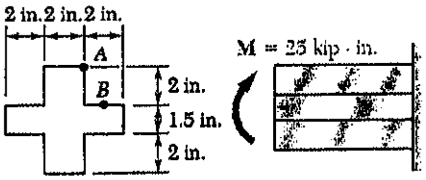
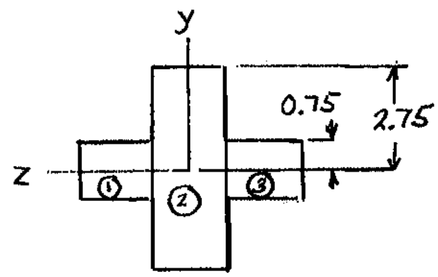
CHAPTER 4

PROBLEM 4.1



4.1 and 4.2 Knowing that the couple shown acts in a vertical plane, determine the stress at (*a*) point *A*, (*b*) point *B*.

SOLUTION



For rectangle $I = \frac{1}{12} b h^3$

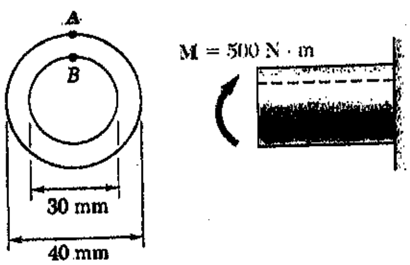
For cross sectional area

$$I = I_1 + I_2 + I_3 = \frac{1}{12}(2)(1.5)^3 + \frac{1}{12}(2)(5.5)^3 + \frac{1}{12}(2)(1.5)^3 = 28.854 \text{ in}^4$$

(a) $y_A = 2.75 \text{ in}$ $\qquad \sigma_A = -\frac{M y_A}{I} = -\frac{(25)(2.75)}{28.854} = -2.38 \text{ ksi}$

(b) $y_B = 0.75 \text{ in.}$ $\qquad \sigma_B = -\frac{M y_B}{I} = -\frac{(25)(0.75)}{28.854} = -0.650 \text{ ksi}$

PROBLEM 4.2



4.1 and 4.2 Knowing that the couple shown acts in a vertical plane, determine the stress at (*a*) point *A*, (*b*) point *B*.

SOLUTION

$r_i = \frac{1}{2} d_i = 15 \text{ mm} \qquad r_o = \frac{1}{2} d_o = 20 \text{ mm}$

$$I = \frac{\pi}{4}(r_o^4 - r_i^4) = \frac{\pi}{4}(20^4 - 15^4)$$

$$= 85.903 \times 10^3 \text{ mm}^4 = 85.903 \times 10^{-9} \text{ m}^4$$

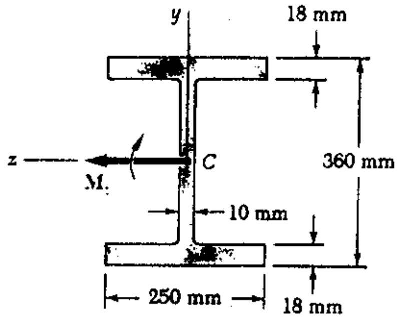
(a) $y_A = 20 \text{ mm} = 0.020 \text{ m}$ $\qquad \sigma_A = -\frac{M y_A}{I} = -\frac{(500)(0.020)}{85.903 \times 10^{9}}$

$$= -116.4 \times 10^6 \text{ Pa} = -116.4 \text{ MPa}$$

(b) $y_B = 15 \text{ mm} = 0.015 \text{ m}$ $\qquad \sigma_B = -\frac{M y_B}{I} = -\frac{(500)(0.015)}{85.903 \times 10^{-9}}$

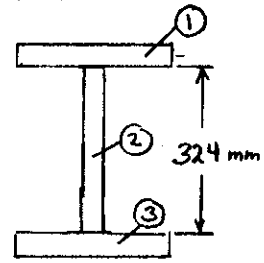
$$= -87.3 \times 10^6 \text{ Pa} = -87.3 \text{ MPa}$$

PROBLEM 4.3



4.3 The wide-flange beam shown is made of a high-strength, low-alloy steel for which $\sigma_Y = 345$ MPa and $\sigma_U = 450$ MPa. Using a factor of safety of 3.0, determine the largest couple that can be applied to the beam when it is bent about the z axis. Neglect the effect of fillets.

SOLUTION



$$I_1 = \frac{1}{12}bh^3 + Ad^2$$
$$= \frac{1}{12}(250)(18^3) + (250)(18)(171)^2$$
$$= 131.706 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12}(10)(324)^3 = 28.344 \times 10^6 \text{ mm}^4$$

$$I_3 = I_1 = 131.706 \times 10^6 \text{ mm}^4$$

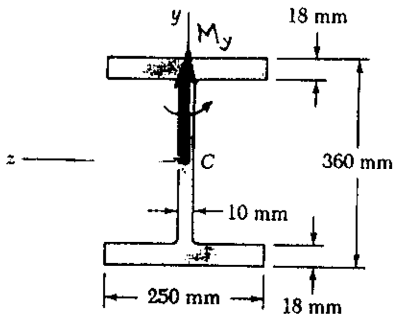
$$I = I_1 + I_2 + I_3 = 291.76 \times 10^6 \text{ mm}^4 = 291.76 \times 10^{-6} \text{ m}^4$$

$$\sigma = \frac{Mc}{I} \quad \text{where} \quad c = \frac{360}{2} = 180 \text{ mm} = 0.180 \text{ m}$$

$$\sigma_{all} = \frac{\sigma_U}{F.S.} = \frac{450 \times 10^6}{3.0} = 150 \times 10^6 \text{ Pa}$$

$$M_{all} = \frac{\sigma_{all} I}{c} = \frac{(150 \times 10^6)(291.76 \times 10^{-6})}{0.180} = 243 \times 10^3 \text{ N·m}$$
$$= 243 \text{ kN·m}$$

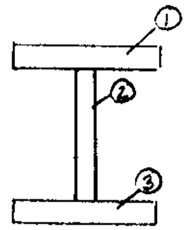
PROBLEM 4.4



4.3 The wide-flange beam shown is made of a high-strength, low-alloy steel for which $\sigma_Y = 345$ MPa and $\sigma_U = 450$ MPa. Using a factor of safety of 3.0, determine the largest couple that can be applied to the beam when it is bent about the z axis. Neglect the effect of fillets.

4.4 **Solve Prob. 4.3, assuming that is bent about the y axis.**

SOLUTION



$$I_1 = \frac{1}{12}(18)(250)^3 = 23.438 \times 10^6 \ \text{mm}^4$$

$$I_2 = \frac{1}{12}(324)(10)^3 = 27 \times 10^3 \ \text{mm}^4$$

$$I_3 = I_1 = 23.438 \ \text{mm}^4$$

$$I_y = I_1 + I_2 + I_3 = 46.903 \times 10^6 \ \text{mm}^4 = 46.903 \times 10^{-6} \ \text{m}^4$$

$$c = \frac{250}{2} \ \text{mm} = 125 \ \text{mm} = 0.125 \ \text{m}$$

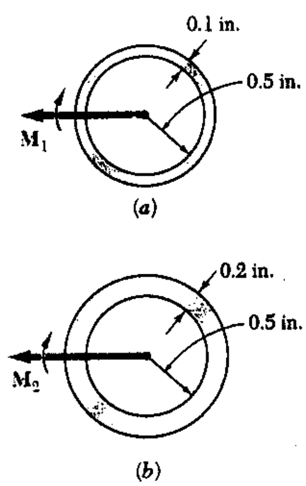
$$\sigma_{all} = \frac{\sigma_U}{F.S.} = \frac{450 \times 10^6}{3.0} = 150 \times 10^6 \ \text{Pa}$$

$$\sigma = \frac{Mc}{I} \qquad M_y = \frac{\sigma_{all} I}{c} = \frac{(150 \times 10^6)(46.903 \times 10^{-6})}{0.125}$$

$$= 56.3 \times 10^3 \ \text{N}\cdot\text{m} = 56.3 \ \text{kN}\cdot\text{m} \quad ◄$$

PROBLEM 4.5

4.5 Using an allowable stress of 16 ksi, determine the largest that can be applied to each pipe.



SOLUTION

(a) $$I = \frac{\pi}{4}(r_o^4 - r_i^4) = \frac{\pi}{4}(0.6^4 - 0.5^4) = 52.7 \times 10^{-3} \ \text{in}^4$$

$$c = 0.6 \ \text{in}$$

$$\sigma = \frac{Mc}{I} \ \therefore \ M = \frac{\sigma I}{c} = \frac{(16)(52.7 \times 10^{-3})}{0.6}$$

$$= 1.405 \ \text{kip}\cdot\text{in.} \quad ◄$$

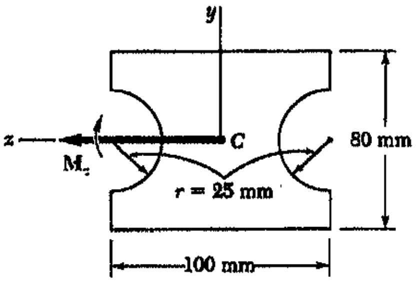
(b) $$I = \frac{\pi}{4}(0.7^4 - 0.5^4) = 139.49 \times 10^{-3} \ \text{in}^4$$

$$c = 0.7 \ \text{in}$$

$$\sigma = \frac{Mc}{I} \ \therefore \ M = \frac{\sigma I}{c} = \frac{(16)(139.49 \times 10^{-3})}{0.7}$$

$$= 3.19 \ \text{kip}\cdot\text{in} \quad ◄$$

PROBLEM 4.6



4.6 A nylon spacing bar has the cross section shown. Knowing that the allowable stress for the grade of nylon used is 24 MPa, determine the largest couple $\mathbf{M}_z$ that can be applied to the bar.

SOLUTION

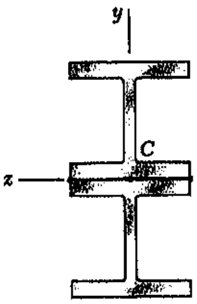
$$I = I_{rect} - I_{circle}$$
$$= \frac{1}{12} b h^3 - \frac{\pi}{4} r^4$$
$$= \frac{1}{12}(100)(80)^3 - \frac{\pi}{4}(25)^4 = 3.9599 \times 10^6 \text{ mm}^4$$
$$= 3.9599 \times 10^{-6} \text{ m}^4$$

$$c = \frac{80}{2} = 40 \text{ mm} = 0.040 \text{ m}$$

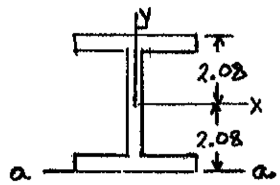
$$\sigma = \frac{Mc}{I} \qquad M = \frac{\sigma I}{c} = \frac{(24 \times 10^6)(3.9599 \times 10^{-6})}{0.040} = 2.38 \times 10^3 \text{ N}\cdot\text{m}$$
$$= 2.38 \text{ kN}\cdot\text{m}$$

PROBLEM 4.7

4.7 and 4.8 Two W 4 × 13 rolled sections are welded together as shown. Knowing that for the steel alloy used $\sigma_Y = 36$ ksi and $\sigma_U = 58$ ksi and using a factor of safety of 3.0, determine the largest couple that can be applied when the assembly is bent about the z axis.



SOLUTION



Properties of W 4×13 rolled section
See Appendix B

Area = 3.83 in^2 Depth = 4.16 in
I_x = 11.3 in^4

For one rolled section, moment of inertia about axis a-a is

$$I_a = I_x + Ad^2 = 11.3 + (3.83)(2.08)^2 = 27.87 \text{ in}^4$$

For both sections $I_z = 2 I_a = 55.74$ in^4

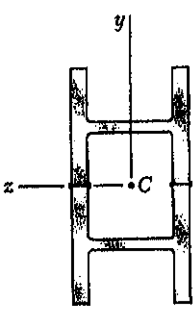
c = depth = 4.16 in

$$\sigma_{all} = \frac{\sigma_U}{F.S.} = \frac{58}{3.0} = 19.333 \text{ ksi} \qquad \sigma = \frac{Mc}{I}$$

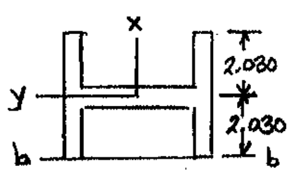
$$M_{all} = \frac{\sigma_{all} I}{c} = \frac{(19.333)(55.74)}{4.16} = 259 \text{ kip}\cdot\text{in.}$$

PROBLEM 4.8

4.7 and 4.8 Two W 4 × 13 rolled sections are welded together as shown. Knowing that for the steel alloy used $\sigma_Y = 36$ ksi and $\sigma_U = 58$ ksi and using a factor of safety of 3.0, determine the largest couple that can be applied when the assembly is bent about the z axis.



SOLUTION



Properties of W 4×13 rolled section
See Appendix B

Area = 3.83 in^2 Width = 4.060 in
I_y = 3.86 in^4

For one rolled section, moment of inertia about axis b-b is

$$I_b = I_y + Ad^2 = 3.86 + (3.83)(2.030)^2 = 19.643 \text{ in}^4$$

For both sections $I_z = 2 I_b = 39.286$ in^4

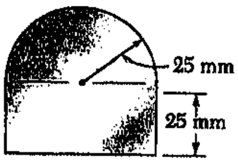
c = width = 4.060 in

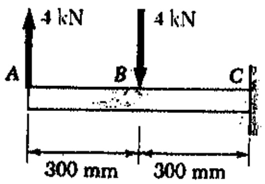
$$\sigma_{all} = \frac{\sigma_U}{F.S.} = \frac{58}{3.0} = 19.333 \text{ ksi} \qquad \sigma = \frac{Mc}{I}$$

$$M_{all} = \frac{\sigma_{all} I}{c} = \frac{(19.333)(39.286)}{4.060} = 187.1 \text{ kip}\cdot\text{in}$$

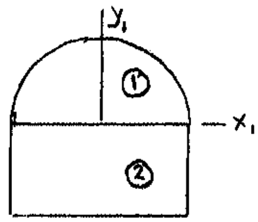
PROBLEM 4.9

4.9 through *4.11* Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in portion *BC* of the beam.





SOLUTION



$$A_1 = \frac{\pi}{2} r^2 = \frac{\pi}{2}(25)^2 = 981.7 \text{ mm}^2$$

$$\bar{y}_1 = \frac{4r}{3\pi} = \frac{(4)(25)}{3\pi} = 10.610 \text{ mm}$$

$$A_2 = bh = (50)(25) = 1250 \text{ mm}^2$$

$$\bar{y}_2 = -\frac{h}{2} = -\frac{25}{2} = -12.5 \text{ mm}$$

$$\bar{y} = \frac{A_1\bar{y}_1 + A_2\bar{y}_2}{A_1 + A_2} = \frac{(981.7)(10.610) + (1250)(-12.5)}{981.7 + 1250}$$

$$= -2.334 \text{ mm}$$

$$\bar{I}_1 = I_{x_1} - A_1\bar{y}_1^2 = \frac{\pi}{8}r^4 - A_1\bar{y}_1^2 = \frac{\pi}{8}(25)^4 - (981.7)(10.610)^2 = 42.886\times10^6 \text{ mm}^4$$

$$d_1 = \bar{y}_1 - \bar{y} = 10.610 - (-2.334) = 12.944 \text{ mm}$$

$$I_1 = \bar{I}_1 + A_1 d_1^2 = 42.866\times10^3 + (981.7)(12.944)^2 = 207.35\times10^3 \text{ mm}^4$$

$$\bar{I}_2 = \frac{1}{12}bh^3 = \frac{1}{12}(50)(25)^3 = 65.104\times10^3 \text{ mm}^4$$

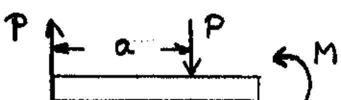
$$d_2 = |\bar{y}_2 - \bar{y}| = |-12.5 - (-2.334)| = 10.166 \text{ mm}$$

$$I_2 = \bar{I}_2 + A_2 d_2^2 = 65.104\times10^3 + (1250)(10.166)^2 = 194.288\times10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 401.16\times10^3 \text{ mm}^4 = 401.16\times10^{-9} \text{ m}^4$$

$$y_{top} = 25 + 2.334 = 27.334 \text{ mm} = 0.027334 \text{ m}$$

$$y_{bot} = -25 + 2.334 = -22.666 \text{ mm} = -0.022666 \text{ m}$$



$$M - Pa = 0 \qquad M = Pa = (4\times10^3)(300\times10^{-3}) = 1200 \text{ N}\cdot\text{m}$$

$$\sigma_{top} = -\frac{My_{top}}{I} = -\frac{(1200)(0.027334)}{401.16\times10^{-9}} = -81.76\times10^6 \text{ Pa}$$

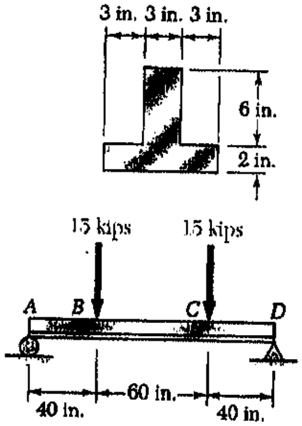
$$= -81.8 \text{ MPa}$$ ◄

$$\sigma_{bot} = -\frac{My_{bot}}{I} = -\frac{(1200)(-0.022666)}{401.16\times10^{-9}} = 67.80\times10^6 \text{ Pa}$$

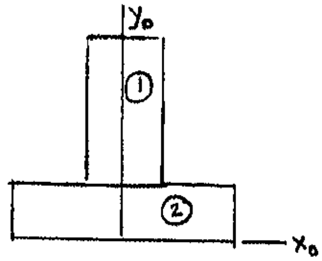
$$= 67.8 \text{ MPa}$$ ◄

PROBLEM 4.10

4.9 through *4.11* Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in portion *BC* of the beam.



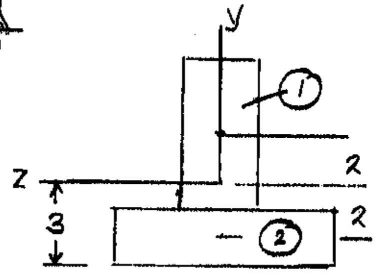
SOLUTION



	A	$\bar{y}_0$	$A\bar{y}_0$
①	18	5	90
②	18	1	18
Σ	36		108

$$\bar{Y}_0 = \frac{108}{36} = 3 \text{ in}$$

Neutral axis lies 3 in. above the base.

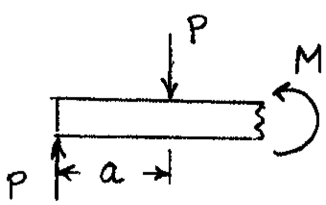


$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(3)(6)^3 + (18)(2)^2 = 126 \text{ in}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12}(9)(2)^3 + (18)(2)^2 = 78 \text{ in}^4$$

$$I = I_1 + I_2 = 126 + 78 = 204 \text{ in}^4$$

$$y_{top} = 5 \text{ in} \qquad y_{bot} = -3 \text{ in.}$$



$$M - Pa = 0$$

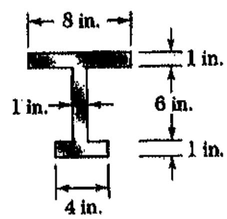
$$M = Pa = (15)(40) = 600 \text{ kip}\cdot\text{in.}$$

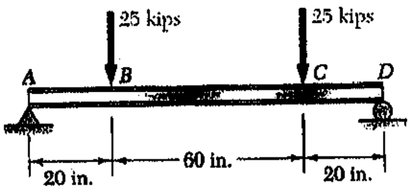
$$\sigma_{top} = -\frac{M y_{top}}{I} = -\frac{(600)(5)}{204} = -14.71 \text{ ksi}$$

$$\sigma_{bot} = -\frac{M y_{bot}}{I} = -\frac{(600)(-3)}{204} = 8.82 \text{ ksi}$$

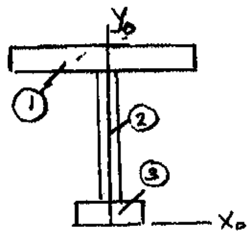
PROBLEM 4.11

4.9 through *4.11* Two vertical forces are applied to a beam of the cross section shown. Determine the maximum tensile and compressive stresses in portion *BC* of the beam.





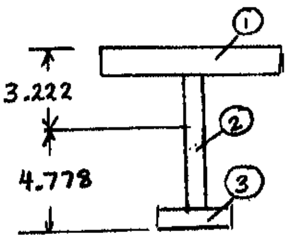
SOLUTION



	A	$\bar{y}_o$	$A\bar{y}_o$
①	8	7.5	60
②	6	4	24
③	4	0.5	2
Σ	18		86

$$\bar{Y}_o = \frac{86}{18} = 4.778 \text{ in}$$

Neutral axis lies 4.778 in above the base.



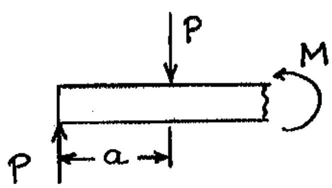
$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(8)(1)^3 + (8)(2.722)^2$$
$$= 59.94 \text{ in}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12}(1)(6)^3 + (6)(0.778)^2$$
$$= 21.63 \text{ in}^4$$

$$I_3 = \frac{1}{12} b_3 h_3^3 + A_3 d_3^2 = \frac{1}{12}(4)(1)^3 + (4)(4.278)^2 = 73.54 \text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 59.94 + 21.63 + 73.54 = 155.16 \text{ in}^4$$

$y_{top} = 3.222$ in $\qquad y_{bot} = -4.778$ in

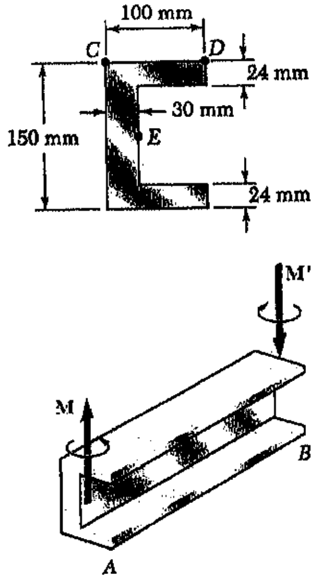


$$M - Pa = 0$$
$$M = Pa = (25)(20) = 500 \text{ kip}\cdot\text{in.}$$

$$\sigma_{top} = -\frac{M y_{top}}{I} = -\frac{(500)(3.222)}{155.16} = -10.38 \text{ ksi}$$

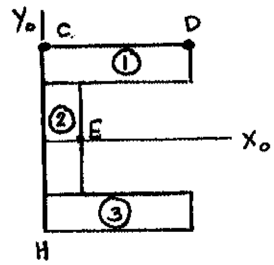
$$\sigma_{bot} = -\frac{M y_{bot}}{I} = -\frac{(500)(-4.778)}{155.16} = 15.40 \text{ ksi}$$

PROBLEM 4.12



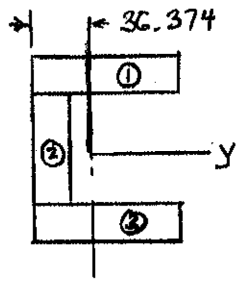
4.12 Two equal and opposite couples of magnitude of $M = 15$ kN·m are applied to the channel-shaped beam AB. Observing that the couples cause the beam to bend in a horizontal plane, determine the stress (*a*) at point C, (*b*) at point D, (*c*) at point E.

SOLUTION



	A, mm^2	$\bar{X}_o$, mm	$A\bar{X}_o$, mm^3
①	2400	50	120×10^3
②	3060	15	45.9×10^3
③	2400	50	120×10^3
Σ	7860		285.9×10^3

$$\bar{X} = \frac{285.9\times10^3}{7860} = 36.374 \text{ mm}$$



$y_C = -36.374 \text{ mm} = -0.036374 \text{ m}$

$y_D = 100 - 36.374 = 63.626 \text{ mm} = 0.63626 \text{ m}$

$y_E = 30 - 36.374 = -6.374 \text{ mm} = -0.006374 \text{ m}$

$d_1 = 50 - 36.374 = 13.626 \text{ mm}$

$d_2 = 36.374 - 15 = 21.374 \text{ mm}$

$d_3 = d_1$

$$I_1 = I_3 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(24)(100)^3 + (2400)(13.626)^2 = 2.4456\times10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12}b_2h_2^3 + A_2d_2^2 = \frac{1}{12}(102)(30)^3 + (3060)(21.374)^2 = 1.6275\times10^6 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 6.5187\times10^6 \text{ mm}^4 = 6.5187\times10^{-6} \text{ m}^4$$

$$M = 15\times10^3 \text{ N-m}$$

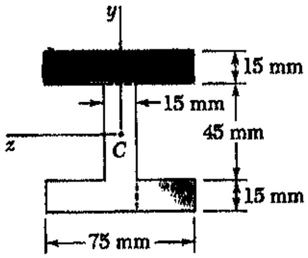
(a) Point C: $$\sigma_C = -\frac{My_C}{I} = -\frac{(15\times10^3)(-0.036374)}{6.5187\times10^{-6}} = 83.7\times10^6 \text{ Pa} = 83.7 \text{ MPa}$$ ◀

(b) Point D: $$\sigma_D = -\frac{My_D}{I} = -\frac{(15\times10^3)(0.063626)}{6.5187\times10^{-6}} = -146.4\times10^6 \text{ Pa} = -146.4 \text{ MPa}$$ ◀

(c) Point E: $$\sigma_E = -\frac{My_E}{I} = -\frac{(15\times10^3)(-0.006374)}{6.5187\times10^{-6}} = 14.67\times10^6 \text{ Pa} = 14.67 \text{ MPa}$$ ◀

PROBLEM 4.13

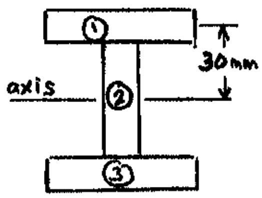
4.13 Knowing that a beam of the cross section shown is bent about a horizontal axis and that the bending moment is 8 kN · m, determine the total force acting on the top flange.



SOLUTION

The stress distribution over the entire cross section is given by the bending stress formula

$$\sigma_x = -\frac{My}{I}$$



where y is a coordinate with its origin on the neutral axis and I is the moment of inertia of the entire cross sectional area. The force on the shaded is calculated from this stress distribution. Over an area element dA the force is

$$dF = \sigma_x \, dA = -\frac{My}{I} dA$$

The total force on the shaded area is then

$$F = \int dF = -\int \frac{My}{I} dA = -\frac{M}{I}\int y \, dA = -\frac{M}{I}\bar{y}^* A^*$$

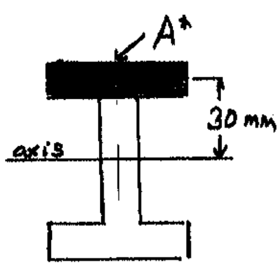
where $\bar{y}^*$ is the centroidal coordinate of the shaded portion and A^* is its area.

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(75)(15)^3 + (75)(15)(30)^2 = 1.0336 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 = \frac{1}{12}(15)(45)^3 = 0.1139 \times 10^6 \text{ mm}^4$$

$$I_3 = I_1 = 1.0336 \times 10^6 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 2.1811 \times 10^6 \text{ mm}^4 = 2.1811 \times 10^{-6} \text{ m}^4$$



$$A^* = (75)(15) = 1125 \text{ mm}^2 = 1125 \times 10^{-6} \text{ m}^2$$

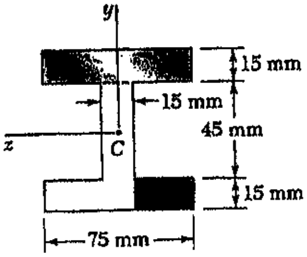
$$\bar{y}^* = 30 \text{ mm} = 0.030 \text{ m}$$

$$F = -\frac{M\bar{y}^* A}{I} = -\frac{(8 \times 10^3)(0.030)(1125 \times 10^{-6})}{2.1811 \times 10^{-6}}$$

$$= -123.8 \times 10^3 \text{ N} = -123.8 \text{ kN}$$

PROBLEM 4.14

4.14 Knowing that a beam of the cross section shown is bent about a vertical axis and that the bending moment is 4 kN · m, determine the total force acting on the shaded portion of the lower flange.

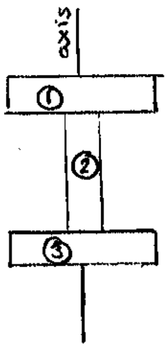


SOLUTION

The stress distribution over the entire cross section is given by the bending stress formula

$$\sigma_x = -\frac{My}{I}$$

where y is a coordinate with its origin on the neutral axis and I is the moment of inertia of the entire cross sectional area. The force on the shaded is calculated from this stress distribution. Over an area element dA the force is



$$dF = \sigma_x \, dA = -\frac{My}{I}\, dA$$

The total force on the shaded area is then

$$F = \int dF = -\int \frac{My}{I}\, dA = -\frac{M}{I}\int y\, dA = -\frac{M}{I}\bar{y}^* A^*$$

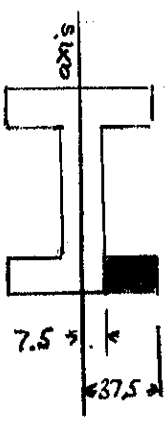
where $\bar{y}^*$ is the centroidal coordinate of the shaded portion and A^* is its area.

$$I_1 = \frac{1}{12} b_1 h_1^3 = \frac{1}{12}(15)(75)^3 = 0.52734\times10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 = \frac{1}{12}(45)(15)^3 = 0.01256\times10^6 \text{ mm}^4$$

$$I_3 = I_1 = 0.5273\times10^{-6}$$

$$I = I_1 + I_2 + I_3 = 1.0672\times10^6 \text{ mm}^4 = 1.0672\times10^{-6} \text{ m}^4$$



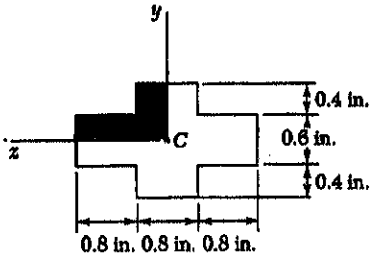
$$A^* = (37.5 - 7.5)(15) = 450 \text{ mm}^2 = 450\times10^{-6} \text{ m}^2$$

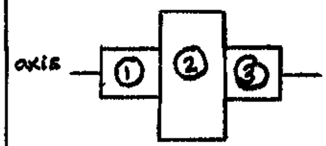
$$\bar{y}^* = \frac{1}{2}(37.5 + 7.5) = 22.5 \text{ mm} = 0.0225 \text{ m}$$

$$F = \frac{M\bar{y}^* A^*}{I} = \frac{(4\times10^3)(0.0225)(450\times10^{-6})}{1.0672\times10^{-6}}$$

$$= 37.9\times10^3 \text{ N} = 37.9 \text{ kN}$$

PROBLEM 4.15





4.15 Knowing that a beam of the cross section shown is bent about a horizontal axis and that the bending moment is 3.5 kip·in., determine the total force acting on the shaded portion of the beam.

SOLUTION

The stress distribution over the entire cross section is given by the bending stress formula

$$\sigma_x = -\frac{My}{I}$$

where y is a coordinate with its origin on the neutral axis and I is the moment of inertia of the entire cross sectional area. The force on the shaded is calculated from this stress distribution. Over an area element dA the force is

$$dF = \sigma_x \, dA = -\frac{My}{I} dA$$

The total force on the shaded area is then

$$F = \int dF = -\int \frac{My}{I} dA = -\frac{M}{I}\int y\,dA = -\frac{M}{I}\bar{y}^* A^*$$

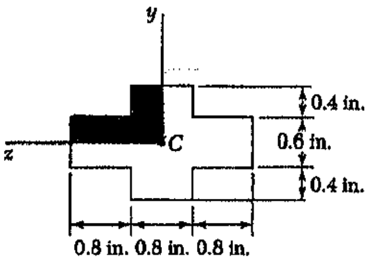
where $\bar{y}^*$ is the centroidal coordinate of the shaded portion and A^* is its area.

$$I = I_1 + I_2 + I_3 = \tfrac{1}{12} b_1 h_1^3 + \tfrac{1}{12} b_2 h_2^3 + \tfrac{1}{12} b_3 h_3^3$$

$$= \tfrac{1}{12}(0.8)(0.6)^3 + \tfrac{1}{12}(0.8)(1.4)^3 + \tfrac{1}{12}(0.8)(0.6)^3 = 0.21173 \text{ in}^4$$

(b)

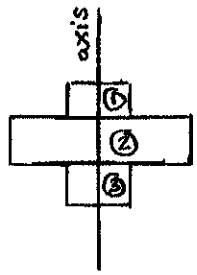
PROBLEM 4.16



4.15 Knowing that a beam of the cross section shown is bent about a horizontal axis and that the bending moment is 3.5 kip·in., determine the total force acting on the shaded portion of the beam.

4.16 Solve Prob. 4.15, assuming that the beam is <u>bent about a vertical axis</u> and that the bending moment is 6 kip·in.

SOLUTION



The stress distribution over the entire cross section is given by the bending stress formula

$$\sigma_x = -\frac{My}{I}$$

where y is a coordinate with its origin on the neutral axis and I is the moment of inertia of the entire cross sectional area. The force on the shaded is calculated from this stress distribution. Over an area element dA the force is

$$dF = \sigma_x\, dA = -\frac{My}{I}\, dA$$

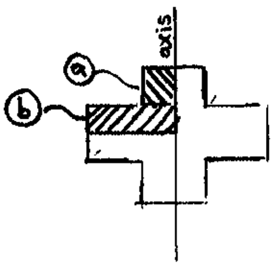
The total force on the shaded area is then

$$F = \int dF = -\int \frac{My}{I} dA = -\frac{M}{I}\int y\, dA = -\frac{M}{I}\bar{y}^* A^*$$

where $\bar{y}^*$ is the centroidal coordinate of the shaded portion and A^* is its area.

$$I = I_1 + I_2 + I_3 = \tfrac{1}{12} b_1 h_1^3 + \tfrac{1}{12} b_2 h_2^3 + \tfrac{1}{12} b_3 h_3^3$$

$$= \tfrac{1}{12}(0.4)(0.8)^3 + \tfrac{1}{12}(0.6)(2.4)^3 + \tfrac{1}{12}(0.4)(0.8)^3 = 0.7253 \text{ in}^4$$



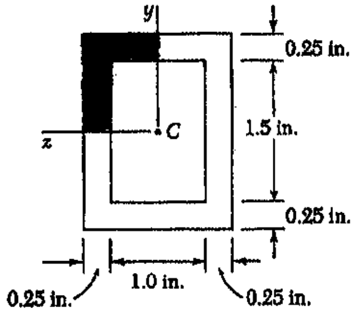
$$\bar{y}^* A^* = \bar{y}_a A_a + \bar{y}_b A_b$$

$$= (0.2)(0.4)(0.4) + (0.6)(0.3)(1.2)$$

$$= 0.248 \text{ in}^3$$

$$F = \frac{M\bar{y}^* A^*}{I} = \frac{(6)(0.248)}{0.7253} = 2.05 \text{ kips}$$

PROBLEM 4.17



4.17 Knowing that a beam of the cross section shown is <u>bent about a horizontal axis</u> and that the bending moment is 6 kip·in., determine the total force acting on the shaded portion of the beam.

SOLUTION

The stress distribution over the entire cross section is given by the bending stress formula

$$\sigma_x = -\frac{My}{I}$$

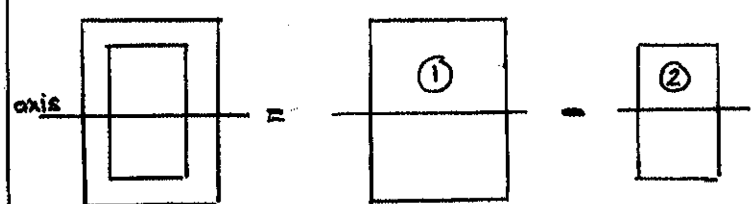
where y is a coordinate with its origin on the neutral axis and I is the moment of inertia of the entire cross sectional area. The force on the shaded is calculated from this stress distribution. Over an area element dA the force is

$$dF = \sigma_x\, dA = -\frac{My}{I}\, dA$$

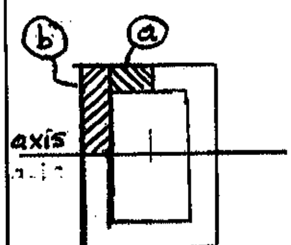
The total force on the shaded area is then

$$F = \int dF = -\int \frac{My}{I}\, dA = -\frac{M}{I}\int y\, dA = -\frac{M}{I}\bar{y}^* A^*$$

where $\bar{y}^*$ is the centroidal coordinate of the shaded portion and A^* is its area.



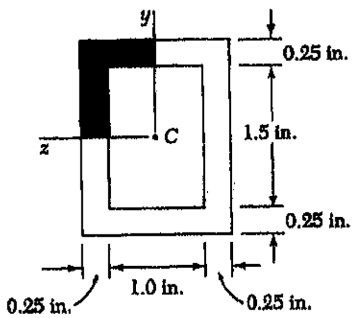
$$I = I_1 - I_2$$
$$= \tfrac{1}{12} b_1 h_1^3 - \tfrac{1}{12} b_2 h_2^3$$
$$= \tfrac{1}{12}(1.5)(2.0)^3 - \tfrac{1}{12}(1.0)(1.5)^3$$
$$= 0.71875 \text{ in}^4$$



$$\bar{y}^* A^* = \bar{y}_a A_a + \bar{y}_b A_b$$
$$= (0.875)(0.5)(0.25) + (0.5)(0.25)(1.0) = 0.23438 \text{ in}^3$$

$$F = \frac{M\bar{y}^* A^*}{I} = \frac{(6)(0.23438)}{0.71875} = 1.957 \text{ kips}$$

PROBLEM 4.18



4.17 Knowing that a beam of the cross section shown is bent about a horizontal axis and that the bending moment is 6 kip·in., determine the total force acting on the shaded portion of the beam.

4.18 Solve Prob. 4.17, assuming that the beam is <u>bent about a vertical axis</u> and that the bending moment is 6 kip·in.

SOLUTION

The stress distribution over the entire cross section is given by the bending stress formula

$$\sigma_x = -\frac{My}{I}$$

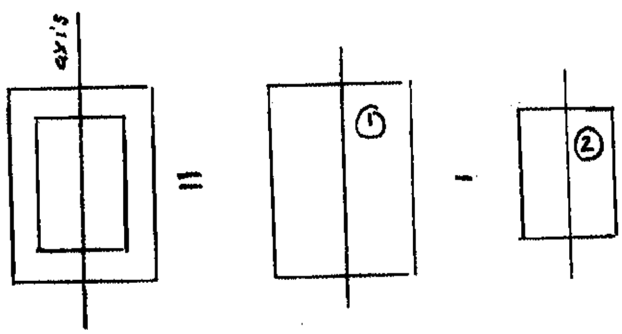
where y is a coordinate with its origin on the neutral axis and I is the moment of inertia of the entire cross sectional area. The force on the shaded is calculated from this stress distribution. Over an area element dA the force is

$$dF = \sigma_x \, dA = -\frac{My}{I} dA$$

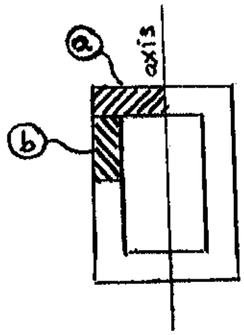
The total force on the shaded area is then

$$F = \int dF = -\int \frac{My}{I} dA = -\frac{M}{I}\int y\,dA = -\frac{M}{I}\bar{y}^* A^*$$

where $\bar{y}^*$ is the centroidal coordinate of the shaded portion and A^* is its area.



$$I = I_1 - I_2$$
$$= \frac{1}{12} b_1 h_1^3 - \frac{1}{12} b_2 h_2^3$$
$$= \frac{1}{12}(2)(1.5)^3 - \frac{1}{12}(1.5)(1.0)^3$$
$$= 0.4375 \text{ in}^4$$

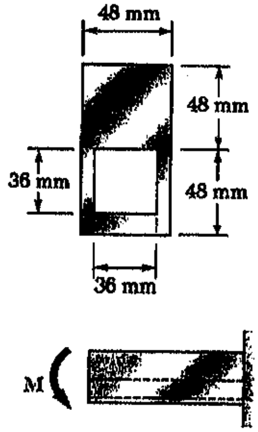


$$\bar{y}^* A^* = \bar{y}_a A_a + \bar{y}_b A_b$$
$$= (0.375)(0.25)(0.75) + (0.625)(0.75)(0.25)$$
$$= 0.1875 \text{ in}^3$$

$$F = \frac{M\bar{y}^* A^*}{I} = \frac{(6)(0.1875)}{0.4375} = 2.57 \text{ kips.}$$

PROBLEM 4.19

4.19 and 4.20 Knowing that for the extruded beam shown the allowable stress is 120 MPa in tension and 150 MPa in compression, determine the largest couple M that can be applied.



SOLUTION

	A, mm²	$\bar{y}_o$, mm	$A\bar{y}_o$, mm³
① solid rectangle	4608	48	221184
② square cutout	-1296	30	-38880
Σ	3312		182304

$$\bar{Y} = \frac{182304}{3312} = 55.04 \text{ mm}$$

Neutral axis lies 55.04 mm above bottom.

$$y_{top} = 96 - 55.04 = 40.96 \text{ mm} = 0.04096 \text{ m}$$
$$y_{bot} = -55.04 \text{ mm} = -0.05504 \text{ m}$$

$$I_1 = \tfrac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \tfrac{1}{12}(48)(96)^3 + (48)(96)(7.04)^2 = 3.7673 \times 10^6 \text{ mm}^4$$

$$I_2 = \tfrac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \tfrac{1}{12}(36)(36)^3 + (36)(36)(25.04)^2 = 0.9526 \times 10^6 \text{ mm}^4$$

$$I = I_1 - I_2 = 2.8147 \times 10^6 \text{ mm}^4 = 2.8147 \times 10^{-6} \text{ m}^4$$

$$|\sigma| = \left|\frac{My}{I}\right| \quad \therefore \quad M = +\left|\frac{\sigma I}{y}\right|$$

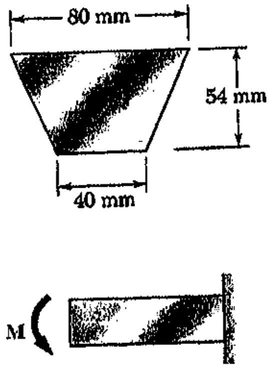
Top: tension side $\quad M = \dfrac{(120\times10^6)(2.8147\times10^{-6})}{0.04096} = 8.25\times10^3 \text{ N}\cdot\text{m}$

Bottom: compression $\quad M = \dfrac{(150\times10^6)(2.8147\times10^{-6})}{0.05504} = 7.67\times10^3 \text{ N}\cdot\text{m}$

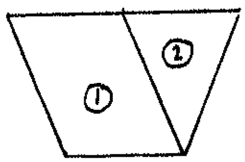
M_{all} is the smaller value $\quad M = 7.67\times10^3 \text{ N}\cdot\text{m} = 7.67 \text{ kN}\cdot\text{m}$ ◄

PROBLEM 4.20

4.19 and 4.20 Knowing that for the extruded beam shown the allowable stress is 120 MPa in tension and 150 MPa in compression, determine the largest couple **M** that can be applied.



SOLUTION



	A, mm^2	$\bar{y}_o$, mm	$A\bar{y}_o$, mm^3
①	2160	27	58320
②	1080	36	38880
Σ	3240		97200

$$\bar{Y} = \frac{97200}{3240} = 30 \text{ mm}$$

The neutral axis lies 30 mm above the bottom.

$y_{top} = 54 - 30 = 24 \text{ mm} = 0.024 \text{ m}$

$y_{bot} = -30 \text{ mm} = -0.030 \text{ m}$

$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(40)(54)^3 + (40)(54)(3)^2 = 544.32 \times 10^3 \text{ mm}^4$

$I_2 = b_2 h_2^2 + A_2 d_2^2 = \frac{1}{36}(40)(54)^3 + \frac{1}{2}(40)(54)(6)^2 = 213.84 \times 10^3 \text{ mm}^4$

$I = I_1 + I_2 = 758.16 \times 10^3 \text{ mm}^4 = 758.16 \times 10^{-9} \text{ m}^4$

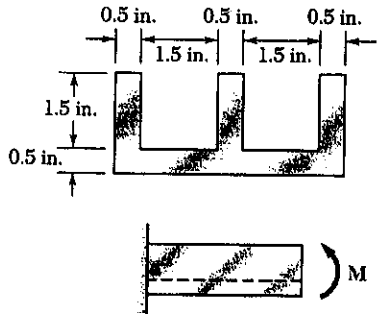
$|\sigma| = \left|\frac{My}{I}\right| \qquad |M| = \left|\frac{\sigma I}{y}\right|$

top: tension side $\quad M = \frac{(120 \times 10^6)(758.16 \times 10^{-9})}{0.024} = 3.7908 \times 10^3 \text{ N}\cdot\text{m}$

bottom: compression $\quad M = \frac{(150 \times 10^6)(758.16 \times 10^{-9})}{0.030} = 3.7908 \times 10^3 \text{ N}\cdot\text{m}$

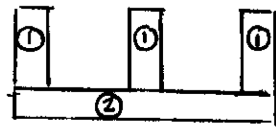
Choose the smaller as M_{all} $\qquad M_{all} = 3.7908 \times 10^3 \text{ N}\cdot\text{m} = 3.79 \text{ kN}\cdot\text{m}$ ◄

PROBLEM 4.21



4.21 Knowing that for the extruded beam shown the allowable stress is 12 ksi in tension and 16 ksi in compression, determine the largest couple **M** that can be applied.

SOLUTION



	A	$\bar{y}_o$	$A\bar{y}_o$
①	2.25	1.25	2.8125
②	2.25	0.25	0.5625
	4.50		3.375

$$\bar{Y} = \frac{3.375}{4.50} = 0.75 \text{ in}$$

The neutral axis lies 0.75 in. above bottom.

$$y_{top} = 2.0 - 0.75 = 1.25 \text{ in}, \qquad y_{bot} = -0.75 \text{ in.}$$

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(1.5)(1.5)^3 + (2.25)(0.5)^2 = 0.984375 \text{ in}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12}(4.5)(0.5)^3 + (2.25)(0.5)^2 = 0.609375 \text{ in}^4$$

$$I = I_1 + I_2 = 1.59375 \text{ in}^4$$

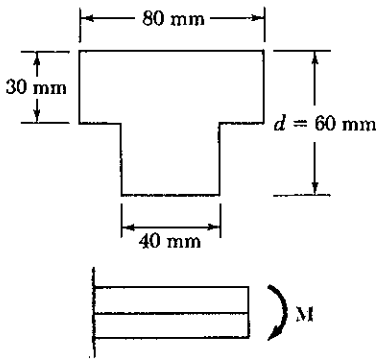
$$|\sigma| = \left|\frac{My}{I}\right| \qquad M = \left|\frac{\sigma I}{y}\right|$$

Top: compression $\quad M = \frac{(16)(1.59375)}{1.25} = 20.4$ kip · in

Bottom: tension $\quad M = \frac{(12)(1.59375)}{0.75} = 25.5$ kip · in

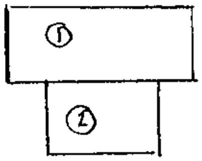
Choose the smaller as M_{all} $\qquad M_{all} = 20.4$ kip. in.

PROBLEM 4.22



4.22 The beam shown is made of a nylon for which the allowable stress 24 MPa in tension and 30 MPa in compression. Determine the largest couple **M** that can be applied to the beam.

SOLUTION



	A, mm^2	$\bar{y}_o$, mm	$A\bar{y}_o$, mm^3
①	2400	45	108000
②	1200	15	18000
Σ	3600		126000

$$\bar{Y}_o = \frac{126000}{3600} = 35 \text{ mm}$$

The neutral axis lies 35 mm above the bottom.

$y_{top} = 60 - 35 = 25 \text{ mm} = 0.025 \text{ m}, \quad y_{bot} = -35 \text{ mm} = -0.035 \text{ m}$

$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(80)(30)^3 + (2400)(10)^2 = 420 \times 10^3 \text{ mm}^4$

$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12}(40)(30)^3 + (1200)(20)^2 = 570 \times 10^3 \text{ mm}^4$

$I = I_1 + I_2 = 990 \times 10^3 \text{ mm}^4 = 990 \times 10^{-9} \text{ m}^4$

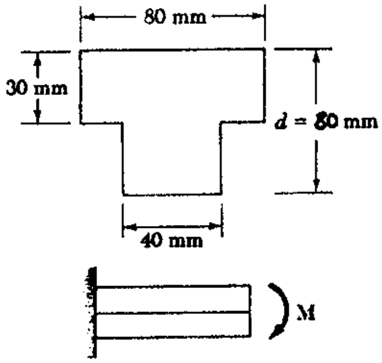
$|\sigma| = \left|\frac{My}{I}\right| \qquad M = \left|\frac{\sigma I}{y}\right|$

Top: tension side $\quad M = \frac{(24 \times 10^6)(990 \times 10^{-9})}{0.025} = 950 \text{ N·m}$

Bottom: compression $\quad M = \frac{(30 \times 10^6)(990 \times 10^{-9})}{0.035} = 849 \text{ N·m}$

Choose smaller value $\qquad M = 849 \text{ N·m}$

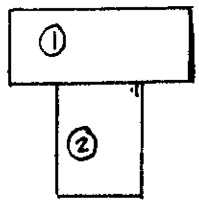
PROBLEM 4.23



4.22 **The beam shown is made of a nylon for which the allowable stress 24 MPa in tension and 30 MPa in compression. Determine the largest couple M that can be applied to the beam.**

4.23 Solve Prob. 4.22, assuming that $d = 80$ mm.

SOLUTION



	A, mm^2	$\bar{y}_o$, mm	$A\bar{y}_o$, mm^3
①	2400	65	156000
②	2000	25	50000
Σ	4400		206000

$$\bar{Y}_o = \frac{206000}{4400} = 46.82 \text{ mm}$$

The neutral axis lies 46.82 mm above the bottom.

$y_{top} = 80 - 46.82 = 33.18 \text{ mm} = 0.03318 \text{ m}$

$y_{bot} = -46.82 \text{ mm} = -0.04682 \text{ m}$

$$I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(80)(30)^3 + (2400)(18.18)^2 = 0.97323 \times 10^6 \text{ mm}^4$$

$$I_2 = \frac{1}{12} b_2 h_2^3 + A_2 d_2^2 = \frac{1}{12}(40)(50)^3 + (2000)(21.82)^2 = 1.36889 \times 10^6 \text{ ''}$$

$$I = I_1 + I_2 = 2.342 \times 10^6 \text{ mm}^4 = 2.342 \times 10^{-6} \text{ m}^4$$

$$|\sigma| = \left|\frac{My}{I}\right| \qquad M = \left|\frac{\sigma I}{y}\right|$$

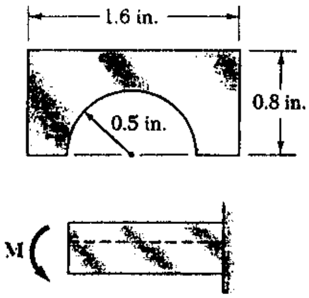
Top: tension side $\qquad M = \dfrac{(24\times 10^6)(2.342\times 10^{-6})}{0.03318} = 1.694 \times 10^3 \text{ N}\cdot\text{m}$

Bottom: compression $\qquad M = \dfrac{(30\times 10^6)(2.342\times 10^{-6})}{0.04682} = 1.501 \times 10^3 \text{ N}\cdot\text{m}$

Choose smaller value $\qquad M = 1.501 \times 10^3 \text{ N}\cdot\text{m} = 1.501 \text{ kN}\cdot\text{m}$ ◄

PROBLEM 4.24

4.24 Knowing that for the beam shown the allowable stress is 12 ksi in tension and 16 ksi in compression, determine the largest couple **M** that can be applied.



SOLUTION

① = rectangle ② = semi-circular cutout

$A_1 = (1.6)(0.8) = 1.28 \text{ in}^2$

$A_2 = \frac{\pi}{2}(0.5)^2 = 0.3927 \text{ in}^2$

$A = 1.28 - 0.3927 = 0.8873 \text{ in}^2$

$\bar{y}_1 = 0.4 \text{ in}$ $\bar{y}_2 = \frac{4r}{3\pi} = \frac{(4)(0.5)}{3\pi} = 0.2122 \text{ in}$

$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{(1.28)(0.4) - (0.3927)(0.2122)}{0.8873} = 0.4831 \text{ in.}$

Neutral axis lies 0.4831 in above the bottom

Moment of inertia about the base

$I_b = \frac{1}{3}bh^3 - \frac{\pi}{8}r^4 = \frac{1}{3}(1.6)(0.8)^3 - \frac{\pi}{8}(0.5)^4 = 0.24852 \text{ in}^4$

Centroidal moment of inertia

$\bar{I} = I_b - A\bar{Y}^2 = 0.24852 - (0.8873)(0.4831)^2$

$= 0.04144 \text{ in}^4$

$y_{top} = 0.8 - 0.4831 = 0.3169 \text{ in},$ $y_{bot} = -0.4831 \text{ in}^4$

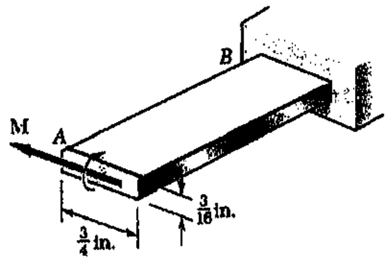
$|\sigma| = \left|\frac{My}{I}\right|$ $M = \left|\frac{\sigma I}{y}\right|$

Top: tension side $M = \frac{(12)(0.04144)}{0.3169} = 1.569 \text{ kip·in}$

Bottom: compression $M = \frac{(16)(0.04144)}{0.4831} = 1.372 \text{ kip·in}$

Choose the smaller value $M = 1.372 \text{ kip·in.}$ ◄

PROBLEM 4.25



4.25 Knowing that $\sigma_{all} = 24$ ksi for the steel strip AB, determine (*a*) the largest couple **M** that can be applied, (*b*) the corresponding radius of curvature. Use $E = 29 \times 10^6$ psi.

SOLUTION

$$I = \frac{1}{12}bh^3 = \frac{1}{12}\left(\frac{3}{4}\right)\left(\frac{3}{16}\right)^3 = 412.0 \times 10^{-6} \text{ in}^4$$

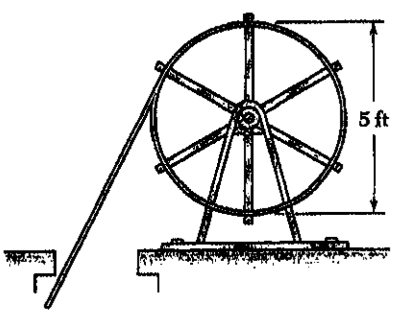
$$\sigma = \frac{Mc}{I} \qquad c = \frac{1}{2}\left(\frac{3}{16}\right) = 0.09375 \text{ in}$$

$$\text{(a)} \quad M = \frac{\sigma I}{c} = \frac{(24 \times 10^3)(412.0 \times 10^{-6})}{0.09375}$$

$$= 105.5 \text{ lb-in}$$

$$\text{(b)} \quad \frac{c}{\rho} = \frac{\sigma_{max}}{E} \qquad \rho = \frac{Ec}{\sigma_{all}} = \frac{(29 \times 10^6)(0.09375)}{24 \times 10^3} = 113.3 \text{ in}$$

PROBLEM 4.26



4.26 Straight rods of 0.30-in. diameter and 200-ft length are sometimes used to clear underground conduits of obstructions or to thread wires through a new conduit. The rods are made of high-strength steel and, for storage and transportation, are wrapped on spools of 5-ft diameter. Assuming that the yield strength is not exceeded, determine (*a*) the maximum stress in a rod, when the rod, which was initially straight, is wrapped on a spool, (*b*) the corresponding bending moment in the rod. Use $E = 29 \times 10^6$ psi.

SOLUTION

$$r = \frac{1}{2}d = \frac{1}{2}(0.30) = 0.15 \text{ in}$$

$$I = \frac{\pi}{4}r^4 = \frac{\pi}{4}(0.15)^4 = 397.61 \times 10^{-6} \text{ in}^4$$

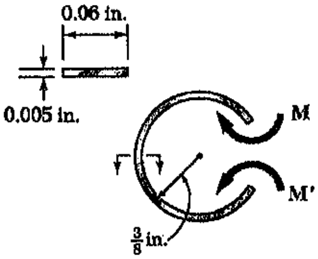
$$D = 5 \text{ ft} = 60 \text{ in} \qquad \rho = \frac{1}{2}D = 30 \text{ in}$$

$$c_1 = r = 0.15 \text{ in.}$$

$$\text{(a)} \quad \sigma_{max} = \frac{Ec}{\rho} = \frac{(29 \times 10^6)(0.15)}{30} = 145 \times 10^3 \text{ psi} = 145 \text{ ksi}$$

$$\text{(b)} \quad M = \frac{EI}{\rho} = \frac{(29 \times 10^6)(397.61 \times 10^{-6})}{30} = 384 \text{ lb} \cdot \text{in}$$

PROBLEM 4.27



4.27 It is observed that a thin steel strip of 0.06-in. width can be bent into a circle of $\frac{3}{4}$ - in. diameter without any resulting permanent deformation. Knowing that $E = 29 \times 10^6$ psi, determine (*a*) the maximum stress in the bent strip, (*b*) the magnitude of the couples required to bend the strip.

SOLUTION

$$I = \frac{1}{12} b h^3 = \frac{1}{12}(0.06)(0.005)^3 = 625 \times 10^{-12} \text{ in}^4$$

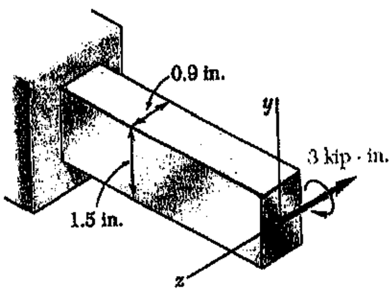
$$\rho = \frac{1}{2} D = \frac{1}{2}\left(\frac{3}{4}\right) = 0.375 \text{ in}$$

$$c = \frac{1}{2} h = 0.0025 \text{ in}$$

(a) $$\sigma_{max} = \frac{Ec}{\rho} = \frac{(29 \times 10^6)(0.0025)}{0.375} = 193.3 \times 10^3 \text{ psi} = 193.3 \text{ ksi}$$ ◄

(b) $$M = \frac{EI}{\rho} = \frac{(29 \times 10^6)(625 \times 10^{-12})}{0.375} = 0.0483 \text{ lb} \cdot \text{in.}$$ ◄

PROBLEM 4.28



4.28 A 3 kip·in. couple is applied to the steel bar shown. (*a*) Assuming that the couple is applied about the *z* axis as shown, determine the maximum stress and the radius of curvature of the bar. (*b*) Solve part *a*, assuming that the couple is applied about the *y* axis. Use $E = 29 \times 10^6$ psi.

SOLUTION

(a) Bending about z-axis.

$$I = \frac{1}{12} b h^3 = \frac{1}{12}(0.9)(1.5)^3 = 0.25313 \text{ in}^4$$

$$c = \frac{1}{2} h = \frac{1}{2}(1.5) = 0.75 \text{ in}$$

$$\sigma = \frac{Mc}{I} = \frac{(3 \times 10^3)(0.75)}{0.25313} = 8.89 \times 10^3 \text{ psi}$$

$$= 8.89 \text{ ksi}$$ ◄

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{3 \times 10^3}{(29 \times 10^6)(0.25313)} = 409 \times 10^{-6} \text{ in}^{-1}$$

$$\rho = 2450 \text{ in} = 204 \text{ ft}$$ ◄

(b) Bending about y-axis

$$I = \frac{1}{12} b h^3 = \frac{1}{12}(1.5)(0.9)^3 = 0.091125 \text{ in}^4$$

$$c = \frac{1}{2} h = \frac{1}{2}(0.9) = 0.45 \text{ in}$$

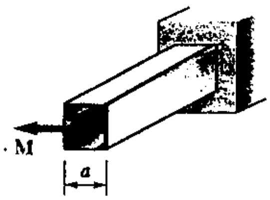
$$\sigma = \frac{Mc}{I} = \frac{(3 \times 10^3)(0.45)}{0.091125} = 14.81 \times 10^3 \text{ psi} = 14.81 \text{ ksi}$$ ◄

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{3 \times 10^3}{(29 \times 10^6)(0.091125)} = 1.135 \times 10^{-5} \text{ in}^{-1}$$

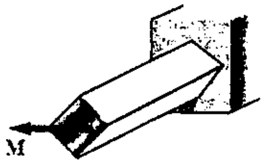
$$\rho = 881 \text{ in} = 73.4 \text{ ft.}$$ ◄

PROBLEM 4.29

4.29 A couple of magnitude M is applied to a square bar of side a. For each of the orientations shown, determine the maximum stress and the curvature of the bar.



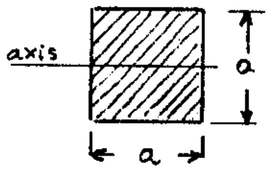
(a)



(b)

SOLUTION

(a)



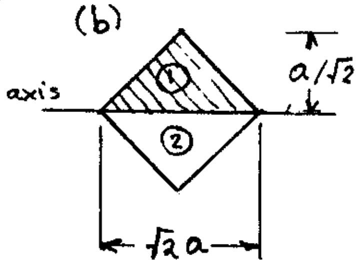
$$I = \frac{1}{12} bh^3 = \frac{1}{12} a a^3 = \frac{a^4}{12}$$

$$c = \frac{a}{2}$$

$$\sigma_{max} = \frac{Mc}{I} = \frac{M \frac{a}{2}}{\frac{a^4}{12}} = \frac{6M}{a^3}$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{M}{E \frac{a^4}{12}} = \frac{12 M}{E a^4}$$

(b)



For one triangle the moment of inertia about its base is

$$I_1 = \frac{1}{12} bh^3 = \frac{1}{12}(\sqrt{2}a)\left(\frac{a}{\sqrt{2}}\right)^3 = \frac{a^4}{24}$$

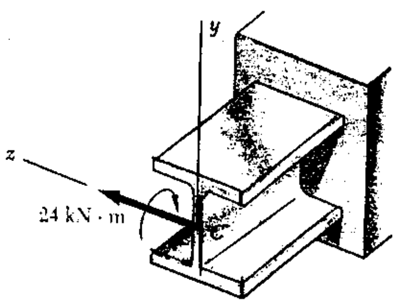
$$I_2 = I_1 = \frac{a^4}{24}$$

$$I = I_1 + I_2 = \frac{a^4}{12}$$

$$c = \frac{a}{\sqrt{2}} \qquad \sigma_{max} = \frac{Mc}{I} = \frac{M\, a/\sqrt{2}}{a^4/12} = \frac{6\sqrt{2} M}{a^3}$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{M}{E \frac{a^4}{12}} = \frac{12 M}{E a^4}$$

PROBLEM 4.30



4.30 A 24 kN·m couple is applied to the W200 × 46.1 rolled-steel beam shown. (*a*) Assuming that the couple is applied about the *z* axis as shown, determine the maximum stress and the radius of curvature of the beam. (*b*) Solve part *a*, assuming that the couple is applied about the *y* axis. Use $E = 200$ GPa.

SOLUTION

For W 200 × 46.1 rolled steel section

$$I_x = 45.5 \times 10^6 \text{ mm}^4 = 45.5 \times 10^{-6} \text{ m}^4$$

$$S_x = 448 \times 10^3 \text{ mm}^3 = 448 \times 10^{-6} \text{ m}^3$$

$$I_y = 15.3 \times 10^6 \text{ mm}^4 = 15.3 \times 10^{-6} \text{ m}^4$$

$$S_y = 151 \times 10^3 \text{ mm}^3 = 151 \times 10^{-6} \text{ m}^3$$

(a) $M_z = 24 \text{ kN}\cdot\text{m} = 24 \times 10^3 \text{ N}\cdot\text{m}$

$$\sigma = \frac{M}{S} = \frac{24 \times 10^3}{448 \times 10^{-6}} = 53.6 \times 10^6 \text{ Pa} = 53.6 \text{ MPa}$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{24 \times 10^3}{(200 \times 10^9)(45.5 \times 10^{-6})} = 2.637 \times 10^{-3} \text{ m}^{-1}$$

$$\rho = 379 \text{ m}$$

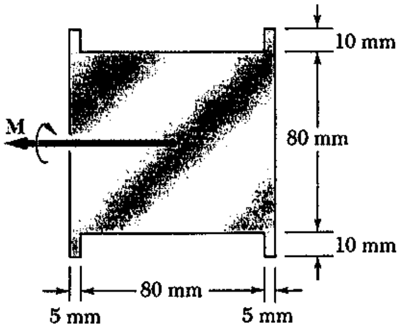
(b) $M_y = 24 \text{ kN}\cdot\text{m} = 24 \times 10^3 \text{ N}\cdot\text{m}$

$$\sigma = \frac{M}{S} = \frac{24 \times 10^3}{151 \times 10^{-6}} = 158.9 \times 10^6 \text{ Pa} = 158.9 \text{ MPa}$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{24 \times 10^3}{(200 \times 10^9)(15.3 \times 10^{-6})} = 7.84 \times 10^{-3} \text{ m}^{-1}$$

$$\rho = 127.5 \text{ m}$$

PROBLEM 4.31



4.31 (*a*) Using an allowable stress of 120 MPa, determine the largest couple **M** that can be applied to a beam of the cross section shown. (*b*) Solve part *a*, assuming that the cross section of the beam is an 80-mm square.

SOLUTION

(a) $I = I_1 + 4I_2$, where I_1 is the moment of inertia of an 80-mm square and I_2 is the moment of inertia of one of the 4 protruding ears.

$$I_1 = \frac{1}{12}bh^3 = \frac{1}{12}(80)(80)^3 = 3.4133\times10^6\ \text{mm}^4$$

$$I_2 = \frac{1}{12}bh^3 + Ad^2 = \frac{1}{12}(5)(10)^3 + (5)(10)(45)^2 = 101.667\times10^3\ \text{mm}^4$$

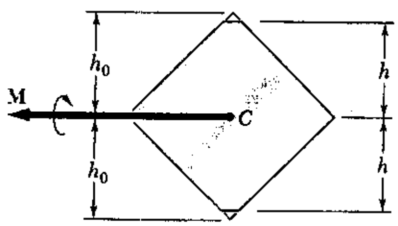
$$I = I_1 + 4I_2 = 3.82\times10^6\ \text{mm}^4 = 3.82\times10^{-6}\ \text{m}^4, \qquad c = 50\ \text{mm} = 0.050\ \text{m}$$

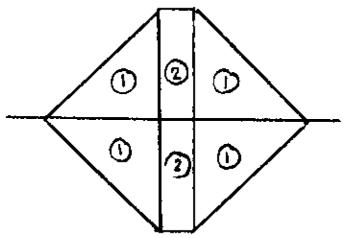
$$\sigma = \frac{Mc}{I} \therefore \quad M = \frac{\sigma I}{c} = \frac{(120\times10^6)(3.82\times10^{-6})}{0.050} = 9.168\times10^3\ \text{N}\cdot\text{m}$$
$$= 9.17\ \text{kN}\cdot\text{m}$$

(b) Without the ears $I = I_1 = 3.4133\times10^{-6}\ \text{m}^2$, $c = 40\ \text{mm} = 0.040\ \text{m}$

$$M = \frac{\sigma I}{c} = \frac{(120\times10^6)(3.4133\times10^{-6})}{0.040} = 10.24\times10^3\ \text{N}\cdot\text{m} = 10.24\ \text{kN}\cdot\text{m}$$

PROBLEM 4.32





4.32 A portion of a square bar is removed by milling, so that its cross section is as shown. The bar is then bent about its horizontal diagonal by a couple **M**. Considering the case where $h = 0.9h_0$, express the maximum stress in the bar in the form $\sigma_m = k\sigma_0$, where σ_0 is the maximum stress that would have occurred if the original square bar had been bent by the same couple **M**, and determine the value of k.

SOLUTION

$$I = 4I_1 + 2I_2$$

$$= (4)(\tfrac{1}{12})h\,h^3 + (2)(\tfrac{1}{3})(2h_0 - 2h)(h^3)$$

$$= \tfrac{1}{3}h^4 + \tfrac{4}{3}h_0 h^3 - \tfrac{4}{3}h\,h^3 = \tfrac{4}{3}h_0 h^3 - h^4$$

$$c = h$$

$$\sigma = \frac{Mc}{I} = \frac{Mh}{\frac{4}{3}h_0 h^3 - h^4} = \frac{3M}{(4h_0 - 3h)h^2}$$

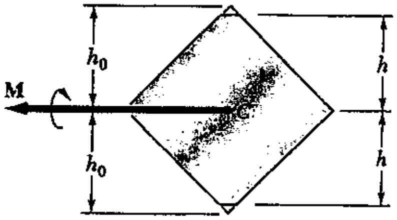
For the original square $h = h_0$, $c = h_0$

$$\sigma_0 = \frac{3M}{(4h_0 - 3h_0)h_0^2} = \frac{3M}{h_0^3}$$

$$\frac{\sigma}{\sigma_0} = \frac{h_0^3}{(4h_0 - 3h)h^2} = \frac{h_0^3}{(4h_0 - (3)(0.9)h_0)(0.9h_0^2)} = 0.950$$

$$\sigma = 0.950\,\sigma_0 \qquad k = 0.950$$

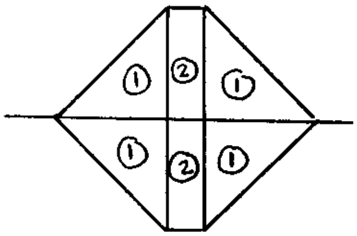
PROBLEM 4.33



4.32 A portion of a square bar is removed by milling, so that its cross section is as shown. The bar is then bent about its horizontal diagonal by a couple **M**. Considering the case where $h = 0.9h_0$, express the maximum stress in the bar in the form $\sigma_m = k\sigma_0$, where σ_0 is the maximum stress that would have occurred if the original square bar had been bent by the same couple **M**, and determine the value of k.

