the line represents their identity.) Because of this identity claim, the heavy line is called the *line of identity* (often abbreviated to *line*).

How do we use the heavy line to convert predicates with blanks into statements? By attaching it to the predicates, as in Figures 14 and 15: Figure 14 can be read "Something (or somebody) was a pupil of Crates," and Figure 15, "Something is the sum of something and something." These sentences do qualify as statements, they do make certain claims, unlike the blank forms (given above) from which they were obtained. Figure 14, for instance, is false if Crates had no pupils at all.

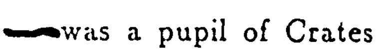


Figure 14.

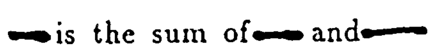


Figure 15.

Predicates whose every blank has been filled by a heavy dot or a line of identity qualify as graphs. Attaching a predicate to a dot or line serves to characterize or describe the individual denoted, serves to describe the "something." We will pretend that predicates have certain *hooks* to which the lines can somehow be fastened.

Figure 16.

The shape and length of a line of identity have no significance, so that each line in Figure 16 asserts "something exists." The three lines taken together do not, however, guarantee the existence of three different objects, for each line might denote the same object. It follows from our interpretation of juxtaposition that Figure 16 means "Something exists and something exists and something exists."

Lines of identity may branch. Figure 17 shows a line with four branches and extremities; Figure 18 shows one way to represent a line with an indefinite, say n, number of branches. In

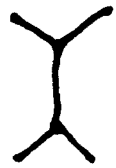


Figure 17.

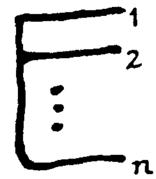


Figure 18.

general, a branching line of identity expresses the identity of the individuals denoted by all its extremities.

Just as there is practically no limit to the way a line may branch, so there is no limit to the number of predicates to which a line may be attached. In Figure 19, there are three distinct lines of identity, but it is their combination with the predicates that enables us to express this true sentence about three prominent ancient Greeks: "There is a Stagirite who teaches a Macedonian conqueror of the world and who is at once a disciple and an opponent of a philosopher admired by Fathers of the Church."

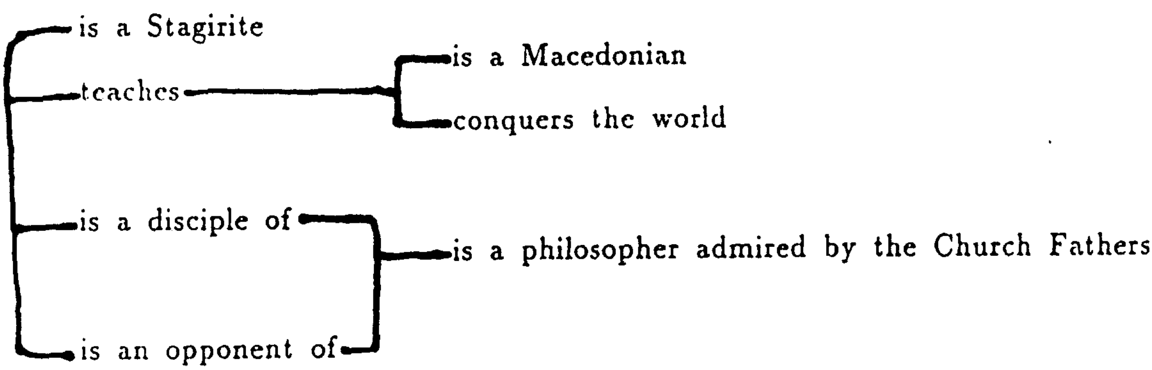


Figure 19.

2.3. *Quantification*

The endoporeutic method of interpretation applies to graphs containing lines of identity. The four categorical statements of Aristotle's syllogistic provide convenient examples. Figure 20 means "There exists in our universe something which is both painful and good," or "Some pain is good."

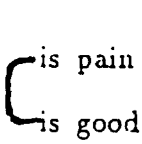


Figure 20.

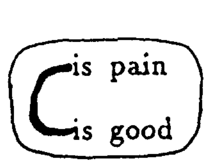


Figure 21.

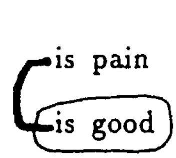


Figure 22.

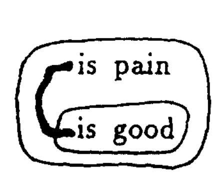


Figure 23.

Figure 21 is the denial of Figure 20. Reading endoporeutically, we begin on SA and interpret the cut first, which yields "It is false that some pain is good." Familiarity with language should make it clear that this reading is equivalent to "No pain is good."

Since a line of identity on SA refers to "some" individual of our universe of discourse, such a line represents the existential quantifier. From Figure 21, we see that a line of identity enclosed in one cut (a level-1 line of identity) refers to "all" individuals, and thus may be taken to represent the universal quantifier.

In Figure 22, we have a line of identity crossing a cut. The continuity of this line means that the part which is outside the cut and the part which is inside the cut denote the same individual. Hence, the graph means "Some pain exists and it is false that this pain is good," that is, "Some pain is not good." Figure 23, the denial of Figure 22, means "It is false that some pain is not good," that is, "Every pain is good."

Although only part of the line in Figure 22 is on **SA**, it is still read as the existential quantifier; and although only part of the line in Figure 23 is once-enclosed, it is still read as the universal quantifier. The general rule of interpretation for lines crossing cuts is this: *a line of identity is as much enclosed as its least enclosed (or outermost) part; and if this outermost part is evenly enclosed the line refers to "some" suitably chosen individual, while if this outermost part is oddly enclosed the line refers to "any" individual you please.*[3] The clause following the semi-colon is a generalization of our earlier remark about quantification, and it will enable us to interpret any line of identity, if we proceed by the endoporeutic method,—if, that is, we first read all unenclosed or level-0 lines, then all once-enclosed or level-1 lines, then all twice-enclosed or level-2 lines, and so on.

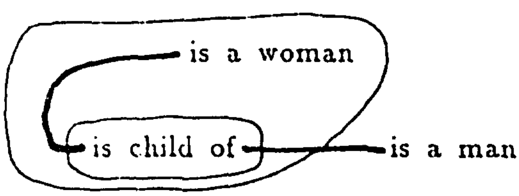


Figure 24.

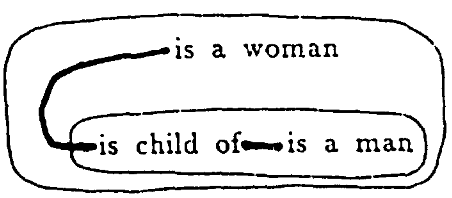


Figure 25.

Thus, in Figure 24, the unenclosed or level-0 line is read first, which gives "There is a man, such that, taking any woman you please, she is a child of that man." In Figure 25, there is no level-0 line, so the level-1 line is read first: "Take any woman you please, she is the child of some man (or other)." The difference between the two graphs consists in the order of selection of the individuals, and this makes all the difference between truth and falsity. For in Figure 24, where the man is chosen first, the claim is that every woman has the *same* father. Figure 25 claims only that every woman *has* a father.

2.4. Special Cases

To make the claim that at least two objects exist, we scribe the graph of Figure 26. Literally, this means "There is an object and there is an object and it is false that these objects"—denoted by the two unenclosed parts of the line—"are identical." By means of this graph, we can say such things as "Some woman has two husbands" (Figure 27). Figure 28 means "There is something, and it is false that there is something else non-identical to it"—that is, "Something is identical with everything." If we attach the predicate "is God" to the unenclosed end of the line only, as in Figure 29, we obtain the statement "God is identical with everything," which perhaps is what some Hindus believe about Brahman. If we attach the predicate to both ends of the line, we obtain (Figure 30) the statement "There is (exactly) one God," which expresses half of the creed of Islam. Figure 31 expresses the Unitarian theology as summarized by Whitehead: "There is one God at most."

