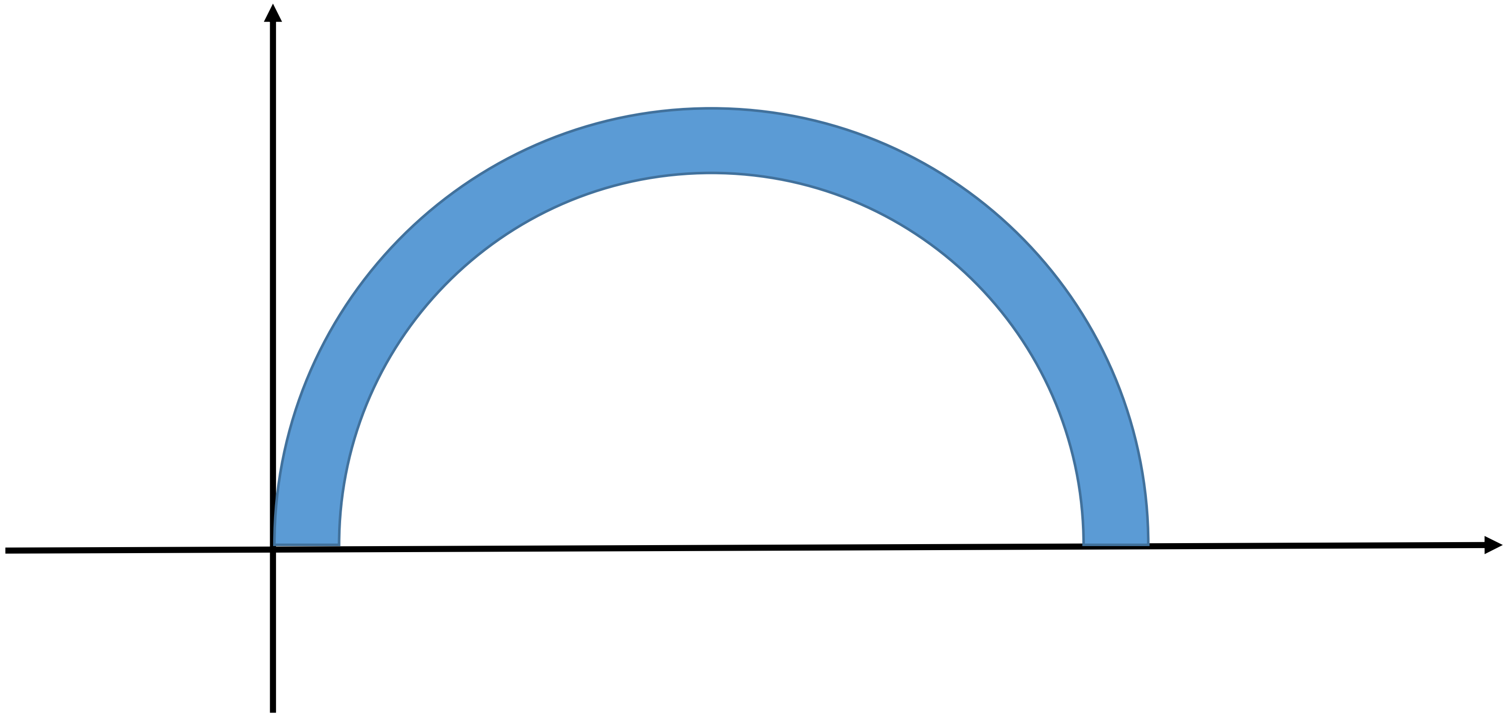
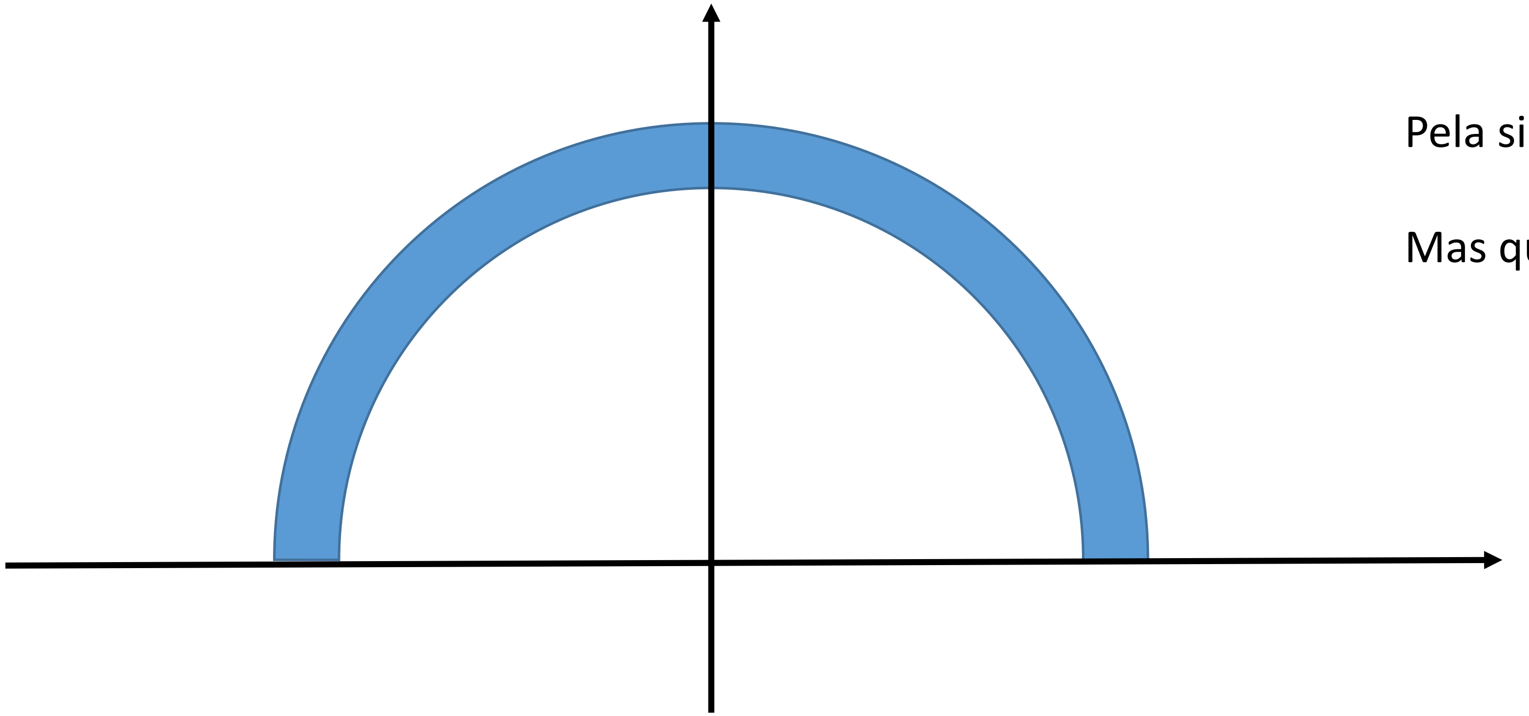
Sistema de Partículas

e-aula (29/04/2020)

Exemplo 2



Exemplo 2

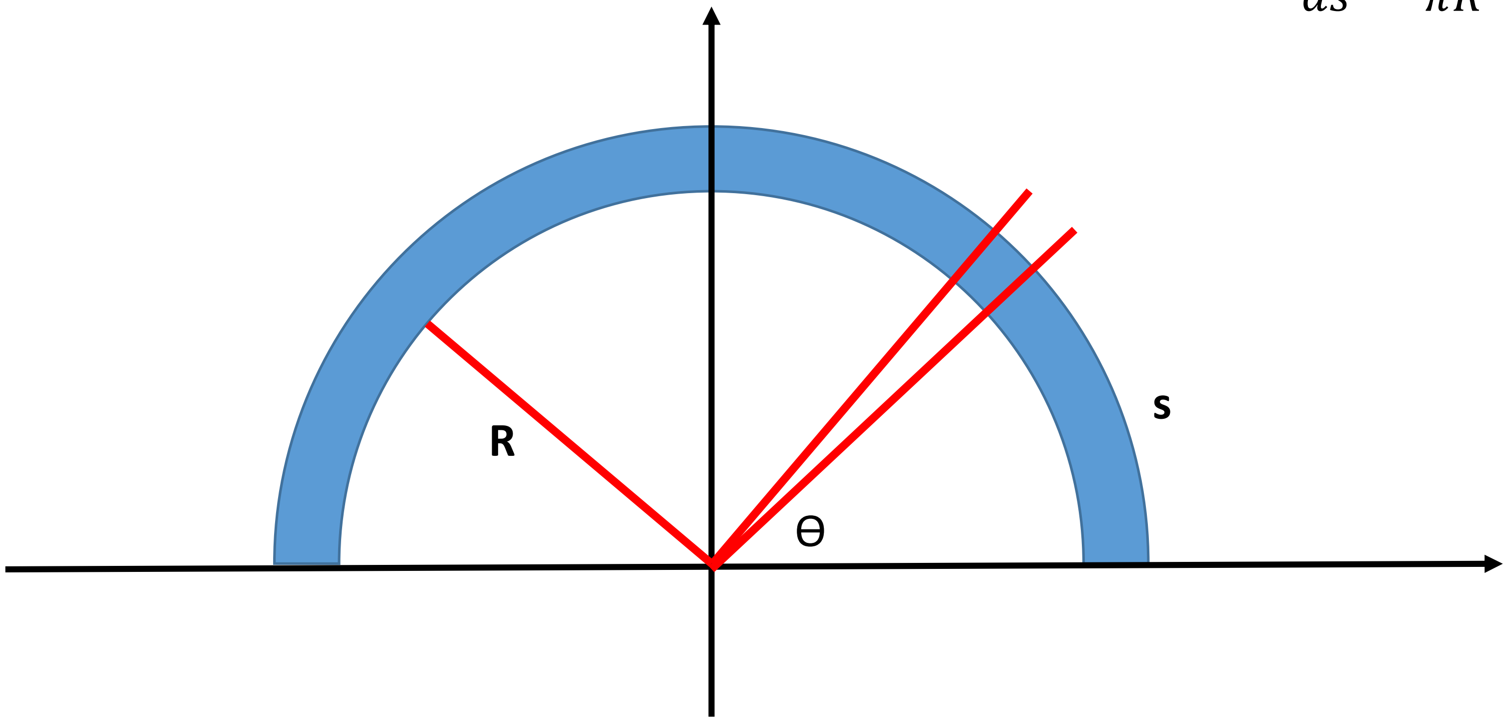


Pela simetria, X_{CM} é zero!

Mas quem é Y_{CM}?