4.33 In Prob. 4.32, determine (*a*) the value of h for which the maximum stress σ_m is as small as possible. (*b*) the corresponding value of k.

SOLUTION



$$I = 4I_1 + 2I_2$$
$$= (4)(\tfrac{1}{12})hh^3 + (2)(\tfrac{1}{3})(2h_0 - 2h)h^3$$
$$= \tfrac{1}{3}h^4 - \tfrac{4}{3}h_0h^3 - \tfrac{4}{3}h^3 \quad = \tfrac{4}{3}h_0h^3 - h^4$$
$$c = h \qquad \frac{I}{c} = \tfrac{4}{3}h_0h^2 - h^3$$

$\frac{I}{c}$ is maximum at $\frac{d}{dh}\left[\tfrac{4}{3}h_0h^2 - h^3\right] = 0$

$$\tfrac{8}{3}h_0h - 3h^2 = 0 \qquad h = \tfrac{8}{9}h_0$$

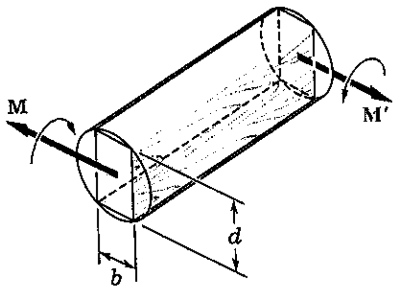
$$\frac{I}{c} = \tfrac{4}{3}h_0\left(\tfrac{8}{9}h_0\right)^2 - \left(\tfrac{8}{9}h_0\right)^3 = \frac{256}{729}h_0^3 \qquad \sigma = \frac{Mc}{I} = \frac{729}{256}\frac{M}{h_0^3}$$

For the original square $\quad h = h_0 \quad c = h_0 \quad \frac{I_0}{c_0} = \frac{1}{3}h_0^3$

$$\sigma_0 = \frac{Mc_0}{I_0} = \frac{3M}{h_0^2}$$

$$\frac{\sigma}{\sigma_0} = \frac{729}{256}\cdot\frac{1}{3} = \frac{729}{768} = 0.949 \qquad k = 0.949$$

PROBLEM 4.34



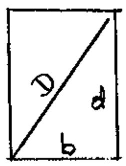
4.34 A couple **M** will be applied to a beam of rectangular cross section which is to be sawed from a log of circular cross section. Determine the ratio d/b, for which (a) the maximum stress σ_m will be as small as possible, (b) the radius of curvature of the beam will be maximum.

SOLUTION

Let D be the diameter of the log.

$$D^2 = b^2 + d^2 \qquad d^2 = D^2 - b^2$$

$$I = \frac{1}{12} b d^3 \qquad c = \frac{1}{2} d \qquad \frac{I}{c} = \frac{1}{6} b d^2$$



(a) σ_m is minimum when $\frac{I}{c}$ is maximum

$$\frac{I}{c} = \frac{1}{6} b (D^2 - b^2) = \frac{1}{6} D^2 b - \frac{1}{6} b^3$$

$$\frac{d}{db}\left(\frac{I}{c}\right) = \frac{1}{6} D^2 - \frac{3}{6} b^2 = 0 \qquad b = \frac{1}{\sqrt{3}} D$$

$$d = \sqrt{D^2 - \frac{1}{3} D^2} = \sqrt{\frac{2}{3}}\, D \qquad \frac{d}{b} = \sqrt{2}$$

$$\rho = \frac{EI}{M} \qquad \rho \text{ is maximum when } I \text{ is maximum.}$$

$\frac{1}{12} b d^3$ is maximum or $b^2 d^6$ is maximum

$(D^2 - d^2) d^6$ is maximum.

$$6 D^2 d^5 - 8 d^7 = 0 \qquad d = \frac{\sqrt{3}}{2} D$$

$$b = \sqrt{D^2 - \frac{3}{4} D^2} = \frac{1}{2} D \qquad \frac{d}{b} = \sqrt{3}$$

PROBLEM 4.35

4.35 For the bar and loading of Example 4.01, determine (*a*) the radius of curvature ρ, (*b*) the radius of curvature ρ' of a transverse cross section, (*c*) the angle between the sides of the bar which were originally vertical. Use $E = 29 \times 10^6$ psi and $\upsilon = 0.29$.

SOLUTION

From Example 4.01 $\quad M = 30$ kip·in, $\quad I = 1.042$ in^4

(a) $\dfrac{1}{\rho} = \dfrac{M}{EI} = \dfrac{(30\times10^3)}{(29\times10^6)(1.042)} = 993\times10^{-6}$ in^{-1} $\qquad \rho = 1007$ in.

(b) $\varepsilon' = \nu\varepsilon = \dfrac{\nu c}{\rho} = \nu\dfrac{c}{\rho'}$

$\dfrac{1}{\rho'} = \nu\dfrac{1}{\rho} = (0.29)(993\times10^{-6})$ in^{-1} $= 288$ in^{-1} $\qquad \rho' = 3470$ in.

(c) $\theta = \dfrac{\text{length of arc}}{\text{radius}} = \dfrac{b}{\rho'} = \dfrac{0.8}{3470} = 230\times10^{-6}$ rad $= 0.01320^\circ$

PROBLEM 4.36

4.36 For the aluminum bar and loading of Sample Prob. 4.1, determine (*a*) the radius of curvature ρ' of a transverse cross section, (*b*) the angle between the sides of the bar which were originally vertical. Use $E = 10.6 \times 10^6$ psi and $\upsilon = 0.33$.

SOLUTION

From Sample Problem 4.1 $\quad I = 12.97$ in^4 $\quad M = 103.8$ kip·in

$\dfrac{1}{\rho} = \dfrac{M}{EI} = \dfrac{103.8\times10^3}{(10.6\times10^6)(12.97)} = 755\times10^{-6}$ in^{-1}

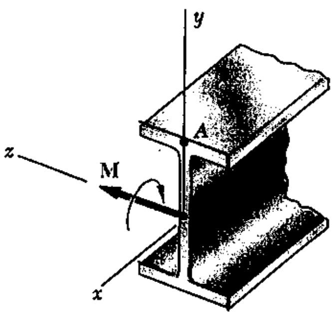
(a) $\dfrac{1}{\rho'} = \nu\dfrac{1}{\rho} = (0.33)(755\times10^{-6}) = 249\times10^{-6}$ in^{-1}

$\rho' = 4010$ in $= 334$ ft

(b) $\theta = \dfrac{\text{length of arc}}{\text{radius}} = \dfrac{b}{\rho'} = \dfrac{3.25}{4010} = 810\times10^{-6}$ rad $= 0.0464^\circ$

PROBLEM 4.37

4.37 A W 200 × 31.3 rolled-steel beam is subjected to a couple **M** of moment 45 kN·m. Knowing that $E = 200$ GPa, $\upsilon = 0.29$, determine (*a*) the radius of curvature ρ, (*b*) the radius of curvature ρ' of a transverse cross section.



SOLUTION

For W 200×31.3 rolled steel section

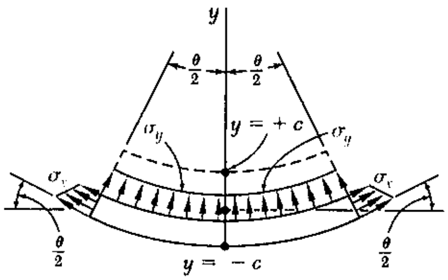
$I = 31.4\times10^6$ mm^4 $= 31.4\times10^{-6}$ m^4

(a) $\dfrac{1}{\rho} = \dfrac{M}{EI} = \dfrac{45\times10^3}{(200\times10^9)(31.4\times10^{-6})} = 7.17\times10^{-3}$ m^{-1}

$\rho = 139.6$ m

(b) $\dfrac{1}{\rho'} = \nu\dfrac{1}{\rho} = (0.29)(7.17\times10^{-3}) = 2.07\times10^{-3}$ m^{-1} $\qquad \rho' = 481$ m

PROBLEM 4.38



4.38 It was assumed in Sec. 4.3 that the normal stresses σ_y in a member in pure bending are negligible. For an initially straight elastic member of rectangular cross section, (*a*) derive an approximate expression for σ_y as a function of *y*, (*b*) show that $(\sigma_y)_{max} \approx -(c/2\rho)(\sigma_x)_{max}$ and, thus, that σ_y can be neglected in all practical situations. (*Hint:* Consider the free-body diagram of the portion of beam located below the surface of ordinate *y* and assume the distribution of the stress σ_x is still linear.)

SOLUTION

Denote the width of the beam by b and the length by L.

$$\theta = \frac{L}{\rho}$$

Using the free body diagram above, with $\cos\frac{\theta}{2} \approx 1$

$$\Sigma F_y = 0 \qquad \sigma_y bL + 2\int_{-c}^{y} \sigma_x b\,dy \sin\frac{\theta}{2} = 0$$

$$\sigma_y = -\frac{2}{L}\sin\frac{\theta}{2}\int_{-c}^{y}\sigma_x\,dy \approx -\frac{\theta}{L}\int_{-c}^{y}\sigma_x\,dy = -\frac{1}{\rho}\int_{-c}^{y}\sigma_x\,dy$$

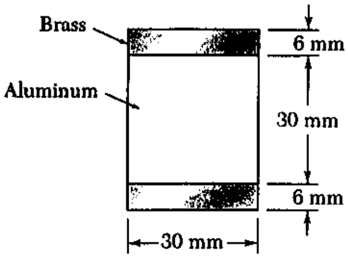
But $\quad \sigma_x = -(\sigma_x)_{max}\dfrac{y}{c}$

$$\sigma_y = \frac{(\sigma_x)_{max}}{\rho c}\int_{-c}^{y} y\,dy = \frac{(\sigma_x)_{max}}{\rho c}\left.\frac{y^2}{2}\right|_{-c}^{y} = \frac{(\sigma_x)_{max}}{2\rho c}(y^2 - c^2)$$

The maximum value σ_y occurs at $y = 0$

$$(\sigma_y)_{max} = -\frac{(\sigma_x)_{max}c^2}{2\rho c} = -\frac{(\sigma_x)_{max}c}{2\rho}$$

PROBLEM 4.39



4.39 and 4.40 Two brass strips are securely bonded to an aluminum bar of 30 × 30-mm square cross section. Using the data given below, determine the largest permissible bending moment when the composite member is bent about a horizontal axis.

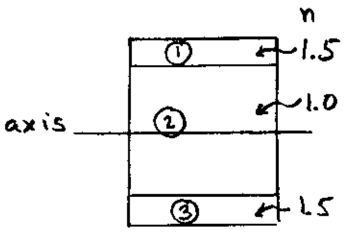
	Aluminum	**Brass**
Modulus of elasticity:	70 GPa	105 GPa
Allowable stress:	100 MPa	160 MPa

SOLUTION

Use aluminum as the reference material

$n = 1.0$ in aluminum

$n = E_b/E_a = 105/70 = 1.5$ in brass



For the transformed section

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2$$

$$= \frac{1.5}{12}(30)(6)^3 + (1.5)(30)(6)(18)^3 = 88.29 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 = \frac{1.0}{12}(30)(30)^3 = 67.5 \times 10^3 \text{ mm}^4, \quad I_3 = I_1 = 88.29 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 244.08 \times 10^3 \text{ mm}^4 = 244.08 \times 10^{-9} \text{ m}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \qquad M = \frac{\sigma I}{n y}$$

Aluminum: $n = 1.0$, $y = 15 \text{ mm} = 0.015 \text{ m}$, $\sigma = 100 \times 10^6$ Pa

$$M = \frac{(100 \times 10^6)(244.08 \times 10^{-9})}{(1.0)(0.015)} = 1.627 \times 10^3 \text{ N·m}$$

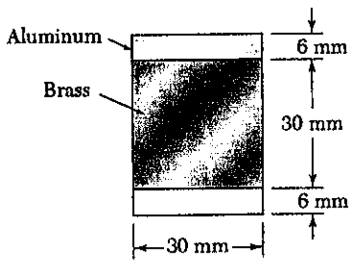
Brass: $n = 1.5$, $y = 21 \text{ mm} = 0.021 \text{ m}$, $\sigma = 160 \times 10^6$ Pa

$$M = \frac{(160 \times 10^6)(244.08 \times 10^{-9})}{(1.5)(0.021)} = 1.240 \times 10^3 \text{ N·m}$$

Choose the smaller value $\quad M = 1.240 \times 10^3 \text{ N·m} = 1.240 \text{ kN·m}$ ◄

PROBLEM 4.40

4.39 and 4.40 Two ~~brass~~ aluminum strips are securely bonded to ~~an~~ a ~~aluminum~~ brass bar of 30 × 30-mm square cross section. Using the data given below, determine the largest permissible bending moment when the composite member is bent about a horizontal axis.



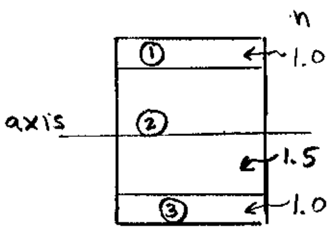
	Aluminum	**Brass**
Modulus of elasticity:	70 GPa	105 GPa
Allowable stress:	100 MPa	160 MPa

SOLUTION

Use aluminum as the reference material

$n = 1.0$ in aluminum

$n = E_b / E_a = 105/70 = 1.5$ in brass



For the transformed section

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2$$

$$= \frac{1.0}{12}(30)(6)^3 + (1.0)(30)(6)(18)^2 = 58.86 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 = \frac{1.5}{12}(30)(30)^3 = 101.25 \times 10^3 \text{ mm}^4, \qquad I_3 = I_1 = 58.86 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 218.97 \times 10^3 \text{ mm}^4 = 218.97 \times 10^{-9} \text{ m}^4$$

$$|\sigma| = \left| \frac{nMy}{I} \right| \quad \therefore \quad M = \frac{\sigma I}{ny}$$

Aluminum: $n = 1.0$, $y = 21 \text{ mm} = 0.021 \text{ m}$ $\quad \sigma = 100 \times 10^6 \text{ Pa}$

$$M = \frac{(100 \times 10^6)(218.97 \times 10^{-9})}{(1.0)(0.021)} = 1.043 \times 10^3 \text{ N}\cdot\text{m}$$

Brass: $n = 1.5$, $y = 15 \text{ mm} = 0.015 \text{m}$, $\sigma = 160 \times 10^6 \text{ Pa}$

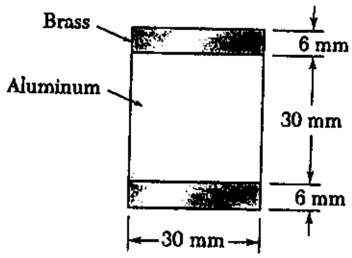
$$M = \frac{(160 \times 10^6)(218.97 \times 10^{-9})}{(1.5)(0.015)} = 1.557 \times 10^3 \text{ N}\cdot\text{m}$$

Choose the smaller value $\quad M = 1.043 \times 10^3 \text{ N}\cdot\text{m} = 1.043 \text{ kN}\cdot\text{m}$ ◄

PROBLEM 4.41

4.41 and 4.42 For the composite bar indicated, determine the permissible bending moment when the bar is bent about a vertical axis.

4.41 Bar of Prob. 4.39



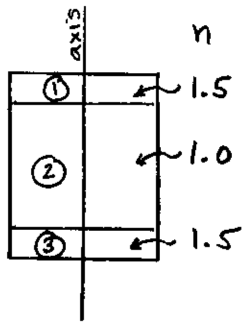
SOLUTION

Use aluminum as reference material

$n = 1.0$ in aluminum

$n = E_b / E_a = 105/70 = 1.5$ in brass

For the transformed section



$$I_1 = \frac{n_1}{12} b_1 h_1^3$$

$$= \frac{1.5}{12}(6)(30)^3 = 20.25 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3$$

$$= \frac{1.0}{12}(30)(30)^3 = 67.5 \times 10^3 \text{ mm}^4$$

$$I_3 = I_1 = 20.25 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 108 \times 10^3 \text{ mm}^4 = 108 \times 10^{-9} \text{ m}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \quad \therefore \quad M = \frac{\sigma I}{ny}$$

Aluminum: $n = 1.0$, $y = 15 \text{ mm} = 0.015 \text{ m}$, $\sigma = 100 \times 10^6$ Pa

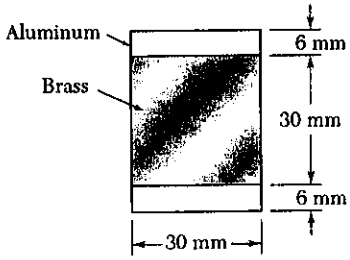
$$M = \frac{(100 \times 10^6)(108 \times 10^{-9})}{(1.0)(0.015)} = 720 \text{ N·m}$$

Brass: $n = 1.5$, $y = 15 \text{ mm} = 0.015 \text{m}$, $\sigma = 160 \times 10^6$ Pa

$$M = \frac{(160 \times 10^6)(108 \times 10^{-9})}{(1.5)(0.015)} = 768 \text{ N·m}$$

Choose the smaller value $\quad M = 720 \text{ N·m}$

PROBLEM 4.42



4.41 and 4.42 **For the composite bar indicated, determine the permissible bending moment when the bar is bent about a vertical axis.**

4.42 Bar of Prob. 4.40

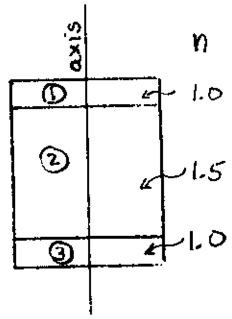
SOLUTION

Use aluminum as reference material

$n = 1.0$ in aluminum

$n = E_b / E_a = 105/70 = 1.5$ in brass

For the transformed section



$$I_1 = \frac{n_1}{12} b_1 h_1^3$$

$$= \frac{1.0}{12}(6)(30)^3 = 13.5 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 = \frac{1.5}{12}(30)(30)^3 = 101.25 \text{ mm}^4$$

$$I_3 = I_1 = 13.5 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 + I_3 = 128.25 \times 10^3 \text{ mm}^4 = 128.25 \times 10^{-9} \text{ m}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \qquad M = \frac{\sigma I}{ny}$$

Aluminum: $n = 1.0$, $y = 15 \text{ mm} = 0.015 \text{ m}$, $\sigma = 100 \times 10^6$ Pa

$$M = \frac{(100 \times 10^6)(128.25 \times 10^{-9})}{(1.0)(0.015)} = 855 \text{ N} \cdot \text{m}$$

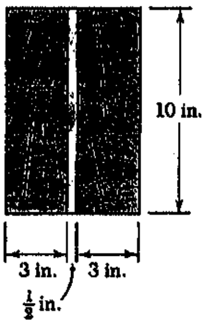
Brass: $n = 1.5$, $y = 15 \text{ mm} = 0.015 \text{ m}$, $\sigma = 160 \times 10^6$ Pa

$$M = \frac{(160 \times 10^6)(128.25 \times 10^{-9})}{(1.5)(0.015)} = 912 \text{ N} \cdot \text{m}$$

Choose the smaller value $\qquad M = 855 \text{ N} \cdot \text{m}$

PROBLEM 4.43

4.43 and 4.44 Wooden beams and steel plates are securely bolted together to form the composite members shown. Using the data given below, determine the largest permissible bending moment when the composite beam is bent about a horizontal axis.



	Wood	Steel
Modulus of elasticity:	2×10^6 psi	30×10^6 psi
Allowable stress:	2000 psi	22 ksi

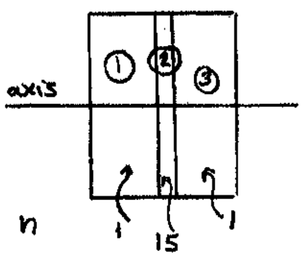
SOLUTION

Use wood as the reference material

$n = 1.0$ in wood

$n = E_s/E_w = 30/2 = 15$ in steel

For the transformed section



$$I_1 = \frac{n_1}{12} b_1 h_1^3 = \frac{1.0}{12}(3)(10)^3 = 250 \text{ in}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 = \frac{15}{12}\left(\tfrac{1}{2}\right)(10)^3 = 625 \text{ in}^4$$

$$I_3 = I_1 = 250 \text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 1125 \text{ in}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \quad \therefore \quad M = \frac{\sigma I}{ny}$$

Wood: $n = 1.0$, $y = 5$ in, $\sigma = 2000$ psi

$$M = \frac{(2000)(1125)}{(1.0)(5)} = 450 \times 10^3 \text{ lb}\cdot\text{in}$$

Steel: $n = 15$, $y = 5$ in, $\sigma = 22$ ksi $= 22 \times 10^3$ psi

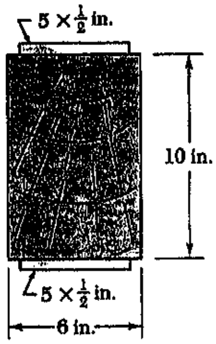
$$M = \frac{(22 \times 10^3)(1125)}{(15)(5)} = 330 \times 10^3 \text{ lb}\cdot\text{in}$$

Choose the smaller value $\quad M = 330 \times 10^3 \text{ lb}\cdot\text{in} = 330 \text{ kip}\cdot\text{in}$ ◀

PROBLEM 4.44

4.43 and 4.44 Wooden beams and steel plates are securely bolted together to form the composite members shown. Using the data given below, determine the largest permissible bending moment when the composite beam is bent about a horizontal axis.

	Wood	Steel
Modulus of elasticity:	2×10^6 psi	30×10^6 psi
Allowable stress:	2000 psi	22 ksi



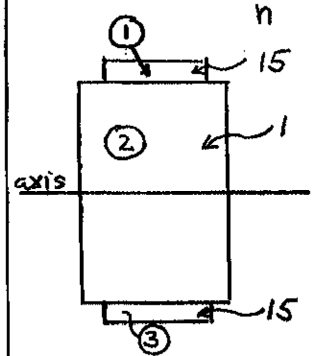
SOLUTION

Use wood as the reference material

$n = 1.0$ in wood

$n = E_s/E_w = 30/2 = 15$ in steel

For the transformed section



$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2$$

$$= \frac{15}{12}(5)\left(\tfrac{1}{2}\right)^3 + (15)(5)\left(\tfrac{1}{2}\right)(5.25)^2 = 1034.4 \text{ in}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^2 = \frac{1.0}{12}(6)(10)^3 = 500 \text{ in}^4$$

$$I_3 = I_1 = 1034.4 \text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 2569 \text{ in}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \quad \therefore \quad M = \frac{\sigma I}{ny}$$

Wood: $n = 1.0$, $y = 5$ in, $\sigma = 2000$ psi

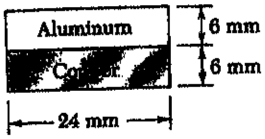
$$M = \frac{(2000)(2569)}{(1.0)(5)} = 1.028 \times 10^6 \text{ lb}\cdot\text{in}$$

Steel $n = 15$, $y = 5.5$ in, $\sigma = 22$ ksi $= 22 \times 10^3$ psi

$$M = \frac{(22 \times 10^3)(2569)}{(15)(5.5)} = 685 \times 10^3 \text{ lb}\cdot\text{in}$$

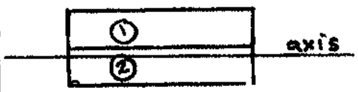
Choose the smaller value $\quad M = 685 \times 10^3$ lb·in $= 685$ kip·in ◄

PROBLEM 4.45



4.45 and 4.46 A copper strip ($E_c = 105$ GPa) and an aluminum strip ($E_a = 75$ GPa) are bonded together to form the composite bar shown. Knowing that the bar is bent about a horizontal axis by a couple of moment 35 N·m, determine the maximum stress in (*a*) the aluminum strip, (*b*) the copper strip.

SOLUTION



Use aluminum as the reference material

$n = 1.0$ in aluminum

$n = E_c/E_a = 105/75 = 1.4$ in copper

Transformed section

	A, mm^2	nA, mm^2	$\bar{y}_o$, mm	$nA\bar{y}_o$, mm^3
1	144	144	9	1296
2	144	201.6	3	604.8
Σ		345.6		1900.8

$$\bar{Y}_o = \frac{1900.8}{345.6} = 5.50 \text{ mm}$$

The neutral axis lies 5.50 mm above the bottom

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2 = \frac{1.0}{12}(24)(6)^3 + (1.0)(24)(6)(3.5)^2 = 2196 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{1.4}{12}(24)(6)^3 + (1.4)(24)(6)(2.5)^2 = 1864.8 \text{ mm}^4$$

$$I = I_1 + I_2 = 4060.8 \text{ mm}^4 = 4.0608 \times 10^{-9} \text{ m}^4$$

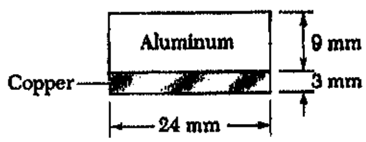
(a) Aluminum $\quad n = 1.0 \quad y = 12 - 5.5 = 6.5 \text{ mm} = 0.0065 \text{ m}$

$$\sigma = -\frac{nMy}{I} = -\frac{(1.0)(35)(0.0065)}{4.0608 \times 10^{-9}} = -56.0 \times 10^6 \text{ Pa} = -56.0 \text{ MPa} \blacktriangleleft$$

(b) Copper $\quad n = (1.4) \quad y = -5.5 \text{ mm} = -0.0055 \text{ m}$

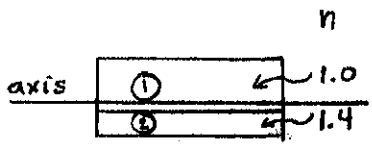
$$\sigma = -\frac{nMy}{I} = -\frac{(1.4)(35)(-0.0055)}{4.0608 \times 10^{-9}} = 66.4 \times 10^6 \text{ Pa} = 66.4 \text{ MPa} \blacktriangleleft$$

PROBLEM 4.46



4.45 and 4.46 A copper strip ($E_c = 105$ GPa) and an aluminum strip ($E_a = 75$ GPa) are bonded together to form the composite bar shown. Knowing that the bar is bent about a horizontal axis by a couple of moment 35 N·m, determine the maximum stress in (*a*) the aluminum strip, (*b*) the copper strip.

SOLUTION



Use aluminum as the reference material

$n = 1.0$ in aluminum

$n = E_c/E_a = 105/75 = 1.4$ in copper

Transformed section

	A, mm²	nA, mm²	$\bar{y}_o$, mm	$nA\bar{y}_o$, mm³
①	216	216	7.5	1620
②	72	100.8	1.5	151.8
Σ		316.8		1771.2

$$\bar{Y}_o = \frac{1771.2}{316.8} = 5.5909 \text{ mm}$$

The neutral axis lies 5.5909 mm above the bottom

$$I_1 = \frac{n_1}{12}b_1h_1^3 + n_1A_1d_1^2 = \frac{1.0}{12}(24)(9)^3 + (1.0)(24)(9)(1.9091)^2 = 2245.2 \text{ mm}^4$$

$$I_2 = \frac{n_2}{12}b_2h_2^3 + n_2A_2d_2^2 = \frac{1.4}{12}(24)(3)^3 + (1.4)(24)(3)(4.0909)^2 = 1762.5 \text{ mm}^4$$

$$I = I_1 + I_2 = 4839 \text{ mm}^4 = 4.008 \times 10^{-9} \text{ m}^4$$

(a) Aluminum: $n = 1.0 \quad y = 12 - 5.5909 = 6.4091 \text{ mm} = 0.0064091$

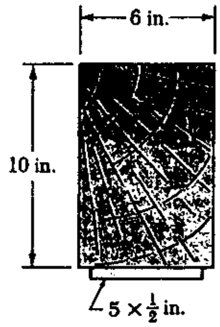
$$\sigma = -\frac{nMy}{I} = -\frac{(1.0)(35)(0.0064091)}{4.008 \times 10^{-9}} = -56.0 \times 10^{6} \text{ Pa} = -56.0 \text{ MPa} \blacktriangleleft$$

(b) Copper: $n = 1.4, \quad y = -5.5909 \text{ mm} = -0.0055909 \text{ m}$

$$\sigma = -\frac{nMy}{I} = -\frac{(1.4)(35)(-0.0055909)}{4.008 \times 10^{-9}} = 68.4 \times 10^{6} \text{ Pa} = 68.4 \text{ MPa} \blacktriangleleft$$

PROBLEM 4.47

4.47 and 4.48 A 6 × 10-in. timber beam has been strengthened by bolting to it the steel straps shown. The modulus of elasticity is 1.5×10^6 psi for the wood and 30×10^6 psi for the steel. Knowing that the beam is bent about a horizontal axis by a couple of moment 200 kip·in., determine the maximum stress in (*a*) the wood, (*b*) the steel.



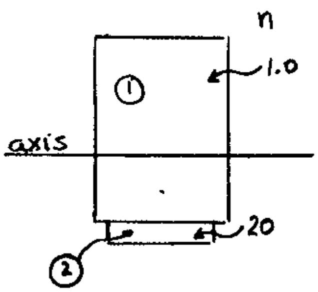
SOLUTION

Use wood as the reference material

$n = 1.0$ in wood

$n = E_s/E_w = 30/1.5 = 20$ for steel

Transformed section



	A	nA	$\bar{y}_o$	$nA\bar{y}_o$
①	60	60	5.5	330
②	2.5	50	0.25	12.5
Σ		110		342.5

$$\bar{Y}_o = \frac{342.5}{110} = 3.114 \text{ in}$$

The neutral axis lies 3.114 in about the bottom.

$$I_1 = \frac{n_1}{12}b_1h_1^3 + n_1A_1d_1^2 = \frac{1.0}{12}(6)(10)^3 + (1.0)(60)(2.386)^2 = 841.6 \text{ in}^4$$

$$I_2 = \frac{n_2}{12}b_2h_2^3 + n_2A_2d_2^2 = \frac{20}{12}(5)(\tfrac{1}{2})^3 + (20)(2.5)(2.864)^2 = 411.2 \text{ in}^4$$

$$I = I_1 + I_2 = 1252.8 \text{ in}^4$$

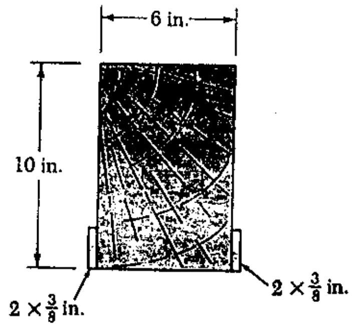
(a) Wood: $n = 1.0$ $y = 10.5 - 3.114 = 7.386$ in

$$\sigma = -\frac{nMy}{I} = -\frac{(1.0)(200)(7.386)}{1252.8} = -1.179 \text{ ksi}$$ ◄

(b) Steel: $n = 20$ $y = -3.114$ in

$$\sigma = -\frac{nMy}{I} = -\frac{(20)(200)(-3.114)}{1252.8} = 9.94 \text{ ksi}$$ ◄

PROBLEM 4.48



4.47 and 4.48 A 6 × 10-in. timber beam has been strengthened by bolting to it the steel straps shown. The modulus of elasticity is 1.5×10^6 psi for the wood and 30×10^6 psi for the steel. Knowing that the beam is bent about a horizontal axis by a couple of moment 200 kip·in., determine the maximum stress in (*a*) the wood, (*b*) the steel.

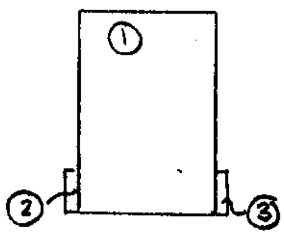
SOLUTION

Use wood as the reference material

$n = 1.0$ in wood

$n = E_s / E_w = 30/1.5 = 20$ in steel

Transformed section



	A	nA	$\bar{y}_0$	$nA\bar{y}_0$
1	60	60	5	300
2	0.75	15	1	15
3	0.75	15	1	15
Σ		90		330

$$\bar{Y}_0 = \frac{330}{90} = 3.667 \text{ in}$$

The neutral axis lies 3.667 in. above the bottom

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2 = \frac{1.0}{12}(6)(10)^3 + (60)(1.333)^2 = 606.7 \text{ in}^4$$

$$I_2 = I_3 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{20}{12}\left(\frac{3}{8}\right)(2)^3 + (15)(2.667)^2 = 111.7 \text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 830 \text{ in}^4$$

(a) Wood: $n = 1.0$, $y = 10 - 3.667 = 6.333$ in

$$\sigma = -\frac{nMy}{I} = -\frac{(1.0)(200)(6.333)}{830} = -1.526 \text{ ksi}$$

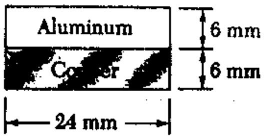
(b) Steel: $n = 20$ $y = -3.667$ in

$$\sigma = -\frac{nMy}{I} = -\frac{(20)(200)(-3.667)}{830} = 17.67 \text{ ksi}$$

PROBLEM 4.49

4.49 and 4.50 For the composite bar indicated, determine the radius of curvature caused by the couple of moment 35N·m.

4.49 Bar of Prob. 4.45



SOLUTION

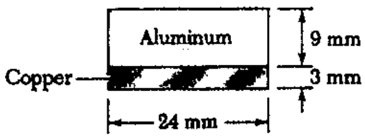
See solution to PROBLEM 4.45 for the calculation of I

$$\frac{1}{\rho} = \frac{M}{E_a I} = \frac{35}{(75\times10^9)(4.0608\times10^{-9})} = 0.1149\ \text{m}^{-1}, \qquad \rho = 8.70\ \text{m}$$

PROBLEM 4.50

4.49 and 4.50 For the composite bar indicated, determine the radius of curvature caused by the couple of moment 35N·m.

4.50 Bar of Prob. 4.46



SOLUTION

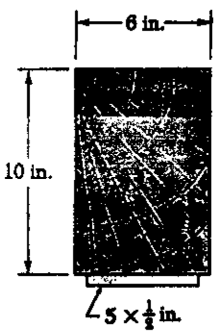
See solution to PROBLEM 4.46 for calculation of I.

$$\frac{1}{\rho} = \frac{M}{E_a I} = \frac{35}{(75\times10^9)(4.008\times10^{-9})} = 0.1164\ \text{m}^{-1}, \qquad \rho = 8.59\ \text{m}$$

PROBLEM 4.51

4.51 and 4.52 For the composite beam indicated, determine the radius of curvature caused by the couple of moment 200 kip·in.

4.51 Beam of Prob. 4.47



SOLUTION

See solution to PROBLEM 4.47 for calculation of I.

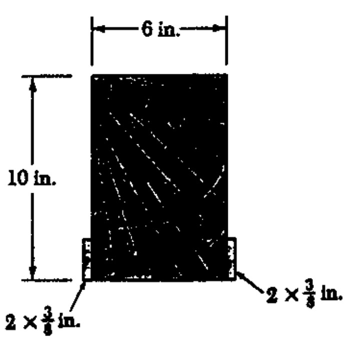
$$\frac{1}{\rho} = \frac{M}{E_w I} = \frac{200\times10^3}{(1.5\times10^6)(1252.8)} = 106.4\times10^{-6}\ \text{in}^{-1}$$

$$\rho = 9396\ \text{in} = 783\ \text{ft.}$$

PROBLEM 4.52

4.51 and 4.52 For the composite beam indicated, determine the radius of curvature caused by the couple of moment 200 kip·in.

4.52 Beam of Prob. 4.48



SOLUTION

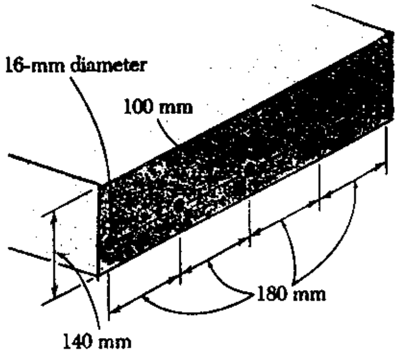
See solution to PROBLEM 4.48 for calculation of I.

$$\frac{1}{\rho} = \frac{M}{E_w I} = \frac{200\times10^3}{(1.5\times10^6)(830)} = 160.6\times10^{-6}\ \text{in}^{-1}$$

$$\rho = 6225\ \text{in} = 519\ \text{ft.}$$

PROBLEM 4.53

4.53 A concrete slab is reinforced by 16-mm-diameter steel rods placed on 180-mm centers as shown. The modulus of elasticity is 20 GPa for concrete and 200 GPA for steel. Using an allowable stress of 9 MPa for the concrete and of 120 MPA for the steel, determine the largest allowable positive bending moment in a portion of slab 1 m wide.



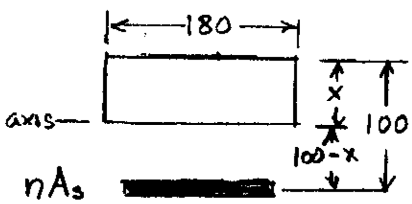
SOLUTION

$$n = \frac{E_s}{E_c} = \frac{200\text{ GPa}}{20\text{ GPa}} = 10$$

Consider a section 180 mm wide with one steel rod.

$$A_s = \frac{\pi}{4}d^2 = \frac{\pi}{4}(16)^2 = 201.06\text{ mm}^2$$

$$nA_s = 2.0106 \times 10^3\text{ mm}^2$$



Locate the neutral axis

$$180\,x\,\frac{x}{2} - (100 - x)(2.0106 \times 10^3) = 0$$

$$90x^2 + 2.0106 \times 10^3 x - 201.06 \times 10^3 = 0$$

Solving for x $\quad x = \dfrac{-2.0106 \times 10^3 + \sqrt{(2.0106 \times 10^3)^2 + (4)(90)(201.06 \times 10^3)}}{(2)(90)}$

$$x = 37.397\text{ mm}, \qquad 100 - x = 62.603\text{ mm}$$

$$I = \tfrac{1}{3}(180)x^3 + (2.0106 \times 10^3)(100 - x)^2$$

$$= \tfrac{1}{3}(180)(37.397)^3 + (2.0106 \times 10^3)(62.603)^2$$

$$= 11.018 \times 10^6\text{ mm}^4 = 11.018 \times 10^{-6}\text{ m}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \quad \therefore \quad M = \frac{\sigma I}{ny}$$

Concrete: $n = 1$, $y = 37.397\text{ mm} = 0.037397\text{ m}$, $\sigma = 9 \times 10^6\text{ Pa}$

$$M = \frac{(9 \times 10^6)(11.018 \times 10^{-6})}{(1.0)(0.037397)} = 2.6516 \times 10^3\text{ N}\cdot\text{m}$$

Steel: $n = 10$, $y = 62.603\text{ mm} = 0.062603\text{ m}$, $\sigma = 120 \times 10^6\text{ Pa}$

$$M = \frac{(120 \times 10^6)(11.018 \times 10^{-6})}{(10)(0.062603)} = 2.1120 \times 10^3\text{ N}\cdot\text{m}$$

Choose the smaller value $\quad M = 2.1120 \times 10^3\text{ N}\cdot\text{m}$

The above is the allowable positive moment for a 180 mm wide section.

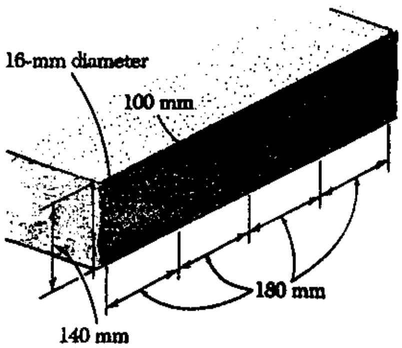
For a 1 m = 1000 mm width, multiply by $\frac{1000}{180} = 5.556$

$$M = (5.556)(2.1120 \times 10^3) = 11.73 \times 10^3\text{ Nm} = 11.73\text{ kN}\cdot\text{m}$$

4.53 A concrete slab is reinforced by 16-mm-diameter steel rods placed on 180-mm centers as shown. The modulus of elasticity is 20 GPa for concrete and 200 GPA for steel. Using an allowable stress of 9 MPa for the concrete and of 120 MPA for the steel, determine the largest allowable positive bending moment in a portion of slab 1 m wide.

4.54 Solve Prob. 4.53, assuming that the spacing of the 16-mm-diameter rods is increased to 225 mm on centers.

PROBLEM 4.54



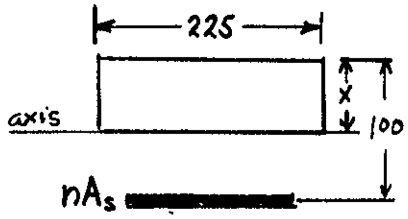
SOLUTION

$$n = \frac{E_s}{E_c} = \frac{200\ \text{GPa}}{20\ \text{GPa}} = 10$$

Consider a section 225 mm wide with one steel rod.

$$A_s = \frac{\pi}{4}d^2 = \frac{\pi}{4}(16)^2 = 201.06\ \text{mm}^2$$

$$nA_s = 2.0106\times10^3\ \text{mm}^2$$



Locate the neutral axis

$$225x\frac{x}{2} - (100-x)(2.0106\times10^3) = 0$$

$$112.5x^2 + 2.0106x - 201.06\times10^3 = 0$$

Solving for x $\quad x = \dfrac{-2.0106\times10^3 + \sqrt{(2.0106\times10^3)^2 + (4)(112.5)(201.06\times10^3)}}{(2)(112.5)}$

$$x = 34.273\ \text{mm} \qquad 100 - x = 65.727$$

$$I = \frac{1}{3}(225)x^3 + 2.0106\times10^3(100-x)^2$$

$$= \frac{1}{3}(225)(34.273)^3 + (2.0106\times10^3)(65.727)^2$$

$$= 11.705\times10^6\ \text{mm}^4 = 11.705\times10^{-6}\ \text{m}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \quad \therefore \quad M = \frac{\sigma I}{ny}$$

Concrete: $n = 1$, $y = 34.273\ \text{mm} = 0.034273\ \text{m}$, $\sigma = 9\times10^6\ \text{Pa}$

$$M = \frac{(9\times10^6)(11.705\times10^{-6})}{(1)(0.034273)} = 3.0738\times10^3\ \text{N}\cdot\text{m}$$

Steel: $n = 10$, $y = 65.727\ \text{m} = 0.065727\ \text{m}$, $\sigma = 120\times10^6\ \text{Pa}$

$$M = \frac{(120\times10^6)(11.705\times10^{-6})}{(10)(0.065727)} = 2.1370\times10^3\ \text{N}\cdot\text{m}$$

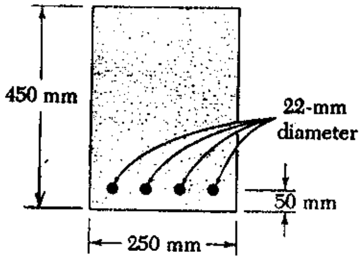
Choose the smaller value $\quad M = 2.1370\times10^3\ \text{N}\cdot\text{m}$

The above is the allowable positive moment for a 225 mm wide section.

For a 1 m = 1000 mm section, multiply by $\frac{1000}{225} = 4.4444$

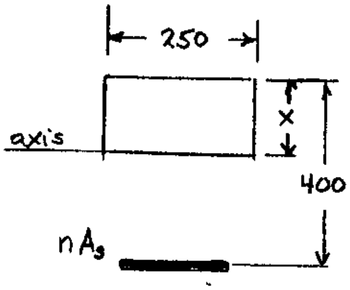
$$M = (4.4444)(2.1370\times10^3) = 9.50\times10^3\ \text{N}\cdot\text{m} = 9.50\ \text{kN}\cdot\text{m}$$

PROBLEM 4.55



4.55 The reinforced concrete beam shown is subjected to a positive bending moment of 175 kN·m. Knowing that the modulus of elasticity is 25 GPa for the concrete and 200 GPa for the steel, determine (*a*) the stress in the steel, (*b*) the maximum stress in the concrete.

SOLUTION



$$n = \frac{E_s}{E_c} = \frac{200 \text{ GPa}}{25 \text{ GPa}} = 8.0$$

$$A_s = 4 \cdot \frac{\pi}{4} d^2 = (4)\left(\frac{\pi}{4}\right)(22)^2 = 1.5205 \times 10^3 \text{ mm}^2$$

$$nA_s = 12.164 \times 10^3 \text{ mm}^2$$

Locate the neutral axis

$$250\, x \frac{x}{2} - (12.164 \times 10^3)(400 - x) = 0$$

$$125 x^2 + 12.164 \times 10^3 x - 4.8657 \times 10^6 = 0$$

Solving for x
$$x = \frac{-12.164 \times 10^3 + \sqrt{(12.164 \times 10^3)^2 + (4)(125)(4.8657 \times 10^6}}{(2)(125)}$$

$$x = 154.55 \text{ mm}, \qquad 400 - x = 245.45 \text{ mm}$$

$$I = \frac{1}{3} 250\, x^3 + (12.164 \times 10^3)(400 - x)^2$$

$$= \frac{1}{3}(250)(154.55)^3 + (12.164 \times 10^3)(245.45)^2$$

$$= 1.0404 \times 10^9 \text{ mm}^4 = 1.0404 \times 10^{-3} \text{ m}^4$$

$$\sigma = -\frac{nMy}{I}$$

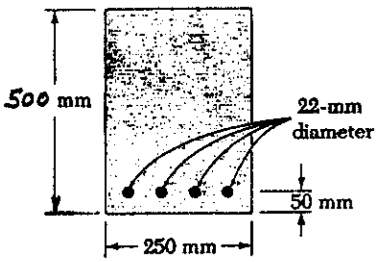
(a) Steel: $y = -245.45 \text{ mm} = -0.24545 \text{ m}$

$$\sigma = -\frac{(8.0)(175 \times 10^3)(-0.24545)}{1.0404 \times 10^{-3}} = 330 \times 10^6 \text{ Pa} = 330 \text{ MPa}$$

(b) Concrete: $y = 154.55 \text{ mm} = 0.15455 \text{ m}$

$$\sigma = -\frac{(1.0)(175 \times 10^3)(0.15455)}{1.0404 \times 10^{-3}} = -26.0 \times 10^6 \text{ Pa} = -26.0 \text{ MPa}$$

PROBLEM 4.56



4.55 The reinforced concrete beam shown is subjected to a positive bending moment of 175 kN·m. Knowing that the modulus of elasticity is 25 GPa for the concrete and 200 GPa for the steel, determine (*a*) the stress in the steel, (*b*) the maximum stress in the concrete.

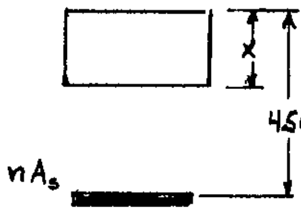
4.56 Solve Prob. 4.55 assuming that the 450-mm depth of the beam is increased to 500 mm.

SOLUTION

$$n = \frac{E_s}{E_c} = \frac{200\text{ GPa}}{25\text{ GPa}} = 8.0$$

$$A_s = 4\,\frac{\pi}{4}d^2 = (4)\left(\frac{\pi}{4}\right)(22)^2 = 1.5205\times10^3\text{ mm}^2$$

$$nA_s = 12.164\times10^3\text{ mm}^2$$



Locate the neutral axis

$$250\times\frac{x}{2} - (12.164\times10^3)(450-x) = 0$$

$$125x^2 + 12.164\times10^3x - 5.4738\times10^6 = 0$$

Solving for x
$$x = \frac{-12.164\times10^3 + \sqrt{(12.164\times10^3)^2 + (4)(125)(5.4738\times10^6)}}{(2)(125)}$$

$$x = 166.19\text{ mm}, \qquad 450 - x = 283.81\text{ mm}$$

$$I = \frac{1}{3}(250)x^3 + (12.164\times10^3)(450-x)^2$$

$$= \frac{1}{3}(250)(166.19)^3 + (12.164\times10^3)(283.81)^2$$

$$= 1.3623\times10^9\text{ mm}^4 = 1.3623\times10^{-3}\text{ m}^4$$

$$\sigma = -\frac{nMy}{I}$$

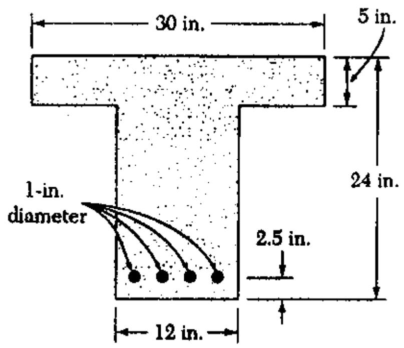
(a) Steel: $y = -283.81\text{ mm} = -0.28381\text{ m}$

$$\sigma = -\frac{(8.0)(175\times10^3)(-0.28381)}{1.3623\times10^{-3}} = 292\times10^6\text{ Pa} = 292\text{ MPa}$$

(b) Concrete: $y = 166.19\text{ mm} = 0.16619\text{ m}$

$$\sigma = -\frac{(1.0)(175\times10^3)(0.16619)}{1.3623\times10^{-3}} = -21.3\times10^6\text{ Pa} = -21.3\text{ MPa}$$

PROBLEM 4.57



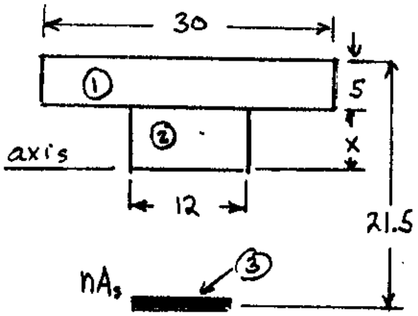
4.57 Knowing that the bending moment in the reinforced concrete beam shown is +150 kip·ft and that the modulus of elasticity is 3.75×10^6 psi for the concrete and 30×10^6 psi for the steel, determine (*a*) the stress in the steel, (*b*) the maximum stress in the concrete.

SOLUTION

$$n = \frac{E_s}{E_c} = \frac{30\times10^6}{3.75\times10^6} = 8.0$$

$$A_s = 4\,\frac{\pi}{4}d^2 = 4\left(\frac{\pi}{4}\right)(1)^2 = 3.1416\text{ in}^2$$

$$nA_s = 25.133\text{ in}^2$$



Locate the neutral axis

$$(30)(5)(x+2.5) + 12\,x\,\frac{x}{2} - (25.133)(16.5-x) = 0$$

$$150x + 375 + 6x^2 - 414.69 + 25.133x = 0$$

$$6x^2 + 175.133x - 39.69 = 0$$

Solve for x $\quad x = \dfrac{-175.133 + \sqrt{(175.133)^2 + (4)(6)(39.69)}}{(2)(6)} = 0.225$ in.

$$16.5 - x = 16.275\text{ in.}$$

$$I_1 = \frac{1}{12}b_1h_1^3 + A_1d_1^2 = \frac{1}{12}(30)(5)^3 + (30)(5)(2.725)^2 = 1426.3\text{ in}^4$$

$$I_2 = \frac{1}{3}b_2x^3 = \frac{1}{3}(12)(0.225)^3 = 0.1\text{ in}^4$$

$$I_3 = nA_sd_3^2 = (25.133)(16.275)^2 = 6657.1\text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 8083.5\text{ in}^4$$

$\sigma = -\dfrac{nMy}{I}$ where $M = 150$ kip·ft $= 1800$ kip·in.

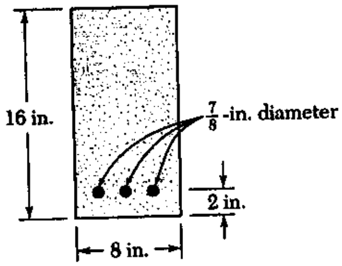
(a) Steel $\quad n = 8.0, \quad y = -16.275$ in

$$\sigma = -\frac{(8.0)(1800)(-16.275)}{8083.5} = 29.0\text{ ksi}$$ ◄

(b) Concrete $\quad n = 1.0, \quad y = 5.225$ in

$$\sigma = -\frac{(1.0)(1800)(5.225)}{8083.5} = -1.163\text{ ksi}$$ ◄

PROBLEM 4.58



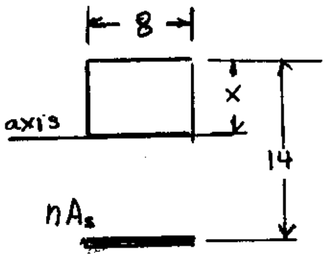
4.58 A concrete beam is reinforced by three steel rods placed as shown. The modulus of elasticity is 3×10^6 psi for the concrete and 30×10^6 psi for the steel. Using an allowable stress of 1350 psi for the concrete and 20 ksi for the steel, determine the largest allowable positive bending moment in the beam.

SOLUTION

$$n = \frac{E_s}{E_c} = \frac{30 \times 10^6}{3 \times 10^6} = 10$$

$$A_s = 3\, \frac{\pi}{4} d^2 = 3\left(\frac{\pi}{4}\right)\left(\frac{7}{8}\right)^2 = 1.8040 \text{ in}^2$$

$$nA_s = 18.040 \text{ in}^2$$



Locate neutral axis.

$$8x\,\frac{x}{2} - (18.040)(14 - x) = 0$$

$$4x^2 + 18.040\,x - 252.56 = 0$$

Solve for x $\quad x = \dfrac{-18.040 + \sqrt{18.040^2 + (4)(4)(252.56)}}{(2)(4)} = 6.005 \text{ in.}$

$$14 - x = 7.995 \text{ in}$$

$$I = \frac{1}{3}\, 8\, x^3 + nA_s(14 - x)^2 = \frac{1}{3}(8)(6.005)^3 + (18.040)(7.995)^2$$

$$= 1730.4 \text{ in}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \quad \therefore \quad M = \frac{\sigma I}{ny}$$

Concrete: $\quad n = 1.0, \quad |y| = 6.005 \text{ in}, \quad |\sigma| = 1350 \text{ psi}$

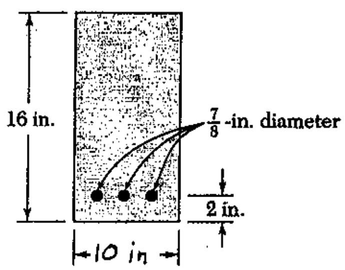
$$M = \frac{(1350)(1730.5)}{(1.0)(6.005)} = 389 \times 10^3 \text{ lb}\cdot\text{in} = 389 \text{ kip}\cdot\text{in}$$

Steel: $\quad n = 10, \quad |y| = 7.995, \quad \sigma = 20 \times 10^3 \text{ psi}$

$$M = \frac{(20 \times 10^3)(1730.5)}{(10)(7.995)} = 433 \times 10^3 \text{ lb}\cdot\text{in} = 433 \text{ kip}\cdot\text{in}$$

Choose the smaller value $\quad M = 389 \text{ kip}\cdot\text{in} = 32.4 \text{ kip}\cdot\text{ft}$

PROBLEM 4.59



4.58 A concrete beam is reinforced by three steel rods placed as shown. The modulus of elasticity is 3×10^6 psi for the concrete and 30×10^6 psi for the steel. Using an allowable stress of 1350 psi for the concrete and 20 ksi for the steel, determine the largest allowable positive bending moment in the beam.

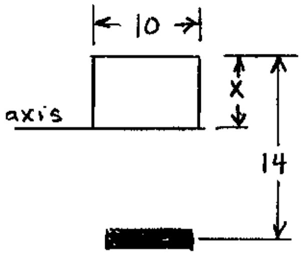
4.59 Solve Prob. 4.58, assuming that the width of the concrete beam is increased to 10 in.

SOLUTION

$$n = \frac{E_s}{E_c} = \frac{30\times10^6}{3\times10^6} = 10$$

$$A_s = 3\,\frac{\pi}{4}d^2 = 3\left(\frac{\pi}{4}\right)\left(\frac{7}{8}\right)^2 = 1.8040\ \text{in}^2$$

$$nA_s = 18.040\ \text{in}^2$$



Locate the neutral axis

$$10x\,\frac{x}{2} - (18.040)(14-x) = 0$$

$$5x^2 + 18.040x - 252.56 = 0$$

Solve for x $$x = \frac{-18.040 + \sqrt{(18.040)^2 + (4)(5)(252.56)}}{(2)(5)} = 5.529\ \text{in}$$

$$14 - x = 8.471\ \text{in}$$

$$I = \tfrac{1}{3}(10)x^3 + nA_s(14-x)^2 = \tfrac{1}{3}(10)(5.529)^3 + (18.040)(8.471)^2$$
$$= 1857.9\ \text{in}^4$$

$$|\sigma| = \left|\frac{nMy}{I}\right| \quad \therefore \quad M = \frac{\sigma I}{ny}$$

Concrete: $n = 1.0$ $\quad |y| = 5.529$ in $\quad |\sigma| = 1350$ psi

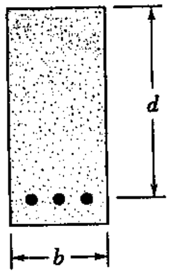
$$M = \frac{(1350)(1857.9)}{(1.0)(5.529)} = 453.6\times10^3\ \text{lb}\cdot\text{in}$$

Steel: $n = 10$ $\quad |y| = 8.471$ in $\quad |\sigma| = 20\times10^3$ psi

$$M = \frac{(20\times10^3)(1857.9)}{(10)(8.471)} = 438.6\times10^3\ \text{lb}\cdot\text{in}$$

Choose the smaller value $\quad M = 438.6\times10^3$ lb·in
$= 438.6$ kip·in
$= 36.6$ kip·ft ◄

PROBLEM 4.60

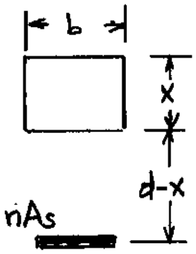


4.60 The design of a reinforced concrete beam is said to be *balanced* if the maximum stresses in the steel and concrete are equal, respectively, to the allowable stresses σ_s and σ_c. Show that to achieve a balanced design the distance x from the top of the beam to the neutral axis must be

$$x = \frac{d}{1 + \dfrac{\sigma_s E_c}{\sigma_c E_s}}$$

where E_c and E_s are the moduli of elasticity of concrete and steel, respectively, and d is the distance from the top of the beam to the reinforcing steel.

SOLUTION



$$\sigma_s = \frac{n M (d-x)}{I} \qquad \sigma_c = \frac{Mx}{I}$$

$$\frac{\sigma_s}{\sigma_c} = \frac{n(d-x)}{x} = n\frac{d}{x} - n$$

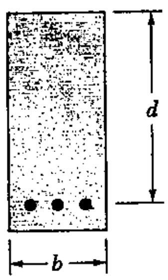
$$\frac{d}{x} = 1 + \frac{1}{n}\frac{\sigma_s}{\sigma_c} = 1 + \frac{E_c \sigma_s}{E_s \sigma_c}$$

$$x = \frac{d}{1 + \dfrac{E_c \sigma_s}{E_s \sigma_c}}$$

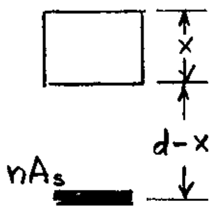
PROBLEM 4.61

4.60 The design of a reinforced concrete beam is said to be *balanced* if the maximum stresses in the steel and concrete are equal, respectively, to the allowable stresses σ_s and σ_c.

4.61 For the concrete beam shown, the modulus of elasticity is 3.5×10^6 psi for the concrete and 29×10^6 psi for the steel. Knowing that $b = 8$ in. and $d = 22$ in., and using an allowable stress of 1800 psi for the concrete and 20 ksi for the steel, determine (*a*) the required area A_s of the steel reinforcement if the design of the beam is to be balanced, (*b*) the largest allowable bending moment. (See Prob. 4.60 for definition of a balanced beam.)



SOLUTION



$$n = \frac{E_s}{E_c} = \frac{29\times10^6}{3.5\times10^6} = 8.2857$$

$$\sigma_s = \frac{nM(d-x)}{I} \qquad \sigma_c = \frac{Mx}{I}$$

$$\frac{\sigma_s}{\sigma_c} = \frac{n(d-x)}{x} = n\frac{d}{x} - n$$

$$\frac{d}{x} = 1 + \frac{1}{n}\frac{\sigma_s}{\sigma_c} = 1 + \frac{1}{8.2857}\cdot\frac{20\times10^3}{1800} = 2.3410$$

$$x = 0.42717\,d = (0.42717)(22) = 9.398 \text{ in}$$

$$d - x = 22 - 9.398 = 12.602 \text{ in}$$

Locate neutral axis

$$bx\frac{x}{2} - nA_s(d-x) = 0$$

(a) $$A_s = \frac{bx^2}{2n(d-x)} = \frac{(8)(9.398)^2}{(2)(8.2857)(12.602)} = 3.3835 \text{ in}^2$$

$$I = \frac{1}{3}bx^3 + nA_s(d-x)^2 = \frac{1}{3}(8)(9.398)^3 + (8.2857)(3.3835)(12.602)^2$$

$$= 6665.6 \text{ in}^4$$

$$\sigma = \frac{nMy}{I} \qquad M = \frac{\sigma I}{ny}$$

Concrete: $n = 1.0 \quad y = 9.398$ in $\quad \sigma = 1800$ psi

$$M = \frac{(1800)(6665.6)}{(1.0)(9.398)} = 1.277\times10^6 \text{ lb}\cdot\text{in}$$

Steel: $n = 8.2857 \quad |y| = 12.602$ in $\quad \sigma = 20\times10^3$ psi

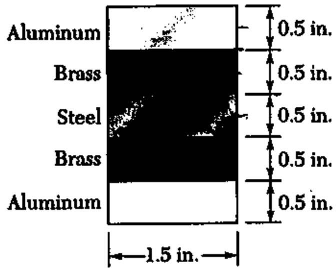
$$M = \frac{(20\times10^3)(6665.6)}{(8.2857)(12.602)} = 1.277\times10^6 \text{ lb}\cdot\text{in}$$

Note that both values are the same for balanced design

$$M = 1.277\times10^3 \text{ kip-in} = 106.4 \text{ kip-ft}$$

PROBLEM 4.62

4.62 and 4.63 Five metal strips, each of 0.5 × 1.5-in. cross section, are bonded together to form the composite beam shown. The modulus of elasticity is 30×10^6 psi for the steel, 15×10^6 psi for the brass, and 10×10^6 psi for the aluminum. Knowing that the beam is bent about a horizontal axis by couples of moment 12 kip·in., determine (*a*) the maximum stress in each of the three metals, (*b*) the radius of curvature of the composite beam.



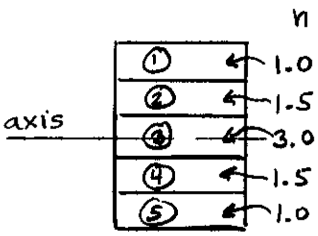
SOLUTION

Use aluminum as the reference material

$$n = \frac{E_s}{E_a} = \frac{30 \times 10^6}{10 \times 10^6} = 3.0 \text{ in steel}$$

$$n = \frac{E_b}{E_a} = \frac{15 \times 10^6}{10 \times 10^6} = 1.5 \text{ in brass}$$

$$n = 1.0 \text{ in aluminum.}$$



For the transformed section

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2 = \frac{1}{12}(1.5)(0.5)^3 + (0.75)(1.0)^2$$

$$= 0.7656 \text{ in}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{1.5}{12}(1.5)(0.5)^3 + (1.5)(0.75)(0.5)^2 = 0.3047 \text{ in}^4$$

$$I_3 = \frac{n_3}{12} b_3 h_3^3 = \frac{3.0}{12}(1.5)(0.5)^3 = 0.0469 \text{ in}^4$$

$$I_4 = I_2 = 0.3047 \text{ in}^4, \qquad I_5 = I_1 = 0.7656 \text{ in}^4$$

$$I = \sum_1^5 I_i = 2.1875 \text{ in}^4$$

(a) Aluminum: $\sigma = \dfrac{nMy}{I} = \dfrac{(1.0)(12)(1.25)}{2.1875} = 6.86 \text{ ksi}$ ◄

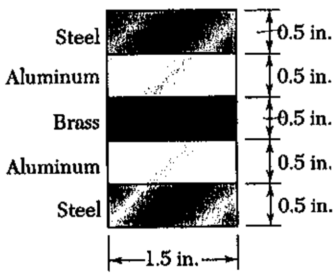
Brass: $\sigma = \dfrac{nMy}{I} = \dfrac{(1.5)(12)(0.75)}{2.1875} = 6.17 \text{ ksi}$ ◄

Steel: $\sigma = \dfrac{nMy}{I} = \dfrac{(3.0)(12)(0.25)}{2.1875} = 4.11 \text{ ksi}$ ◄

(b) $\dfrac{1}{\rho} = \dfrac{M}{E_a I} = \dfrac{12 \times 10^3}{(10 \times 10^6)(2.1875)} = 548.57 \times 10^{-6} \text{ in}^{-1}$ ◄

$$\rho = 1823 \text{ in} = 151.9 \text{ ft}$$ ◄

PROBLEM 4.63



4.62 and 4.63 Five metal strips, each of 0.5 × 1.5-in. cross section, are bonded together to form the composite beam shown. The modulus of elasticity is 30×10^6 psi for the steel, 15×10^6 psi for the brass, and 10×10^6 psi for the aluminum. Knowing that the beam is bent about a horizontal axis by couples of moment 12 kip·in., determine (*a*) the maximum stress in each of the three metals, (*b*) the radius of curvature of the composite beam.

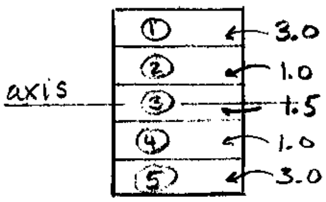
SOLUTION

Use aluminum as the reference material

$$n = \frac{E_s}{E_a} = \frac{30 \times 10^6}{10 \times 10^6} = 3.0 \quad \text{in steel}$$

$$n = \frac{E_b}{E_a} = \frac{15 \times 10^6}{10 \times 10^6} = 1.5 \quad \text{in brass}$$

$$n = 1.0 \quad \text{in aluminum}$$



For the transformed section

$$I_1 = \frac{n_1}{12} b_1 h_1^3 + n_1 A_1 d_1^2$$

$$= \frac{3.0}{12}(1.5)(0.5)^3 + (3.0)(0.75)(1.0)^2 = 2.2969 \text{ in}^4$$

$$I_2 = \frac{n_2}{12} b_2 h_2^3 + n_2 A_2 d_2^2 = \frac{1.0}{12}(1.5)(0.5)^3 + (1.0)(0.75)(0.5)^2 = 0.2031 \text{ in}^4$$

$$I_3 = \frac{n_3}{12} b_3 h_3^3 = \frac{1.5}{12}(1.5)(0.5)^3 = 0.0234 \text{ in}^4$$

$$I_4 = I_2 = 0.2031 \text{ in}^4, \qquad I_5 = I_1 = 2.2969 \text{ in}^4$$

$$I = \sum_1^5 I_i = 5.0234 \text{ in}^4$$

(a) Steel: $\sigma = \dfrac{nMy}{I} = \dfrac{(3.0)(12)(1.25)}{5.0234} = 8.96 \text{ ksi}$ ◄

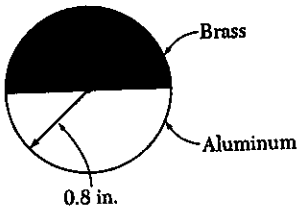
Aluminum: $\sigma = \dfrac{nMy}{I} = \dfrac{(1.0)(12)(0.75)}{5.0234} = 1.792 \text{ ksi}$ ◄

Brass: $\sigma = \dfrac{nMy}{I} = \dfrac{(1.5)(12)(0.25)}{5.0234} = 0.896 \text{ ksi}$ ◄

(b) $\dfrac{1}{\rho} = \dfrac{M}{E_a I} = \dfrac{12 \times 10^3}{(10 \times 10^6)(5.0234)} = 238.89 \times 10^{-6} \text{ in}^{-1}$

$\rho = 4186 \text{ in.} = 349 \text{ ft}$ ◄

PROBLEM 4.64



4.64 The composite beam shown is formed by bonding together a brass rod and an aluminum rod of semicircular cross sections. The modulus of elasticity is 15×10^6 psi for the brass and 10×10^6 psi for the aluminum. Knowing that the composite beam is bent about a horizontal axis by couples of moment 8 kip·in., determine the maximum stress (*a*) in the brass, (*b*) in the aluminum.