A special and important class of scrolis consists of those whose outer area *either* is blank *or* contains only portions of lines of identity which pass from inside the inner area to outside the

[3] In some graphs, such as that of Figure 26, a line of identity may represent more than one idividual; such a line can be recognized by its having more than one least enclosed part.

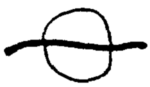


Figure 26.

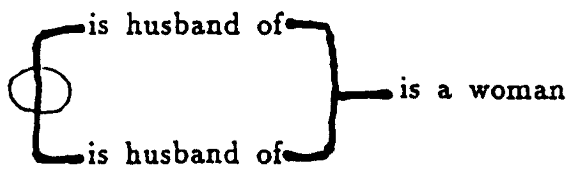


Figure 27.

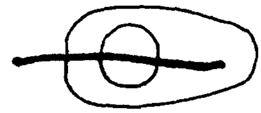


Figure 28.

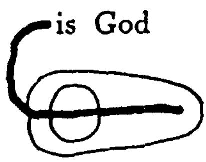


Figure 29.

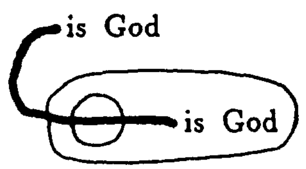


Figure 30.

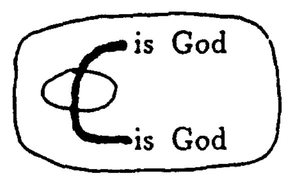


Figure 31.

outer area. Any such scroll is called a *double cut* (DC). Figures 32, 33, and 34 are examples of such scrolls; Figures 32 and 33 qualify as double cuts because their outer areas are blank; Figure 34 qualifies because its line of identity extends from outside the outer cut to inside the inner cut, there being no other graph on the outer area of the scroll. Figure 35 fails to qualify as a DC, because the line of identity terminates on its outer area, and the termination counts as a graph. It is readily seen that the double cut is the graphical equivalent of double negation.

Figure 32.

Figure 33.

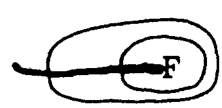


Figure 34.

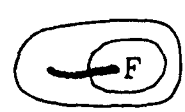


Figure 35.

Should a line of identity terminate on a cut, it is to be interpreted according to this convention: points on a cut shall be considered to lie outside the area of that cut. Thus, Figure 36 has the same meaning as Figure 37 ("Something is F and Q is false"), and Figure 38 the same meaning as Figure 39 ("Something is not F and Q is true").

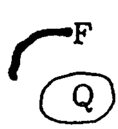


Figure 36.

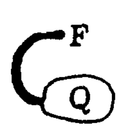


Figure 37.

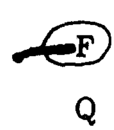


Figure 38.

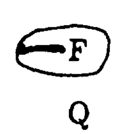


Figure 39.

3. DEDUCTION

3.1. Axioms and Rules

We have introduced the three special symbols of EG, namely SA, the cut, and the line of identity. For a formal development of the system we need, in addition, an infinite supply of predicates with no hooks, predicates with one hook, predicates with two hooks, and so on.

Figure 40.

Figure 41.

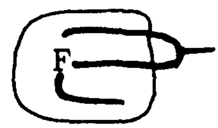


Figure 42.

The class of graphs is specified as follows: (1) Any part of the blank SA is a graph. (2) Any unattached line of identity is a graph. (3) If P and Q are graphs, then their simple juxtaposition PQ is a graph. (4) If P is a graph, then the enclosure consisting of a single cut with P alone scribed on its area (Figure 40) is a graph. (5) A predicate F with n hooks, $n = 0, 1, 2, \ldots$, is a graph iff each hook is attached to some line of identity. Such lines need not be enclosed by every cut that encloses F, nor is it required that all n lines be distinct (for $n > 1$). Thus, for $n = 3$, Figures 41 and 42 are graphs. It follows from (5), when $n = 0$, that any predicate having no hooks is a graph.

There are two axioms in EG: the blank sheet of assertion, SA, and the unenclosed, unattached line of identity. To make the line of identity an axiom is to assume that there is at least one object in any universe of discourse we choose.

The following five rules of transformation are the only inference rules required for elementary logic. We show below that none of them can change a true graph into a false one.

- R1. *The rule of erasure.* Any evenly enclosed graph may be erased, except for lines of identity which cross cuts—only evenly enclosed portions of such lines may be erased.
- R2. *The rule of insertion.* Any graph may be scribed on any oddly enclosed area, and two lines of identity (or portions of lines) oddly enclosed on the same area, may be joined.
- R3. *The rule of iteration.* A graph which already occurs may be scribed again within the same or additional cuts.
- R4. *The rule of deiteration.* Any graph whose occurrence is, or could be, the result of iteration, may be erased.
- R5. *The rule of the double cut.* The double cut may be inserted around or removed (where it occurs) from any graph on any area.

In R3, the word "same" does not mean the same number of cuts, but the identically same cuts. And in R4, the phrase "or could be" is required to emphasize that this rule can be applied even when no previous use of R3 has occurred; it is sufficient that the graph to be deiterated *might* have come about by iteration. In fact, it will facilitate our work with the graphs, if we elaborate a bit on R3 and R4, beginning with a restatement that makes use of the notion of "same nest of cuts":

R3 restated: A graph G may be scribed again on $\{G\}$ (the area of its original occurrence), or on any more-times-enclosed area of the same nest of cuts, but not on an area which is part of G itself.

R4 restated: If two or more instances of the same graph G occur on the same area, all but one may be erased; and if two or more instances of G occur on different areas of the same nest of cuts, those that are more-times-enclosed may be erased.

To clarify the application of these rules to lines of identity, we expand their statement into several clauses each:

R3 permits (a) a branch with a loose end to be added to any line of identity, provided that no crossing of cuts results from this addition; (b) any loose end of a line to be extended *inwards* through cuts; (c) any line thus extended to be joined to the corresponding line of an iterated instance of a graph; and (d) the two loose ends that are the *innermost* parts of a line to be joined by inward extensions, forming a *cycle*: a self-returning line of identity.

R4 permits (a) a branch with a loose end to be retracted into any line of identity, provided that no crossing of cuts occurs in the retraction; (b) any loose end of a line to be retracted *outwards* through cuts; and (c) any cycle to be cut at its *in*most part.

3.2. Elementary Inferences

The rule of insertion (R2) enables us to transform the graph of Figure 43 into that of Figure 44, since this is to introduce Q onto an oddly-enclosed area. Two applications of the rule of erasure (R1) transform Figure 45 into Figure 46: first erase the evenly-enclosed (because unenclosed) cut-P, and next erase the level-2 R. Note that the removal of the cut from around the P would not be a legitimate application of erasure, since what occurs evenly enclosed in Figure 45 is not a cut, but an enclosure.

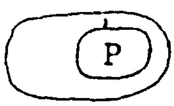


Figure 43.

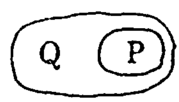


Figure 44.

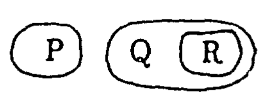


Figure 45.

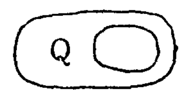


Figure 46.