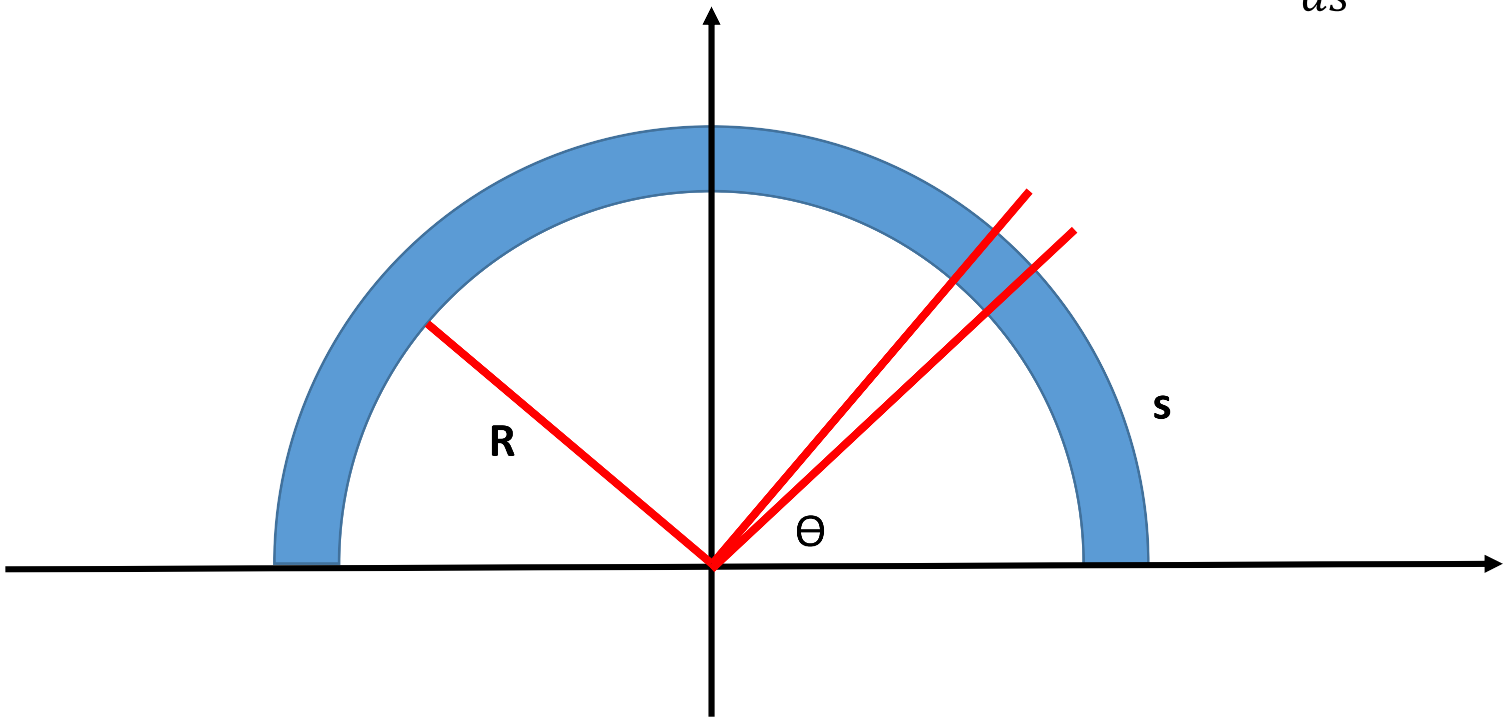
Exemplo 2

$$\lambda = \frac{dm}{ds} = \frac{M}{\pi R}$$



Exemplo 2

$$\lambda = \frac{dm}{ds}$$

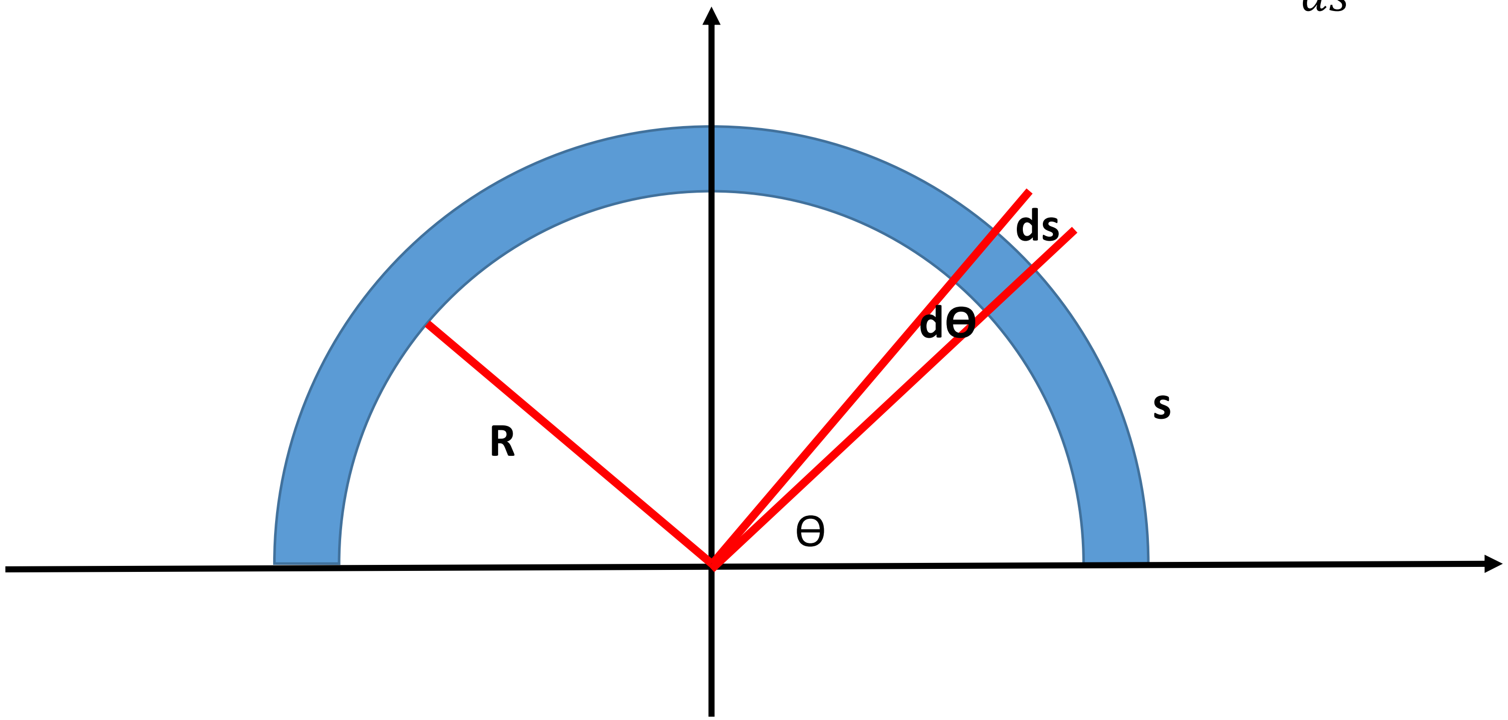


Medida do arco:

$$s = R.\theta$$

Exemplo 2

$$\lambda = \frac{dm}{ds}$$



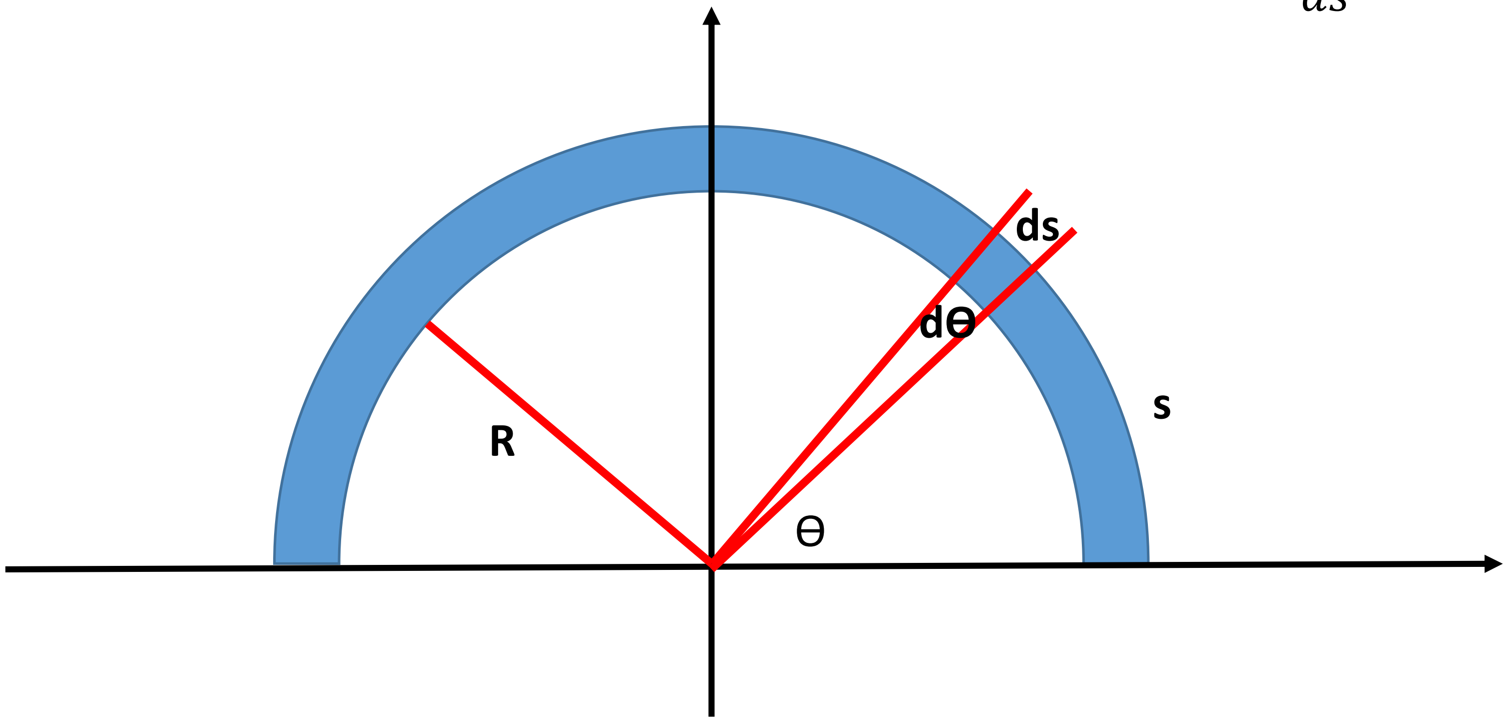
Medida do arco:

$$s = R.\theta$$

$$ds = R.d\theta$$

Exemplo 2

$$\lambda = \frac{dm}{ds}$$



Medida do arco:

