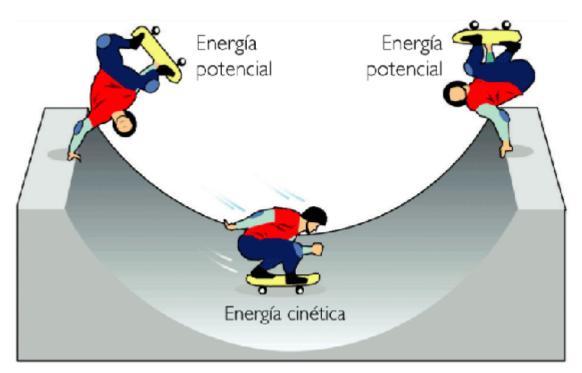
#### Física A para Engenharia Ambiental - 2020

#### Vídeo-aula 11 – Trabalho e energia

Prof. Dr. Marcos de Oliveira Junior



#### Na última aula

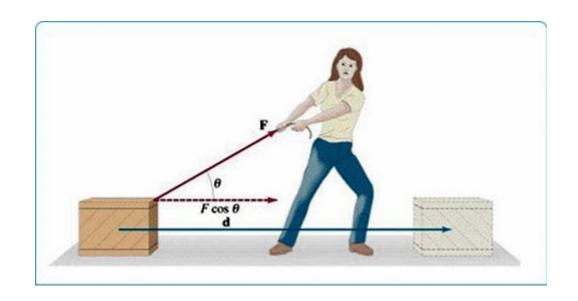


U = mgh

$$K = \frac{1}{2}mv^2$$

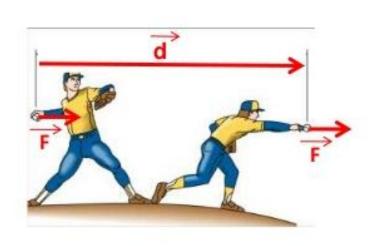
 $E_{total} = K + U = cte$ 

## Trabalho de uma força constante



$$W = \vec{F} \cdot \Delta \vec{r}$$

#### Teorema da Energia Cinética



$$F_{x} = ma_{x}$$

$$v^{2} = v_{0}^{2} + 2a_{x}\Delta x$$

$$v^{2} = v_{0}^{2} + 2\frac{F_{x}}{m}\Delta x$$

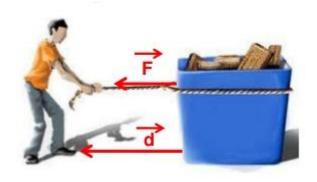
$$F_x \Delta x = \frac{1}{2} m v^2 - \frac{1}{2} m v_0^2$$

$$W = \vec{F} \cdot \Delta \vec{r} = \frac{1}{2} m v^2 - \frac{1}{2} m v_0^2$$

#### Conceito

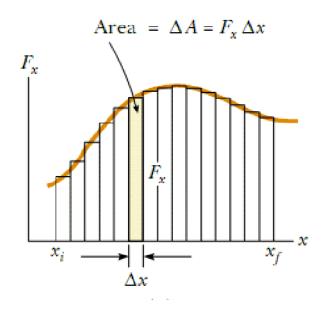
#### MEDIDA DA TRANSFORMAÇÃO/ VARIAÇÃO/TRANSFERÊNCIA DE ENERGIA

Quem ganhou energia: recebeu trabalho Quem perdeu energia: realizou trabalho



TRABALHO foi realizado pela pessoa sobre a caixa: pessoa perde energia química (processos biológicos internos) e caixa ganha energia cinética e energia térmica por causa do atrito.

