PRO 5859

Statistical Process Monitoring

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2020

Outline

Attribute+Variable control chart

Attribute+Variable control chart: Introduction

- The aim: Improve the performance of the attribute chart in monitoring of the process mean and/or variability
- Features:
  - The monitoring always starts using an attribute chart
  - If this chart does not signal, the process is declared in-control
  - Otherwise, depending on the proposal, the process may be declared out-of-control or measurements of quality characteristics are taken on the sample units and the decision on the process depends on the value of some statistic (like sample mean/variance of these measures)

attribute+variable control charts: Introduction

GCC by attributes

- Mean:
  - np
  - np_x
  - $\overline{X}_{rec}$ and $\overline{X}^{att}$
- Variance: np_{S^2}

CC by attribute+variable

- Mean:
  - np-$\overline{X}$
  - $np_x - \overline{X}$
  - ATTRIVAR1 and ATTRIVAR2
- Variance: MIXS^2

np-Xbar (MIXED) chart proposed by Aslam et al. (2015)

- Attribute Chart:
  - The items are classified as non-conforming if they do not meet the specification limits: $Y = \#$ of non-conforming items in a sample of n
  - 2 sets of control limits: UCL_i and LCL_i, i=1, 2.
  - If $Y \in [LCL_2; UCL_2]$ - the process is in-control
  - If $Y \notin [LCL_1; UCL_1]$ - the process is out-of-control
  - Otherwise, use the VARIABLE chart
- Variable chart:
  - Measure all n items and obtain $\overline{X}$
  - If $\overline{X} \in [LCL_3; UCL_3]$ - the process is in control, otherwise out-of-control

np-Xbar chart proposed by Aslam et al. (2015)

Table 1. ARLs of the proposed control charts when $r_0 = 200$ and $p = 0.10$.

c	$n = 20$ $k_1 = 3.23$ $k_2 = 0.87$ $k_3 = 2.56$ ARL_1	$n = 30$ $k_1 = 3.49$ $k_2 = 0.04$ $k_3 = 2.88$ ARL_1	$n = 40$ $k_1 = 3.68$ $k_2 = 0.70$ $k_3 = 2.64$ ARL_1	$n = 5$ $k_1 = 3.5$ $k_2 = 0.2$ $k_3 = 2.8$ ARL_1
0	200.00	200.00	200.00	200.0
0.1	87.11	70.65	61.72	51.6
0.2	31.94	21.15	15.53	11.5
0.3	12.64	7.58	5.09	3.7
0.4	5.74	3.39	2.31	1.8
0.5	3.06	1.91	1.43	1.2
0.6	1.93	1.34	1.12	1.0
0.7	1.42	1.11	1.03	1.0
0.8	1.183	1.02	1.00	1.0
0.9	1.074	1.00	1.00	1.0
1	1.02	1.00	1.00	1.0

Figure 1: Plans for n=20, 30 and 40 with $ARL_0 = 200$ and USL=1.28

np-Xbar chart proposed by Aslam et al. (2015)

Table 7. Comparisons ARL_1 of the proposed control charts when $r_0 = 300$ and $p = 0.10$.

	$n = 40$			$n = 50$		
	Proposed control chart	Attribute chart $k = 3.19$	Variable chart $k = 2.93$	Proposed control chart	Attribute chart $k = 3.01$	Variable chart $k = 2.93$
c	ARL_1	ARL_1	ARL_1	ARL_1	ARL_1	ARL_1
0	300.00	680.40	300.00	300.00	310.56	300.00
0.1	91.52	176.54	92.36	72.35	80.26	76.49
0.2	20.53	54.00	21.07	14.99	25.18	15.59
0.3	6.17	19.51	6.68	4.44	9.61	4.81
0.4	2.61	8.33	2.91	1.97	4.45	2.18
0.5	1.52	4.20	1.69	1.27	2.49	1.37
0.6	1.16	2.48	1.24	1.06	1.65	1.10
0.7	1.04	1.69	1.07	1.01	1.27	1.02
0.8	1.00	1.31	1.01	1.00	1.10	1.00
0.9	1.00	1.13	1.00	1.00	1.03	1.00
1	1.00	1.04	1.00	1.00	1.00	1.00