$$s = R.\theta$$

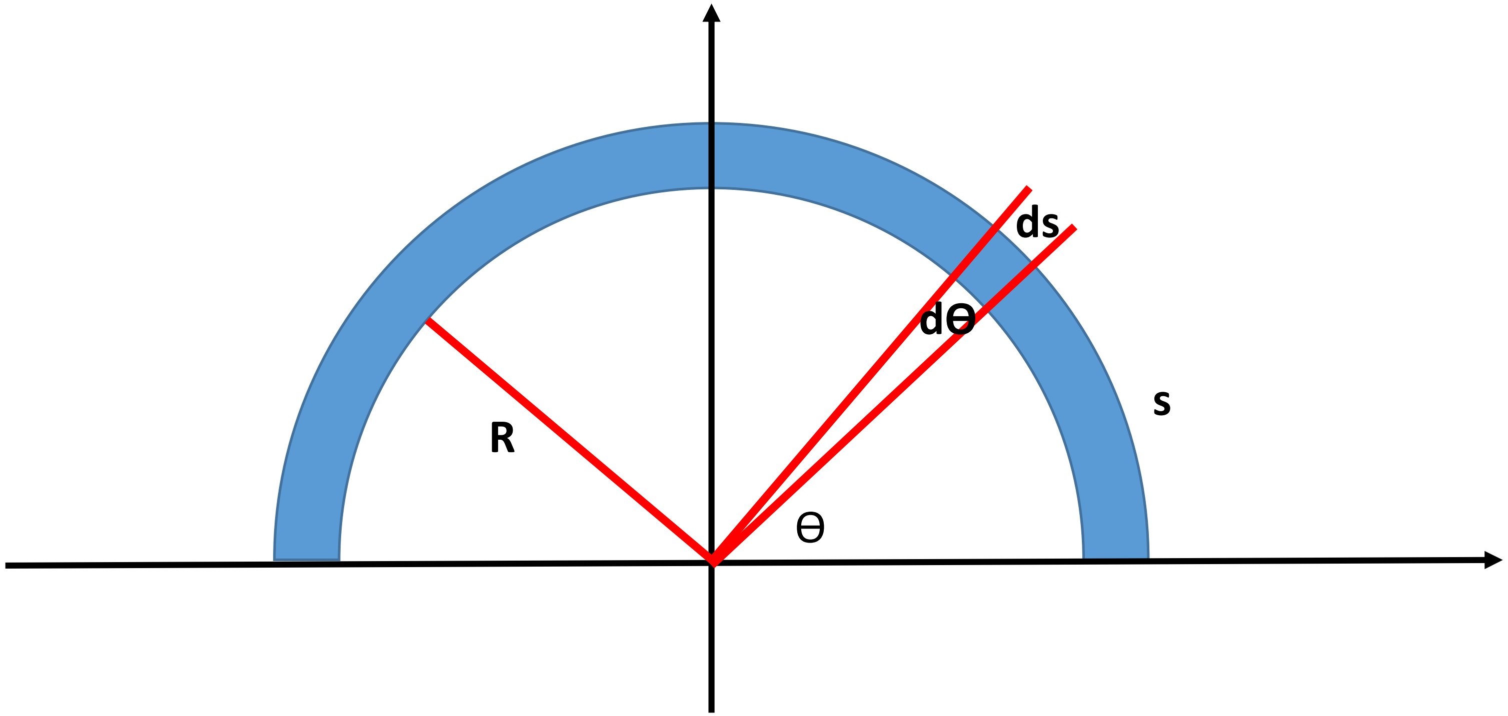
$$ds = R.d\theta$$

$$\lambda = \frac{dm}{ds} \rightarrow \frac{dm}{R.d\theta}$$

$$dm = R.\lambda.d\theta$$

Exemplo 2

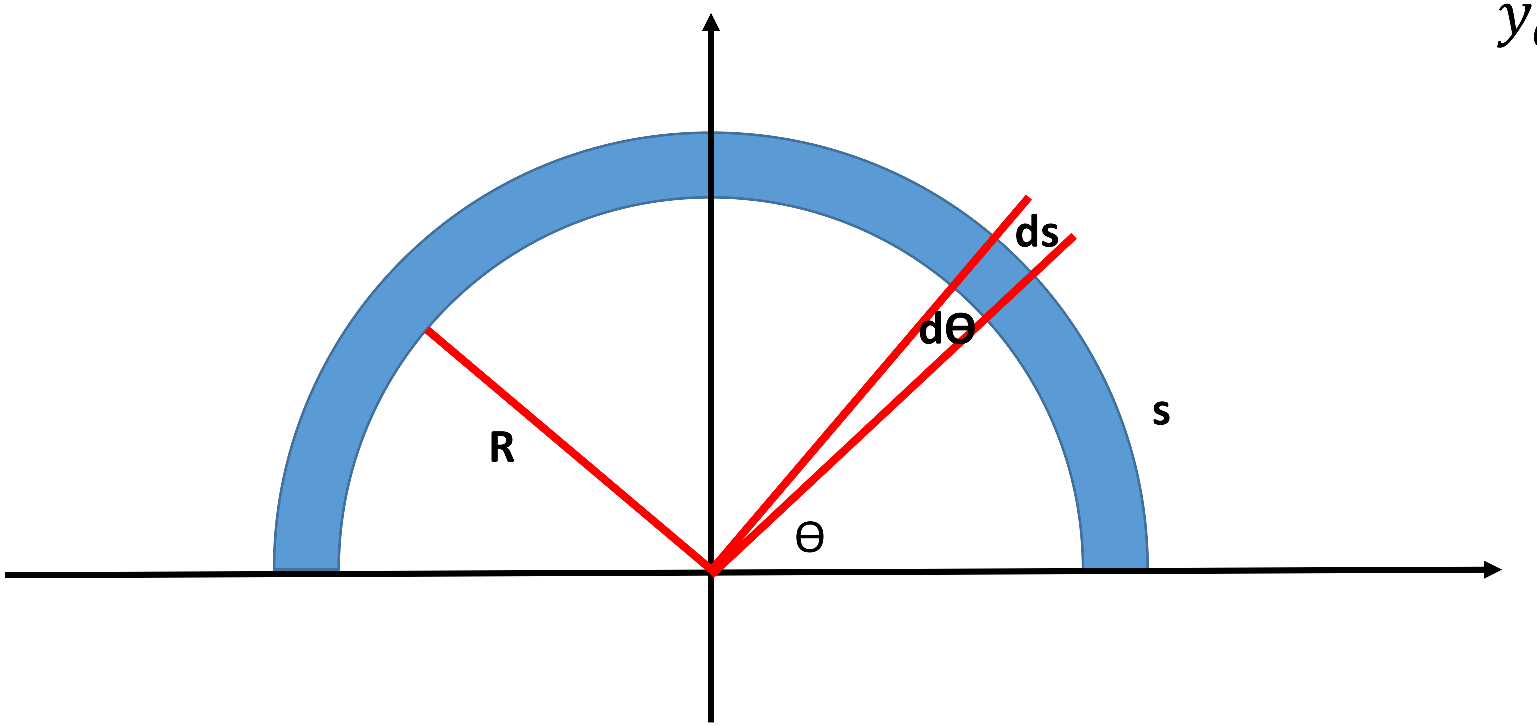
$$dm = R.\lambda.d\theta$$



Exemplo 2

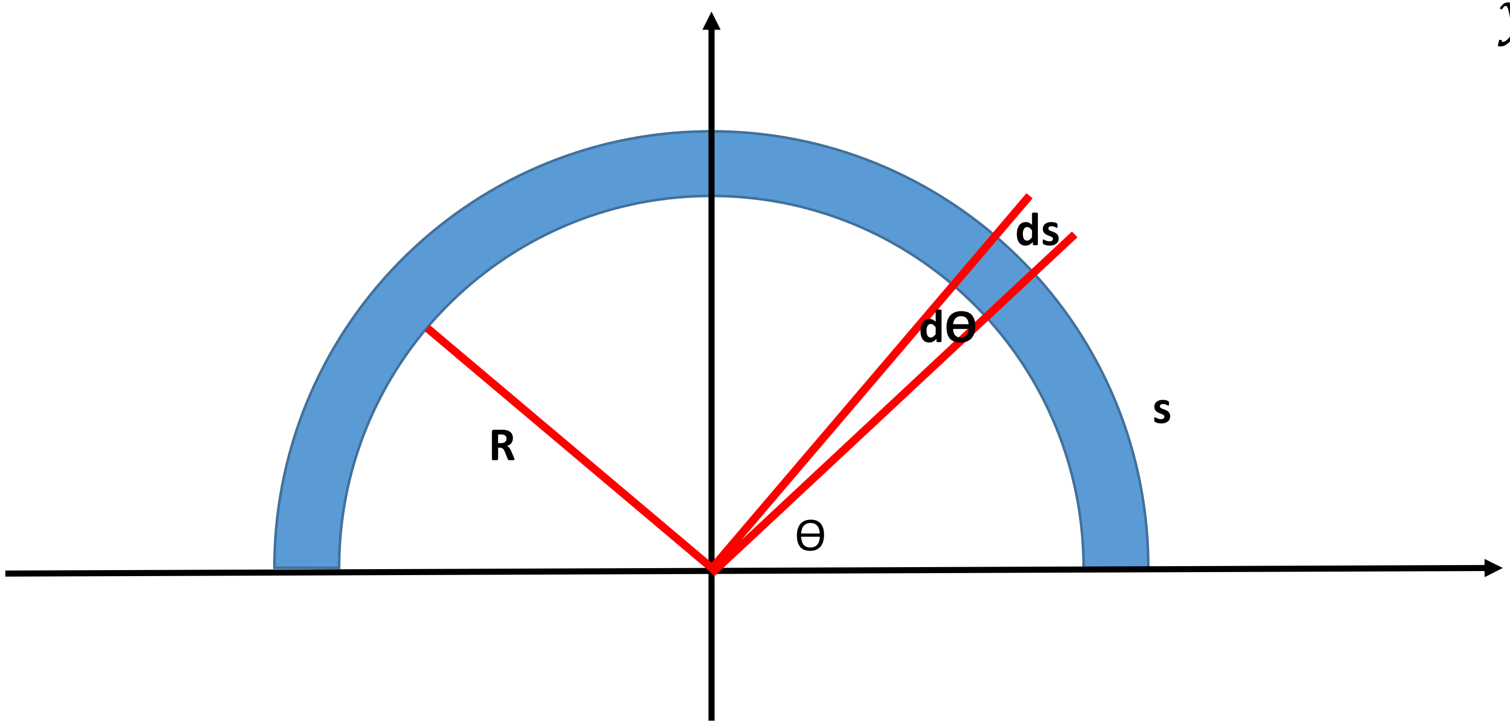
$$dm = R.\lambda.d\theta$$

$$y_{CM} = \frac{1}{M}\int y dm$$



Exemplo 2

$$dm = R.\lambda.d\theta$$



$$y_{CM} = \frac{1}{M}\int y dm$$

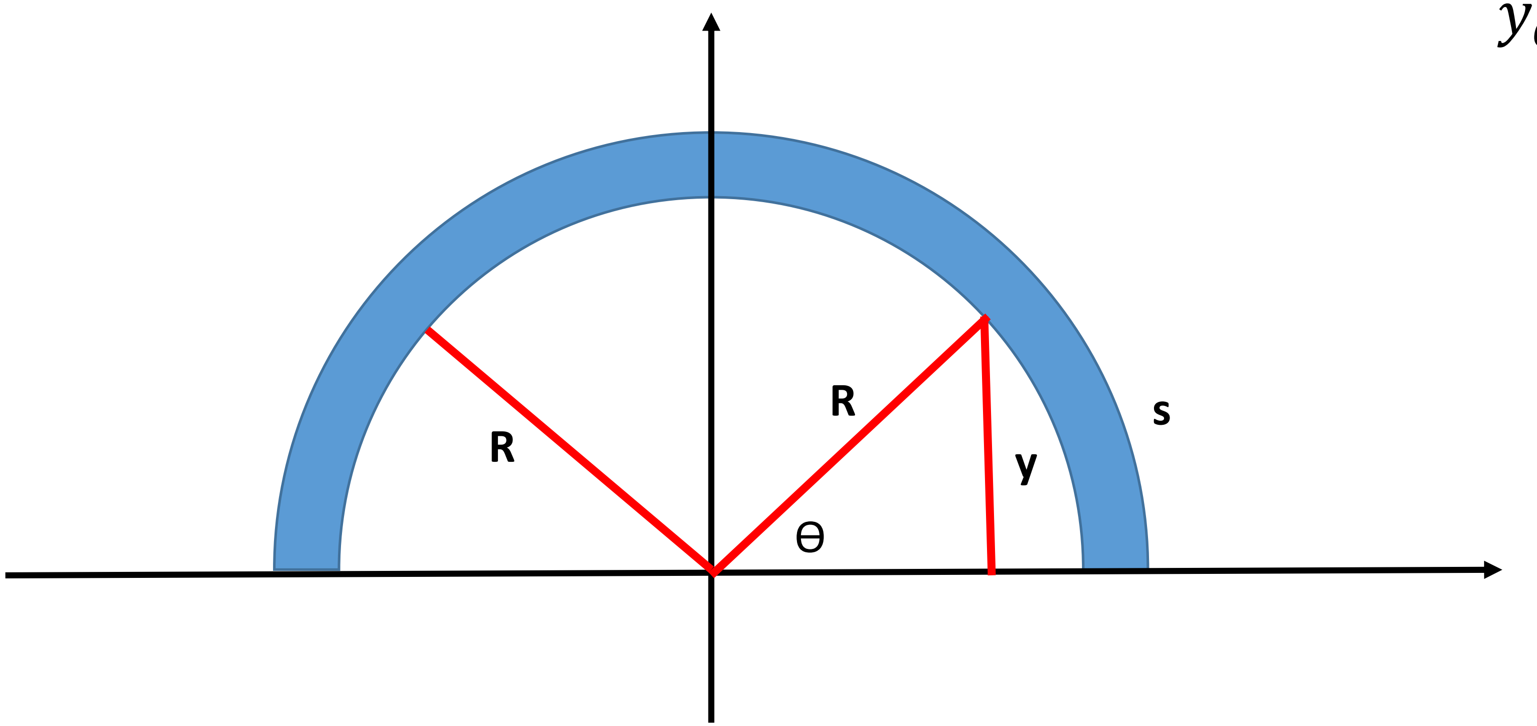
?

Exemplo 2

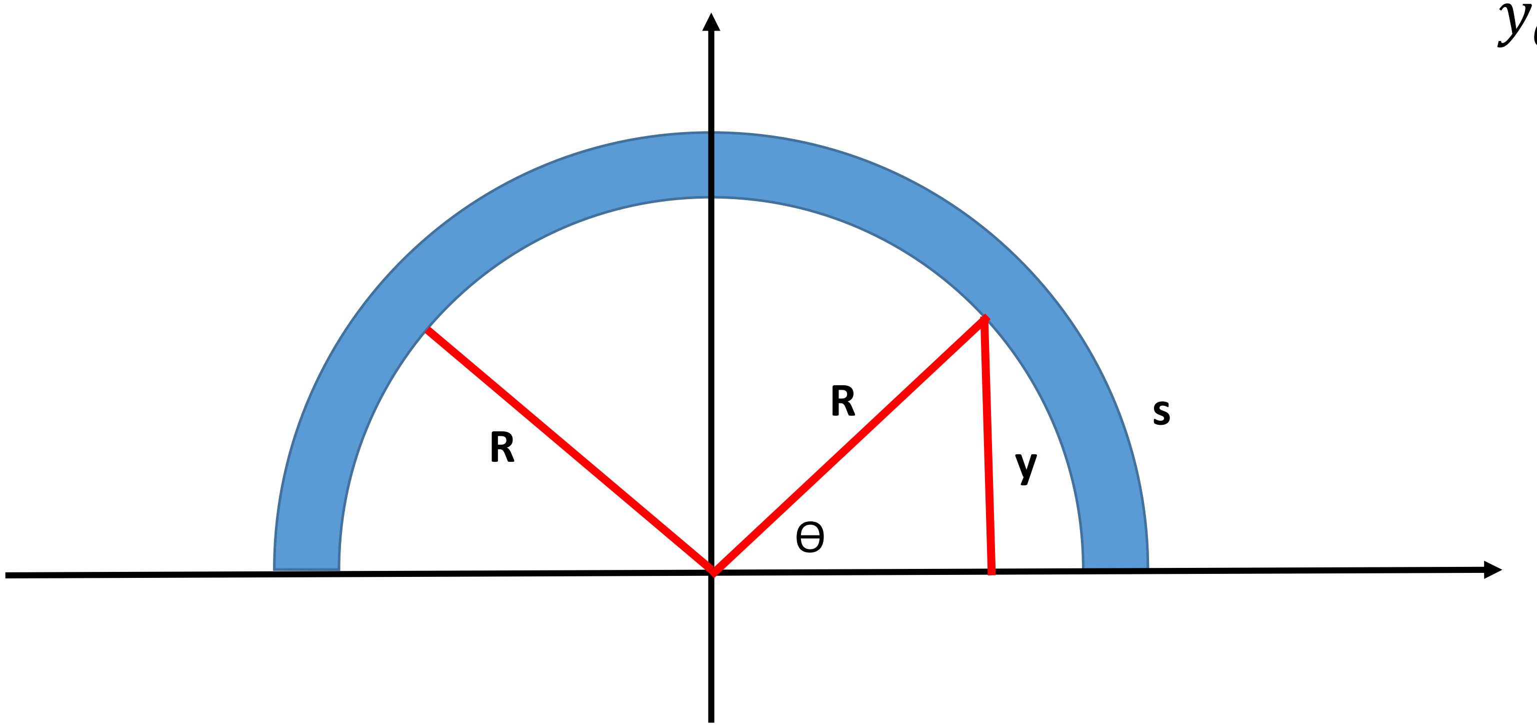
$$dm = R.\lambda.d\theta$$

$$y_{CM} = \frac{1}{M}\int y dm$$

?



Exemplo 2



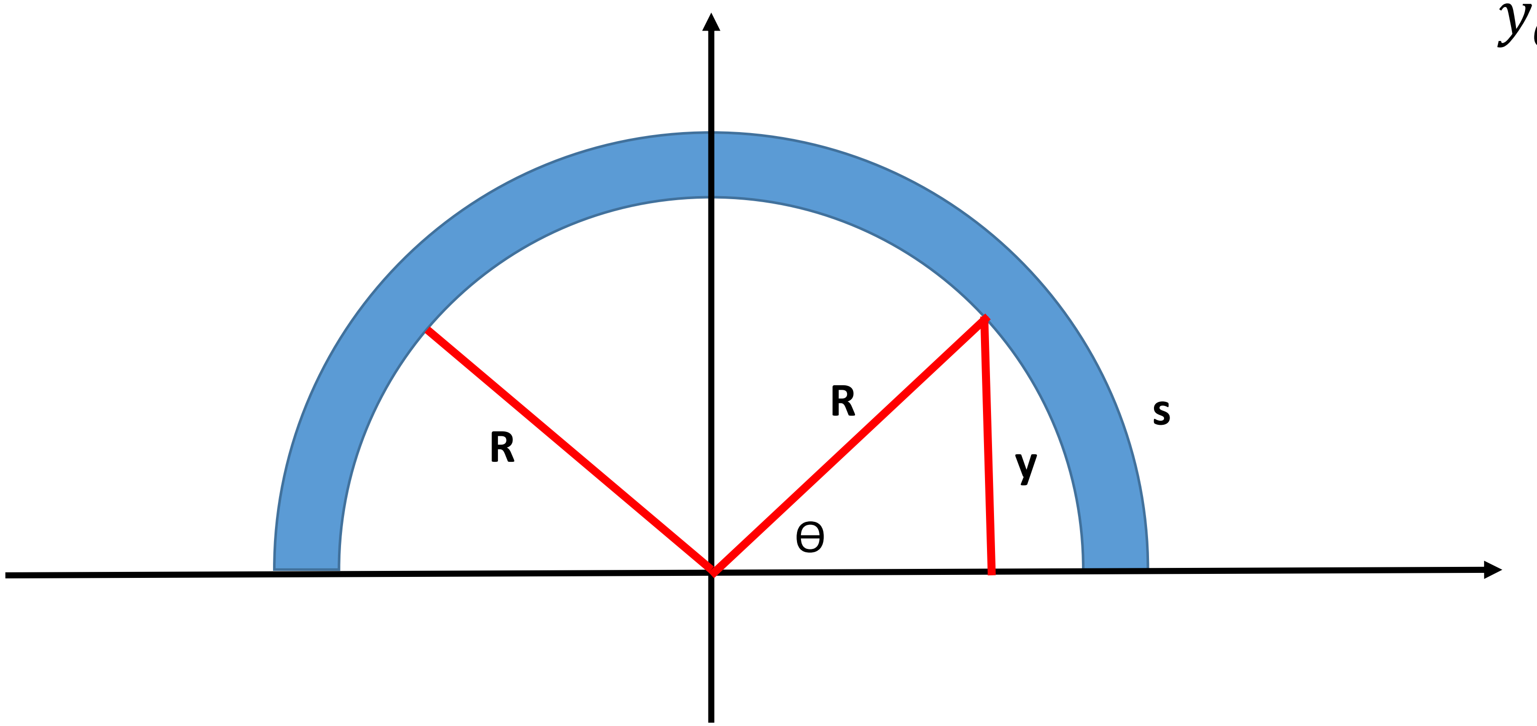
$$dm = R.\lambda.d\theta$$

$$y_{CM} = \frac{1}{M}\int y dm$$

?