The simplest applications of iteration (R3) and deiteration (R4) enable us to transform Figures 47 and 48 into each other. Iteration across cuts permits us to transform Figure 49 into Figure 50, since the new occurrences of P and cut-R are in a nest of cuts located on {P} and {cut-R} ("the place of cut-R"). Note that R itself cannot be iterated onto the level-2 area, which is {cut-Q}, because in Figure 49 {R} is not in the same nest of cuts as is {cut-Q}. In Figure 51, there are two occurrences of P in the same nest of cuts. Deiteration allows the transformation of Figure 51 into Figure 52, since the P, which is eliminated, was more-times-enclosed than the P which remains.

Figure 47.

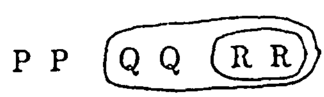


Figure 48.

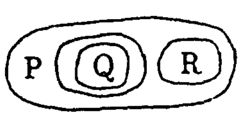


Figure 49.

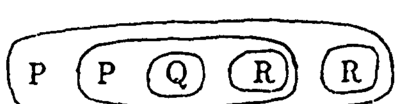


Figure 50.

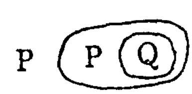


Figure 51.

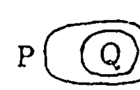


Figure 52.

Figure 53.

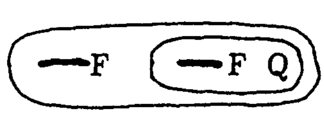


Figure 54.

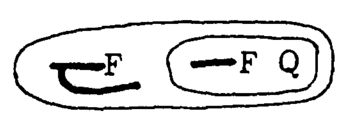


Figure 55.

Graphs in which lines of identity occur can, of course, undergo all the transformations illustrated thus far, but there are some transformations peculiar to lines of identity. For example iteration (R3) transforms Figure 53 into Figure 54; by clause (a) of the same rule, Figure 54 can be transformed into Figure 55, in which a branch has been added to the oddly enclosed line; by clause (b), the line can be extended into the cut, yielding Figure 56; and clause (c) permits the joining of the lines on the evenly enclosed area, as in Figure 57. Deiteration (R4) can reverse each of these steps, giving us Figure 53 again. (The transformation of Figure 57 into Figure 56 consists in the erasure on a twice-enclosed area of a connection or join which also occurs on a once-enclosed area of the same nest of cuts.) An alternative way to obtain Figure 53 from Figure 57 would be to begin by applying deiteration to Figure 57 in order to obtain Figure 58, and then retracting the line by clauses (c) and (b) of the rule. Variations on the steps from Figure 53 to Figure 57 occur so frequently that it is convenient to omit the intermediate steps, scribing only the first and the last, and labelling the inference "R3/R3(a, b, c)." If, for reasons of clarity, the initial iteration is shown (by a graph corresponding to that of Figure 54), the inference exemplified in Figures 54 to 57 (omitting steps corresponding to Figures 55 and 56) will be labelled "R3(a, b, c)."

Figure 56.

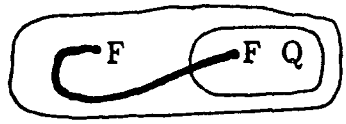


Figure 57.

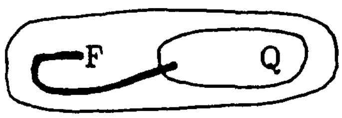


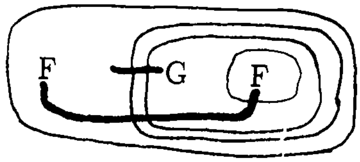
Figure 58.

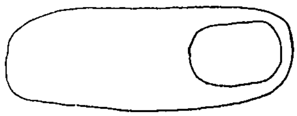
3.3. Theorems and Metatheorems

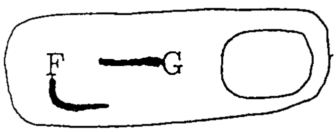
Peirce sometimes said that EG provides a "motion picture of thought." Animated drawings might best depict the gradual changes taking place as an inference proceeds. At the start, we would see the blank SA or a graph representing the premises of an argument; we would then see the graph change as the rules of transformation are applied; the final result would be a theorem, or a conclusion of the argument. Note, however, that if no film or videotape were made of a graphical inference, everything except the conclusion would disappear; for the process itself does not provide a record of the premises or of the intermediate stages of the transformation.

To remedy this inconvenience, we represent a proof in EG by a set of graphs each of which can be thought of as a "frame" taken from a film or a videotape of the inference. This makes our proof format much like that used in other systems of logic. Each graph is numbered as it enters the proof set, and to the right of each we indicate how its inclusion in the set is justified.

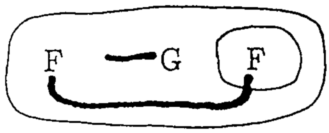
As a first illustration we present the proof of a graphical analogue of the theorem $(\forall x)(\forall y)$ $[Fx \supset (Gy \supset Fx)]$:



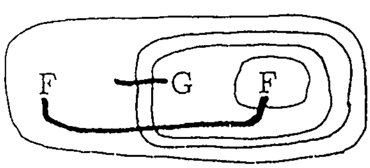
1.  R5 (Double cut)

2. 

1, R2 (Insertion on odd)

3. 

2, R3/R3(a, b, c)

4. 

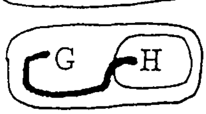
3, R5

At Step 3, we make use of the abbreviation introduced in the final paragraph of the previous section.

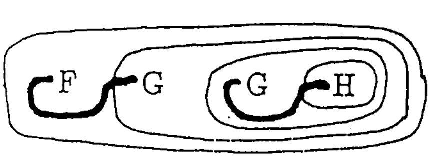
Another standard move in **EG** is to iterate an entire graph onto some area of another graph. We illustrate it in Step 3 of the following proof of the categorical syllogism known as "Barbara": From "All F is G" and "All G is H," we are entitled to infer "All F is H"; that is, from $(\forall x)[Fx \supset Gx]$ and $(\forall x)[Gx \supset Hx]$, we may infer $(\forall x)[Fx \supset Hx]$.

1. 

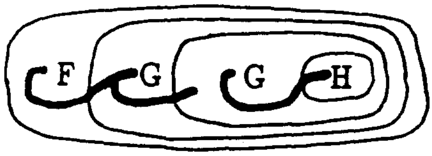
Premise

2. 

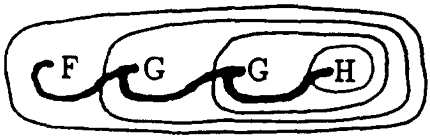
Premise

3. 

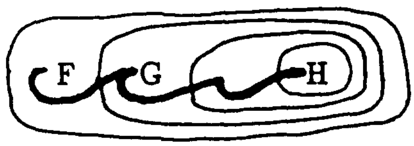
1, 2, R3

4. 

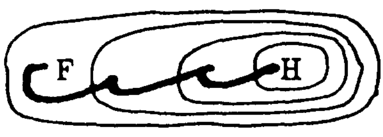
3, R3(a, b)

5. 

4, R2

6. 

5, R4: Deit. level-3 line-G

7. 

6, R1

8. 

7, R5

In Step 3, the second premise has been iterated onto the second area of the first premise. The inward extension of the line is, as usual, justified by R3; but the joining of the lines in Step 5 cannot be justified by R3(c) because the level-3 line-G is not an iteration of the level-2 line-G, but of the line-G of Premise 2. Since the lines to be joined occur on an oddly enclosed area, they may be joined by R2. This common sequence of inferences will often be abbreviated by the omission of the graph corresponding to Step 6; the label "R3/XJ/R2" will call attention to this extension by R3 and subsequent joining by R2. The Step 6 deiteration of the level-3 line-G is justified because, as a result of Step 5, the two occurrences of the predicate G are attached to the same line of identity; this qualifies the innermost G as a "could have been" iteration of the level-2 G (see the initial statement of R4).