SOLUTION

For each semicircle $r = 0.8$ in,

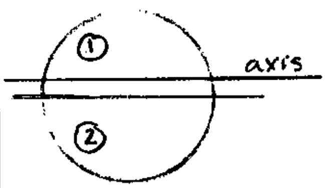
$A = \frac{\pi}{2} r^2 = 1.00531 \text{ in}^2$, $\quad \bar{y}_0 = \frac{4r}{3\pi} = \frac{(4)(0.8)}{3\pi} = 0.33953$ in

$I_{base} = \frac{\pi}{8} r^4 = 0.160850 \text{ in}^4$

$\bar{I} = I_{base} - A\bar{y}_0^2 = 0.160850 - (1.00531)(0.33953)^2$

$= 0.044957 \text{ in}^4$

Use aluminum as the reference material



$n = 1.0$ in aluminum

$n = \frac{E_b}{E_a} = \frac{15 \times 10^6}{10 \times 10^6} = 1.5$ in brass

Locate neutral axis

	A, in^2	nA, in^2	$\bar{y}_0$, in	$nA\bar{y}_0$, in^3
①	1.00531	1.50796	0.33953	0.51200
②	1.0053	1.00531	−0.33953	−0.34133
Σ		2.51327		0.17067

$\bar{Y}_0 = \frac{0.17067}{2.51327} = 0.06791$ in

The neutral axis lies 0.06791 in above the material interface.

$d_1 = 0.33953 - 0.06791 = 0.27162$ in, $\quad d_2 = 0.33953 + 0.06791 = 0.40744$ in

$I_1 = n_1\bar{I} + n_1 A d_1^2 = (1.5)(0.044957) + (1.5)(1.00531)(0.27162)^2 = 0.17869 \text{ in}^4$

$I_2 = n_2\bar{I} + n_2 A d_2^2 = (1.0)(0.044957) + (1.0)(1.00531)(0.40744)^2 = 0.21185 \text{ in}^4$

$I = I_1 + I_2 = 0.39054 \text{ in}^4$

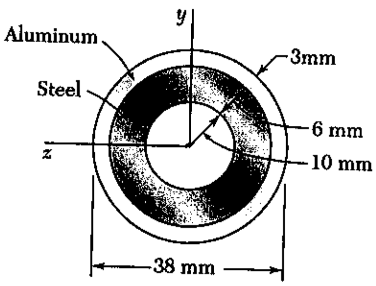
(a) Brass: $n = 1.5$, $\quad y = 0.8 - 0.06791 = 0.73209$ in

$$\sigma = -\frac{nMy}{I} = -\frac{(1.5)(8)(0.73209)}{0.39054} = -22.5 \text{ ksi} \quad ◀$$

(b) Aluminum: $n = 1.0 \quad y = -0.8 - 0.06791 = -0.86791$ in

$$\sigma = -\frac{nMy}{I} = -\frac{(1.0)(8)(-0.86791)}{0.39054} = 17.78 \text{ ksi} \quad ◀$$

PROBLEM 4.65



4.65 A steel pipe and an aluminum pipe are securely bonded together to form the composite beam shown. The modulus of elasticity is 210 GPa for the steel and 70 GPa for the aluminum. Knowing that the composite beam is bent by couples of moment 500 N·m, determine the maximum stress (*a*) in the aluminum, (*b*) in the steel.

SOLUTION

Use aluminum as the reference material

$n = 1.0$ in aluminum

$n = \frac{E_s}{E_a} = \frac{210}{70} = 3.0$ in steel

Steel: $I_1 = n_1 \frac{\pi}{4}(r_o^4 - r_i^4) = (3.0)\frac{\pi}{4}(16^4 - 10^4) = 130.85 \times 10^3 \text{ mm}^4$

Aluminum: $I_2 = n_2 \frac{\pi}{4}(r_o^4 - r_i^4) = (1.0)\frac{\pi}{4}(19^4 - 16^4) = 50.88 \times 10^3 \text{ mm}^4$

$I = I_1 + I_2 = 181.73 \times 10^3 \text{ mm}^4 = 181.73 \times 10^{-9} \text{ m}^4$

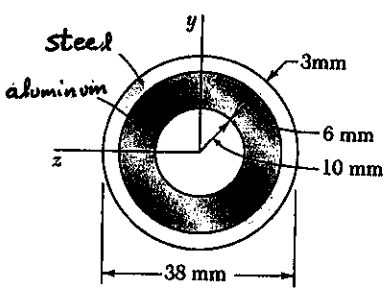
(a) Aluminum: $c = 19 \text{ mm} = 0.019 \text{ m}$

$\sigma = \frac{nMc}{I} = \frac{(1.0)(500)(0.019)}{181.73 \times 10^{-9}} = 52.3 \times 10^6 \text{ Pa} = 52.3 \text{ MPa}$ ◄

(b) Steel: $c = 16 \text{ mm} = 0.016 \text{ m}$

$\frac{nMc}{I} = \frac{(3.0)(500)(0.016)}{181.73 \times 10^{-9}} = 132.1 \times 10^6 \text{ Pa} = 132.1 \text{ MPa}$ ◄

PROBLEM 4.66



4.65 A steel pipe and an aluminum pipe are securely bonded together to form the composite beam shown. The modulus of elasticity is 210 GPa for the steel and 70 GPa for the aluminum. Knowing that the composite beam is bent by couples of moment 500 N·m, determine the maximum stress (*a*) in the aluminum, (*b*) in the steel.

4.66 Solve Prob. 4.65, assuming that the 6-mm-thick inner pipe is made of aluminum and that the 3-mm-thick outer pipe is made of steel.

SOLUTION

Use aluminum as the reference material

$n = 1.0$ in aluminum

$n = \frac{E_s}{E_a} = \frac{210}{70} = 3.0$ in steel

Steel: $I_1 = n_1 \frac{\pi}{4}(r_o^4 - r_i^4) = (3.0)\frac{\pi}{4}(19^4 - 16^4) = 152.65 \times 10^3 \text{ mm}^4$

Aluminum: $I_2 = n_2 \frac{\pi}{4}(r_o^4 - r_i^4) = (1.0)\frac{\pi}{4}(16^4 - 10^4) = 43.62 \times 10^3 \text{ mm}^4$

$I = I_1 + I_2 = 196.27 \times 10^3 \text{ mm}^4 = 196.27 \times 10^{-9} \text{ m}^4$

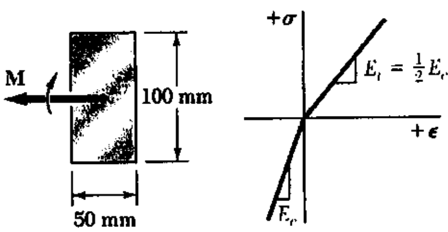
(a) Aluminum: $c = 16 \text{ mm} = 0.016 \text{ m}$

$\sigma = \frac{nMc}{I} = \frac{(1.0)(500)(0.016)}{196.27 \times 10^{-9}} = 40.8 \times 10^6 \text{ Pa} = 40.8 \text{ MPa}$ ◄

(b) Steel: $c = 19 \text{ mm} = 0.019 \text{ m}$

$\sigma = \frac{nMc}{I} = \frac{(3.0)(500)(0.019)}{196.27 \times 10^{-9}} = 145.2 \times 10^6 \text{ Pa} = 145.2 \text{ MPa}$ ◄

PROBLEM 4.67

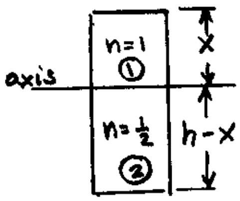


4.67 The rectangular beam shown is made of a plastic for which the value of the modulus of elasticity in tension is one half of its value in compression. For a bending moment $M = 600$ N·m, determine the maximum (*a*) tensile stress, (*b*) compressive stress.

SOLUTION

$n = \frac{1}{2}$ on the tension side of neutral axis

$n = 1$ on the compression side



Locate neutral axis.

$$n_1 b x \frac{x}{2} - n_2 b(h-x)\frac{h-x}{2} = 0$$

$$\frac{1}{2}bx^2 - \frac{1}{4}b(h-x)^2 = 0$$

$$x^2 = \frac{1}{2}(h-x)^2 \qquad x = \frac{1}{\sqrt{2}}(h-x)$$

$$x = \frac{1}{\sqrt{2}+1}h = 0.41421\,h = 41.421 \text{ mm}$$

$$h - x = 58.579 \text{ mm}$$

$$I_1 = n_1 \frac{1}{3}bx^3 = (1)(\tfrac{1}{3})(50)(41.421)^3 = 1.1844\times 10^6 \text{ mm}^4$$

$$I_2 = n_2 \frac{1}{3}b(h-x)^3 = (\tfrac{1}{2})(\tfrac{1}{3})(50)(58.579)^3 = 1.6751\times 10^6 \text{ mm}^4$$

$$I = I_1 + I_2 = 2.8595\times 10^6 \text{ mm}^4 = 2.8595\times 10^{-6} \text{ m}^4$$

(a) tensile stress: $n = \frac{1}{2}$, $y = -58.579$ mm $= -0.058579$ m

$$\sigma = -\frac{nMy}{I} = -\frac{(0.5)(600)(-0.058579)}{2.8595\times 10^{-6}} = 6.15\times 10^6 \text{ Pa}$$

$$= 6.15 \text{ MPa}$$

(b) compressive stress: $n = 1$, $y = 41.421$ mm $= 0.041421$ m

$$\sigma = -\frac{nMy}{I} = -\frac{(1.0)(600)(0.041421)}{2.8595\times 10^{-6}} = -8.69\times 10^6 \text{ Pa}$$

$$= -8.69 \text{ MPa}$$

PROBLEM 4.68

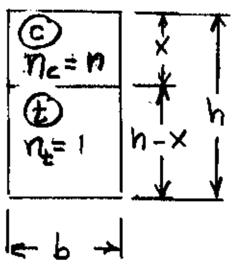
*4.68 A rectangular beam is made of a material for which the modulus of elasticity is E_t in tension and E_c in compression. Show that the curvature of the beam in pure bending is

$$\frac{1}{\rho} = \frac{M}{E_r I}$$

where

$$E_r = \frac{4E_t E_c}{(\sqrt{E_t} + \sqrt{E_c})^2}$$

SOLUTION



Use E_t as the reference modulus.

Then $E_c = n E_t$

Locate neutral axis

$$n b x \frac{x}{2} - b(h-x)\frac{h-x}{2} = 0$$

$$n x^2 - (h-x)^2 = 0 \qquad \sqrt{n}\, x = (h-x)$$

$$x = \frac{h}{\sqrt{n}+1} \qquad h - x = \frac{\sqrt{n}\, h}{\sqrt{n}+1}$$

$$I_{trans} = \frac{n}{3} b x^3 + \frac{1}{3} b (h-x)^3 = \left[\frac{n}{3}\left(\frac{1}{\sqrt{n}+1}\right)^3 + \left(\frac{\sqrt{n}}{\sqrt{n}+1}\right)^3\right] b h^3$$

$$= \frac{1}{3}\frac{n + n^{3/2}}{(\sqrt{n}+1)^3} b h^3 = \frac{1}{3}\frac{n(1+\sqrt{n})}{(\sqrt{n}+1)^3} b h^3 = \frac{1}{3}\cdot\frac{n}{(\sqrt{n}+1)^2} b h^3$$

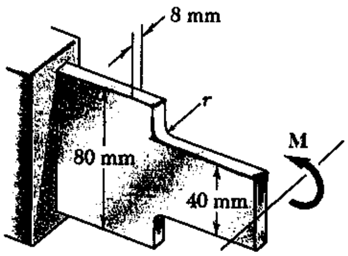
$$\frac{1}{\rho} = \frac{M}{E_t I_{trans}} = \frac{M}{E_r I} \qquad \text{where } I = \frac{1}{12} b h^3$$

$$E_r I = E_t I_{trans}$$

$$E_r = \frac{E_t I_{trans}}{I} = \frac{12}{bh^3}\cdot E_t \cdot \frac{n}{3(\sqrt{n}+1)^2} b h^3$$

$$= \frac{4 E_t E_c / E_t}{(\sqrt{E_c/E_t} + 1)^2} = \frac{4 E_t E_c}{(\sqrt{E_c} + \sqrt{E_t})^2}$$

PROBLEM 4.69



4.69 Knowing that $M = 250$ N·m, determine the maximum stress in the beam shown when the radius r of the fillets is (*a*) 4 mm, (*b*) 8 mm.

SOLUTION

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(8)(40)^3 = 42.667 \times 10^3\ mm^4 = 42.667 \times 10^{-9}\ m^4$$

$$c = 20\ mm = 0.020\ m$$

$$\frac{D}{d} = \frac{80\ mm}{40\ mm} = 2.00$$

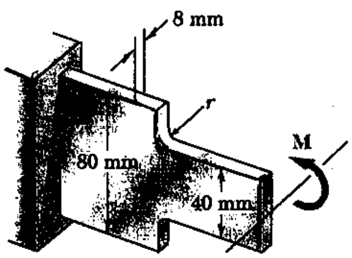
(a) $\frac{r}{d} = \frac{4\ mm}{40\ mm} = 0.10$ From Fig. 4.31 $K = 1.87$

$$\sigma_{max} = K\frac{Mc}{I} = \frac{(1.87)(250)(0.020)}{42.667 \times 10^{-9}} = 219 \times 10^6\ Pa = 219\ MPa$$

(b) $\frac{r}{d} = \frac{8\ mm}{40\ mm} = 0.20$ From Fig. 4.31 $K = 1.52$

$$\sigma_{max} = K\frac{Mc}{I} = \frac{(1.52)(250)(0.020)}{42.667 \times 10^{-9}} = 176 \times 10^6\ Pa = 176\ MPa$$

PROBLEM 4.70



4.70 Knowing that the allowable stress for the beam shown is 90 MPa, determine the allowable bending moment M when the radius r of the fillets is (*a*) 8 mm, (*b*) 12 mm.

SOLUTION

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(8)(40)^3 = 42.667 \times 10^3\ mm^4 = 42.667 \times 10^{-9}\ m^4$$

$$c = 20\ mm = 0.020\ m$$

$$\frac{D}{d} = \frac{80\ mm}{40\ mm} = 2.00$$

(a) $\frac{r}{d} = \frac{8\ mm}{40\ mm} = 0.2$ From Fig. 4.31 $K = 1.50$

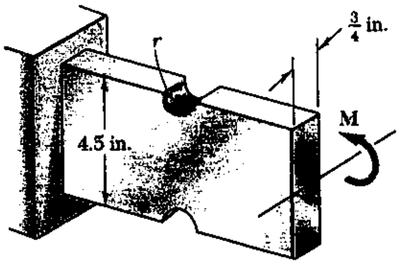
$$\sigma_{max} = K\frac{Mc}{I} \quad \therefore \quad M = \frac{\sigma_{max} I}{Kc} = \frac{(90 \times 10^6)(42.667 \times 10^{-9})}{(1.50)(0.020)}$$

$$= 128\ N \cdot m$$

(b) $\frac{r}{d} = \frac{12\ mm}{40\ mm} = 0.3$ From Fig 4.31 $K = 1.35$

$$M = \frac{(90 \times 10^6)(42.667 \times 10^{-9})}{(1.35)(0.020)} = 142\ N \cdot m$$

PROBLEM 4.71



4.71 Semicircular grooves of radius r must be milled as shown in the sides of a steel member. Using an allowable stress of 8 ksi, determine the largest bending moment
that can be applied to the member when the radius r of the semicircular grooves is (*a*) $\frac{3}{8}$ in., (*b*) $\frac{3}{4}$ in.

SOLUTION

(a) $d = D - 2r = 4.5 - (2)(\frac{3}{8}) = 3.75$ in.

$\frac{D}{d} = \frac{4.5}{3.75} = 1.20, \quad \frac{r}{d} = \frac{0.375}{3.75} = 0.10$

From Fig. 4.32 $K = 2.07$

$I = \frac{1}{12}bh^3 = \frac{1}{12}(\frac{3}{4})(3.75)^3 = 3.296 \text{ in}^4, \qquad c = \frac{1}{2} = 1.875 \text{ in}$

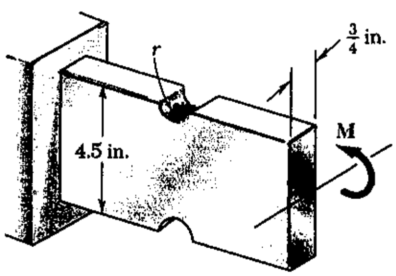
$\sigma = K\frac{Mc}{I} \therefore M = \frac{\sigma I}{Kc} = \frac{(8)(3.296)}{(2.07)(1.875)} = 6.79 \text{ kip}\cdot\text{in}$ ◄

(b) $d = D - 2r = 4.5 - (2)(\frac{3}{4}) = 3.0, \quad \frac{D}{d} = \frac{4.5}{3.0} = 1.5, \quad \frac{r}{d} = \frac{0.75}{3.0} = 0.25$

From Fig. 4.32 $K = 1.61, \quad I = \frac{1}{12}bh^3 = \frac{1}{12}(\frac{3}{4})(3.0)^3 = 1.6875 \text{ in}^4$

$c = \frac{1}{2}d = 1.5 \text{ in.} \qquad M = \frac{\sigma I}{Kc} = \frac{(8)(1.6875)}{(1.61)(1.5)} = 5.59 \text{ kip}\cdot\text{in.}$ ◄

PROBLEM 4.72



4.72 Semicircular grooves of radius r must be milled as shown in the sides of a steel member. Knowing that $M = 4$ kip·in., determine the maximum stress in the member when (*a*) $r = \frac{3}{8}$ in., (*b*) $r = \frac{3}{4}$ in.

SOLUTION

(a) $d = D - 2r = 4.5 - (2)(\frac{3}{8}) = 3.75$ in.

$\frac{D}{d} = \frac{4.5}{3.75} = 1.20, \quad \frac{r}{d} = \frac{0.375}{3.75} = 0.10$

From Fig. 4.32 $K = 2.07$

$I = \frac{1}{12}bh^3 = \frac{1}{12}(\frac{3}{4})(3.75)^3 = 3.296 \text{ in}^4, \qquad c = \frac{1}{2}d = 1.875 \text{ in.}$

$\sigma = K\frac{Mc}{I} = \frac{(2.07)(4)(1.875)}{3.296} = 4.71 \text{ ksi}$ ◄

(b) $d = D - 2r = 4.5 - (2)(\frac{3}{4}) = 3.00 \text{ in}, \quad \frac{D}{d} = \frac{4.5}{3.00} = 1.50, \quad \frac{r}{d} = \frac{0.75}{3.00} = 0.25$

From Fig 4.32 $K = 1.61$

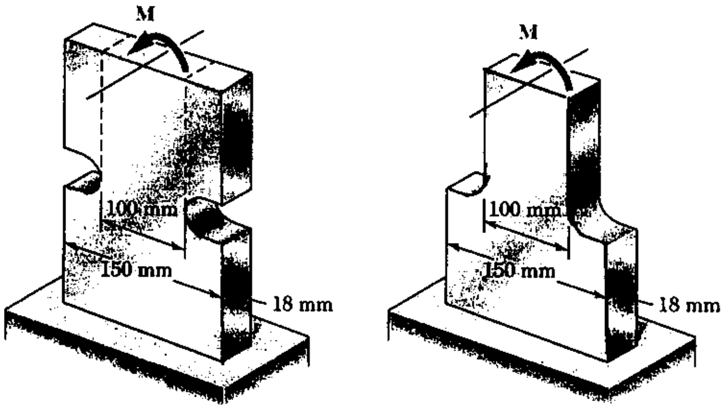
$I = \frac{1}{12}bh^3 = \frac{1}{12}(\frac{3}{4})(3.00)^3 = 1.6875 \text{ in}^4, \qquad c = \frac{1}{2}d = 1.5 \text{ in}$

$\sigma = K\frac{Mc}{I} = \frac{(1.61)(4)(1.5)}{1.6875} = 5.72 \text{ ksi}$ ◄

PROBLEM 4.73

SOLUTION

4.73 The allowable stress used in the design of a steel bar is 80 MPa. Determine the largest couple **M** that can be applied to the bar (*a*) if the bar is designed with grooves having semicircular portions of radius $r = 15$ mm, as shown in Fig. *a*, (*b*) if the bar is redesigned by removing the material above the grooves as shown in Fig. *b*.



(*a*) (*b*)

For both configuations

$D = 150$ mm, $d = 100$ mm,

$r = 15$ mm.

$$\frac{D}{d} = \frac{150}{100} = 1.50$$

$$\frac{r}{d} = \frac{15}{100} = 0.15$$

For configuration (a), Fig 4.32 gives $K_a = 1.92$.

For configuration (b) Fig 4.31 gives $K_b = 1.57$.

$$I = \frac{1}{12} b h^3 = \frac{1}{12}(18)(100)^3 = 1.5 \times 10^6 \text{ mm}^4 = 1.5 \times 10^{-6} \text{ m}^4$$

$$c = \frac{1}{2} d = 50 \text{ mm} = 0.050 \text{ m}$$

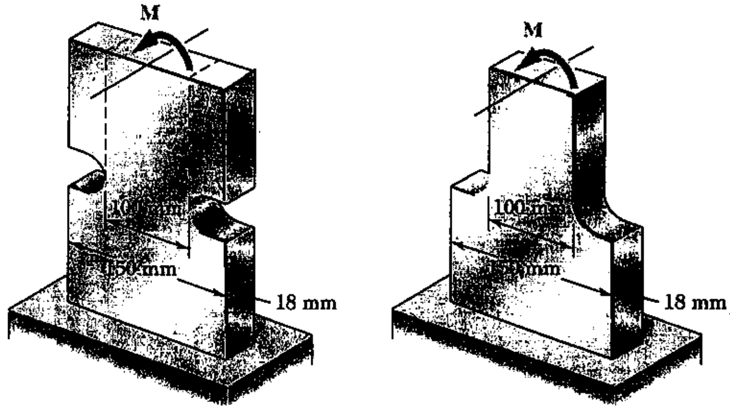
(a) $\sigma = \frac{KMc}{I} \therefore \quad M = \frac{\sigma I}{Kc} = \frac{(80 \times 10^6)(1.5 \times 10^{-6})}{(1.92)(0.05)} = 1.25 \times 10^3 \text{ N}\cdot\text{m}$

$= 1.25 \text{ kN}\cdot\text{m}$ ◄

(b) $M = \frac{\sigma I}{Kc} = \frac{(80 \times 10^6)(1.5 \times 10^{-6})}{(1.57)(0.050)} = 1.53 \times 10^3 \text{ N}\cdot\text{m} = 1.53 \text{ kN}\cdot\text{m}$ ◄

PROBLEM 4.74

4.74 A couple of moment $M = 2$ kN·m is to be applied to the end of a steel bar. Determine the maximum stress in the bar (*a*) if the bar is designed with grooves having semicircular portions of radius $r = 10$ mm, as shown in Fig. *a*, (*b*) if the bar is redesigned by removing the material above the grooves as shown in Fig. *b*.



(*a*) (*b*)

SOLUTION

For both configurations

$D = 150$ mm, $d = 100$ mm

$r = 10$ mm.

$$\frac{D}{d} = \frac{150}{100} = 1.50$$

$$\frac{r}{d} = \frac{10}{100} = 0.10$$

For configuration (a),
Fig 4.32 give $K_a = 2.21$

For configuration (b), Fig. 4.31 gives $K_b = 1.79$

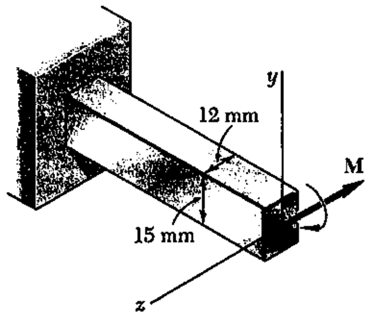
$$I = \frac{1}{12}bh^3 = \frac{1}{12}(18)(100)^3 = 1.5 \times 10^6 \text{ mm}^4 = 1.5 \times 10^{-6} \text{ m}^4$$

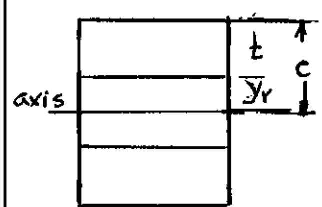
$$c = \frac{1}{2}d = 50 \text{ mm} = 0.05 \text{ m}$$

(a) $$\sigma = \frac{KMc}{I} = \frac{(2.21)(2 \times 10^3)(0.05)}{1.5 \times 10^{-6}} = 147 \times 10^6 \text{ Pa} = 147 \text{ MPa}$$ ◄

(b) $$\sigma = \frac{KMc}{I} = \frac{(1.79)(2 \times 10^3)(0.05)}{1.5 \times 10^{-6}} = 119 \times 10^6 \text{ Pa} = 119 \text{ MPa}$$ ◄

PROBLEM 4.75





4.75 A bar of rectangular cross section, made of a steel assumed to be elastoplastic with $\sigma_Y = 320$ MPa. is subjected to a couple **M** parallel to the z axis. Determine the moment M of the couple for which (a) yield first occurs, (b) the plastic zones at the top and bottom of the bar are 5 mm thick.

SOLUTION

(a) $I = \frac{1}{12}bh^3 = \frac{1}{12}(12)(15)^3 = 3.375\times10^3\ \text{mm}^4 = 3.375\times10^{-9}\ \text{m}^4$

$c = \frac{1}{2}h = 7.5\ \text{mm} = 0.0075\ \text{m}$

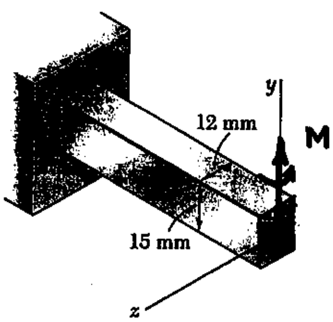
$M_Y = \frac{\sigma_Y I}{c} = \frac{(320\times10^6)(3.375\times10^{-9})}{0.0075} = 144\ \text{N}\cdot\text{m}$ ◄

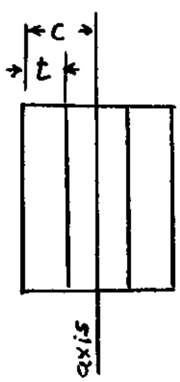
(b) $t = 5\ \text{mm}$ $\quad y_Y = c - t = 7.5 - 5\ \text{mm} = 2.5\ \text{mm} = 0.0025\ \text{m}$

$M = \frac{3}{2}M_Y\left[1 - \frac{1}{3}\left(\frac{y_Y}{c}\right)^2\right]$

$= \frac{3}{2}(144)\left[1 - \frac{1}{3}\left(\frac{2.5}{7.5}\right)^2\right] = 208\ \text{N}\cdot\text{m}$ ◄

PROBLEM 4.76





4.75 A bar of rectangular cross section, made of a steel assumed to be elastoplastic with $\sigma_Y = 320$ MPa. is subjected to a couple **M** parallel to the z axis. Determine the moment M of the couple for which (a) yield first occurs, (b) the plastic zones at the top and bottom of the bar are 5 mm thick.

4.76 Solve Prob. 4.75, assuming that the couple **M** is parallel to the y axis.

SOLUTION

(a) $I = \frac{1}{12}bh^3 = \frac{1}{12}(15)(12)^3 = 2.16\times10^3\ \text{mm}^4 = 2.16\times10^{-9}\ \text{m}^4$

$c = \frac{1}{2}h = 6\ \text{mm} = 0.006\ \text{m}$

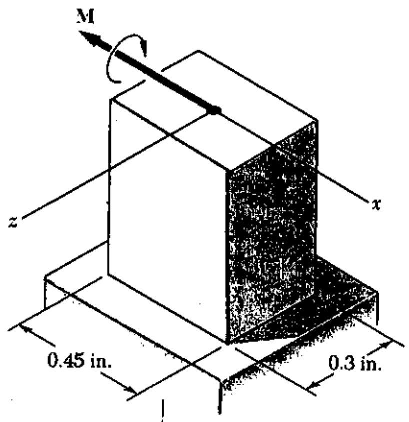
$M_Y = \frac{\sigma_Y I}{c} = \frac{(320\times10^6)(2.16\times10^{-9})}{0.006} = 115.2\ \text{N}\cdot\text{m}$ ◄

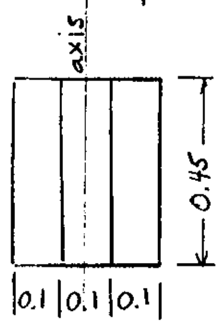
(b) $t = 5\ \text{mm}$ $\quad z_Y = c - t = 6 - 5 = 1\ \text{mm}$

$M = \frac{3}{2}M_Y\left[1 - \frac{1}{3}\left(\frac{z_Y}{c}\right)^2\right]$

$\frac{3}{2}(115.2)\left[1 - \frac{1}{3}\left(\frac{1}{6}\right)^2\right] = 171.2\ \text{N}\cdot\text{m}$ ◄

PROBLEM 4.77





4.77 The prismatic bar shown, made of a steel assumed to be elastoplastic with $\sigma_Y = 42$ ksi, is subjected to a couple **M** parallel to the *x* axis. Determine the moment *M* of the couple for which (*a*) yield first occurs, (*b*) the elastic core of the bar is 0.1 in. thick.

SOLUTION

(a) $I = \frac{1}{12} b h^3 = \frac{1}{12}(0.45)(0.3)^3 = 1.0125 \times 10^{-3}\ \text{in}^4$

$c = \frac{1}{2} h = 0.15\ \text{in}$

$M_Y = \dfrac{\sigma_Y I}{c} = \dfrac{(42)(1.0125\times10^{-3})}{0.15} = 0.2835\ \text{kip}\cdot\text{in}$

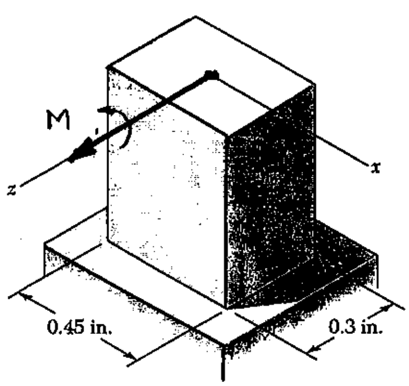
$= 283.5\ \text{lb}\cdot\text{in}$ ◄

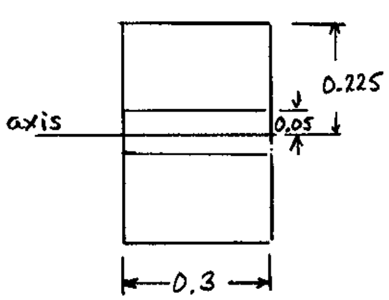
(b) $z_Y = \frac{1}{2} t_E = \frac{1}{2}(0.1) = 0.05\ \text{in}$

$M_p = \frac{3}{2} M_Y \left[1 - \frac{1}{3}\left(\frac{z_Y}{c}\right)^2 \right]$

$= \frac{3}{2}(283.5)\left[1 - \frac{1}{3}\left(\frac{0.05}{0.15}\right)^2\right] = 409.5\ \text{lb}\cdot\text{in}$ ◄

PROBLEM 4.78





4.77 The prismatic bar shown, made of a steel assumed to be elastoplastic with $\sigma_Y = 42$ ksi, is subjected to a couple **M** parallel to the *x* axis. Determine the moment *M* of the couple for which (*a*) yield first occurs, (*b*) the elastic core of the bar is 0.1 in. thick.

4.78 Solve Prob. 4.77, assuming that the couple **M** is parallel to the *z* axis.

SOLUTION

(a) $I = \frac{1}{12} b h^3 = \frac{1}{12}(0.3)(0.45)^3 = 2.2781 \times 10^{-3}\ \text{in}^4$

$c = \frac{1}{2} h = 0.225\ \text{in}$

$M_Y = \dfrac{\sigma_Y I}{c} = \dfrac{(42)(2.2781\times10^{-3})}{0.225} = 0.425\ \text{kip}\cdot\text{in}$

$425\ \text{lb}\cdot\text{in}$ ◄

(b) $x_Y = \frac{1}{2} t_E = \frac{1}{2}(0.1) = 0.05\ \text{in.}$

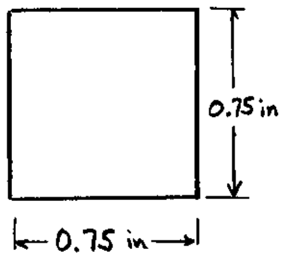
$M_p = \frac{3}{2} M_Y \left[1 - \frac{1}{3}\left(\frac{x_Y}{c}\right)^2 \right]$

$= \frac{3}{2}(425)\left[1 - \frac{1}{3}\left(\frac{0.05}{0.225}\right)^2\right] = 627\ \text{lb}\cdot\text{in.}$ ◄

PROBLEM 4.79

4.79 A solid square rod of side 0.75 in. is made of a steel that is assumed to be elastoplastic with $E = 29 \times 10^6$ psi and $\sigma_Y = 40$ ksi. Determine the maximum stress and the radius of curvature caused by a 4 kip·in. couple applied and maintained about an axis parallel to a side of the cross section.

SOLUTION



$$I = \frac{1}{12}bh^3 = \frac{1}{12}(0.75)(0.75)^3 = 0.026367 \text{ in}^4$$

$$c = \frac{1}{2}h = 0.375 \text{ in}$$

$$M_Y = \frac{\sigma I}{c} = \frac{(40)(0.026367)}{0.375} = 2.8125 \text{ kip}\cdot\text{in.}$$

$$M = \frac{3}{2}M_Y\left(1 - \frac{1}{3}\frac{y_Y^2}{c^2}\right) \quad \text{or} \quad \frac{y_Y}{c} = \sqrt{3 - 2\frac{M}{M_Y}}$$

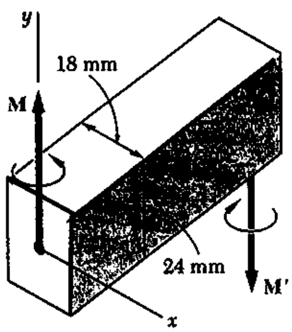
$$\frac{y_Y}{c} = \sqrt{3 - 2\frac{M}{M_Y}} = \sqrt{3 - \frac{(2)(4)}{2.8125}} = 0.39441$$

$$\frac{c}{\rho_Y} = \varepsilon_Y = \frac{\sigma_Y}{E} \quad \therefore \quad \rho_Y = \frac{Ec}{\sigma_Y} = \frac{(29\times10^6)(0.375)}{40\times10^3} = 271.88 \text{ in}$$

$$\frac{\rho}{\rho_Y} = \frac{y_Y}{c} \quad \therefore \quad \rho = \rho_Y\frac{y_Y}{c} = (271.88)(0.39441) = 107.2 \text{ in} = 8.94 \text{ ft}$$

PROBLEM 4.80

4.80 The prismatic rod shown is made of a steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 280$ MPa. Knowing that couples **M** and **M′** of moment 525 N·m are applied and maintained about axes parallel to the y axis, determine (*a*) the thickness of the elastic core, (*b*) the radius of curvature of the bar.



SOLUTION

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(24)(18)^3 = 11.664\times10^3 \text{ mm}^4 = 11.664\times10^{-9} \text{ m}^4$$

$$c = \frac{1}{2}h = 9 \text{ mm} = 0.009 \text{ m}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(280\times10^6)(11.664\times10^{-9})}{0.009} = 362.88 \text{ N}\cdot\text{m}$$

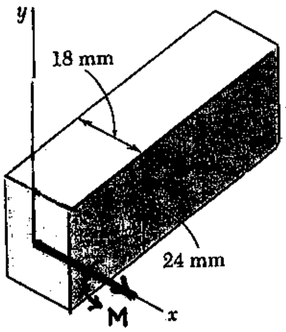
$$M = \frac{3}{2}M_Y\left(1 - \frac{1}{3}\frac{y_Y^2}{c^2}\right) \quad \text{or} \quad \frac{y_Y}{c} = \sqrt{3 - 2\frac{M}{M_Y}}$$

$$\frac{y_Y}{c} = \sqrt{3 - \frac{(2)(525)}{362.88}} = 0.32632, \quad y_Y = 0.32632\,c = 2.9368 \text{ mm}$$

(a) $t_{core} = 2y_Y = 5.87$ mm

(b) $\varepsilon_Y = \frac{y_Y}{\rho} = \frac{\sigma_Y}{E} \quad \therefore \quad \rho = \frac{Ey_Y}{\sigma_Y} = \frac{(200\times10^9)(2.9368\times10^{-3})}{280\times10^6} = 2.09 \text{ m}$

PROBLEM 4.81



4.80 The prismatic rod shown is made of a steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 280$ MPa. Knowing that couples **M** and **M′** of moment 525 N·m are applied and maintained about axes parallel to the y axis, determine (*a*) the thickness of the elastic core, (*b*) the radius of curvature of the bar.

4.81 Solve Prob. 4.80, assuming that the couples **M** and **M′** are applied and maintained about axes parallel to the x axis.

SOLUTION

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(18)(24)^3 = 20.736\times10^3\ \text{mm}^4 = 20.736\times10^{-9}\ \text{m}^4$$

$$c = \frac{1}{2}h = 12\ \text{mm} = 0.012\ \text{m}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(280\times10^6)(20.736\times10^{-9})}{0.012} = 483.84\ \text{N-m}$$

$$M = \frac{3}{2}M_Y\left(1 - \frac{1}{3}\frac{y_Y^2}{c^2}\right) \quad \text{or} \quad \frac{y_Y}{c} = \sqrt{3 - 2\frac{M}{M_Y}}$$

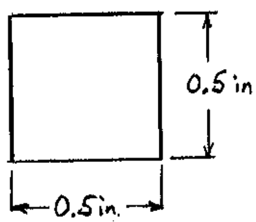
$$\frac{y_Y}{c} = \sqrt{3 - \frac{(2)(525)}{483.84}} = 0.91097, \qquad y_Y = 0.91097\,c = 10.932\ \text{mm}$$

(a) $$t_{core} = 2y_Y = 21.9\ \text{mm}$$

(b) $$\varepsilon_Y = \frac{y_Y}{\rho} = \frac{\sigma_Y}{E} \therefore \quad \rho = \frac{E y_Y}{\sigma_Y} = \frac{(200\times10^9)(10.932\times10^{-3})}{280\times10^6} = 7.81\ \text{m}$$

PROBLEM 4.82

4.82 A solid square rod of side 0.5 in. is made of a steel that is assumed to be elastoplastic with $E = 29 \times 10^6$ psi and $\sigma_Y = 42$ ksi. Knowing that a couple **M** is applied and maintained about an axis parallel to a side of the cross section, determine the moment M of the couple for which the radius of curvature is (*a*) 5 ft, (*b*) 2 ft.



SOLUTION

$$I = \frac{1}{12}bh^3 = \frac{1}{12}(0.5)(0.5)^3 = 5.2083\times10^{-3}\ \text{in}^4$$

$$c = \frac{1}{2}h = 0.25\ \text{in.}$$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(42\times10^3)(5.2083\times10^{-3})}{0.25} = 875\ \text{lb·in.}$$

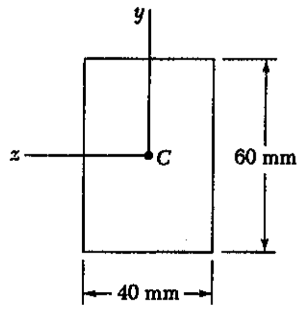
$$\varepsilon_Y = \frac{c}{\rho_Y} = \frac{\sigma_Y}{E} \therefore \quad \rho_Y = \frac{Ec}{\sigma_Y} = \frac{(29\times10^6)(0.25)}{42\times10^3} = 172.62\ \text{in.}$$

$$M = \frac{3}{2}M_Y\left[1 - \frac{1}{3}\left(\frac{\rho}{\rho_Y}\right)^2\right]$$

(a) $\rho = 5\ \text{ft} = 60\ \text{in.}$ $$M = \frac{3}{2}(875)\left[1 - \frac{1}{3}\left(\frac{60}{172.62}\right)^2\right] = 1260\ \text{lb·in}$$

(b) $\rho = 2\ \text{ft.} = 24\ \text{in.}$ $$M = \frac{3}{2}(875)\left[1 - \frac{1}{3}\left(\frac{24}{172.62}\right)^2\right] = 1304\ \text{lb·in.}$$

PROBLEM 4.83



4.83 and *4.84* A bar of the cross section shown is made of a steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 240$ MPa. For bending about the z axis, determine the bending moment at which (*a*) yield first occurs, (*b*) the plastic zones at the top and bottom of the bar are 20 mm thick.

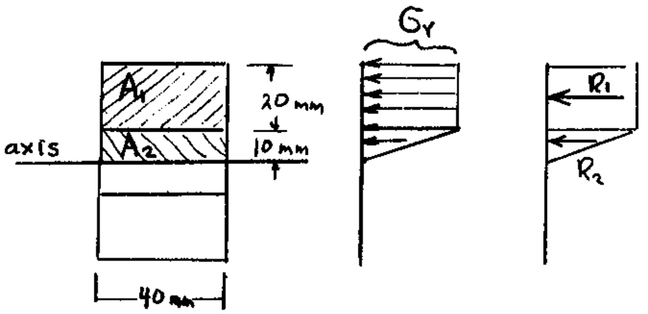
SOLUTION

(a) $I = \frac{1}{12} b h^3 = \frac{1}{12}(40)(60)^3 = 720 \times 10^3 \text{ mm}^4 = 720 \times 10^{-9} \text{ m}^4$

$c = \frac{1}{2} h = 30 \text{ mm} = 0.030 \text{ m}$

$M_Y = \dfrac{\sigma_Y I}{c} = \dfrac{(240 \times 10^6)(720 \times 10^{-9})}{0.030} = 5.76 \times 10^3 \text{ N}\cdot\text{m}$

$= 5.76 \text{ kN}\cdot\text{m}$



$R_1 = \sigma_Y A_1 = (240 \times 10^6)(0.040)(0.020)$

$= 192 \times 10^3 \text{ N}$

$y_1 = 10 \text{ mm} + 10 \text{ mm} = 0.020 \text{ m}$

$R_2 = \frac{1}{2} \sigma_Y A_2 = (\frac{1}{2})(240 \times 10^6)(0.040)(0.010)$

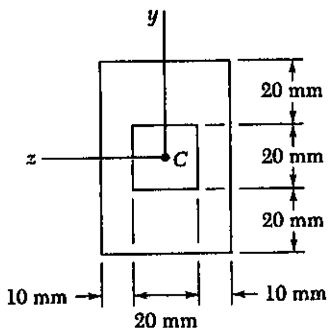
$= 48 \times 10^3 \text{ N}$

$y_2 = \frac{2}{3}(10 \text{ mm}) = 6.667 \text{ mm} = 0.006667 \text{ m}$

(b) $M = 2(R_1 y_1 + R_2 y_2) = 2[(192 \times 10^3)(0.020) + (48 \times 10^3)(0.006667)]$

$= 8.32 \times 10^3 \text{ N}\cdot\text{m} = 8.32 \text{ kN}\cdot\text{m}$

PROBLEM 4.84



4.83 and *4.84* A bar of the cross section shown is made of a steel that is assumed to be elastoplastic with $E = 200$ GPa and $\sigma_Y = 240$ MPa. For bending about the z axis, determine the bending moment at which (*a*) yield first occurs, (*b*) the plastic zones at the top and bottom of the bar are 20 mm thick.

SOLUTION

(a) $I_{rect} = \frac{1}{12}bh^3 = \frac{1}{12}(40)(60)^3 = 720 \times 10^3 \text{ mm}^4$

$I_{cutout} = \frac{1}{12}bh^3 = \frac{1}{12}(20)(20)^3 = 13.33 \times 10^3 \text{ mm}^4$

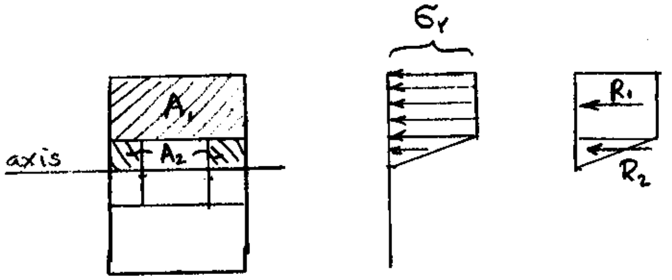
$I = 720 \times 10^3 - 13.33 \times 10^3 = 706.67 \times 10^3 \text{ mm}^4$

$= 706.67 \times 10^{-9} \text{ m}^4$

$c = \frac{1}{2}h = 30 \text{ mm} = 0.030 \text{ m}$

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(240 \times 10^6)(706.67 \times 10^{-9})}{0.030}$$

$= 5.6533 \times 10^3 \text{ N}\cdot\text{m} = 5.65 \text{ kN}\cdot\text{m}$



$R_1 = \sigma_Y A_1 = (240 \times 10^6)(0.040)(0.020) = 192 \times 10^3 \text{ N}$

$y_1 = 10\text{mm} + 10\text{mm} = 20 \text{ mm} = 0.020 \text{ m}$

$R_2 = \frac{1}{2}\sigma_Y A_2 = \frac{1}{2}(240 \times 10^6)(0.020)(0.010) = 24 \times 10^3 \text{ N}$

$y_2 = \frac{2}{3}(10\text{mm}) = 6.667 \text{ mm} = 0.006667 \text{ m}$

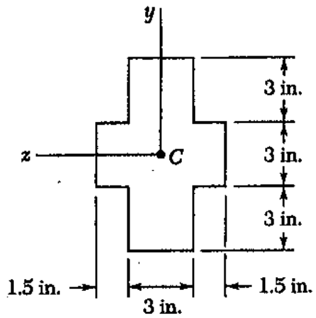
(b) $M = 2(R_1 y_1 + R_2 y_2)$

$= 2[(192 \times 10^3)(0.020) + (24 \times 10^3)(0.006667)]$

$= 8.00 \times 10^3 \text{ N}\cdot\text{m} = 8.00 \text{ kN}\cdot\text{m}$

4.85 and 4.86 A bar of the cross section shown is made of a steel that is assumed to be elastoplastic with $E = 29 \times 10^6$ psi and $\sigma_Y = 42$ ksi. For bending about the z axis, determine the bending moment at which (*a*) yield first occurs, (*b*) the plastic zones at the top and bottom of the bar are 3 in. thick.

PROBLEM 4.85



SOLUTION

(a) $I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(3)(3)^3 + (3)(3)(3)^2 = 87.75 \text{ in}^4$

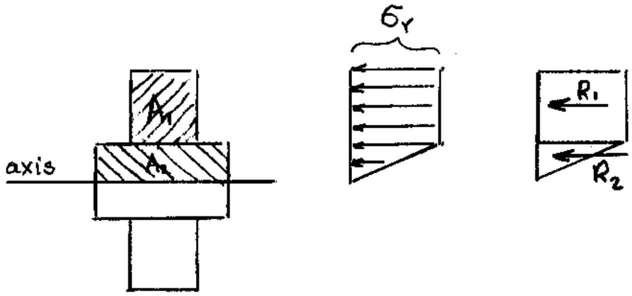
$I_2 = \frac{1}{12} b_2 h_2^3 = \frac{1}{12}(6)(3)^3 = 13.5 \text{ in}^4$

$I_3 = I_1 = 87.75 \text{ in}^4$

$I = I_1 + I_2 + I_3 = 188.5 \text{ in}^4$

$c = 4.5$ in.

$M_y = \frac{\sigma_Y I}{c} = \frac{(42)(188.5)}{4.5} = 1759$ kip·in



$R_1 = \sigma_Y A_1 = (42)(3)(3) = 378$ kip

$y_1 = 1.5 + 1.5 = 3.0$ in

$R_2 = \frac{1}{2}\sigma_Y A_2 = \frac{1}{2}(42)(6)(1.5)$

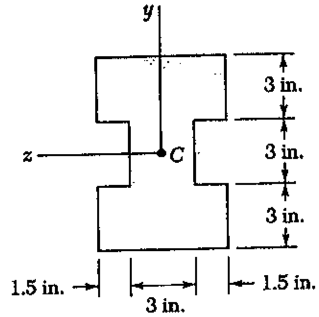
$= 189$ kip

$y_2 = \frac{2}{3}(1.5) = 1.0$ in.

(b) $M = 2(R_1 y_1 + R_2 y_2) = 2[(378)(3.0) + (189)(1.0)] = 2646$ kip·in

PROBLEM 4.86

4.85 and 4.86 A bar of the cross section shown is made of a steel that is assumed to be elastoplastic with $E = 29 \times 10^6$ psi and $\sigma_Y = 42$ ksi. For bending about the z axis, determine the bending moment at which (*a*) yield first occurs, (*b*) the plastic zones at the top and bottom of the bar are 3 in. thick.



SOLUTION

(a) $I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(6)(3)^3 + (6)(3)(3)^2 = 175.5 \text{ in}^4$

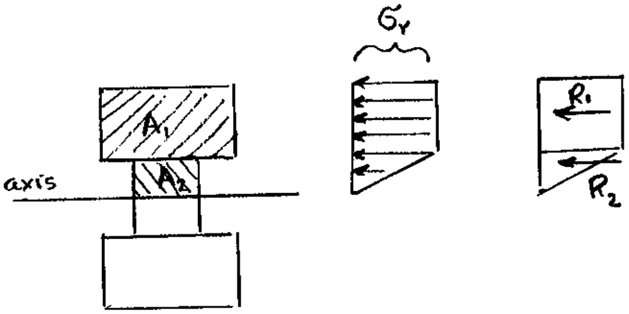
$I_2 = \frac{1}{12} b_2 h_2^3 = \frac{1}{12}(3)(3)^3 = 6.75 \text{ in}^4$

$I_3 = I_1 = 175.5 \text{ in}^4$

$I = I_1 + I_2 + I_3 = 357.75 \text{ in}^4$

$c = 4.5 \text{ in}$

$M_Y = \frac{\sigma_Y I}{c} = \frac{(42)(357.75)}{4.5} = 3339 \text{ kip} \cdot \text{in}$ ◄



$R_1 = \sigma_Y A_1 = (42)(6)(3) = 756 \text{ kip}$

$y_1 = 1.5 + 1.5 = 3 \text{ in}$

$R_2 = \frac{1}{2}\sigma_Y A_2 = \frac{1}{2}(42)(3)(1.5)$

$= 94.5 \text{ kip}$

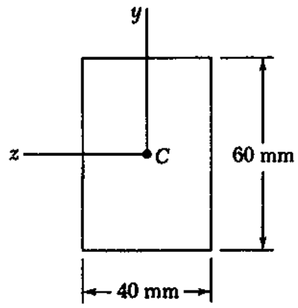
$y_2 = \frac{2}{3}(1.5) = 1.0 \text{ in.}$

(b) $M = 2(R_1 y_1 + R_2 y_2) = 2[(756)(3) + (94.5)(1.0)] = 4725 \text{ kip} \cdot \text{in}$ ◄

PROBLEM 4.87

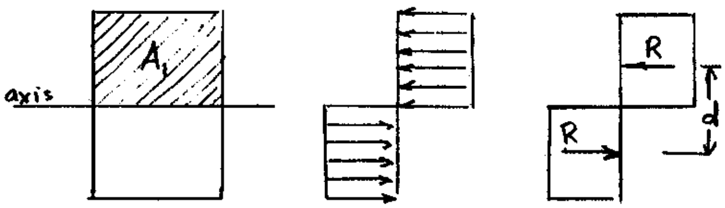
4.87 through 4.90 For the bar indicated, determine (*a*) the fully plastic moment M_p, (*b*) the shape factor of the cross section.

4.87 Bar of Prob. 4.83



SOLUTION

From PROBLEM 4.83 $E = 200$ GPa and $\sigma_Y = 240$ MPa.



$A_1 = (40)(30) = 1200 \text{ mm}^2$

$= 1200 \times 10^{-6} \text{ m}^2$

$R = \sigma_Y A_1$

$= (240 \times 10^6)(1200 \times 10^{-6})$

$= 288 \times 10^3 \text{ N}$

$d = 30 \text{ mm} = 0.030 \text{ m}$

(a) $M_p = R\,d = (288 \times 10^3)(0.030) = 8.64 \times 10^3 \text{ N}\cdot\text{m} = 8.64 \text{ kN}\cdot\text{m}$ ◄

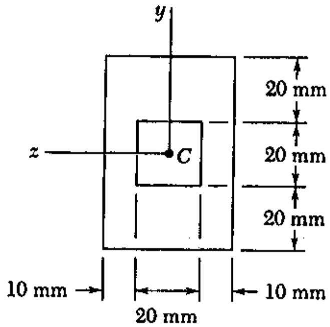
(b) $I = \frac{1}{12} b h^3 = \frac{1}{12}(40)(60)^3 = 720 \times 10^3 \text{ mm}^4 = 720 \times 10^{-9} \text{ m}^4$

$c = 30 \text{ mm} = 0.030 \text{ m}$

$M_Y = \frac{\sigma_Y I}{c} = \frac{(240 \times 10^6)(720 \times 10^{-9})}{0.030} = 5.76 \text{ kN}\cdot\text{m}$

$k = \frac{M_p}{M_Y} = \frac{8.64}{5.76} = 1.5$ ◄

PROBLEM 4.88

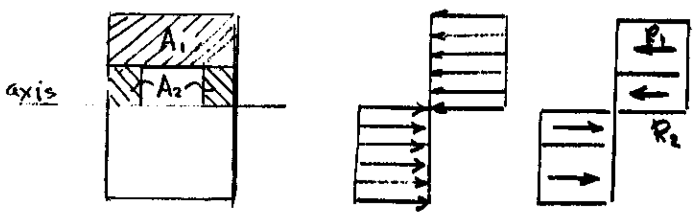


4.87 through 4.90 For the bar indicated, determine (*a*) the fully plastic moment M_p, (*b*) the shape factor of the cross section.

4.88 Bar of Prob. 4.84

SOLUTION

From PROBLEM 4.84 $E = 200$ GPa and $\sigma_Y = 240$ MPa.



$R_1 = \sigma_Y A_1$

$= (240 \times 10^6)(0.040)(0.020)$

$= 192 \times 10^3$ N

$y_1 = 10\text{ mm} + 10\text{ mm} = 20\text{ mm}$
$= 0.020$ m

$R_2 = \sigma_Y A_2$

$= (240 \times 10^6)(0.020)(0.010)$

$= 48 \times 10^3$ N

$y_2 = \frac{1}{2}(10) = 5\text{ mm} = 0.005$ m

$M_p = 2(R_1 y_1 + R_2 y_2) = 2[(192 \times 10^3)(0.020) + (48 \times 10^3)(0.005)]$

$= 8.16 \times 10^3$ N·m $= 8.16$ kN·m ◄

(b) $I_{rect} = \frac{1}{12} b h^3 = \frac{1}{12}(40)(60)^3 = 720 \times 10^3$ mm^4

$I_{cutout} = \frac{1}{12} b h^3 = \frac{1}{12}(20)(20)^3 = 13.33 \times 10^3$ mm^4

$I = I_{rect} - I_{cutout} = 720 \times 10^3 - 13.33 \times 10^3 = 706.67 \times 10^3$ mm^4

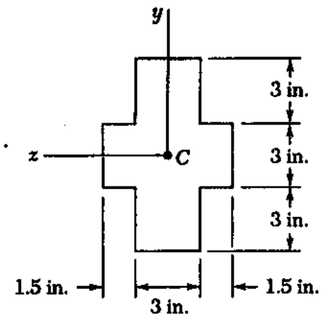
$= 706.67 \times 10^{-9}$ m^4

$c = \frac{1}{2} h = 30$ mm $= 0.030$ m

$M_Y = \dfrac{\sigma_Y I}{c} = \dfrac{(240 \times 10^6)(706.67 \times 10^{-9})}{0.030} = 5.6533$ N·m

$k = \dfrac{M_p}{M_Y} = \dfrac{8.16}{5.6533} = 1.443$ ◄

PROBLEM 4.89

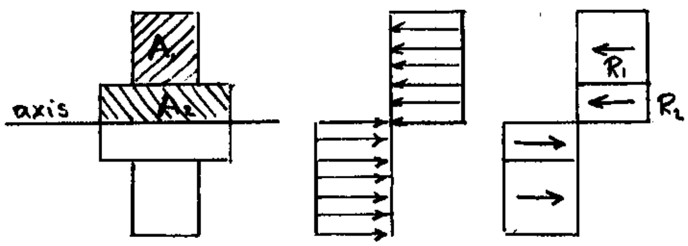


4.87 through 4.90 For the bar indicated, determine (*a*) the fully plastic moment M_p, (*b*) the shape factor of the cross section.

4.89 Bar of Prob. 4.85

SOLUTION

From PROBLEM 4.85 $E = 29 \times 10^6$ psi and $\sigma_Y = 42$ ksi.



$R_1 = \sigma_Y A_1 = (42)(3)(3) = 378$ kip

$y_1 = 1.5 + 1.5 = 3.0$ in

$R_2 = \sigma_Y A_2 = (42)(6)(1.5) = 378$ kip.

$y_2 = \frac{1}{2}(1.5) = 0.75$ in.

$$M_p = 2(R_1 y_1 + R_2 y_2) = 2[(378)(3.0) + (378)(0.75)] = 2835 \text{ kip}\cdot\text{in}$$

(b) $I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(3)(3)^3 + (3)(3)(3)^2 = 87.75$ in^4

$I_2 = \frac{1}{12} b_2 h_2^3 = \frac{1}{12}(6)(3)^3 = 13.5$ in^4

$I_3 = I_1 = 87.75$ in^4

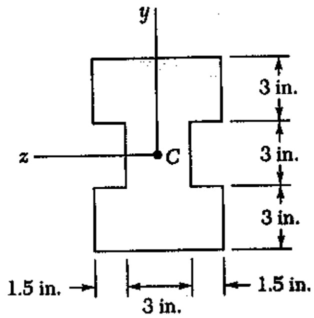
$I = I_1 + I_2 + I_3 = 188.5$ in^4

$c = 4.5$ in.

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(42)(188.5)}{4.5} = 1759.3 \text{ kip}\cdot\text{in.}$$

$$k = \frac{M_p}{M_Y} = \frac{2835}{1759.3} = 1.611$$

PROBLEM 4.90

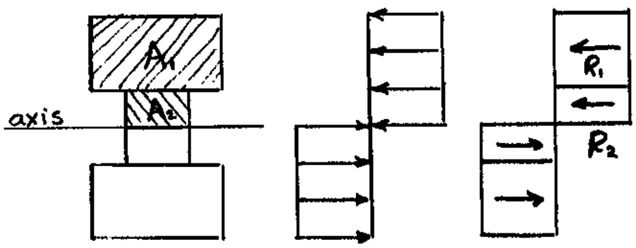


4.87 through 4.90 For the bar indicated, determine (*a*) the fully plastic moment M_p, (*b*) the shape factor of the cross section.

4.90 Bar of Prob. 4.86

SOLUTION

From PROBLEM 4.86 $E = 29 \times 10^6$ and $\sigma_Y = 42$ ksi.



$R_1 = \sigma_Y A_1 = (42)(6)(3) = 756$ kip

$y_1 = 1.5 + 1.5 = 3.0$ in

$R_2 = \sigma_Y A_2 = (42)(3)(1.5) = 189$ kip

$y_2 = \frac{1}{2}(1.5) = 0.75$ in.

$$M_p = 2(R_1 y_1 + R_2 y_2) = 2[(756)(3.0) + (189)(0.75)] = 4819.5 \text{ kip}\cdot\text{in}$$

(b) $I_1 = \frac{1}{12} b_1 h_1^3 + A_1 d_1^2 = \frac{1}{12}(6)(3)^3 + (6)(3)(3)^2 = 175.5$ in^4

$I_2 = \frac{1}{12} b_2 h_2^3 = \frac{1}{12}(3)(3)^3 = 6.75$ in^4

$I_3 = I_1 = 175.5$ in^4

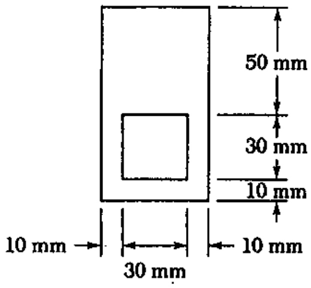
$I = I_1 + I_2 + I_3 = 357.75$ in^4

$c = 4.5$ in

$$M_Y = \frac{\sigma_Y I}{c} = \frac{(42)(357.75)}{4.5} = 3339 \text{ kip}\cdot\text{in}$$

$$k = \frac{M_p}{M_Y} = \frac{4819.5}{3339} = 1.443$$

PROBLEM 4.91



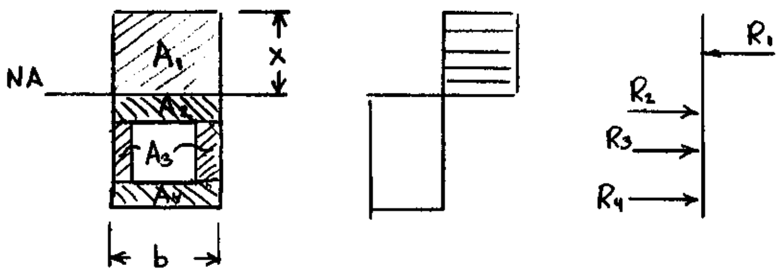
4.91 and 4.92 Determine the plastic moment M_p of a steel beam of the cross section shown, assuming the steel to be elastoplastic with a yield strength of 240 MPa.

SOLUTION

Total area $A = (50)(90) - (30)(30) = 3600 \text{ mm}^2$

$\frac{1}{2}A = 1800 \text{ mm}^2$

$x = \frac{\frac{1}{2}A}{b} = \frac{1800}{50} = 36 \text{ mm}$



$A_1 = (50)(36) = 1800 \text{ mm}^2, \quad \bar{y}_1 = 18 \text{ mm} \qquad A_1\bar{y}_1 = 32.4 \times 10^3 \text{ mm}^3$

$A_2 = (50)(14) = 700 \text{ mm}^2, \quad \bar{y}_2 = 7 \text{ mm} \qquad A_2\bar{y}_2 = 4.9 \times 10^3 \text{ mm}^3$

$A_3 = (20)(30) = 600 \text{ mm}^2, \quad \bar{y}_3 = 29 \text{ mm} \qquad A_3\bar{y}_3 = 17.4 \times 10^3 \text{ mm}^3$

$A_4 = (50)(10) = 500 \text{ mm}^2, \quad \bar{y}_4 = 49 \text{ mm} \qquad A_4\bar{y}_4 = 24.5 \times 10^3 \text{ mm}^3$

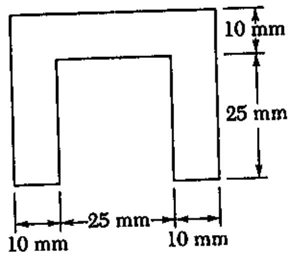
$A_1\bar{y}_1 + A_2\bar{y}_2 + A_3\bar{y}_3 + A_4\bar{y}_4 = 79.2 \times 10^3 \text{ mm}^3 = 79.2 \times 10^{-6} \text{ m}^3$

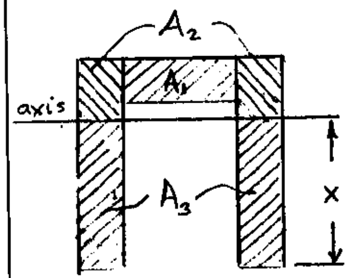
$M_p = \sigma_Y \Sigma A_i\bar{y}_i = (240 \times 10^6)(79.2 \times 10^{-6}) = 19.008 \times 10^3 \text{ N}\cdot\text{m}$

$= 19.01 \text{ kN}\cdot\text{m}$

PROBLEM 4.92

4.91 and 4.92 Determine the plastic moment M_p of a steel beam of the cross section shown, assuming the steel to be elastoplastic with a yield strength of 240 MPa.





SOLUTION

Total area $A = (25)(10) + (2)(10)(35) = 950\ mm^2$

$\frac{1}{2}A = 475\ mm^2$

$x = \dfrac{\frac{1}{2}A}{2b} = \dfrac{475}{20} = 23.75\ mm = 0.02375\ m$

$R_1 = \sigma_Y A_1 = (240\times10^6)(0.025)(0.010) = 60\times10^3\ N$

$\bar{y}_1 = 30 - 23.75 = 6.25\ m = 0.00625\ m$

$R_2 = \sigma_Y A_2 = (240\times10^6)(0.020)(0.01125) = 54\times10^3\ N$

$\bar{y}_2 = \frac{1}{2}(0.01125) = 0.005625\ m$

$R_3 = \sigma_Y A_3 = (240\times10^6)(0.020)(0.02375) = 114\times10^3\ N$

$\bar{y}_3 = \frac{1}{2}x = 0.011875\ m$

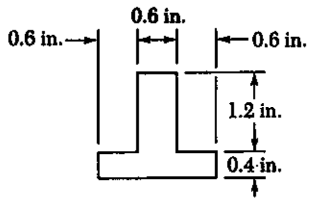
$M_p = R_1\bar{y}_1 + R_2\bar{y}_2 + R_3\bar{y}_3$

$= (60\times10^3)(0.00625) + (54\times10^3)(0.005625) + (114\times10^3)(0.011875)$

$= 2.0325\times10^3\ N\cdot m = 2.03\ kN\cdot m$ ◀

PROBLEM 4.93

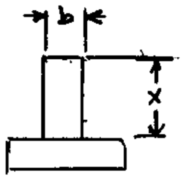
4.93 and 4.94 Determine the plastic moment M_p of a steel beam of the cross section shown, assuming the steel to be elastoplastic with a yield strength of 36 ksi.



SOLUTION

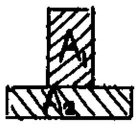
Total area $A = (1.8)(0.4) + (0.6)(1.2) = 1.44 \text{ in}^2$

$\frac{1}{2}A = 0.72 \text{ in}^2$



$$x = \frac{\frac{1}{2}A}{b} = \frac{0.72}{0.6} = 1.2 \text{ in.}$$

Neutral axis lies 1.2 in below the top



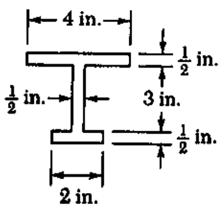
$A_1 = \frac{1}{2}A = 0.72 \text{ in}^2, \quad \bar{y}_1 = \frac{1}{2}(1.2) = 0.6 \text{ in}$

$A_2 = \frac{1}{2}A = 0.72 \text{ in}^2, \quad \bar{y}_2 = \frac{1}{2}(0.4) = 0.2 \text{ in.}$

$$M_p = \sigma_Y(A_1\bar{y}_1 + A_2\bar{y}_2)$$
$$= (36)[(0.72)(0.6) + (0.72)(0.2)]$$
$$= 20.7 \text{ kip}\cdot\text{in}$$

PROBLEM 4.94

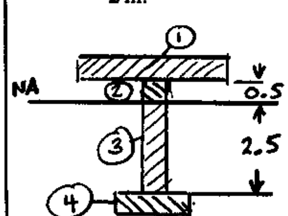
4.93 and 4.94 Determine the plastic moment M_p of a steel beam of the cross section shown, assuming the steel to be elastoplastic with a yield strength of 36 ksi.



SOLUTION

Total area: $A = (4)(\frac{1}{2}) + (\frac{1}{2})(3) + (2)(\frac{1}{2}) = 4.5 \text{ in}^2$

$\frac{1}{2}A = 2.25 \text{ in}^2$



$A_1 = 2.00 \text{ in}^2, \quad \bar{y}_1 = 0.75, \quad A_1\bar{y}_1 = 1.50 \text{ in}^3$

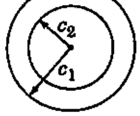
$A_2 = 0.25 \text{ in}^2, \quad \bar{y}_2 = 0.25, \quad A_2\bar{y}_2 = 0.0625 \text{ in}^3$

$A_3 = 1.25 \text{ in}^2, \quad \bar{y}_3 = 1.25, \quad A_3\bar{y}_3 = 1.5625 \text{ in}^3$

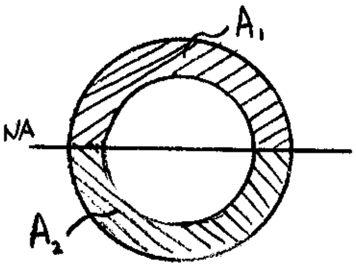
$A_4 = 1.00 \text{ in}^2, \quad \bar{y}_4 = 2.75, \quad A_4\bar{y}_4 = 2.75 \text{ in}^3$

$$M_p = \sigma_Y(A_1\bar{y}_1 + A_2\bar{y}_2 + A_3\bar{y}_3 + A_4\bar{y}_4)$$
$$= (36)(1.50 + 0.0625 + 1.5625 + 2.75) = 211.5 \text{ Kip}\cdot\text{in}$$

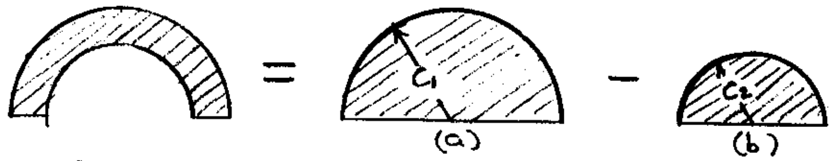
PROBLEM 4.95



4.95 A thick-walled pipe of the cross section shown is made of a steel that is assumed to be elastoplastic with a yield strength σ_Y. Derive an expression for the plastic moment M_p of the pipe in terms of c_1, c_2, and σ_Y.

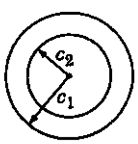


SOLUTION



$$A_1\bar{y}_1 = A_a\bar{y}_a - A_b\bar{y}_b$$
$$= \left(\frac{\pi}{2}c_1^2\right)\left(\frac{4c_1}{3\pi}\right) - \left(\frac{\pi}{2}c_2^2\right)\left(\frac{4c_2}{3\pi}\right)$$
$$= \frac{2}{3}(c_1^3 - c_2)^3$$
$$A_2\bar{y}_2 = A_1\bar{y}_1 = \frac{2}{3}(c_1^3 - c_2^2)$$
$$M_p = \sigma_Y(A_1\bar{y}_1 + A_2\bar{y}_2) = \frac{4}{3}\sigma_Y(c_1^3 - c_2^3)$$

PROBLEM 4.96



4.96 . Determine the plastic moment M_p of a thick-walled pipe of the cross section shown, knowing that $c_1 = 60$mm, $c_2 = 40$ mm, and $\sigma_Y = 240$ MPa.