$$y = R.sen\theta$$

Exemplo 2



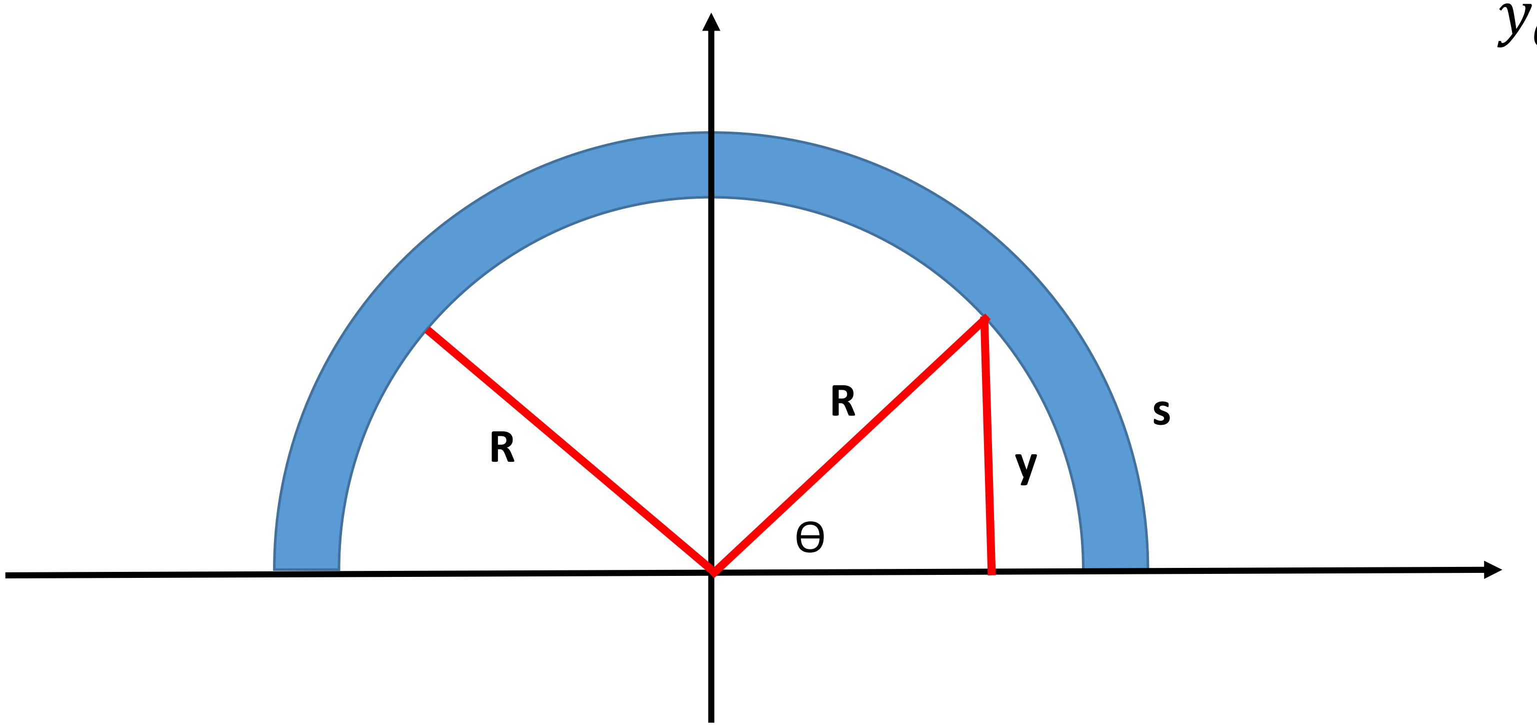
$$dm = R.\lambda.d\theta$$

$$y_{CM} = \frac{1}{M}\int y dm$$

$$y = R.sen\theta$$

$$y_{CM} = \frac{1}{M}\int R.sen\theta \, dm$$

Exemplo 2



$$dm = R.\lambda.d\theta$$

$$y_{CM} = \frac{1}{M}\int ydm$$

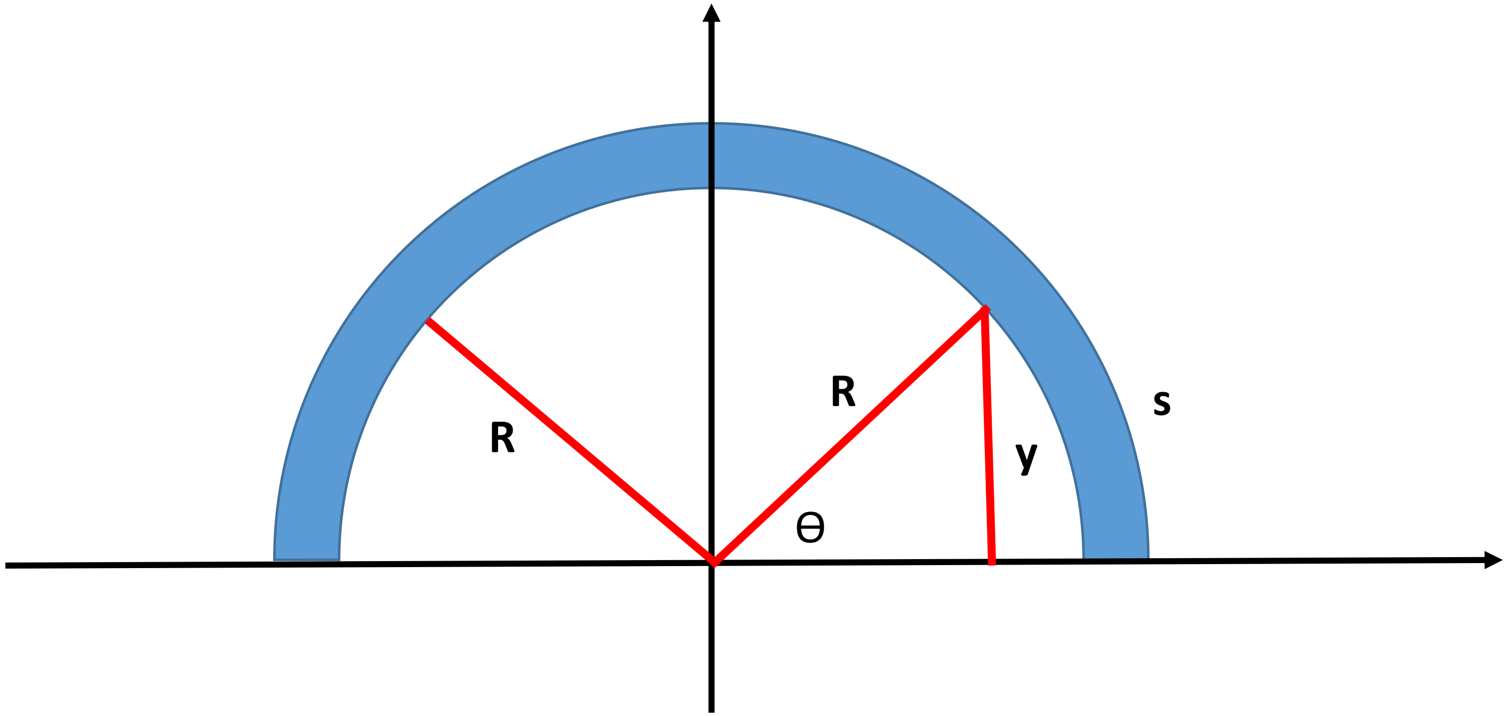
$$y = R.sen\theta$$

$$y_{CM} = \frac{1}{M}\int R.sen\theta\, dm$$

$$y_{CM} = \frac{1}{M}\int R.sen\theta R.\lambda.d\theta$$

Exemplo 2

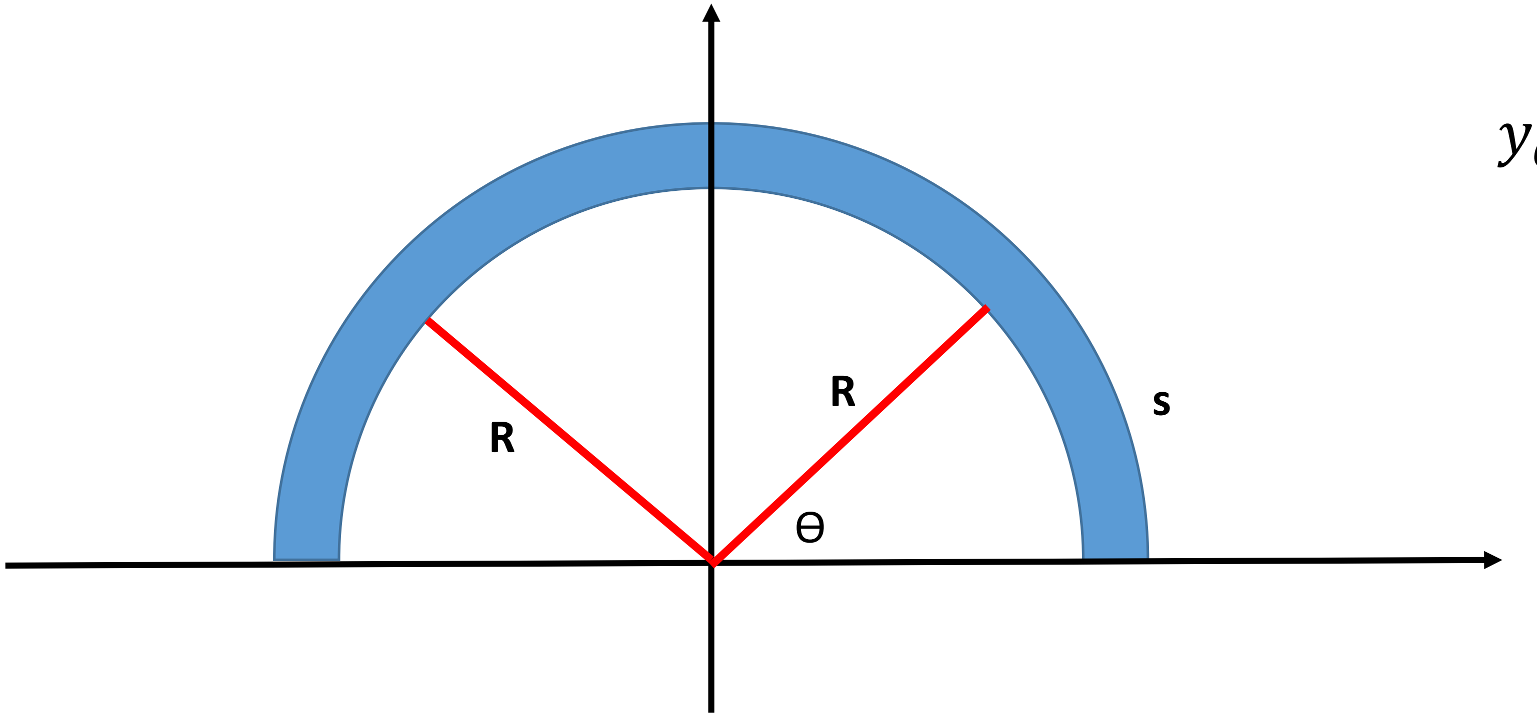
$$y_{CM} = \frac{1}{M}\int R.sen\theta R.\lambda.d\theta$$