Figure 2: Comparing with *np* and $\overline{X}$ charts

np-Xbar chart proposed by Aslam et al. (2015)

- For each sample n=30

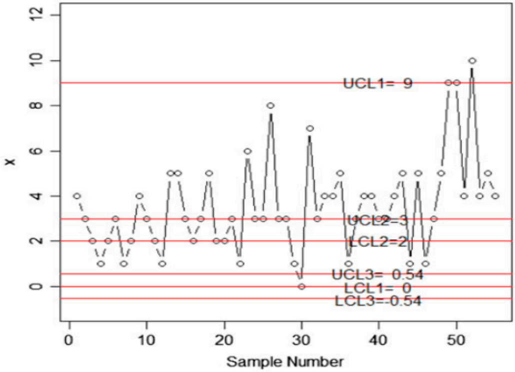
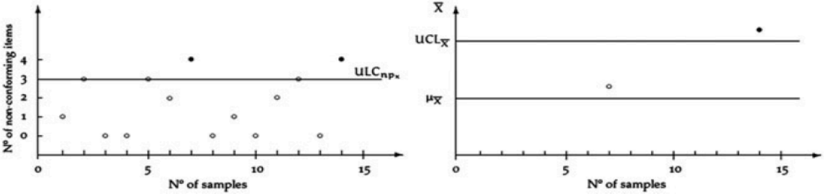


Figure 3: Example of chart proposed by Aslam et al. (2015)

The combined $np_x - \overline{X}$ chart proposed by Sampaio et al. (2013)

- Use a two stages sampling scheme
- The sample of size n is split into two sub-samples: n_{np_x} e $n_{\overline{X}} = n - n_{np_x}$
- Phase 1: np_x chart is built plotting the results of the classification (using a gauge go-nogo) of items of the first sub-sample
- Phase 2: If np_x chart signals, then values of the quality characteristic are taken from the units of second sub-sample and $\overline{X}$ is calculated
- If $\overline{X}$ also signals, then the process is declared out-of-control. An adjustment on the process is required, otherwise, the process goes on

The combined $np_x - \overline{X}$ chart proposed by por Sampaio et al. (2013)



The combined $np_x - \overline{X}$ control chart

The combined $np_x - \overline{X}$ chart proposed by Sampaio et al. (2013)

- Parameters of the combined $np_x - \overline{X}$ chart:
  - np_x chart: warning limits of the device (LWL;UWL), control limit (UCL_{np_x})
  - $\overline{X}$ chart: Control limits:$LCL_{\overline{X}}$ and $UCL_{\overline{X}}$
- Objective Function:

$$LWL^o; UWL^o; UCL^o_{np_x}; UCL^o_{\overline{X}} = argmin(ARL_1 = \frac{1}{1-\beta})$$

$$\text{suject to } ARL_0 = \frac{1}{\alpha} = \tau$$

The combined $np_x - \overline{X}$ chart proposed by Sampaio et al. (2013)

ATTENTION: Expression considering only increases in the process mean from μ_o to μ_1; LWL set at $-\infty$, UWL is searched

$$\alpha = P(Y > UCL_{np_x} | n_{np_x}, p_0) \times P(\overline{X} > UCL_{\overline{X}} | \mu_0, n_{\overline{X}})$$

$$\alpha = \alpha_{np_x} \times \alpha_{\overline{X}}$$

$p_0 = P(X > \mu_0 + k_w \sigma | \mu_0)$, the probability of an item be disapproved.

$$\beta = 1 - P(Y > UCL_{np_x} | n_{np_x}, p_1) \times P(\overline{X} > UCL_{\overline{X}} | \mu_1 = \mu_0 + \delta\sigma, n_{\overline{X}})$$

where $p_1 = P(X > \mu_0 + k_w \sigma | \mu_1 = \mu_0 + \delta\sigma)$,

The combined $np_x - \overline{X}$ chart proposed by Sampaio et al. (2013)