SOLUTION

See the solution to PROBLEM 4.95 for derivation of the following expression for M_p.

$$M_p = \frac{4}{3}\sigma_Y(c_1^3 - c_2^3)$$

Data: $\sigma_Y = 240 \text{ MPa} = 240 \times 10^6 \text{ Pa}$

$c_1 = 60 \text{ mm} = 0.060 \text{ m}$

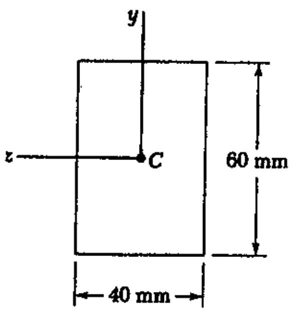
$c_2 = 40 \text{ mm} = 0.040 \text{ m}$

$$M_p = \frac{4}{3}(240 \times 10^6)(0.060^3 - 0.040^3) = 48.64 \times 10^3 \text{ N}\cdot\text{m}$$
$$= 48.6 \text{ kN}\cdot\text{m}$$

PROBLEM 4.97

4.97 and 4.98 For the beam indicated a couple of moment equal to the fully plastic moment M_p is applied and then removed. Using a yield strength of 240 MPa, determine the residual stress at $y = 30$ mm.

4.97 Beam of Prob. 4.83

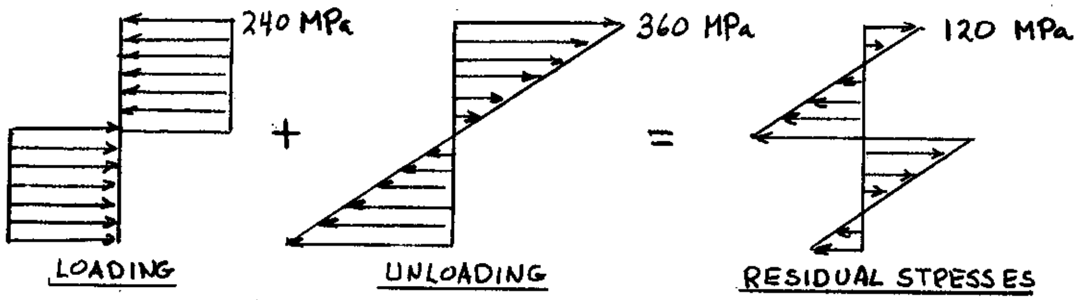


SOLUTION

$M_p = 8.64$ kN·m (See SOLUTION to PROBLEM 4.87)

$I = 720 \times 10^{-9}\ m^4, \quad c = 0.030$ m

$\sigma' = \dfrac{M_{max}\, y}{I} = \dfrac{M_p c}{I}$ at $y = c = 30$ mm.



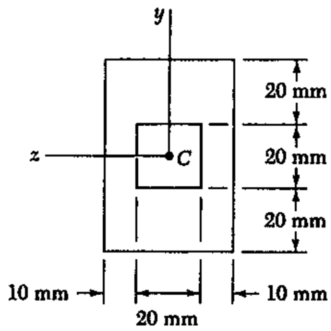
$$\sigma' = \frac{(8.64\times10^3)(0.030)}{720\times10^{-9}} = 360\times10^6 \text{ Pa}$$

$$\sigma_{res} = \sigma' - \sigma_Y = 360\times10^6 - 240\times10^6 = 120\times10^6 \text{ Pa} = 120 \text{ MPa}$$

PROBLEM 4.98

4.97 and 4.98 For the beam indicated a couple of moment equal to the fully plastic moment M_p is applied and then removed. Using a yield strength of 240 MPa, determine the residual stress at $y = 30$ mm.

4.98 Beam of Prob. 4.84



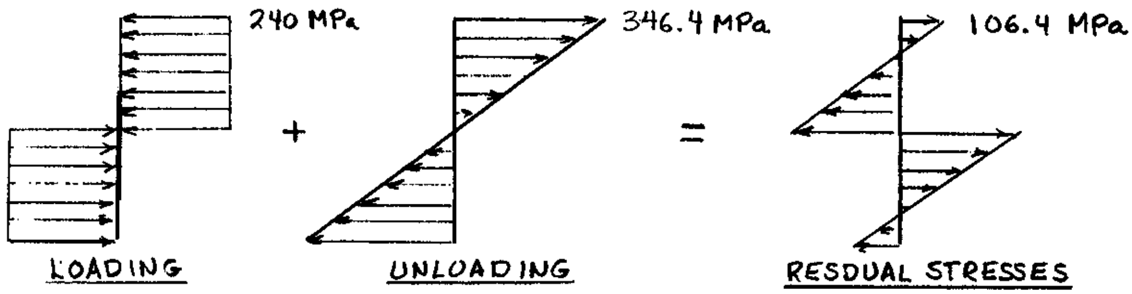
SOLUTION

$M_p = 8.16$ kN·m (See SOLUTION to PROBLEM 4.88)

$I = 706.67 \times 10^{-9}$ m^4, $\quad c = 0.030$ m

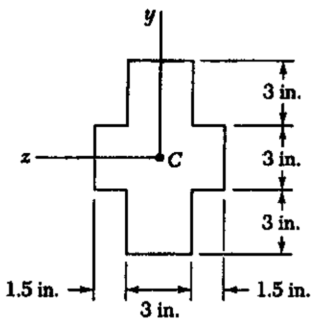
$\sigma' = \frac{M_{max} y}{I} = \frac{M_p c}{I}$ at $y = c$.

$\sigma' = \frac{(8.16 \times 10^3)(0.030)}{706.67 \times 10^{-9}} = 346.4 \times 10^6$ Pa



$\sigma_{res} = \sigma' - \sigma_Y = 346.4 \times 10^6 - 240 \times 10^6 = 106.4 \times 10^6$ Pa $= 106.4$ MPa

PROBLEM 4.99



4.99 and 4.100 For the beam indicated a couple of moment equal to the fully plastic moment M_p is applied and then removed. Using a yield strength of 42 ksi, determine the residual stress at $y = 4.5$ in.

4.99 Beam of Prob. 4.85

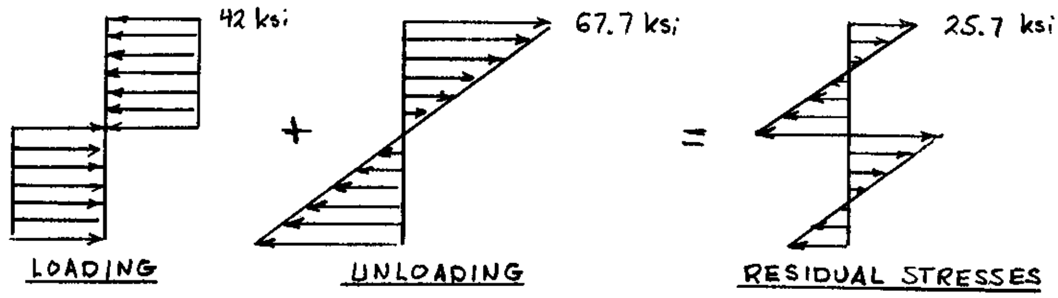
SOLUTION

$M_p = 2835$ kip·in (See SOLUTION to PROBLEM 4.89)

$I = 188.5 \text{ in}^4, \quad c = 4.5 \text{ in}$

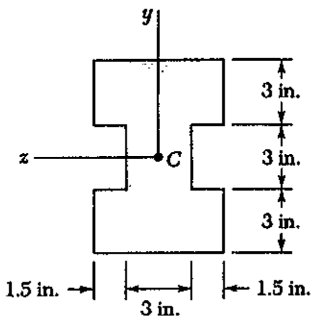
$\sigma' = \dfrac{M_{max}\, y}{I} = \dfrac{M_p c}{I}$ at $y = c$.

$\sigma' = \dfrac{(2835)(4.5)}{188.5} = 67.7 \text{ ksi}$



$\sigma_{res} = \sigma' - \sigma_Y = 67.7 - 42 = 25.7 \text{ ksi}$

PROBLEM 4.100



4.99 and 4.100 For the beam indicated a couple of moment equal to the fully plastic moment M_p is applied and then removed. Using a yield strength of 42 ksi, determine the residual stress at $y = 4.5$ in.

4.100 Beam of Prob. 4.86

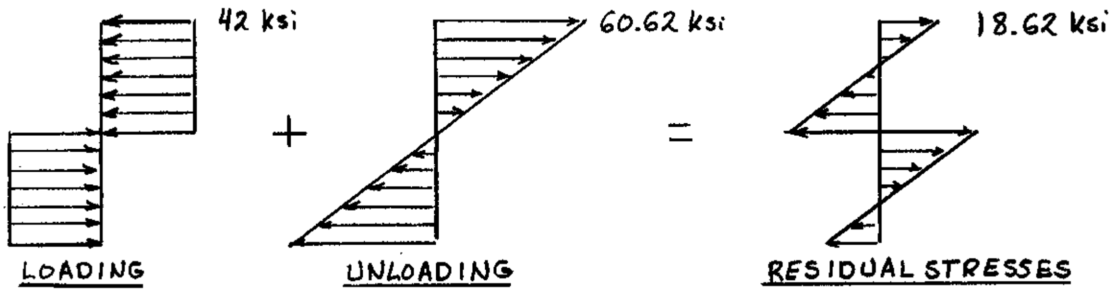
SOLUTION

$M_p = 4819.5$ kip·in (See SOLUTION to PROBLEM 4.90)

$I = 357.75 \text{ in}^4, \qquad c = 4.5 \text{ in.}$

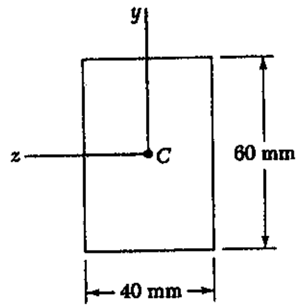
$\sigma' = \dfrac{M_{max} y}{I} = \dfrac{M_p c}{I}$ for $y = c$

$\sigma' = \dfrac{(4819.5)(4.5)}{357.75} = 60.62 \text{ ksi}$



$\sigma_{res} = \sigma' - \sigma_Y = 60.62 - 42 = 18.62 \text{ ksi}$

PROBLEM 4.101



4.101 and 4.102 A bending couple is applied to the bar indicated, causing plastic zones 20-mm thick to develop at the top and bottom of the bar. After the couple has been removed, determine (*a*) the residual stress at $y = 30$ mm, (*b*) the points where the residual stress is zero, (*c*) the radius of curvature corresponding to the permanent deformation of the bar.

4.101 Bar of Prob. 4.83

SOLUTION

See SOLUTION to PROBLEM 4.83 for bending couple and stress distribution during loading.

$M = 8.32$ kN $\qquad y_Y = 10$ mm $= 0.010$ m

$E = 200$ GPa $\qquad \sigma_Y = 240$ MPa

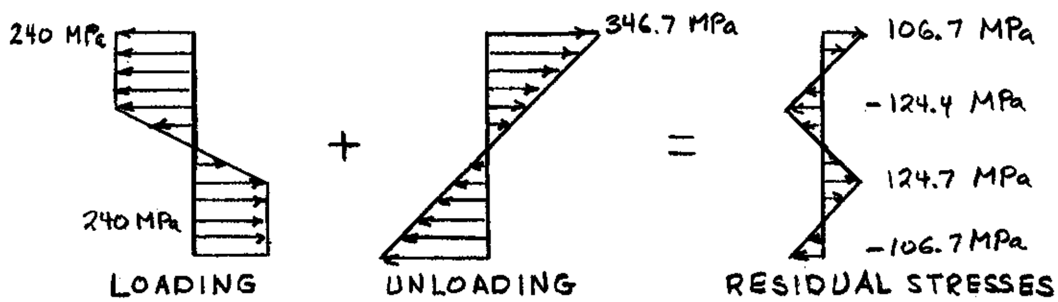
$I = 720 \times 10^{-9}$ m^4 $\qquad c = 0.030$ m

(a) $\sigma' = \dfrac{Mc}{I} = \dfrac{(8.32 \times 10^3)(0.030)}{720 \times 10^{-9}} = 346.7 \times 10^6$ Pa $= 346.7$ MPa

$\sigma'' = \dfrac{My_Y}{I} = \dfrac{(8.32 \times 10^3)(0.010)}{720 \times 10^{-9}} = 115.6 \times 10^6$ Pa $= 115.6$ MPa

At $y = c$ $\quad \sigma_{res} = \sigma' - \sigma_Y = 346.7 - 240 = 106.7$ MPa

At $y = y_Y$ $\quad \sigma_{res} = \sigma'' - \sigma_Y = 115.6 - 240 = -124.4$ MPa



(b) $\sigma_{res} = 0 \quad \therefore \quad \dfrac{My_0}{I} - \sigma_Y = 0$

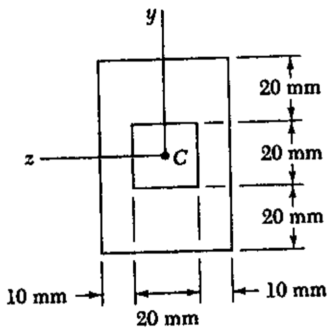
$y_0 = \dfrac{I\sigma_Y}{M} = \dfrac{(720 \times 10^{-9})(240 \times 10^6)}{8.32 \times 10^3} = 20.77 \times 10^{-3}$ m $= 20.77$ mm

ans. $y_0 = -20.77$ mm, 0, 20.77 mm

(c) At $y = y_Y$, $\quad \sigma_{res} = -124.4 \times 10^6$ Pa

$\sigma = -\dfrac{Ey}{\rho} \quad \therefore \quad \rho = -\dfrac{Ey}{\sigma} = \dfrac{(200 \times 10^9)(0.010)}{-124.4 \times 10^6} = 16.08$ m

PROBLEM 4.102



4.101 and 4.102 A bending couple is applied to the bar indicated, causing plastic zones 20-mm thick to develop at the top and bottom of the bar. After the couple has been removed, determine (*a*) the residual stress at $y = 30$ mm, (*b*) the points where the residual stress is zero, (*c*) the radius of curvature corresponding to the permanent deformation of the bar.

4.102 Bar of Prob. 4.84

SOLUTION

See SOLUTION to PROBLEM 4.84 for bending couple and stress distribution during loading.

$M = 8.00\ \text{kN}\cdot\text{m}$ $\qquad y_Y = 10\ \text{mm} = 0.010\ \text{m}$

$E = 200\ \text{GPa}$ $\qquad \sigma_Y = 240\ \text{MPa}$

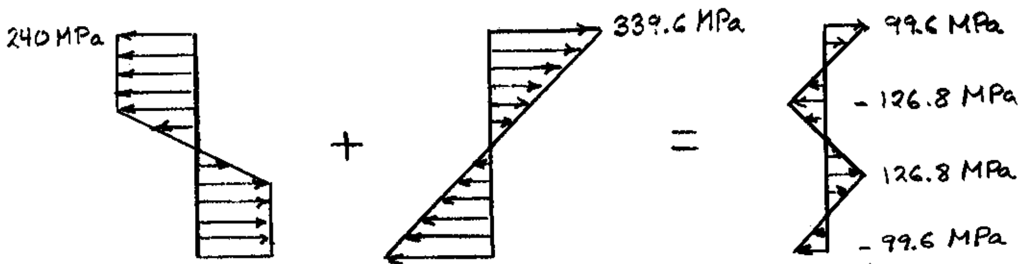
$I = 706.67\times10^{-9}\ \text{m}^4$ $\qquad c = 0.030\ \text{m}$

(a) $$\sigma' = \frac{Mc}{I} = \frac{(8.00\times10^3)(0.030)}{706.67\times10^{-9}} = 339.6\times10^6\ \text{Pa} = 339.6\ \text{MPa}$$

$$\sigma'' = \frac{My_Y}{I} = \frac{(8.00\times10^3)(0.010)}{706.67\times10^{-9}} = 113.2\times10^6\ \text{Pa} = 113.2\ \text{MPa}$$

At $y = c$ $\qquad \sigma_{res} = \sigma' - \sigma_Y = 339.6 - 240 = 99.6\ \text{MPa}$

At $y = y_Y$ $\qquad \sigma_{res} = \sigma'' - \sigma_Y = 113.2 - 240 = -126.8\ \text{MPa}$



(b) $\sigma_{res} = 0 \quad \therefore \quad \dfrac{My_0}{I} - \sigma_Y = 0$

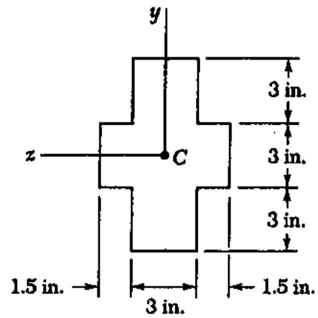
$$y_0 = \frac{I\sigma_Y}{M} = \frac{(706.67\times10^{-9})(240\times10^6)}{8.00\times10^3} = 21.2\times10^{-3}\ \text{m} = 21.2\ \text{mm}$$

ans. $y_0 = -21.2\ \text{mm},\ 0,\ 21.2\ \text{mm}$

(c) At $y = y_Y$ $\qquad \sigma_{res} = -126.8\times10^6\ \text{Pa}$

$$\sigma = -\frac{Ey}{\rho} \quad \therefore \quad \rho = -\frac{Ey}{\sigma} = \frac{(200\times10^9)(0.010)}{126.8\times10^6} = 15.77\ \text{m}$$

PROBLEM 4.103



4.103* and *4.104 A bending couple is applied to the bar indicated, causing plastic zones 3-in. thick to develop at the top and bottom of the bar. After the couple has been removed, determine (*a*) the residual stress at $y = 4.5$ in., (*b*) the points where the residual stress is zero, (*c*) the radius of curvature corresponding to the permanent deformation of the bar.

4.103 Bar of Prob. 4.85

SOLUTION

See SOLUTION to PROBLEM 4.85 for bending couple and stress distribution during loading

$M = 2646$ kip · in $\qquad y_Y = 1.5$ in.

$E = 29 \times 10^6$ psi $= 29 \times 10^3$ ksi $\qquad \sigma_Y = 42$ ksi

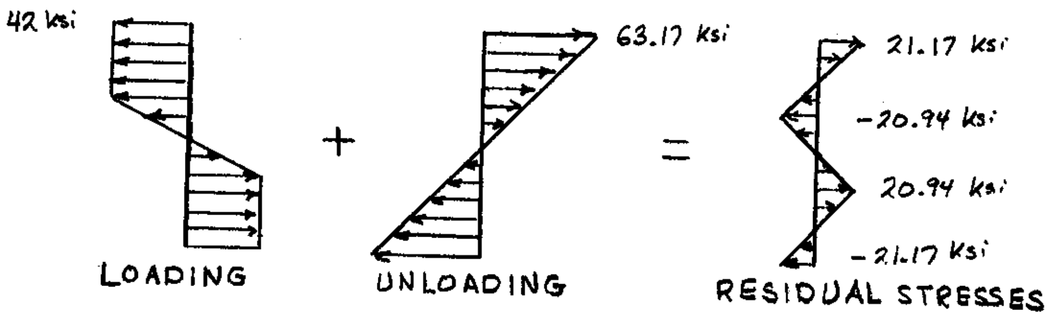
$I = 188.5$ in^4 $\qquad c = 4.5$ in.

(a) $\sigma' = \dfrac{Mc}{I} = \dfrac{(2646)(4.5)}{188.5} = 63.17$ ksi

$\sigma'' = \dfrac{My_Y}{I} = \dfrac{(2646)(1.5)}{188.5} = 21.06$ ksi

At $y = c$ $\quad \sigma_{res} = \sigma' - \sigma_Y = 63.17 - 42 = 21.17$ ksi ◄

At $y = y_Y$ $\quad \sigma_{res} = \sigma'' - \sigma_Y = 21.06 - 42 = -20.94$ ksi



(b) $\sigma_{res} = 0 \quad \therefore \quad \dfrac{My_0}{I} = \sigma_Y$

$y_0 = \dfrac{I\sigma_Y}{M} = \dfrac{(188.5)(42)}{2646} = 2.992$ in

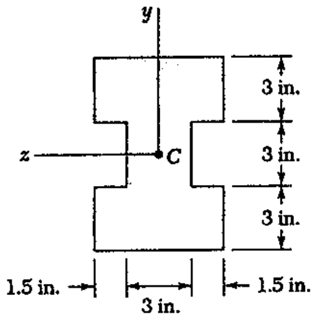
ans. $\quad y_0 = -2.992$ in., 0, 2.992 in ◄

(c) At $y = y_Y$, $\quad \sigma_{res} = -20.94$ ksi

$\sigma = -\dfrac{Ey}{\rho} \quad \therefore \quad \rho = -\dfrac{Ey}{\sigma} = \dfrac{(29\times10^3)(1.5)}{20.94} = 2077$ in

$= 173.1$ ft ◄

PROBLEM 4.104



4.103* and *4.104 A bending couple is applied to the bar indicated, causing plastic zones 3-in. thick to develop at the top and bottom of the bar. After the couple has been removed, determine (*a*) the residual stress at $y = 4.5$ in., (*b*) the points where the residual stress is zero, (*c*) the radius of curvature corresponding to the permanent deformation of the bar.

4.104 Bar of Prob. 4.86

SOLUTION

See SOLUTION to PROBLEM 4.86 for bending couple and stress distribution

$M = 4725$ kip·in $\qquad y_Y = 1.5$ in.

$E = 29 \times 10^6$ psi $= 29 \times 10^3$ ksi $\qquad \sigma_Y = 42$ ksi

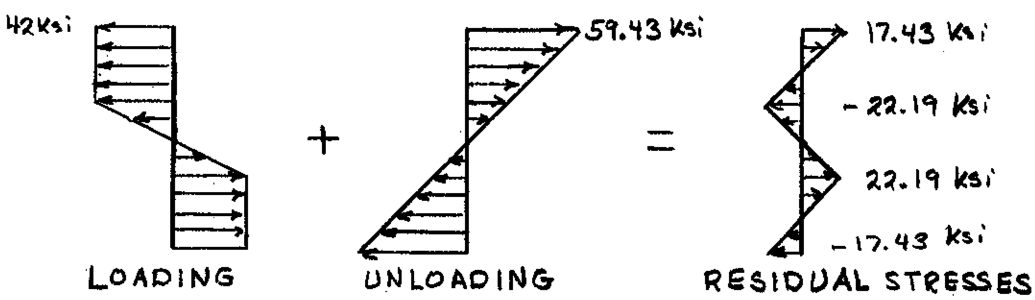
$I = 357.75$ in^4 $\qquad c = 4.5$ in.

(a) $\sigma' = \dfrac{Mc}{I} = \dfrac{(4725)(4.5)}{357.75} = 59.43$ ksi

$\sigma'' = \dfrac{My_Y}{I} = \dfrac{(4725)(1.5)}{357.75} = 19.81$ ksi

At $y = c$ $\qquad \sigma_{res} = \sigma' - \sigma_Y = 59.43 - 42 = 17.43$ ksi ◄

At $y = y_Y$ $\qquad \sigma_{res} = \sigma'' - \sigma_Y = 19.81 - 42 = -22.19$ ksi



(b) $\sigma_{res} = 0 \quad \therefore \quad \dfrac{My_0}{I} - \sigma_Y = 0$

$y_0 = \dfrac{I\sigma_Y}{M} = \dfrac{(357.75)(42)}{4725} = 3.18$ in

ans. $y_0 = -3.18$ in, 0, 3.18 in. ◄

(c) At $y = y_Y$, $\quad \sigma_{res} = -22.19$ ksi

$\sigma = -\dfrac{Ey}{\rho} \quad \therefore \quad \rho = -\dfrac{Ey}{\sigma} = \dfrac{(29 \times 10^3)(1.5)}{22.19} = 1960$ in

$= 163.4$ ft. ◄

PROBLEM 4.105

***4.105** A rectangular bar that is straight and unstressed is bent into an arc of circle of radius ρ by two couples of moment M. After the couples are removed, it is observed that the radius of curvature of the bar is ρ_R. Denoting by ρ_Y the radius of curvature of the bar at the onset of yield, show that the radii of curvature satisfy the following relation

$$\frac{1}{\rho_R} = \frac{1}{\rho}\left\{1 - \frac{3}{2}\frac{\rho}{\rho_Y}\left[1 - \frac{1}{3}\left(\frac{\rho}{\rho_Y}\right)^2\right]\right\}$$

SOLUTION

$$\frac{1}{\rho_Y} = \frac{M_Y}{EI}, \qquad M = \frac{3}{2}M_Y\left(1 - \frac{1}{3}\frac{\rho^2}{\rho_Y^2}\right) \qquad \text{Let } m \text{ denote } \frac{M}{M_Y}$$

$$m = \frac{M}{M_Y} = \frac{3}{2}\left(1 - \frac{\rho^2}{\rho_Y^2}\right) \quad \therefore \quad \frac{\rho^2}{\rho_Y^2} = 3 - 2m$$

$$\frac{1}{\rho_R} = \frac{1}{\rho} - \frac{M}{EI} = \frac{1}{\rho} - \frac{mM_Y}{EI} = \frac{1}{\rho} - \frac{m}{\rho_Y}$$

$$= \frac{1}{\rho}\left\{1 - \frac{\rho}{\rho_Y}m\right\} = \frac{1}{\rho}\left\{1 - \frac{3}{2}\frac{\rho}{\rho_Y}\left(1 - \frac{1}{3}\frac{\rho^2}{\rho_Y^2}\right)\right\}$$

PROBLEM 4.106

4.106 A solid bar of rectangular cross section is made of a material that is assumed to be elastoplastic. Denoting by M_Y and ρ_Y, respectively, the bending moment and radius of curvature at the onset of yield, determine (*a*) the radius of curvature when a couple of moment $M = 1.25M_Y$ is applied to the bar, (*b*) the radius of curvature after the couple is removed. Check the results obtained by using the relatioin derived in Prob. 4.105.

SOLUTION

(a) $$\frac{1}{\rho_Y} = \frac{M_Y}{EI}, \qquad M = \frac{3}{2}M_Y\left(1 - \frac{1}{3}\frac{\rho^2}{\rho_Y^2}\right) \qquad \text{Let } m = \frac{M}{M_Y} = 1.25$$

$$m = \frac{M}{M_Y} = \frac{3}{2}\left(1 - \frac{1}{3}\frac{\rho^2}{\rho_Y^2}\right) \qquad \frac{\rho}{\rho_Y} = \sqrt{3 - 2m} = 0.70711$$

$$\rho = 0.70711\,\rho_Y$$

(b) $$\frac{1}{\rho_R} = \frac{1}{\rho} - \frac{M}{EI} = \frac{1}{\rho} - \frac{mM_Y}{EI} = \frac{1}{\rho} - \frac{m}{\rho_Y} = \frac{1}{0.70711\,\rho_Y} - \frac{1.25}{\rho_Y}$$

$$= \frac{0.16421}{\rho_Y} \qquad \therefore \qquad \rho_R = 6.09\,\rho_Y$$

PROBLEM 4.107

4.106 A solid bar of rectangular cross section is made of a material that is assumed to be elastoplastic. Denoting by M_Y and ρ_Y, respectively, the bending moment and radius of curvature at the onset of yield, determine (*a*) the radius of curvature when a couple of moment $M = 1.25M_Y$ is applied to the bar, (*b*) the radius of curvature after the couple is removed. Check the results obtained by using the relatioin derived in Prob. 4.105.

4.107 Solve Prob. 4,106, assuming that the moment of the couple applied to the bar is $1.40M_Y$.

SOLUTION

(a) $\frac{1}{\rho_Y} = \frac{M_Y}{EI}$, $\quad M = \frac{3}{2}M_Y\left(1 - \frac{1}{3}\frac{\rho^2}{\rho_Y^2}\right) \qquad$ Let $m = \frac{M}{M_Y} = 1.40$

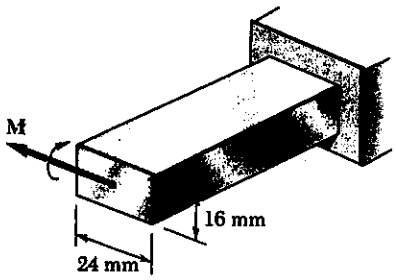
$$m = \frac{M}{M_Y} = \frac{3}{2}\left(1 - \frac{1}{3}\frac{\rho^2}{\rho_Y^2}\right) \qquad \frac{\rho}{\rho_Y} = \sqrt{3 - 2m} = 0.44721$$

$$\rho = 0.44721\,\rho_Y$$

(b) $$\frac{1}{\rho_R} = \frac{1}{\rho} - \frac{M}{EI} = \frac{1}{\rho} - \frac{mM_Y}{EI} = \frac{1}{\rho} - \frac{m}{\rho_Y} = \frac{1}{0.44721\,\rho_Y} - \frac{1.40}{\rho_Y}$$

$$= \frac{0.83607}{\rho_Y} \qquad \therefore \quad \rho_R = 1.196\,\rho_Y$$

PROBLEM 4.108



4.108 The prismatic bar shown is made of a steel that is assumed to be elastoplastic and for which $E = 200$ GPa. Knowing that the radius of curvature of the bar is 2.4 m when a couple of moment $M = 420$ N·m is applied as shown, determine (*a*) the yield strength σ_Y of the steel, (*b*) the thickness of the elastic core of the bar.

SOLUTION

$$M = \frac{3}{2} M_Y \left(1 - \frac{1}{3}\frac{\rho^2}{\rho_Y^2}\right)$$
$$= \frac{3}{2}\frac{\sigma_Y I}{c}\left(1 - \frac{1}{3}\frac{\rho^2 \sigma_Y^2}{E^2 c^2}\right)$$
$$= \frac{3}{2}\frac{\sigma_Y b(2c)^3}{12c}\left(1 - \frac{1}{3}\frac{\rho^2\sigma_Y^2}{E^2c^2}\right)$$
$$= \sigma_Y bc^2\left(1 - \frac{1}{3}\frac{\rho^2\sigma_Y^2}{E^2c^2}\right)$$

(a) $bc^2\sigma_Y\left(1 - \frac{\rho^2\sigma_Y^2}{3E^2c^2}\right) = M$ Cubic equation for σ_Y

Data: $E = 200 \times 10^9$ Pa, $M = 420$ N·m $\rho = 2.4$ m

$b = 24$ mm $= 0.024$ m $c = \frac{1}{2}h = 8$ mm $= 0.008$ m

$$(1.536 \times 10^{-6})\sigma_Y\left[1 - 750 \times 10^{-21}\sigma_Y^2\right] = 420$$

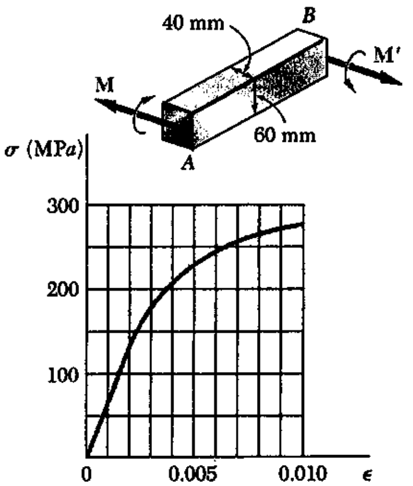
$$\sigma_Y\left[1 - 750 \times 10^{-21}\sigma_Y^2\right] = 273.44 \times 10^6$$

Solving by trial $\sigma_Y = 292 \times 10^6$ Pa $= 292$ MPa ◄

(b) $y_Y = \frac{\sigma_Y \rho}{E} = \frac{(292 \times 10^6)(2.4)}{200 \times 10^9} = 3.504 \times 10^{-3}$ m $= 3.504$ mm

thickness of elastic core $= 2y_Y = 7.01$ mm ◄

PROBLEM 4.109



4.109 The prismatic bar *AB* is made of an aluminum alloy for which the tensile stress-strain diagram is as shown. Assuming that the σ-ϵ diagram is the same in compression as in tension, determine (*a*) the radius of curvature of the bar when the maximum stress is 250 MPa, (*b*) the corresponding value of the bending moment. (*Hint:* For part *b*, plot σ versus y and use an approximate method of integration.)

SOLUTION

(a) $\sigma_m = 250 \text{ MPa} = 250 \times 10^6 \text{ Pa}$

$\varepsilon_m = 0.0064$ from curve

$c = \frac{1}{2}h = 30 \text{ mm} = 0.030 \text{ m}$

$b = 40 \text{ mm} = 0.040 \text{ m}$

$$\frac{1}{\rho} = \frac{\varepsilon_m}{c} = \frac{0.0064}{0.030} = 0.21333 \text{ m}^{-1}$$

$\rho = 4.69 \text{ m}$ ◄

(b) Strain distribution $\varepsilon = -\varepsilon_m \frac{y}{c} = -\varepsilon_m u$ where $u = \frac{y}{c}$

Bending couple

$$M = -\int_{-c}^{c} y\sigma b\,dy = 2b\int_0^c y|\sigma|\,dy = 2bc^2 \int_0^1 u|\sigma|\,du = 2bc^2 J$$

where the integral J is given by $\int_0^1 u|\sigma|\,du$

Evaluate J using a method of numerial integration. If Simpson's rule is used, the integration formula is

$$J = \frac{\Delta u}{3} \sum w u |\sigma|$$

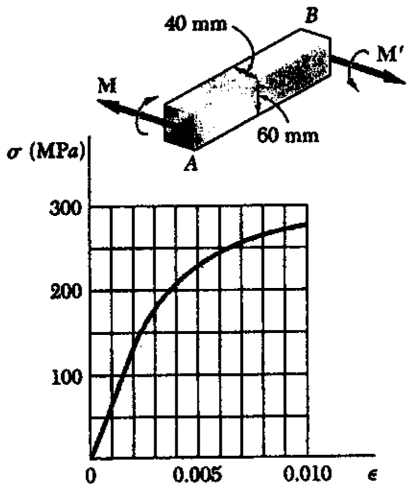
where w is a weighting factor, Using $\Delta u = 0.25$ we get the values given in the table below:

u	$\lvert\varepsilon\rvert$	$\lvert\sigma\rvert$, (MPa)	$u\lvert\sigma\rvert$, (MPa)	w	$wu\lvert\sigma\rvert$, (MPa)
0	0	0	0	1	0
0.25	0.0016	110	27.5	4	110
0.5	0.0032	180	90	2	180
0.75	0.0048	225	168.75	4	675
1.00	0.0064	250	250	1	250
					1215 ← $\sum wu\lvert\sigma\rvert$

$$J = \frac{(0.25)(1215)}{3} = 101.25 \text{ MPa} = 101.25 \times 10^6 \text{ Pa}$$

$M = (2)(0.040)(0.030)^2(101.25 \times 10^6) = 7.29 \times 10^3 \text{ N}\cdot\text{m} = 7.29 \text{ kN}\cdot\text{m}$ ◄

PROBLEM 4.110



4.110 For the bar of Prob. 4.109, determine (*a*) the maximum stress when the radius of curvature of the bar is 3 m, (*b*) the corresponding value of the bending moment. (See hint given in Prob. 4.109.)

SOLUTION

(a) $\rho = 3 \text{ m}$, $c = 0.030 \text{ mm} = 0.030 \text{ m}$

$b = 40 \text{ mm} = 0.040 \text{ m}$

$\varepsilon_m = \frac{c}{\rho} = \frac{0.030}{3} = 0.010$

From curve $\sigma_m = 275 \text{ MPa}$ ◄

(b) Strain distribution $\varepsilon = -\varepsilon_m \frac{y}{c} = -\varepsilon_m u$ where $u = \frac{y}{c}$

Bending couple

$$M = -\int_{-c}^{c} y\sigma b\,dy = 2b\int_0^c y|\sigma|\,dy = 2bc^2\int_0^1 u|\sigma|\,du = 2bc^2 J$$

where the integral J is given by $\int_0^1 u|\sigma|\,du$

Evaluate J using a method of numerial integration. If Simpson's rule is used, the integration formula is

$$J = \frac{\Delta u}{3}\sum w u|\sigma|$$

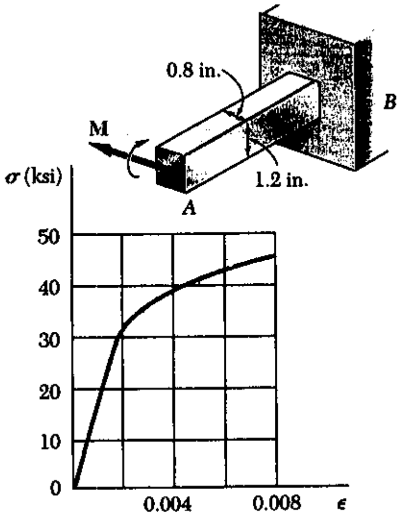
where w is a weighting factor. Using $\Delta u = 0.25$ we get the values given in the table below:

u	$\|\varepsilon\|$	$\|\sigma\|$ (MPa)	$u\|\sigma\|$ (MPa)	w	$wu\|\sigma\|$ (MPa)
0	0	0	0	1	0
0.25	0.0025	160	40	4	160
0.5	0.0050	254	127	2	254
0.75	0.0075	266	199.5	4	798
1.00	0.0100	275	275	1	275
					1487 ← $\Sigma wu\|\sigma\|$

$$J = \frac{(0.25)(1487)}{3} = 123.9 \text{ MPa} = 123.9\times10^6 \text{ Pa}$$

$$M = (2)(0.040)(0.030)^2(123.9\times10^6) = 8.92\times10^3 \text{ N}\cdot\text{m} = 8.92 \text{ kN}\cdot\text{m}$$ ◄

PROBLEM 4.111



4.111 The prismatic bar *AB* is made of a bronze alloy for which the tensile stress-strain diagram is as shown. Assuming that the σ-ϵ diagram is the same in compression as in tension, determine (*a*) the maximum stress in the bar when the radius of curvature of the bar is 100 in., (*b*) the corresponding value of the bending moment. (See hint given in Prob. 4.109.)

SOLUTION

(a) $\rho = 100$ in, $b = 0.8$ in, $c = 0.6$ in.

$$\varepsilon_m = \frac{c}{\rho} = \frac{0.6}{100} = 0.006$$

From the curve $\sigma_m = 43$ ksi

(b) Strain distribution $\varepsilon = -\varepsilon_m \frac{y}{c} = -\varepsilon_m u$ where $u = \frac{y}{c}$

Bending couple

$$M = -\int_{-c}^{c} y\sigma b\,dy = 2b\int_0^c y|\sigma|\,dy = 2bc^2\int_0^1 u|\sigma|\,du = 2bc^2 J$$

where the integral J is given by $\int_0^1 u|\sigma|\,du$

Evaluate J using a method of numerical integration. If Simpson's rule is used, the integration formula is

$$J = \frac{\Delta u}{3}\sum w u|\sigma|$$

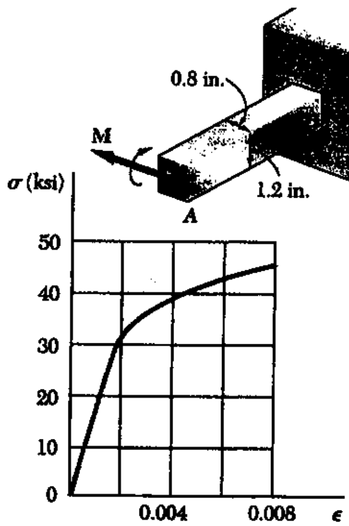
where w is a weighting factor. Using $\Delta u = 0.25$ we get the values given in the table below:

u	$\lvert\varepsilon\rvert$	$\lvert\sigma\rvert$, ksi	$u\lvert\sigma\rvert$, ksi	w	$wu\lvert\sigma\rvert$, ksi
0	0	0	0	1	0
0.25	0.0015	25	6.25	4	25
0.5	0.003	36	18	2	36
0.75	0.0045	40	30	4	120
1.00	0.006	43	43	1	43
					224 ← $\Sigma wu\lvert\sigma\rvert$

$$J = \frac{(0.25)(224)}{3} = 18.67 \text{ ksi}$$

$$M = (2)(0.8)(0.6)^2(18.67) = 10.75 \text{ kip}\cdot\text{in.}$$

PROBLEM 4.112



4.112 For the bar of Prob. 4.111, determine (*a*) the radius of curvature of the bar when the maximum stress is 45 ksi, (*b*) the corresponding value of the bending moment. (See hint given in Prob. 4.109.)

SOLUTION

(a) $b = 0.8$ in $\quad c = 0.6$ in

$\sigma_m = 45$ ksi

From the curve $\quad \varepsilon_m = 0.008$

$$\frac{1}{\rho} = \frac{\varepsilon_m}{c} = \frac{0.008}{0.6} = 0.013333 \text{ in}^{-1}$$

$\rho = 75$ in.

(b) Strain distribution $\quad \varepsilon = -\varepsilon_m \frac{y}{c} = -\varepsilon_m u \quad$ where $u = \frac{y}{c}$

Bending couple

$$M = -\int_{-c}^{c} y\sigma b\,dy = 2b\int_0^c y|\sigma|\,dy = 2bc^2\int_0^1 u|\sigma|\,du = 2bc^2 J$$

where the integral J is given by $\int_0^1 u|\sigma|\,du$

Evaluate J using a method of numerical integration. If Simpson's rule is used, the integration formula is

$$J = \frac{\Delta u}{3}\sum w u|\sigma|$$

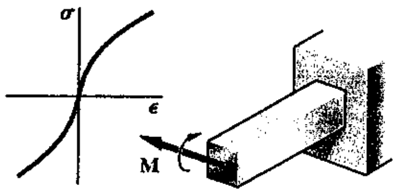
where w is a weighting factor. Using $\Delta u = 0.25$ we get the values given in the table below:

u	$\lvert\varepsilon\rvert$	$\lvert\sigma\rvert$, ksi	$u\lvert\sigma\rvert$, ksi	w	$wu\lvert\sigma\rvert$, ksi
0	0.	0	0	1	0
0.25	0.002	32	8.	4	32
0.5	0.004	38	19	2	38
0.75	0.006	43	32.25	4	129
1.0	0.008	45	45	5	45
					244 $\leftarrow \sum wu\lvert\sigma\rvert$

$$J = \frac{(0.25)(244)}{3} = 20.33 \text{ ksi}$$

$$M = (2)(0.8)(0.6)^2(20.33) = 11.7 \text{ kip}\cdot\text{in}$$

PROBLEM 4.113



4.113 A prismatic bar of rectangular cross section is made of an alloy for which the stress-strain diagram can be represented by the relation, $\epsilon = k\sigma^n$ for $\sigma > 0$, and $\epsilon = -|k\sigma^n|$ for $\sigma < 0$. If a couple **M** is applied to the bar, show that the maximum stress is

$$\sigma_m = \frac{1+2n}{3n}\frac{Mc}{I}$$

SOLUTION

Strain distribution $\quad \varepsilon = -\varepsilon_m \frac{y}{c} = -\varepsilon_m \upsilon \quad$ where $\upsilon = \frac{y}{c}$

Bending couple

$$M = -\int_{-c}^{c} y\sigma\, b\, dy = 2b\int_0^c y|\sigma|\,dy = 2bc^2\int_0^1 \frac{y}{c}|\sigma|\frac{dy}{c}$$

$$= 2bc^2\int_0^1 \upsilon|\sigma|\,d\upsilon$$

For $\quad \varepsilon = K\sigma^n, \qquad \varepsilon_m = K\sigma_m$

$$\frac{\varepsilon}{\varepsilon_m} = \upsilon = \left(\frac{\sigma}{\sigma_m}\right)^n \qquad \therefore\ |\sigma| = \sigma_m \upsilon^{\frac{1}{n}}$$

Then $\quad M = 2bc^2\int_0^1 \upsilon\,\sigma_m \upsilon^{\frac{1}{n}}\,d\upsilon = 2bc^2\sigma_m\int_0^1 \upsilon^{1+\frac{1}{n}}\,d\upsilon$

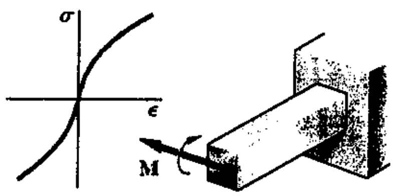
$$= 2bc^2\sigma_m \left.\frac{\upsilon^{2+\frac{1}{n}}}{2+\frac{1}{n}}\right|_0^1 = \frac{2n}{2n+1}\,bc^2\sigma_m$$

$$\sigma_m = \frac{2n+1}{2}\,\frac{M}{bc^2}$$

Recall $\quad \frac{I}{c} = \frac{1}{12}\frac{b(2c)^3}{c} = \frac{2}{3}bc^2 \qquad \therefore\ \frac{1}{bc^2} = \frac{2}{3}\frac{c}{I}$

$$\sigma_m = \frac{2n+1}{3n}\,\frac{Mc}{I}$$

PROBLEM 4.114



4.114 A prismatic bar of rectangular cross section is made of an alloy for which the stress-strain diagram can be represented by the relation $\epsilon = k\sigma^3$. If a couple **M** is applied to the bar, show that the maximum stress is

$$\sigma_m = \frac{7}{9}\frac{Mc}{I}$$

SOLUTION

Strain distribution $\varepsilon = -\varepsilon_m \frac{y}{c} = -\varepsilon_m u$ where $u = \frac{y}{c}$

Bending couple

$$M = -\int_{-c}^{c} y\sigma\, b\,dy = 2b\int_0^c y|\sigma|\,dy = 2bc^2\int_0^1 \frac{y}{c}|\sigma|\frac{dy}{c}$$

$$= 2bc^2\int_0^1 u|\sigma|\,du$$

For $\varepsilon = K\sigma^n$, $\varepsilon_m = K\sigma_m$

$$\frac{\varepsilon}{\varepsilon_m} = u = \left(\frac{\sigma}{\sigma_m}\right)^n \qquad \therefore\ |\sigma| = \sigma_m u^{\frac{1}{n}}$$

Then $M = 2bc^2\int_0^1 u\,\sigma_m u^{\frac{1}{n}}\,du = 2bc^2\sigma_m\int_0^1 u^{1+\frac{1}{n}}\,du$

$$= 2bc^2\sigma_m \left.\frac{u^{2+\frac{1}{n}}}{2+\frac{1}{n}}\right|_0^1 = \frac{2n}{2n+1}bc^2\sigma_m$$

$$\sigma_m = \frac{2n+1}{2}\frac{M}{bc^2}$$

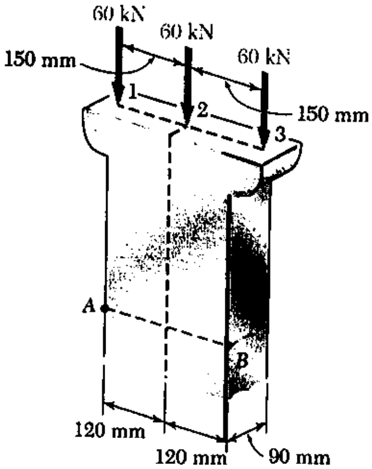
Recall $\frac{I}{c} = \frac{1}{12}\frac{b(2c)^3}{c} = \frac{2}{3}bc^2 \qquad \therefore\ \frac{1}{bc^2} = \frac{2}{3}\frac{c}{I}$

$$\sigma_m = \frac{2n+1}{3n}\frac{Mc}{I}$$

With $n = 3$ $\qquad \sigma_m = \frac{(2)(3)+1}{(3)(3)}\frac{Mc}{I} = \frac{7}{9}\frac{Mc}{I}$

PROBLEM 4.115

4.115 Determine the stress at points A and B ... 60-kN loads are applied at points 1 and 2 only



SOLUTION

(a) Loading is centric.

$$P = 180 \text{ kN} = 180 \times 10^3 \text{ N}$$

$$A = (90)(240) = 21.6 \times 10^3 \text{ mm}^2 = 21.6 \times 10^{-6} \text{ m}^2$$

At A and B $\quad \sigma = -\frac{P}{A} = -\frac{180 \times 10^3}{21.6 \times 10^{-3}} = -8.33 \times 10^6 \text{ Pa}$

$= -8.33 \text{ MPa}$ ◄

(b) Eccentric loading

$$P = 120 \text{ kN} = 120 \times 10^3$$

$$M = (60 \times 10^3)(150 \times 10^{-3}) = 9.0 \times 10^3 \text{ N}\cdot\text{m}$$

$$I = \frac{1}{12} b h^3 = \frac{1}{12}(90)(240)^3 = 103.68 \times 10^6 \text{ mm}^4 = 103.68 \times 10^{-6} \text{ m}^4$$

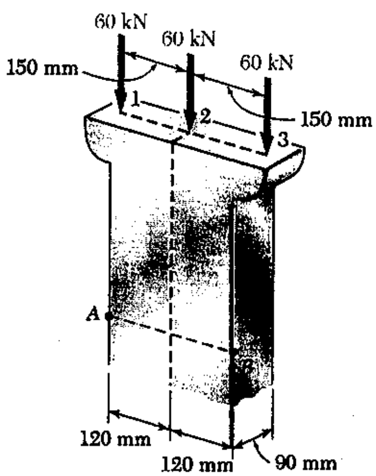
$$c = 120 \text{ mm} = 0.120 \text{ m}$$

At A $\quad \sigma_A = -\frac{P}{A} - \frac{Mc}{I} = -\frac{120 \times 10^3}{21.6 \times 10^{-3}} - \frac{(9.0 \times 10^3)(0.120)}{103.68 \times 10^{-6}} = -15.97 \times 10^6 \text{ Pa} = -15.97 \text{ MPa}$ ◄

At B $\quad \sigma_B - \frac{P}{A} + \frac{Mc}{I} = -\frac{120 \times 10^3}{21.6 \times 10^{-3}} + \frac{(9.0 \times 10^3)(0.120)}{103.68 \times 10^{-6}} = 4.86 \times 10^6 \text{ Pa} = 4.86 \text{ MPa}$ ◄

PROBLEM 4.116

4.116 Determine the stress at points A and B, (*a*) for the loading shown, (*b*) if the 60-kN loads are applied at points 2 and 3 are removed.



SOLUTION

(a) Loading is centric.

$$P = 180 \text{ kN} = 180 \times 10^3 \text{ N}$$

$$A = (90)(240) = 21.6 \times 10^3 \text{ mm}^2 = 21.6 \times 10^{-3} \text{ m}^2$$

At A and B $\quad \sigma = -\frac{P}{A} = -\frac{180 \times 10^3}{21.6 \times 10^{-3}} = -8.33 \times 10^6 \text{ Pa}$

$= -8.33 \text{ MPa}$ ◄

(b) Eccentic loading

$$P = 160 \text{ kN} = 60 \times 10^3 \text{ N}$$

$$M = (60 \times 10^3)(150 \times 10^{-3}) = 9.0 \times 10^3 \text{ N}\cdot\text{m}$$

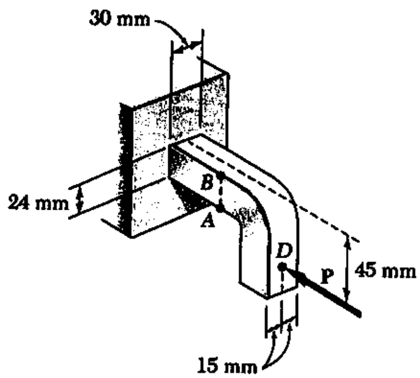
$$I = \frac{1}{12} b h^3 = \frac{1}{12}(90)(240)^3 = 103.68 \times 10^6 \text{ mm}^4 = 103.68 \times 10^{-6} \text{ m}^4$$

$$c = 120 \text{ mm} = 0.120 \text{ m}$$

At A $\quad \sigma_A = -\frac{P}{A} - \frac{Mc}{I} = -\frac{60 \times 10^3}{21.6 \times 10^{-3}} - \frac{(9.0 \times 10^3)(0.120)}{103.68 \times 10^{-6}} = -13.19 \times 10^6 \text{ Pa} = -13.19 \text{ MPa}$ ◄

At B $\quad \sigma_B = -\frac{P}{A} + \frac{Mc}{I} = -\frac{60 \times 10^3}{21.6 \times 10^{-3}} + \frac{(9.0 \times 10^3)(0.120)}{103.68 \times 10^{-6}} = 7.64 \times 10^6 \text{ Pa} = 7.64 \text{ MPa}$ ◄

PROBLEM 4.117



4.117 Knowing that the magnitude of the horizontal force **P** is 8-kN, determine the stress at (*a*) point *A*, (*b*) point *B*.

SOLUTION

$A = (30)(24) = 720 \text{ mm}^2 = 720 \times 10^{-6} \text{ m}^2$

$e = 45 - 12 = 33 \text{ mm} = 0.033 \text{ m}$

$I = \frac{1}{12} bh^3 = \frac{1}{12}(30)(24)^3 = 34.56 \times 10^3 \text{mm}^4 = 34.56 \times 10^{-9} \text{m}^4$

$c = 24 \text{ mm} = 0.12 \text{ m} \qquad P = 8 \times 10^3 \text{ N}$

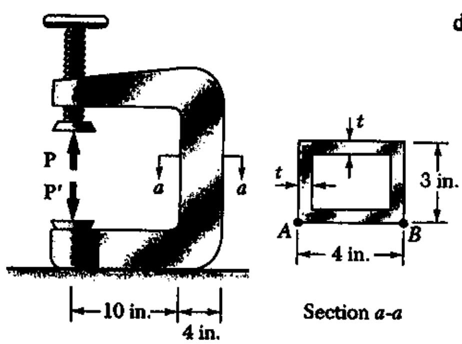
$M = Pe = (8 \times 10^3)(0.033) = 264 \text{ N}\cdot\text{m}$

At A $\quad \sigma_A = -\frac{P}{A} - \frac{Mc}{I} = -\frac{8 \times 10^3}{720 \times 10^{-6}} - \frac{(264)(0.12)}{34.56 \times 10^{-9}}$

$= -102.8 \times 10^6 \text{ Pa} = -102.8 \text{ MPa}$

At B $\quad \sigma_B = -\frac{P}{A} + \frac{Mc}{I} = -\frac{8 \times 10^3}{720 \times 10^{-6}} + \frac{(264)(0.12)}{34.56 \times 10^{-9}} = 80.6 \times 10^6 \text{ Pa} = 80.6 \text{ MPa}$

PROBLEM 4.118



Section *a-a*

4.118 The vertical portion of the press shown consists of a rectangular tube having a wall thickness $t = \frac{1}{2}$ in. Knowing that the press has been tightened until $P = 6$ kips, determine the stress (*a*) at point *A*, (*b*) at point *B*.

SOLUTION

$t = \frac{1}{2} \text{ in.} \qquad P = 6 \text{ kips}$

$A = (3)(4) - (2)(3) = 6 \text{ in}^2$

$I = \frac{1}{12}(3)(4)^3 - \frac{1}{12}(2)(3)^3 = 11.5 \text{ in}^4$

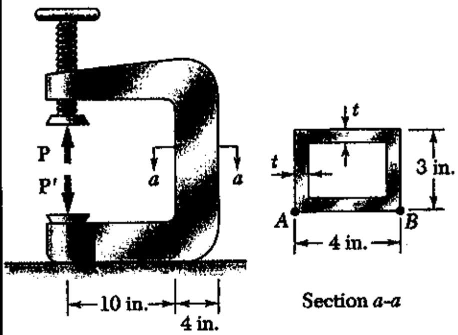
$c = 2 \text{ in}, \qquad e = 10 + 2 = 12 \text{ in.}$

$M = Pe = (6)(12) = 72 \text{ kip}\cdot\text{in.}$

(a) $\sigma_A = \frac{P}{A} + \frac{Mc}{I} = \frac{6}{6} + \frac{(72)(2)}{11.5} = 13.52 \text{ ksi}$

(b) $\sigma_B = \frac{P}{A} - \frac{Mc}{I} = \frac{6}{6} - \frac{(72)(12)}{11.5} = -11.52 \text{ ksi}$

PROBLEM 4.119



4.118 The vertical portion of the press shown consists of a rectangular tube having a wall thickness $t = \frac{1}{2}$ in. Knowing that the press has been tightened until $P = 6$ kips, determine the stress (*a*) at point *A*, (*b*) at point *B*.

4.119 Solve Prob. 4.118, assuming that the wall thickness of the vertical portion of the press is $t = \frac{3}{8}$ in.

SOLUTION

Rectangular cutout is $2\frac{1}{4}$ in. x $3\frac{1}{4}$ in.

$$A = (3)(4) - (2.25)(3.25) = 4.6875 \text{ in}^2$$

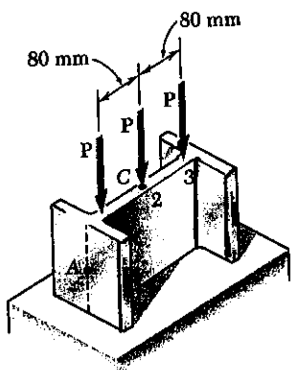
$$I = \tfrac{1}{12}(3)(4)^3 - \tfrac{1}{12}(2.25)(3.25)^3 = 9.5635 \text{ in}^4$$

$c = 2$ in., $\quad e = 10 + 2 = 12$ in., $\quad M = Pe = (6)(12) = 72$ kip·in

(a) $$\sigma_A = \frac{P}{A} + \frac{Mc}{I} = \frac{6}{4.6875} + \frac{(72)(2)}{9.5635} = 16.34 \text{ ksi}$$

(b) $$\sigma_B = \frac{P}{A} - \frac{Mc}{I} = \frac{6}{4.6875} - \frac{(72)(2)}{9.5635} = -13.78 \text{ ksi}$$

PROBLEM 4.120



4.120 As many as three axial loads each of magnitude $P =$ 50 kN can be applied to the end of a ~~W8×21~~ rolled-steel shape. Determine the stress at point *A*, (*a*) for the loading shown, (*b*) if loads are applied at points 1 and 2 only.

W 200 × 31.3

SOLUTION

For W 200 × 31.3 rolled steel shape

$$A = 4000 \text{ mm}^2 = 4.000 \times 10^{-3} \text{ m}^2$$

$$c = \tfrac{1}{2} d = \tfrac{1}{2}(210) = 105 \text{ mm} = 0.105 \text{ m}$$

$$I = 31.4 \times 10^6 \text{ mm}^4 = 31.4 \times 10^{-6} \text{ m}^4$$

(a) Centric load

$$3P = 50 + 50 + 50 = 150 \text{ kN} = 150 \times 10^3 \text{ N}$$

$$\sigma = -\frac{3P}{A} = -\frac{150 \times 10^3}{4.0 \times 10^{-3}} = -37.5 \times 10^6 \text{ Pa} = -37.5 \text{ MPa}$$

(b) Eccentric loading $\qquad e = 80 \text{ mm} = 0.080 \text{ m}$

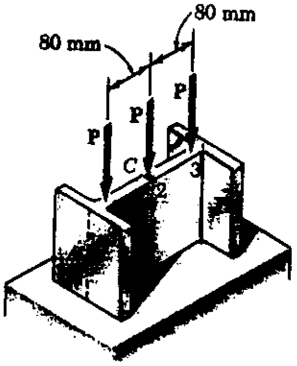
$$2P = 50 + 50 = 100 \text{ kN} = 100 \times 10^3 \text{ N}$$

$$M = Pe = (50 \times 10^3)(0.080) = 4.0 \times 10^3 \text{ N·m}$$

$$\sigma_A = -\frac{2P}{A} - \frac{Mc}{I} = -\frac{100 \times 10^3}{4.0 \times 10^{-3}} - \frac{(4.0 \times 10^3)(0.105)}{31.4 \times 10^{-6}} = -38.4 \times 10^6 \text{ Pa}$$

$$= -38.4 \text{ MPa}$$

PROBLEM 4.121



4.121 As many as three axial loads, each of magnitude $P = 50\ kN$, can be applied to the end of a ~~W8×21~~ rolled-steel shape. Determine the stress at point A, (*a*) for the loading shown, (*b*) if loads are applied at points 2 and 3 only.

SOLUTION

W 200 × 31.3

For a W 200 × 31.3 rolled steel shape

$A = 4000\ mm^2 = 4.00 \times 10^{-3}\ m^2$

$c = \frac{1}{2}d = \frac{1}{2}(210) = 105\ mm = 0.105\ m$

$I = 31.4 \times 10^6\ mm^4 = 31.4 \times 10^{-6}\ m^4$

(a) Centric loading

$3P = 50 + 50 + 50 = 150\ kN = 150 \times 10^3\ N$

$$\sigma = -\frac{3P}{A} = -\frac{150 \times 10^3}{4.0 \times 10^{-3}} = -37.5 \times 10^6\ Pa = -37.5\ MPa$$ ◄

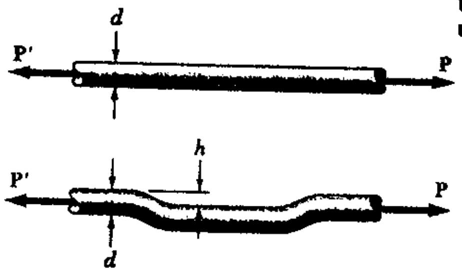
(b) Eccentric loading $\quad e = 80\ mm = 0.080\ m$

$M = Pe = (50 \times 10^3)(0.080) = 4.0 \times 10^3\ N \cdot m$

$$\sigma_A = -\frac{2P}{A} + \frac{Mc}{I} = -\frac{100 \times 10^3}{4.0 \times 10^{-3}} + \frac{(4.0 \times 10^3)(0.105)}{31.4 \times 10^{-6}} = -11.62 \times 10^6\ Pa$$

$= -11.62\ MPa$ ◄

PROBLEM 4.122



4.122 An offset h must be introduced into a solid circular rod of diameter d. Knowing that the maximum stress after the offset is introduced must not exceed four times the stress in the rod when it was straight, determine the largest offset that can be used.

SOLUTION

For centric loading $\quad \sigma_c = \frac{P}{A}$

For eccentric loading $\quad \sigma_e = \frac{P}{A} + \frac{Phc}{I}$

Given $\quad \sigma_e = 4\sigma_c$

$$\frac{P}{A} + \frac{Phc}{I} = 4\frac{P}{A}$$

$$\frac{Phc}{I} = 3\frac{P}{A} \quad \therefore \quad h = \frac{3I}{cA} = \frac{(3)(\frac{\pi}{64}d^4)}{(\frac{d}{2})(\frac{\pi}{4}d^2)} = \frac{3}{8}d = 0.375\ d$$ ◄

PROBLEM 4.123

4.123 An offset h must be introduced into a metal tube of 18-mm outer diameter and 2-mm wall thickness. Knowing that the maximum stress after the offset is introduced must not exceed four times the stress in the rod when it was straight, determine the largest offset that can be used.

SOLUTION

$d_i = d_o - 2t = 18 - (2)(2) = 14 \text{ mm}$ $\qquad c = \frac{1}{2} d_o = 9 \text{ mm}$

$A = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(18^2 - 14^2) = 100.53 \text{ mm}^2$

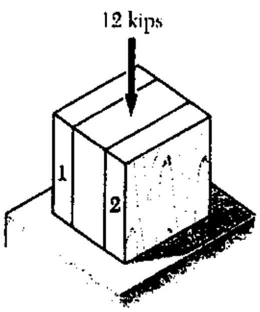
$I = \frac{\pi}{64}(d_o^4 - d_i^4) = \frac{\pi}{64}(18^4 - 14^4) = 3.2673 \times 10^3 \text{ mm}^4$

For centric loading $\sigma_c = \frac{P}{A}$; For eccentric loading $\sigma_e = \frac{P}{A} + \frac{Phc}{I}$

Given $\sigma_e = 4\sigma_c \therefore \frac{P}{A} + \frac{Phc}{I} = 4\frac{P}{A} \quad \therefore \quad \frac{Phc}{I} = 3\frac{P}{A}$

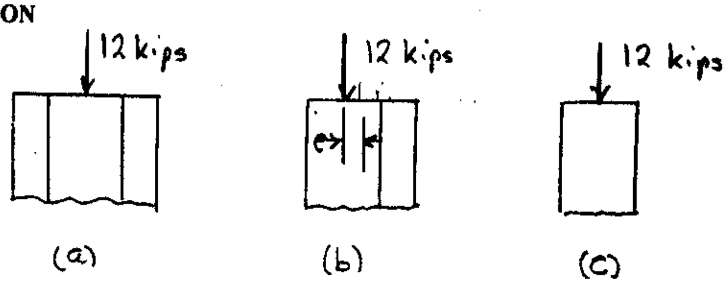
$$h = \frac{3I}{cA} = \frac{(3)(3.2673 \times 10^3)}{(9)(100.53)} = 10.83 \text{ mm}$$ ◄

PROBLEM 4.124



4.124 A short column is made by nailing two 1 × 4-in. planks to a 2 × 4-in. timber. Determine the largest compressive stress created in the column by a 12-kip load applied as shown at the center of the top section of the timber if (*a*) the column is as described, (*b*) plank 1 is removed, (*c*) both planks are removed.

SOLUTION



(a) Centric loading: 4 in × 4 in cross section $\qquad A = (4)(4) = 16 \text{ in}^2$

$$\sigma = -\frac{P}{A} = -\frac{12}{16} = -0.75 \text{ ksi}$$ ◄

(b) Eccentric loading: 4 in × 3 in cross section $\qquad A = (4)(3) = 12 \text{ in}^2$

$c = (\frac{1}{2})(3) = 1.5 \text{ in}$ $\qquad e = 1.5 - 1.0 = 0.5 \text{ in.}$

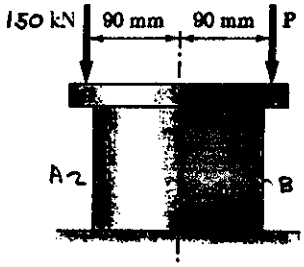
$I = \frac{1}{12} bh^3 = \frac{1}{12}(4)(3)^3 = 9 \text{ in}^4$

$$\sigma = -\frac{P}{A} - \frac{Pec}{I} = -\frac{12}{12} - \frac{(12)(0.5)(1.5)}{9} = -2.00 \text{ ksi}$$ ◄

(c) Centric loading: 4 in. × 2 in cross section $\qquad A = (4)(2) = 8 \text{ in}^2$

$$\sigma = -\frac{P}{A} = -\frac{12}{8} = -1.50 \text{ ksi}$$ ◄

PROBLEM 4.125



4.125 The two forces shown are applied to a rigid plate supported by a steel pipe of 140-mm outer diameter and 120-mm inner diameter. Knowing that the allowable compressive stress is 100 MPa, determine the range of allowable values of *P*.

SOLUTION

$$A = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(140^2 - 120^2) = 4.084\times10^3\ \text{mm}^2 = 4.084\times10^{-3}\ \text{m}^2$$

$$I = \frac{\pi}{64}(d_o^4 - d_i^4) = \frac{\pi}{64}(140^4 - 120^4) = 8.679\times10^6\ \text{mm}^4 = 8.679\times10^{-6}\ \text{m}^4$$

$$c = \tfrac{1}{2}d_o = 70\ \text{mm} = 0.070\ \text{m}$$

$$F = 150\times10^3 + P, \qquad M = (0.090)(150\times10^3) - (0.090)P = 13.5\times10^3 - 0.09P$$

At A $\quad \sigma_A = -\frac{F}{A} - \frac{Mc}{I} = -\frac{(150\times10^3) + P}{4.084\times10^{-3}} - \frac{(13.5\times10^3 - 0.09P)(0.070)}{8.679\times10^{-6}}$

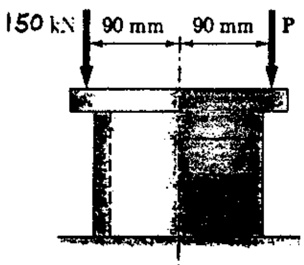
$$= -145.61\times10^6 + 481.03P = -100\times10^6 \quad \therefore \quad P = 94.8\times10^3\ \text{N}$$

At B $\quad \sigma_B = -\frac{F}{A} + \frac{Mc}{I} = -\frac{(150\times10^3) + P}{4.084\times10^{-3}} + \frac{(13.5\times10^3 - 0.09P)(0.070)}{8.679\times10^{-6}}$

$$= 72.155\times10^6 - 970.75P = -100\times10^6 \quad \therefore \quad P = 177.3\times10^3\ \text{N}$$

$$94.8\ \text{kN} < P < 177.3\ \text{kN}$$

PROBLEM 4.126



4.126 The two forces shown are applied to a rigid plate supported by a steel pipe of 140-mm outer diameter and 120-mm inner diameter. Determine the range of allowable values of *P* for which all stresses in the pipe are compressive and less than 100 MPa..