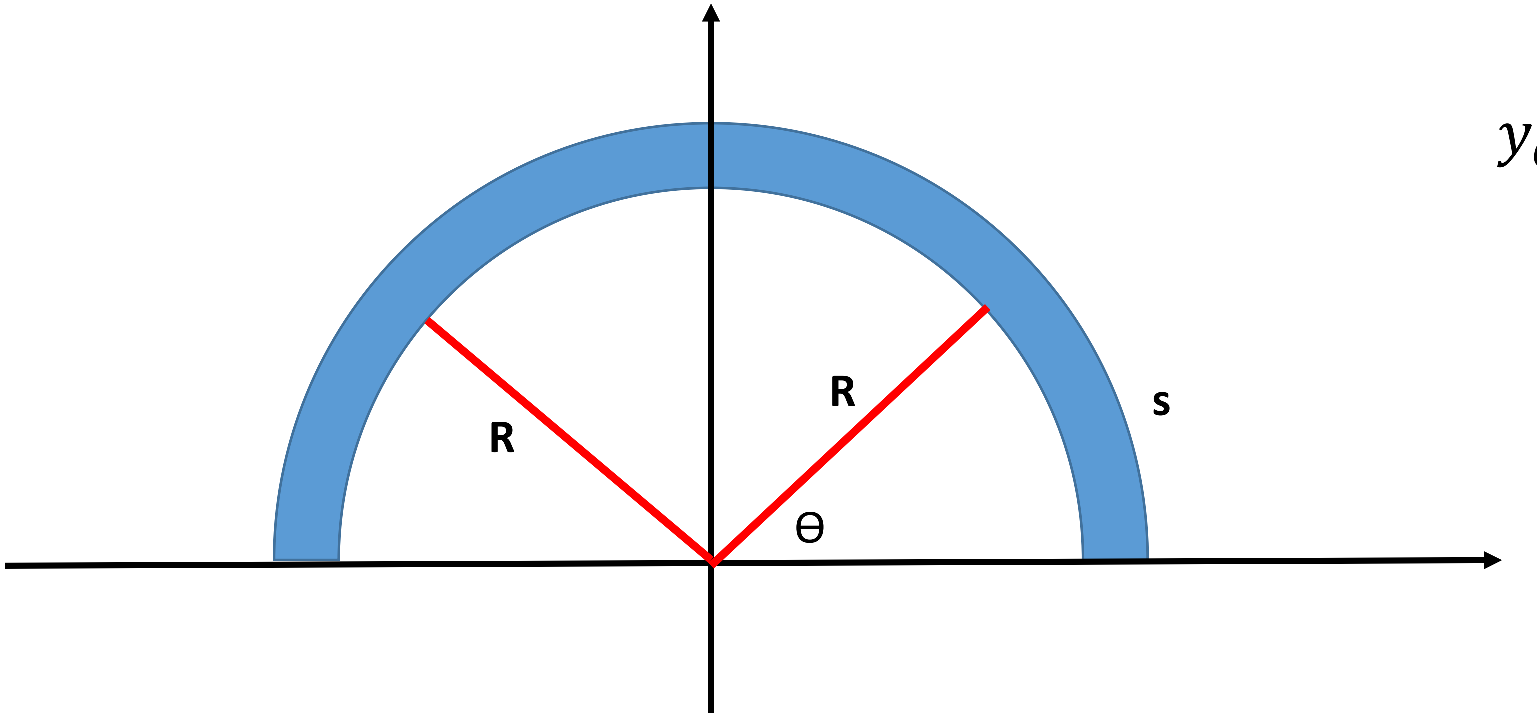
Exemplo 2

$$y_{CM} = \frac{1}{M}\int R.\,sen\theta R.\,\lambda.\,d\theta$$

$$y_{CM} = \frac{\lambda R^2}{M}\int sen\theta d\theta$$



Exemplo 2

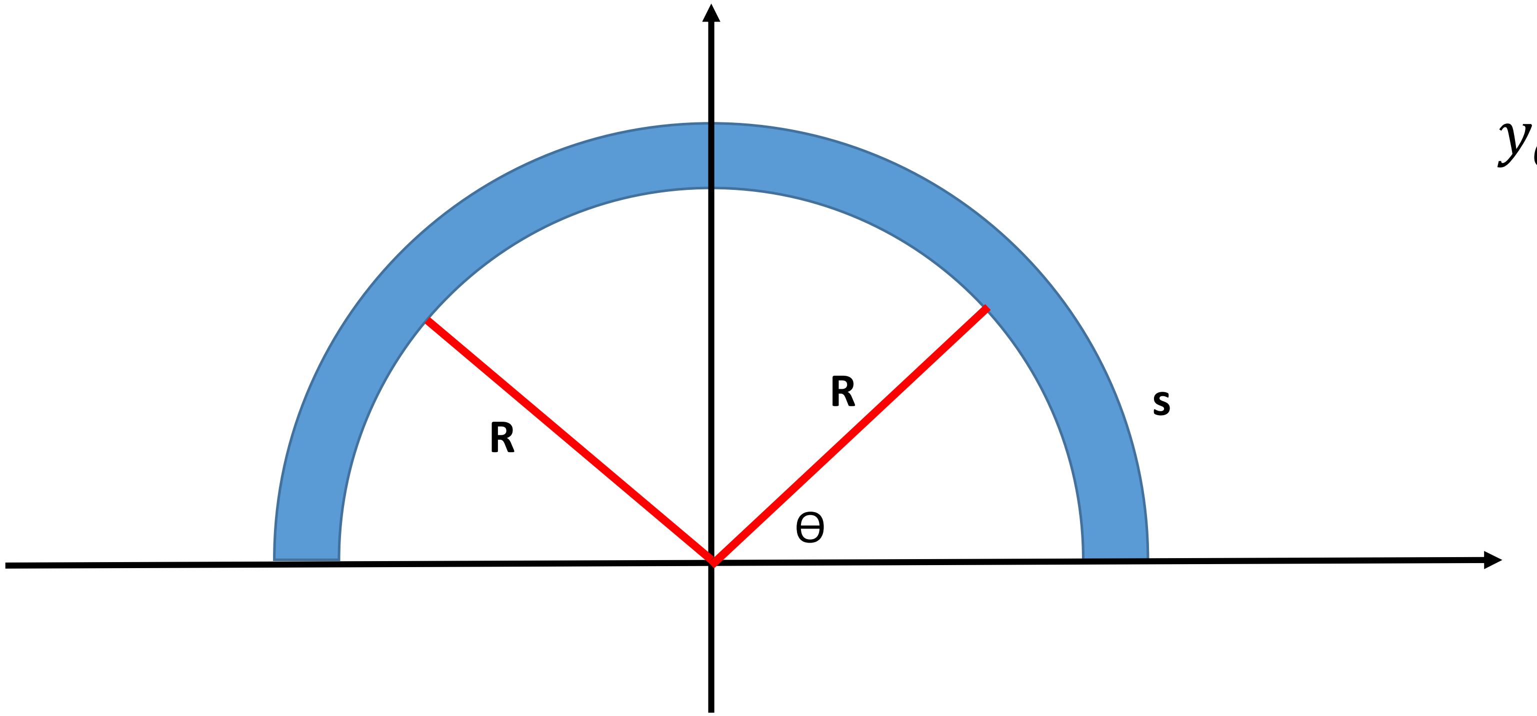


$$y_{CM} = \frac{1}{M}\int R.sen\theta R.\lambda.d\theta$$

$$y_{CM} = \frac{\lambda R^2}{M}\int sen\theta d\theta$$

$$y_{CM} = \frac{\lambda R^2}{M}\int_0^{\pi} sen\theta d\theta$$

Exemplo 2



$$y_{CM} = \frac{1}{M}\int R.sen\theta R.\lambda.d\theta$$

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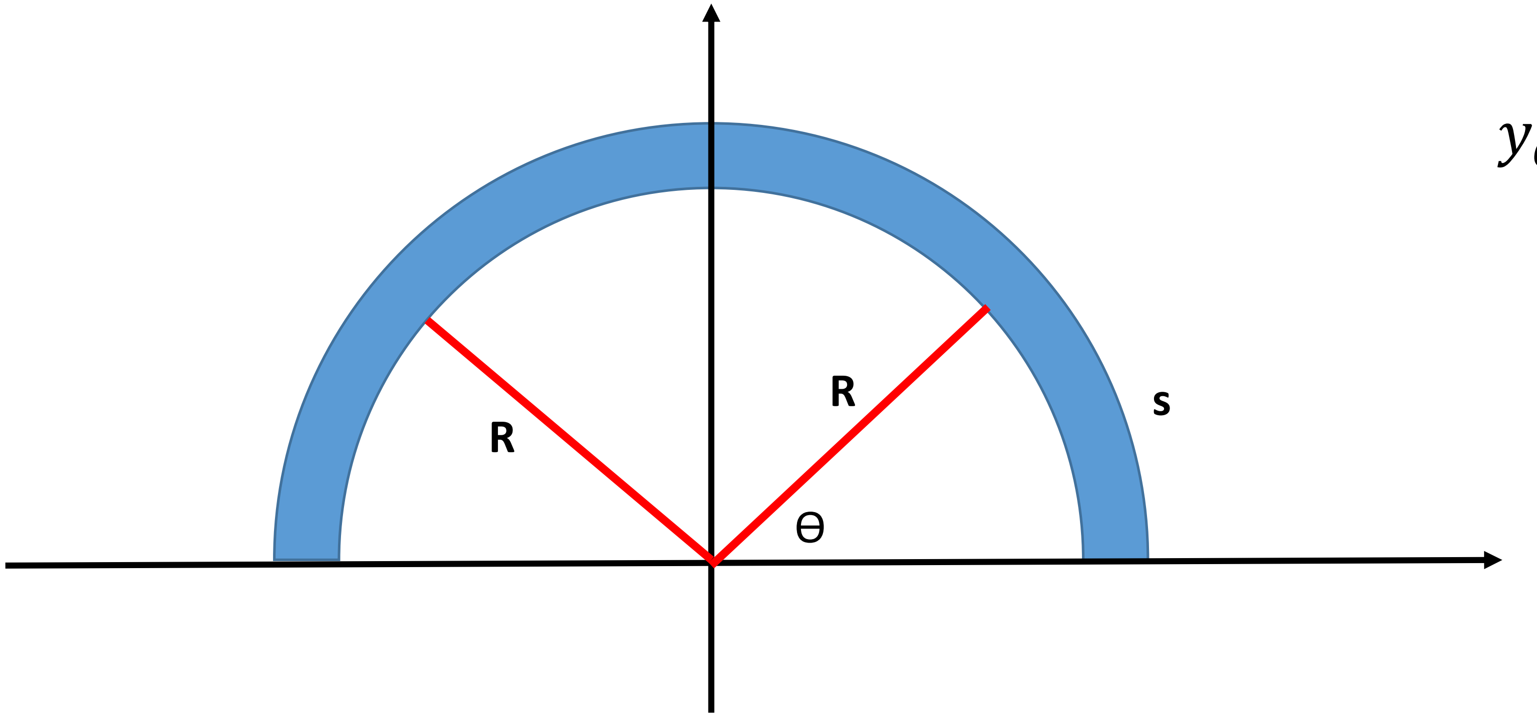
Exemplo 2

$$y_{CM} = \frac{1}{M}\int R.sen\theta R.\lambda.d\theta$$



$$y_{CM} = \frac{\lambda R^2}{M}\int sen\theta d\theta$$

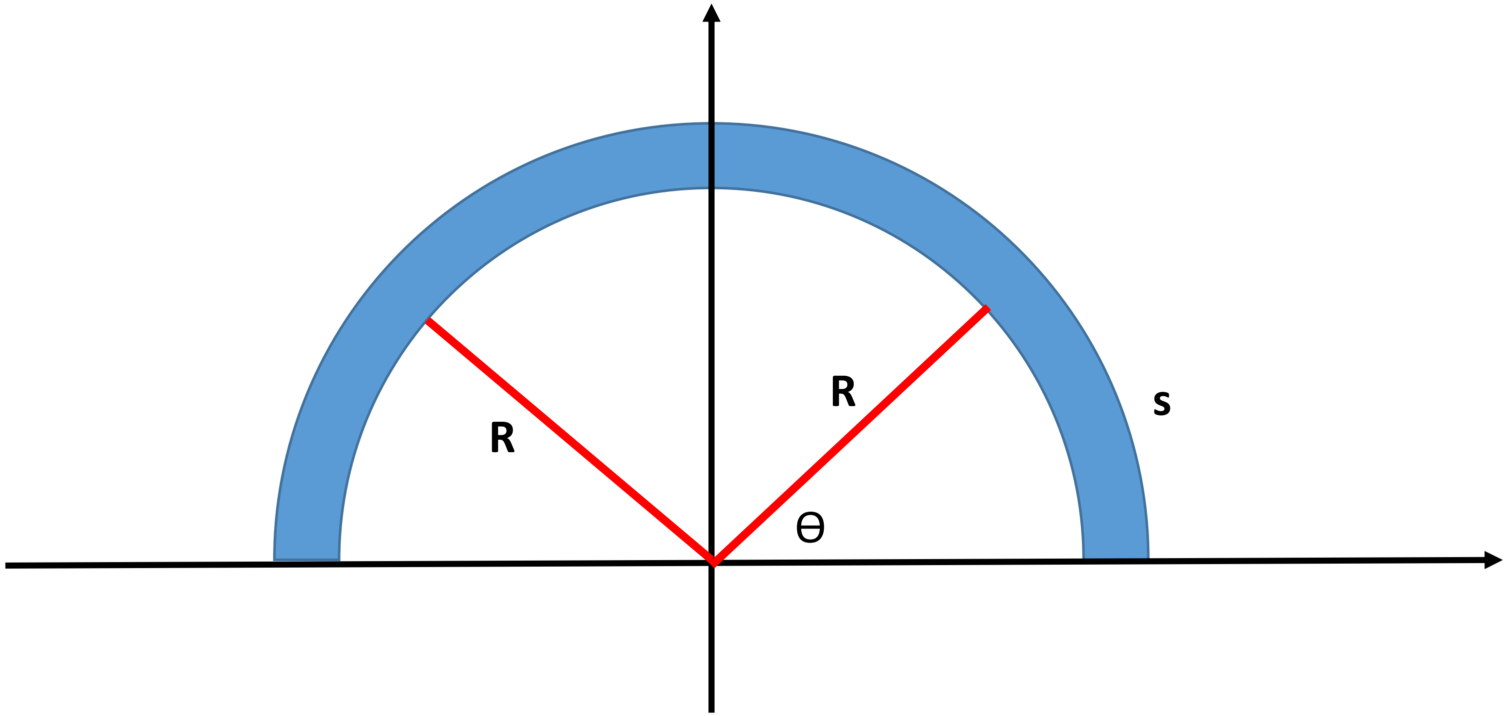
$$y_{CM} = \frac{\lambda R^2}{M}\int_0^{\pi} sen\theta d\theta$$

$$y_{CM} = \frac{\lambda R^2}{M}\int_0^{\pi} sen\theta d\theta$$

$$y_{CM} = \frac{\lambda R^2}{M}(-cos\theta)_0^{\pi} = \frac{\lambda R^2}{M}[-(cos\pi) - (-cos0)] = \frac{2\lambda R^2}{M}$$