- Some plans - with ARL_0=370 and shift δ=0.25

Table I. ATS of some competitors for $\overline{X}$ control chart: $ARL_0 = 370$ and $\delta = 0.25$

$\overline{X}$		np_x		Combined $np_x - \overline{X}$			
$n_{\overline{x}}$	$ATS_{\overline{x}}$	n_{np_x}	ATS_{np_x}	n_1	n_2	ASS	$ATS_{np_x-\overline{x}}$
3	105.7	5	93.95	2	2	2.12	101.76
				3	2	3.06	93.36
4	88.4	6	84.86	2	3	2.32	87.51
				4	2	4.05	83.92
				3	3	3.27	80.79
				4	3	4.16	73.94
5	75.75	8	70.27	4	3	4.16	73.94
				3	4	3.54	70.88
				5	3	5.10	68.69
				4	4	4.30	65.74
				5	4	5.21	61.68
6	66.06	9	64.91	4	4	4.30	65.74
				3	5	3.90	62.90
				6	3	6.09	63.76
				5	4	5.21	61.68
				4	5	4.54	58.93
				6	4	6.19	57.50
				5	5	5.45	55.70
				6	5	6.33	52.16

The combined $np_x - \overline{X}$ chart proposed by Sampaio et al. (2013)

- Some plans - comparing AIC, unit cost equal for attribute and variable inspection

Table V. AIC of some competitors for $\overline{X}$ control chart: $ARL_0 = 370$ and $\delta = 0.25$

$\overline{X}$		np_x		Combined $np_x - \overline{X}$			
$n_{\overline{X}}$	$AIC_{\overline{X}}$	n_{np_x}	AIC_{np_x}	n_1	n_2	ASS	$AIC_{np_x - \overline{X}}$
3	15	5	25	2	2	2.12	10.60
				3	2	3.06	18.40
4	20	6	30	2	3	2.32	11.62
				4	2	4.05	20.27
				3	3	3.27	16.35
				4	3	4.16	20.81
5	25	8	40	4	3	4.16	20.81
				3	4	3.54	17.70
				5	3	5.10	25.54
				4	4	4.30	21.54
				5	4	5.21	26.08
6	30	9	45	4	4	4.30	21.54
				3	5	3.90	16.92
				6	3	6.09	30.45
				5	4	5.21	26.08
				4	5	4.54	27.70
				6	4	6.19	30.98
				5	5	5.45	27.25

ATTRIVAR charts proposed by Ho & Aparisi (2016)

- Features of ATTRIVAR chart:
  - Development of a chart with a similar performance of $\overline{X}$ chart
  - Low operational cost
  - 2 stages of inspection: by attributes and variables
  - The most of times the decision is taken considering only the results of the attribute inspection
  - A restriction for the proportion of times of variable inspection is included

ATTRIVAR charts proposed by Ho & Aparisi (2016)

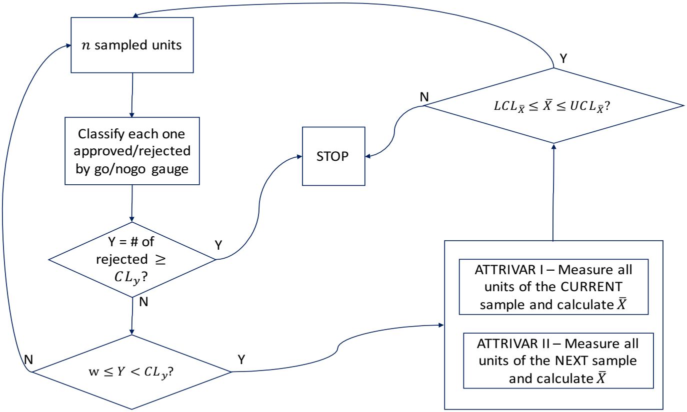


Figure 7: Inspection Procedure

ATTRIVAR charts proposed by Ho & Aparisi (2016)