SOLUTION

$$A = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(140^2 - 120^2) = 4.084\times10^3\ \text{mm}^2 = 4.084\times10^{-3}\ \text{m}^2$$

$$I = \frac{\pi}{64}(d_o^4 - d_i^4) = \frac{\pi}{64}(140^4 - 120^4) = 8.679\times10^6\ \text{mm}^4 = 8.679\times10^{-6}\ \text{m}^2$$

$$c = \tfrac{1}{2}d_o = 70\ \text{mm} = 0.070\ \text{m}$$

$$F = 150\times10^3 + P, \qquad M = (0.090)(150\times10^3) - 0.090P = 13.5\times10^3 - 0.09P$$

At A $\quad \sigma_A = -\frac{F}{A} - \frac{Mc}{I} = -\frac{(150\times10^3) + P}{4.084\times10^{-3}} - \frac{(13.5\times10^3 - 0.09P)(0.070)}{8.679\times10^{-6}}$

$$= -145.61\times10^6 + 481.03P = -100\times10^6 \quad \therefore \quad P = 94.8\times10^3\ \text{N}$$

$$\sigma_A = -145.61\times10^6 + 481.03P = 0 \qquad P = 303\times10^3\ \text{N}$$

Based on stress limits at A $\quad 94.8\ \text{kN} \le P \le 303\ \text{kN}$

At B $\quad \sigma_B = -\frac{F}{A} + \frac{Mc}{I} = -\frac{(150\times10^3) + P}{4.084\times10^{-3}} + \frac{(13.5\times10^3 - 0.09P)(0.070)}{8.679\times10^{-6}}$

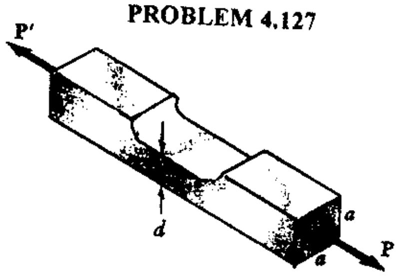
$$= 72.155\times10^6 - 970.75P = -100\times10^6 \qquad P = 177.3\times10^3\ \text{N}$$

$$\sigma_B = 72.155\times10^6 - 970.75P = 0 \qquad P = 74.3\times10^3\ \text{N}$$

Based on stress limits at B $\quad 74.3\ \text{kN} \le P \le 177.3\ \text{kN}$

Based on both limits $\quad 94.8\ \text{kN} \le P \le 177.3\ \text{kN}$

PROBLEM 4.127



4.127 A milling operation was used to remove a portion of a solid bar of square cross section. Knowing that $a = 1.2$ in., $d = 0.8$ in., and $\sigma_{all} = 8$ ksi, determine the largest magnitude P of the forces that can be safely applied at the centers of the ends of the bar.

SOLUTION

$$A = ad, \quad I = \frac{1}{12}ad^3, \quad c = \frac{1}{2}d$$

$$e = \frac{a}{2} - \frac{d}{2}$$

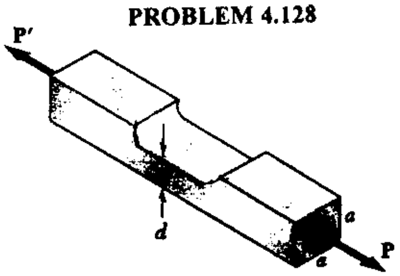
$$\sigma = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{ad} + \frac{6Ped}{ad^3}$$

$$\sigma = \frac{P}{ad} + \frac{3P(a-d)}{ad^2} = KP$$

$$K = \frac{1}{ad} + \frac{3(a-d)}{ad^2} = \frac{1}{(1.2)(0.8)} + \frac{(3)(1.2-0.8)}{(1.2)(0.8)^2} = 2.604 \text{ in}^{-1}$$

$$P = \frac{\sigma}{K} = \frac{8}{2.604} = 3.07 \text{ kips}$$

PROBLEM 4.128



4.128 A milling operation was used to remove a portion of a solid bar of square cross section. Forces of magnitude $P = 4$ kips are applied at the centers of the ends of the bar. Knowing that $a = 1.2$ in. and $\sigma_{all} = 8$ ksi, determine the smallest allowable depth d of the milled portion of the rod.

SOLUTION

$$A = ad, \quad I = \frac{1}{12}ad^3, \quad c = \frac{1}{2}d$$

$$e = \frac{a}{2} - \frac{d}{2}$$

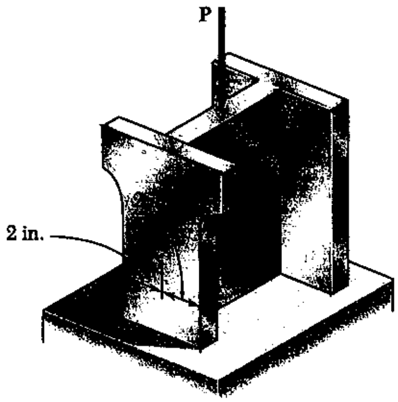
$$\sigma = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{ad} + \frac{Pec}{I} = \frac{P}{ad} + \frac{P\frac{1}{2}(a-d)\frac{1}{2}d}{\frac{1}{12}ad^3} = \frac{P}{ad} + \frac{3P(a-d)}{ad^2}$$

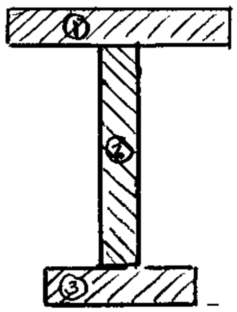
$$\sigma = \frac{3P}{d^2} - \frac{2P}{ad} \quad \text{or} \quad \sigma d^2 + \frac{2P}{a}d - 3P = 0$$

Solving for d $\quad d = \frac{1}{2\sigma}\left\{\sqrt{\left(\frac{2P}{a}\right)^2 + 12P\sigma} - \frac{2P}{a}\right\}$

$$d = \frac{1}{(2)(8)}\left\{\sqrt{\left[\frac{(2)(4)}{1.2}\right]^2 + (12)(4)(8)} - \frac{(2)(4)}{1.2}\right\} = 0.877 \text{ in.}$$

PROBLEM 4.129





4.129 Three steel plates, each of 1 × 6-in. cross section, are welded together to form a short H-shaped column. Later, for architectural reasons, a 1-in. strip is removed from each side of one of the flanges. Knowing that the load remains centric with respect to the original cross section, and that the allowable stress is 15 ksi, determine the largest force **P**, (*a*) which could be applied to the original column, (*b*) which can be applied to the modified column.

SOLUTION

(a) Centric loading

$$A = (3)(1)(6) = 18 \text{ in}^2$$

$$\sigma = -\frac{P}{A} \quad \therefore \quad P = -\sigma A = -(-15)(18) = 270 \text{ kips}$$

(b) Eccentric loading

Reduced cross section

	A, in^2	$\bar{y}_o$, in.	$A\bar{y}_o$, in^3
1	6	3.5	21.0
2	6	0	0
3	4	-3.5	-14.0
Σ	16		7.0

$$\bar{Y}_o = \frac{\Sigma A\bar{y}_o}{\Sigma A} = \frac{7.0}{16} = 0.4375 \text{ in}$$

The centroid lies 0.4375 in from the midpoint of the web.

$$I_1 = \frac{1}{12}(6)(1)^3 + (6)(3.0625)^2 = 56.773 \text{ in}^4$$

$$I_2 = \frac{1}{12}(1)(6)^3 + (6)(0.4375)^2 = 19.148 \text{ in}^4$$

$$I_3 = \frac{1}{12}(4)(1)^3 + (4)(3.9375)^2 = 62.349 \text{ in}^4$$

$$I = I_1 + I_2 + I_3 = 138.27 \text{ in}^4, \qquad c = 4.4375 \text{ in}$$

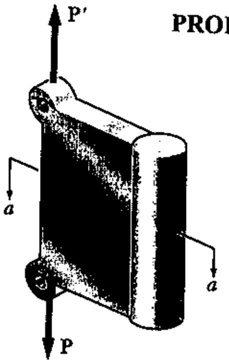
$$M = Pe \quad \text{where} \quad e = 0.4375 \text{ in}$$

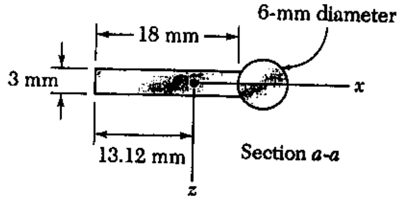
$$\sigma = -\frac{P}{A} - \frac{Mc}{I} = -\frac{P}{A} + \frac{Pec}{I} = -KP$$

$$K = \frac{1}{A} + \frac{ec}{I} = \frac{1}{16} + \frac{(0.4375)(4.4375)}{138.27} = 0.076541 \text{ in}^{-2}$$

$$P = -\frac{\sigma}{K} = -\frac{-15}{0.076541} = 196.0 \text{ kips}$$

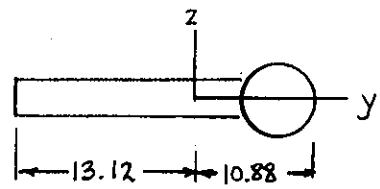
PROBLEM 4.130





4.130 A steel rod is welded to a steel plate to form the machine element shown. Knowing that the allowable stress is 135 MPa, determine (*a*) the largest force **P** that can be applied to the element, (*b*) the corresponding location of the neutral axis. *Given:* Centroid of the cross section is at *C* and $I_z = 4195\ \text{mm}^4$.

SOLUTION



(a) $A = (3)(18) + \frac{\pi}{4}(6)^2 = 82.27\ \text{mm}^2 = 82.27 \times 10^{-6}\ \text{m}^2$

$I = 4195\ \text{mm}^4 = 4195 \times 10^{-12}\ \text{m}^4$

$e = 13.12\ \text{mm} = 0.01312\ \text{m}$

Based on tensile stress at $y = -13.12\ \text{mm} = -0.01312\ \text{m}$

$$\sigma = \frac{P}{A} + \frac{Pec}{I} = \left(\frac{1}{A} + \frac{ec}{I}\right)P = KP$$

$$K = \frac{1}{A} + \frac{ec}{I} = \frac{1}{82.27 \times 10^{-6}} + \frac{(0.01312)(0.01312)}{4195 \times 10^{-12}} = 53.188 \times 10^3\ \text{m}^{-2}$$

$$P = \frac{\sigma}{K} = \frac{135 \times 10^6}{53.188 \times 10^3} = 2.538 \times 10^3\ \text{N} = 2.54\ \text{kN}$$ ◄

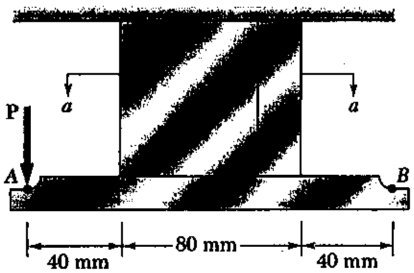
(b) Location neutral axis. $\sigma = 0$

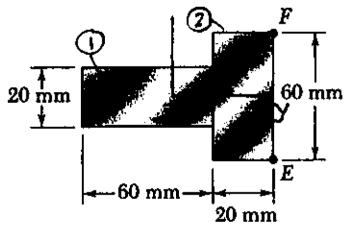
$$\sigma = \frac{P}{A} - \frac{My}{I} = \frac{P}{A} - \frac{Pey}{I} = 0 \qquad \frac{ey}{I} = \frac{1}{A}$$

$$y = \frac{I}{Ae} = \frac{4195 \times 10^{-12}}{(82.27 \times 10^{-6})(0.01312)} = 3.89 \times 10^{-3}\ \text{m} = 3.89\ \text{mm}$$ ◄

The neutral axis lies 3.89 mm to the right of the centroid or 17.01 mm to the right of the line of action of the loads.

PROBLEM 4.131





Section *a-a*

4.131 Knowing that the allowable stress is 150 MPa in section *a-a* of the hanger shown, determine (*a*) the largest vertical force **P** that can be applied at point *A*, (*b*) the corresponding location of the neutral axis of section *a-a*.

SOLUTION

Locate centroid

	A, mm^2	$\bar{y}_o$, mm	$A\bar{y}_o$, mm^3
①	1200	30	36×10^3
②	1200	70	84×10^3
Σ	2400		120×10^3

$$\bar{Y}_o = \frac{\Sigma A\bar{y}_o}{\Sigma A} = \frac{120 \times 10^3}{2400} = 50 \text{ mm}$$

The centroid lies 50 mm to the right of the left edge of the section.

Bending couple $M = Pe$

$e = 40 + 50 = 90 \text{ mm} = 0.090 \text{ m}$

$$I_1 = \frac{1}{12}(20)(60)^3 + (1200)(20)^2 = 840 \times 10^3 \text{ mm}^4$$

$$I_2 = \frac{1}{12}(60)(20)^3 + (1200)(20)^2 = 520 \times 10^3 \text{ mm}^4$$

$$I = I_1 + I_2 = 1.360 \times 10^6 \text{ mm}^4 = 1.360 \times 10^{-6} \text{ m}^4, \qquad A = 2400 \times 10^{-6} \text{ m}^2$$

(a) Based on tensile stress at left edge: $y = -50 \text{ mm} = -0.050 \text{ m}$

$$\sigma = \frac{P}{A} - \frac{Pey}{I} = KP$$

$$K = \frac{1}{A} - \frac{ey}{I} = \frac{1}{2400 \times 10^{-6}} - \frac{(0.090)(-0.050)}{1.360 \times 10^{-6}} = 3.7255 \times 10^3 \text{ m}^{-2}$$

$$P = \frac{\sigma}{K} = \frac{150 \times 10^6}{3.7255 \times 10^3} = 40.3 \times 10^3 \text{ N} = 40.3 \text{ kN}$$ ◄

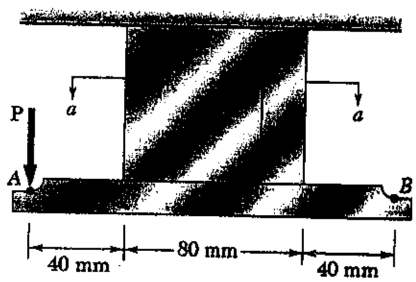
(b) Location of neutral axis: $\sigma = 0$

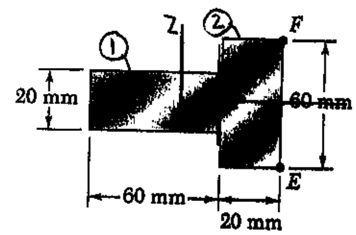
$$\sigma = \frac{P}{A} - \frac{Pey}{I} = 0 \qquad \frac{ey}{I} = \frac{1}{A}$$

$$y = \frac{I}{Ae} = \frac{1.360 \times 10^{-6}}{(2400 \times 10^{-6})(0.090)} = 6.30 \times 10^{-3} \text{ m} = 6.30 \text{ mm}$$ ◄

The neutral axis lies 6.30 mm to the right of the centroid or 56.30 mm from the left edge

PROBLEM 4.132





Section *a-a*

4.131 Knowing that the allowable stress is 150 MPa in section *a-a* of the hanger shown, determine (*a*) the largest vertical force **P** that can be applied at point *A*, (*b*) the corresponding location of the neutral axis of section *a-a*.

4.132 Solve Prob. 4.131, assuming that the vertical force **P** is applied at point *B*.

SOLUTION

Locate centroid

	A, mm^2	$\bar{y}_o$, mm	$A\bar{y}_o$, mm^3
①	1200	30	36×10^3
②	1200	70	84×10^3
Σ	2400		120×10^3

$$\bar{Y}_o = \frac{\Sigma A\bar{y}_o}{\Sigma A} = \frac{120\times10^3}{2400} = 50 \text{ mm}$$

The centroid lies 50 mm to the right of the left edge of the section.

Bending couple $M = Pe$

$e = 50 - 120 = -70 \text{ mm} = -0.070 \text{ m}$

$$I_1 = \tfrac{1}{12}(20)(60)^3 + (1200)(20)^2 = 840\times10^3 \text{ mm}^4$$
$$I_2 = \tfrac{1}{12}(60)(20)^3 + (1200)(20)^2 = 520\times10^3 \text{ mm}^4$$
$$I = I_1 + I_2 = 1.360\times10^6 \text{ mm}^4 = 1.360\times10^{-6} \text{ m}^4, \quad A = 2400\times10^{-6} \text{ m}^2$$

(a) Based on stress at left edge of section: $y = -50 \text{ mm} = -0.050 \text{ m}$

$$\sigma = \frac{P}{A} - \frac{Pey}{I} = K_L P$$
$$K_L = \frac{1}{A} - \frac{ey}{I} = \frac{1}{2400\times10^{-6}} - \frac{(-0.070)(-0.050)}{1.360\times10^{-6}} = -2.1569\times10^3 \text{ m}^{-2}$$
$$P = \frac{\sigma}{K_L} = \frac{-150\times10^6}{-2.1569\times10^3} = 69.6\times10^3 \text{ N}$$

Based on stress at right edge of section: $y = 30 \text{ mm} = 0.030 \text{ m}$

$$\sigma = \frac{P}{A} - \frac{Pey}{I} = K_R P$$
$$K_R = \frac{1}{A} - \frac{ey}{I} = \frac{1}{2400\times10^{-6}} - \frac{(-0.070)(0.030)}{1.360\times10^{-6}} = 1.9608\times10^3 \text{ m}^{-2}$$
$$P = \frac{\sigma}{K} = \frac{150\times10^6}{1.9608\times10^3} = 76.5\times10^3 \text{ N}$$

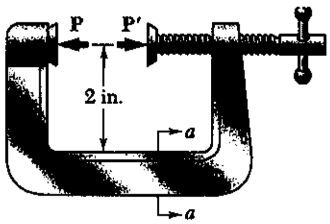
Choose the smaller value $P = 69.6\times10^3 \text{ N} = 69.6 \text{ kN}$ ◄

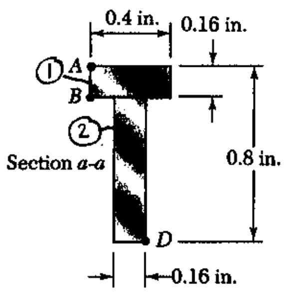
(b) Location of neutral axis: $\sigma = 0$

$$\sigma = \frac{P}{A} - \frac{Pey}{I} = 0 \qquad \frac{ey}{I} = \frac{1}{A}$$
$$y = \frac{I}{Ae} = \frac{1.360\times10^{-6}}{(2400\times10^{-6})(-0.070)} = -8.10\times10^{-3} \text{ m} = -8.10 \text{ mm}$$

Neutral axis lies $50 - 8.10 = 41.9$ mm from left edge. ◄

PROBLEM 4.133





4.133 Knowing that the clamp shown has been tightened until $P = 75$ lb, determine in section *a-a* (*a*) the stress at point *A*, (*b*) the stress at point , (*c*) the location of the neutral axis.

SOLUTION

Locate centroid

Part	A, in^2	$\bar{y}_o$, in	$A\bar{y}_o$, in^3
①	0.064	0.72	0.04608
②	0.1024	0.32	0.03277
Σ	0.1664		0.07885

$$\bar{Y}_o = \frac{\Sigma A\bar{y}_o}{\Sigma A} = \frac{0.07885}{0.1664} = 0.4739 \text{ in.}$$

The centroid lies 0.4739 in. above point D.

Bending couple $\quad M = Pe$

$$e = -(2 + 0.8 - 0.4739) = -2.3261 \text{ in}$$

$$I_1 = \tfrac{1}{12}(0.4)(0.16)^3 + (0.064)(0.72 - 0.4739)^2 = 4.013 \times 10^{-3} \text{ in}^4$$

$$I_2 = \tfrac{1}{12}(0.16)(0.64)^3 + (0.1024)(0.4739 - 0.32)^2 = 5.921 \times 10^{-3} \text{ in}^4$$

$$I = I_1 + I_2 = 9.934 \times 10^{-3} \text{ in}^4$$

(a) Stress at point A: $y = 0.8 - 0.4739 = 0.3261$ in

$$\sigma_A = \frac{P}{A} - \frac{My}{I} = \frac{P}{A} - \frac{Pey}{I} = \frac{75}{0.1664} - \frac{(75)(-2.3261)(0.3261)}{9.934 \times 10^{-3}}$$

$$= 6.18 \times 10^3 \text{ psi} = 6.18 \text{ ksi}$$

(b) Stress at point D: $\quad y = -0.4739$ in. $= 0.1661$ in

$$\sigma_D = \frac{P}{A} - \frac{My}{I} = \frac{P}{A} - \frac{Pey}{I} = \frac{75}{0.1664} - \frac{(75)(-2.3261)(-0.4739)}{9.934 \times 10^{-3}}$$

$$= -7.87 \times 10^3 \text{ psi} = -7.87 \text{ ksi}$$

(c) Location of neutral axis $\qquad \sigma = 0$

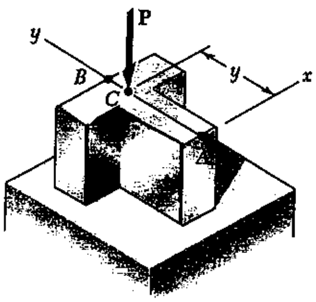
$$\sigma = \frac{P}{A} - \frac{My}{I} = \frac{P}{A} - \frac{Pey}{I} = 0 \qquad \frac{ey}{I} = \frac{1}{A}$$

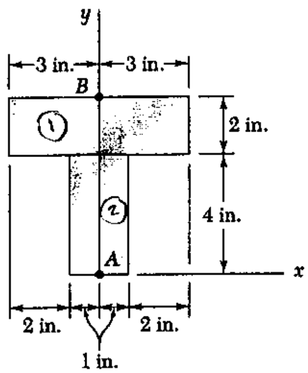
$$y = \frac{I}{Ae} = \frac{9.934 \times 10^{-3}}{(0.1664)(-2.3261)} = -0.0257 \text{ in}$$

The neutral axis lies $0.4739 - 0.0257 = 0.448$ in. above point D.

PROBLEM 4.134

4.134 A vertical force **P** of magnitude 20 kips is applied at a point C located on the line of symmetry of the cross section of a short column. Knowing that $y = 5$ in., determine (*a*) the stress at point A, (*b*) the stress at point B, (*c*) the location of the neutral axis.





(*b*)

SOLUTION

Locate centroid

Part	A, in^2	$\bar{y}$, in	$A\bar{y}$, in^3
1	12	5	60
2	8	2	16
Σ	20		76

$$\bar{Y} = \frac{\Sigma A_i \bar{y}_i}{\Sigma A_i} = \frac{76}{20} = 3.8 \text{ in}$$

Eccentricity of load $e = 5 - 3.8 = 1.2$ in.

$$I_1 = \frac{1}{12}(6)(2)^3 + (12)(1.2)^2 = 21.28 \text{ in}^4$$

$$I_2 = \frac{1}{12}(2)(4)^3 + (8)(1.8)^2 = 36.587 \text{ in}^4$$

$$I = I_1 + I_2 = 57.867 \text{ in}^4$$

(a) Stress at A $\quad c_A = 3.8$ in

$$\sigma_A = -\frac{P}{A} + \frac{Pec_A}{I} = -\frac{20}{20} + \frac{(20)(1.2)(3.8)}{57.867} = 0.576 \text{ ksi}$$

(b) Stress at B $\quad c_B = 6 - 3.8 = 2.2$ in

$$\sigma_B = -\frac{P}{A} - \frac{Pec_B}{I} = -\frac{20}{20} - \frac{(20)(1.2)(2.2)}{57.867} = -1.912 \text{ ksi}$$

(c) Location of neutral axis: $\quad \sigma = 0$

$$\sigma = -\frac{P}{A} + \frac{Pea}{I} = 0 \quad \therefore \quad \frac{ea}{I} = \frac{1}{A}$$

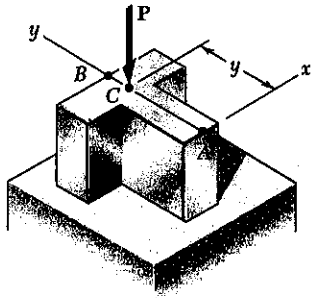
$$a = \frac{I}{Ae} = \frac{57.867}{(20)(1.2)} = 2.411 \text{ in}$$

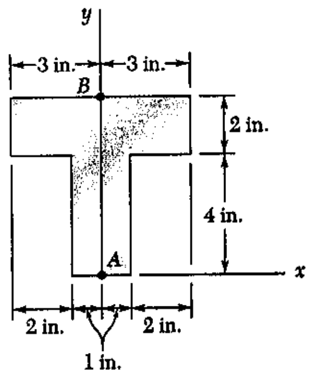
Neutral axis lies 2.411 in. below centroid or $3.8 - 2.411$
$= 1.389$ in above point A.

Answer 1.389 in from point A

PROBLEM 4.135

4.135 A vertical force **P** is applied at a point *C* located on the line of symmetry of the cross section of a short column. Determine the range of values of *y* for which tensile stresses do not occur in the column.





SOLUTION

Locate centroid

	A, in²	$\bar{y}$, in	$A\bar{y}$, in³
①	12	5	60
②	8	2	16
Σ	20		76

$$\bar{Y} = \frac{\Sigma A_i \bar{y}_i}{\Sigma A_i} = \frac{76}{20} = 3.8 \text{ in}$$

Eccentricity of load $e = y - 3.8$ in.

$$y = e + 3.8 \text{ in.}$$

$$I_1 = \tfrac{1}{12}(6)(2)^3 + (12)(1.2)^2 = 21.28 \text{ in}^4$$

$$I_2 = \tfrac{1}{12}(2)(4)^3 + (8)(1.8)^2 = 36.587 \text{ in}^4$$

$$I = I_1 + I_2 = 57.867 \text{ in}^4$$

If stress at A equals zero. $c_A = 3.8$ in

$$\sigma_A = -\frac{P}{A} + \frac{Pec_A}{I} = 0 \quad \therefore \quad \frac{ec_A}{I} = \frac{1}{A}$$

$$e = \frac{I}{Ac_A} = \frac{57.867}{(20)(3.8)} = 0.761 \text{ in} \qquad y = 0.761 + 3.8 = 4.561 \text{ in.}$$

If stress at B equals zero. $c_B = 6 - 3.8 = 2.2$ in

$$\sigma_B = -\frac{P}{A} - \frac{Pec_B}{I} = 0 \quad \therefore \quad \frac{ec_B}{I} = -\frac{1}{A}$$

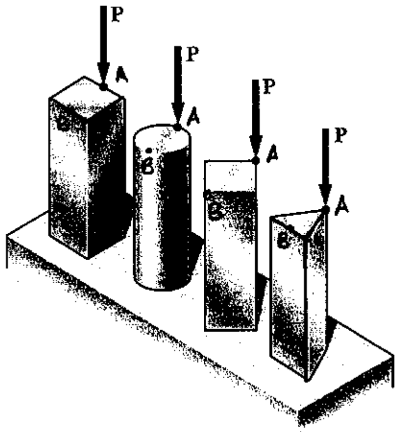
$$e = -\frac{I}{Ac_B} = -\frac{57.867}{(20)(2.2)} = -1.315 \text{ in}$$

$$y = -1.315 + 3.8 = 2.485 \text{ in.}$$

Answer: $2.485 \text{ in} < y < 4.561 \text{ in.}$

PROBLEM 4.136

4.136 The four bars shown have the same cross-sectional area. For the given loadings, show that (*a*) the maximum compressive stresses are in the ratio 4:5:7:9, (*b*) the maximum tensile stresses are in the ratio 2:3:5:3. (*Note:* the cross section of the triangular bar is an equilateral triangle.)



SOLUTION

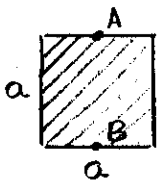
Stresses

At A $\quad \sigma_A = -\frac{P}{A} - \frac{Pec_A}{I}$

$$= -\frac{P}{A}\left(1 + \frac{Aec_A}{I}\right)$$

At B $\quad \sigma_B = -\frac{P}{A} + \frac{Pec_B}{I}$

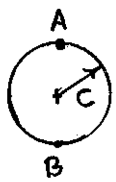
$$= \frac{P}{A}\left(\frac{Aec_B}{I} - 1\right)$$



$A_1 = a^2, \quad I_1 = \frac{1}{12}a^4, \quad c_A = c_B = \frac{1}{2}a, \quad e = \frac{1}{2}a$

$\sigma_A = -\frac{P}{A}\left(1 + \frac{(a^2)(\frac{1}{2}a)(\frac{1}{2}a)}{\frac{1}{12}a^2}\right) = -4\frac{P}{A_1}$ ◄

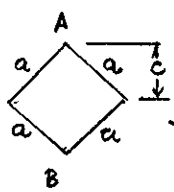
$\sigma_B = \frac{P}{A}\left(\frac{(a^2)(\frac{1}{2}a)(\frac{1}{2}a)}{\frac{1}{12}a^2} - 1\right) = 2\frac{P}{A_1}$ ◄



$A_2 = \pi c^2 = a^2 \quad \therefore c = \frac{a}{\sqrt{\pi}}, \quad I_2 = \frac{\pi}{4}c^4$

$\sigma_A = -\frac{P}{A_2}\left(1 + \frac{(\pi c^2)(c)(c)}{\frac{\pi}{4}c^4}\right) = -5\frac{P}{A_2}$ ◄

$\sigma_B = \frac{P}{A_2}\left(\frac{(\pi c^2)(c)(c)}{\frac{\pi}{4}c^4} - 1\right) = 3\frac{P}{A_2}$ ◄



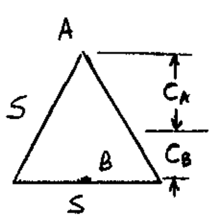
$A_3 = a^2 \qquad c = \frac{\sqrt{2}}{2}a \qquad I_3 = \frac{1}{12}a^4 \qquad e = c$

$\sigma_A = -\frac{P}{A_3}\left(1 + \frac{(a^2)(\frac{\sqrt{2}}{2}a)(\frac{\sqrt{2}}{2}a)}{\frac{1}{12}a^4}\right) = -7\frac{P}{A_3}$ ◄

$\sigma_B = \frac{P}{A_3}\left(\frac{(a^2)(\frac{\sqrt{2}}{2}a)(\frac{\sqrt{2}}{2}a)}{\frac{1}{12}a^4} - 1\right) = 5\frac{P}{A_3}$ ◄

$A_4 = \frac{1}{2}(s)(\frac{\sqrt{3}}{2}s) = \frac{\sqrt{3}}{4}s^2 \qquad I_4 = \frac{1}{36}s(\frac{\sqrt{3}}{2}s)^3 = \frac{\sqrt{3}}{96}s^4$

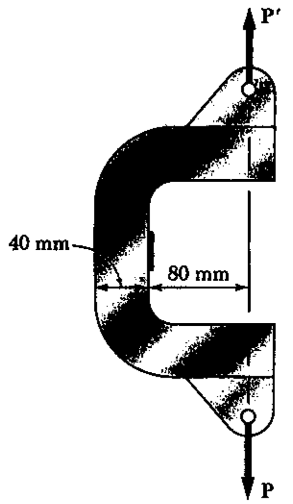
$c_A = \frac{2}{3}\frac{\sqrt{3}}{2}s = \frac{s}{\sqrt{3}} = e \qquad c_B = \frac{s}{2\sqrt{3}}$



$\sigma_A = -\frac{P}{A_4}\left(1 + \frac{(\frac{\sqrt{3}}{4}s^2)(\frac{s}{\sqrt{3}})(\frac{s}{\sqrt{3}})}{\frac{\sqrt{3}}{96}s^4}\right) = -9\frac{P}{A_4}$ ◄

$\sigma_B = \frac{P}{A_4}\left(\frac{(\frac{\sqrt{3}}{4}s^2)(\frac{s}{\sqrt{3}})(\frac{s}{2\sqrt{3}})}{\frac{\sqrt{3}}{96}s^4} - 1\right) = 3\frac{P}{A_4}$ ◄

PROBLEM 4.137



4.137 The C-shaped steel bar is used as a dynamometer to determine the magnitude P of the forces shown. Knowing that the cross section of the bar is a square of side 40 mm and that strain on the inner edge was measured and found to be 450 μ, determine the magnitude P of the forces. Use $E = 200$ GPa.

SOLUTION

At the strain gage location

$$\sigma = E\varepsilon = (200\times10^{9})(450\times10^{-6}) = 90\times10^{6}$$

$$A = (40)(40) = 1600\ \text{mm}^2 = 1600\times10^{-6}\ \text{m}^2$$

$$I = \tfrac{1}{12}(40)(40)^3 = 213.33\times10^{3}\ \text{mm}^4 = 213.33\times10^{-9}\ \text{m}^4$$

$$e = 80 + 20 = 100\ \text{mm} = 0.100\ \text{m}$$

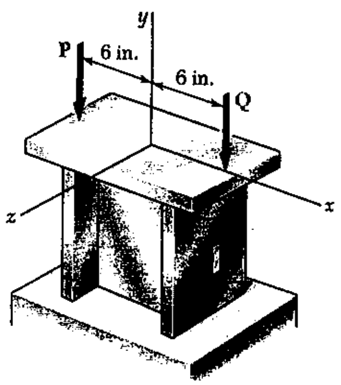
$$c = 20\ \text{mm} = 0.020\ \text{m}$$

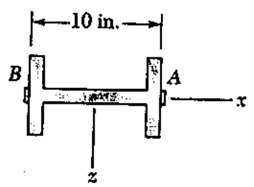
$$\sigma = \frac{P}{A} + \frac{Mc}{I} = \frac{P}{A} + \frac{Pec}{I} = KP$$

$$K = \frac{1}{A} + \frac{ec}{I} = \frac{1}{1600\times10^{-6}} + \frac{(0.100)(0.020)}{213.33\times10^{-9}} = 10.00\times10^{3}\ \text{m}^{-2}$$

$$P = \frac{\sigma}{K} = \frac{90\times10^{6}}{10.00\times10^{3}} = 9.00\times10^{3}\ \text{N} = 9.00\ \text{kN}$$

PROBLEM 4.138





$A = 10.0\ \text{in}^2$
$I_z = 273\ \text{in}^4$

4.138 A short length of a rolled-steel column supports a rigid plate on which two loads **P** and **Q** are applied as shown. The strains at two points A and B on the center lines of the outer faces of the flanges have been measured and found to be

$$\epsilon_A = -400 \times 10^{-6}\ \text{in./in.} \qquad \epsilon_B = -300 \times 10^{-6}\ \text{in./in.}$$

Knowing that $E = 29 \times 10^6$ psi, determine the magnitude of each load.

SOLUTION

Stresses at A and B from strain gages

$$\sigma_A = E\varepsilon_A = (29\times10^6)(-400\times10^{-6}) = -11.6\times10^3\ \text{psi}$$

$$\sigma_B = E\varepsilon_B = (29\times10^6)(-300\times10^{-6}) = -8.7\times10^3\ \text{psi}$$

Centric force $\quad F = P + Q$

Bending couple $\quad M = 6P - 6Q$

$c = 5$ in.

$$\sigma_A = -\frac{F}{A} + \frac{Mc}{I} = -\frac{P+Q}{10.0} + \frac{(6P-6Q)(5)}{273}$$

$$-11.6\times10^3 = +0.00989\,P - 0.20989\,Q \qquad (1)$$

$$\sigma_B = -\frac{F}{A} - \frac{Mc}{I} = -\frac{P+Q}{10.0} - \frac{(6P-6Q)(5)}{273}$$

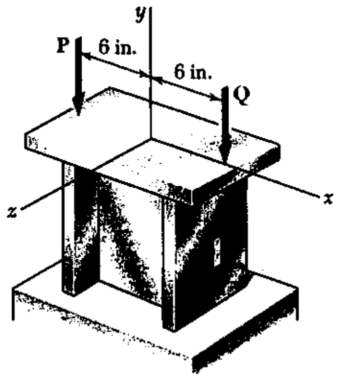
$$-8.7\times10^3 = -0.20989\,P + 0.00989\,Q \qquad (2)$$

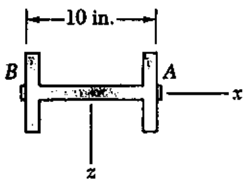
Solving (1) and (2) simultaneously

$$P = 44.2\times10^3\ \text{lb} = 44.2\ \text{kips}$$

$$Q = 57.3\times10^3\ \text{lb} = 57.3\ \text{kips}$$

PROBLEM 4.139





$A = 10.0\text{ in}^2$
$I_z = 273\text{ in}^4$

4.138 A short length of a rolled-steel column supports a rigid plate on which two loads **P** and **Q** are applied as shown. The strains at two points A and B on the center lines of the outer faces of the flanges have been measured and found to be

$\epsilon_A = -400 \times 10^{-6}$ in./in. $\qquad \epsilon_B = -300 \times 10^{-6}$ in./in.

Knowing that $E = 29 \times 10^6$ psi, determine the magnitude of each load.

4.139 Solve Prob. 4.138, assuming that the measured strains are

$\epsilon_A = -350 \times 10^{-6}$ in./in. $\qquad \epsilon_B = -50 \times 10^{-6}$ in./in.

SOLUTION

Stresses at A and B from strain gages

$$\sigma_A = E\varepsilon_A = (29\times10^6)(-350\times10^{-6}) = -10.15\times10^3 \text{ psi}$$

$$\sigma_B = E\varepsilon_B = (29\times10^6)(-50\times10^{-6}) = -1.45\times10^3 \text{ psi}$$

Centric force $\quad F = P + Q$

Bending couple $\quad M = 6P - 6Q$

$c = 5$ in

$$\sigma_A = -\frac{E}{A} + \frac{Mc}{I} = -\frac{P+Q}{10.0} + \frac{(6P-6Q)(5)}{273}$$

$$-10.15\times10^3 = 0.00989\,P - 0.20989\,Q \qquad (1)$$

$$\sigma_B = -\frac{E}{A} - \frac{Mc}{I} = -\frac{P+Q}{10.0} - \frac{(6P-6Q)(5)}{273}$$

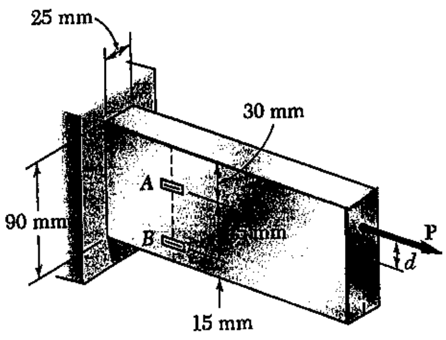
$$-1.45\times10^3 = -0.20989\,P + 0.00989\,Q \qquad (2)$$

Solving (1) and (2) simultaneously

$P = 9.21\times10^3$ lb $= 9.21$ kips ◄

$Q = 48.8\times10^3$ lb. $= 48.8$ kips ◄

PROBLEM 4.140



4.140 An eccentric axial force **P** is applied as shown to a steel bar of 25 × 90-mm cross section. The strains at A and B have been measured and found to be

$$\epsilon_A = +350\ \mu \qquad \epsilon_B = -70\ \mu$$

Knowing that $E = 200$ GPa, determine (a) the distance d, (b) the magnitude of the force **P**.

SOLUTION

$h = 15 + 45 + 30 = 90$ mm

$b = 25$ mm $\qquad c = \frac{1}{2}h = 45\text{mm} = 0.045$ m

$A = bh = (25)(90) = 2.25\times10^3\ \text{mm}^2 = 2.25\times10^{-3}\ \text{m}^2$

$I = \frac{1}{12}bh^3 = \frac{1}{12}(25)(90)^3 = 1.51875\times10^6\ \text{mm}^4$

$= 1.51875\times10^{-6}\ \text{m}^4$

$y_A = 60 - 45 = 15\ \text{mm} = 0.015\text{m}$, $\quad y_B = 15 - 45 = -30\ \text{mm} = -0.030$ m

Stresses from strain gages at A and B

$\sigma_A = E\varepsilon_A = (200\times10^9)(350\times10^{-6}) = 70\times10^6$ Pa

$\sigma_B = E\varepsilon_B = (200\times10^9)(-70\times10^{-6}) = -14\times10^6$ Pa

$$\sigma_A = \frac{P}{A} - \frac{My_A}{I} \qquad (1)$$

$$\sigma_B = \frac{P}{A} - \frac{My_B}{I} \qquad (2)$$

Subtracting $\qquad \sigma_A - \sigma_B = -\frac{M(y_A - y_B)}{I}$

$$M = -\frac{I(\sigma_A - \sigma_B)}{y_A - y_B} = -\frac{(1.51875\times10^{-6})(84\times10^6)}{0.045} = -2835\ \text{N}\cdot\text{m}$$

Multiplying (2) by y_A and (1) by y_B and subtracting

$$y_A\sigma_B - y_B\sigma_A = (y_A - y_B)\frac{P}{A}$$

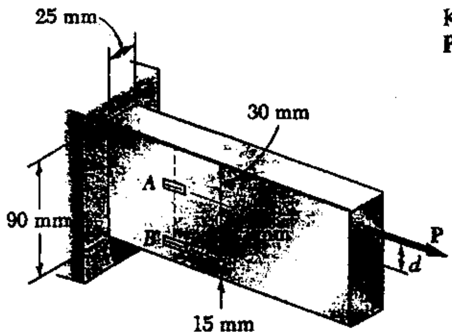
$$P = \frac{A(y_A\sigma_B - y_B\sigma_A)}{y_A - y_B} = \frac{(2.25\times10^{-3})[(0.015)(-14\times10^6) - (-0.030)(70\times10^6)]}{0.045}$$

$$= 94.5\times10^3\ \text{N}$$

(a) $\quad M = -Pd \quad \therefore \quad d = -\frac{M}{P} = -\frac{-2835}{94.5\times10^3} = 0.030\ \text{m} = 30\ \text{mm}$ ◄

(b) $\qquad P = 94.5\ \text{kN}\cdot\text{m}$ ◄

PROBLEM 4.141



4.140 An eccentric axial force **P** is applied as shown to a steel bar of 25 × 90-mm cross section. The strains at A and B have been measured and found to be

$$\epsilon_A = +350\ \mu \qquad \epsilon_B = -70\ \mu$$

Knowing that $E = 200$ GPa, determine (*a*) the distance d, (*b*) the magnitude of the force **P**.

4.141 Solve Prob. 4.140, assuming that the measured strains are

$$\epsilon_A = +600\ \mu \qquad \epsilon_B = +420\ \mu$$

SOLUTION

$$h = 15 + 45 + 30 = 90 \text{ mm}$$

$$b = 25 \text{ mm} \qquad c = \tfrac{1}{2}h = 45 \text{ mm} = 0.045 \text{ m}$$

$$A = bh = (25)(90) = 2.25\times10^3 \text{ mm}^2 = 2.25\times10^{-3} \text{ m}^2$$

$$I = \tfrac{1}{12}bh^3 = \tfrac{1}{12}(25)(90)^3 = 1.51875\times10^6 \text{ mm}^4 = 1.51875\times10^{-6} \text{ m}^4$$

$$y_A = 60 - 45 = 15 \text{ mm} = 0.015 \text{ m}, \qquad y_B = 15 - 45 = -30 \text{ mm} = -0.030 \text{ m}$$

Stresses from strain gages at A and B

$$\sigma_A = E\varepsilon_A = (200\times10^9)(600\times10^{-6}) = 120\times10^6 \text{ Pa}$$

$$\sigma_B = E\varepsilon_B = (200\times10^9)(420\times10^{-6}) = 84\times10^6 \text{ Pa}$$

$$\sigma_A = \frac{P}{A} - \frac{My_A}{I} \qquad (1)$$

$$\sigma_B = \frac{P}{A} - \frac{My_B}{I} \qquad (2)$$

Subtracting $\qquad \sigma_A - \sigma_B = -\dfrac{M(y_A - y_B)}{I}$

$$M = -\frac{I(\sigma_A - \sigma_B)}{y_A - y_B} = -\frac{(1.51875\times10^{-6})(36\times10^6)}{0.045} = -1215 \text{ N·m}$$

Multiplying (2) by y_A and (1) by y_B and subtracting

$$y_A\sigma_B - y_B\sigma_A = (y_A - y_B)\frac{P}{A}$$

$$P = \frac{A(y_A\sigma_B - y_B\sigma_A)}{y_A - y_B} = \frac{(2.25\times10^{-3})[(0.015)(84\times10^6) - (-0.030)(120\times10^6)]}{0.045}$$

$$= 243\times10^3 \text{ N}$$

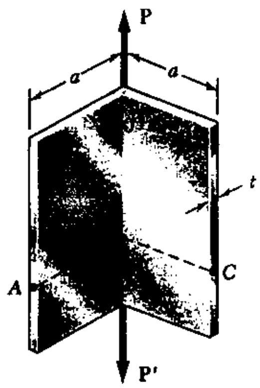
$$M = -Pd$$

(a) $\qquad \therefore\ d = -\dfrac{M}{P} = -\dfrac{-1215}{243\times10^3} = 5\times10^{-3} \text{ m} = 5 \text{ mm}$ ◄

(b) $\qquad P = 243 \text{ kN}$ ◄

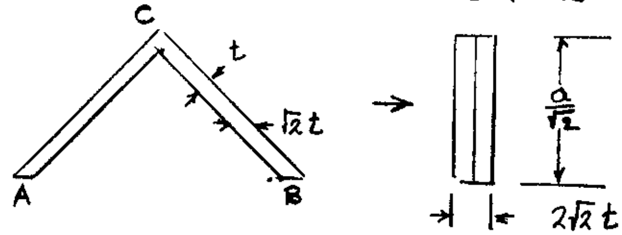
PROBLEM 4.142

4.142 The shape shown was formed by bending a thin steel plate. Assuming that the thickness t is small compared to the length a of a side of the shape, determine the stress (*a*) at A, (*b*) at B, (*c*) at C.



SOLUTION

Moment of inertia about centroid



$$I = \frac{1}{12}(2\sqrt{2}t)\left(\frac{a}{\sqrt{2}}\right)^3 = \frac{1}{12}ta^3$$

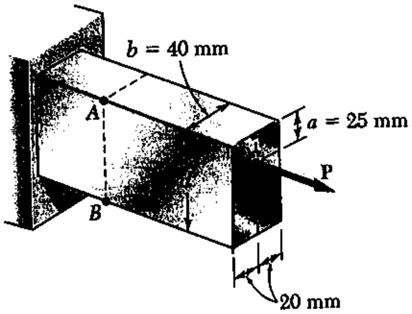
Area $A = (2\sqrt{2}t)\left(\frac{a}{\sqrt{2}}\right) = 2at$, $c = \frac{a}{2\sqrt{2}}$

(a) $\sigma_A = \frac{P}{A} - \frac{Pec}{I} = \frac{P}{2at} - \frac{P\left(\frac{a}{2\sqrt{2}}\right)\left(\frac{a}{2\sqrt{2}}\right)}{\frac{1}{12}ta^3} = -\frac{P}{2at}$ ◄

(b) $\sigma_B = \frac{P}{A} + \frac{Pec}{I} = \frac{P}{2at} + \frac{P\left(\frac{a}{\sqrt{2}}\right)\left(\frac{a}{2\sqrt{2}}\right)}{\frac{1}{12}ta^3} = \frac{2P}{at}$ ◄

(c) $\sigma_C = \sigma_A = -\frac{P}{2at}$ ◄

PROBLEM 4.143



4.143 The eccentric axial force **P** acts at point D, which must be located 25 mm below the top surface of the steel bar shown. For $P = 60$ kN, determine (a) the depth d of the bar for which the tensile stress at point A is maximum, (b) the corresponding stress at point A.

SOLUTION

$$A = bd \qquad I = \frac{1}{12} b d^3$$

$$c = \frac{1}{2} d \qquad e = \frac{1}{2} d - a$$

$$\sigma_A = \frac{P}{A} + \frac{Pec}{I}$$

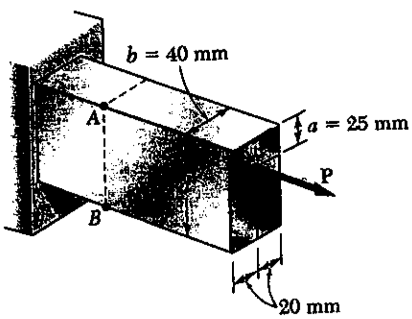
$$\sigma_A = \frac{P}{b}\left\{\frac{1}{d} + \frac{12(\frac{1}{2}d - a)(\frac{1}{2}d)}{d^3}\right\} = \frac{P}{b}\left\{\frac{4}{d} - \frac{6a}{d^2}\right\}$$

(a) Depth d for maximum σ_A. Differentiate with respect to d.

$$\frac{d\sigma_A}{dd} = \frac{P}{b}\left\{-\frac{4}{d^2} + \frac{12a}{d^3}\right\} = 0 \qquad d = 3a = 75 \text{ mm}$$

(b) $$\sigma_A = \frac{60 \times 10^3}{40 \times 10^{-3}}\left\{\frac{4}{75 \times 10^{-3}} - \frac{(6)(25 \times 10^{-3})}{(75 \times 10^{-3})^2}\right\} = 40 \times 10^6 \text{ Pa} = 40 \text{ MPa}$$

PROBLEM 4.144



4.143 The eccentric axial force **P** acts at point D, which must be located 25 mm below the top surface of the steel bar shown. For $P = 60$ kN, determine (a) the depth d of the bar for which the tensile stress at point A is maximum, (b) the corresponding stress at point A.

4.144 For the bar and loading of Prob. 4.143, determine (a) the depth d of the bar for which the compressive stress at point B is maximum, (b) the corresponding stress at point B.

SOLUTION

$$A = bd \qquad I = \frac{1}{12} b d^3$$

$$c = \frac{1}{2} d \qquad e = \frac{1}{2} d - a$$

$$\sigma_B = \frac{P}{A} - \frac{Pec}{I}$$

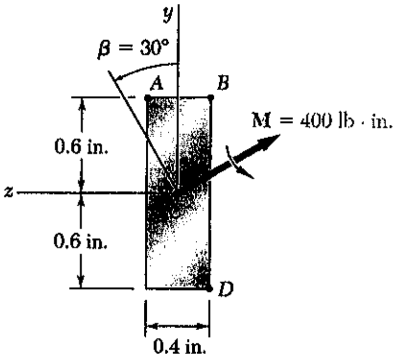
$$\sigma_B = \frac{P}{b}\left\{\frac{1}{d} - \frac{(12)(\frac{1}{2}d - a)(\frac{1}{2}d)}{d^2}\right\} = \frac{P}{b}\left\{-\frac{2}{d} + \frac{6a}{d^2}\right\}$$

(a) Depth d for maximum σ_B: Differentiate with respect to d

$$\frac{d\sigma_B}{dd} = \frac{P}{b}\left\{\frac{2}{d^2} - \frac{12a}{d^3}\right\} = 0 \qquad d = 6a = 150 \text{ mm}$$

(b) $$\sigma_B = \frac{60 \times 10^3}{40 \times 10^{-3}}\left\{-\frac{2}{150 \times 10^{-3}} + \frac{(6)(25 \times 10^{-3})}{(150 \times 10^{-3})^2}\right\} = -10 \times 10^6 \text{ Pa} = -10 \text{ MPa}$$

PROBLEM 4.145



4.145 through 4.147 The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point D

SOLUTION

$$I_z = \frac{1}{12}(0.4)(1.2)^3 = 57.6 \times 10^{-3} \text{ in}^4$$

$$I_y = \frac{1}{12}(1.2)(0.4)^3 = 6.40 \times 10^{-3} \text{ in}^4$$

$$y_A = y_B = -y_D = 0.6 \text{ in}$$

$$z_A = -z_B = -z_D = \left(\tfrac{1}{2}\right)(0.4) = 0.2 \text{ in.}$$

$$M_y = 400 \cos 60° = 200 \text{ lb}\cdot\text{in} \,, \quad M_z = -400 \sin 60° = -346.41 \text{ lb}\cdot\text{in}$$

(a) $$\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(-346.41)(0.6)}{57.6 \times 10^{-3}} + \frac{(200)(0.2)}{6.40 \times 10^{-3}}$$

$$= 9.86 \times 10^3 \text{ psi} = 9.86 \text{ ksi}$$ ◄

(b) $$\sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(-346.41)(0.6)}{57.6 \times 10^{-6}} + \frac{(200)(-0.2)}{6.4 \times 10^{-3}}$$

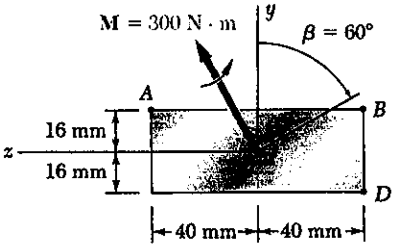
$$= -2.64 \times 10^3 \text{ psi} = -2.64 \text{ ksi}$$ ◄

(c) $$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(-346.41)(-0.6)}{57.6 \times 10^{-3}} + \frac{(200)(-0.2)}{6.40 \times 10^{-3}}$$

$$= -9.86 \times 10^3 \text{ psi} = -9.86 \text{ ksi}$$ ◄

PROBLEM 4.146

4.145 through 4.147 The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point D.



SOLUTION

$$I_z = \frac{1}{12}(80)(32)^3 = 218.45\times10^3\ \text{mm}^4 = 218.45\times10^{-9}\ \text{m}^4$$

$$I_y = \frac{1}{12}(32)(80)^3 = 1.36533\times10^6\ \text{mm}^4 = 1.36533\times10^{-6}\ \text{m}^4$$

$$y_A = y_B = -y_D = 16\ \text{mm}$$

$$z_A = -z_B = -z_D = 40\ \text{mm}$$

$$M_y = 300\cos 30^\circ = 259.81\ \text{N}\cdot\text{m},\quad M_z = 300\sin 30^\circ = 150\ \text{N}\cdot\text{m}$$

(a) $$\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(150)(16\times10^{-3})}{218.45\times10^{-9}} + \frac{(259.81)(40\times10^{-3})}{1.36533\times10^{-6}}$$

$$= -3.37\times10^6\ \text{Pa} = -3.37\ \text{MPa}$$

(b) $$\sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(150)(16\times10^{-3})}{218.45\times10^{-9}} + \frac{(259.81)(-40\times10^{-3})}{1.36533\times10^{-6}}$$

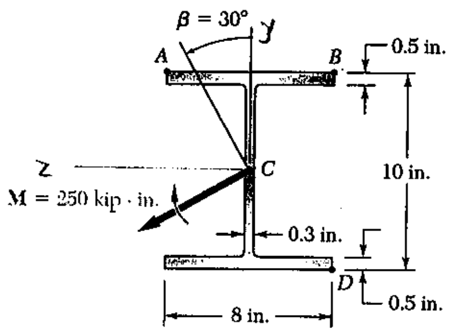
$$= -18.60\times10^6\ \text{Pa} = -18.60\ \text{MPa}$$

(c) $$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(150)(-16\times10^{-3})}{218.45\times10^{-9}} + \frac{(259.81)(-40\times10^{-3})}{1.36533\times10^{-6}}$$

$$= 3.37\times10^6\ \text{Pa} = 3.37\ \text{MPa}$$

PROBLEM 4.147

4.145 through 4.147 The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point D.



SOLUTION

Flange: $I_z = \frac{1}{12}(8)(0.5)^3 + (8)(0.5)(4.75)^2$

$= 90.333 \text{ in}^4$

$I_y = \frac{1}{12}(0.5)(8)^3 = 21.333 \text{ in}^4$

Web: $I_z = \frac{1}{12}(0.3)(9)^3 = 18.225 \text{ in}^4$

$I_y = \frac{1}{12}(9)(0.3)^3 = 0.02025 \text{ in}^4$

Total: $I_z = (2)(90.333) + 18.225 = 198.89 \text{ in}^4$

$I_y = (2)(21.333) + 0.02025 = 42.687 \text{ in}^4$

$y_A = y_B = -y_D = 5 \text{ in.} \quad ; \quad z_A = -z_B = -z_c = 4 \text{ in.}$

$M_z = 250 \cos 30° = 216.51 \text{ kip-in}$

$M_y = -250 \sin 30° = -125 \text{ kip-in}$

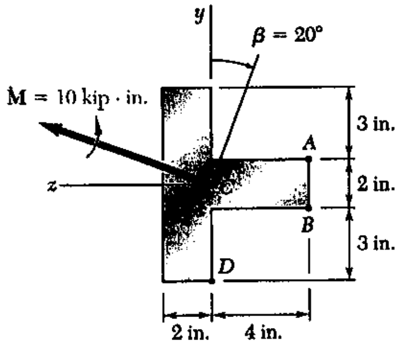
(a) $\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(216.51)(5)}{198.89} + \frac{(-125)(4)}{42.687} = -17.16 \text{ ksi}$ ◄

(b) $\sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(216.51)(5)}{198.89} + \frac{(-125)(-4)}{42.687} = 6.27 \text{ ksi}$ ◄

(c) $\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(216.51)(-5)}{198.89} + \frac{(-125)(-4)}{42.687} = 17.16 \text{ ksi}$ ◄

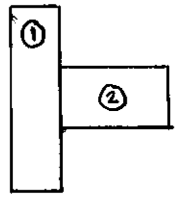
PROBLEM 4.148

4.148 through 4.150 The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point D.



SOLUTION

Locate centroid



	A, in²	$\bar{z}$, in	$A\bar{z}$, in³
①	16	-1	-16
②	8	2	16
Σ	24		0

The centroid lies at point C

$$I_z = \frac{1}{12}(2)(8)^3 + \frac{1}{12}(4)(2)^3 = 88 \text{ in}^4$$

$$I_y = \frac{1}{3}(8)(2)^3 + \frac{1}{3}(2)(4)^3 = 64 \text{ in}^4$$

$$y_A = -y_B = 1 \text{ in}, \qquad y_D = -4 \text{ in}$$

$$z_A = z_B = -4 \text{ in}, \qquad z_D = 0$$

$$M_z = 10 \cos 20° = 9.3969 \text{ kip-in}$$

$$M_y = 10 \sin 20° = 3.4202 \text{ kip-in.}$$

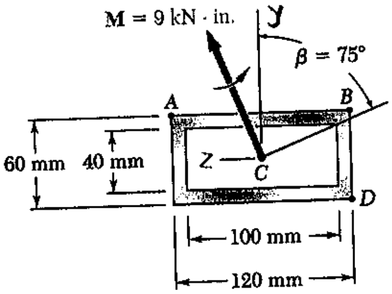
(a) $$\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(9.3969)(1)}{88} + \frac{(3.4202)(-4)}{64} = 0.321 \text{ ksi}$$ ◀

(b) $$\sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(9.3969)(-1)}{88} + \frac{(3.4202)(-4)}{64} = -0.107 \text{ ksi}$$ ◀

(c) $$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(9.3969)(-4)}{88} + \frac{(3.4202)(0)}{64} = 0.427 \text{ ksi}$$ ◀

PROBLEM 4.149

4.148 through 4.150 The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point D.



SOLUTION

$$I_z = \tfrac{1}{12}(120)(60)^3 - \tfrac{1}{12}(100)(40)^3 = 1.62667 \times 10^6 \text{ mm}^4$$
$$= 1.62667 \times 10^{-6} \text{ m}^4$$

$$I_y = \tfrac{1}{12}(60)(120)^3 - \tfrac{1}{12}(40)(100)^3 = 5.3067 \times 10^6 \text{ mm}^4$$
$$= 5.3067 \times 10^{-6} \text{ m}^4$$

$$y_A = y_B = -y_D = 30 \text{ mm}$$

$$z_A = -z_B = -z_D = 60 \text{ mm}$$

$$M_z = (9 \times 10^3) \sin 15° = 2.3294 \times 10^3 \text{ N}\cdot\text{m}$$

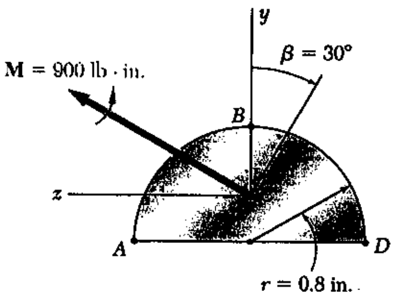
$$M_y = (9 \times 10^3) \cos 15° = 8.6933 \times 10^3 \text{ N}\cdot\text{m}$$

(a) $$\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(2.3294 \times 10^3)(30 \times 10^{-3})}{1.62667 \times 10^{-6}} + \frac{(8.6933 \times 10^3)(60 \times 10^{-3})}{5.3067 \times 10^{-6}}$$
$$= 55.3 \times 10^6 \text{ Pa} = 55.3 \text{ MPa}$$

(b) $$\sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(2.3294 \times 10^3)(30 \times 10^{-3})}{1.62667 \times 10^{-6}} + \frac{(8.6933 \times 10^3)(-60 \times 10^{-3})}{5.3067 \times 10^{-6}}$$
$$= -141.2 \times 10^6 \text{ Pa} = -141.2 \text{ MPa}$$

(c) $$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(2.3294 \times 10^3)(-30 \times 10^{-3})}{1.62667 \times 10^{-6}} + \frac{(8.6933 \times 10^3)(-60 \times 10^{-3})}{5.3067 \times 10^{-6}}$$
$$= -55.3 \times 10^6 \text{ Pa} = -55.3 \text{ MPa}$$

PROBLEM 4.150



4.148 through 4.150 The couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) point D.

SOLUTION

$$I_z = \frac{\pi}{8}r^4 - \left(\frac{\pi}{2}r^2\right)\left(\frac{4r}{3\pi}\right)^2 = \left(\frac{\pi}{8} - \frac{8}{9\pi}\right)r^4$$

$$= (0.109757)(0.8)^4 = 44.956\times10^{-3} \text{ in}^4$$

$$I_y = \frac{\pi}{8}r^4 = \frac{\pi}{8}(0.8)^4 = 160.85\times10^{-3} \text{ in}^4$$

$$y_A = y_D = -\frac{4r}{3\pi} = -\frac{(4)(0.8)}{3\pi} = -0.33953 \text{ in.}$$

$$y_B = 0.8 - 0.33953 = 0.46047 \text{ in.}$$

$$z_A = -z_D = 0.8 \text{ in.}, \qquad z_B = 0$$

$$M_y = 900 \sin 30° = 450 \text{ lb·in}$$

$$M_z = 900 \cos 30° = 779.42 \text{ lb·in.}$$

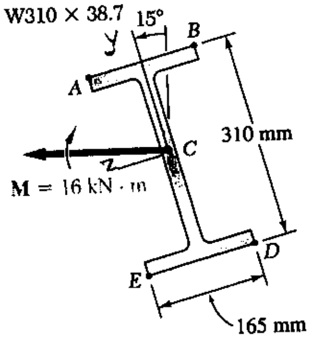
(a) $$\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(779.42)(-0.33953)}{44.956\times10^{-3}} + \frac{(450)(0.8)}{160.85\times10^{-3}} = 8.12\times10^3 \text{ psi}$$
$$= 8.12 \text{ ksi}$$

(b) $$\sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(779.42)(0.46047)}{44.956\times10^{-3}} + \frac{(450)(0)}{160.85\times10^{-3}} = -7.98\times10^3 \text{ psi}$$
$$= -7.98 \text{ ksi}$$

(c) $$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(779.42)(-0.33953)}{44.956\times10^{-3}} + \frac{(450)(-0.8)}{160.85\times10^{-3}} = 3.65\times10^3 \text{ psi}$$
$$= 3.65 \text{ ksi}$$

PROBLEM 4.151

4.151 through 4.153 The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine (*a*) the angle that the neutral axis forms with the horizontal plane, (*b*) the maximum tensile stress in the beam.



SOLUTION

For W 310 × 38.7 rolled steel shape

$$I_z = 85.1 \times 10^6 \ mm^4 = 85.1 \times 10^{-6} \ m^4$$

$$I_y = 7.27 \times 10^6 \ mm^4 = 7.27 \times 10^{-6} \ m^4$$

$$y_A = y_B = -y_D = -y_E = (\tfrac{1}{2})(310) = 155 \ mm$$

$$z_A = z_E = -z_B = -z_D = (\tfrac{1}{2})(165) = 82.5 \ mm$$

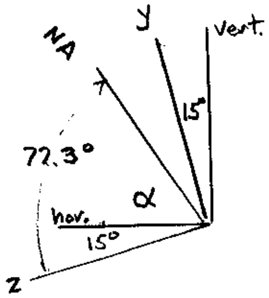
$$M_z = (16 \times 10^3) \cos 15° = 15.455 \times 10^3 \ N \cdot m$$

$$M_y = (16 \times 10^3) \sin 15° = 4.1411 \times 10^3 \ N \cdot m$$

(a) $$\tan \varphi = \frac{I_z}{I} \tan \theta = \frac{85.1 \times 10^{-6}}{7.27 \times 10^{-6}} \tan 15° = 3.1365$$

$$\varphi = 72.3°$$

$$\alpha - 72.3 - 15 = 57.3°$$



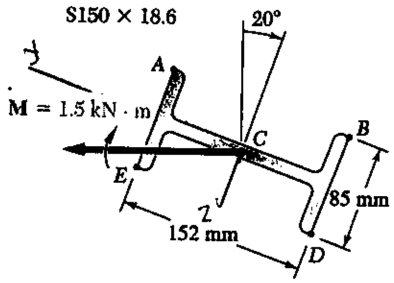
(b) Maximum tensile stress occurs at point E

$$\sigma_E = -\frac{M_z y_E}{I_z} + \frac{M_y z_E}{I_y} = -\frac{(15.455 \times 10^3)(-155 \times 10^{-3})}{85.1 \times 10^{-6}} + \frac{(4.1411 \times 10^3)(82.5 \times 10^{-3})}{7.27 \times 10^{-6}}$$

$$= 75.1 \times 10^6 \ Pa = 75.1 \ MPa$$

PROBLEM 4.152

4.151 through 4.153 The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine (*a*) the angle that the neutral axis forms with the horizontal plane, (*b*) the maximum tensile stress in the beam.



SOLUTION

For S 150 × 18.6 rolled steel shape

$I_z = 9.11 \times 10^6 \text{ mm}^4 = 9.11 \times 10^{-6} \text{ m}^4$

$I_y = 0.782 \times 10^6 \text{ mm}^4 = 0.782 \times 10^{-6} \text{ m}^4$

$z_E = -z_A = -z_B = z_D = (\frac{1}{2})(85) = 42.5 \text{ mm}$

$y_A = y_B = -y_D = -y_E = (\frac{1}{2})(152) = 76 \text{ mm}$

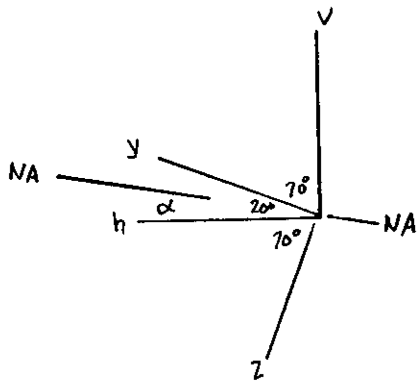
$M_z = 1.5 \times 10^3) \sin 20° = 0.51303 \times 10^3 \text{ N·m}$

$M_y = (1.5 \times 10^3) \cos 20° = 1.4095 \times 10^3 \text{ N·m}$

(a) $\tan \varphi = \frac{I_z}{I_y} \tan \theta = \frac{9.11 \times 10^{-6}}{0.782 \times 10^{-6}} \tan (90° - 20°) = 32.007$

$\varphi = 88.21$

$\alpha = 88.21 - 70° = 18.21°$

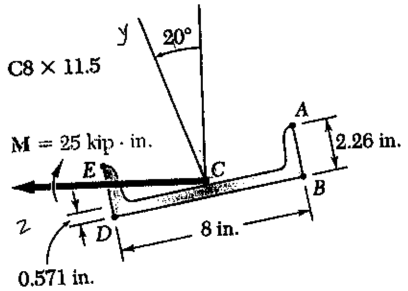


(b) Maximum tensile stress occurs at point D

$$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(0.51303 \times 10^3)(-76 \times 10^{-3})}{9.11 \times 10^{-6}} + \frac{(1.4095 \times 10^3)(42.5 \times 10^{-3})}{0.782 \times 10^{-6}}$$

$= 80.9 \times 10^6 \text{ Pa} = 80.9 \text{ MPa}$

PROBLEM 4.153



4.151 through 4.153 The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine (*a*) the angle that the neutral axis forms with the horizontal plane, (*b*) the maximum tensile stress in the beam.

SOLUTION

For C8×11.5 rolled steel shape

$I_z = 1.32 \text{ in}^4$, $I_y = 32.6 \text{ in}^4$

$z_E = z_D = 0.4 \text{ in}$, $z_B = z_A = -0.4 \text{ in}$

$y_D = y_B = -0.571 \text{ in.}$

$y_E = y_A = 2.26 - 0.571 = 1.689 \text{ in.}$

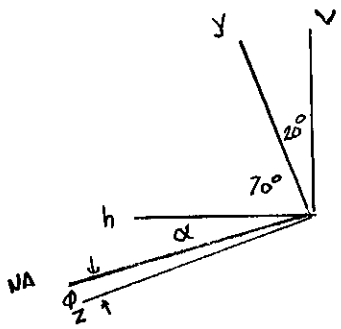
$M_y = 25 \sin 20° = 8.5505 \text{ kip·in}$

$M_z = 25 \cos 20° = 23.492 \text{ kip·in.}$

(a) $\tan \varphi = \dfrac{I_z}{I_y} \tan \theta = \dfrac{1.32}{32.6} \tan 20° = 0.014737$

$\varphi = 0.844°$

$\alpha = 20 - 0.844 = 19.16°$ ◀

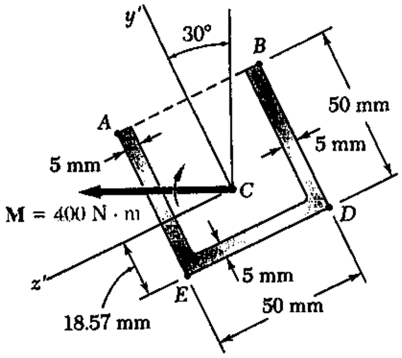


(b) Maximum tensile stress occurs at point D

$$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(23.492)(-0.571)}{1.32} + \frac{(8.5505)(4)}{32.6}$$

$$= 10.162 + 1.049 = 11.21 \text{ ksi}$$ ◀

PROBLEM 4.154



$I_{y'} = 281 \times 10^3\ \text{mm}^4$
$I_{z'} = 176.9 \times 10^3\ \text{mm}^4$

4.154 through 4.156 The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine (*a*) the angle that the neutral axis forms with the horizontal plane, (*b*) the maximum tensile stress in the beam.