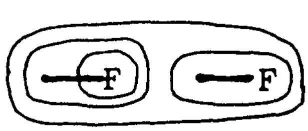


Figure 59.

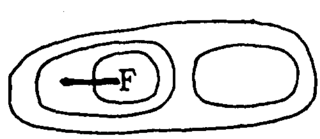


Figure 60.

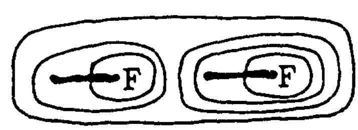


Figure 61.

The Beta axiom, the unenclosed and unattached line of identity, enables us to prove the graphical version of $(\forall x)\, Fx \supset (\exists x)\, Fx$, given in Figure 59, which asserts "If everything is F, then there exists something which is F." We start by inserting "Everything is F" onto the oddly enclosed outer area of a double cut (use R5 first, then R2), which yields Figure 60. Iteration (R3) gives us Figure 61. Now we scribe the Beta axiom on SA (Figure 62), iterate it by R3 onto the level-2 area (Figure 63), and use R3/XJ/R2 (see preceding paragraph) to obtain Figure 64.

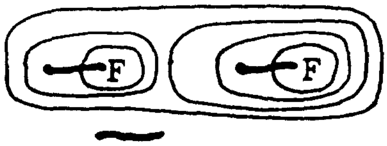


Figure 62.

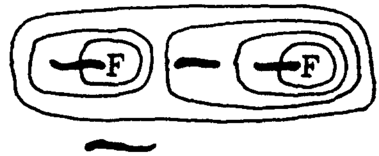


Figure 63.

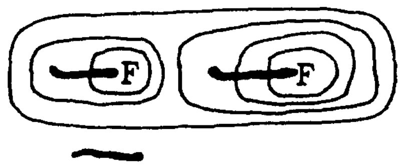


Figure 64.

Removal of the resulting double cut by R5, and the level-0 line by R1, yields the desired theorem (Figure 59).

Here is a more complex proof. The graphs of Figures 65 and 66 can each be derived from the other. This equivalence is the theorem $[(\exists x)\, Fx \vee (\exists x)\, Gx] \equiv (\exists x)\, [Fx \vee Gx]$.

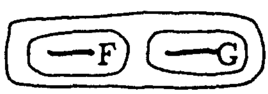


Figure 65.

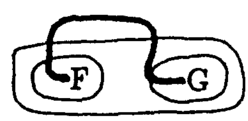


Figure 66.

The inference from Figure 66 to Figure 65 is straightforward, but the converse inference requires subtlety. It is tempting to say that the initial addition of double cuts is the "key" to the proof, but the proper insertions onto level-3 and the strategy of constructing an opportunity for the Step 7 use of deiteration are also essential. Taking the graph of Figure 65 as our premise, we begin by adding a double cut:

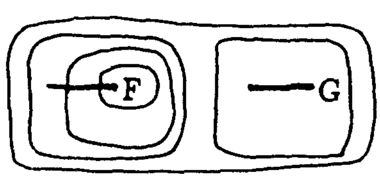
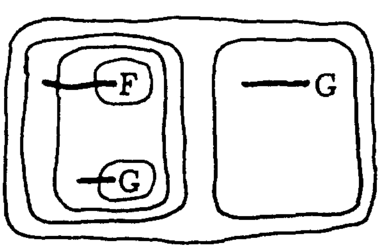
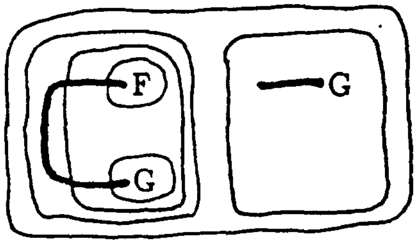
1. 

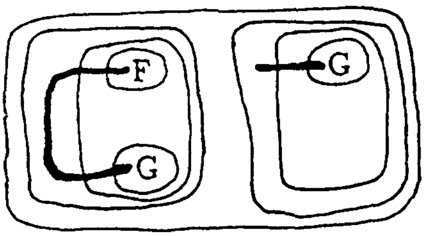
Figure 65, R5

2. 

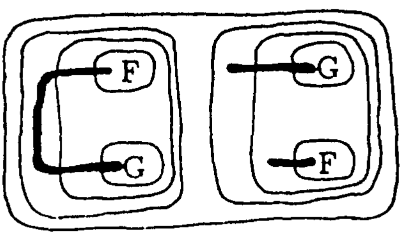
1, R2

3. 

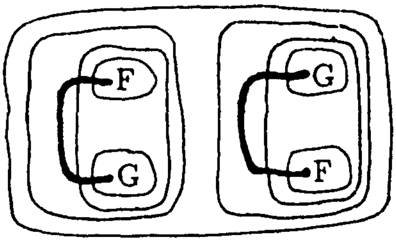
2, R3/XJ/R2

4. 

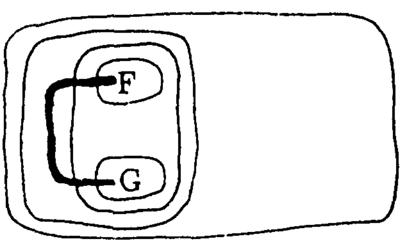
3, R5

5. 

4, R2

6. 

5, R3/XJ/R2

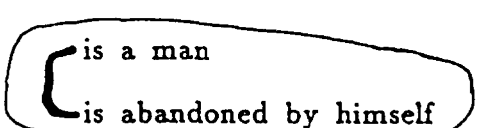
7. 

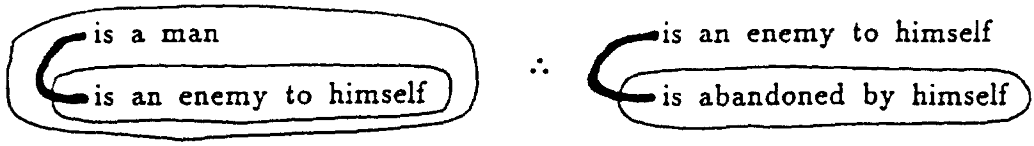
6, R4

From Step 7, Figure 66 follows by R5.

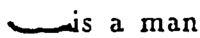
3.4. Analysis of an Argument

To illustrate the application of **EG** to arguments stated in ordinary language, we take an example from Antoine Arnauld's *Port Royal Logic* of 1662: "No man can abandon himself. Every man is an enemy to himself. Therefore, there are some enemies whom we cannot abandon." This is an EAO Aristotelian syllogism in the third figure, which is expressed as follows in **EG**:

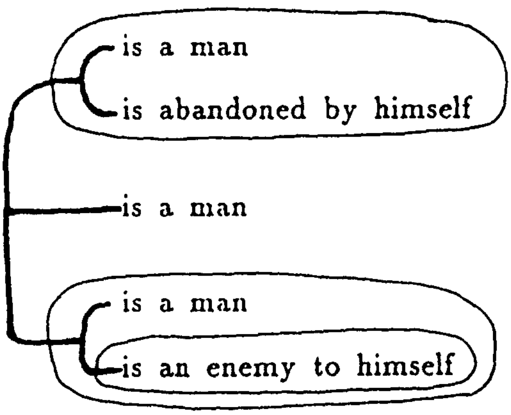
1. 

2. 

This is a valid argument in traditional logic where the existential import of universal statements is assumed; but in modern logic the universal statement "No man is abandoned by himself" is not taken as a guarantee that any man exists. In **EG**, therefore, we add this guarantee as an additional premise (Step 3), and the inference now is practically immediate.