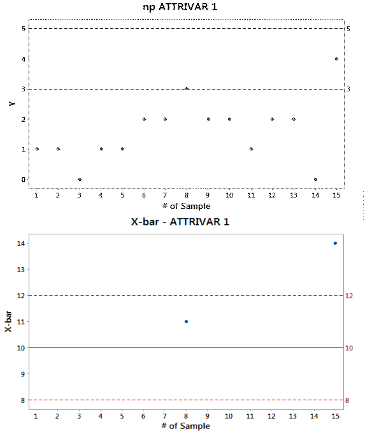


Figure 8: Example - ATTRIVAR1

ATTRIVAR charts proposed by Ho & Aparisi (2016)

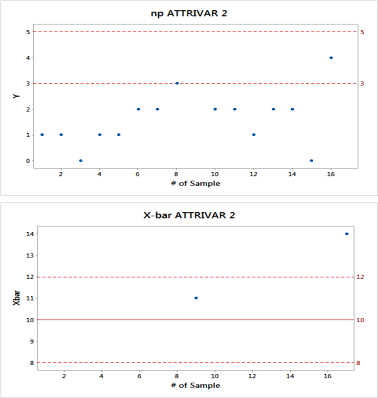


Figure 9: Example - ATTRIVAR2

ATTRIVAR charts proposed by Ho & Aparisi (2016)

- Parameters of the chart:
  - Device: Discriminant limits LDL, UDL
  - Attribute chart: Control limit CL_y; Warning limit w
  - Variable chart: Control limits $UCL_{\overline{X}}$ and $LCL_{\overline{X}}$
- Restrictions:
  - $ARL_0 = \tau$
  - % of time that measurements of X is taken when the process is in-control $\%\overline{X}_{Max} \leq \pi$
- Objective Function:
  $(LDL^o, UDL^o, CL_y^o, w^o, UCL_{\overline{X}}^o, LCL_{\overline{X}}^o) = arg\ min\ (\ ARL_1)$
- Parameters searched by genetic algorithm due to the complexity

ATTRIVAR charts proposed by Ho & Aparisi (2016)

Table 1
ARL comparison for two ATTRIVAR-2 charts optimized for different mean shifts.

Shift (δ)	0	0.2	0.5	0.7	1.0	1.5	2.0	2.5	3.0
Chart A, $\delta^* = 0.5$	370	206.7	**43.5**	17.6	6.4	2.7	2.1	1.9	1.6
Chart B, $\delta^* = 2$	370	278.3	84.7	32.9	9.5	2.6	**1.5**	1.1	1.0

Figure 10: Some plans - ATTRIVAR2

ATTRIVAR charts proposed by Ho & Aparisi (2016)

Table 2
ARL comparison of ATTRIVAR control chart versus standard $\overline{X}$ chart.

Sample size, n	$\%\overline{X}_{max}$	Shift, δ^*	$\overline{X}$ chart	ATTRIVAR-1	ATTRIVAR-2
3	25	0.25	184.06	189.84 (3.1%)	209.53 (13.8%)
3	25	0.5	60.64	61.29 (1.1%)	72.39 (19.4%)
3	25	1	9.76	10.19 (4.4%)	11.49 (17.7%)
3	25	1.25	4.95	5.16 (4.2%)	6.06 (22.4%)
3	25	1.5	2.91	3 (3.1%)	3.78 (29.9%)
3	25	2	1.47	1.51 (2.7%)	1.98 (34.7%)
3	15	0.25	184.06	189.84 (3.1%)	209.53 (13.8%)
3	15	0.5	60.64	61.29 (1.1%)	72.39 (19.4%)
3	15	1	9.76	10.19 (4.4%)	11.49 (17.7%)
3	15	1.25	4.95	5.2 (5.1%)	6.06 (22.4%)
3	15	1.5	2.91	3.03 (4.1%)	3.78 (29.9%)
3	15	2	1.47	1.61 (9.5%)	1.98 (34.7%)
3	5	0.25	184.06	192.85 (4.8%)	219.49 (19.2%)
3	5	0.5	60.64	64.78 (6.8%)	78.1 (28.8%)
3	5	1	9.76	11.65 (19.4%)	12.38 (26.8%)
3	5	1.25	4.95	5.83 (17.8%)	6.49 (31.1%)
3	5	1.5	2.91	3.36 (15.5%)	3.87 (33%)
3	5	2	1.47	1.73 (17.7%)	2.07 (40.8%)