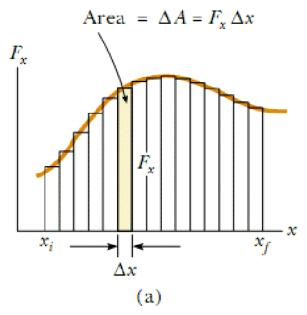
# Trabalho de uma força variável

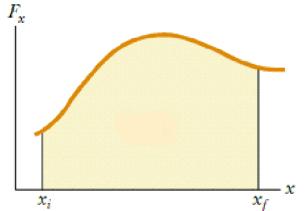


$$W = F_{x} \Delta x$$

$$W = \sum_{x_{i}}^{x_{f}} F_{x} \Delta x$$

# Trabalho de uma força variável





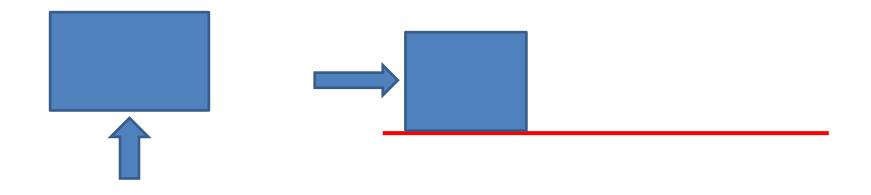
$$W = F_x \Delta x$$

$$W = \sum_{x_i}^{x_f} F_x \Delta x$$

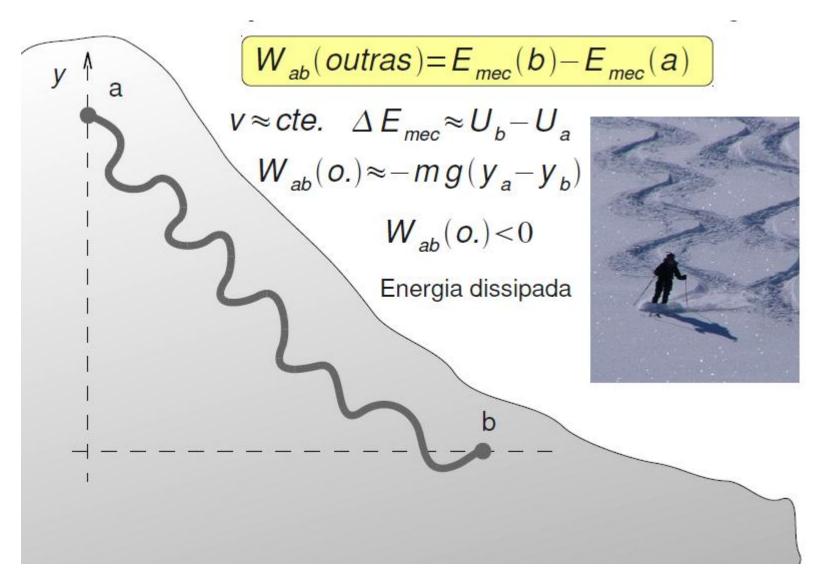
$$\lim_{\Delta x \to 0} \sum_{x_i}^{x_f} F_x \Delta x = \int_{x_i}^{x_f} F_x dx$$

$$W = \int_{x_i}^{x_f} F_x dx$$

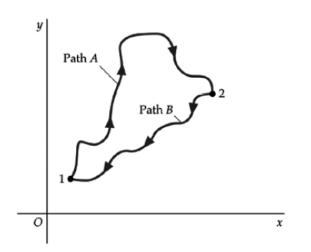
### Forças conservativas e nãoconservativas



# Energia dissiada



## Trabalho com forças conservativas

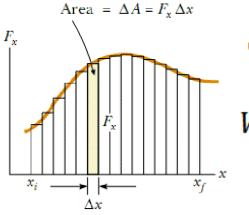


$$E = K_i + U_i = K_f + U_f$$

$$K_f - K_i = U_i - U_f$$

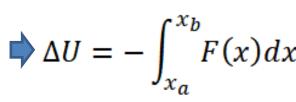
$$W = -\Lambda U$$

O trabalho de uma força conservativa é igual à variação da energia potencial associada à força.

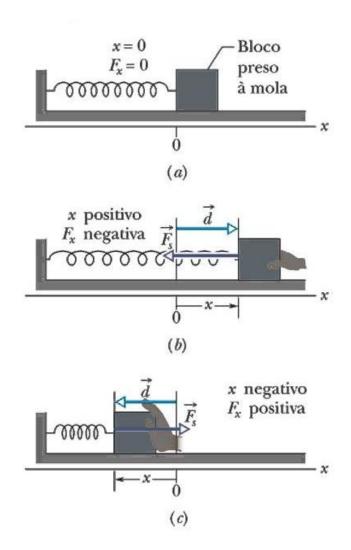


$$dW = -dU$$

$$W = -\int_{a}^{b} dU = \int_{x_a}^{x_b} F(x) dx \Rightarrow \Delta U = -\int_{x_a}^{x_b} F(x) dx$$