Exemplo 2

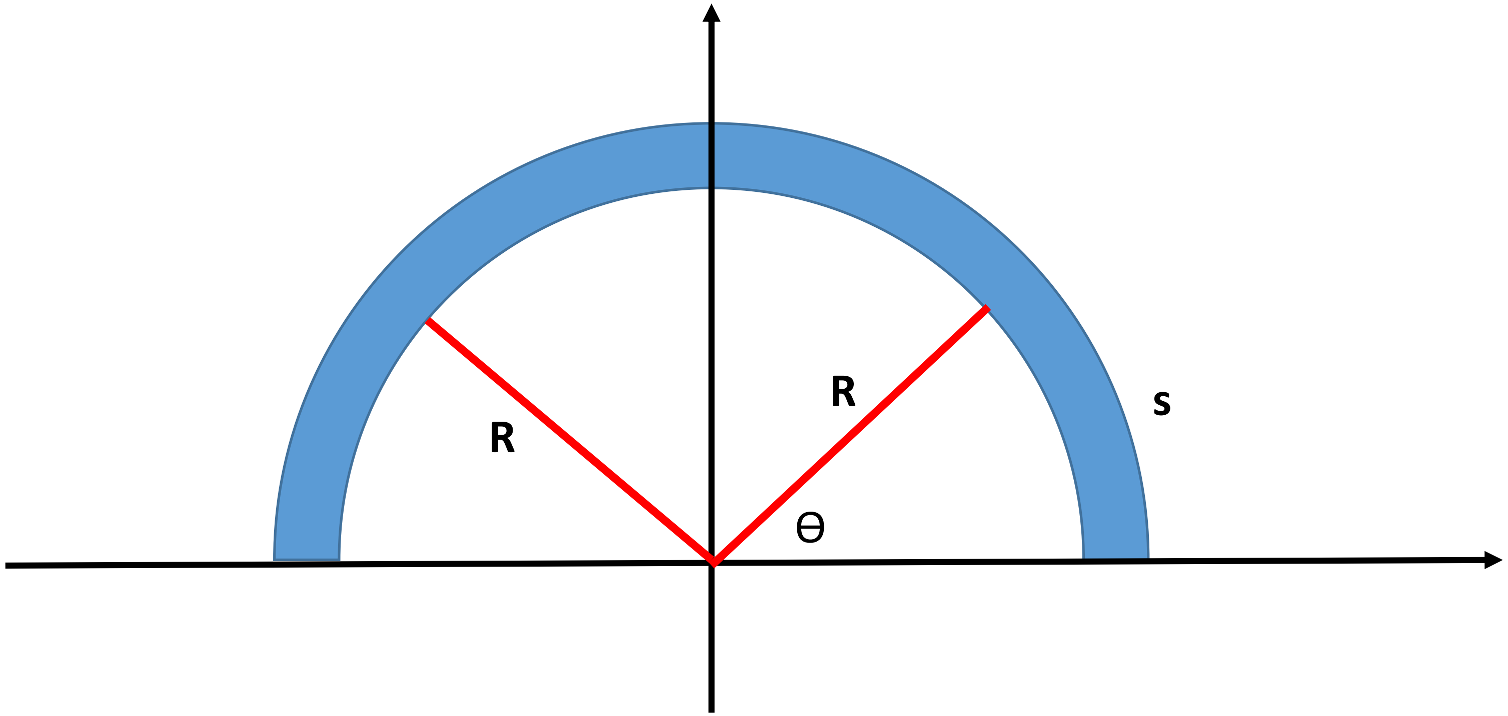
$$y_{CM} = \frac{2\lambda R^2}{M}$$



Exemplo 2

$$y_{CM} = \frac{2\lambda R^2}{M}$$

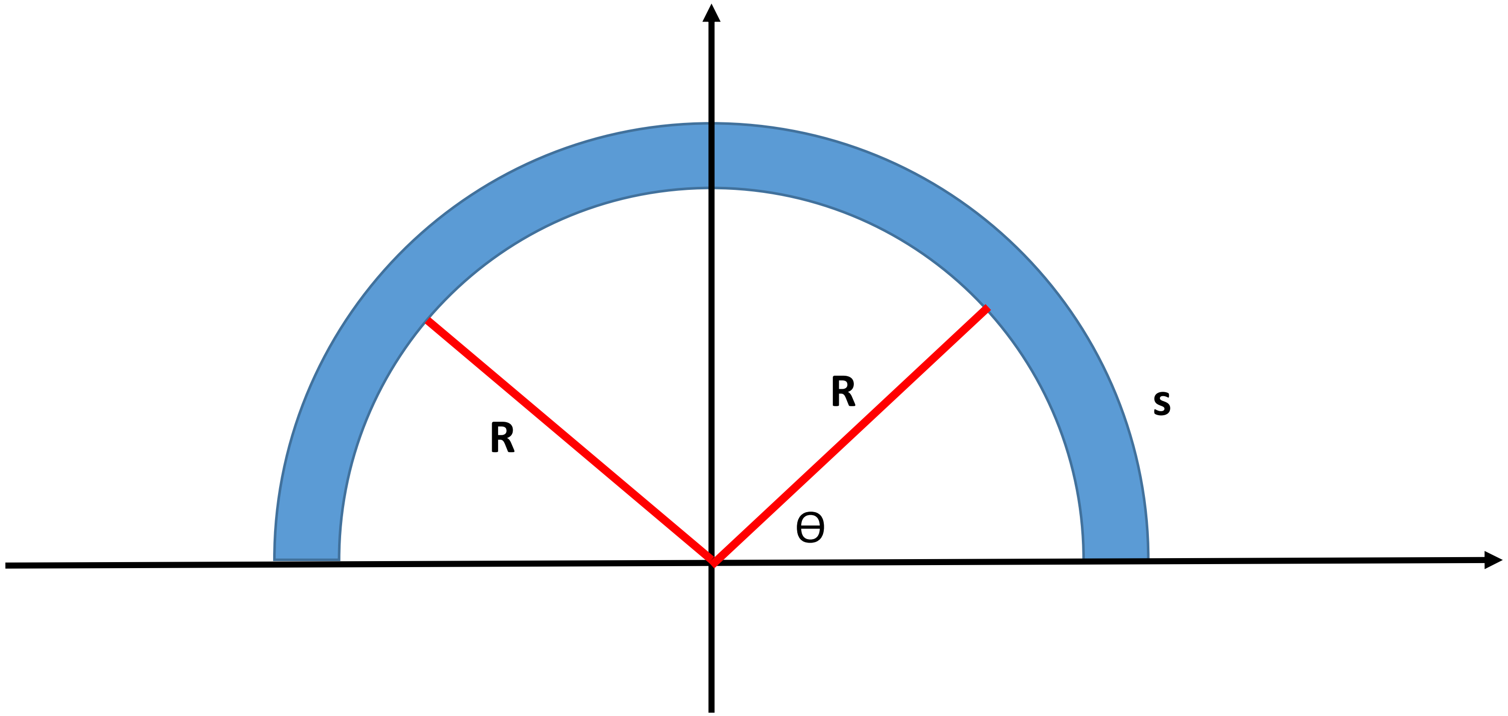
Mas quem é λ?



Exemplo 2

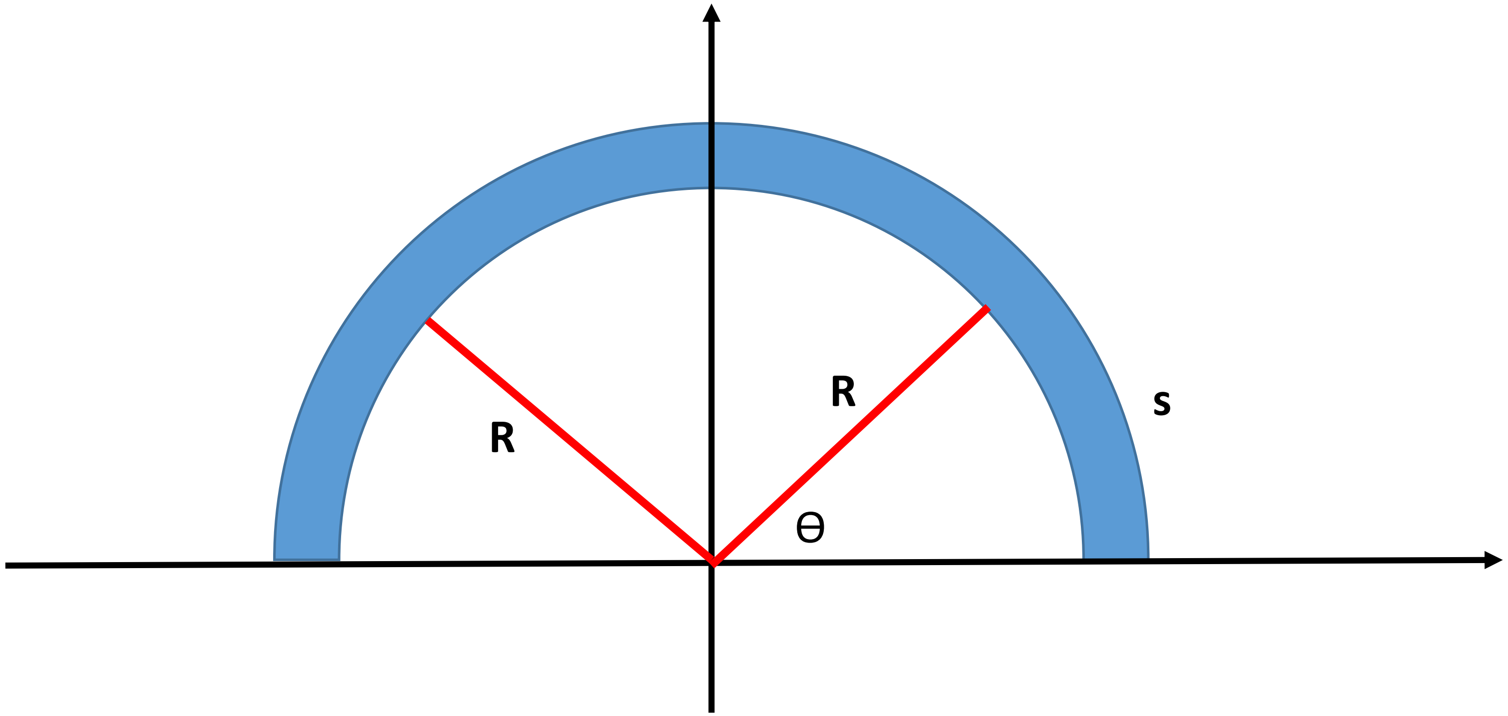
$$y_{CM} = \frac{2\lambda R^2}{M}$$

Mas quem é λ?