Figure 11: ATTRIVAR versus $\overline{X}$, n=3 and $\%\overline{X}_{Max} = 25; 15; 5$

ATTRIVAR charts proposed by Ho & Aparisi (2016)

Sample size, n	$\%X_{max}$	Shift, δ^*	$\bar{X}$ chart	ATTRIVAR-1	ATTRIVAR-2
10	25	0.25	73.21	79.55 (8.7%)	92.39 (26.2%)
10	25	0.5	12.82	14.18 (10.6%)	18.11 (41.3%)
10	25	1	1.77	1.99 (12.4%)	3.04 (71.8%)
10	25	1.25	1.21	1.36 (12.4%)	1.99 (64.5%)
10	25	1.5	1.04	1.06 (1.9%)	1.63 (56.7%)
10	25	2	1	1.02 (2%)	1.09 (9%)
10	15	0.25	73.21	77.82 (6.3%)	99.76 (36.3%)
10	15	0.5	12.82	14.31 (11.6%)	20.28 (58.2%)
10	15	1	1.77	2.01 (13.6%)	3.11 (75.7%)
10	15	1.25	1.21	1.36 (12.4%)	2.07 (71.1%)
10	15	1.5	1.04	1.08 (3.8%)	1.63 (56.7%)
10	15	2	1	1 (0%)	1.09 (9%)
10	5	0.25	73.21	89.55 (22.3%)	119.62 (63.4%)
10	5	0.5	12.82	19.27 (50.3%)	24.77 (93.2%)
10	5	1	1.77	2.66 (50.3%)	3.65 (106.2%)
10	5	1.25	1.21	1.76 (45.5%)	2.36 (95%)
10	5	1.5	1.04	1.3 (25%)	1.66 (59.6%)
10	5	2	1	1.04 (4%)	1.16 (16%)

Figure 12: ATTRIVAR versus $\overline{X}$, n=10 and $\%\overline{X}_{Max} = 25; 15; 5$

ATTRIVAR charts proposed by Ho & Aparisi (2016)

Table 5
ATTRIVAR versus the np_x-$\bar{X}$ control chart.

	Designs of the np_x-$\bar{X}$; ATTRIVAR-1 and ATTRIVAR-2 control charts							
	np_x-$\bar{X}$ control chart				ATTRIVAR-1: RC_1		ATTRIVAR-2: RC_2	
Cases	n_1	n_2	ASS	$\%\bar{X}_c$	$\%\bar{X}_{max}=5$	$\%\bar{X}_{max}=15$	$\%\bar{X}_{max}=5$	$\%\bar{X}_{max}=15$
$\delta^* = 0.25$; $n = 3$					$\%\bar{X}_1$=3.64	$\%\bar{X}_1$=7.74	$\%\bar{X}_2$=2.38	$\%\bar{X}_2$=2.38
	2	3	2.32	10.7	**61.71**	**91.00**	**86.44**	**84.16**
	3	2	3.06	0.3	123.43	182.00	172.88	168.33
$\delta^* = 1.0$; $n = 5$					$\%\bar{X}_1$=3.93	$\%\bar{X}_1$=7.18	$\%\bar{X}_2$=4.47	$\%\bar{X}_2$=13.98
	4	5	4.54	10.80	**60.34**	**82.03**	**62.48**	223.52
	5	4	5.21	5.30	**97.07**	131.96	100.51	198.71

Figure 13: ATTRIVAR versus $np_x - \overline{X}$

ATTRIVAR charts proposed by Ho & Aparisi (2016)

Table 4
ATTRIVAR versus mixed control chart, $n = 20$.