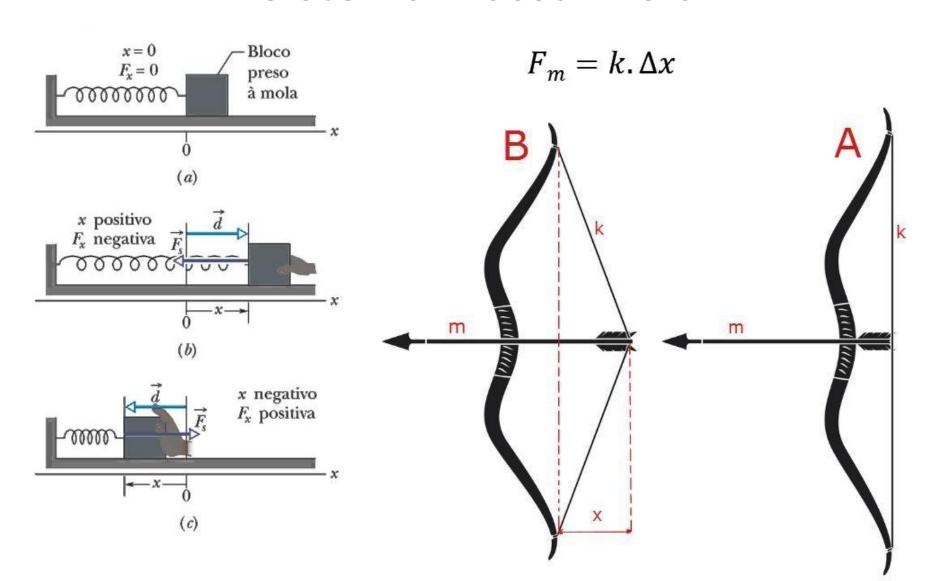


#### Exemplo de força variável Sistema massa-mola

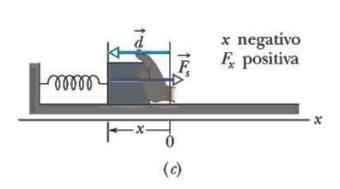


$$F_m = k.\Delta x$$

#### Exemplo de força variável Sistema massa-mola



## Energia potencial elástica



$$F = -kx$$

$$W = \int_0^x -kx dx$$

$$\Delta U = U - U_0 = -\int_0^x -kx dx \qquad \Longrightarrow \qquad U = -\int_0^x -kx dx$$

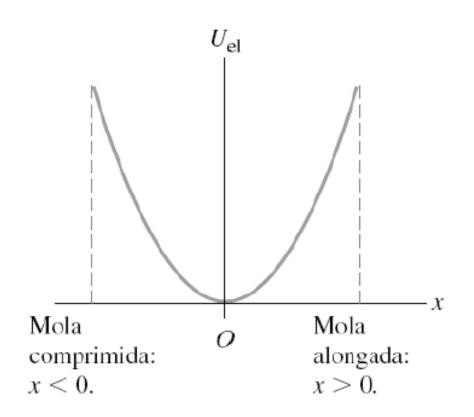
$$U = -\int_0^x -kx dx$$

$$U = \frac{1}{2}kx^2\Big]_0^x$$

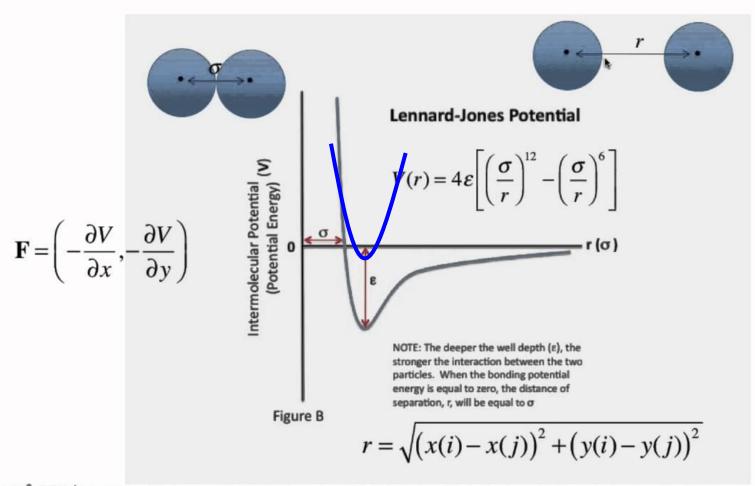
$$U = \frac{1}{2}kx^2$$

$$U = \frac{1}{2}kx^2$$

# Gráfico energia elástica



# Aproximação de sistemas físicos para o sistema massa-mola



http://cherteriki.ucoavis.edu/Core/Physical\_Chemistry/Physical\_Properties\_of\_Matter/Atomic\_and\_Molecular\_Properties/Intermolecular\_Forces/Specific\_Interactions/Lennard-Jones\_Potential