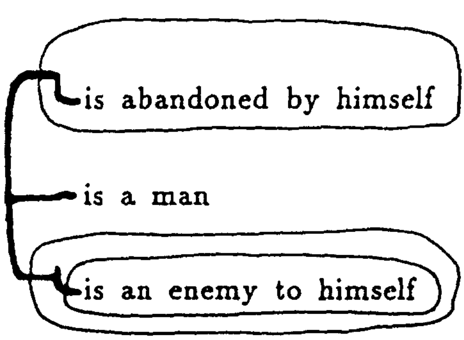
3. 

By what we have been calling R3/XJ/R2, we cause the line of identity attached to "is a man" to branch, we extend it into the level-1 areas of Steps 1 and 2, and join it to the lines already on those areas; this produces Step 4.

4. 

3, 1, 2, R3/XJ/R2

Because each occurrence of the predicate "is a man" is connected to the same line of identity, the more-times-enclosed occurrences can be deiterated, which yields Step 5. Now remove the double cut (by R5) and erase the level-0 line-(is a man) to obtain the conclusion.

5. 

4, R4

It is somewhat inaccurate, however, to press these sentences into categorical form. A more natural way to express them is readily available in **EG**. Thus, to express the statement "No man can abandon himself" we introduce the two-hook predicate "can abandon," and we attach the same line of identity to both its hooks as in Figure 67. This says "something can abandon itself." To identify that something as a man, we simply attach to the same line of identity the predicate "is a man," as in Figure 68. This asserts that some man can abandon himself; but we need, as first premise, the negation of this, which we obtain by putting a cut around the graph, obtaining Figure 69. The second premise, which also makes use of a two-hook predicate,

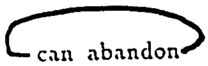


Figure 67.

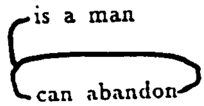


Figure 68.

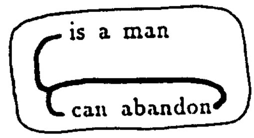


Figure 69.

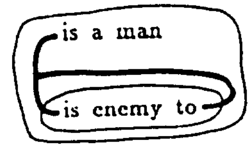


Figure 70.

is given in Figure 70. Let us take the conclusion to mean "Every man has some enemy whom he cannot abandon," which seems to be a fair rendering of the original. (The original, taken by itself, appears to suggest that everybody has the same enemy, which is not supported by the premises and is probably false as well.) The graph is given in Figure 71. A slightly more literal reading is "Take any [this reads the level-1 line of identity] man you please, there is at least one [this reads the level-2 line] enemy of his whom he cannot abandon."

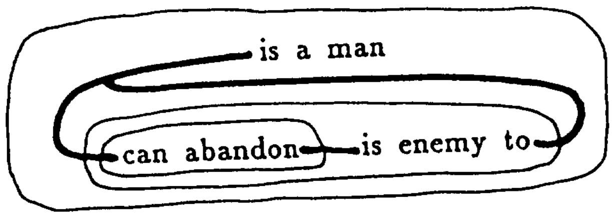


Figure 71.

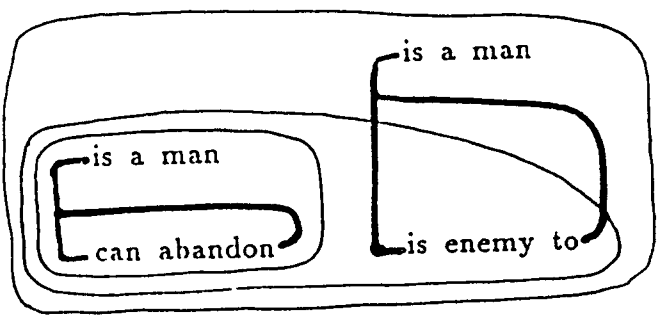


Figure 72.

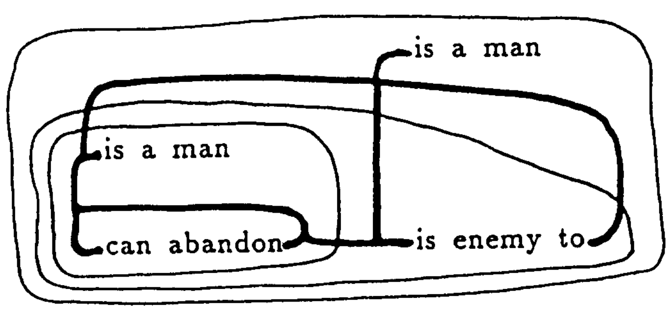


Figure 73.

The first step in deriving this conclusion from its premises is to iterate the first premise (Figure 69) onto the level-2 area of the second premise (Figure 70), which gives Figure 72. Next, we make two uses of our abbreviation labelled "R3/XJ/R2," first on the level-1 line of identity, attaching it to the level-3 line; second on the level-2 line, attaching it to a different portion of that same level-3 line. This gives us Figure 73. Now we deiterate the level-3 predicate "is a man," and by R4(c) break the cyclic line of identity on level-3 (its "inmost part"), which results in Figure 74. We then use R4(a) to retract the loose ends of the line on level-3, and erase a portion of the level-2 line by R1 in such a way as to produce Figure 75. From this graph the conclusion follows immediately, by retracting onto level-1 the loose end of the line produced by R1.

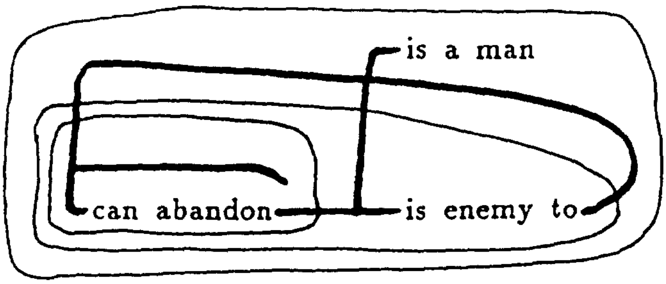


Figure 74.

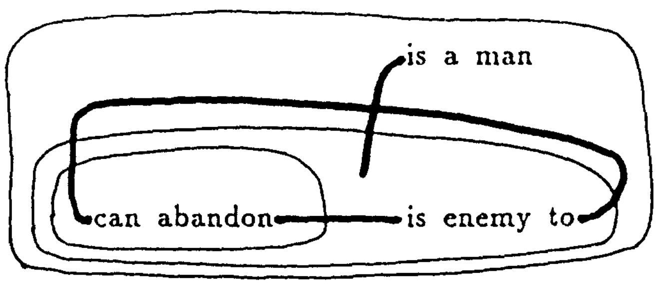


Figure 75.

4. CONSISTENCY

4.1. The Valuation Procedure

Deductive reasoning is good if it yields only true conclusions from true premises, and not otherwise. To show that this is the case for reasoning carried out in EG, we analyze the effects of our rules of inference, in terms of the following method for evaluating graphs.

Let 1 and 2 represent the truth-values truth and falsehood, respectively. By *value of an area* is meant the value of the juxtaposition, that is, the conjunction, of all the graphs scribed on that area. This value is calculated by the rule that *a conjunction has the value 1 (true) iff each conjunct has the value 1.* This means that a single 2 on an area is sufficient to give the area the value 2. The value of a graph is indicated by placing a 1 or a 2 next to the graph. The value of an area is indicated by placing a 1 or a 2 inside square brackets on that area. The *value of an enclosure*, indicated by a 1 or a 2 placed just outside its cuts, *is 2 (false) if the value of its area is 1 (true), and it is 1 (true) if the value of its area is 2 (false).* By *value of the cut K* is meant the value of the enclosure whose outermost cut is K. And by *n-th area* of a given nest of cuts is meant the area enclosed by n number of cuts.