$$\lambda = \frac{dm}{ds} = \frac{M}{\pi R}$$

Exemplo 2



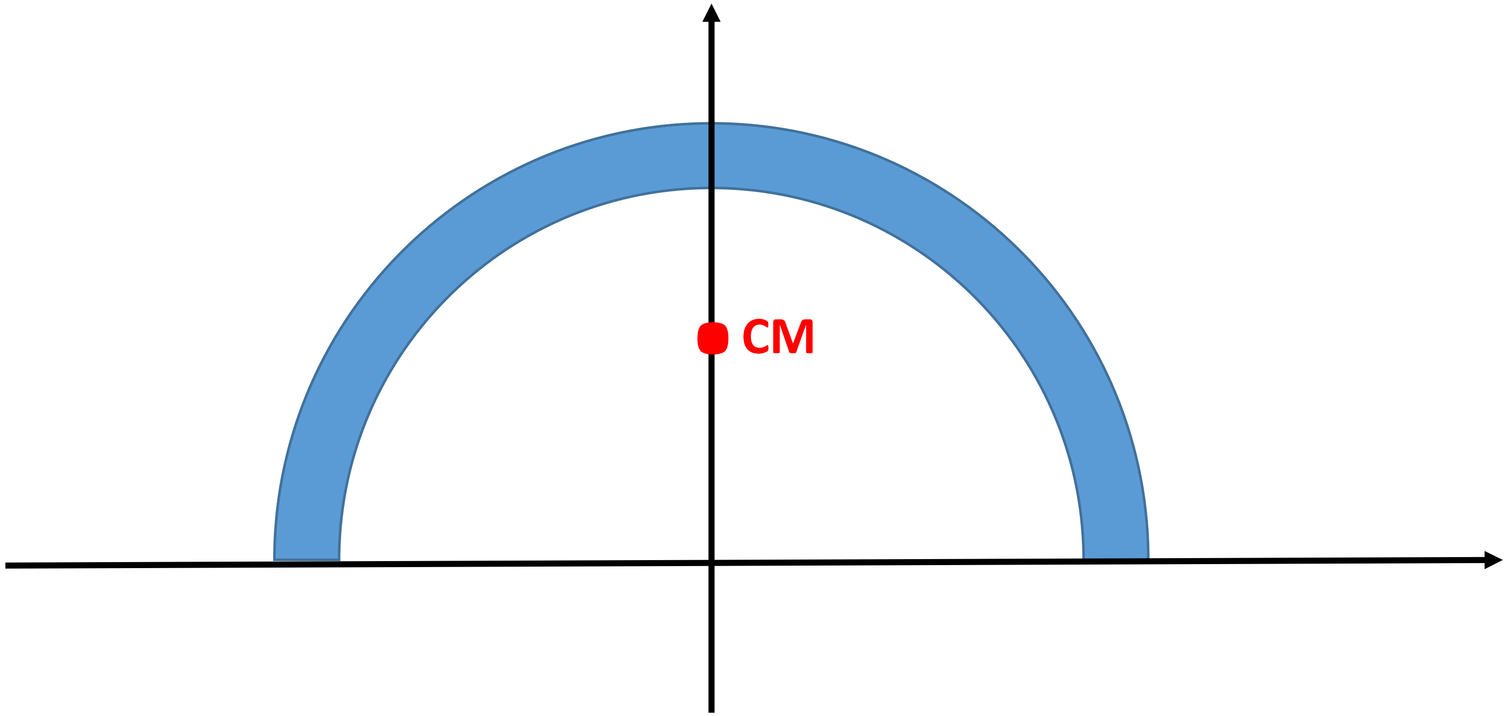
$$y_{CM} = \frac{2\lambda R^2}{M}$$

$$\lambda = \frac{dm}{ds} = \frac{M}{\pi R}$$

$$y_{CM} = \frac{2R^2 M}{M\pi R}$$

$$y_{CM} = \frac{2R}{\pi}$$

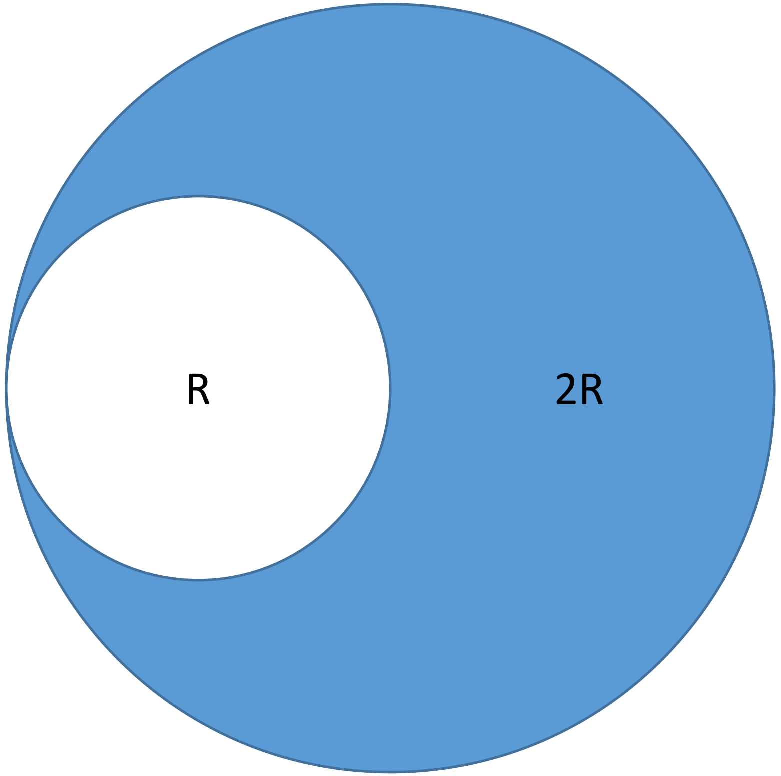
Exemplo 2



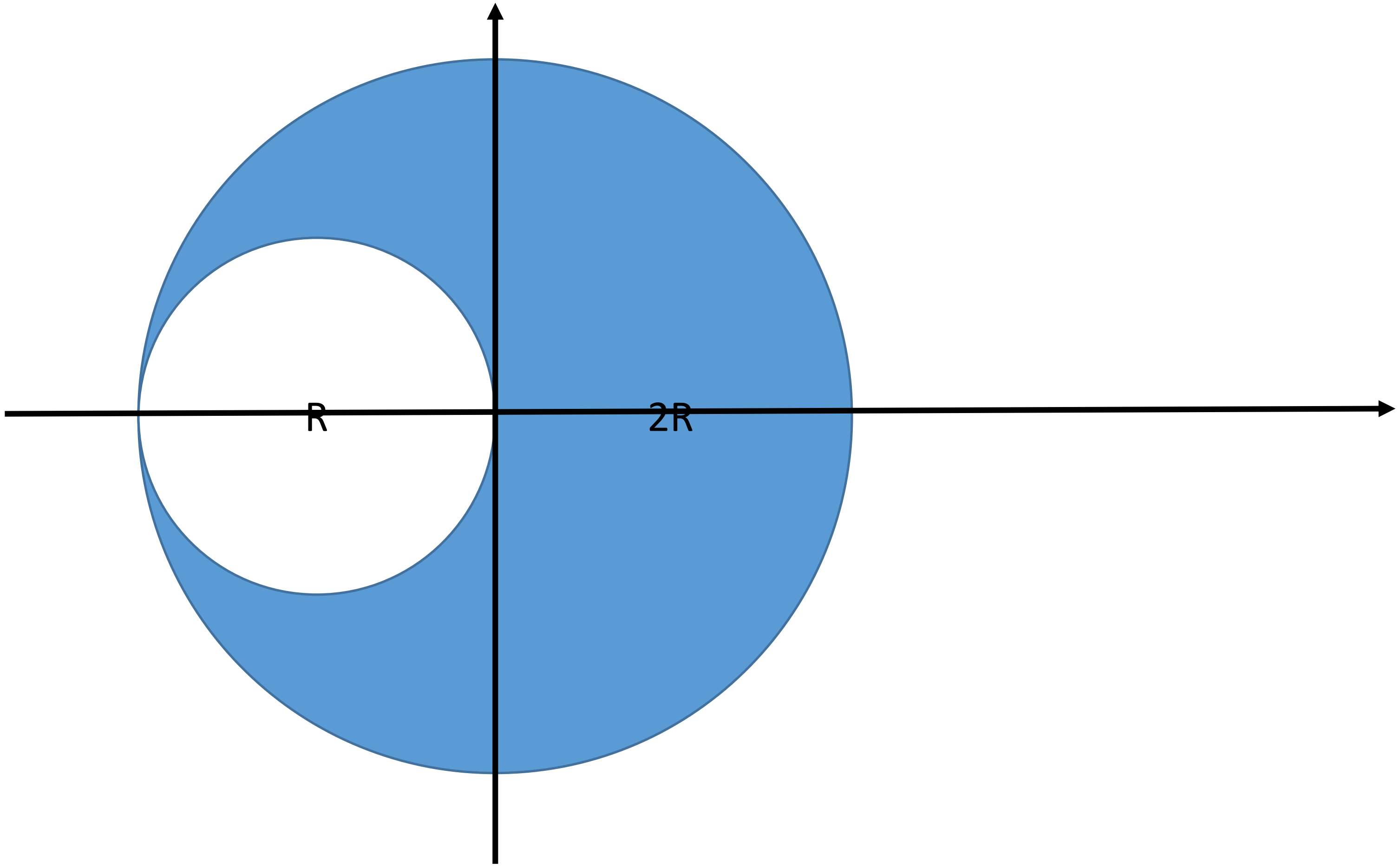
$$y_{CM} = \frac{2R}{\pi}$$

$$x_{CM} = 0$$

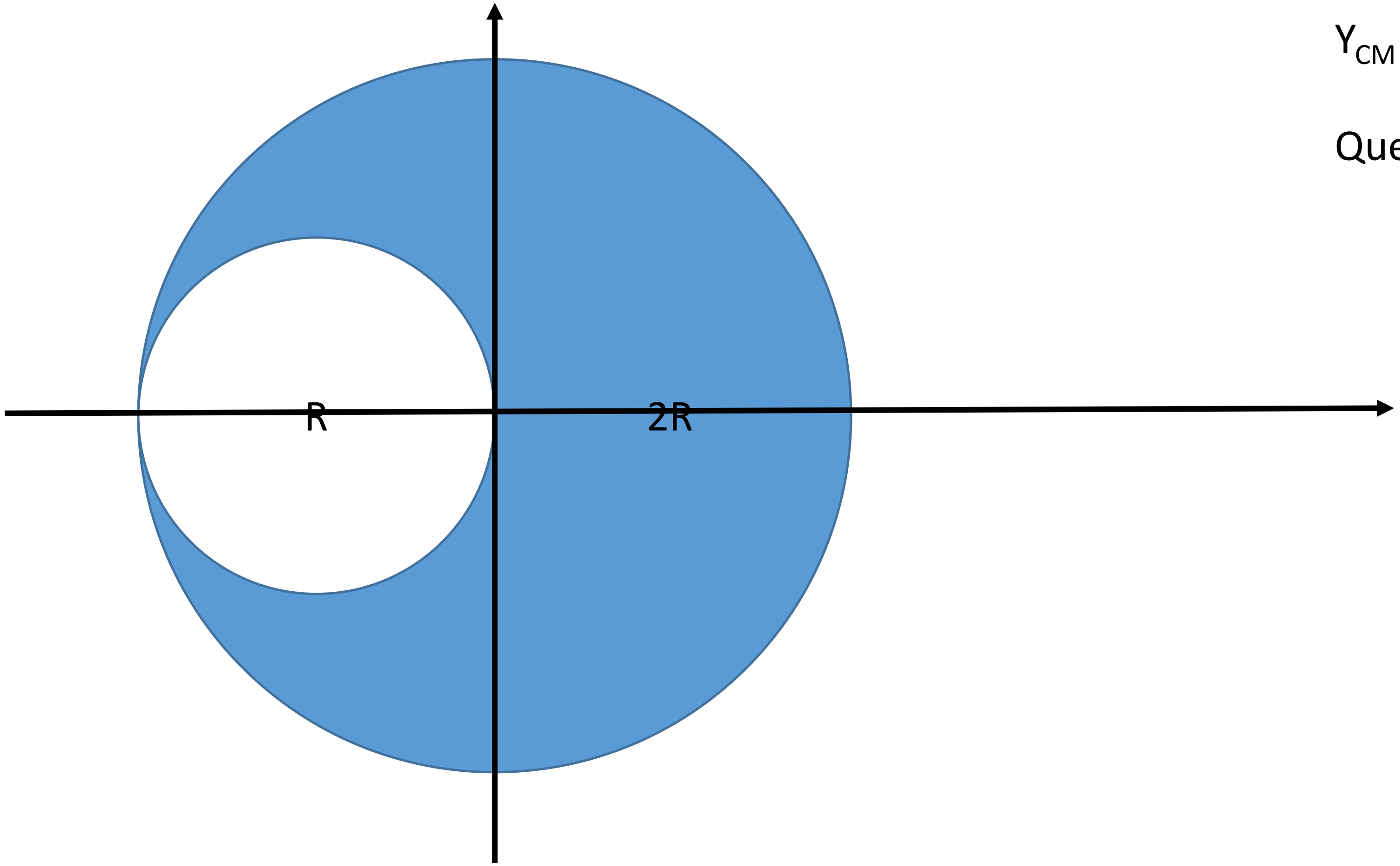
Exemplo 3



Exemplo 3



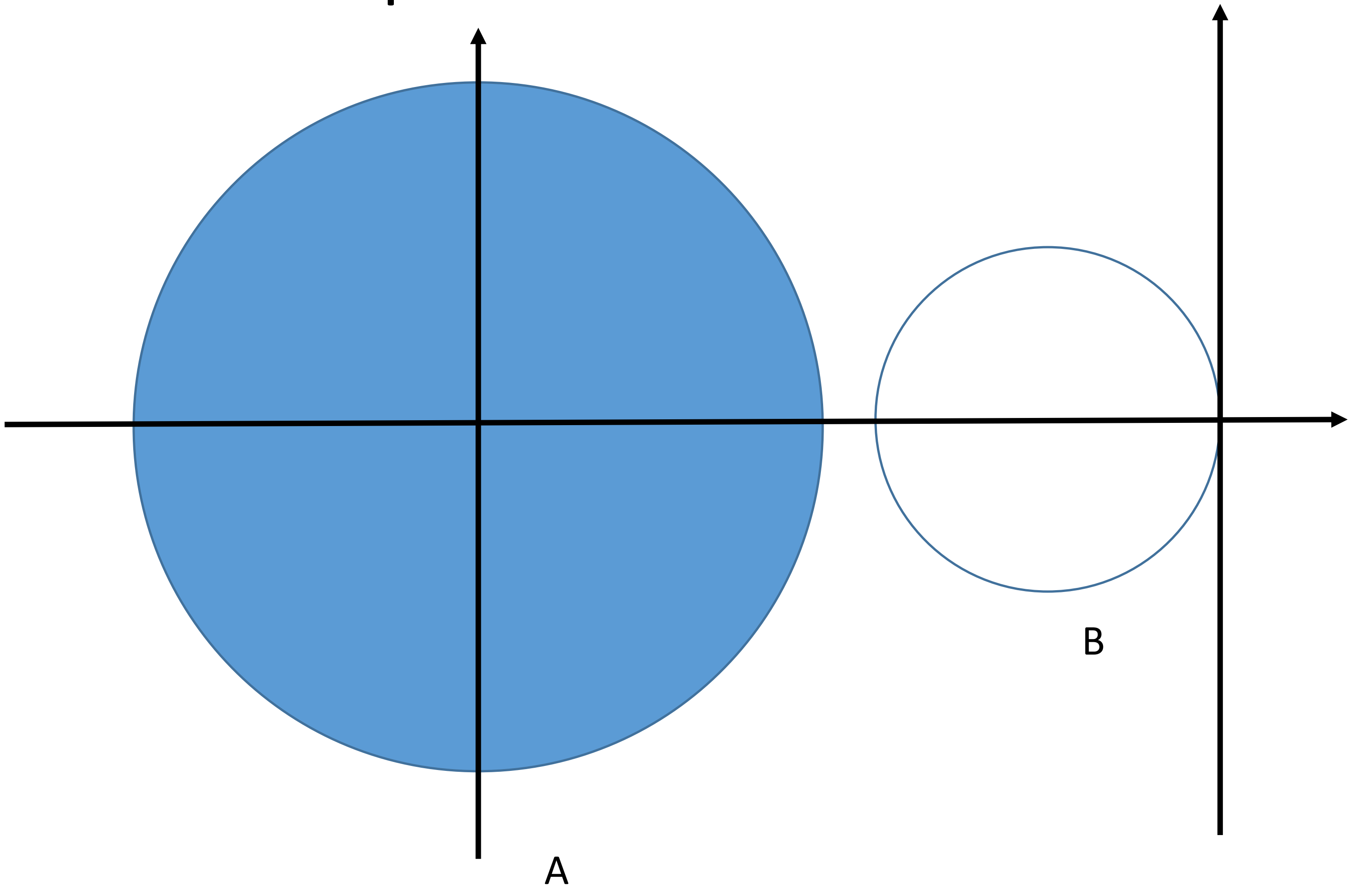
Exemplo 3



Y_{CM} é zero!

Quem é o X_{CM}?