			Values of Shifts δ^*			
			0.2	0.4	0.6	1.0
MIXED		$\%\overline{X}_M$	71.44	71.44	71.44	71.44
		ARL	53.31	8.05	2.34	1.03
ATTRIVAR1	$\%\overline{X}_{max}$=25	$\%\overline{X}_1$	10.57	19.43	10.44	11.08
		ARL	56.97	10.28	3.78	1.28
		RC_{M1}	**20.37**	**31.96**	**20.20**	**21.04**
	$\%\overline{X}_{max}$=15	$\%\overline{X}_1$	10.57	10.61	10.66	10.78
		ARL	56.97	11.89	4.42	1.35
		RC_{M1}	**20.37**	**20.42**	**20.49**	**20.64**
	$\%\overline{X}_{max}$=5	$\%\overline{X}_1$	0.85	1.58	3.34	3.29
		ARL	145.91	30.06	6.34	1.90
		RC_{M1}	**7.65**	**8.61**	**10.91**	**10.85**
ATTRIVAR2	$\%\overline{X}_{max}$=25	$\%\overline{X}_2$	19.43	19.43	19.43	24.43
		ARL	79.23	17.43	4.84	1.90
		RC_{M2}	**30.69**	**30.69**	**30.69**	**36.90**
	$\%\overline{X}_{max}$=15	$\%\overline{X}_2$	10.66	10.65	10.66	12.04
		ARL	89.16	17.45	5.46	2.00
		RC_{M2}	**19.79**	**19.78**	**19.79**	**21.50**
	$\%\overline{X}_{max}$=5	$\%\overline{X}_2$	3.67	3.66	4.28	3.61
		ARL	119.63	24.40	7.68	2.53
		RC_{M2}	**11.10**	**11.09**	**11.86**	**11.03**

Figure 14: ATTRIVAR versus MIXED

About MIX S^2 chart: Introduction

- The aim: to develop a chart to monitor the variability with a better performance than np_{S^2} chart
- MIX S^2 chart: Inspection in two stages
- Stage 1: Attribute chart is used
- Stage 2: If this chart signals, a second variable chart is built
- Decision Criterion: If both charts signal, the process is said out-of-control

MIX S^2 chart proposed by Ho & Quinino (2016)

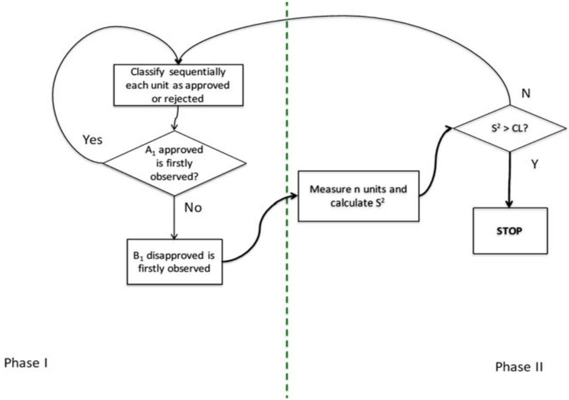


Figure 15: Inspection Procedure

MIX S^2 chart proposed by Ho & Quinino (2016)

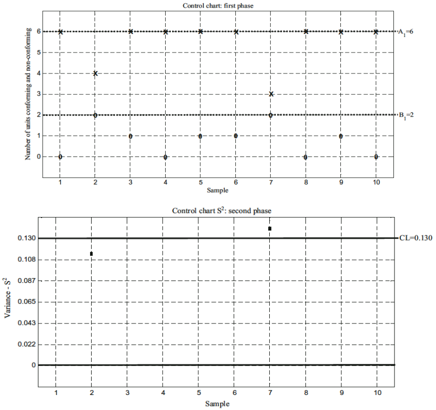


Figure 16: An example of MIX S^2 chart

MIX S^2 chart proposed by Ho & Quinino (2016)