To illustrate the procedure, we show that "P scrolls Q" has the value 2 when P = 1 and Q = 2; the evaluation, outlined in Figure 76, takes 5 steps. If we wish to determine the value of "P scrolls Q" for all possible values of P and Q, we can use a *truth-table* as in Figure 77. The two left-most columns, the *reference columns*, set out the four possible combinations of truth-values for the graphs P and Q. The values are arranged in columns beneath the letters, beneath the square brackets, and beneath the left-most edges of the cuts to which they belong. Each of the values in the table proper, beneath the graph being evaluated, is obtained according to the method explained above. The calculations in a given row of the table are based on the values assigned to P and Q in the reference columns for that row.

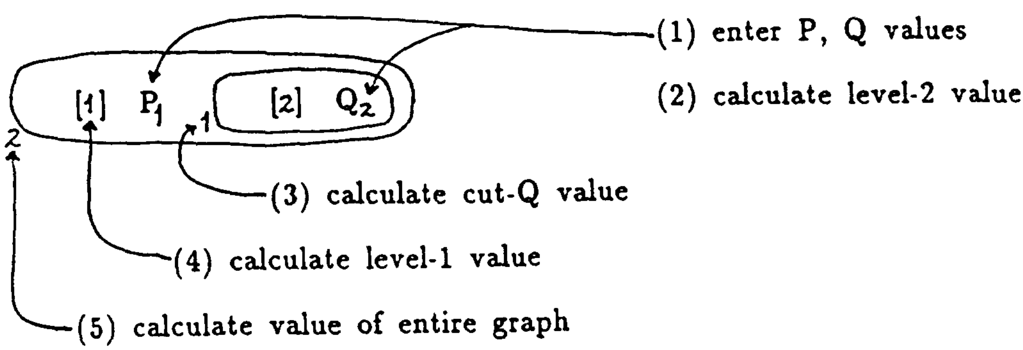


Figure 76.

P	Q		[ ]	P		[ ]	Q
1	1	1	2	1	2	1	1
1	2	2	1	1	1	2	2
2	1	1	2	2	2	1	1
2	2	1	2	2	1	2	2

Figure 77.

Truth-tables are convenient as long as lines of identity are not involved in the graphs being evaluated; the presence of lines complicates matters: the size of the universe becomes relevant, the number of hooks belonging to each predicate must be noted, and the procedure is subject to certain limitations which do not concern us here. Nevertheless, whenever the procedure can be applied to a graph G, the resulting truth-table must show as values for G either (a) a 1 in every row, or (b) a 2 in every row, or (c) a 1 in some rows and a 2 in the others. It is the first case (a) which is of interest to us, because that is the case in which G is said to be valid; and the consistency of **EG** depends upon the proof that every theorem is valid.

To be more precise, if a graph G is true for all interpretations of its predicates, for all combinations of values of its partial graphs, and for a non-empty universe of a given size—say, k number of objects—then: G is said to be *valid in* that universe of k objects. And G is said to be *valid* if it is valid in every non-empty universe. The **EG** axioms, **SA** and the unenclosed, unattached line of identity (Section 9 above), are examples of valid graphs, but valid by stipulation rather than by calculation. What we have now to prove is that the **EG** rules of transformation preserve validity, and our proof requires the following five lemmas in which the effects of erasure and insertion are made clear.

LEMMA 1. *Let α and β be areas of the nest of cuts N, such that $\alpha \supseteq \beta$. If the value of β has no effect on the value of α, then the value of β has no effect on the value of N.*

PROOF. This follows from the fact that, according to the valuation procedure just introduced, the calculation of the value of a nest of cuts begins from inside the nest and proceeds outwards. (For the value of a cut depends upon the value of its area; the value of an area depends upon the values of all the graphs scribed on it; and so on.) And, in this calculation, the value of each area is used exactly once.

NOTE. The following lemmas consider the erasure and insertion of *graphs in general*. This includes, among other things, portions of lines of identity. Thus, to erase a portion of a line of identity is to erase a graph (a graph asserting identity); and to join lines is to insert a graph (again, a graph asserting identity). Notice that to erase a portion of a line—to break or to sever it—is *not* to assert the non-identity of the individuals denoted by the newly separated lines; it is simply to drop or eliminate a previous claim of identity, so that the resulting graph asserts less than the original graph.

LEMMA 2. *To erase a graph from an area α can change the value of α from 2 to 1 but not from 1 to 2.*

PROOF. That the erasure of a graph G from α cannot change the value of α from 1 to 2 follows from the way in which the valuation procedure assigns a value to juxtaposition (conjunction), and from the stipulation that the blank has the value 1 (which is relevant in case the erasure empties α). But the erasure can change the value of α from 2 to 1, and it will do so in case G = 2 and no other graph on α has the value 2.

LEMMA 3. *To insert a graph onto an area α can change the value of α from 1 to 2 but not from 2 to 1.*

PROOF. That the insertion of G onto α cannot change the value of α from 2 to 1 follows from the way juxtaposition is evaluated. But such an insertion can change the value of α from 1 to 2, and will do so in case G = 2 and no other graph on α has the value 2.

LEMMA 4. *To erase a graph from an area β which is contained by an area α and enclosed by two more cuts than α can change the value of α from 2 to 1 but not from 1 to 2.*

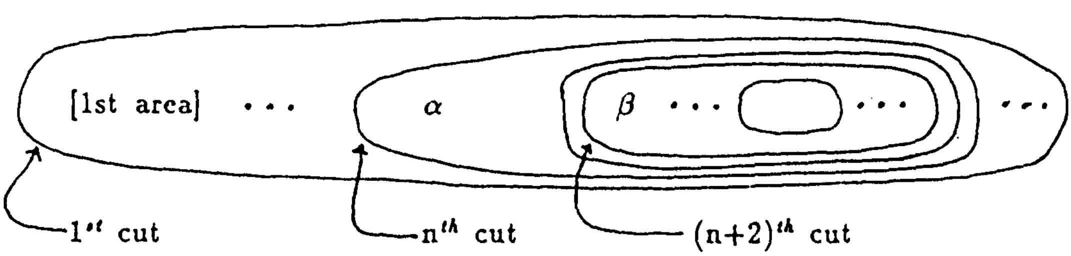


Figure 78.

PROOF. To fix our ideas, consider Figure 78, and suppose that α is enclosed by n cuts, β by $n+2$ cuts; we distinguish two cases.

Case 1. The value of α is 1. By Lemma 2, to erase a graph from β will either make no change in the value of β or will change it from 2 to 1. If the former, then, of course, no change will occur in the value of α. And if the latter, again no change will be effected in the value of α; for when the value of β is 1—no other value changes being made in the nest of cuts—straightforward calculation shows that α must retain the value 1. Hence, erasures performed on β cannot change the value of α from 1 to 2.

Case 2. The value of α is 2. Then any change in the value of α would be a change from 2 to 1, and this change can result from a value change on β. For suppose the value 2 of α is a result of the value 2 of the $(n+1)^{th}$ cut; then the value of the $(n+1)^{th}$ area must be 1. To change the value of β from 2 to 1 would change all of this, as straightforward calculation will show.

LEMMA 5. *To insert a graph onto an area β which is contained by α and enclosed by two more cuts than α can change the value of α from 1 to 2 but not from 2 to 1.*

PROOF. Consider Figure 78 again, with α and β identified as in Lemma 4. We distinguish two cases.