SOLUTION

$I_{z'} = 176.9 \times 10^3\ \text{mm}^4 = 176.9 \times 10^{-9}\ \text{m}^4$

$I_y' = 281 \times 10^3\ \text{mm}^4 = 281 \times 10^{-9}\ \text{m}^4$

$y_E' = -18.57\ \text{mm}, \qquad z_E = 25\ \text{mm}$

$M_z' = 400 \cos 30° = 346.41\ \text{N}\cdot\text{m}$

$M_{y'} = 400 \sin 30° = 200\ \text{N}\cdot\text{m}$

(a) $\tan\varphi = \dfrac{I_{z'}}{I_{y'}} \tan\theta = \dfrac{176.9 \times 10^{-9}}{281 \times 10^{-9}} \cdot \tan 30° = 0.36346$

$\varphi = 19.97°$

$\alpha = 30° - 19.97° = 10.03°$ ◄

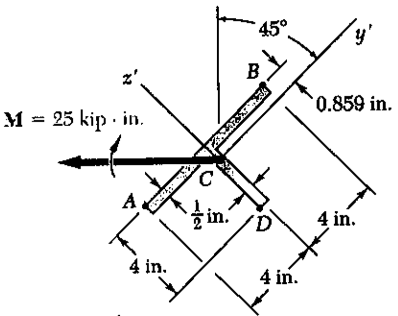
(b) Maximum tensile stress occurs at point E

$$\sigma_D = -\frac{M_z y_E}{I_z} + \frac{M_y z_E}{I_y} = -\frac{(346.41)(-18.57 \times 10^{-3})}{176.9 \times 10^{-9}} + \frac{(200)(25 \times 10^{-3})}{281 \times 10^{-9}}$$

$$= 36.36 \times 10^6 + 17.79 \times 10^6 = 54.2 \times 10^6\ \text{Pa}$$

$$= 54.2\ \text{MPa}$$ ◄

PROBLEM 4.155



$I_{y'} = 6.74\text{ in}^4$
$I_{z'} = 21.4\text{ in}^4$

4.154 through 4.156 The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine (*a*) the angle that the neutral axis forms with the horizontal plane, (*b*) the maximum tensile stress in the beam.

SOLUTION

$I_{z'} = 21.4\ \text{in}^4 \quad , \quad I_{y'} = 6.74\ \text{in}^4$

$z_A' = z_B' = 0.859\ \text{in}$

$z_D = -4 + 0.859\ \text{in} = -3.141\ \text{in}$

$y_A = -4\ \text{in}, \quad y_B = 4\ \text{in}, \quad y_D = -0.25\ \text{in}$

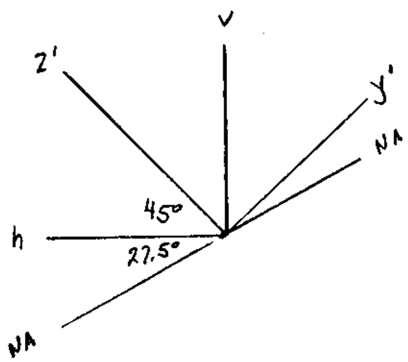
$M_y' = -25 \sin 45° = -17.678\ \text{kip}\cdot\text{in}$

$M_z' = 25 \cos 45° = 17.678\ \text{kip}\cdot\text{in}$

(a) $\tan\varphi = \dfrac{I_{z'}}{I_{y'}} \tan\theta = \dfrac{21.4}{6.74}\tan(-45°) = 3.1751$

$\varphi = -72.5°$

$\alpha = 72.5° - 45° = 27.5°$

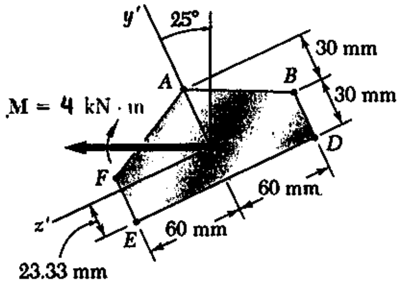


(b) Maximum tensile stress occurs at point D.

$$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(17.678)(-0.25)}{21.4} + \frac{(-17.678)(-3.141)}{6.74}$$

$$= 0.2065 + 8.238 = 8.44\ \text{ksi}$$

PROBLEM 4.156



4.154 through 4.156 The couple **M** acts in a vertical plane and is applied to a beam oriented as shown. Determine (*a*) the angle that the neutral axis forms with the horizontal plane, (*b*) the maximum tensile stress in the beam.

SOLUTION

$$I_z = \frac{1}{36}(120)(30)^3 + \left(\frac{1}{2}\right)(120)(30)(40 - 23.33)^2$$
$$+ \frac{1}{12}(120)(30)^3 + (120)(30)(23.33 - 15)^2$$
$$= 1.11 \times 10^6 \text{ mm}^4 = 1.11 \times 10^{-6} \text{ m}^4$$

$$I_y = 2\left\{\frac{1}{12}(30)(60)^3 + \frac{1}{3}(30)(60)^3\right\} = 5.40 \times 10^6 \text{ mm}^4 = 5.40 \times 10^{-6} \text{ m}^4$$

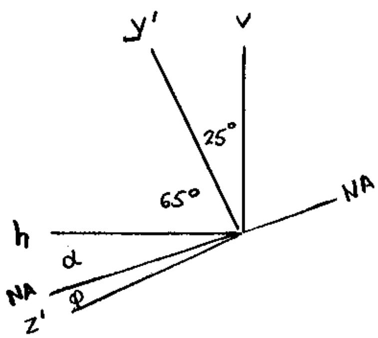
$y_E = -23.33$ mm $\qquad\qquad z_E = 60$ mm

$$M_{z'} = (4 \times 10^3)\cos 25° = 3.6252 \times 10^3 \text{ N-m}$$
$$M_{y'} = (4 \times 10^3)\sin 25° = 1.6905 \times 10^3 \text{ N}\cdot\text{m}$$

(a) $\tan\varphi = \dfrac{I_{z'}}{I_{y'}}\tan\theta = \dfrac{1.11 \times 10^{-6}}{5.40 \times 10^{-6}}\tan 25° = 0.095822$

$\varphi = 5.475°$

$\alpha = 25° - 5.475° = 19.52°$ ◄



(b) Maximum tensile stress occurs at point E

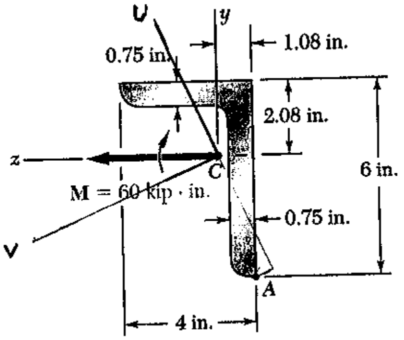
$$\sigma_E = -\frac{M_z y_E}{I_z} + \frac{M_y z_E}{I_y} = -\frac{(3.6252 \times 10^3)(-23.33 \times 10^{-3})}{1.11 \times 10^{-6}} + \frac{(1.6905 \times 10^3)(60 \times 10^{-3})}{5.40 \times 10^{-6}}$$

$$= 76.195 \times 10^6 + 18.783 \times 10^6 = 95.0 \times 10^6 \text{ Pa}$$

$= 95.0$ MPa ◄

PROBLEM 4.157

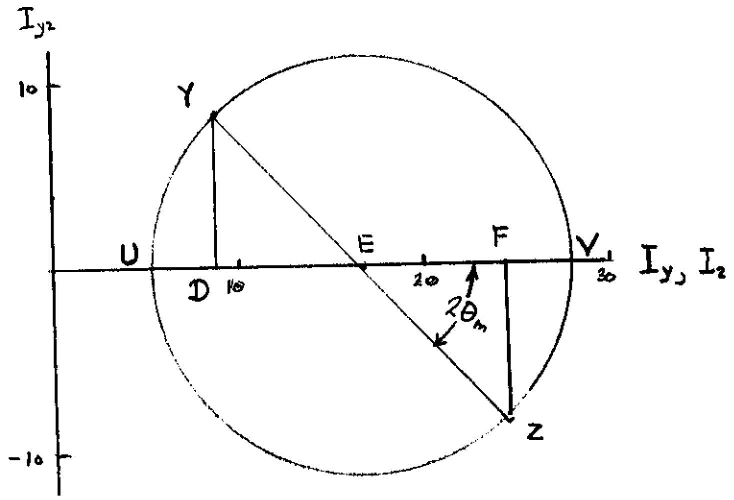
***4.157 and 4.158** The couple **M** acts in a vertical plane and is applied to a beam of the cross section shown. Determine the stress at point A.



$I_y = 8.7\text{ in}^4$
$I_z = 24.5\text{ in}^4$
$I_{yz} = +8.3\text{ in}^4$

SOLUTION

Using Mohr's circle, determine the principal axes and principal moments of inertia



$Y\ (8.7,\ 8.3)\ \text{in}^4$
$Z\ (24.5,\ -8.3)\ \text{in}^4$
$E\ (16.6,\ 0)\ \text{in}^4$

$EF = 7.9\ \text{in}^4$
$FZ = 8.3\ \text{in}^4$

$R = \sqrt{7.9^2 + 8.3^2} = 11.46\ \text{in}^4 \qquad \tan 2\theta_m = \dfrac{FZ}{EF} = \dfrac{8.3}{7.9} = 1.0506$

$\theta_m = 23.2^\circ \qquad I_u = 16.6 - 11.46 = 5.14\ \text{in}^4, \quad I_v = 16.6 + 11.46 = 28.06\ \text{in}^4$

$M_u = M \sin\theta_m = (60)\sin 23.2^\circ = 23.64\ \text{kip}\cdot\text{in}$

$M_v = M\cos\theta_m = (60)\cos 23.2^\circ = 55.15\ \text{kip}\cdot\text{in}$

$u_A = y_A\cos\theta_m + z_A\sin\theta_m = -3.92\cos 23.2^\circ - 1.08\sin 23.2^\circ = -4.03\ \text{in.}$

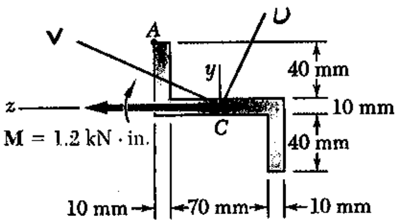
$v_A = z_A\cos\theta_m - y_A\sin\theta_m = -1.08\cos 23.2^\circ + 3.92\sin 23.2^\circ = 0.552\ \text{in.}$

$$\sigma_A = -\frac{M_v u_A}{I_v} + \frac{M_u v_A}{I_u} = -\frac{(55.15)(-4.03)}{28.06} + \frac{(23.64)(0.552)}{5.14}$$

$$= 10.46\ \text{ksi}$$

PROBLEM 4.158

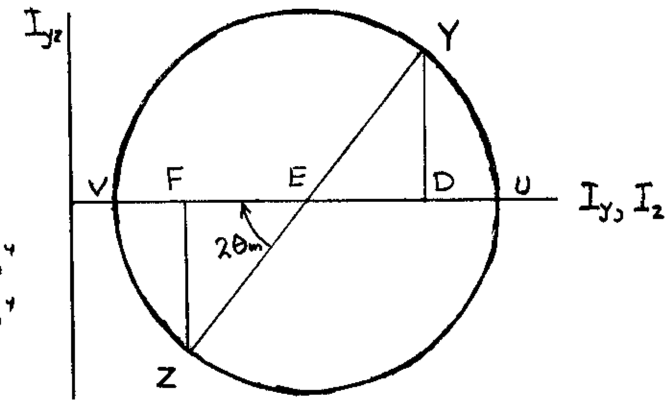
***4.157 and 4.158** The couple **M** acts in a vertical plane and is applied to a beam of the cross section shown. Determine the stress at point *A*.



$I_y = 1.894 \times 10^6\ \text{mm}^4$
$I_z = 0.614 \times 10^6\ \text{mm}^4$
$I_{yz} = +0.800 \times 10^6\ \text{mm}^4$

SOLUTION

Using Mohr's circle, determine the principal axes and the principal moments of inertia.



$Y\,(1.894,\ 0.800) \times 10^6\ \text{mm}^4$

$Z\,(0.614,\ 0.800) \times 10^6\ \text{mm}^4$

$E\,(1.254,\ 0\ ) \times 10^6\ \text{mm}^4$

$$R = \sqrt{\overline{EF}^2 + \overline{FZ}^2} = \sqrt{0.640^2 + 0.800^2} \times 10^{-6} = 1.0245 \times 10^6\ \text{mm}^4$$

$$I_v = (1.254 - 1.0245) \times 10^6\ \text{mm}^4 = 0.2295 \times 10^6\ \text{mm}^4 = 0.2295 \times 10^{-6}\ \text{m}^4$$

$$I_u = (1.254 + 1.0245) \times 10^6\ \text{mm}^4 = 2.2785 \times 10^6\ \text{mm}^4 = 2.2785 \times 10^{-6}\ \text{m}^4$$

$$\tan 2\theta_m = \frac{FZ}{FE} = \frac{0.800 \times 10^6}{0.640 \times 10^6} = 1.25 \qquad \theta_m = 25.67°$$

$$M_v = M \cos\theta_m = (1.2 \times 10^3)\cos 25.67° = 1.0816 \times 10^3\ \text{N·m}$$

$$M_u = -M \sin\theta_m = -(1.2 \times 10^3)\sin 25.67° = -0.5198 \times 10^3\ \text{N·m}$$

$$u_A = y_A \cos\theta_m - z_A \sin\theta_m = 45\cos 25.67° - 45 \sin 25.67° = 21.07\ \text{mm}$$

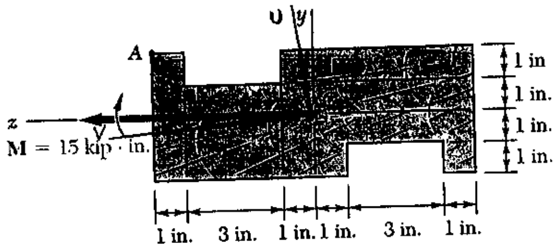
$$v_A = z_A \cos\theta_m + y_A \sin\theta_m = 45 \cos 25.67° + 45 \sin 25.67° = 60.05\ \text{mm}$$

$$\sigma_A = -\frac{M_v u_A}{I_v} + \frac{M_u v_A}{I_u} = -\frac{(1.0816 \times 10^3)(21.07 \times 10^{-3})}{0.2295 \times 10^{-6}} + \frac{(-0.5198 \times 10^3)(60.05 \times 10^{-3})}{2.2785 \times 10^{-6}}$$

$$= 113.0 \times 10^6\ \text{Pa} = 113.0\ \text{MPa}$$

PROBLEM 4.159

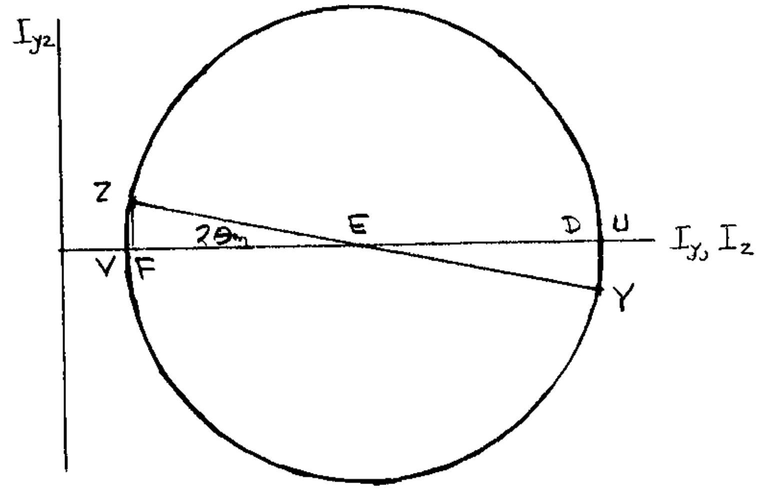
***4.159** A 4 × 10-in. timber has been trimmed to form a beam of the cross section shown. Knowing that the couple **M** acts in a vertical plane, determine the stress at point A.



$I_y = 291\ \text{in}^4$
$I_z = 39.3\ \text{in}^4$
$I_{yz} = -22.5\ \text{in}^4$

SOLUTION

Using Mohr's circle determine the principal axes and principal moments of inertia.



$Y\ (291, -22.5)\ \text{in}^4$

$Z\ (39.3, 22.5)\ \text{in}^4$

$E\ (165.15, 0)\ \text{in}^4$

$$\tan 2\theta_m = \frac{FZ}{EF} = \frac{22.5}{125.85} = 0.17878$$

$$\theta_m = 5.07^\circ$$

$$R = \sqrt{\overline{EF}^2 + \overline{FZ}^2} = \sqrt{125.85^2 + 22.5^2} = 127.85\ \text{in}^4$$

$$I_v = 165.15 - 127.85 = 37.30\ \text{in}^4$$

$$I_u = 165.15 + 127.85 = 293.0\ \text{in}^4$$

$$u_A = y_A \cos\theta_m + z_A \sin\theta_m = 2\cos 5.07^\circ + 5\sin 5.07^\circ = 2.434\ \text{in}$$

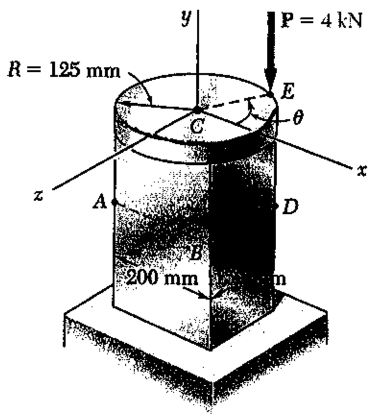
$$v_A = z_A \cos\theta_m - y_A \sin\theta_m = 5\cos 5.07^\circ - 2\sin 5.07^\circ = 4.804\ \text{in}$$

$$M_v = 15\cos 5.07^\circ = 14.94\ \text{kip}\cdot\text{in}$$

$$M_u = 15\sin 5.07^\circ = 1.326\ \text{kip}\cdot\text{in}$$

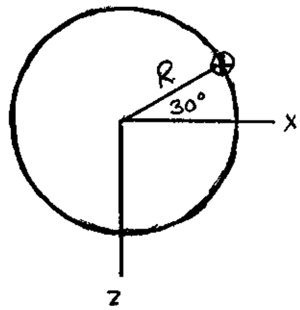
$$\sigma_A = -\frac{M_v u_A}{I_v} + \frac{M_u v_A}{I_u} = -\frac{(14.94)(2.434)}{37.30} + \frac{(1.326)(4.804)}{293.0} = -0.953\ \text{ksi}$$

PROBLEM 4.160



4.160 A rigid plate of 125-mm diameter is attached to a solid 150 × 200-mm rectangular post, with the center of the plate directly above the center of the post. If a 4-kN force **P** is applied at *E* with $\theta = 30°$, determine (*a*) the stress at point *A*, (*b*) the stress at point *B*, (*c*) the point where the neutral axis intersects line *ABD*.

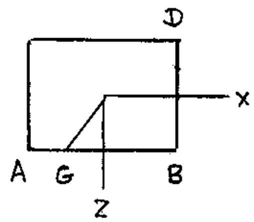
SOLUTION



$P = 4 \times 10^3$ N (compression)

$$M_x = -PR \sin 30°$$
$$= -(4\times10^3)(125\times10^{-3}) \sin 30°$$
$$= -250 \text{ N}\cdot\text{m}$$

$$M_z = -PR \cos 30°$$
$$= -(4\times10^3)(125\times10^{-3}) \cos 30°$$
$$= -433 \text{ N}\cdot\text{m}$$



$$I_x = \tfrac{1}{12}(200)(150)^3 = 56.25\times10^6 \text{ mm}^4 = 56.25\times10^{-6} \text{ m}^4$$

$$I_z = \tfrac{1}{12}(150)(200)^3 = 100\times10^6 \text{ mm}^4 = 100\times10^{-6} \text{ m}^4$$

$$-x_A = x_B = 100 \text{ mm}$$
$$z_A = z_B = 75 \text{ mm}$$
$$A = (200)(150) = 30\times10^3 \text{ mm}^2 = 30\times10^{-3} \text{ m}^2$$

(a) $$\sigma_A = -\frac{P}{A} + \frac{M_x z_A}{I_x} + \frac{M_z x_A}{I_z} = -\frac{4\times10^3}{30\times10^{-3}} - \frac{(-250)(75\times10^{-3})}{56.25\times10^{-6}} + \frac{(-433)(-100\times10^{-3})}{100\times10^{-6}}$$

$$= 633\times10^3 \text{ Pa} = 633 \text{ kPa}$$

(b) $$\sigma_B = -\frac{P}{A} - \frac{M_x z_B}{I_x} + \frac{M_z x_B}{I_z} = -\frac{4\times10^3}{30\times10^{-3}} - \frac{(-250)(75\times10^{-3})}{56.25\times10^{-6}} + \frac{(-433)(100\times10^{-3})}{100\times10^{-6}}$$

$$= -233\times10^3 \text{ Pa} = -233 \text{ kPa}$$

(c) Let G be the point on AB where neutral axis intersects.

$\sigma_G = 0 \qquad z_G = 75$ mm $\qquad x_G = ?$

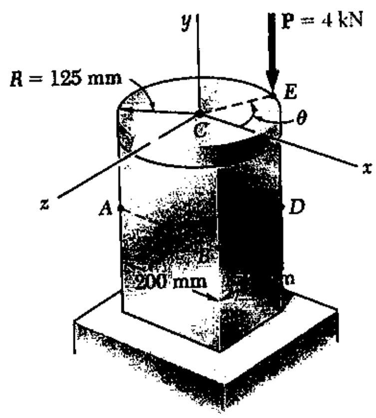
$$\sigma_G = -\frac{P}{A} - \frac{M_x z_G}{I_x} + \frac{M_z x_G}{I_z} = 0$$

$$x_G = \frac{I_z}{M_z}\left\{\frac{P}{A} + \frac{M_x z_G}{I_x}\right\} = \frac{100\times10^{-6}}{-433}\left\{\frac{4\times10^3}{30\times10^{-3}} + \frac{(-250)(75\times10^{-3})}{56.25\times10^{-6}}\right\}$$

$$= 46.2\times10^{-3} \text{ m} = 46.2 \text{ mm}$$

Point G lies 146.2 mm from point A

PROBLEM 4.161

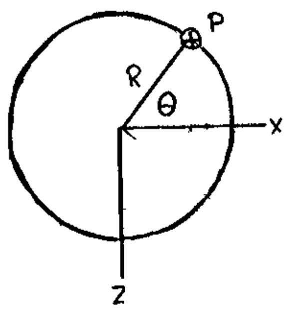


4.160 A rigid plate of 125-mm diameter is attached to a solid 150 × 200-mm rectangular post, with the center of the plate directly above the center of the post. If a 4-kN force **P** is applied at *E* with $\theta = 30°$, determine (*a*) the stress at point *A*, (*b*) the stress at point *B*, (*c*) the point where the neutral axis intersects line *ABD*.

4.161 In Prob. 4.160, determine (*a*) the value of θ for which the stress at *D* reaches its largest value, (*b*) the corresponding values of the stress at *A*, *B*, *C*, and *D*.

SOLUTION

(a)

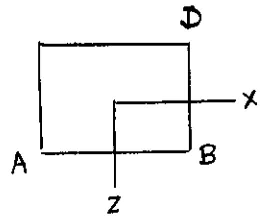


$P = 4 \times 10^3$ N

$PR = (4\times10^3)(125\times10^{-3}) = 500$ N·m

$M_x = -PR \sin\theta = -500 \sin\theta$

$M_x = -PR \cos\theta = -500 \cos\theta$



$I_x = \frac{1}{12}(200)(150)^3 = 56.25\times10^6 \text{ mm}^4 = 56.25\times10^{-6} \text{ m}^4$

$I_z = \frac{1}{12}(150)(200)^3 = 100\times10^6 \text{ mm}^4 = 100\times10^{-6} \text{ m}^4$

$x_D = 100$ mm $\qquad z_D = -75$ mm

$A = (200)(150) = 30\times10^3 \text{ mm}^2 = 30\times10^{-3} \text{ m}^2$

$$\sigma = -\frac{P}{A} - \frac{M_x z}{I_x} + \frac{M_z x}{I_z} = -P\left\{\frac{1}{A} - \frac{Rz\sin\theta}{I_x} + \frac{Rx\cos\theta}{I_z}\right\}$$

For σ to be a maximum $\frac{d\sigma}{d\theta} = 0$ with $z = z_D$, $x = x_D$

$$\frac{d\sigma_D}{d\theta} = -P\left\{0 - \frac{Rz_D\cos\theta}{I_x} + \frac{Rx_D\sin\theta}{I_z}\right\} = 0$$

$$\frac{\sin\theta}{\cos\theta} = \tan\theta = -\frac{I_z z_D}{I_x x_D} = -\frac{(100\times10^{-6})(-75\times10^{-3})}{(56.25\times10^{-6})(100\times10^{-3})} = \frac{4}{3}$$

$\sin\theta = 0.8, \quad \cos\theta = 0.6, \quad \theta = 53.1°$ ◀

(b) $$\sigma_A = -\frac{P}{A} - \frac{M_x z_A}{I_x} + \frac{M_z x_A}{I_z} = -\frac{4\times10^3}{30\times10^{-3}} + \frac{(500)(0.8)(75\times10^{-3})}{56.25\times10^{-6}} - \frac{(500)(0.6)(-100\times10^{-3})}{100\times10^{-6}}$$

$= (-0.13333 + 0.53333 + 0.300)\times10^6$ Pa $= 0.700\times10^6$ Pa $= 700$ kPa ◀

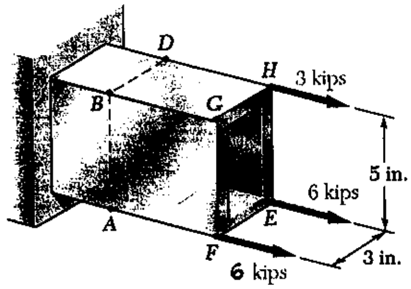
$\sigma_B = (-0.13333 + 0.53333 - 0.300)\times10^6$ Pa $= 0.100\times10^6$ Pa $= 100$ kPa ◀

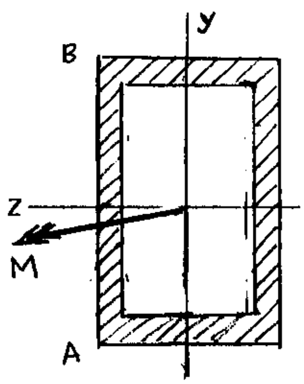
$\sigma_C = (-0.13333 + 0 + 0)\times10^6$ Pa $= -133.3$ kPa ◀

$\sigma_D = (-0.13333 - 0.53333 - 0.300)\times10^6$ Pa $= -0.967\times10^6$ Pa $= -967$ kPa ◀

PROBLEM 4.162

4.162 The tube shown has a uniform wall thickness of 0.5 in. For the given loading, determine *(a)* the stress at points *A* and *B*, *(b)* the point where the neutral axis intersects line *ABD*.





SOLUTION

Add y- and z-axes as shown.

$$I_z = \frac{1}{12}(3)(5)^3 - \frac{1}{12}(2)(4)^3 = 20.583 \text{ in}^4$$

$$I_y = \frac{1}{12}(5)(3)^3 - \frac{1}{12}(4)(2)^3 = 8.5833 \text{ in}^4$$

$$A = (3)(5) - (2)(4) = 7.0 \text{ in}^2$$

Resultant force and bending couples

$$P = 3 + 6 + 6 = 15 \text{ kips}$$

$$M_z = -(2.5)(3) + (2.5)(6) + (2.5)(6) = 22.5 \text{ kip-in.}$$

$$M_y = -(1.5)(3) - (1.5)(6) + (1.5)(6) = -4.5 \text{ kip-in}$$

(a) $$\sigma_A = \frac{P}{A} - \frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = \frac{15}{7} - \frac{(22.5)(-2.5)}{20.583} + \frac{(-4.5)(1.5)}{8.5833} = 4.09 \text{ ksi}$$

$$\sigma_B = \frac{P}{A} - \frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = \frac{15}{7} - \frac{(22.5)(2.5)}{20.583} + \frac{(-4.5)(1.5)}{8.5833} = -1.376 \text{ ksi}$$

(b) Let point H be the point where the neutral axis intersects AB.

$$z_H = 1.5, \quad y_H = ?, \quad \sigma_H = 0$$

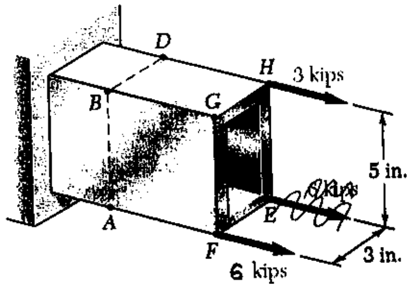
$$0 = \frac{P}{A} - \frac{M_z y_H}{I_z} + \frac{M_y z_H}{I_y}$$

$$y_H = \frac{I_z}{M_z}\left(\frac{P}{A} + \frac{M_y z_H}{I_y}\right) = \frac{20.583}{22.5}\left\{\frac{15}{7} + \frac{(-4.5)(1.5)}{8.5833}\right\} = 1.241 \text{ in.}$$

$$2.5 + 1.241 = 3.741 \text{ in.}$$

Answer: 3.741 in. above point A.

PROBLEM 4.163



4.162 The tube shown has a uniform wall thickness of 0.5 in. For the given loading, determine *(a)* the stress at points *A* and *B*, *(b)* the point where the neutral axis intersects line *ABD*.

4.163 Solve Prob. 4.162, assuming that the 6-kip force at point *E* is removed.

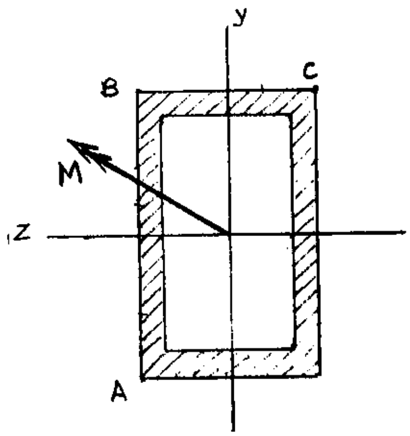
SOLUTION

Add y- and z-axes as shown.

$I_z = \frac{1}{12}(3)(5)^3 - \frac{1}{12}(2)(4)^3 = 20.583 \text{ in}^4$

$I_y = \frac{1}{12}(5)(3)^3 - \frac{1}{12}(4)(2)^3 = 8.5833 \text{ in}^4$

$A = (3)(5) - (2)(4) = 7 \text{ in}^2$



Resultant force and couples

$P = 3 + 6 = 9 \text{ kips}$

$M_z = -(2.5)(3) + (2.5)(6) = 7.5 \text{ kip}\cdot\text{in.}$

$M_y = -(1.5)(3) + (1.5)(6) = 4.5 \text{ kip}\cdot\text{in.}$

(a) $\sigma_A = \frac{P}{A} - \frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = \frac{9}{7} - \frac{(7.5)(-2.5)}{20.583} + \frac{(4.5)(1.5)}{8.5833} = 2.98 \text{ ksi}$ ◄

$\sigma_B = \frac{P}{A} - \frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = \frac{9}{7} - \frac{(7.5)(2.5)}{20.583} + \frac{(4.5)(1.5)}{8.5833} = 1.161 \text{ ksi}$ ◄

(b) Let point K be the point where the neutral axis intersects BC.

$y_K = 2.5 \text{ in}, \quad z_K = ?, \quad \sigma_K = 0$

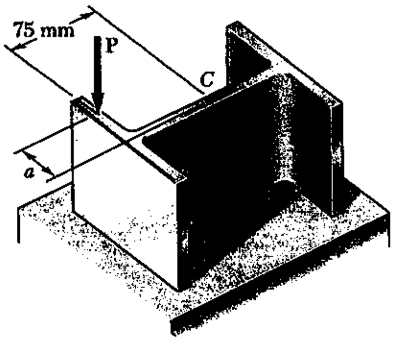
$0 = \frac{P}{A} - \frac{M_z y_K}{I_z} + \frac{M_y z_K}{I_y}$

$z_K = \frac{I_y}{M_y}\left(\frac{M_z y_K}{I_z} - \frac{P}{A}\right) = \frac{8.5833}{4.5}\left\{\frac{(7.5)(2.5)}{20.583} - \frac{9}{7}\right\} = -0.715 \text{ in.}$

$1.5 + 0.715 = 2.215 \text{ in}$

Answer: 2.215 in. to the right of point B. ◄

PROBLEM 4.164



4.164 An axial load P of magnitude 50 kN is applied as shown to a short section of a W 150 × 24 rolled-steel member. Determine the largest distance a for which the maximum compressive stress does not exceed 90 MPa.

SOLUTION

Add y- and z- axes.

For W 150 × 24 rolled-steel section

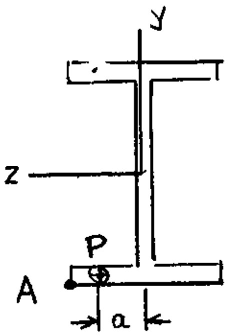
$A = 3060 \text{ mm}^2 = 3060 \times 10^{-6} \text{ m}^2$

$I_z = 13.4 \times 10^6 \text{ mm}^4 = 13.4 \times 10^{-6} \text{ m}^4$

$I_y = 1.83 \times 10^6 \text{ mm}^4 = 1.83 \times 10^{-6} \text{ m}^4$

$d = 160 \text{ mm}, \quad b_f = 102 \text{ mm}$

$y_A = -\frac{d}{2} = -80 \text{ mm}, \quad z_A = \frac{b_f}{2} = 51 \text{ mm}.$



$P = 50 \times 10^3 \text{ N}$

$M_z = -(50 \times 10^3)(75 \times 10^{-3}) = -3.75 \times 10^3 \text{ N·m}$

$M_y = -Pa$

$$\sigma_A = -\frac{P}{A} - \frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} \qquad \sigma_A = -90 \times 10^6 \text{ Pa}$$

$$M_y = \frac{I_y}{z_A}\left\{\frac{M_z y_A}{I_z} + \frac{P}{A} + \sigma_A\right\}$$

$$= \frac{1.83 \times 10^{-6}}{51 \times 10^{-3}}\left\{\frac{(-3.75 \times 10^3)(-80 \times 10^{-3})}{13.4 \times 10^{-6}} + \frac{50 \times 10^3}{3060 \times 10^{-6}} + (-90 \times 10^6)\right\}$$

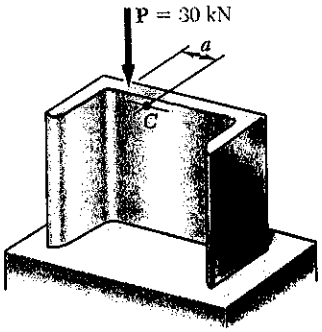
$$= \frac{1.83 \times 10^{-6}}{51. \times 10^{-3}}\left\{+22.388 + 16.340 - 90\right\} \times 10^6$$

$$= -1.8398 \times 10^3 \text{ N·m}$$

$$a = -\frac{M_y}{P} = -\frac{-1.8398 \times 10^3}{50 \times 10^3} = 36.8 \times 10^{-3} \text{ m} = 36.8 \text{ mm}$$

PROBLEM 4.165

4.165 An axial load **P** of magnitude 30 kN is applied as shown to a short section of a C 150 × 12.2 rolled-steel channel. Determine the largest distance a for which the maximum compressive stress is 60 MPa.

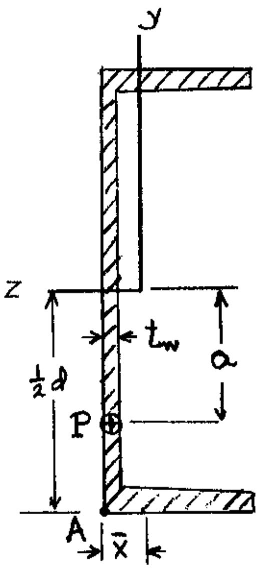


SOLUTION

Add y- and z- axes as shown

For C 150 × 12.2 rolled steel section

$A = 1540 \text{ mm}^2 = 1540 \times 10^{-6} \text{ m}^2$
$d = 152 \text{ mm}$
$b_f = 48 \text{ mm}$
$t_w = 5.1 \text{ mm}$
$I_z = 5.35 \times 10^6 \text{ mm}^4 = 5.35 \times 10^{-6} \text{ m}^4$
$I_y = 0.276 \times 10^6 \text{ mm}^4 = 0.276 \times 10^{-6} \text{ m}^4$
$\bar{x} = 12.7 \text{ mm}$



Line of action of force P

$y_P = -a \qquad z_P = \bar{x} - \frac{1}{2}t_w = 10.15 \text{ mm}$

$P = 30 \times 10^3 \text{ N}$

$M_y = -Pz_P = -(30 \times 10^3)(10.15 \times 10^{-3}) = -304.5 \text{ N}\cdot\text{m}$

$M_z = -Pa \qquad \sigma_A = -60 \times 10^6 \text{ Pa}$

$y_A = -\frac{1}{2}d = -76 \text{ mm} \qquad z_A = \bar{x} = 12.7 \text{ mm}$

$$\sigma_A = -\frac{P}{A} - \frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y}$$

$$M_z = \frac{I_z}{y_A}\left\{\frac{M_y z_A}{I_y} + \frac{P}{A} - \sigma_A\right\}$$

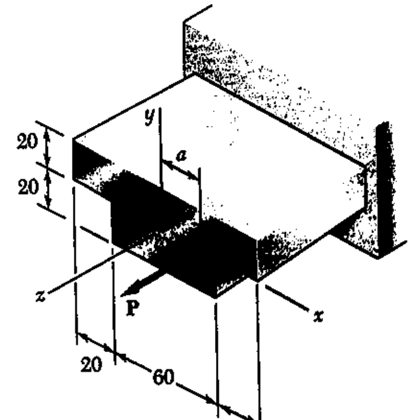
$$= \frac{5.35 \times 10^{-6}}{-76 \times 10^{-3}}\left\{\frac{(-304.5)(12.7 \times 10^{-3})}{0.276 \times 10^{-6}} - \frac{30 \times 10^3}{1540 \times 10^{-6}} + 60 \times 10^6\right\}$$

$$-\frac{5.35 \times 10^{-6}}{76 \times 10^{-3}}\{-14.011 - 19.481 + 60\} \times 10^6 = -1.866 \times 10^3 \text{ N}\cdot\text{m}$$

$$a = -\frac{M_z}{P} = -\frac{(-1.866 \times 10^3)}{30 \times 10^3} = 62.2 \times 10^{-3} \text{ m} = 62.2 \text{ mm}$$

PROBLEM 4.166

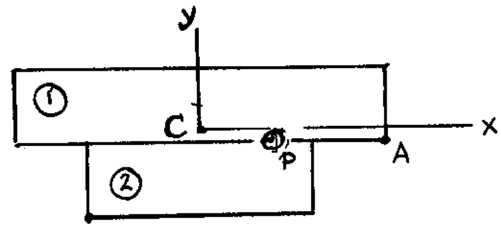
4.166 A horizontal load **P** is applied to the beam shown. Knowing that $a = 20$ mm and that the tensile stress in the beam is not to exceed 75 MPa, determine the largest permissible load **P**.



Dimensions in mm

SOLUTION

Locate the centroid



	A, mm^2	$\bar{y}$, mm	$A\bar{y}$, mm^3
1	2000	10	20×10^3
2	1200	-10	-12×10^3
Σ	3200		8×10^3

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{8 \times 10^3}{3200} = 2.5 \text{ mm}$$

Move coordinate origin to the centroid.
Coordinates of load point: $x_P = a$, $y_P = -2.5$ mm

Bending couples $\quad M_x = y_P P \qquad M_y = -aP$

$$I_x = \tfrac{1}{12}(100)(20)^3 + (2000)(7.5)^2 + \tfrac{1}{12}(60)(20)^3 + (1200)(12.5)^2 = 0.40667 \times 10^6 \text{ mm}^4 = 0.40667 \times 10^{-6} \text{ m}^4$$

$$I_y = \tfrac{1}{12}(20)(100)^3 + \tfrac{1}{12}(20)(60)^3 = 2.0267 \times 10^6 \text{ mm}^4 = 2.0267 \times 10^{-6} \text{ m}^4$$

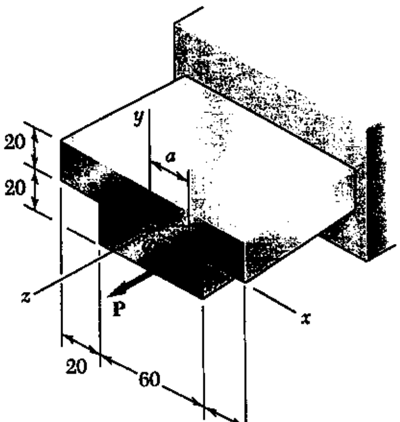
$$\sigma = \frac{P}{A} + \frac{M_x y}{I_x} - \frac{M_y x}{I_y} = P\left\{\frac{1}{A} + \frac{y_P y}{I_x} + \frac{a x}{I_y}\right\} = KP$$

For point A $\quad K_A = \dfrac{1}{3200 \times 10^{-6}} + \dfrac{(-2.5 \times 10^{-3})(-2.5 \times 10^{-3})}{0.40667 \times 10^{-6}} + \dfrac{(20 \times 10^{-3})(50 \times 10^{-3})}{2.0267 \times 10^{-6}}\Big\}$

$$= 821.28 \text{ m}^{-2}$$

$$P = \frac{\sigma_A}{K_A} = \frac{75 \times 10^6}{821.28} = 91.3 \times 10^3 \text{ N} = 91.3 \text{ kN}$$

PROBLEM 4.167

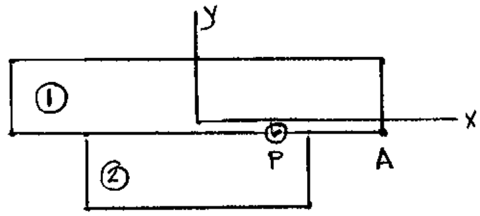


Dimensions in mm

4.167 A horizontal load **P** of magnitude 100 kN is applied to the beam shown. Determine the largest distance *a* for which the maximum tensile stress in the beam does not exceed 75 MPa.

SOLUTION

Locate the centroid



	A, mm^2	$\bar{y}$, mm	$A\bar{y}$, mm^3
1	2000	10	20×10^3
2	1200	−10	-12×10^3
Σ	3200		8×10^3

$$\bar{Y} = \frac{\Sigma A\bar{y}}{\Sigma A} = \frac{8\times10^3}{3200} = 2.5 \text{ mm}$$

Move coordinate origin to the centroid

Coordinates of load point: $x_P = a$, $y_P = -2.5$ mm

Bending couples $M_x = y_P P$ $M_y = -aP$

$$I_x = \tfrac{1}{12}(100)(20)^3 + (2000)(7.5)^2 + \tfrac{1}{12}(60)(20)^3 + (1200)(12.5)^2 = 0.40667\times10^6 \text{ mm}^4 = 0.40667\times10^{-6} \text{ m}^4$$

$$I_y = \tfrac{1}{12}(20)(100)^3 + \tfrac{1}{12}(20)(60)^3 = 2.0267\times10^6 \text{ mm}^4 = 2.0267\times10^{-6} \text{ m}^4$$

$$\sigma = \frac{P}{A} + \frac{M_x y}{I_x} - \frac{M_y x}{I_y} \qquad \sigma_A = 75\times10^6 \text{ Pa}, \quad P = 100\times10^3 \text{ N}$$

$$M_y = \frac{I_y}{x}\left\{\frac{P}{A} + \frac{M_x y}{I_x} - \sigma\right\}$$

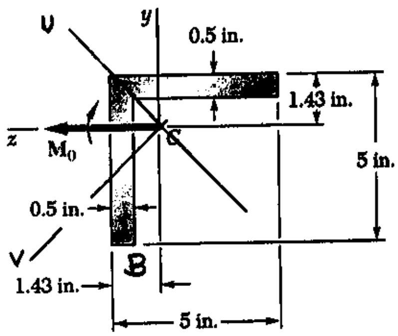
For point A $x = 50$ mm, $y = -2.5$ mm

$$M_y = \frac{2.0267\times10^{-6}}{50\times10^{-3}}\left\{\frac{100\times10^3}{3200\times10^{-6}} + \frac{(-2.5)(100\times10^3)(-2.5\times10^{-3})}{0.40667\times10^{-6}} - 75\times10^6\right\}$$

$$= \frac{2.0267\times10^{-6}}{50\times10^{-3}}\left\{31.25 + 1.537 - 75\right\}\times10^6 = -1.7111\times10^3 \text{ N}\cdot\text{m}$$

$$a = -\frac{M_y}{P} = -\frac{(1.7111\times10^3)}{100\times10^3} = 17.11\times10^3 \text{ m} = 17.11 \text{ mm}$$

PROBLEM 4.168



4.168 A beam having the cross section shown is subjected to a couple $\mathbf{M}_0$ which acts in a vertical plane. Determine the largest permissible value of the moment M_0 of the couple if the maximum stress in the beam is not to exceed 12 ksi. *Given:* $I_y = I_z = 11.3$ in^4, $A = 4.75$ in^2, $k_{min} = 0.983$ in. (*Hint:* By reason of symmetry, the principal axes form an angle of 45° with the coordinate axes. Use the relations $I_{min} = Ak_{min}^2$ and $I_{min} + I_{max} = I_y + I_z$.

SOLUTION

$$M_U = M_o \sin 45^\circ = 0.70711\, M_o$$

$$M_V = M_o \cos 45^\circ = 0.7071\, M_o$$

$$I_{min} = A k_{min}^2 = (4.75)(0.983)^2 = 4.59 \text{ in}^4$$

$$I_{max} = I_y + I_z - I_{min} = 11.3 + 11.3 - 4.59 = 18.01 \text{ in}^4$$

$$U_B = y_B \cos 45^\circ + z_B \sin 45^\circ = -3.57 \cos 45^\circ + 0.93 \sin 45^\circ = -1.866 \text{ in.}$$

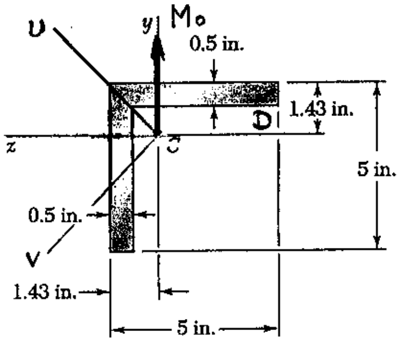
$$V_B = z_B \cos 45^\circ - y_B \sin 45^\circ = -0.93 \cos 45^\circ - (-3.57) \sin 45^\circ = 3.182 \text{ in}$$

$$\sigma_B = -\frac{M_V U_B}{I_V} + \frac{M_U V_B}{I_U} = 0.70711\, M_o \left[-\frac{U_B}{I_{min}} + \frac{V_B}{I_{max}} \right]$$

$$= 0.70711\, M_o \left[-\frac{(-1.866)}{4.59} + \frac{3.182}{18.01} \right] = 0.4124\, M_o$$

$$M_o = \frac{\sigma_B}{0.4124} = \frac{12}{0.4124} = 29.1 \text{ kip} \cdot \text{in}$$

PROBLEM 4.169



4.168 A beam having the cross section shown is subjected to a couple $\mathbf{M}_0$ which acts in a vertical plane. Determine the largest permissible value of the moment M_0 of the couple if the maximum stress in the beam is not to exceed 12 ksi. *Given:* $I_y = I_z = 11.3$ in^4, $A = 4.75$ in^2, $k_{min} = 0.983$ in. (*Hint*: By reason of symmetry, the principal axes form an angle of 45° with the coordinate axes. Use the relations $I_{min} = Ak_{min}^2$ and $I_{min} + I_{max} = I_x + I_z$.

4.169 Solve Prob. 4.168, assuming that the couple $\mathbf{M}_0$ acts in a horizontal plane.

SOLUTION

$$M_U = M_o \cos 45° = 0.70711\, M_o$$

$$M_V = -M_o \sin 45° = -0.70711\, M_o$$

$$I_{min} = A k_{min}^2 = (4.75)(0.983)^2 = 4.59 \text{ in}^4$$

$$I_{max} = I_y + I_z - I_{min} = 11.3 + 11.3 - 4.59 = 18.01 \text{ in}^4$$

$$U_D = y_D \cos 45° + z_D \sin 45° = 0.93 \cos 45° + (-3.57 \sin 45°) = -1.866 \text{ in.}$$

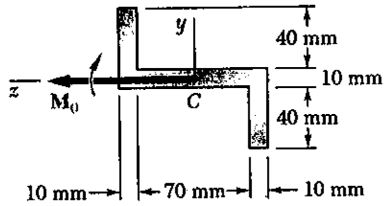
$$V_D = z_D \cos 45° - y_D \sin 45° = (-3.57)\cos 45° - (0.93)\sin 45° = 3.182 \text{ in.}$$

$$\sigma_D = -\frac{M_V U_D}{I_V} + \frac{M_U V_D}{I_U} = 0.70711\, M_o \left[-\frac{U_D}{I_{min}} + \frac{V_D}{I_{max}} \right]$$

$$= 0.70711\, M_o \left[-\frac{(-1.866)}{4.59} + \frac{3.182}{18.01} \right] = 0.4124\, M_o$$

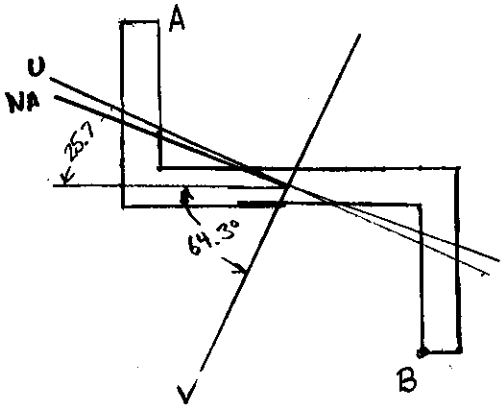
$$M_o = \frac{\sigma_D}{0.4124} = \frac{12}{0.4124} = 29.1 \text{ kip}\cdot\text{in}$$ ◄

PROBLEM 4.170



4.170 The Z section shown is subjected to a couple $\mathbf{M}_0$ acting in a vertical plane. Determine the largest permissible value of the moment M_0 of the couple if the maximum stress is not to exceed 80 MPa. *Given*: $I_{max} = 2.28 \times 10^6$ mm^4, $I_{min} = 0.23 \times 10^6$ mm^4, principal axes 25.7° ↘ and 64.3° ↗.

SOLUTION



$$I_v = I_{max} = 2.28 \times 10^6 \text{ mm}^4 = 2.28 \times 10^{-6} \text{ m}^4$$

$$I_u = I_{min} = 0.23 \times 10^6 \text{ mm}^4 = 0.23 \times 10^{-6} \text{ m}^4$$

$$M_v = M_o \cos 64.3°$$

$$M_u = M_o \sin 64.3°$$

$$\theta = 64.3°$$

$$\tan\varphi = \frac{I_v}{I_u}\tan\theta$$

$$= \frac{2.28\times10^{-6}}{0.23\times10^{-6}}\tan 64.3° = 20.597$$

$$\varphi = 87.22°$$

Points A and B are farthest from the neutral axis.

$$u_B = y_B \cos 64.3° + z_B \sin 64.3° = (-45)\cos 64.3° + (-35)\sin 64.3°$$

$$= -51.05 \text{ mm}$$

$$v_B = z_B \cos 64.3° - y_B \sin 64.3° = (-35)\cos 64.3° - (-45)\sin 64.3°$$

$$= +25.37 \text{ mm}$$

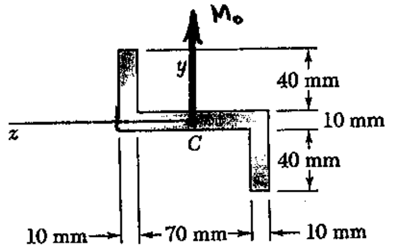
$$\sigma_B = -\frac{M_v u_B}{I_v} + \frac{M_u v_B}{I_u}$$

$$80\times10^6 = -\frac{(M_o\cos 64.3°)(-51.05\times10^{-3})}{2.28\times10^{-6}} + \frac{(M_o \sin 64.3°)(25.37\times10^{-3})}{0.23\times10^{-6}}$$

$$= 93.81 \times 10^3 \; M_o$$

$$M_o = \frac{80\times10^6}{109.1\times10^3} = 733 \text{ N}\cdot\text{m}$$

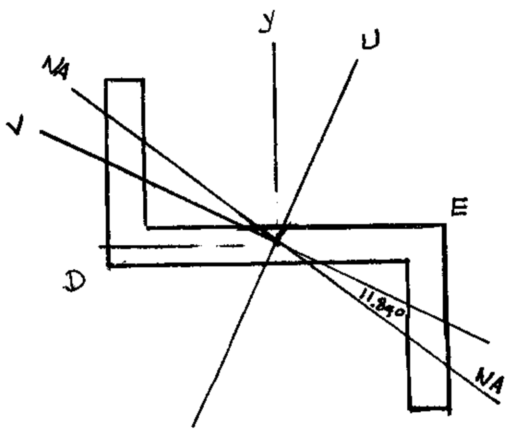
PROBLEM 4.171



4.170 The Z section shown is subjected to a couple $\mathbf{M}_0$ acting in a vertical plane. Determine the largest permissible value of the moment M_0 of the couple if the maximum stress is not to exceed 80 MPa. *Given:* $I_{max} = 2.28 \times 10^6$ mm^4, $I_{min} = 0.23 \times 10^6$ mm^4, principal axes 25.7° ↘ and 64.3° ↗.

4.171 Solve Prob. 4.170, assuming that the couple $\mathbf{M}_0$ acts in a horizontal plane

SOLUTION



$I_V = I_{min} = 0.23 \times 10^6 \text{ mm}^4 = 0.23 \times 10^6 \text{ m}^4$

$I_U = I_{max} = 2.23 \times 10^6 \text{ mm}^4 = 2.23 \times 10^6 \text{ m}^4$

$M_V = M_0 \cos 64.3°$

$M_U = M_0 \sin 64.3°$

$\theta = 64.3°$

$\tan \varphi = \dfrac{I_V}{I_U} \tan \theta$

$= \dfrac{0.23 \times 10^{-6}}{2.28 \times 10^{-6}} \tan 64.3° = 0.20961$

$\varphi = 11.84°$

Points D and E are farthest from the neutral axis.

$U_D = y_D \cos 25.7° - z_D \sin 25.7° = (-5)\cos 25.7° - 45 \sin 25.7°$

$= -24.02$ mm

$V_D = z_D \cos 25.7° + y_D \sin 25.7° = 45 \cos 25.7° + (-5) \sin 25.7°$

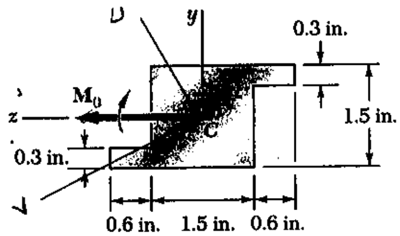
$= 38.38$ mm

$$\sigma_D = -\frac{M_V U_D}{I_V} + \frac{M_U V_D}{I_U} = -\frac{(M_0 \cos 64.3°)(-24.02 \times 10^{-3})}{0.23 \times 10^{-6}} + \frac{(M_0 \sin 64.3°)(38.38 \times 10^{-3})}{2.28 \times 10^{-6}}$$

$80 \times 10^6 = 60.48 \times 10^3 \; M_0$

$M_0 = 1.323 \times 10^3 \text{ N·m} = 1.323 \text{ kN·m}$

PROBLEM 4.172



4.172 An extruded aluminum member having the cross section shown is subjected to a couple $\mathbf{M}_0$ acting in a vertical plane. Determine the largest permissible value of the moment M_0 of the couple if the maximum stress is not to exceed 12 ksi. *Given*: $I_{max} =$ 0.957 in^4, $I_{min} = 0.427$ in^4, principal axes 29.4° ↗ and 60.6° ↘.

SOLUTION

$I_u = I_{max} = 0.957 \text{ in}^4$

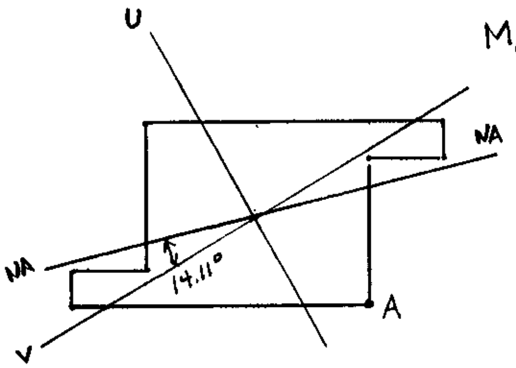
$I_v = I_{min} = 0.427 \text{ in}^4$

$M_u = M_0 \sin 29.4°$, $M_v = M_0 \cos 29.4°$

$\theta = 29.4°$

$\tan\varphi = \frac{I_v}{I_u}\tan\theta = \frac{0.427}{0.957}\tan 29.4°$

$= 0.2514 \qquad \varphi = 14.11°$



Point A is farthest from the neutral axis.

$y_A = -0.75 \text{ in}$, $z_A = -0.75 \text{ in.}$

$u_A = y_A \cos 29.4° + z_A \sin 29.4° = -1.0216 \text{ in}$

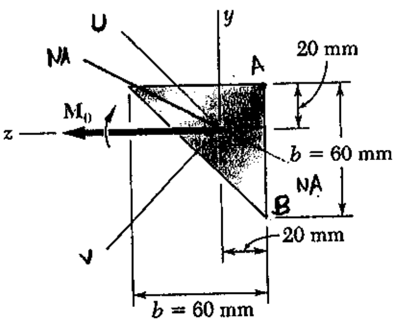
$v_A = z_A \cos 29.4° - y_A \sin 29.4° = -0.2852 \text{ in.}$

$$\sigma_A = -\frac{M_v u_A}{I_v} + \frac{M_u v_A}{I_u} = -\frac{(M_0 \cos 29.4°)(-1.0216)}{0.427} + \frac{(M_0 \sin 29.4°)(-0.2852)}{0.957}$$

$$= 1.9381\ M_0$$

$$M_0 = \frac{\sigma_A}{1.9381} = \frac{12}{1.9381} = 6.19 \text{ ksi}$$

PROBLEM 4.173



4.173 A beam having the cross section shown is subjected to a couple $\mathbf{M}_0$ acting in a vertical plane. Determine the largest permissible value of the moment M_0 of the couple if the maximum stress is not to exceed 100 MPa. *Given:* $I_y = I_z = b^4/36$ and $I_{yz} = b^4/72$.

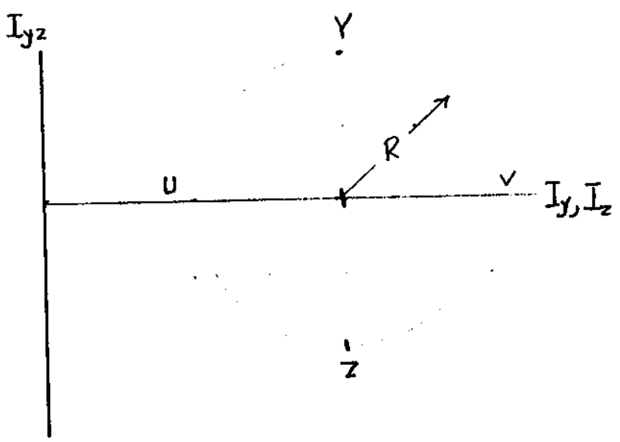
SOLUTION

$$I_y = I_z = \frac{b^4}{36} = \frac{60^4}{36} = 0.360 \times 10^6 \text{ mm}^4$$

$$I_{yz} = \frac{b^4}{72} = \frac{60^4}{72} = 0.180 \times 10^6 \text{ mm}^4$$

Principal axes are symmetry axes.

Using Mohr's circle determine the principal moments of inertia.



$$R = |I_{yz}| = 0.180 \times 10^6 \text{ mm}^4$$

$$I_v = \frac{I_y + I_z}{2} + R$$

$$= 0.540 \times 10^6 \text{ mm}^4 = 0.540 \times 10^{-6} \text{ m}^4$$

$$I_u = \frac{I_y + I_z}{2} - R$$

$$= 0.180 \times 10^6 \text{ mm}^4 = 0.180 \times 10^{-6} \text{ m}^4$$

$$M_u = M_o \sin 45° = 0.70711\, M_o, \quad M_v = M_o \cos 45° = 0.70711\, M_o$$

$$\theta = 45° \qquad \tan\varphi = \frac{I_v}{I_u}\tan\theta = \frac{0.540 \times 10^{-6}}{0.180 \times 10^{-6}} \tan 45° = 3$$

$$\varphi = 71.56°$$

Point A: $u_A = 0$. $v_A = -20\sqrt{2}$ mm

$$\sigma_A = -\frac{M_v u_A}{I_v} + \frac{M_u v_A}{I_u} = 0 + \frac{(0.70711\, M_o)(-20\sqrt{2} \times 10^{-3})}{0.180 \times 10^{-6}} = -111.11 \times 10^3\, M_o$$

$$M_o = -\frac{\sigma_A}{111.11 \times 10^3} = -\frac{-100 \times 10^6}{111.11 \times 10^3} = 900 \text{ N}\cdot\text{m}$$ ◄

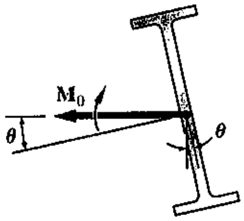
Point B: $u_B = -\frac{60}{\sqrt{2}}$ mm, $v_B = \frac{20}{\sqrt{2}}$ mm

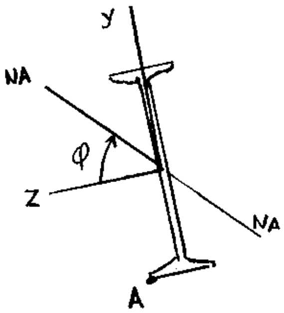
$$\sigma_B = -\frac{M_v u_B}{I_v} + \frac{M_u v_B}{I_u} = -\frac{(0.70711\, M_o)(-\frac{60}{\sqrt{2}} \times 10^{-3})}{0.540 \times 10^{-6}} + \frac{(0.70711\, M_o)(\frac{20}{\sqrt{2}} \times 10^{-3})}{0.180 \times 10^{-6}}$$

$$= 111.11 \times 10^3\, M_o$$

$$M_o = \frac{\sigma_B}{111.11 \times 10^{-3}} = \frac{100 \times 10^6}{111.11 \times 10^{-3}} = 900 \text{ N}\cdot\text{m}$$ ◄

PROBLEM 4.174





4.174 A couple $\mathbf{M}_0$ acting in a vertical plane is applied to a W 12 × 16 rolled-steel beam, whose web forms an angle θ with the vertical. Denoting by σ_0 the maximum stress in the beam when $\theta = 0$, determine the angle of inclination θ of the beam for which the maximum stress is $2\sigma_0$.

SOLUTION

For W 12 × 16 rolled steel section

$$I_z = 103 \text{ in}^4 \qquad I_y = 2.82 \text{ in}^4$$

$$d = 11.99 \text{ in} \qquad b_f = 3.990 \text{ in.}$$

$$y_A = -\frac{d}{2} \qquad z_A = \frac{b_f}{2}$$

$$\tan\varphi = \frac{I_z}{I_y}\tan\theta = \frac{103}{2.82}\tan\theta = 36.52\tan\theta$$

Point A is farthest from the neutral axis.

$$M_y = M_0 \sin\theta \qquad M_z = M_0 \cos\theta$$

$$\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = \frac{M_0 d}{2I_z}\cos\theta + \frac{M_0 b_f}{2I_y}\sin\theta$$

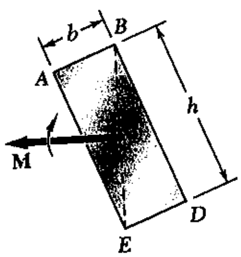
$$= \frac{M_0 d}{2I_z}\left(1 + \frac{I_z b_f}{I_y d}\tan\theta\right)$$

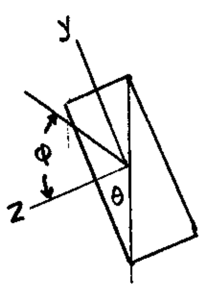
For $\theta = 0$ $\qquad \sigma_0 = \dfrac{M_0 d}{2I_z}$

$$\sigma_A = \sigma_0\left(1 + \frac{I_z b_f}{I_y d}\tan\theta\right) = 2\sigma_0$$

$$\tan\theta = \frac{I_y d}{I_z b_f} = \frac{(2.82)(11.99)}{(103)(3.990)} = 0.08273 \qquad \theta = 4.70^\circ$$

PROBLEM 4.175





4.175 Show that, if a solid rectangular beam is bent by a couple applied in a plane containing one diagonal of the rectangular cross section, the neutral axis will lie along the other diagonal.

SOLUTION

$$\tan\theta = \frac{b}{h}$$

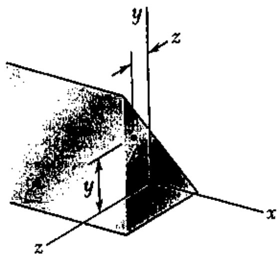
$$M_z = M\cos\theta, \qquad M_z = M\sin\theta$$

$$I_z = \frac{1}{12}bh^3 \qquad I_y = \frac{1}{12}hb^3$$

$$\tan\varphi = \frac{I_z}{I_y}\tan\theta = \frac{\frac{1}{12}bh^3}{\frac{1}{12}hb^3}\cdot\frac{b}{h} = \frac{h}{b}$$

Thus neutral axis passes through corner A

PROBLEM 4.176



4.176 A beam of unsymmetric cross section is subjected to a couple $\mathbf{M}_0$ acting in the vertical xy plane. Show that the stress at point A, of coordinates y and z, is

$$\sigma_A = -\frac{yI_y - zI_{yz}}{I_y I_z - I_{yz}^2} M_z$$

where I_y, I_z, and I_{yz} denote the moments and product of inertia of the cross section with respect to centroidal axes, and M_z the moment of the couple.

SOLUTION

The stress σ_A varies linearly with the coordinates y and z. Since the axial force is zero, the y- and z-axes are centroidal axes.

$\sigma_A = C_1 y + C_2 z$ where C_1, and C_2 are constants.

$$M_y = \int z\sigma_A \, dA = C_1 \int yz \, dA + C_2 \int z^2 \, dA$$

$$= I_{yz} C_1 + I_y C_2 = 0$$

$$C_2 = -\frac{I_{yz}}{I_y} C_1$$

$$M_z = -\int y \sigma_A \, dz = -C_1 \int y^2 \, dA + C_2 \int yz \, dA$$

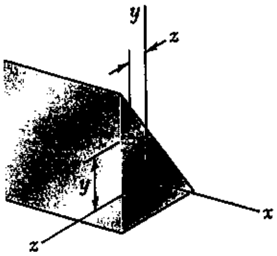
$$= -I_z C_1 - I_{yz} \frac{I_{yz}}{I_y} C_1$$

$$I_y M_z = -(I_y I_z - I_{yz}^2) C_1$$

$$C_1 = -\frac{I_y M_z}{I_y I_z - I_{yz}^2} \qquad C_2 = +\frac{I_{yz} M_z}{I_y I_z - I_{yz}^2}$$

$$\sigma_A = -\frac{I_y z - I_{yz} y}{I_y I_z - I_{yz}^2} M_z$$

PROBLEM 4.177



4.177 A beam of unsymmetric cross section is subjected to a couple $\mathbf{M}_0$ acting in the horizontal xz plane. Show that the stress at point A is

$$\sigma_A = -\frac{zI_z - yI_{yz}}{I_yI_z - I_{yz}^2} M_y$$

where I_y, I_z, and I_{yz} denote the moments and product of inertia of the cross section with respect to centroidal axes, and M_y the moment of the couple.

SOLUTION

The stress σ_A varies linearly with the coordinates y and z. Since the axial force is zero, the y- and z-axes are centroidal axes.