- Parameters of the chart: # of approved items A_1; # of disapproved items B_1; Discriminating limits UDL and LDL
- Objective Function:

$$(A_1^o, B_1^o, UDL^o, LDL^o) = argminARL_1(A_1, B_1, UDL, LDL)$$

subject to:

$$ARL_0 = \frac{1}{\alpha} = 370$$

$$ARL_1^{MixS^2} < ARL_1^{S^2}$$

$$\alpha_2 = \frac{\alpha}{\alpha_1}$$

$$ASC^{MixS^2} < ASC^{S^2}$$

ASC=average sampling cost; α_1 and α_2, errors of type I of charts at stages 1 and 2; values of α, α_1 and α_2 in the interval [0; 1]

MIX S^2 chart proposed by Ho & Quinino (2016)

Table 1 Some plans of MIX S^2 control chart with no more than $(a+b-1)$ inspected units and shift $\delta = 1.5$

Plans	A_1	B_1	LDL	UDL	α_1	α_2	ARL_1	ANI_0	ANI_1	Number	Cost($)
A	20	10	8.49	11.51	0.0028	0.9763	2.391	23.011	26.547	2	29.12
B	14	4	8.00	12.00	0.0063	0.4266	6.217	14.654	16.229	4	20.75
C	5	2	7.72	12.28	0.0072	0.3741	8.889	5.113	5.574	2	6.17
D	6	2	7.48	12.52	0.0028	0.9708	7.773	6.069	6.457	2	7.25
E	13	9	8.59	11.41	0.0300	0.9025	3.891	15.449	19.133	3	21.70

Figure 17: Examples of plans of MIX S^2 chart

MIX S^2 chart proposed by Ho & Quinino (2016)

Table 2 Optimal plans of MIX S^2 control chart

Δ	A_1	B_1	LDC	UDC	α_1	α_2	LC	ARL_1	ANI_0	ANI_1	Number	ARL_1 of S^2 ($n=5$)	Max ASC MIX S^2
1.1	6	3	7.93	12.07	0.00280	0.9810	0.0006	106.616	6.2388	6.3772	2	106.9340	$6.434
1.3	6	3	7.93	12.07	0.00280	0.9810	0.0006	20.293	6.2388	6.7128	2	21.0908	$7.014
1.5	6	2	7.48	12.52	0.00278	0.9708	0.0013	7.773	6.0692	6.4574	2	8.0275	$7.253
1.7	6	2	7.48	12.52	0.00278	0.9708	0.0013	4.108	7.9959	7.5864	2	4.3649	$8.091
2.0	7	1	6.46	13.54	0.00280	0.9651	0.0019	2.420	6.9972	6.4359	2	2.5153	$9.000

Figure 18: Best plans of MIX S^2 chart

MIX S^2 chart proposed by Ho & Quinino (2016)

Table 3 Average inspection cost of MIX S^2 and S^2 ($n = 5$) as functions of c_a and c_m

c_a	c_m	A_1	B_1	MIX S^2	S^2 ($n = 5$)
1	4	7	1	\$9.000	\$20.000
1	3.5	7	1	\$8.580	\$17.500
1	3	7	1	\$8.150	\$15.000
1	2.5	7	1	\$7.720	\$12.500
1	2	7	1	\$7.290	\$10.000
1	1.5	7	1	\$7.000	\$7.500
1	1.45	7	1	\$7.000	\$7.250
1	1.4	7	1	\$6.999	\$7.000
1	1.35	6	2	\$6.963	\$6.750
1	1.25	6	2	\$6.870	\$6.250

Figure 19: Comparing the plans MIX S^2 chart with its main competitor - economical scenario: c_a=inspection cost by attribute; c_m=measurement

Partial list of papers: CC by attribute or mixture (attribute+variable) in the monitoring of the process mean and/or variability

Only attribute

- Haridy et al. (2014)
- Ho & Costa (2011)
- Ho & Quinino (2013)
- Ho & Costa (2015)
- Melo et al. (2017)
- Quinino et al. (2015)
- Stevens (1948)
- Steiner et al. (1994, 1996)
- Wu & Jiao (2008)
- Wu et al. (2009)