Case 1. The value of α is 2. By Lemma 3, to insert a graph onto β will either make no change in the value of β, or will change it from 1 to 2. If the former, then no change will result in the value of α. If the latter, then again no change will occur in the value of α; for if the value of β changes from 1 to 2—no other value changes being made in the nest of cuts—straightforward calculation shows that α must retain the value 2. Hence, insertions onto β cannot change the value of α from 2 to 1.

Case 2. The value of α is 1. Then any change in the value of α would be a change from 1 to 2, which can result from an insertion onto β, as the reader can verify for himself (using an argument similar to that in Case 2 of Lemma 4).

4.2. Preserving Validity

We now turn to the proof that reasoning in **EG** cannot yield false conclusions from true premises. We show, in fact, that the **EG** rules of transformation (Section 9 above) *preserve validity* in the sense that when the rules are applied to a valid graph as premise, the conclusion must also be valid. We use the term *total graph* to denote **SA** taken together with all the graphs scribed on **SA**. By *partial graph* is meant any graph scribed in the presence of other graphs.

R1, *the rule of erasure, preserves validity.*

PROOF. In Section 4, we defined a graph to be evenly enclosed iff it is enclosed either by an even number of cuts or by no cuts at all. We consider the latter case first.

Case 1. R1 is applied to some graph unenclosed on **SA**. It follows immediately from Lemma 2 that if the premise, the total graph, has the value 1, so does the conclusion. Hence, if the premise is valid, so is the conclusion.

Case 2. R1 is applied to some graph which is evenly enclosed in a nest of cuts scribed on **SA**. Now Lemma 2 states that erasure of a graph from any area can change the value of that area from 2 to 1, but not from 1 to 2. Let us call such a change of value a *validating change*. By Lemma 4, validating changes on any area of a nest of cuts produce only validating changes on the area of that nest enclosed by exactly two fewer cuts. Hence, validating changes on some k^{th} area of a nest can produce only validating changes on the $(k-2)^{th}$ area; validating changes on the $(k-2)^{th}$ area can produce only such changes on the $(k-4)^{th}$ area; and so on. So if k is any even number (our supposition for this Case 2), the production of changes will terminate on **SA**—and the result is this: validating changes on the k^{th} area of the nest can produce only validating changes on **SA**. By Case 1, such changes on **SA** preserve validity. Hence, in Case 2 also, if the premise, the total graph, has the value 1, so does the conclusion; and if the premise is valid, so is the conclusion. This complete the proof.

R2, *the rule of insertion, preserves validity.*

PROOF. Again we consider two cases.

Case 1. R2 is applied to the first area of a nest of cuts. Now if the nest of cuts has the value 1, the value of its first area must be 2; and by Lemma 3, insertion cannot change this value. Hence, if the premise, the total graph, has the value 1, so does the conclusion. And if the premise is valid, so is the conclusion.

Case 2. R2 is applied to an area enclosed by some odd number of cuts greater than one. Now Lemma 3 states that insertion of a graph onto any area can change the value of that area from 1 to 2, but not from 2 to 1. Let us call such a change of value a *falsifying change*. By Lemma 5, falsifying changes on any area of a nest of cuts produce only falsifying changes on the area of that nest enclosed by exactly two fewer cuts. Hence, falsifying changes on some k^{th} area of a nest can produce only such changes on the $(k-2)^{th}$ area; falsifying changes on the $(k-2)^{th}$ area can produce only such changes on the $(k-4)^{th}$ area; and so on. So if k is any odd number greater than one (our supposition for this Case 2), the production of changes will terminate on the first area of the nest—and the result is this: falsifying changes on the k^{th} area of the nest can produce only falsifying changes on the first area of the nest. By Case 1, such changes on the first area preserve validity. Hence, in Case 2 also, if the premise is valid, so is the conclusion. This completes the proof.

R3 and R4, *the rules of iteration and deiteration, preserve validity.*

PROOF. Let N be a nest of cuts in which the transformations are imagined to take place. Let α be the area of the original occurrence of a graph which is to be iterated, or the remaining occurrence of a graph which is to be deiterated; and let β be the area onto which the graph is to be iterated by R3, or from which it is to be deiterated by R4. It follows that $\alpha \supseteq \beta$. Two cases are distinguished.

Case 1. The graph G to be iterated or deiterated has the value 1. Regardless of the value of β before the use of R3 or R4, it follows from the way juxtaposition is evaluated that the insertion of G onto β or its removal from β cannot change the value of β and, therefore, cannot change the value of N. Hence, if the premise, the total graph, has the value 1, so does the conclusion; and if the premise is valid, so is the conclusion.

Case 2. The graph G to be iterated or deiterated has the value 2. Then the value of α is 2 regardless of the value of β before and after the use of R3 or R4, and it follows from this by Lemma 1 that the value of β has no effect on the value of N. Hence, if the premise, the total graph, has the value 1, so does the conclusion; and, hence, if the premise is valid, so is the conclusion.

From Cases 1 and 2, it follows that both R3 and R4 preserve validity.

To see that this proof covers applications of R3 and R4 to lines of identity, it is sufficient to recall that the interpretation—and, therefore, the evaluation—of a line of identity depends upon the position of its least enclosed part. Thus, (a) the adding or removing of *unattached* branches of a line, without crossing cuts, has no effect on the interpretation or evaluation of the line; and (b) the extension inwards or retraction outwards through cuts, of *unattached* branches of a line, has no effect on the interpretation or evaluation of the line. Furthermore, (c) to join corresponding lines of a graph and its iteration, or to break such a join, is to insert or erase an identity claim whose value is 1 (precisely because the one graph is an iteration of the other), and by Case 1 of the proof for R3 and R4, we know that such transformations preserve validity. Finally, (d) the formation or disruption of a cycle by R3 or R4 is to insert or erase a graph (an identity claim) whose value is 1; and again Case 1 of the proof applies.

R5, *the rule of the double cut, preserves validity.*

PROOF. This follows immediately from the valuation procedure as applied to the cut.

Since the **EG** axioms are valid and the **EG** rules of transformation preserve validity, it follows that any graph proved in **EG** from the axioms and rules alone is valid. That is:

THE VALIDITY THEOREM. *Every* **EG** *theorem is valid.*

Among the several senses of consistency that may be distinguished we define two: (1) A system of logic Σ is said to be *consistent with respect to* a given transformation, by which each expression P of Σ is transformed into an expression P′, if there is no expression P such that both P and P′ are theorems of Σ. (2) A system Σ is said to be *absolutely consistent* if not all of its expressions are theorems.

COROLLARY 1. **EG** *is consistent with respect to the transformation of P into cut-P.*

PROOF. By the definition of validity and the way in which the cut is evaluated, not both P and cut-P can be valid. Hence, by the Validity Theorem, not both P and cut-P can be theorems of **EG**.

COROLLARY 2. **EG** *is absolutely consistent.*

PROOF. The empty cut is not valid (in fact, it always has the value 2), and therefore by the Validity Theorem it is not a theorem of **EG**.

5. BEYOND ALPHA AND BETA

5.1. Gamma and the Tinctures

So far, we have restricted our attention to the Alpha and Beta parts of **EG** only; we have illustrated reasoning *in* the system and, with the consistency proof, reasoning *about* the system. Peirce, however, wanted to provide a way to reason *about* **EG** *in* **EG**, and he also wanted to use the system to investigate such things as qualities and relations and logical possibilities.[4] At the present time, these topics are routinely handled in standard second and higher order logics,

[4] Beta, for example, cannot express the sentence "Aristotle has all the virtues of a philosopher" because Beta does not quantify over predicates.

and in modal and many-valued logics. In Peirce's time, however, the idea of incorporating them into a formal system was relatively new. He claimed he "first broke ground" in this part of logic in 1885, as a consequence of his study of a paper published in 1882 by one of his students, O. H. Mitchell; and within two years of his discovery of EG, that is, by 1898, he had begun to incorporate these developments into a "Gamma" part of the system.[5] In both 1903 and 1906, Peirce worked intensively on these graphs, and he continued working on them in the following years at a more leisurely pace. The history of his accomplishments along these lines is given in Roberts 1973 [1]; here we present the briefest possible sketch.