$\sigma_A = C_1 y + C_2 z$ where C_1 and C_2 are constants.

$$M_z = -\int y \sigma_A \, dA = -C_1 \int y^2 \, dA - C_2 \int yz \, dA$$

$$= -I_z C_1 - I_{yz} C_2 = 0$$

$$C_1 = -\frac{I_{yz}}{I_z} C_2$$

$$M_y = \int z \sigma_A \, dA = C_1 \int yz \, dA + C_2 \int z^2 \, dA$$

$$= I_{yz} C_1 + I_y C_2$$

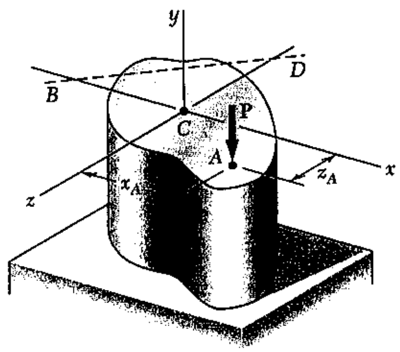
$$-I_{yz} \frac{I_{yz}}{I_z} C_2 + I_y C_2$$

$$I_z M_y = (I_y I_z - I_{yz}^2) C_2$$

$$C_2 = \frac{I_z M_y}{I_y I_z - I_{yz}^2} \qquad C_1 = -\frac{I_{yz} M_y}{I_y I_z - I_{yz}^2}$$

$$\sigma_A = \frac{I_z y - I_{yz} z}{I_y I_z - I_{yz}^2} M_y$$

PROBLEM 4.178



4.178 (*a*) Show that, if a vertical force **P** is applied at point *A* of the section shown, the equation of the neutral axis *BD* is

$$\left(\frac{x_A}{k_z^2}\right)x + \left(\frac{z_A}{k_x^2}\right)z = -1$$

where k_z and k_x denote the radius of gyration of the cross section with respect to the *z* axis and the *x* axis, respectively. (*b*) Further show that, if a vertical force **Q** is applied at any point located on line *BD*, the stress at point *A* will be zero.

SOLUTION

Definitions $\quad k_x^2 = \dfrac{I_x}{A}, \quad k_z^2 = \dfrac{I_z}{A}$

(a) $\quad M_x = P z_A \qquad M_z = -P x_A$

$$\sigma_E = -\frac{P}{A} + \frac{M_z x_E}{I_z} - \frac{M_x z_E}{I_x} = -\frac{P}{A} - \frac{P x_A x_E}{A k_z^2} - \frac{P z_A z_E}{A k_x^2}$$

$$= -\frac{P}{A}\left[1 + \left(\frac{x_A}{k_z^2}\right)x_E + \left(\frac{z_A}{k_x^2}\right)z_E\right] = 0 \quad \text{if } E \text{ lies on neutral axis.}$$

$$1 + \left(\frac{x_A}{k_z^2}\right)x + \left(\frac{z_A}{k_x^2}\right)z = 0, \qquad \left(\frac{x_A}{k_z^2}\right)x + \left(\frac{z_A}{k_x^2}\right)z = -1$$

(b) $\quad M_x = P z_E \qquad M_z = -P x_E$

$$\sigma_A = -\frac{P}{A} + \frac{M_z x_A}{I_z} - \frac{M_x z_A}{I_x} = -\frac{P}{A} - \frac{P x_E x_A}{A k_z^2} - \frac{P z_E z_A}{A k_x^2}$$

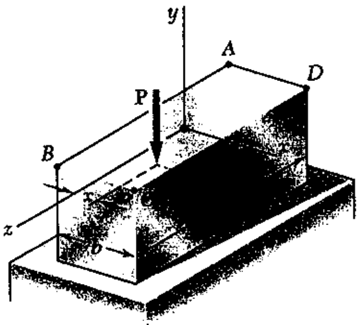
$= 0$ by equation from Part (a)

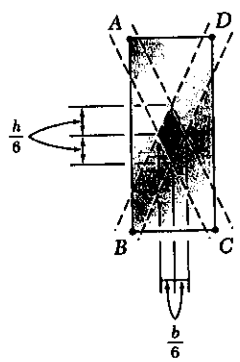
PROBLEM 4.179

4.179 (*a*) Show that the stress at corner *A* of the prismatic member shown in Fig. (*a*) will be zero if the vertical force **P** is applied at a point located on the line

$$\frac{x}{b/6}+\frac{z}{h/6}=1$$

(*b*) Further show that, if no tensile stress is to occur in the member, the force **P** must be applied at a point located within the area bounded by the line found in part *a* and the three similar lines corresponding to the condition of zero stress at *B, C,* and *D,* respectively. This area, shown in Fig.(*b*), is known as the *kern* of the cross section.





SOLUTION

$$I_z = \frac{1}{12}hb^3 \qquad I_x = \frac{1}{12}bh^3 \qquad A = bh$$

$$z_A = -\frac{h}{2} \qquad x_A = -\frac{b}{2}$$

Let P be the load point

$$M_z = -Px_P \qquad M_x = Pz_P$$

$$\sigma_A = -\frac{P}{A} + \frac{M_z x_A}{I_z} - \frac{M_x z_A}{I_x}$$

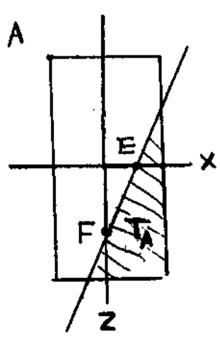
$$= -\frac{P}{bh} + \frac{(-Px_P)(-\frac{b}{2})}{\frac{1}{12}hb^3} - \frac{Pz_P(-\frac{h}{2})}{\frac{1}{12}bh^3}$$

$$= -\frac{P}{bh}\left[1 - \frac{x_P}{b/6} - \frac{z_P}{h/6}\right]$$

For $\sigma_A = 0$ $\qquad 1 - \frac{x}{b/6} - \frac{z}{h/6} = 0, \quad \frac{x}{b/6} + \frac{z}{h/6} = 1$

At point E $\qquad z = 0 \quad \therefore \; x_E = b/6$

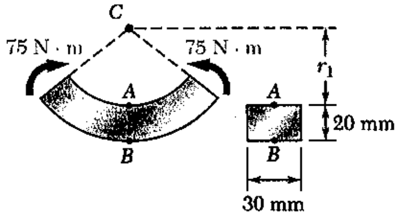
At point F $\qquad x = 0 \quad \therefore \; z_F = h/6$



If the line of action (x_P, z_P) lies within the portion marked T_A, a tensile will occur at corner A.

By considering $\sigma_B = 0$, $\sigma_C = 0$, and $\sigma_D = 0$, the other portions producing tensile stresses are identified.

PROBLEM 4.180



4.180 For the curved bar and loading shown, determine the stress at point A when (*a*) $r_1 = 30$ mm, (*b*) $r_1 = 50$ mm.

SOLUTION

(a) $r_1 = 30 \text{ mm} \qquad r_2 = 30 + 20 = 50 \text{ mm}$

$$R = \frac{h}{\ln \frac{r_2}{r_1}} = \frac{20}{\ln \frac{50}{30}} = 39.1523 \text{ mm}$$

$$\bar{r} = \tfrac{1}{2}(r_1 + r_2) = 40 \text{ mm}$$

$$e = \bar{r} - R = 0.8477 \text{ mm}$$

$$A = (20)(30) = 600 \text{ mm}^2 = 600 \times 10^{-6} \text{ m}^2$$

$$y_A = 39.1523 - 30 = 9.1523 \text{ mm}$$

$$\sigma_A = -\frac{My}{Aer} = -\frac{(75)(9.1523 \times 10^{-3})}{(600 \times 10^{-6})(0.8477 \times 10^{-3})(30 \times 10^{-3})} = -45.0 \times 10^6 \text{ Pa}$$

$$= -45.0 \text{ MPa}$$ ◄

(b) $r_1 = 50 \text{ mm}, \qquad r_2 = 50 + 20 = 70 \text{ mm}$

$$R = \frac{h}{\ln \frac{r_2}{r_1}} = \frac{20}{\ln \frac{70}{50}} = 59.44027$$

$$\bar{r} = \tfrac{1}{2}(r_1 + r_2) = 60 \text{ mm}$$

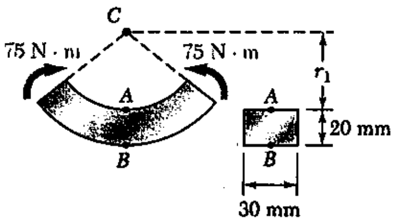
$$e = \bar{r} - R = 0.55973 \text{ mm}$$

$$y_A = 59.44027 - 50 = 9.44027 \text{ mm}$$

$$\sigma_A = -\frac{My_A}{Aer_A} = -\frac{(75)(9.44027)}{(600 \times 10^{-6})(0.55973 \times 10^{-3})(50 \times 10^{-3})} = -42.2 \times 10^6$$

$$= -42.2 \text{ MPa}$$ ◄

PROBLEM 4.181



4.181 For the curved bar and loading shown, determine the stress at points *A* and *B* when $r_1 = 40$ mm.

SOLUTION

$h = 20 \text{ mm} \qquad r_1 = 40 \text{ mm} \qquad r_2 = 40 + 20 = 60 \text{ mm}$

$A = (30)(20) = 600 \text{ mm}^2 = 600 \times 10^{-6} \text{ m}^2$

$R = \dfrac{h}{\ln \frac{r_2}{r_1}} = \dfrac{20}{\ln \frac{60}{40}} = 49.3261 \text{ mm}$

$\bar{r} = \frac{1}{2}(r_1 + r_2) = 50 \text{ mm}$

$e = \bar{r} - R = 0.6739 \text{ mm}$

$y_A = 49.3261 - 40 = 9.3261 \text{ mm} \qquad r_A = 40 \text{ mm}$

$$\sigma_A = -\frac{M y_A}{A e r_A} = -\frac{(75)(9.3261 \times 10^{-3})}{(600 \times 10^{-6})(0.6739 \times 10^{-3})(40 \times 10^{-3})} = -43.2 \times 10^6 \text{ Pa}$$

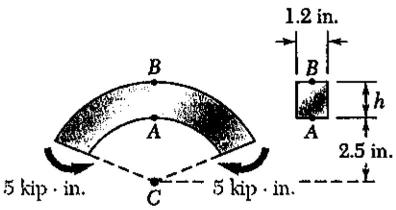
$= -43.2$ MPa ◄

$y_B = 49.3261 - 60 = -10.6739 \text{ mm}$

$$\sigma_B = -\frac{M y_B}{A e r_B} = -\frac{(75)(-10.6739 \times 10^{-3})}{(600 \times 10^{-6})(0.6739 \times 10^{-3})(60 \times 10^{-3})} = 33.0 \times 10^6 \text{ Pa}$$

$= 33.0$ MPa ◄

PROBLEM 4.182



4.182 For the curved bar and loading shown, determine the stress at point A when (*a*) $h = 2.5$ in., (*b*) $h = 3$ in.

SOLUTION

(a) $h = 2.5$ in, $r_1 = 2.5$ in. $r_2 = 5$ in.

$A = (1.2)(2.5) = 3.00 \text{ in}^2$, $M = 5$ kip·in.

$$R = \frac{h}{\ln \frac{r_2}{r_1}} = \frac{2.5}{\ln \frac{5}{2.5}} = 3.6067$$

$$\bar{r} = \frac{1}{2}(r_1 + r_2) = 3.75$$

$$e = \bar{r} - R = 0.1433 \text{ in.}$$

$y_A = 3.6067 - 2.5 = 1.1067$ in $\qquad r_A = 2.5$ in

$$\sigma_A = -\frac{My_A}{Aer_A} = -\frac{(5)(1.1067)}{(3.00)(0.1433)(2.5)} = -5.15 \text{ ksi}$$

(b) $h = 3$ in, $r_1 = 2.5$ in $r_2 = 5.5$ in. $A = (1.2)(3) = 3.6 \text{ in}^2$

$$R = \frac{h}{\ln \frac{r_2}{r_1}} = \frac{3}{\ln \frac{5.5}{2.5}} = 3.8049 \text{ in.}$$

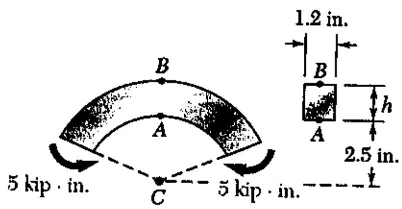
$\bar{r} = \frac{1}{2}(r_1 + r_2) = 4.0000$ in, $\qquad e = \bar{r} - R = 0.1951$ in.

$y_A = 3.8049 - 2.5 = 1.3049$ in $\qquad r_A = 2.5$ in

$$\sigma_A = -\frac{My_A}{Aer_A} = -\frac{(5)(1.3049)}{(3.6)(0.1951)(2.5)} = -3.72 \text{ ksi}$$

PROBLEM 4.183

4.183 For the curved bar and loading shown, determine the stress at points A and B when $h = 2.75$ in.



SOLUTION

$h = 2.75$ in. $\quad r_1 = 2.5$ in, $\quad r_2 = 5.25$ in

$A = (1.2)(2.75) = 3.30 \text{ in}^2, \quad M = 5$ kip·in.

$$R = \frac{h}{\ln \frac{r_2}{r_1}} = \frac{2.75}{\ln \frac{5.25}{2.5}} = 3.7065 \text{ in.}$$

$\bar{r} = \frac{1}{2}(r_1 + r_2) = 3.875$ in. $\quad e = \bar{r} - R = 0.1685$ in.

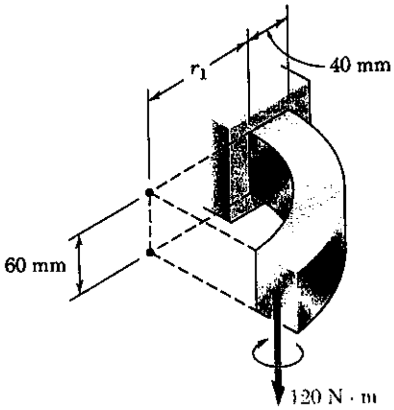
$y_A = 3.7065 - 2.5 = 1.2065$ in. $\quad r_A = 2.5$ in.

$$\sigma_A = -\frac{M y_A}{A e r_A} = -\frac{(5)(1.2065)}{(3.30)(0.1685)(2.5)} = -4.34 \text{ ksi}$$ ◄

$y_B = 3.7065 - 5.25 = -1.5435$ in. $\quad r_B = 5.25$ in

$$\sigma_B = -\frac{M y_B}{A e r_B} = -\frac{(5)(-1.5435)}{(3.30)(0.1685)(5.25)} = 2.64 \text{ ksi}$$ ◄

PROBLEM 4.184



4.184 The curved bar shown has a cross section of 40 × 60 mm and an inner radius $r_1 = 15$ mm. For the loading shown determine the largest tensile and compressive stresses.

SOLUTION

$h = 40 \text{ mm}, \quad r_1 = 15 \text{ mm}, \quad r_2 = 55 \text{ mm}$

$A = (60)(40) = 2400 \text{ mm}^2 = 2400 \times 10^{-6} \text{ m}^2$

$R = \dfrac{h}{\ln \frac{r_2}{r_1}} = \dfrac{40}{\ln \frac{55}{40}} = 30.786 \text{ mm}$

$\bar{r} = \frac{1}{2}(r_1 + r_2) = 35 \text{ mm}$

$e = \bar{r} - R = 4.214 \text{ mm} \qquad \sigma = -\dfrac{My}{Aer}$

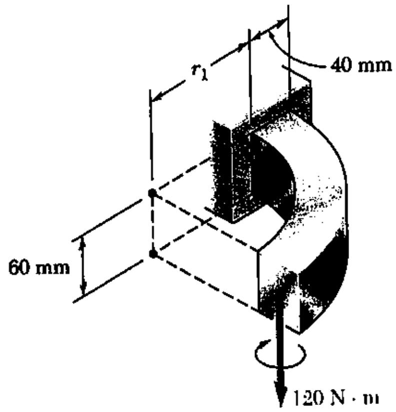
At $r = 15$ mm $\quad y = 30.786 - 15 = 15.786$ mm

$$\sigma = -\frac{(120)(15.786 \times 10^{-3})}{(2400 \times 10^{-6})(4.214 \times 10^{-3})(15 \times 10^{-3})} = -12.49 \times 10^6 \text{ Pa} = -12.49 \text{ MPa}$$

At $r = 55$ mm $\quad y = 30.786 - 55 = -24.214$ mm

$$\sigma = -\frac{(120)(-24.214 \times 10^{-3})}{(2400 \times 10^{-6})(4.214 \times 10^{-3})(55 \times 10^{-3})} = 5.22 \times 10^6 \text{ Pa} = 5.22 \text{ MPa}$$

PROBLEM 4.185



4.185 For the curved bar and loading shown, determine the percent error introduced in the computation of the maximum stress by assuming that the bar is straight. Consider the case when (*a*) $r_1 = 20$ mm, (*b*) $r_1 = 200$ mm, (*c*) $r_1 = 2$ m.

SOLUTION

$h = 40\text{ mm}, \quad A = (60)(40) = 2400\text{ mm}^2 = 2400\times10^{-6}\text{ m}^2$

$M = 120\text{ N}\cdot\text{m}$

$I = \frac{1}{12}bh^3 = \frac{1}{12}(60)(40)^3 = 0.32\times10^6\text{ mm}^4 = 0.32\times10^{-6}\text{ m}^4$

$c = \frac{1}{2}h = 20\text{ mm}$

Assuming that the bar is straight

$$\sigma_s = -\frac{Mc}{I} = -\frac{(120)(20\times10^{-3})}{0.32\times10^{-6}} = 7.5\times10^6\text{ Pa} = 7.5\text{ MPa}$$

(a) $r_1 = 20\text{ mm} \qquad r_2 = 60\text{ mm}$

$$R = \frac{h}{\ln\frac{r_2}{r_1}} = \frac{40}{\ln\frac{60}{20}} = 36.4096\text{ mm} \qquad r_1 - R = -16.4096\text{ mm}$$

$$\bar{r} = \frac{1}{2}(r_1 + r_2) = 40\text{ mm} \qquad e = \bar{r} - R = 3.5904\text{ mm}$$

$$\sigma_a = \frac{M(r_1-R)}{Aer} = \frac{(120)(-16.4096\times10^{-3})}{(2400\times10^{-6})(3.5904\times10^{-3})(20\times10^{-3})} = -11.426\times10^6\text{ Pa} = -11.426\text{ MPa}$$

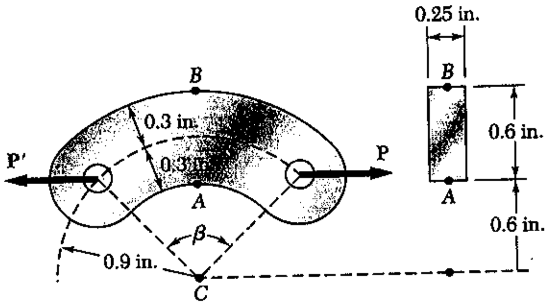
$$\%\text{ error} = \frac{-11.426 - (-7.5)}{-11.426}\times100\% = -34.4\%$$

For parts (b) and (c) we get the values in the table below:

	r_1, mm	r_2, mm	R, mm	$\bar{r}$, mm	e, mm	σ, MPa	% error
(a)	20	60	36.4096	40	3.5904	−11.426	−34.4 %
(b)	200	240	219.3926	220	0.6074	−7.982	6.0 %
(c)	2000	2040	2019.9340	2020	0.0660	−7.546	0.6 %

PROBLEM 4.186

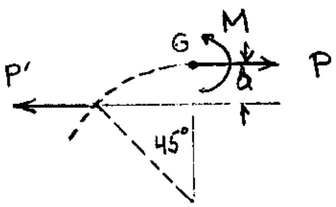
4.186 Steel links having the cross section shown are available with different central angles β. Knowing that the allowable stress is 15 ksi, determine the largest force **P** that can be applied to a link for which $\beta = 90°$.



SOLUTION

Reduce section force to a force-couple system at G, the centroid of the cross section AB.

$$a = \bar{r}\left(1 - \cos\frac{\beta}{2}\right)$$



The bending couple is $M = -Pa$

For the rectangular section, the neutral axis for bending couple only lies at

$$R = \frac{h}{\ln\frac{r_2}{r_1}}. \qquad \text{Also} \quad e = \bar{r} - R$$

At point A the tensile stress is

$$\sigma_A = \frac{P}{A} - \frac{My_A}{Aer_1} = \frac{P}{A} + \frac{Pay_A}{Aer_1} = \frac{P}{A}\left(1 + \frac{ay_A}{er_1}\right) = K\frac{P}{A}$$

where $K = 1 + \frac{ay_A}{er_1}$ and $y_A = R - r_1$

$$P = \frac{A\sigma_A}{K}$$

Data: $\bar{r} = 0.9$ in, $r_1 = 0.6$ in, $r_2 = 1.2$ in, $h = 0.6$ in, $b = 0.25$ in.

$A = (0.25)(0.6) = 0.15\ \text{in}^2$, $\quad R = \frac{0.6}{\ln\frac{1.2}{0.6}} = 0.86562$ in

$e = 0.9 - 0.86562 = 0.03438$ in., $\quad y_A = 0.86562 - 0.6 = 0.26562$ in

$a = 0.9(1 - \cos 45°) = 0.26360$ in

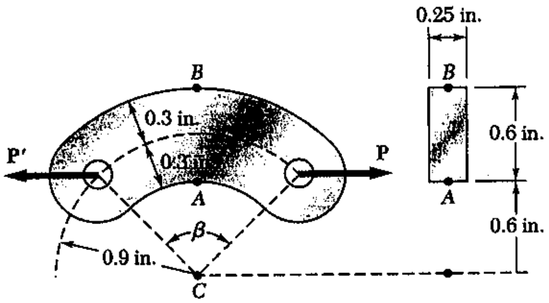
$$K = 1 + \frac{(0.26360)(0.26562)}{(0.03438)(0.6)} = 4.3943$$

$$P = \frac{(0.15)(15)}{4.3943} = 0.512 \text{ kips} = 512 \text{ lb}$$

PROBLEM 4.187

4.186 Steel links having the cross section shown are available with different central angles β. Knowing that the allowable stress is 15 ksi, determine the largest force **P** that can be applied to a link for which $\beta = 90°$.

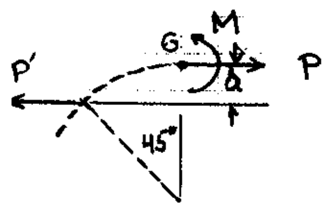
4.187 Solve Prob. 4.186, assuming that $\beta = 60°$.



SOLUTION

Reduce section force to a force-couple system at G, the centroid of the cross section AB.

$$a = \bar{r}\left(1 - \cos\frac{\beta}{2}\right)$$



The bending couple is $M = -Pa$

For the rectangular section, the neutral axis for bending couple only lies at

$$R = \frac{h}{\ln \frac{r_2}{r_1}}. \qquad \text{Also } e = \bar{r} - R$$

At point A the tensile stress is

$$\sigma_A = \frac{P}{A} + \frac{My_A}{Aer_1} = \frac{P}{A} + \frac{Pa\,y_A}{Aer_1} = \frac{P}{A}\left(1 + \frac{ay_A}{er_1}\right) = K\frac{P}{A}$$

where $K = 1 + \frac{ay_A}{er_1}$ and $y_A = R - r_1$

$$P = \frac{A\sigma_A}{K}$$

Data: $\bar{r} = 0.9$ in, $r_1 = 0.6$ in, $r_2 = 1.2$ in, $h = 0.6$ in, $b = 0.25$

$A = (0.25)(0.6) = 0.15 \text{ in}^2$, $R = \frac{0.6}{\ln \frac{1.2}{0.6}} = 0.86562$ in.

$e = 0.9 - 0.86562 = 0.03438$ in., $y_A = 0.86562 - 0.6 = 0.26562$

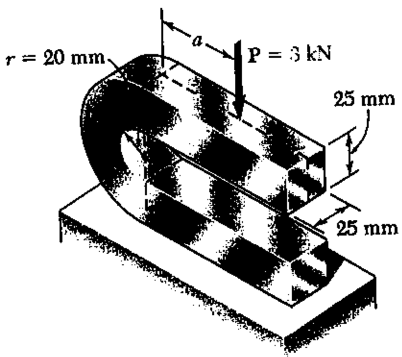
$a = 0.9(1 - \cos 30°) = 0.12058$ in.

$$K = 1 + \frac{(0.12058)(0.26562)}{(0.03438)(0.6)} = 2.5526$$

$$P = \frac{(0.15)(15)}{2.5526} = 0.881 \text{ kips} = 881 \text{ lb.}$$

PROBLEM 4.188

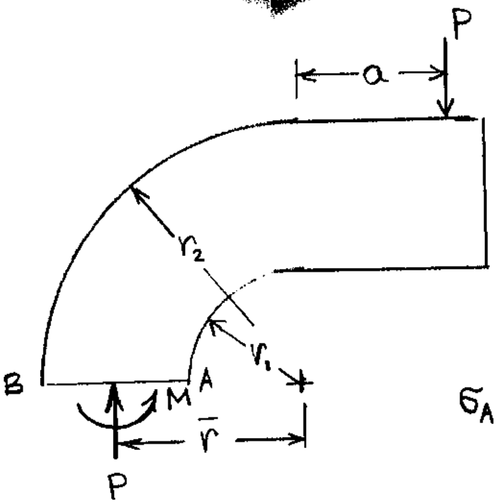
4.188 The curved portion of the bar shown has an inner radius of 20 mm. Knowing that the line of action of the 3-kN force is located at a distance $a = 60$ mm from the vertical plane containing the center of curvature of the bar, determine the largest compressive stress in the bar.



SOLUTION

Reduce the internal forces transmitted across section AB to a force-couple system at the centroid of the section. The bending couple is

$$M = P(a + \bar{r})$$



For the rectangular section, the neutral axis for bending couple only lies at

$$R = \frac{h}{\ln \frac{r_2}{r_1}}. \qquad \text{Also} \quad e = \bar{r} - R$$

The maximum compressive stress occurs at point A. It is given by

$$\sigma_A = -\frac{P}{A} - \frac{M y_A}{A e r_1} = -\frac{P}{A} - \frac{P(a+\bar{r}) y_A}{A e r_1}$$

$$= -K\frac{P}{A} \qquad \text{with } y_A = R - r_1$$

Thus, $$K = 1 + \frac{(a+\bar{r})(R - r_1)}{e r_1}$$

Data: $h = 25$ mm, $r_1 = 20$ mm, $r_2 = 45$ mm, $\bar{r} = 32.5$ mm

$$R = \frac{25}{\ln \frac{45}{20}} = 30.8288 \text{ mm}, \quad e = 32.5 - 30.8288 = 1.6712 \text{ mm}$$

$b = 25$ mm, $A = bh = (25)(25) = 625 \text{ mm}^2 = 625 \times 10^{-6} \text{ m}^2$

$a = 60$ mm, $a + \bar{r} = 92.5$ mm, $R - r_1 = 10.8288$ mm

$$K = 1 + \frac{(92.5)(10.8288)}{(1.6712)(20)} = 30.968$$

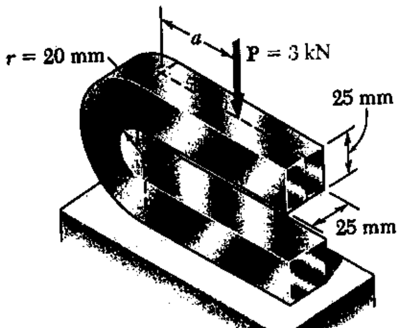
$P = 3 \times 10^3$ N

$$\sigma_A = -\frac{KP}{A} = -\frac{(30.968)(3\times10^3)}{625\times10^{-6}} = -148.6 \times 10^6 \text{ Pa}$$

$$= -148.6 \text{ MPa}$$

PROBLEM 4.189

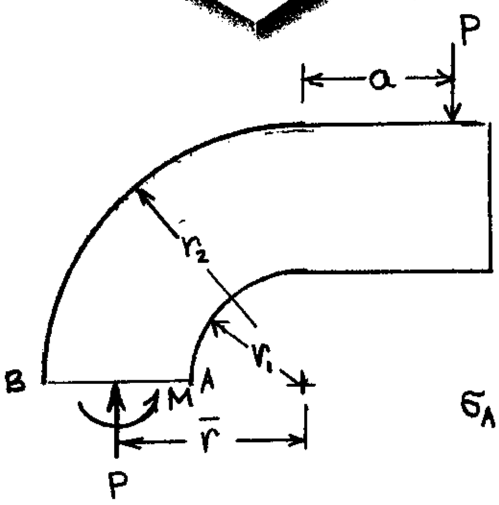
4.189 Knowing that the allowable stress in the bar is 150 MPa, determine the largest permissible distance a from the line of action of the 3-kN force to the vertical plane containing the center of curvature of the bar.



SOLUTION

Reduce the internal forces transmitted across section AB to a force-couple system at the centroid of the section. The bending couple is

$$M = P(a + \bar{r})$$



For the rectangular section, the neut axis for bending couple only lies

$$R = \frac{h}{\ln \frac{r_2}{r_1}} \qquad \text{Also} \quad e = \bar{r} - R$$

The maximum compressive stress occ at point A. It is given by

$$\sigma_A = -\frac{P}{A} - \frac{My_A}{Aer_1} = -\frac{P}{A} - \frac{P(a+\bar{r})y_A}{Aer_1}$$

$$= -K\frac{P}{A} \qquad \text{with } y_A = R - r_1$$

Thus, $$K = 1 + \frac{(a+\bar{r})(R-r_1)}{er_1}$$

Data: $h = 25\ \text{mm}, \quad r_1 = 20\ \text{mm}, \quad r_2 = 45\ \text{mm}, \quad \bar{r} = 32.5\ \text{mm}$

$$R = \frac{25}{\ln \frac{45}{20}} = 30.8288\ \text{mm}, \quad e = 32.5 - 30.8288 = 1.6712\ \text{mm}$$

$$b_1 = 25\ \text{mm}, \quad A = bh = (25)(25) = 625\ \text{mm}^2 = 625 \times 10^{-6}\ \text{m}^2$$

$$R - r_1 = 10.8288\ \text{mm}$$

$$P = 3 \times 10^3\ \text{N}\cdot\text{m} \qquad \sigma_A = -150 \times 10^6\ \text{Pa}$$

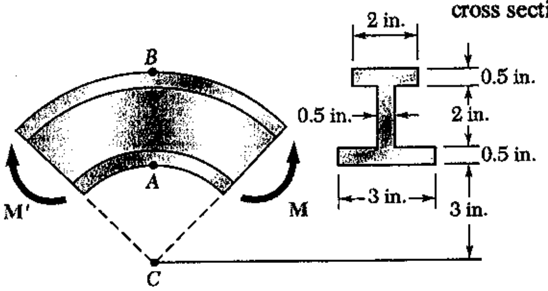
$$K = -\frac{\sigma_A A}{P} = -\frac{(-150 \times 10^6)(625 \times 10^{-6})}{3 \times 10^3} = 31.25$$

$$a + \bar{r} = \frac{(K-1)er_1}{R - r_1} = \frac{(30.25)(1.6712)(20)}{10.8288} = 93.37\ \text{mm}$$

$$a = 93.37 - 32.5 = 60.9\ \text{mm}$$

PROBLEM 4.190

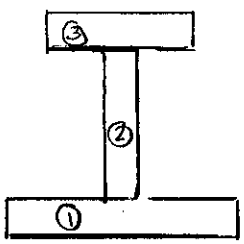
4.190 Three plates are welded together to form the curved beam shown. For the given loading, determine the distance e between the neutral axis and the centroid of the cross section.



SOLUTION

$$R = \frac{\Sigma A}{\Sigma \int \frac{1}{r} dA} = \frac{\Sigma b_i h_i}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}} = \frac{\Sigma A}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}}$$

$$\bar{r} = \frac{\Sigma A \bar{r}_i}{\Sigma A}$$



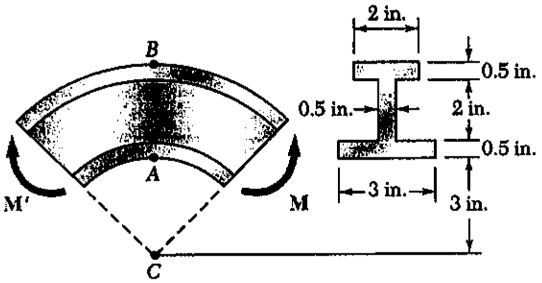
r	part	b	h	A	$b \ln \frac{r_{i+1}}{r_i}$	$\bar{r}$	$A\bar{r}$
3	①	3.	0.5	1.5	0.462452	3.25	4.875
3.5	②	0.5	2	1.0	0.225993	4.5	4.5
5.5	③	2	0.5	1.0	0.174023	5.75	5.75
6	Σ			3.5	0.862468		15.125

$$R = \frac{3.5}{0.862468} = 4.05812 \text{ in}, \quad \bar{r} = \frac{15.125}{3.5} = 4.32143 \text{ in.}$$

$$e = \bar{r} - R = 0.26331 \text{ in.}$$

PROBLEM 4.191

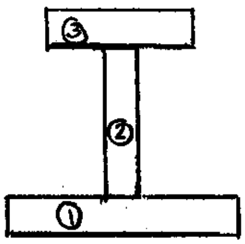
4.191 Three plates are welded together to form the curved beam shown. For $M = 8$ kip·in., determine the stress at (*a*) point *A*, (*b*) point *B*, (*c*) the centroid of the cross section.



SOLUTION

$$R = \frac{\Sigma A}{\Sigma \int \frac{1}{r} dA} = \frac{\Sigma b_i h_i}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}} = \frac{\Sigma A}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}}$$

$$\bar{r} = \frac{\Sigma A \bar{r}_i}{\Sigma A}$$



r	part	b	h	A	$b \ln \frac{r_{i+1}}{r_i}$	$\bar{r}$	$A\bar{r}$
3	①	3	0.5	1.5	0.462452	3.25	4.875
3.5	②	0.5	2	1.0	0.225993	4.5	4.5
5.5	③	2	0.5	1.0	0.174023	5.75	5.75
6	Σ			3.5	0.862468		15.125

$R = \dfrac{3.5}{0.862468} = 4.05812$ in., $\bar{r} = \dfrac{15.125}{3.5} = 4.32143$ in.

$e = \bar{r} - R = 0.26331$ in. $\qquad M = -8$ kip-in

(a) $y_A = R - r_1 = 4.05812 - 3 = 1.05812$ in.

$$\sigma_A = -\frac{M y_A}{A e r_1} = -\frac{(-8)(1.05812)}{(3.5)(0.26331)(3)} = 3.06 \text{ ksi}$$

(b) $y_B = R - r_2 = 4.05812 - 6 = -1.94188$ in.

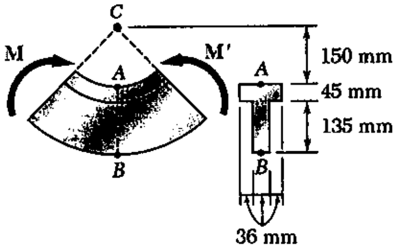
$$\sigma_B = -\frac{M y_B}{A e r_2} = -\frac{(-8)(-1.94188)}{(3.5)(0.26331)(6)} = -2.81 \text{ ksi}$$

(c) $y_C = R - \bar{r} = -e$

$$\sigma_C = -\frac{M y_C}{A e \bar{r}} = -\frac{Me}{Ae\bar{r}} = -\frac{M}{A\bar{r}} = -\frac{-8}{(3.5)(4.32143)} = 0.529 \text{ ksi}$$

PROBLEM 4.192

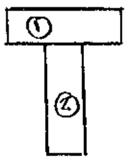
4.192 and 4.193 Knowing that $M = 20$ kN·m, determine the stress at (*a*) point *A*, (*b*) point *B*.



SOLUTION

$$R = \frac{\Sigma A}{\Sigma \int \frac{1}{r} dA} = \frac{\Sigma b_i h_i}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}} = \frac{\Sigma A_i}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}}$$

$$\bar{r} = \frac{\Sigma A_i \bar{r}_i}{\Sigma A_i}$$



r, mm	Part	b, mm	h, mm	A, mm^2	$b_i \ln \frac{r_{i+1}}{r_i}$, mm	$\bar{r}$, mm	$A\bar{r}$, mm^3
150							
	1	108	45	4860	28.3353	172.5	838.35×10^3
195							
	2	36	135	4860	18.9394	262.5	1275.75×10^3
330							
	Σ			9720	47.2747		2114.1×10^3

$$R = \frac{9720}{47.2747} = 205.606 \text{ mm} \qquad \bar{r} = \frac{2114.1 \times 10^3}{9720} = 217.5 \text{ mm}$$

$$e = \bar{r} - R = 11.894 \text{ mm.} \qquad M = 20 \times 10^3 \text{ N·m}$$

(a) $$y_A = R - r_1 = 205.606 - 150 = 55.606 \text{ mm}$$

$$\sigma_A = -\frac{M y_A}{A e r_1} = -\frac{(20 \times 10^3)(55.606 \times 10^{-3})}{(9720 \times 10^{-6})(11.894 \times 10^{-3})(150 \times 10^{-3})}$$

$$= -64.1 \times 10^6 \text{ Pa} = -64.1 \text{ MPa}$$ ◄

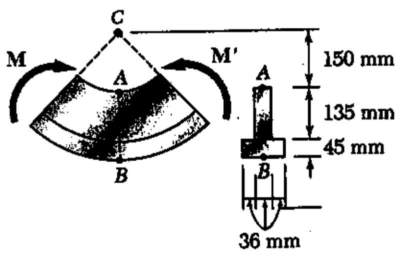
(b) $$y_B = R - r_2 = 205.606 - 330 = -124.394 \text{ mm}$$

$$\sigma_B = -\frac{M y_B}{A e r_2} = -\frac{(20 \times 10^3)(-124.394 \times 10^{-3})}{(9720 \times 10^{-6})(11.894 \times 10^{-3})(330 \times 10^{-3})}$$

$$= 65.2 \times 10^6 \text{ Pa} = 65.2 \text{ MPa}$$ ◄

PROBLEM 4.193

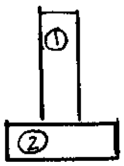
4.192 and 4.193 Knowing that $M = 20$ kN·m, determine the stress at (*a*) point *A*, (*b*) point *B*.



SOLUTION

$$R = \frac{\Sigma A}{\Sigma \int \frac{1}{r} dA} = \frac{\Sigma b_i h_i}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}} = \frac{\Sigma A_i}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}}$$

$$\bar{r} = \frac{\Sigma A_i \bar{r}_i}{\Sigma A_i}$$



r		b, mm	h, mm	A, mm^2	$b_i \ln \frac{r_{i+1}}{r_i}$, mm	$\bar{r}$, mm	$A\bar{r}$, mm^3
150	①	36	135	4860	23.1067	217.5	1.05705×10^6
285	②	108	45	4860	15.8332	307.5	1.49445×10^6
330	Σ			9720	38.9399		2.5515×10^6

$$R = \frac{9720}{38.9399} = 249.615 \text{ mm}, \qquad \bar{r} = \frac{2.5515 \times 10^6}{9720} = 262.5 \text{ mm}$$

$$e = \bar{r} - R = 12.885 \text{ mm}, \qquad M = 20 \times 10^3 \text{ N·m}$$

(a) $y_A = R - r_1 = 249.615 - 150 = 99.615$ mm

$$\sigma_A = -\frac{M y_A}{A e r_1} = -\frac{(20 \times 10^3)(99.615 \times 10^{-3})}{(9720 \times 10^{-6})(12.885 \times 10^{-3})(150 \times 10^{-3})}$$

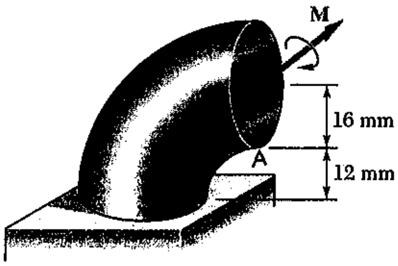
$$= -106.1 \times 10^6 \text{ Pa} = -106.1 \text{ MPa}$$ ◄

(b) $y_B = R - r_2 = 249.615 - 330 = -80.385$ mm

$$\sigma_B = -\frac{M y_B}{A e r_2} = -\frac{(20 \times 10^3)(-80.385 \times 10^{-3})}{(9720 \times 10^{-6})(12.885 \times 10^{-3})(330 \times 10^{-6})}$$

$$= 38.9 \times 10^6 \text{ Pa} = 38.9 \text{ MPa}$$ ◄

PROBLEM 4.194



4.194 The curved bar shown has a circular cross section of 32-mm diameter. Determine the largest couple **M** that can be applied to the bar about a horizontal axis if the maximum stress is not to exceed 60 MPa.

SOLUTION

$c = 16 \text{ mm} \qquad \bar{r} = 12 + 16 = 28 \text{ mm}$

$$R = \frac{1}{2}\left[\bar{r} + \sqrt{\bar{r}^2 - c^2}\right]$$

$$= \frac{1}{2}\left[28 + \sqrt{28^2 - 16^2}\right] = 25.4891 \text{ mm}$$

$e = \bar{r} - R = 28 - 25.4891 = 2.5109 \text{ mm.}$

σ_{max} occurs at A is given by $|\sigma_{max}| = \left|\frac{M y_A}{A e r_1}\right|$ from which

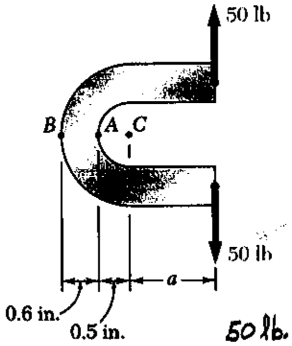
$M = \frac{A e r_1 |\sigma_{max}|}{y_A}$ Also $A = \pi c^2 = \pi (16)^2 = 804.25 \text{ mm}^2$

Data: $y_A = R - r_1 = 25.4891 - 12 = 13.4891 \text{ mm}$

$$M = \frac{(804.25 \times 10^{-6})(2.5109 \times 10^{-3})(12 \times 10^{-3})(60 \times 10^{6})}{13.4891 \times 10^{-3}} = 107.8 \text{ N-m}$$

PROBLEM 4.195

4.195 The bar shown has a circular cross section of 0.6-in. diameter. Knowing that $a = 1.2$ in., determine the stress at (*a*) point *A*, (*b*) point *B*.

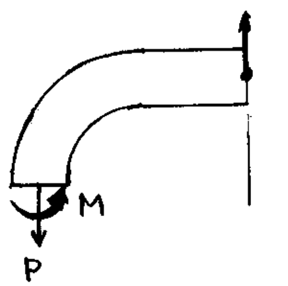


SOLUTION

$c = \frac{1}{2}d = 0.3$ in. $\quad \bar{r} = 0.5 + 0.3 = 0.8$ in.

$R = \frac{1}{2}\left[\bar{r} + \sqrt{\bar{r}^2 - c^2}\right] = \frac{1}{2}\left[0.8 + \sqrt{0.8^2 - 0.3^2}\right]$

$= 0.77081$ in



$e = \bar{r} - R = 0.02919$ in.

$A = \pi c^2 = \pi(0.3)^2 = 0.28274$ in^2

$M = -P(a + \bar{r}) = -50(1.2 + 0.8) = -100$ lb·in

$y_A = R - r_1 = 0.77081 - 0.5 = 0.27081$ in.

$y_B = R - r_2 = 0.77081 - 1.1 = -0.32919$ in.

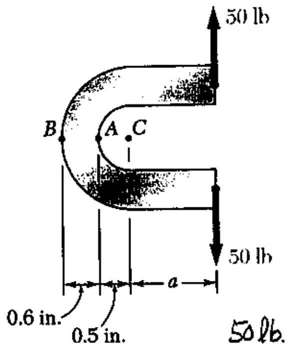
(a) $\sigma_A = \frac{P}{A} - \frac{My_A}{Aer_1} = \frac{50}{0.28274} - \frac{(-100)(0.27081)}{(0.28274)(0.02919)(0.5)} = 6.74 \times 10^3$ psi

$= 6.74$ ksi ◄

(b) $\sigma_B = \frac{P}{A} - \frac{My_B}{Aer_2} = \frac{50}{0.28274} - \frac{(-100)(-0.32919)}{(0.28274)(0.02919)(1.1)} = -3.45 \times 10^3$ psi

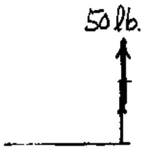
$= -3.45$ ksi ◄

PROBLEM 4.196



4.196 The bar shown has a circular cross section of 0.6-in. diameter. Knowing that the allowable tensile stress is 8 ksi, determine the largest permissible distance a from the line of action of the 50-lb forces to the plane containing the center of curvature of the bar.

SOLUTION



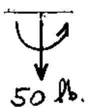
$c = \frac{1}{2}d = 0.3 \text{ in}, \qquad \bar{r} = 0.5 + 0.3 = 0.8 \text{ in}$

$R = \frac{1}{2}\left[\bar{r} + \sqrt{\bar{r}^2 - c^2}\right] = \frac{1}{2}\left[0.8 + \sqrt{0.8^2 - 0.3^2}\right]$

$= 0.77081 \text{ in.} \qquad e = \bar{r} - R = 0.02919 \text{ in.}$

$A = \pi c^2 = \pi(0.3)^2 = 0.28274 \text{ in}^2$

$M = -P(a + \bar{r})$



$y_A = R - r_1 = 0.77081 - 0.5 = 0.27081 \text{ in.}$

$\sigma_A = \frac{P}{A} - \frac{M y_A}{A e r_1} = \frac{P}{A} + \frac{P(a+\bar{r})y_A}{A e r_1} = \frac{P}{A}\left[1 + \frac{(a+\bar{r})y_A}{e r_1}\right]$

$= \frac{KP}{A} \quad \text{where} \quad K = 1 + \frac{(a+\bar{r})y_A}{e r_1}$

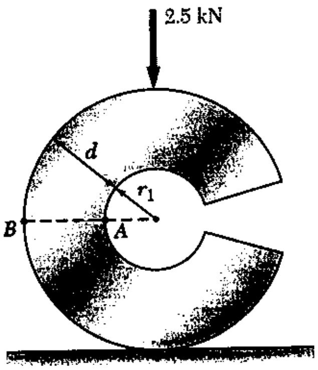
$K = \frac{\sigma_A A}{P} = \frac{(8\times10^3)(0.28274)}{50} = 45.238$

$a + \bar{r} = \frac{(K-1)e r_1}{y_A} = \frac{(44.238)(0.02919)(0.5)}{0.27081} = 2.384 \text{ in}$

$a = 2.384 - 0.8 = 1.584 \text{ in.}$

PROBLEM 4.197

4.197 The split ring shown has an inner radius $r_1 = 20$ mm and a circular cross section of diameter $d = 32$ mm. For the loading shown, determine the stress at (*a*) point *A*, (*b*) point *B*.



SOLUTION

$$c = \tfrac{1}{2}d = 16\text{ mm} \qquad r_1 = 20\text{ mm}; \quad r_2 = r_1 + d = 52\text{ mm}$$

$$\bar{r} = r_1 + c = 36\text{ mm}$$

$$R = \tfrac{1}{2}\left[\bar{r} + \sqrt{\bar{r}^2 - c^2}\right] = \tfrac{1}{2}\left[36 + \sqrt{36^2 - 16^2}\right]$$

$$= 34.1245\text{ mm}$$

$$e = \bar{r} - R = 1.8755\text{ mm}$$

$$A = \tfrac{\pi}{4}d^2 = \tfrac{\pi}{4}(32)^2 = 804.25\text{ mm}^2 = 804.25 \times 10^{-6}\text{ m}^2$$

$$P = 2.5 \times 10^3\text{ N} \qquad M = P\bar{r} = (2.5 \times 10^3)(36 \times 10^{-3}) = 90\text{ N}\cdot\text{m}$$

(a) Point A: $\quad y_A = R - r_1 = 34.1245 - 20 = 14.1245\text{ mm}$

$$\sigma_A = -\frac{P}{A} - \frac{My_A}{Aer_1} = -\frac{2.5 \times 10^3}{804.25 \times 10^{-6}} - \frac{(90)(14.1245 \times 10^{-3})}{(804.25 \times 10^{-6})(1.8755 \times 10^{-3})(20 \times 10^{-3})}$$

$$= -45.2 \times 10^6\text{ Pa} = -45.2\text{ MPa}$$

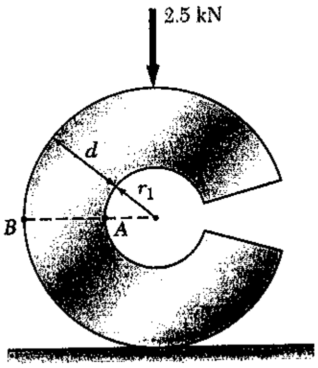
(b) Point B: $\quad y_B = R - r_2 = 34.1245 - 52 = -17.8755\text{ mm}$

$$\sigma_B = -\frac{P}{A} - \frac{My_B}{Aer_2} = -\frac{2.5 \times 10^3}{804.25 \times 10^{-6}} - \frac{(90)(-17.8755 \times 10^{-3})}{(804.25 \times 10^{-6})(1.8755 \times 10^{-3})(52 \times 10^{-3})}$$

$$= 17.40 \times 10^6\text{ Pa} = 17.40\text{ MPa}$$

PROBLEM 4.198

4.198 The split ring shown has an inner radius $r_1 = 16$ mm and a circular cross section of diameter $d = 32$ mm. For the loading shown, determine the stress at (*a*) point *A*, (*b*) point *B*.



SOLUTION

$$c = \tfrac{1}{2}d = 16\text{ mm}, \quad r_1 = 16\text{ mm}, \quad r_2 = r_1 + d = 48\text{ mm}$$

$$\bar{r} = r_1 + c = 32\text{ mm}$$

$$R = \tfrac{1}{2}\left[\bar{r} + \sqrt{\bar{r}^2 - c^2}\right] = \tfrac{1}{2}\left[32 + \sqrt{32^2 - 16^2}\right]$$

$$= 29.8564\text{ mm}$$

$$e = \bar{r} - R = 2.1436\text{ mm}$$

$$A = \tfrac{\pi}{4}d^2 = \tfrac{\pi}{4}(32)^2 = 804.25\text{ mm}^2 = 804.25 \times 10^{-6}\text{ m}^2$$

$P = 2.5 \times 10^3$ N $\qquad M = P\bar{r} = (2.5\times10^3)(32\times10^{-3}) = 80$ N·m

(a) Point A: $\quad y_A = R - r_1 = 29.8564 - 16 = 13.8564$ mm

$$\sigma_A = -\frac{P}{A} - \frac{My_A}{Aer_1} = -\frac{2.5\times10^3}{804.25\times10^{-6}} - \frac{(80)(13.8564\times10^{-3})}{(804.25\times10^{-6})(2.1436\times10^{-3})(16\times10^{-3})}$$

$$= -43.3\times10^6\text{ Pa} = -43.3\text{ MPa}$$

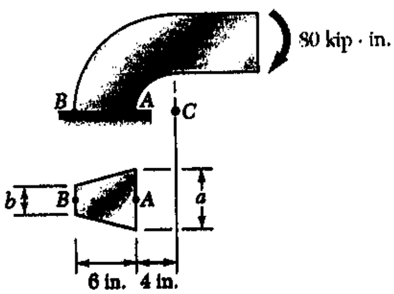
(b) Point B: $\quad y_B = R - r_2 = 29.8564 - 48 = -18.1436$ mm.

$$\sigma_B = -\frac{P}{A} - \frac{My_B}{Aer_2} = -\frac{2.5\times10^6}{804.25\times10^{-6}} - \frac{(80)(-18.1436\times10^{-3})}{(804.25\times10^{-6})(2.1436\times10^{-3})(48\times10^{-3})}$$

$$= 14.43\times10^6\text{ Pa} = 14.43\text{ MPa}$$

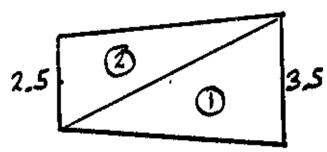
PROBLEM 4.199

4.199 Knowing that the machine component shown has a trapezoidal cross section with $a = 3.5$ in. and $b = 2.5$ in., determine the stress at (*a*) point *A*, (*b*) point *B*.



SOLUTION

Locate centroid



	A, in^2	$\bar{r}$, in	$A\bar{r}$, in^3
1	10.5	6	63
2	7.5	8	60
Σ	18		123

$\bar{r} = \frac{123}{18} = 6.8333$ in.

$$R = \frac{\frac{1}{2}h^2(b_1 + b_2)}{(b_1 r_2 - b_2 r_1)\ln\frac{r_2}{r_1} - h(b_1 - b_2)}$$

$$= \frac{(0.5)(6)^2(3.5 + 2.5)}{[(3.5)(10) - (2.5)(4)]\ln\frac{10}{4} - (6)(3.5 - 2.5)} = 6.3878 \text{ in}$$

$e = \bar{r} - R = 0.4452$ in $\qquad M = 80$ kip·in.

(a) $y_A = R - r_1 = 6.3878 - 4 = 2.3878$ in.

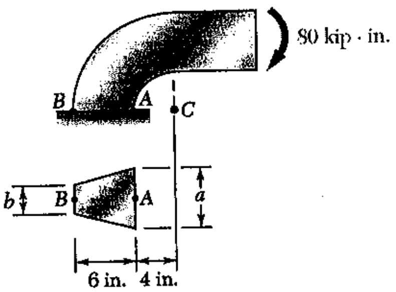
$$\sigma_A = -\frac{M y_A}{A e r_1} = -\frac{(80)(2.3878)}{(18)(0.4452)(4)} = -5.96 \text{ ksi}$$

(b) $y_B = R - r_2 = 6.3878 - 10 = -3.6122$ in.

$$\sigma_B = -\frac{M y_B}{A e r_2} = -\frac{(80)(-3.6122)}{(18)(0.4452)(10)} = 3.61 \text{ ksi}$$

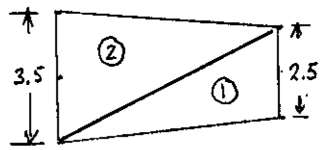
PROBLEM 4.200

4.200 Knowing that the machine component shown has a trapezoidal cross section with $a = 2.5$ in. and $b = 3.5$ in., determine the stress at (*a*) point *A*, (*b*) point *B*.



SOLUTION

Locate centroid



	A, in^2	$\bar{r}$, in.	$A\bar{r}$, in^3
①	7.5	6	45
②	10.5	8	84
Σ	18		129

$$\bar{r} = \frac{129}{18} = 7.1667 \text{ in.}$$

$$R = \frac{\frac{1}{2}h^2(b_1+b_2)}{(b_1 r_2 - b_2 r_1)\ln\frac{r_2}{r_1} - h(b_1 - b_2)}$$

$$= \frac{(0.5)(6)^2(2.5+3.5)}{[(2.5)(10) - (3.5)(4)]\ln\frac{10}{4} - (6)(2.5-3.5)} = 6.7168 \text{ in}$$

$$e = \bar{r} - R = 0.4499 \text{ in.} \qquad M = 80 \text{ kip·in.}$$

(a) $y_A = R - r_1 = 2.7168$ in.

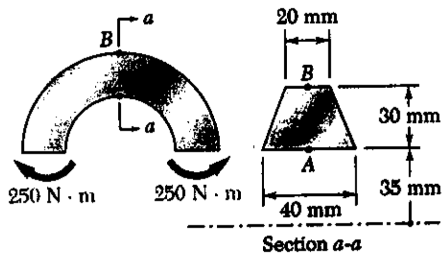
$$\sigma_A = -\frac{M y_A}{A e r_1} = -\frac{(80)(2.7168)}{(18)(0.4499)(4)} = -6.71 \text{ ksi}$$

(b) $y_B = R - r_2 = -3.2832$ in.

$$\sigma_B = -\frac{M y_B}{A e r_2} = -\frac{(80)(-3.2832)}{(18)(0.4499)(10)} = 3.24 \text{ ksi}$$

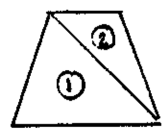
PROBLEM 4.201

4.201 For the curved beam and loading shown, determine the stress at (*a*) point *A*, (*b*) point *B*.



SOLUTION

Locate centroid



	A, mm^2	$\bar{r}$, mm	$A\bar{r}$, mm^3
1	600	45	27×10^3
2	300	55	16.5×10^3
Σ	900		43.5×10^3

$$\bar{r} = \frac{43.5 \times 10^3}{900} = 48.333 \text{ mm.}$$

$$R = \frac{\frac{1}{2} h^2 (b_1 + b_2)}{(b_1 r_2 - b_2 r_1) \ln \frac{r_2}{r_1} - h(b_2 - b_1)}$$

$$= \frac{(0.5)(30)^2 (40 + 20)}{[(40)(65) - (20)(35)] \ln \frac{65}{35} - (30)(40 - 20)} = 46.8608 \text{ mm}$$

$$e = \bar{r} - R = 1.4725 \text{ mm} \qquad M = -250 \text{ N·m}$$

(a) $y_A = R - r_1 = 11.8608$ mm.

$$\sigma_A = -\frac{M y_A}{A e r_1} = -\frac{(-250)(11.8608 \times 10^{-3})}{(900 \times 10^{-6})(1.4725 \times 10^{-3})(35 \times 10^{-3})} = 63.9 \times 10^6 \text{ Pa}$$

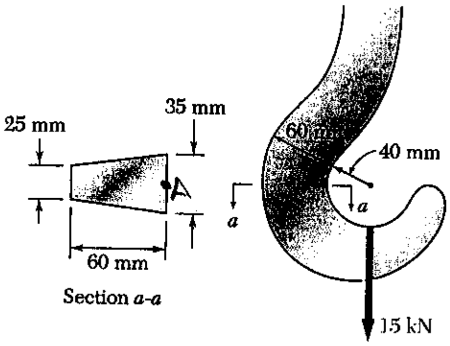
$$= 63.9 \text{ MPa}$$

(b) $y_B = R - r_2 = -18.1392$ mm

$$\sigma_B = -\frac{M y_B}{A e r_2} = -\frac{(-250)(-18.1392 \times 10^{-3})}{(900 \times 10^{-6})(1.4725 \times 10^{-3})(65 \times 10^{-3})} = -52.6 \times 10^6 \text{ Pa} = -52.6 \text{ MPa}$$

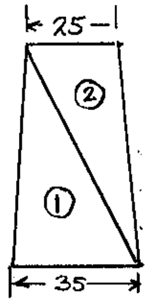
PROBLEM 4.202

4.202 For the crane hook shown, determine the largest tensile stress in section *a-a*.



SOLUTION

Locate centroid.



	A, mm^2	$\bar{r}$, mm	$A\bar{r}$, mm^3
①	1050	60	63×10^3
②	750	80	60×10^3
Σ	1800		103×10^3

$$\bar{r} = \frac{103 \times 10^3}{1800} = 68.333 \text{ mm.}$$

Foce-couple system at centroid: $P = 15 \times 10^3$ N

$$M = -P\bar{r} = -(15 \times 10^3)(68.333 \times 10^{-3}) = -1.025 \times 10^3 \text{ N}\cdot\text{m}$$

$$R = \frac{\frac{1}{2}h^2(b_1 + b_2)}{(b_1 r_2 - b_2 r_1)\ln\frac{r_2}{r_1} - h(b_1 - b_2)}$$

$$= \frac{(0.5)(60)^2(35 + 25)}{[(35)(100) - (25)(40)]\ln\frac{100}{40} - (60)(35+25)} = 63.878 \text{ mm.}$$

$$e = \bar{r} - R = 4.452 \text{ mm.}$$

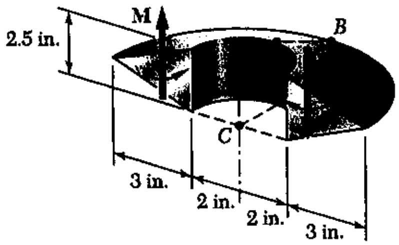
Maximum tensile stress occurs at point A

$$y_A = R - r_1 = 23.878 \text{ mm.}$$

$$\sigma_A = \frac{P}{A} - \frac{My_A}{Aer_1} = \frac{15 \times 10^3}{1800 \times 10^{-6}} - \frac{-(1.025 \times 10^3)(23.878 \times 10^{-3})}{(1800 \times 10^{-6})(4.452 \times 10^{-3})(40 \times 10^{-3})}$$

$$= 84.7 \times 10^6 \text{ Pa} = 84.7 \text{ MPa}$$

PROBLEM 4.203



4.203 and 4.204 Knowing that $M = 5$ kip·in., determine the stress at (*a*) point *A*, (*b*) point *B*.

SOLUTION

$A = \frac{1}{2}bh = \frac{1}{2}(2.5)(3) = 3.75 \text{ in}^2$

$\bar{r} = 2 + 1 = 3.00000 \text{ in}$

$b_1 = 2.5$ in., $r_1 = 2$ in, $b_2 = 0$, $r_2 = 5$ in.

Use formula for trapezoid

$$R = \frac{\frac{1}{2}h^2(b_1 + b_2)}{(b_1 r_2 - b_2 r_1)\ln\frac{r_2}{r_1} - h(b_1 - b_2)}$$

$$= \frac{(0.5)(3)^2(2.5+0)}{[(2.5)(5) - (0)(2)]\ln\frac{5}{2} - (3)(2.5-0)} = 2.84548 \text{ in.}$$

$e = \bar{r} - R = 0.15452$ in. $\qquad M = 5$ kip·in.

(a) $y_A = R - r_1 = 0.84548$ in.

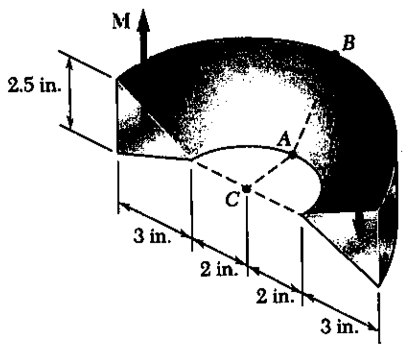
$$\sigma_A = -\frac{My_A}{Aer_1} = -\frac{(5)(0.84548)}{(3.75)(0.15452)(2)} = -3.65 \text{ ksi}$$

(b) $y_B = R - r_2 = -2.15452$ in.

$$\sigma_B = -\frac{My_B}{Aer_2} = -\frac{(5)(-2.15452)}{(3.75)(0.15452)(5)} = 3.72 \text{ ksi}$$

PROBLEM 4.204

4.203 and 4.204 Knowing that $M = 5$ kip·in., determine the stress at (*a*) point *A*, (*b*) point *B*.



SOLUTION

$$A = \frac{1}{2}(2.5)(3) = 3.75 \text{ in}^2$$

$$\bar{r} = 2 + 2 = 4.00000 \text{ in}$$

$b_1 = 0$, $r_1 = 2$ in., $b_2 = 2.5$ in. $r_2 = 5$ in.

Use formula for trapezoid.

$$R = \frac{\frac{1}{2}h^2(b_1 + b_2)}{(b_1 r_2 - b_2 r_1)\ln\frac{r_2}{r_1} - h(b_1 - b_2)}$$

$$= \frac{(0.5)(3)^2(0 + 2.5)}{[(0)(5) - (2.5)(2)]\ln\frac{5}{2} - (3)(0 - 2.5)} = 3.85466 \text{ in.}$$

$$e = \bar{r} - R = 0.14534 \text{ in.} \qquad M = 5 \text{ kip}\cdot\text{in.}$$

(a) $y_A = R - r_1 = 1.85466$ in.

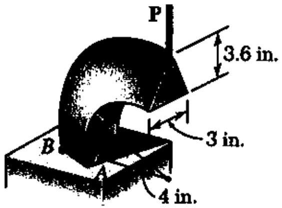
$$\sigma_A = -\frac{M y_A}{A e r_1} = -\frac{(5)(1.85466)}{(3.75)(0.14534)(2)} = -8.51 \text{ ksi}$$

(b) $y_B = R - r_2 = -1.14534$ in.

$$\sigma_B = -\frac{M y_B}{A e r_2} = -\frac{(5)(-1.14534)}{(3.75)(0.14534)(5)} = 2.10 \text{ ksi}$$

PROBLEM 4.205

4.205 Knowing that $P = 3.5$ kips, determine the stress at (*a*) point *A*, (*b*) point *B*.



SOLUTION

$b = 3$ in, $h = 3.6$ in., $r_1 = 4$ in

$A = \frac{1}{2} bh = \frac{1}{2}(3)(3.6) = 5.4 \text{ in}^2$

$r_2 = r_1 + h = 7.6$ in. $\quad \bar{r} = r_1 + \frac{1}{3}h = 5.2$ in

Reduce section forces to a force-couple system at the centroid

$P = 3.5$ kips $\qquad M = P\bar{r} = (3.5)(5.2) = 18.2$ kip-in

For a triangular section $\quad R = \dfrac{\frac{1}{2}h}{\frac{r_2}{h}\ln\frac{r_2}{r_1} - 1}$

$$= \frac{(0.5)(3.6)}{\frac{7.6}{3.6}\ln\frac{7.6}{4} - 1} = 5.07007 \text{ in}$$

$e = \bar{r} - R = 0.12993$ in

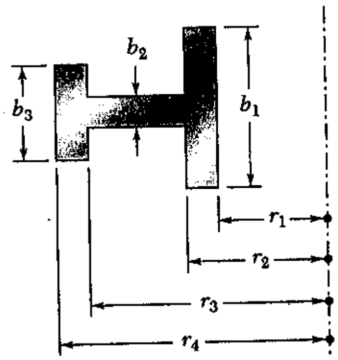
(a) $y_A = R - r_1 = 1.07007$ in.

$$\sigma_A = -\frac{P}{A} - \frac{My_A}{Aer_1} = -\frac{3.5}{5.4} - \frac{(18.2)(1.07007)}{(5.4)(0.12993)(4)} = -7.59 \text{ ksi}$$

(b) $y_B = R - r_2 = -2.52993$ in

$$\sigma_B = -\frac{P}{A} - \frac{My_B}{Aer_2} = -\frac{3.5}{5.4} - \frac{(18.2)(-2.52993)}{(5.4)(0.12993)(7.6)} = 7.99 \text{ ksi}$$

PROBLEM 4.206



4.206 Show that if the cross section of a curved beam consists of two or more rectangles, the radius R of the neutral surface can be expressed as

$$R = \frac{A}{\ln\left[\left(\frac{r_2}{r_1}\right)^{b_1}\left(\frac{r_3}{r_2}\right)^{b_2}\left(\frac{r_4}{r_3}\right)^{b_3}\right]}$$

where A is the total area of the cross section.