#### Potência

$$P = \frac{dW}{dt} \qquad \xrightarrow{dW = \vec{F} \cdot d\vec{r}} \qquad P = \frac{d}{dt} (\vec{F} \cdot d\vec{r}) = \vec{F} \cdot \vec{v}$$

$$P = F_x v_x$$

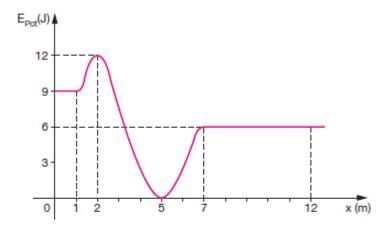
$$P = ma_x v_x$$

$$a_x = \frac{mv_x}{P}$$

$$P = ma_x v_x = mv_x \frac{dv_x}{dt} = \frac{d}{dt} \left( \frac{1}{2} mv_x^2 \right) = \frac{dK}{dt}$$

$$P = \frac{dK}{dt}$$

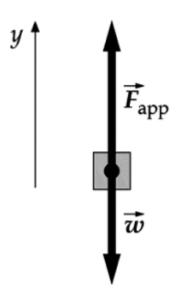
**193** (UFGO) A energia potencial de um carrinho em uma montanha-russa varia, como mostra a figura a seguir.



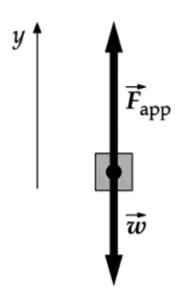
Sabe-se que em x=2 m, a energia cinética é igual a 2 J, e que não há atrito, sobre o carrinho, entre as posições x=0 e x=7 m. Desprezando a resistência do ar, determine:

- a) a energia mecânica total do carrinho
- b) a energia cinética e potencial do carrinho na posição  $x=7\ m$
- c) a força de atrito que deve atuar no carrinho, a partir do posição x = 7 m, para levá-lo ao repouso em 5 m

A truck of mass 3000 kg is to be loaded onto a ship by a crane that exerts an upward force of 31 kN on the truck. This force, which is just strong enough to get the truck started upward, is applied over a distance of 2 m. Find (a) the work done by the crane, (b) the work done by gravity, and (c) the upward speed of the truck after 2 m.

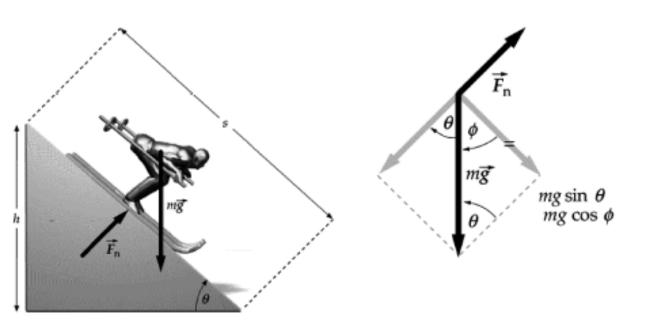


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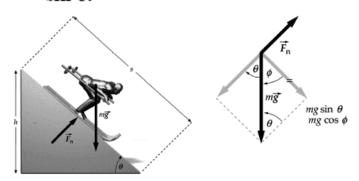


$$\begin{split} W_{\rm app} &= F_{\rm app} \cos 0^\circ \, \Delta y = (31 \text{ kN})(1)(2 \text{ m}) = 62 \text{ kJ} \\ W_{\rm g} &= mg \, \cos 180^\circ \, \Delta y \\ &= (3000 \text{ kg})(9.81 \text{ N/kg})(-1)(2 \text{ m}) = -59 \text{ kJ} \\ K_{\rm f} &= \frac{1}{2} m v_{\rm f}^2 \\ v_{\rm f} &= \sqrt{\frac{2K_{\rm f}}{m}} \\ W_{\rm total} &= \Delta K = K_{\rm f} - K_{\rm i} = K_{\rm f} \\ \end{split}$$

You ski downhill on waxed skis that are nearly frictionless. (a) What work is done on you as you ski a distance s down the hill? (b) What is your speed on reaching the bottom of the run? Assume the length of the ski run is s, its angle of incline is  $\theta$ , and your mass is m. The height of the hill is then  $h = s \sin \theta$ .



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$$W = m\vec{g} \cdot \vec{s} = mgs \cos \phi = mgs \sin \theta$$

$$\sin \theta = \frac{h}{s}$$

$$W = mgh$$

$$W = mgh = \frac{1}{2}mv^2 - 0$$
 or  $v = \sqrt{2gh}$ 

(Fuvest) No desenvolvimento do sistema amortecedor de queda de um elevador de massa m, o engenheiro projetista impõe que a mola deve se contrair de um valor máximo d, quando o elevador cai, a partir do repouso, de uma altura h, como ilustrado na figura a seguir. Para que a exigência do projetista seja satisfeita, a mola a ser empregada deve ter constante elástica dada por