Mixtures: attribute+variable

- Aslam et al. (2015)
- Ho & Aparisi (2016)
- Sampaio et al. (2013)
- Ho & Quinino (2016)

Questions for Seminars: Attribute CC to monitor process mean and variability

- Some contributors of attribute CC to monitor process mean and variability

  - Steiner, S., Geyer, P., and Wesolowsky, G. (1994). Control charts based on grouped data.International Journal of Production Research, 32(1):75–91.
  - Steiner, S., Geyer, P., and Wesolowsky, G. (1996). Shewhart control charts to detect mean and standard deviation shifts based on grouped data. Quality and reliability engineering international, 12:345–353.
  - Stevens, W. L. (1948). Control by gauging. Journal of the Royal Statistical Society, 10(1):54–108.
  - Wu, Z. and Jiao, J. (2008). A control chart for monitoring process mean based on attribute inspection. International Journal of Production Research, 46:4331–4337.
  - Wu, Z., Khoo, M., Shu, L., and Jiang, W. (2009). An np control chart for monitoring the mean of a variable based on

Question for Seminars: Attribute CC to monitor process mean

- List the common points and differences of these papers
- How to plan attribute chart to have equal performance (in term of ARL, Sampling Cost) of a variable chart?

Aslam, M., Azam, M., N, K. & Jun, C. (2015), 'A mixed control chart to monitor the process', **53**(15), 4684–4693.

Haridy, S., Wu, Z., Lee, K. & A, R. (2014), 'An attribute chart for monitoring the process mean and variance', *International Journal of Production Research* **52**(11), 3366–3380.

Ho, L. & Costa, A. (2011), 'Monitoring a wandering mean with an np chart', *Production* **21**, 254–258.

Ho, L. L. & Aparisi, F. (2016), 'Attrivar: Optimized control charts to monitor process mean with lower operational cost', *International Journal of Production Economics* **182**, 472–483.

Ho, L. L. & Costa, A. F. B. (2015), 'Attribute charts for monitoring the mean vector of bivariate processes', *Quality and Reliability Engineering International* **31**(4), 683–693. DOI: 10.1002/qre.1628.

Ho, L. L. & Quinino, R. C. (2013), 'An attribute control chart for monitoring the variability of a process', *International Journal of Production Economics* **45**, 263–267.

Ho, L. L. & Quinino, R. d. C. (2016), 'Combining attribute and variable data to monitor process variability: MIX s^2 control chart', *Int J Adv Manuf Technol* **87**. DOI 10.1007/s00170-016-8702-5.

Melo, M. S., Ho, L. L. & Medeiros, P. G. (2017), 'Max D: An attribute control chart to monitor a bivariate process mean', *International Journal of Advanced Manufacturing Technology* **DOI 10.1007/500170-016-9368-8**, to appear.

Quinino, R. C., Ho, L. L. H. & Trindade, A. L. G. (2015), 'Monitoring the process mean based on attribute inspection when a small sample is available', *Journal of the Operational Research Society* **66**(11), 1860–1867.

Sampaio, E., Ho, L. & Medeiros, P. (2013), 'A combined $np_x\overline{X}$-control chart to monitor the process mean in two-stage sampling', *Quality and Reliability Engineering International* **30**, 1003–1013.

Steiner, S., Geyer, P. & Wesolowsky, G. (1994), 'Control charts based on grouped data', *International Journal of Production Research* **32**(1), 75–91.

Steiner, S., Geyer, P. & Wesolowsky, G. (1996), 'Shewhart control charts to detect mean and standard deviation shifts based on grouped data', *Quality and Reliability Engineering International* **12**, 345–353.

Stevens, W. L. (1948), 'Control by gauging', *Journal of the Royal Statistical Society* **10**(1), 54–108.

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Wu, Z., Khoo, M., Shu, L. & Jiang, W. (2009), 'An np control chart for monitoring the mean of a variable based on an attribute inspection', *International Journal of Production Economics* **121**, 141–147.