Figure 79.

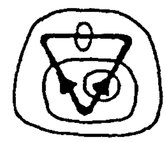


Figure 80.

The Gamma signs are of the same five varieties we have found in Alpha and Beta: graphs making or representing assertions, the sheet of assertion, the cut, the line of identity, and *spots*—Peirce's general term for unanalyzed expressions of predicates. Each of these five varieties takes on new forms in Gamma.

Figure 81.

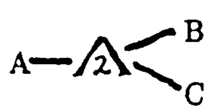


Figure 82.

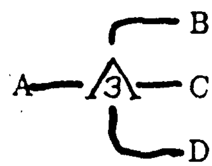


Figure 83.

One of his early special symbols looks simply like a line of identity which has been thickened into a node, as in Figure 79, which Peirce translated "A belongs to the general unordered collection B" or "A possesses the character B." By means of this symbol Peirce could diagram the statement "Given any two things, there is some character which one possesses and the other does not" (this is Figure 80, reading from top to bottom). A later version of symbols to express logical possibilities provided, as in Figures 81–83, characterizations of his three categories, Firstness, Secondness, and Thirdness. The figures mean "B possesses the quality A," "B is in the dyadic relation A to C," and "B is in the triadic relation A to C for D." These, and other sysmbols of the same sort, he called "the potentials." At the same time that he introduced the potentials Peirce was using colored lines of identity as well as lines placed between two rows of dots in order to represent abstractions.

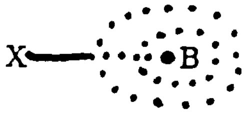


Figure 84.

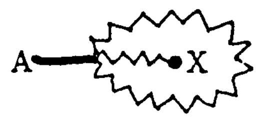


Figure 85.

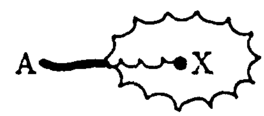


Figure 86.

[5] These remarks occur in MS 467, p. 12 (1903) [2], in a lengthy passage omitted by Hartshorne and Weiss at the very end of [5, 4.511]. Mitchell's essay appeared in *Studies in Logic*, a book edited by Peirce, containing essays by four other students of his along with several things by Peirce himself [9].

Peirce sometimes used Greek letters to represent predicates needed in scribing graphs of graphs. Thus, the lower case gamma represented graph-instances, kappa represented enclosures, and alpha represented areas. He altered the look of the cut in various ways and for several years experimented with dotted cuts or "rims," saw cuts, and wavy cuts. Figure 84 means "The individual X has the character of being a B," Figure 85 "A is the collection of all X's," and Figure 86 "A is one of the collections of X's."

His most succesful new cut, however, was the broken cut, which is illustrated in the graphs of Figures 87–90. They express the following statements: "It is possible that it does not rain," "It is necessary that it rains," "It is possible that it rains," and "It is possible that it necessarily rains." This is the only Gamma innovation that has caught the attention of more recent logicians. Zeman developed a Gamma version of Lewis' modal systems S4, S4.2, and S5 by tinkering, in a clever way, with the rules of iteration and deiteration [7]. And Butterworth developed a Gamma version of the system T in a similar way [10].

As early as 1898, Peirce had the idea of using his sheet of assertion to represent various universes of discourse and, in 1903, he spoke of replacing the single sheet with a book of separate sheets representing different kinds of possibility. The most ambitious development appeared in 1906, when Peirce published an account of his "tinctured" EG: the system was now provided with a large supply of tinctured or colored sheets, which were intended to allow the diagramming of questions, commands, and resolutions in addition to statements of fact and possibility. Peirce continued to work on these ideas after 1906 (as he continued working on the Gamma devices) but no further systematic additions were made apart from a very modest beginning in Roberts [1].

The range of ideas and the difficulties considered by Peirce are suggested in a draft of a letter he wrote to Lady Welby, date March 9, 1906 [3]:

> The system of Existential Graphs (at least, so far as it is at present developed) does not represent every kind of Sign. For example, a piece of concerted music is a sign; for it is a medium for the conveyance of Form. But I know not how to make a graph equivalent to it. So the command of a military officer to his men: "Halt!" "Ground arms!" which is interpreted in their action, is a sign beyond the competence of existential graphs. All that existential graphs can represent is *propositions*, on a single sheet, and arguments on a succession of sheets, presented in temporal succession.

My guess is that Peirce hoped the tinctured graphs or some other improved version *would* do the job even for such things as music and commands. This would explain the optimism expressed in the following remark from a later unpublished manuscript, MS 499s [2]: "There are countless Objects of consciousness that words cannot express; such as the feelings a symphony inspires or that which is in the soul of a furiously angry man in [the] presence of his enemy. But all these can perfectly be expressed in Graphs." It may still be premature to conclude that Peirce was simply daydreaming here, but I confess that this last claim seems mysterious.

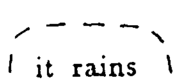


Figure 87.

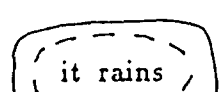


Figure 88.

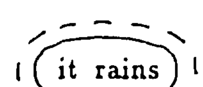


Figure 89.

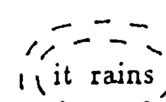


Figure 90.

5.2. Work in Progress: Pronovost's Computational Model for EG

In the 1970's, Roberts developed a method for determining the truth value of certain Beta graphs. Although the decision problem for the whole of first order logic is unsolvable, many interesting subsets are decidable [11]. And just as the method of trees is useful for these subsets when logic is treated in one of the standard algebraic notations, so Roberts' method, which he calls "weeds," has proved useful for Beta graphs.

When Dan Pronovost of Watcom Systems Inc. (Waterloo, Ontario) was introduced to weeds, he thought that a full implementation of the method would be greatly facilitated if the work could be done by a computer. The first problem was to develop a representation of **EG** by means of a model which is (1) isomorphic to **EG**, (2) faithful to the unusual features of **EG**,[6] (3) easy to program, and (4) computationally efficient.

It is not necessarily best to model a system such as **EG** directly. For one thing, although the topological properties of the graphs yield an elegant and productive representation of logic for human interpretation, they produce many difficulties for a computer model. While it might be possible to represent the topology of the graphs in a program, the discrete nature of computer programs does not suggest an effective or reliable representation. Instead, Pronovost based his model on a tree oriented data structure. The result, which he calls "Tree Existential Graphs" (**TEG**), satisfies the four constraints listed above.

So far, Pronovost and Roberts have developed translation algorithms from **EG** to **TEG** and from **TEG** to **EG**, and have verified that these transformations preserve the isomorphism of the two systems. These algorithms, programmed in the C programmer's language, have allowed us to represent the full variety of Beta graphs on the computer.

We have begun translating the **EG** rules of transformation into their **TEG** equivalents, but work in this area remains to be done. The next job is to translate weeds into **TEG**. Once this is done, we shall use **TEG** to complete the job of formalizing weeds and to confirm that the method is sound.

Our intention in programming weeds is to investigate whether or not a decision algorithm for logic based on the features of **EG** (as translated into **TEG**) is sufficiently computationally efficient to be of some practical use. As is well known, many of the standard decision proofs are based on methods whose order of computation is so high as to render them nearly useless from a practical point of view.[7]

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[6]Notice that a standard treatment of first order logic is isomorphic to **EG** but contains none of the topological features of **EG**.

[7]Ackermann [11], pp. 35–47.