SOLUTION

$$R = \frac{\Sigma A}{\Sigma \int \frac{1}{r} dA} = \frac{A}{\Sigma b_i \ln \frac{r_{i+1}}{r_i}}$$

$$= \frac{A}{\Sigma \ln \left(\frac{r_{i+1}}{r_i}\right)^{b_i}} = \frac{A}{\ln\left[\left(\frac{r_2}{r_1}\right)^{b_1}\left(\frac{r_3}{r_2}\right)^{b_2}\left(\frac{r_4}{r_3}\right)^{b_3}\right]}$$

Note that for each rectangle $\int \frac{1}{r} dA = \int_{r_i}^{r_{i+1}} b_i \frac{dr}{r}$

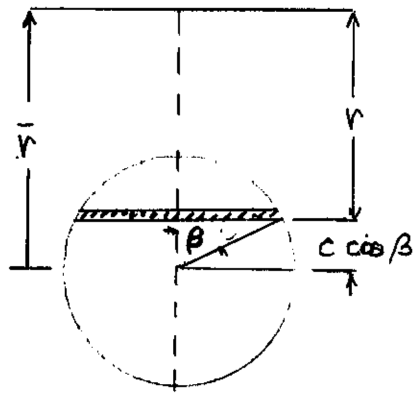
$$= b_i \int_{r_i}^{r_{i+1}} \frac{dr}{r} = b_i \ln \frac{r_{i+1}}{r_i}$$

PROBLEM 4.207

4.207 through 4.209 Using Eq.. (4.66), derive the expression for R given in Fig. 4.79 for

***4.207** A circular cross section

SOLUTION



Use polar coordinate β as shown.

width $\quad w = 2c\sin\beta$

$$r = \bar{r} - c\cos\beta$$

$$dr = c\sin\beta\, d\beta$$

$$dA = w\, dr = 2c^2\sin^2\beta\, d\beta$$

$$\int \frac{dA}{r} = \int_0^{\pi} \frac{2c^2\sin^2\beta}{\bar{r} - c\cos\beta}\, d\beta$$

$$\int \frac{dA}{r} = \int_0^{\pi} \frac{c^2(1-\cos^2\beta)}{\bar{r} - c\cos\beta}\, d\beta = 2\int_0^{\pi} \frac{\bar{r}^2 - c^2\cos^2\beta - (\bar{r}^2 - c^2)}{\bar{r} - c\cos\beta}\, d\beta$$

$$= 2\int_0^{\pi} (\bar{r} + c\cos\beta)\, d\beta - 2(\bar{r}^2 - c^2)\int_0^{\pi} \frac{dr}{\bar{r} - c\cos\beta}$$

$$= 2\bar{r}\,\beta\Big|_0^{\pi} + 2c\sin\beta\Big|_0^{\pi}$$

$$- 2(\bar{r}^2 - c^2)\frac{2}{\sqrt{\bar{r}^2 - c^2}}\tan^{-1}\frac{\sqrt{\bar{r}^2 - c^2}\,\tan\frac{1}{2}\beta}{\bar{r} + c}\Bigg|_0^{\pi}$$

$$= 2\bar{r}(\pi - 0) + 2c(0 - 0) - 4\sqrt{\bar{r}^2 - c^2}\cdot\left(\frac{\pi}{2} - 0\right)$$

$$2\pi\bar{r} - 2\pi\sqrt{\bar{r}^2 - c^2}$$

$$A = \pi c^2$$

$$R = \frac{A}{\int \frac{dA}{r}} = \frac{\pi c^2}{2\pi\bar{r} - 2\pi\sqrt{\bar{r}^2 - c^2}}$$

$$= \frac{1}{2}\,\frac{c^2}{\bar{r} - \sqrt{\bar{r}^2 - c^2}} \cdot \frac{\bar{r} + \sqrt{\bar{r}^2 - c^2}}{\bar{r} + \sqrt{\bar{r}^2 - c^2}}$$

$$= \frac{1}{2}\,\frac{c^2(\bar{r} + \sqrt{\bar{r}^2 - c^2})}{\bar{r}^2 - (\bar{r}^2 - c^2)} = \frac{1}{2}\,\frac{c^2(\bar{r} + \sqrt{\bar{r}^2 - c^2})}{c^2}$$

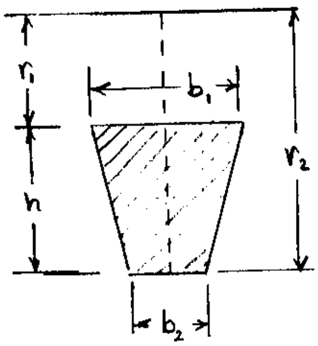
$$= \frac{1}{2}\left(\bar{r} + \sqrt{\bar{r}^2 - c^2}\right)$$

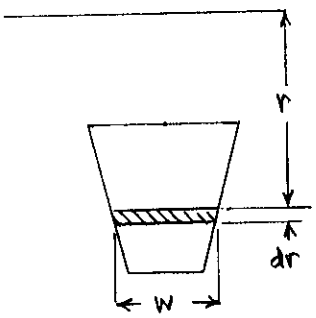
PROBLEM 4.208

4.207 through 4.209 Using Eq.. (4.66), derive the expression for R given in Fig. 4.79 for

4.208 A trapezoidal section

SOLUTION





The section width w varies linearly with r

$$w = c_o + c_1 r$$

$w = b_1$ at $r = r_1$ and $w = b_2$ at $r = r_2$

$$b_1 = c_o + c_1 r_1$$

$$b_2 = c_o + c_1 r_2$$

$$b_1 - b_2 = c_1(r_1 - r_2) = -c_1 h$$

$$c_1 = -\frac{b_1 - b_2}{h}$$

$$r_2 b_1 - r_1 b_2 = (r_2 - r_1) c_o = h c_o$$

$$c_o = \frac{r_2 b_1 - r_1 b_2}{h}$$

$$\int \frac{dA}{r} = \int_{r_1}^{r_2} \frac{w}{r} dr = \int_{r_1}^{r_2} \frac{c_o + c_1 r}{r} dr$$

$$= c_o \ln r \Big|_{r_1}^{r_2} + c_1 r \Big|_{r_1}^{r_2}$$

$$= c_o \ln \frac{r_2}{r_1} + c_1 (r_2 - r_1)$$

$$= \frac{r_2 b_1 - r_1 b_2}{h} \ln \frac{r_2}{r_1} - \frac{b_1 - b_2}{h} h$$

$$= \frac{r_2 b_1 - r_1 b_2}{h} \ln \frac{r_2}{r_1} - (b_1 - b_2)$$

$$A = \frac{1}{2}(b_1 + b_2) h$$

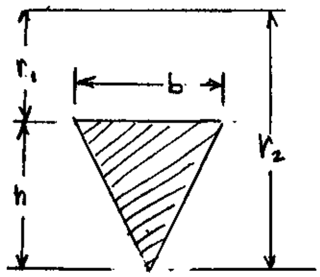
$$R = \frac{A}{\int \frac{dA}{r}} = \frac{\frac{1}{2} h^2 (b_1 + b_2)}{(r_2 b_1 - r_1 b_2) \ln \frac{r_2}{r_1} - h(b_1 - b_2)}$$

PROBLEM 4.209

4.207 through 4.209 Using Eq.. (4.66), derive the expression for R given in Fig. 4.79 for

4.209 A triangular cross section

SOLUTION



The section width w varies linearly with r

$$w = c_0 + c_1 r$$

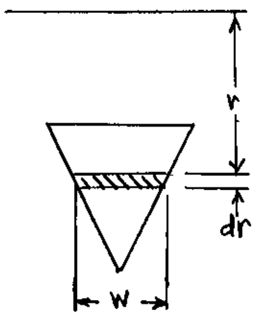
$$w = b \text{ at } r = r_1 \text{ and } w = 0 \text{ at } r = r_2$$

$$b = c_0 + c_1 r_1$$

$$0 = c_0 + c_1 r_2$$

$$b = c_1(r_1 - r_2) = -c_1 h$$

$$c_1 = -\frac{b}{h} \quad \text{and} \quad c_0 = -c_1 r_2 = \frac{b r_2}{h}$$



$$\int \frac{dA}{r} = \int_{r_1}^{r_2} \frac{w}{r}\,dr = \int_{r_1}^{r_2} \frac{c_0 + c_1 r}{r}\,dr$$

$$= c_0 \ln r \Big|_{r_1}^{r_2} + c_1 r \Big|_{r_1}^{r_2}$$

$$= c_0 \ln \frac{r_2}{r_1} + c_1(r_2 - r_1)$$

$$= \frac{b r_2}{h} \ln \frac{r_2}{r_1} - \frac{b}{h} h$$

$$= \frac{b r_2}{h} \ln \frac{r_2}{r_1} - b = b\left(\frac{r_2}{h} \ln \frac{r_2}{r_1} - 1\right)$$

$$A = \frac{1}{2} b h$$

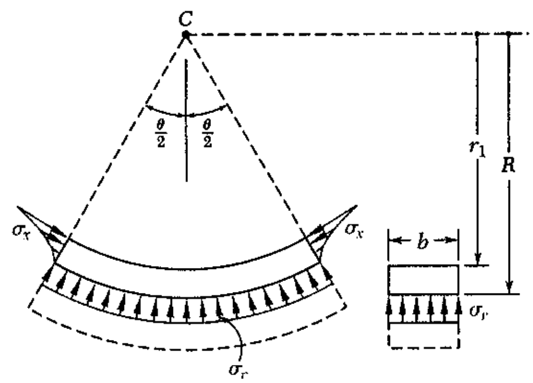
$$R = \frac{A}{\int \frac{dA}{r}} = \frac{\frac{1}{2} b h}{b\left(\frac{r_2}{h} \ln \frac{r_2}{r_1} - 1\right)} = \frac{\frac{1}{2} h}{\frac{r_2}{h} \ln \frac{r_2}{r_1} - 1}$$

PROBLEM 4.210

***4.210** For a curved bar of rectangular cross section subjected to a bending couple **M**, show that the radial stress at the neutral surface is

$$\sigma_r = \frac{M}{Ae}\left(1 - \frac{r_1}{R} - \ln\frac{R}{r_1}\right)$$

and compute the value of σ_r for the curved bar of Examples 4.10 and 4.11. (*Hint:* consider the free-body diagram of the portion of the beam located above the neutral surface.)



SOLUTION

At radial distance r

$$\sigma_\theta = \frac{M(r-R)}{Aer} = \frac{M}{Ae} - \frac{MR}{Aer}$$

For portion above the neutral axis, the resultant force is

$$H = \int \sigma_\theta \, dA = \int_{r_1}^{R} \sigma_\theta \, b \, dr$$

$$= \frac{Mb}{Ae}\int_{r_1}^{R} dr - \frac{MRb}{Ae}\int_{r_1}^{R}\frac{dr}{r}$$

$$= \frac{Mb}{Ae}(R - r_1) - \frac{MRb}{Ae}\ln\frac{R}{r_1} = \frac{MbR}{Ae}\left(1 - \frac{r_1}{R} - \ln\frac{R}{r_1}\right)$$

Resultant of σ_r

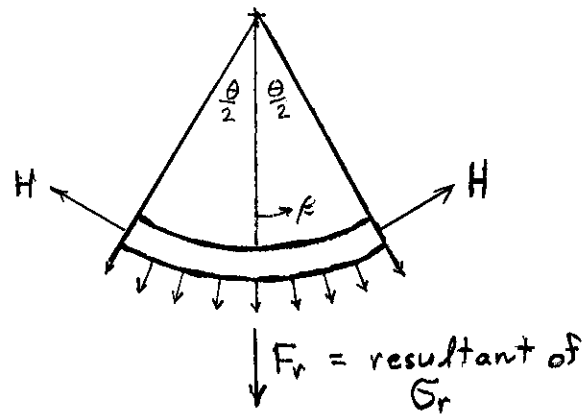
$$F_r = \int \sigma_r \cos\beta \, dA$$

$$= \int_{-\frac{\theta}{2}}^{\frac{\theta}{2}} \sigma_r \cos\beta \, b \, R \, d\beta$$

$$= \sigma_r bR \int_{-\frac{\theta}{2}}^{\frac{\theta}{2}} \cos\beta \, d\beta$$

$$= \sigma_r bR \sin\beta \Big|_{-\frac{\theta}{2}}^{\frac{\theta}{2}}$$

$$= 2\sigma_r bR \sin\frac{\theta}{2}$$

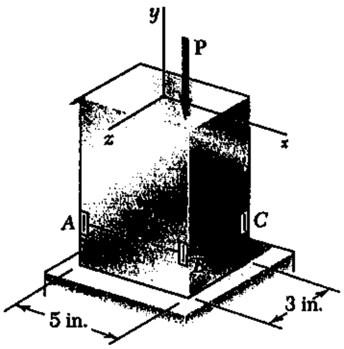


For equilibrium $\quad F_r - 2H\sin\frac{\theta}{2} = 0$

$$2\sigma_r bR \sin\frac{\theta}{2} - 2\frac{MbR}{Ae}\left(1 - \frac{r_1}{R} - \ln\frac{R}{r_1}\right) = 0$$

$$\sigma_r = \frac{M}{Ae}\left(1 - \frac{r_1}{R} - \ln\frac{R}{r_1}\right)$$

PROBLEM 4.211



4.211 A single vertical force **P** is applied to a short steel post as shown. Gages located at *A*, *B*, and *C* indicate the following strains:

$\epsilon_A = -500\ \mu$ $\qquad \epsilon_B = -1000\ \mu$ $\qquad \epsilon_C = -200\ \mu$

Knowing that $E = 29 \times 10^6$ psi, determine (*a*) the magnitude of **P**, (*b*) the line of action of **P**, (*c*) the corresponding strain at the hidden edge of the post, where $x = -2.5$ in. and $z = -1.5$ in.

SOLUTION

$$I_x = \frac{1}{12}(5)(3)^3 = 11.25 \text{ in}^4$$

$$I_z = \frac{1}{12}(3)(5)^3 = 31.25 \text{ in.}^4$$

$$A = (5)(3) = 15 \text{ in}^2$$

$$M_x = Pz \qquad M_z = -Px$$

$$x_A = -2.5 \text{ in}, \quad x_B = 2.5 \text{ in}, \quad x_C = 2.5 \text{ in}, \quad x_D = -2.5 \text{ in.}$$

$$z_A = 1.5 \text{ in}, \quad z_B = 1.5 \text{ in}, \quad z_C = -1.5 \text{ in}, \quad z_D = -1.5 \text{ in}$$

$$\sigma_A = E\varepsilon_A = (29\times10^6)(-500\times10^{-6}) = -14500 \text{ psi} = -14.5 \text{ ksi}$$

$$\sigma_B = E\varepsilon_B = (29\times10^6)(-1000\times10^{-6}) = -29000 \text{ psi} = -29 \text{ ksi}$$

$$\sigma_C = E\varepsilon_C = (29\times10^6)(-200\times10^{-6}) = -5800 \text{ psi} = -5.8 \text{ ksi}$$

$$\sigma_A = -\frac{P}{A} - \frac{M_x z_A}{I_x} + \frac{M_z x_A}{I_z} = -0.06667\,P - 0.13333\,M_x - 0.08\,M_z \qquad (1)$$

$$\sigma_B = -\frac{P}{A} - \frac{M_x z_B}{I_x} + \frac{M_z x_B}{I_z} = -0.06667\,P - 0.13333\,M_x + 0.08\,M_z \qquad (2)$$

$$\sigma_C = -\frac{P}{A} - \frac{M_x z_C}{I_x} + \frac{M_z x_C}{I_z} = -0.06667\,P + 0.13333\,M_x + 0.08\,M_z \qquad (3)$$

Substituting the values for σ_A, σ_B, and σ_C into (1), (2), and (3) and solving the simultaneous equations gives

$$M_x = 87 \text{ kip·in}, \quad M_z = -90.625 \text{ kip·in}, \quad P = 152.25 \text{ kips}$$

$$x = -\frac{M_z}{P} = -\frac{-90.625}{152.25} = 0.595 \text{ in.}$$

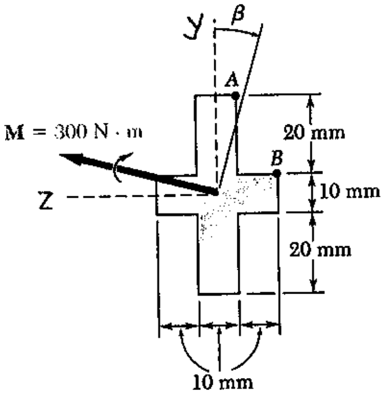
$$z = \frac{M_x}{P} = \frac{87}{152.25} = 0.571 \text{ in.}$$

$$\sigma_D = -\frac{P}{A} - \frac{M_x z_D}{I_x} + \frac{M_z x_D}{I_z} = -0.06667\,P + 0.13333\,M_x - 0.08\,M_z$$

$$= -(0.06667)(152.25) + (0.13333)(87) - (0.08)(-90.625)$$

$$= 8.70 \text{ ksi}$$

PROBLEM 4.212



4.212 The couple **M**, which acts in a vertical plane ($\beta = 0$), is applied to an aluminum beam of the cross section shown. Determine (*a*) the stress at point *A*, (*b*) the stress at point *B*, (*c*) the radius of curvature of the beam. Use $E = 72$ GPa.

SOLUTION

Label axes y and z as shown on the sketch.

$$I_z = \frac{1}{12}(10)(50)^3 + 2\cdot\frac{1}{12}(10)(10)^3$$

$$= 0.105833\times10^{-6}\ \text{mm}^4 = 0.105833\times10^{-6}\ \text{m}^4$$

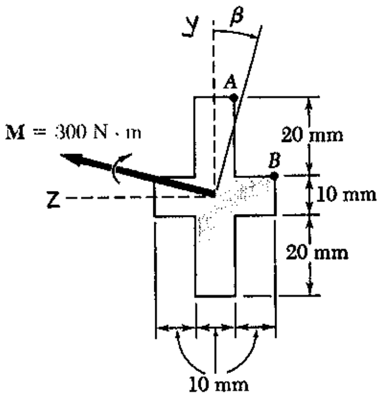
$$M_z = 300\ \text{N}\cdot\text{m} \qquad M_y = 0$$

(a) $y_A = 25$ mm $\qquad \sigma_A = -\frac{M_z y_A}{I_z} = -\frac{(300)(25\times10^{-3})}{0.105833\times10^{-6}} = -70.9\times10^6$ Pa $= -70.9$ MPa ◄

(b) $y_B = 5$ mm $\qquad \sigma_B = -\frac{M_z y_B}{I_z} = -\frac{(300)(5\times10^{-3})}{0.105833\times10^{-6}} = -14.17\times10^6$ Pa $= -14.17$ MPa ◄

(c) $\frac{1}{\rho} = \frac{M_z}{EI_z} \quad \therefore \quad \rho = \frac{EI_z}{M_z} = \frac{(72\times10^9)(0.105833\times10^{-6})}{300} = 25.4$ m ◄

PROBLEM 4.213



4.213 The couple **M** is applied to a beam of the cross section shown in a plane forming an angle $\beta = 15°$ with the vertical. Determine (*a*) the stress at point *A*, (*b*) the stress at point *B*, (*c*) the angle that the neutral axis forms with the horizontal.

SOLUTION

Label axes y and z as shown on the sketch.

$$I_z = \frac{1}{12}(10)(50)^3 + 2 \cdot \frac{1}{12}(10)(10)^3$$

$$= 0.105833 \times 10^6 \text{ mm}^4 = 0.105833 \times 10^{-6} \text{ m}^4$$

$$I_y = \frac{1}{12}(10)(30)^3 + 2 \cdot \frac{1}{12}(20)(10)^3$$

$$= 0.025833 \times 10^6 \text{ mm}^4 = 0.025833 \times 10^{-6} \text{ m}^4$$

For $\beta = 15°$ $\quad M_z = 300 \cos 15° = 289.78 \text{ N-m}$

$M_y = 300 \sin 15° = 77.65 \text{ N-m}$

(a) $y_A = 25 \text{ mm}, \quad z_A = -5 \text{ mm}$

$$\sigma_A = -\frac{M_z y_A}{I_z} + \frac{M_y z_A}{I_y} = -\frac{(289.78)(25\times10^{-3})}{0.105833\times10^{-6}} + \frac{(77.65)(-5\times10^{-3})}{0.025833\times10^{-6}}$$

$$= -83.5\times10^6 \text{ Pa} = -83.5 \text{ MPa}$$

(b) $y_B = 5 \text{mm}, \quad z_B = -15 \text{ mm}$

$$\sigma_B = -\frac{M_z y_B}{I_z} + \frac{M_y z_B}{I_y} = -\frac{(289.78)(5\times10^{-3})}{0.105833\times10^{-6}} + \frac{(77.65)(-15\times10^{-3})}{0.025833\times10^{-6}}$$

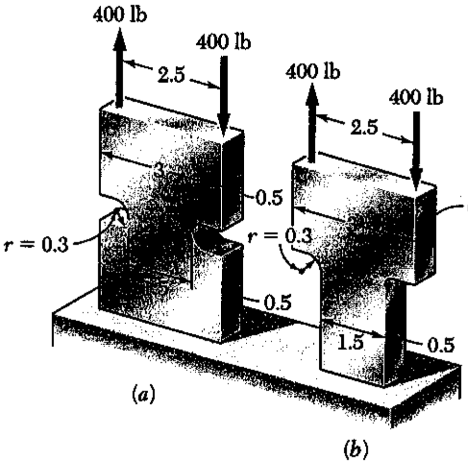
$$= -58.8\times10^6 \text{ Pa} = -58.8 \text{ MPa}$$

(c) $$\tan\varphi = \frac{I_z}{I_y}\tan\theta = \frac{0.105833\times10^{-6}}{0.025833\times10^{-6}}\tan 15° = 1.0977$$

$$\varphi = 47.7°$$

PROBLEM 4.214

4.214 Determine the maximum stress in each of the two machine elements shown.



All dimensions in inches

SOLUTION

For each case $M = (400)(2.5) = 1000$ lb·in

At the minimum section

$I = \frac{1}{12}(0.5)(1.5)^3 = 0.140625$ in^4

$c = 0.75$ in.

(a) $D/d = 3/1.5 = 2$

$r/d = 0.3/1.5 = 0.2$

From Fig 4.32 $K = 1.75$

$$\sigma_{max} = \frac{KMc}{I} = \frac{(1.75)(1000)(0.75)}{0.140625} = 9.33 \times 10^3 \text{ psi} = 9.33 \text{ ksi}$$ ◄

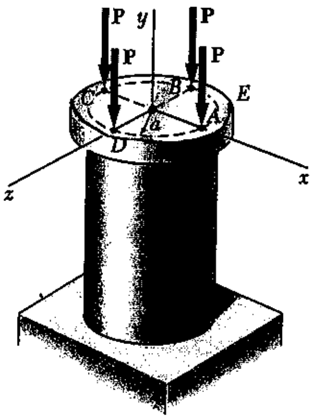
(b) $D/d = 3/1.5 = 2$ $r/d = 0.3/1.5 = 0.2$

From Fig 4.31 $K = 1.50$

$$\sigma_{max} = \frac{KMc}{I} = \frac{(1.50)(1000)(0.75)}{0.140625} = 8.00 \times 10^3 \text{ psi} = 8.00 \text{ ksi}$$ ◄

PROBLEM 4.215

4.215 The four forces shown are applied to a rigid plate supported by a solid steel post of radius a. Determine the maximum stress in the post when (a) all four forces are applied, (b) the force at D is removed, (c) the forces at C and D are removed.



SOLUTION

For a solid circular section of radius a

$$A = \pi a^2 \qquad I = \frac{\pi}{4} a^4$$

(a) Centric force $\quad F = 4P, \quad M_x = M_z = 0$

$$\sigma = -\frac{F}{A} = -\frac{4P}{\pi a^2} = -1.273\ P/a^2$$

(b) Force at D is removed.

$$F = 3P, \quad M_x = -Pa, \quad M_z = 0$$

$$\sigma = -\frac{F}{A} - \frac{M_x z}{I} = -\frac{3P}{\pi a^2} - \frac{(-Pa)(-a)}{\frac{\pi}{4} a^2} = -\frac{7P}{\pi a^2} = -2.228\ P/a^2$$

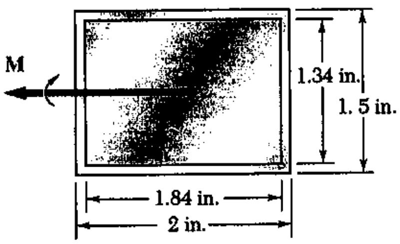
(c) Forces at C and D are removed

$$F = 2P \quad M_x = -Pa, \quad M_z = -Pa$$

Resultant bending couple $\quad M = \sqrt{M_x^2 + M_z^2} = \sqrt{2}\,Pa$

$$\sigma = -\frac{F}{A} - \frac{Mc}{I} = -\frac{2P}{\pi a^2} - \frac{\sqrt{2}\,Pa\,a}{\frac{\pi}{4} a^2} = -\frac{2+4\sqrt{2}}{\pi}\frac{P}{a^2} = -2.437\ P/a^2$$

PROBLEM 4.216



4.216 In order to increase corrosion resistance, a 0.08-in.-thick cladding of aluminum has been added to a steel bar as shown. The modulus of elasticity is 29×10^6 psi for steel and 10.4×10^6 psi for aluminum. For a bending moment of 12 kip·in., determine (*a*) the maximum stress in the steel, (*b*) the maximum stress in the aluminum, (*c*) the radius of curvature of the bar.

SOLUTION

Use steel as the reference material

$$n_{steel} = 1 \qquad n_{alum} = \frac{E_a}{E_s} = \frac{10.4}{29} = 0.3586$$

$$I_{trans} = I_{steel} + n_{alum} I_{alum}$$

$$= \frac{1}{12}(1.84)(1.34)^3 + 0.3586 \cdot \frac{1}{12}\left[(2)(1.5)^3 - (1.84)(1.34)^3\right] = 0.43835 \text{ in}^4$$

(a) $y_s = \frac{1.34}{2} = 0.67$ in

$$\sigma_s = \frac{M y_s}{I_{trans}} = \frac{(12)(0.67)}{0.43835} = 18.35 \text{ ksi}$$

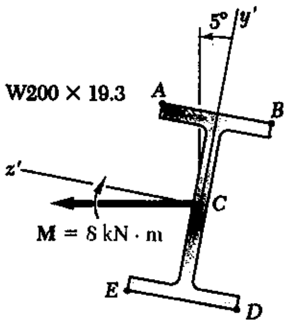
(b) $y_a = \frac{1.5}{2} = 0.75$ in.

$$\sigma_a = n_a \frac{M y_a}{I} = 0.3586 \frac{(12)(0.75)}{0.43835} = 7.36 \text{ ksi}$$

(c) $$\frac{1}{\rho} = \frac{M}{E_{ref} I_{trans}} = \frac{12 \times 10^3}{(29 \times 10^6)(0.43835)} = 944 \times 10^{-6} \text{ in}^{-1}$$

$$\rho = 1059 \text{ in} = 88.3 \text{ ft.}$$

PROBLEM 4.217



4.217 A couple **M** of moment 8 kN·m acting in a vertical plane is applied to a W 200 × 19.3 rolled-steel beam as shown. Determine (*a*) the angle that the neutral axis forms with the horizontal plane, (*b*) the maximum stress in the beam.

SOLUTION

For W 200 × 19.3 rolled steel section

$$I_{z'} = 16.6 \times 10^{6}\ \text{mm}^4 = 16.6 \times 10^{-6}\ \text{m}^4$$

$$I_{y'} = 1.15 \times 10^{6}\ \text{mm}^4 = 1.15 \times 10^{-6}\ \text{m}^4$$

$$y_A = y_B = -y_D = -y_E = \frac{203}{2} = 101.5\ \text{mm}$$

$$z_A = -z_B = -z_D = z_E = \frac{102}{2} = 51\ \text{mm}$$

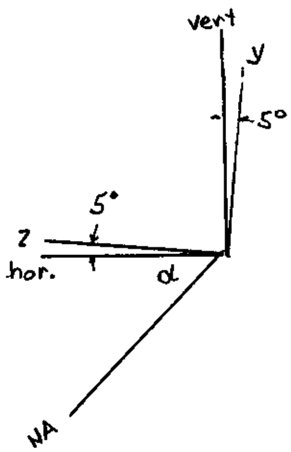
$$M_z = (8 \times 10^3)\cos 5° = 7.9696 \times 10^3\ \text{N}\cdot\text{m}$$

$$M_y = -(8 \times 10^3)\sin 5° = -0.6972 \times 10^3\ \text{N}\cdot\text{m}$$

(a) $$\tan\varphi = \frac{I_z}{I_y}\tan\theta = \frac{16.6 \times 10^{-6}}{1.15 \times 10^{-6}}\tan(-5°) = -1.2629$$

$$\varphi = -51.6°$$

$$\alpha = 51.6° - 5° = 46.6°$$



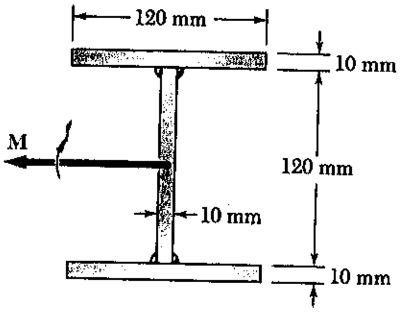
(b) Maximum tensile stress occurs at point D

$$\sigma_D = -\frac{M_z y_D}{I_z} + \frac{M_y z_D}{I_y} = -\frac{(7.9696 \times 10^3)(-101.5 \times 10^{-3})}{16.6 \times 10^{-6}} + \frac{(0.6972 \times 10^3)(51 \times 10^{-3})}{1.15 \times 10^{-6}}$$

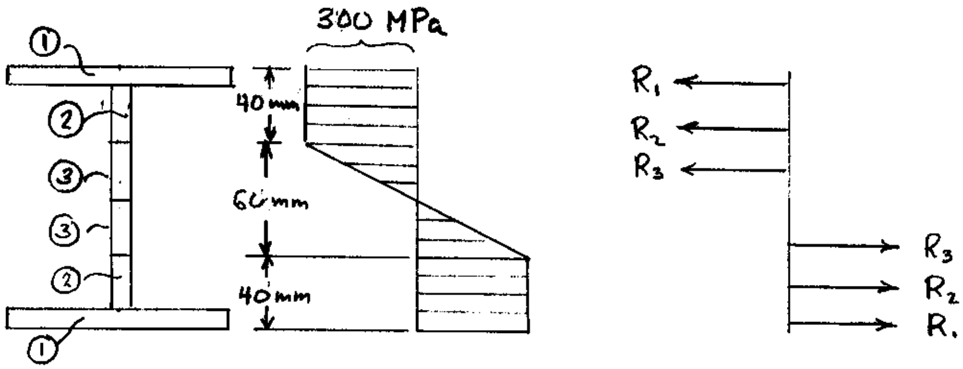
$$= 79.6 \times 10^{6}\ \text{Pa} = 79.6\ \text{MPa}$$

PROBLEM 4.218

4.218 Three 120 × 10-mm steel plates have been welded together to form the beam shown. Assuming that the steel is elastoplastic with $E = 200$ GPa and $\sigma_Y = 300$ MPa, determine (*a*) the bending moment for which the plastic zones at the top and bottom of the beam are 40 mm thick, (*b*) the corresponding radius of curvature of the beam.



SOLUTION



$A_1 = (120)(10) = 1200 \text{ mm}^2$ $\qquad R_1 = \sigma_Y A_1 = (300 \times 10^6)(1200 \times 10^{-6}) = 360 \times 10^3$ N

$A_2 = (30)(10) = 300 \text{ mm}^2$ $\qquad R_2 = \sigma_Y A_2 = (300 \times 10^6)(300 \times 10^{-6}) = 90 \times 10^3$ N

$A_3 = (30)(10) = 300 \text{ mm}^2$ $\qquad R_3 = \frac{1}{2}\sigma_Y A_2 = \frac{1}{2}(300 \times 10^6)(300 \times 10^{-6})$

$= 45 \times 10^3$ N

$y_1 = 65 \text{ mm} = 65 \times 10^{-3}$ m

$y_2 = 45 \text{ mm} = 45 \times 10^{-3}$ m

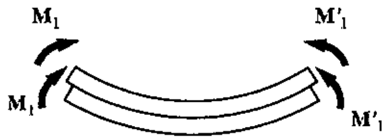
$y_3 = 20 \text{ mm} = 20 \times 10^{-3}$ m

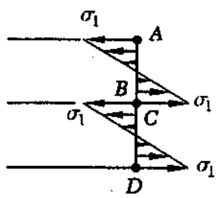
(a) $M = 2(R_1 y_1 + R_2 y_2 + R_3 y_3) = 2\{(360)(65) + (90)(45) + (45)(20)\}$

$= 56.7 \times 10^3 \text{ N}\cdot\text{m} = 56.7 \text{ kN}\cdot\text{m}$ ◄

(b) $\dfrac{y_Y}{\rho} = \dfrac{\sigma_Y}{E}$ $\qquad \rho = \dfrac{E y_Y}{\sigma_Y} = \dfrac{(200 \times 10^9)(30 \times 10^{-3})}{300 \times 10^6} = 20$ m ◄

PROBLEM 4.219





4.219 Two thin strips of the same material and same cross section are bent by couples of the same magnitude and glued together. After the two surfaces of contact have been securely bonded, the couples are removed. Denoting by σ_1 the maximum stress and by ρ_1 the radius of curvature of each strip while the couples were applied, determine (*a*) the final stresses at points *A*, *B*, *C*, and *D*, (*b*) the final radius of curvature

SOLUTION

Let b = width and t = thickness of one strip.

Loading one strip $\quad M = M_1$

$$I_1 = \frac{1}{12} b t^3, \qquad c = \frac{1}{2} t$$

$$\sigma_1 = \frac{M_1 c}{I} = \frac{6 M_1}{b t^2}$$

$$\frac{1}{\rho_1} = \frac{M_1}{E I_1} = \frac{12 M_1}{E t^3}$$

After M_1 is applied to each of the strips, the stresses are those given in the sketch above. They are

$$\sigma_A = -\sigma_1, \quad \sigma_B = \sigma_1, \quad \sigma_C = -\sigma_1, \quad \sigma_D = \sigma_1$$

The total bending couple is $2M_1$.

After gluing, this couple is removed.

$$M' = 2M_1, \qquad I' = \frac{1}{12} b (2t)^3 = \frac{2}{3} b t^3$$

$c = t$. The stresses removed are

$$\sigma' = -\frac{M' y}{I} = -\frac{2 M_1 y}{\frac{2}{3} b t^3} = -\frac{3 M_1 y}{b t^2}$$

$$\sigma_A' = -\frac{3M_1}{bt^2} = -\frac{1}{2}\sigma_1, \quad \sigma_B' = \sigma_C' = 0, \quad \sigma_D' = \frac{3M_1}{bt^2} = \frac{1}{2}\sigma_1$$

(a) Final stresses:

$$\sigma_A = -\sigma_1 - \left(-\frac{1}{2}\sigma_1\right) = -\frac{1}{2}\sigma_1$$

$$\sigma_B = \sigma_1$$

$$\sigma_C = -\sigma_1$$

$$\sigma_D = \sigma_1 - \frac{1}{2}\sigma_1 = \frac{1}{2}\sigma_1$$

$$\frac{1}{\rho'} = \frac{M'}{EI'} = \frac{2M_1}{E\frac{2}{3}bt^3} = \frac{3M_1}{Et^3} = \frac{1}{4}\frac{1}{\rho_1}$$

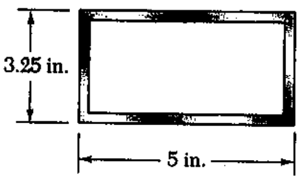
(b) Final radius

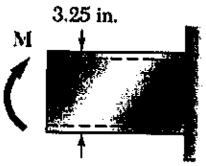
$$\frac{1}{\rho} = \frac{1}{\rho_1} - \frac{1}{\rho'} = \frac{1}{\rho_1} - \frac{1}{4\rho_1} = \frac{3}{4}\frac{1}{\rho_1}$$

$$\rho = \frac{4}{3}\rho_1$$

PROBLEM 4.220

4.220 Knowing that the hollow beam shown has a uniform wall thickness of 0.25 in. determine (*a*) the largest couple that can be applied without exceeding the allowable stress of 20 ksi, (*b*) the corresponding radius of curvature of the beam.





SOLUTION

$E = 10.6 \times 10^6$ psi

$$I = \frac{1}{12} b_o h_o^3 - \frac{1}{12} b_i h_i^3 = \frac{1}{12}(5)(3.25)^3 - \frac{1}{12}(4.5)(2.75)^3 = 6.5046 \text{ in}^4$$

$$c = \frac{3.25}{2} = 1.625 \text{ in.}$$

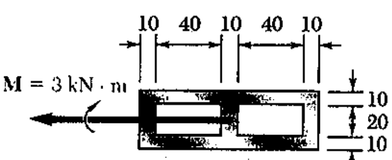
(a) $\sigma_{max} = \dfrac{Mc}{I} \quad \therefore \quad M = \dfrac{\sigma_{max} I}{c} = \dfrac{(20)(6.5046)}{1.625} = 80.1$ kip·in. ◄

(b) $\varepsilon_{max} = \dfrac{c}{\rho} = \dfrac{\sigma_{max}}{E} \quad \therefore \quad \rho = \dfrac{Ec}{\sigma_{max}} = \dfrac{(10.6 \times 10^6)(1.625)}{20 \times 10^3}$

$= 861$ in $= 71.8$ ft. ◄

PROBLEM 4.221

4.221 A beam of the cross section shown is extruded from an aluminum alloy for which $E = 72$ GPa. Knowing that the couple shown acts in a vertical plane, determine (*a*) the maximum stress in the beam, (*b*) the corresponding radius of curvature.



Dimensions in mm

SOLUTION

For outer rectangle: $b = 110$ mm, $h = 40$ mm

$$I_1 = \frac{1}{12} bh^3 = \frac{1}{12}(110)(40)^3 = 0.58667 \times 10^6 \text{ mm}^4$$

For one cutout rectangle: $b = 40$ mm, $h = 20$ mm

$$I_2 = \frac{1}{12} bh^3 = \frac{1}{12}(40)(20)^3 = 0.02667 \times 10^6 \text{ mm}^4$$

$$I = I_1 - 2I_2 = 0.53333 \times 10^6 \text{ mm}^4 = 0.53333 \times 10^{-6} \text{ m}^4$$

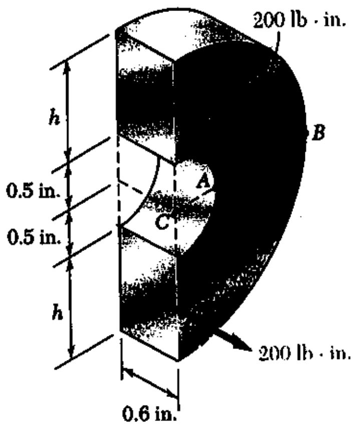
$M = 3 \times 10^3$ N·m $\qquad c = 20$ mm $= 20 \times 10^{-3}$ m

(a) $\sigma = \dfrac{Mc}{I} = \dfrac{(3 \times 10^3)(20 \times 10^{-3})}{0.53333 \times 10^{-6}} = 112.5 \times 10^6$ Pa $= 112.5$ MPa ◄

(b) $\dfrac{1}{\rho} = \dfrac{M}{EI} \quad \therefore \quad \rho = \dfrac{EI}{M} = \dfrac{(72 \times 10^9)(0.53333 \times 10^{-6})}{3 \times 10^3} = 12.80$ m ◄

PROBLEM 4.222

4.222 For the machine element and loading shown, determine the stress at point *A*, knowing that (*a*) $h = 0.9$ in., (*b*) $h = 1.5$ in.



SOLUTION

(a) $h = 0.9\ \text{in}, \quad r_1 = 0.5\ \text{in.} \quad r_2 = 1.4\ \text{in}$

$$A = (0.6)(0.9) = 0.54\ \text{in}^2$$

$$R = \frac{h}{\ln\frac{r_2}{r_1}} = \frac{0.9}{\ln\frac{1.4}{0.5}} = 0.87411\ \text{in.}$$

$$\bar{r} = \tfrac{1}{2}(r_1 + r_2) = 0.95\ \text{in}$$

$$e = \bar{r} - R = 0.07589\ \text{in}$$

$$M = -200\ \text{lb}\cdot\text{in}, \quad y_A = R - r_1 = 0.37411$$

$$\sigma_A = -\frac{M y_A}{A e r_1} = -\frac{(-200)(0.37411)}{(0.54)(0.07589)(0.5)} = 3.65\times10^3\ \text{psi} = 3.65\ \text{ksi}$$

(b) $h = 1.5\ \text{in}, \quad r_1 = 0.5\ \text{in}, \quad r_2 = 2.0\ \text{in.}$

$$A = (0.6)(1.5) = 0.90\ \text{in}^2$$

$$R = \frac{h}{\ln\frac{r_2}{r_1}} = \frac{1.5}{\ln\frac{2.0}{0.5}} = 1.08202\ \text{in.}$$

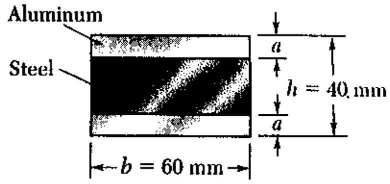
$$\bar{r} = \tfrac{1}{2}(r_1 + r_2) = 1.25\ \text{in.}$$

$$e = \bar{r} - R = 0.16798\ \text{in.}$$

$$M = -200\ \text{lb}\cdot\text{in.} \qquad y_A = R - r_1 = 0.58202\ \text{in}$$

$$\sigma_A = -\frac{M y_A}{A e r} = -\frac{(-200)(0.58202)}{(0.90)(0.16798)(0.5)} = 1.540\times10^3\ \text{psi} = 1.540\ \text{ksi}$$

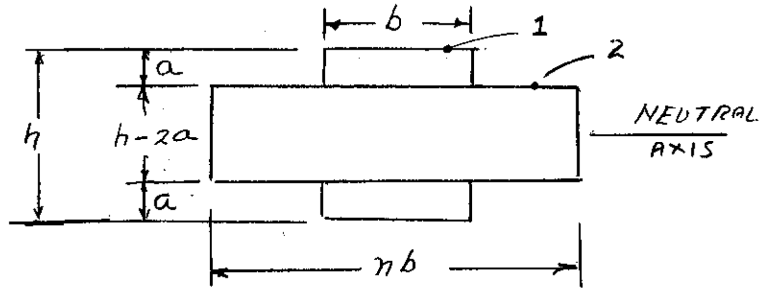
PROBLEM 4.C1

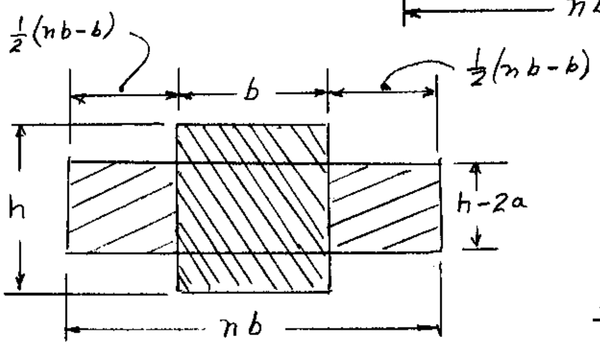


4.C1 Two aluminum strips and a steel strip are to be bonded together to form a composite member of width $b = 60$ mm and depth $h = 40$ mm. The modulus of elasticity is 200 GPa for the steel and 75 GPa for the aluminum. Knowing that $M = 1500$ N · m, write a computer program to calculate the maximum stress in the aluminum and in the steel for values of a from 0 to 20 mm using 2-mm increments. Using appropriate smaller increments, determine (*a*) the largest stress that can occur in the steel, (*b*) the corresponding value of a.

SOLUTION

TRANSFORMED SECTION (ALL STEEL) $n = \dfrac{E_{STEEL}}{E_{ALUM}}$





$$\bar{I} = \frac{1}{12} b h^3 + \frac{1}{12}\left[2\left(\tfrac{1}{2}\right)(nb-b)\right](h-2a)^3$$

AT POINT 1: $\sigma_{ALUM} = \dfrac{M\left(\frac{h}{2}\right)}{\bar{I}}$

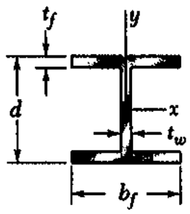
AT POINT 2: $\sigma_{STEEL} = n\,\dfrac{M\left(\frac{h}{2}-a\right)}{\bar{I}}$

FOR $a = 0$ TO 20 mm USING 2-mm INTERVALS COMPUTE: n, $\bar{I}$, σ_{ALUM}, σ_{STEEL}.

PROGRAM OUTPUT

```
 b = 60 mm    h = 40 mm     M = 1500 N.m
  Moduli of elasticity:  Steel = 200 GPa  Aluminum = 75 GPa

    a        I          sigma       sigma
                       aluminum     steel
   mm     m^4/10^6       MPa         MPa

  0.000    0.8533     35.156      93.750
  2.000    0.7088     42.325     101.580
  4.000    0.5931     50.585     107.914
  6.000    0.5029     59.650     111.347
  8.000    0.4352     68.934     110.294
 10.000    0.3867     77.586     103.448
 12.000    0.3541     84.714      90.361
 14.000    0.3344     89.713      71.770
 16.000    0.3243     92.516      49.342
 18.000    0.3205     93.594      24.958
 20.000    0.3200     93.750       0.000
====================================================
 Find 'a' for max steel stress
     and the corresponding aluminum stress

  6.600    0.4804     62.447     111.572083
  6.610    0.4800     62.494     111.572159
  6.620    0.4797     62.540     111.572113

   Max Steel Stress = 111.6 MPa occurs when a = 6.61 mm
   Corresponding Aluminum stress = 62.5 MPa
```

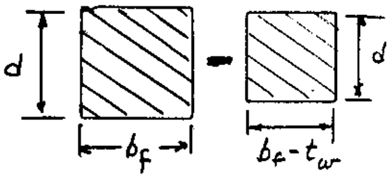
PROBLEM 4.C2



4.C2 A beam of the cross section shown, made of a steel that is assumed to be elastoplastic with a yield strength σ_Y and a modulus of elasticity E, is bent about the x axis. (*a*) Denoting by y_Y the half thickness of the elastic core, write a computer program to calculate the bending moment M and the radius of curvature ρ for values of y_Y from $\frac{1}{2}d$ to $\frac{1}{6}d$ using decrements equal to $\frac{1}{2}t_f$. Neglect the effect of fillets. (*b*) Use this program to solve Prob. 4.218.

SOLUTION

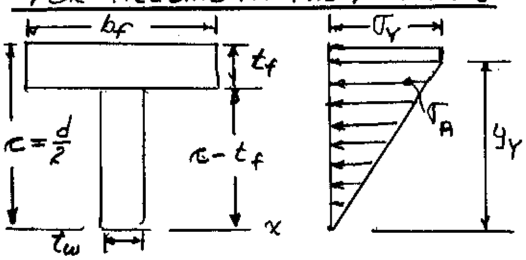
COMPUTE MOMENT OF INERTIA I_x



$$I_x = \frac{1}{12} b_f d^3 - \frac{1}{12}(b_f - t_w)(d - 2t_f)^3$$

MAXIMUM ELASTIC MOMENT: $M_Y = \sigma_Y \dfrac{I_x}{(d/2)}$

FOR YIELDING IN THE FLANGES: (CONSIDER UPPER HALF OF CROSS SECTION) $c = \frac{d}{2}$

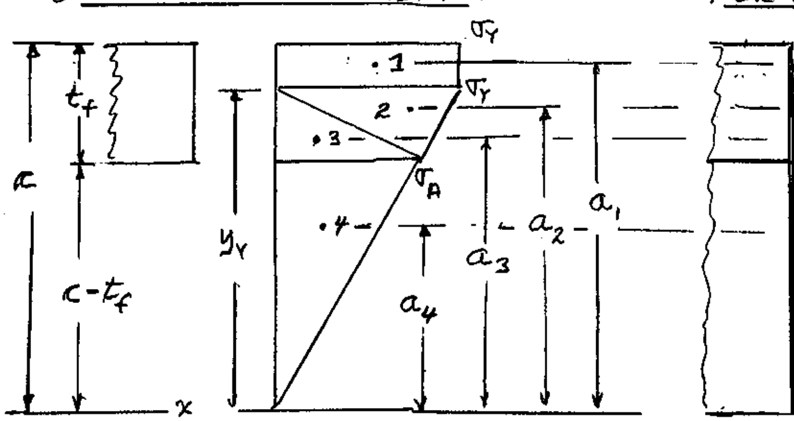


STRESS AT JUNCTION OF WEB AND FLANGE

$$\sigma_A = \frac{(d/2) - t_f}{y_Y}\,\sigma_Y$$

DETAIL OF STRESS DIAGRAM

RESULTANT FORCES



$R_1 = \sigma_Y \left[b_f (c - \sigma_Y) \right]$

$R_2 = \frac{1}{2} \sigma_Y \, b_f \left[y_Y - (c - t_f) \right]$

$R_3 = \frac{1}{2} \sigma_A \, b_f \left[y_Y - (c - t_f) \right]$

$R_3 = \frac{1}{2} \sigma_A \, t_w (c - t_f)$

$a_1 = \frac{1}{2}(c + y_Y)$

$a_2 = y_Y - \frac{1}{3}\left[y_Y - (c - t_f) \right]$

$a_3 = y_Y - \frac{2}{3}\left[y_Y - (c - t_f) \right]$

$a_4 = \frac{2}{3}(c - t_f)$

BENDING MOMENT

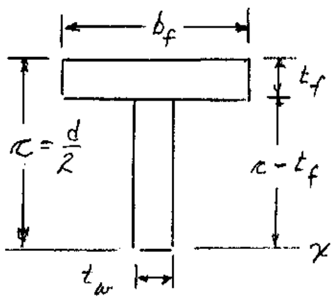
$$M = 2 \sum_{n=1}^{4} R_n a_n$$

RADIUS OF CURVATURE

$$y_Y = \varepsilon_Y \rho = \frac{\sigma_Y}{E} \rho \,; \quad \rho = \frac{y_Y E}{\sigma_Y}$$

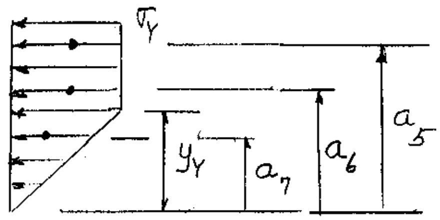
CONTINUED

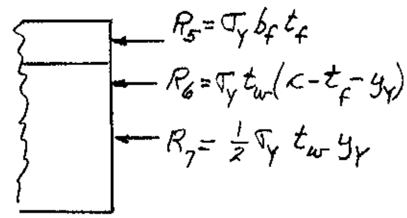
PROBLEM 4.C2 - CONTINUED



FOR YIELDING IN THE WEB (CONSIDER UPPER HALF OF CROSS SECTION)

$c = d/2$





$$a_5 = c - \tfrac{1}{2} t_f$$

$$a_6 = \tfrac{1}{2}\left[y_Y + (c - t_f)\right]$$

$$a_7 = \tfrac{2}{3} y_Y$$

BENDING MOMENT $\quad M = 2 \sum_{n=5}^{7} R_n a_n$

RADIUS OF CURVATURE $\quad y_Y = \epsilon_Y \rho = \dfrac{\sigma_Y}{E}\rho \qquad \rho = \dfrac{y_Y E}{\sigma_Y}$

PROGRAM: KEY IN EXPRESSIONS FOR a_n and R_n FOR $n = 1$ TO 7.

FOR $y_Y = c$ TO $(c - t_f)$ AT $-t_f/2$ DECREMENTS

COMPUTE $M = 2 \sum R_n a_n$, FOR $n = 1$ TO 4 AND $\rho = \dfrac{y_Y E}{\sigma_Y}$, THEN PRINT

FOR $y_Y = (c - t_w)$ TO $c/3$ AT $-t_f/2$ DECREMENTS

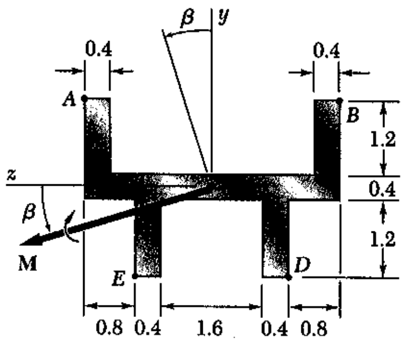
COMPUTE $M = 2 \sum R_n a_n$ FOR $n = 5$ TO 7. AND $\rho = \dfrac{y_Y E}{\sigma_Y}$, THEN PRINT

INPUT NUMERICAL VALUES AND RUN PROGRAM

PROGRAM OUTPUT

```
 For a beam of Prob 4.218
Depth d = 140.00 mm                      Width of flange bf = 120.00 mm
Thickness of flange tf =  10.00 mm       Thickness of web tw =  10.00 mm

I = 0.000011600 m to the 4th
Yield strength of Steel sigmaY = 300 MPa
Yield Moment MY = 49.71 kip.in.

          yY(mm)              M(kN.m)     rho(m)
 For yielding still in the flange.
        70.000              49.71        46.67
        65.000              52.59        43.33
        60.000              54.00        40.00
 For yielding in the web
        60.000              54.00        40.00
        55.000              54.58        36.67
        50.000              55.10        33.33
        45.000              55.58        30.00
        40.000              56.00        26.67
        35.000              56.38        23.33
        30.000              56.70        20.00
        25.000              56.97        16.67
```

PROBLEM 4.C3



Dimensions in inches

4.C3 An 8 kip · in. couple **M** is applied to a beam of the cross section shown in a plane forming an angle β with the vertical. Noting that the centroid of the cross section is located at C and that the y and z axes are principal axes, write a computer program to calculate the stress at A, B, C, and D for values of β from 0 to 180° using 10° increments. (*Given*: $I_y = 6.23 \text{ in}^4$ and $I_z = 1.481 \text{ in}^4$.)

SOLUTION INPUT COORDINATES OF A, B, C, D

$z_A = z(1) = 2$ $\quad y_A = y(1) = 1.4$

$z_B = z(2) = -2$ $\quad y_B = y(2) = 1.4$

$z_C = z(3) = -1$ $\quad y_C = y(3) = -1.4$

$z_D = z(4) = 1$ $\quad y_D = y(4) = -1.4$

COMPONENTS OF M.

$M_y = -M \sin\beta \qquad M_z = M\cos\beta$

Eq. 4.55 page 273: $\sigma(n) = -\dfrac{M_z\, y(n)}{I_z} + \dfrac{M_y\, z(n)}{I_y}$

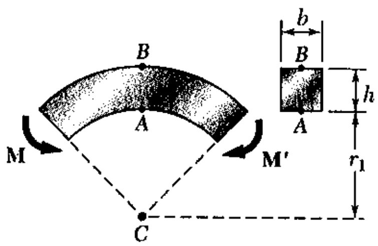
PROGRAM: FOR $\beta = 0$ TO 180° USING 10° INCREMENTS.
FOR $n = 1$ TO 4 USING UNIT INCREMENTS.
EVALUATE EQ 4.55 AND PRINT STRESSES
RETURN
RETURN

PROGRAM OUTPUT

```
Moment of couple M =  8.00 kip·in.
Moments of inertia: Iy = 6.23  in^4     Iz  = 1.481 in^4

      Coordinates of points A, B, D, and E
     Point A:  z(1)  =  2:      y(1)  =  1.4
     Point B:  z(2)  = -2:      y(2)  =  1.4
     Point D:  z(3)  = -1:      y(3)  = -1.4
     Point E:  z(4)  =  1:      y(4)  = -1.4
              - - - Stress at Points - - -
```

beta °	A ksi	B ksi	D ksi	E ksi
0	-7.565	-7.565	7.565	7.565
10	-7.896	-7.004	7.673	7.227
20	-7.987	-6.230	7.548	6.669
30	-7.836	-5.267	7.193	5.909
40	-7.446	-4.144	6.621	4.970
50	-6.830	-2.895	5.846	3.879
60	-6.007	-1.558	4.895	2.670
70	-5.001	-0.174	3.794	1.381
80	-3.843	1.216	2.578	0.049
90	-2.569	2.569	1.284	-1.284
100	-1.216	3.843	-0.049	-2.578
110	0.174	5.001	-1.381	-3.794
120	1.558	6.007	-2.670	-4.895
130	2.895	6.830	-3.879	-5.846
140	4.144	7.446	-4.970	-6.621
150	5.267	7.836	-5.909	-7.193
160	6.230	7.987	-6.669	-7.548
170	7.004	7.896	-7.227	-7.673
180	7.565	7.565	-7.565	-7.565

PROBLEM 4.C4



4.C4 Couples of moment $M = 2\text{ kN}\cdot\text{m}$ are applied as shown to a curved bar having a rectangular cross section with $h = 100\text{ mm}$ and $b = 25\text{ mm}$. Write a computer program and use it to calculate the stresses at points A and B for values of the ratio r_1/h from 10 to 1 using decrements of 1, and from 1 to 0.1 using decrements of 0.1. Using appropriate smaller increments, determine the ratio r_1/h for which the maximum stress in the curved bar is 50 percent larger than the maximum stress in a strainght bar of the same cross section.

SOLUTION INPUT: $h = 100\text{ mm}$, $b = 25\text{ mm}$, $M = 2\text{ kN}\cdot\text{m}$

FOR STRAIGHT BAR: $\sigma_{STRAIGHT} = \frac{M}{S} = \frac{6M}{h^2 b} = 48\text{ MPa}$

FOLLOWING NOTATION OF SEC. 4.15, KEY IN THE FOLLOWING:

$r_2 = h + r_1$; $R = h/\ln(r_2 - r_1)$; $\bar{r} = r_1 + r_2$: $e = \bar{r} - R$; $A = bh = 2500$ (I)

STRESSES: $\sigma_A = \sigma_1 = M(r_1 - R)/(Aer_1)$ $\quad \sigma_B = \sigma_2 = M(r_2 - R)/(Aer_2)$ (II)

SINCE $h = 100\text{ mm}$, FOR $r_1/h = 10$, $r_1 = 1000\text{ mm}$. ALSO $r_1/h = 10$, $r_1 = 100$

PROGRAM: FOR $r_1 = 1000$ TO 100 AT -100 DECREMENTS

USING EQUATIONS OF LINES I AND II EVALUATE r_2, R, $\bar{r}$, e, σ_1 AND σ_2

ALSO EVALUATE: $\text{ratio} = \sigma_1/\sigma_{STRAIGHT}$

RETURN AND REPEAT FOR $r_1 = 100$ TO 10 AT -10 DECREMENT

PROGRAM OUTPUT

```
M = Bending Moment = 2.  kN.m    h = 100.000 in.    A = 2500.00 mm^2
Stress in straight beam  =  48.00 MPa
```

r1	rbar	R	e	sigma1	sigma2	r1/h	ratio
mm	mm	mm	mm	MPa	MPa	-	-
1000	1050	1049	0.794	-49.57	46.51	10.000	-1.033
900	950	949	0.878	-49.74	46.36	9.000	-1.036
800	850	849	0.981	-49.95	46.18	8.000	-1.041
700	750	749	1.112	-50.22	45.95	7.000	-1.046
600	650	649	1.284	-50.59	45.64	6.000	-1.054
500	550	548	1.518	-51.08	45.24	5.000	-1.064
400	450	448	1.858	-51.82	44.66	4.000	-1.080
300	350	348	2.394	-53.03	43.77	3.000	-1.105
200	250	247	3.370	-55.35	42.24	2.000	-1.153
100	150	144	5.730	-61.80	38.90	1.000	-1.288
100	150	144	5.730	-61.80	38.90	1.000	-1.288
90	140	134	6.170	-63.15	38.33	0.900	-1.316
80	130	123	6.685	-64.80	37.69	0.800	-1.350
70	120	113	7.299	-66.86	36.94	0.700	-1.393
60	110	102	8.045	-69.53	36.07	0.600	-1.449
50	100	91	8.976	-73.13	35.04	0.500	-1.523
40	90	80	10.176	-78.27	33.79	0.400	-1.631
30	80	68	11.803	-86.30	32.22	0.300	-1.798
20	70	56	14.189	-100.95	30.16	0.200	-2.103
10	60	42	18.297	-138.62	27.15	0.100	-2.888

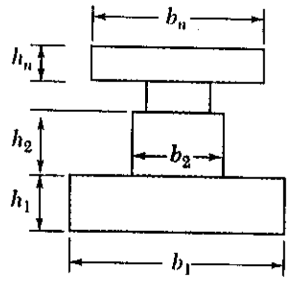
Find r1/h for (sigma max)/(sigma straight) = 1.5

52.70	103	94	8.703	-72.036	35.34	0.527	-1.501
52.80	103	94	8.693	-71.998	35.35	0.528	-1.500
52.90	103	94	8.683	-71.959	35.36	0.529	-1.499

Ratio of stresses is 1.5 for r1 = 52.8 mm or r1/h = 0.529

[Note: The desired ratio r1/h is valid for any beam having a rectangular cross section.]

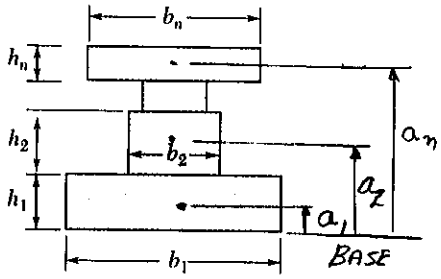
PROBLEM 4.C5



4.C5 The couple **M** is applied to a beam of the cross section shown. (*a*) Write a computer program that, for loads expressed in either SI or U.S. customary units, can be used to calculate the maximum tensile and compressive stresses in the beam. (*b*) Use this program to solve Probs. 4.1, 4.10, and 4.11.

SOLUTION

INPUT: BENDING MOMENT M



FOR $n = 1$ TO n: ENTER b_n AND h_n

$\Delta AREA = b_n h_n$ (PRINT)

$a_n = a_{n-1} + (h_{n-1})/2 + h_n/2$

[MOMENT OF RECTANGLE ABOUT BASE]

$\Delta m = (\Delta AREA)\, a_n$

[FOR WHOLE CROSS SECTION]

$m = m + \Delta m$; $AREA = AREA + \Delta AREA$

LOCATION OF CENTROID ABOVE BASE

$\bar{y} = m/AREA$ (PRINT)

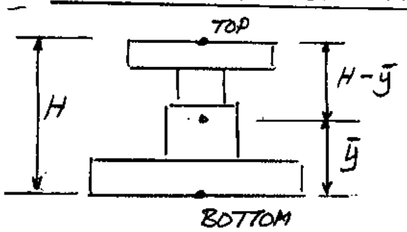
MOMENT OF INERTIA ABOUT HORIZONTAL CENTROIDAL AXIS

FOR $n = 1$ TO n: $a_n = a_{n-1} + (h_{n-1})/2 + h_n/2$

$\Delta I = b_n h_n^3/12 + (b_n h_n)(\bar{y} - a_n)^2$

$I = I + \Delta I$ (PRINT)

COMPUTATION OF STRESSES



TOTAL HEIGHT: FOR $n = 1$ TO n

$H = H + h_n$

STRESS AT TOP

$$M_{TOP} = -M \frac{H - \bar{y}}{I}$$ (PRINT)

STRESS AT BOTTOM

$$M_{BOTTOM} = M \frac{\bar{y}}{I}$$ (PRINT)

SEE NEXT PAGE FOR PRINTOUTS FOR PROBLEMS 4.1, 4.10, 4.11

CONTINUED

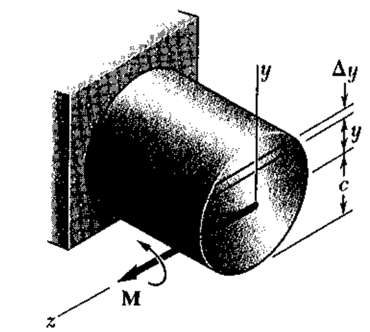
PROBLEM 4.C5 - CONTINUED

```
          Problem 4.1
Summary of Cross Section Dimensions
     Width (in.)       Height (in.)
        2.00              2.00
        6.00              1.50
        2.00              2.00

Bending Moment =  25.000 kip.in.
Centroid is    2.750 in. above lower edge
Centroidal Moment of Inertia is    28.854 in^4

Stress at top of beam    =   -2.383 ksi
Stress at bottom of beam =    2.383 ksi
```

```
          Problem 4.10
Summary of Cross Section Dimensions
     Width (in.)       Height (in.)
        9.00              2.00
        3.00              6.00

Bending Moment = 600.000 kip.in.
Centroid is     3.000 in. above lower edge
Centroidal Moment of Inertia is   204.000 in^4

Stress at top of beam    =  -14.706 ksi
Stress at bottom of beam =    8.824 ksi
```

```
          Problem 4.11
Summary of Cross Section Dimensions
     Width (in.)       Height (in.)
        4.00              1.00
        1.00              6.00
        8.00              1.00

Bending Moment = 500.000 kip.in.
Centroid is    4.778 in. above lower edge
Centroidal Moment of Inertia is   155.111 in^4

Stress at top of beam    =  -10.387 ksi
Stress at bottom of beam =   15.401 ksi
```

PROBLEM 4.C6



4.C6 A solid rod of radius $c = 1.2$ in. is made of a steel that is assumed to be elastoplastic with $E = 29{,}000$ ksi and $\sigma_Y = 42$ ksi. The rod is subjected to a couple of moment M that increases from zero to the maximum elastic moment M_Y and then to the plastic moment M_p. Denoting by y_Y the half thickness of the elastic core, write a computer program and use it to calculate the bending moment M and the radius of curvature ρ for values of y_Y from 1.2 in. to 0 using 0.2-in. decrements. (*Hint*: Divide the cross section into 80 horizontal elements of 0.03-in. height.)

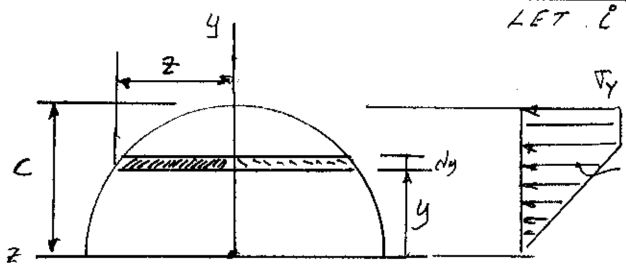
SOLUTION $M_Y = \sigma_Y \frac{\pi}{4} c^3 = (42 \text{ ksi}) \frac{\pi}{4} (1.2 \text{ in.})^3 = 57 \text{ kip}\cdot\text{in}$

$$M_p = \sigma_Y \frac{4}{3} c^3 = (42 \text{ ksi}) \frac{4}{3} (1.2 \text{ in})^3 = 96.8 \text{ kip}\cdot\text{in.}$$

CONSIDER TOP HALF OF ROD

LET i = NUMBER OF ELEMENTS IN TOP HALF

Δh = HEIGTH OF EACH ELEMENT: $\Delta h = \frac{c}{i}$



σ_E = STRESS ON ELEMENT

FOR $n = 0$ TO $i-1$ STEP 1

$y = n(\Delta h)$

$z = \left[c^2 - \{(n+0.5)\Delta h\}^2 \right]$ ← z AT MIDHEIGHT OF ELEMENT

IF $y \geq y_Y$ GO TO 100

$\sigma_E = \sigma_Y \dfrac{(n+0.5)\Delta h}{\sigma_y}$ ← STRESS IN ELASTIC CORE

GO TO 200

100 $\sigma_E = \sigma_Y$ ← STRESS IN PLASTIC ZONE

200 $\Delta AREA = 2z(\Delta h)$

$\Delta FORCE = \sigma_E (\Delta AREA)$

$\Delta MOMENT = \Delta FORCE (n+.5)\Delta h$

$M = M + \Delta MOMENT$

$\rho = y_Y E / \sigma_Y$

PRINT y_Y, M, AND ρ.

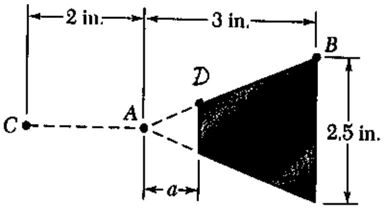
NEXT

REPEAT FOR $y_Y = 1.2$ in. TO $y_Y = 0$ AT -0.2-in. DECREMENTS

PROGRAM OUTPUT

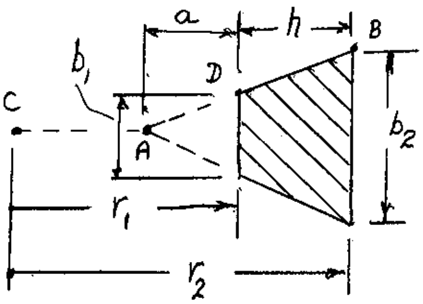
```
Radius of rod = 1.2 in.
Yield point of steel =  42 ksi
Yield moment = 57.0  kip·in        Plastic moment = 96.8  kip.in.
Number of elements in half of the rod =  40

For yY =  1.20 in.     M = 57.1 kip·in.    Radius of curvature = 828.57 in.
For yY =  1.00 in.     M = 67.2 kip·in.    Radius of curvature = 690.48 in.
For yY =  0.80 in.     M = 76.9 kip·in.    Radius of curvature = 552.38 in.
For yY =  0.60 in.     M = 85.2 kip·in.    Radius of curvature = 414.29 in.
For yY =  0.40 in.     M = 91.6 kip·in.    Radius of curvature = 276.19 in.
For yY =  0.20 in.     M = 95.5 kip·in.    Radius of curvature = 138.10 in.
For yY =  0.00 in.     M = infinite        Radius of curvature = zero
```

PROBLEM 4.C7



4.C7 The machine element of Prob. 4.204 is to be redesigned by removing part of the triangular cross section. It is believed that the removal of a small triangular area of width a will lower the maximum stress in the element. In order to verify this design concept, write a computer program to calculate the maximum stress in the element for values of a from 0 to 1 in. using 0.1-in. increments. Using appropriate smaller increments, determine the distance a for which the maximum stress is as small as possible and the corresponding value of the maximum stress.

SOLUTION SEE FIG 4.79 PAGE 289



$M = 5$ kip·in. $r_2 = 5$ in. $b_2 = 2.5$ in

FOR $a = 0$ TO 1.0 AT 0.1 INTERVALS

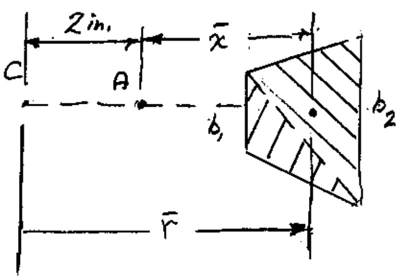
$h = 3 - a$

$r_1 = 2 + a$

$b_1 = b_2\,(a/(h+a))$

$AREA = (b_1 + b_2)(h/2)$

$\bar{x} = a + \left[\frac{1}{2} b_1 h (h/3) + \frac{1}{2} b_2 h (2h/3)\right] / AREA$



$\bar{r} = r_2 - (h - \bar{x})$

$$R = \frac{\frac{1}{2} h^2 (b_1 + b_2)}{(b_1 r_2 - b_2 r_1) \ln \frac{r_2}{r_1} - h(b_1 - b_2)}$$

$e = \bar{r} - R$

$\sigma_D = M(r_1 - R) / [AREA(e)(r_1)]$

$\sigma_B = M(r_2 - R) / [AREA(e)(r_2)]$

PRINT AND RETURN

PROGRAM OUTPUT

a	R	sigmaD	sigmaB	b1	rbar	e
in.	in.	ksi	ksi			
0.00	3.855	-8.5071	2.1014	0.00	4.00	0.145
0.10	3.858	-7.7736	2.1197	0.08	4.00	0.144
0.20	3.869	-7.2700	2.1689	0.17	4.01	0.140
0.30	3.884	-6.9260	2.2438	0.25	4.02	0.134
0.40	3.904	-6.7004	2.3423	0.33	4.03	0.127
0.50	3.928	-6.5683	2.4641	0.42	4.05	0.119
0.60	3.956	-6.5143	2.6102	0.50	4.07	0.111
0.70	3.985	-6.5296	2.7828	0.58	4.09	0.103
0.80	4.018	-6.6098	2.9852	0.67	4.11	0.094
0.90	4.052	-6.7541	3.2220	0.75	4.14	0.086
1.00	4.089	-6.9647	3.4992	0.83	4.17	0.078

Determination of the maximum compressive stress that is as small as possible

a	R	sigmaD	sigmaB	b1	rbar	e
in.	in.	ksi	ksi			
0.620	3.961	-6.51198	2.6425	0.52	4.07	0.109
0.625	3.963	-6.51185	2.6507	0.52	4.07	0.109
0.630	3.964	-6.51188	2.6591	0.52	4.07	0.109

ANSWER: When a = 625 in. the compressive stress is 6.51 ksi