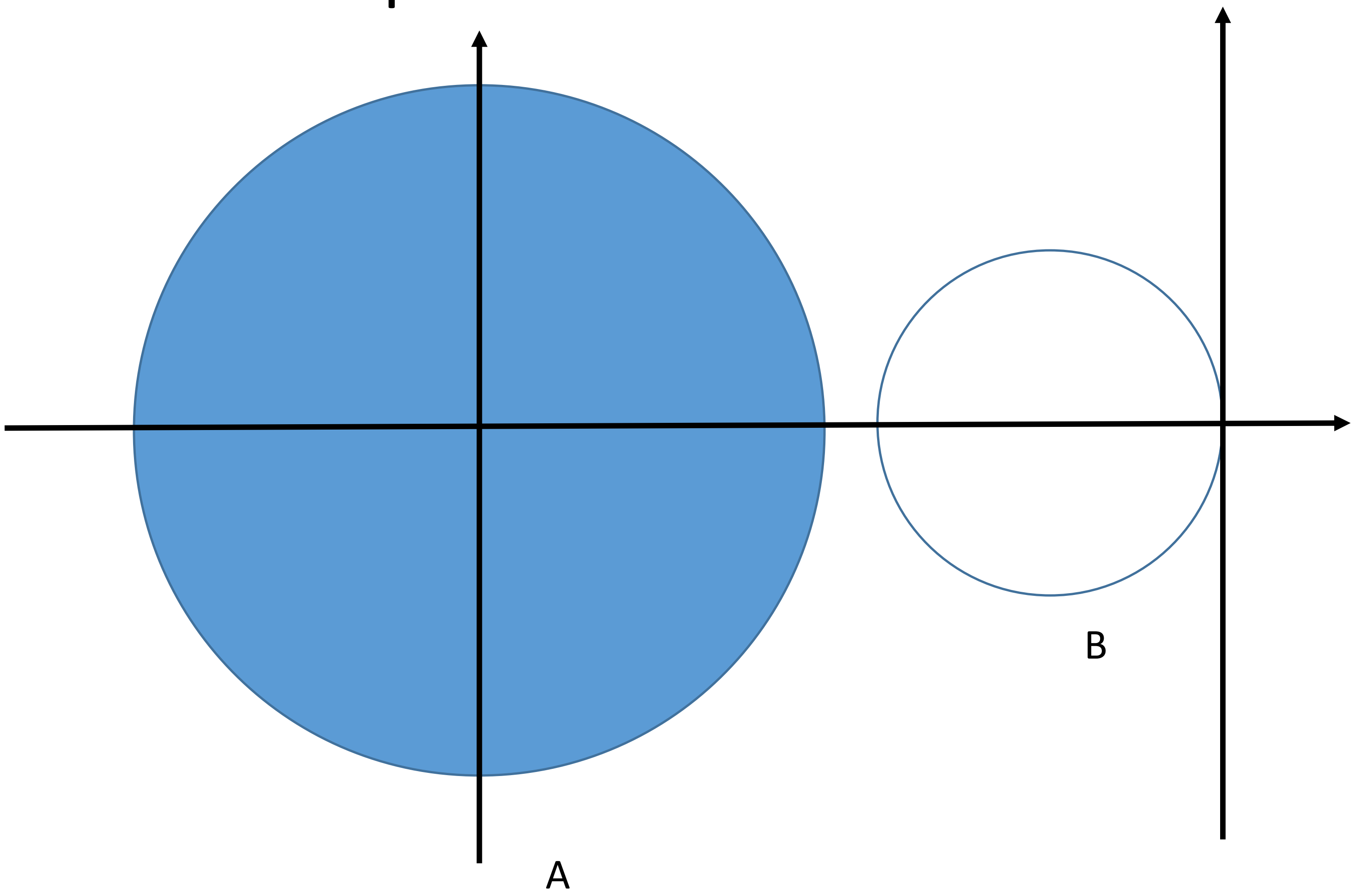
Exemplo 3



Quem é o X_{CM} de A?

Quem é o X_{CM} de B?

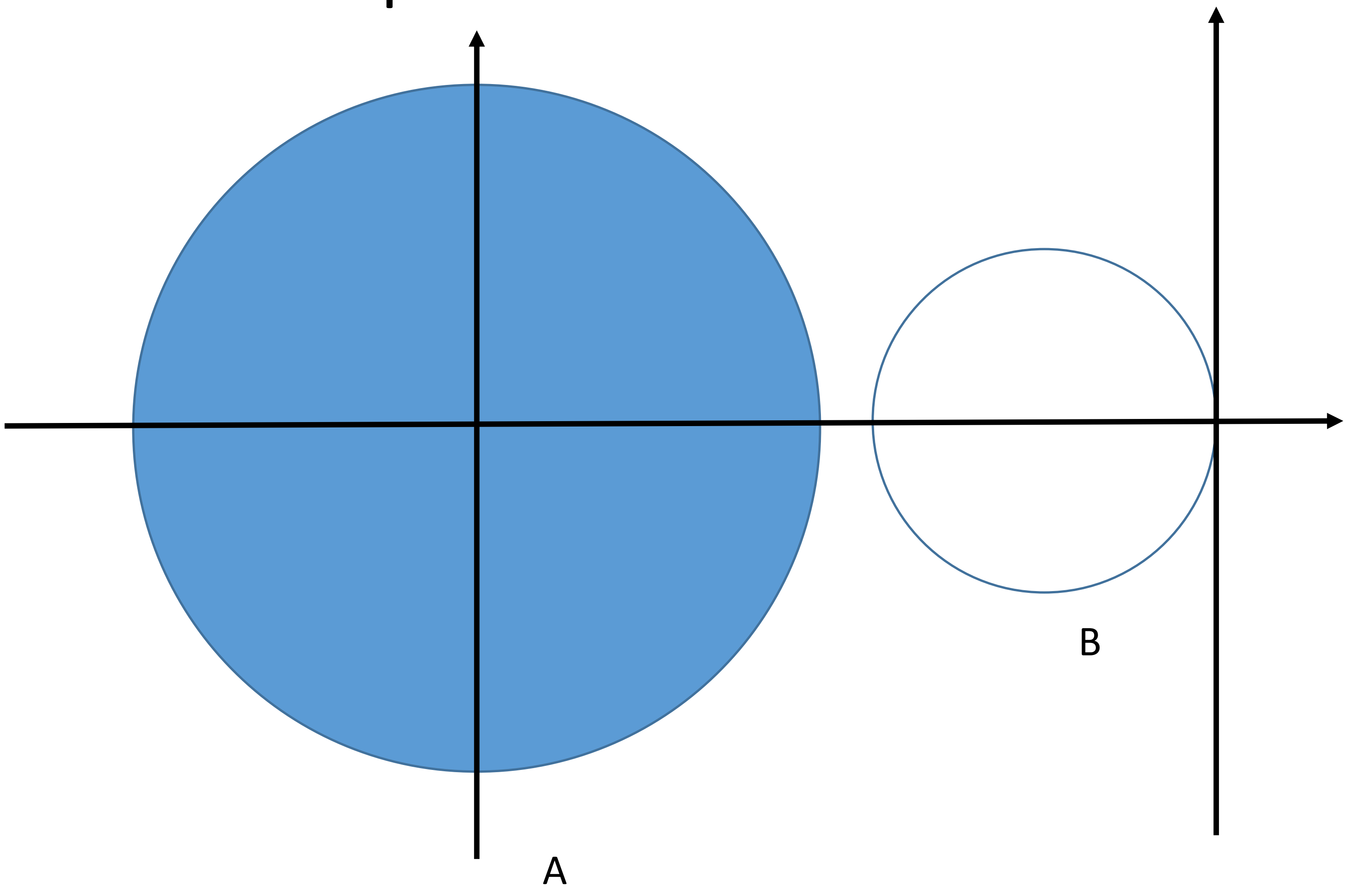
Exemplo 3



Quem é o X_{CM} de A? =0

Quem é o X_{CM} de B? =-R

Exemplo 3

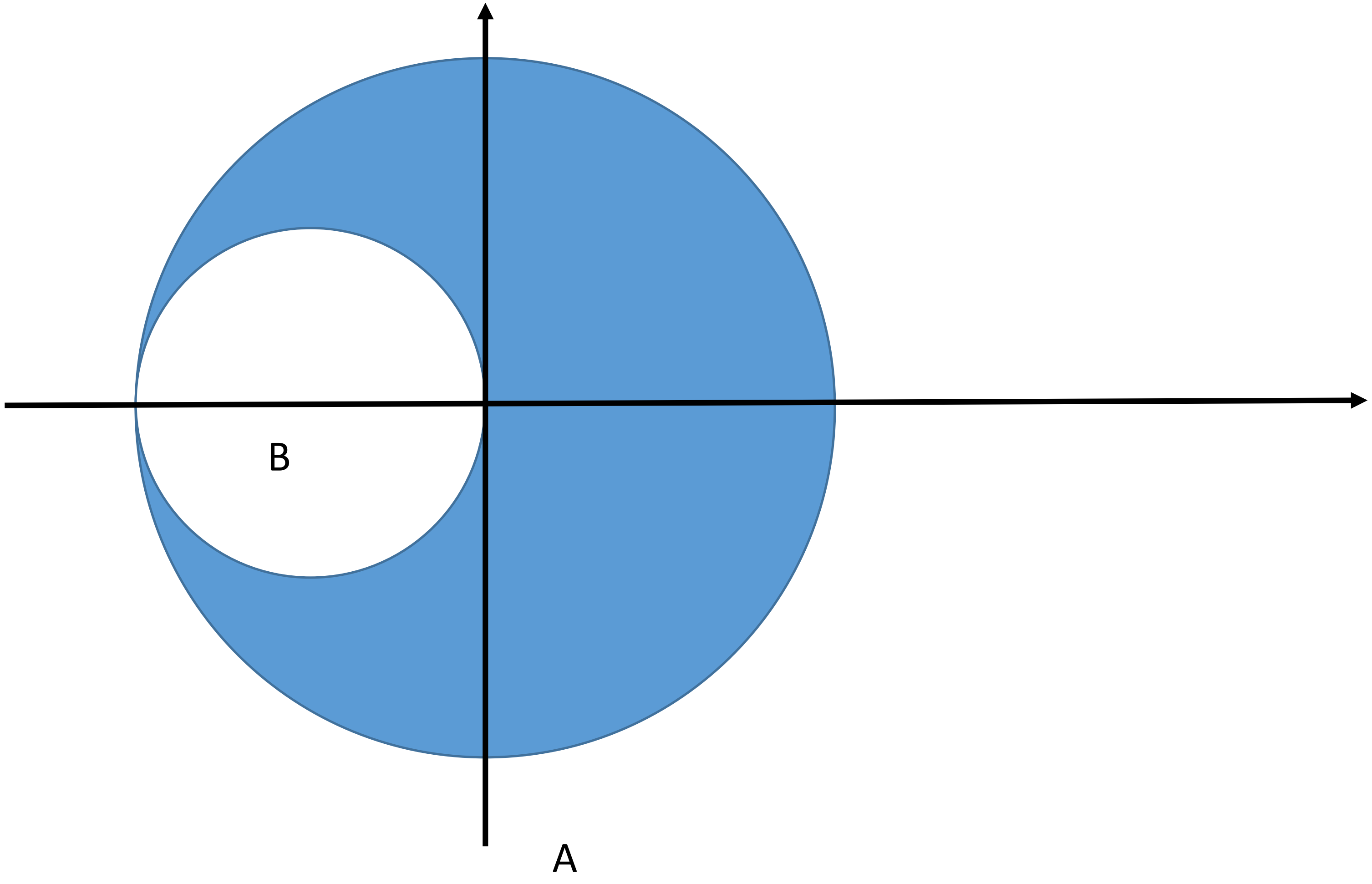


Quem é o X_{CM} de A? =0

Quem é o X_{CM} de B? =-R

$$M_A \rightarrow \alpha = \frac{M_A}{\pi(2R)^2}$$

Exemplo 3



Quem é o X_{CM} de A? =0

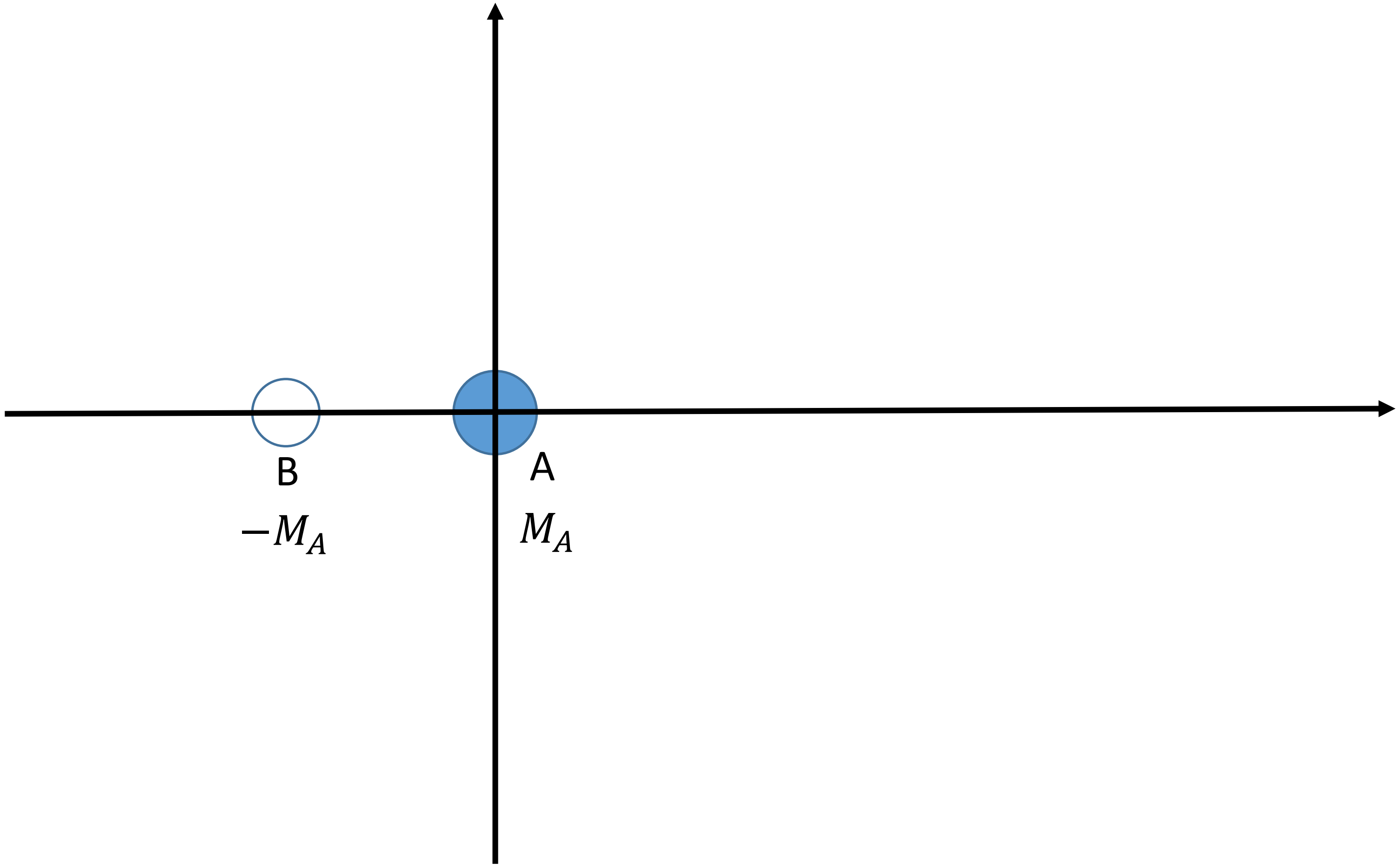
Quem é o X_{CM} de B? =-R

$$M_A \rightarrow \alpha = \frac{M_A}{\pi(2R)^2}$$

$$M_B \rightarrow \alpha = \frac{M_B}{\pi(R)^2}$$

$$M = M_A - M_B$$

Exemplo 3



Quem é o X_{CM} de A? =0

Quem é o X_{CM} de B? =-R

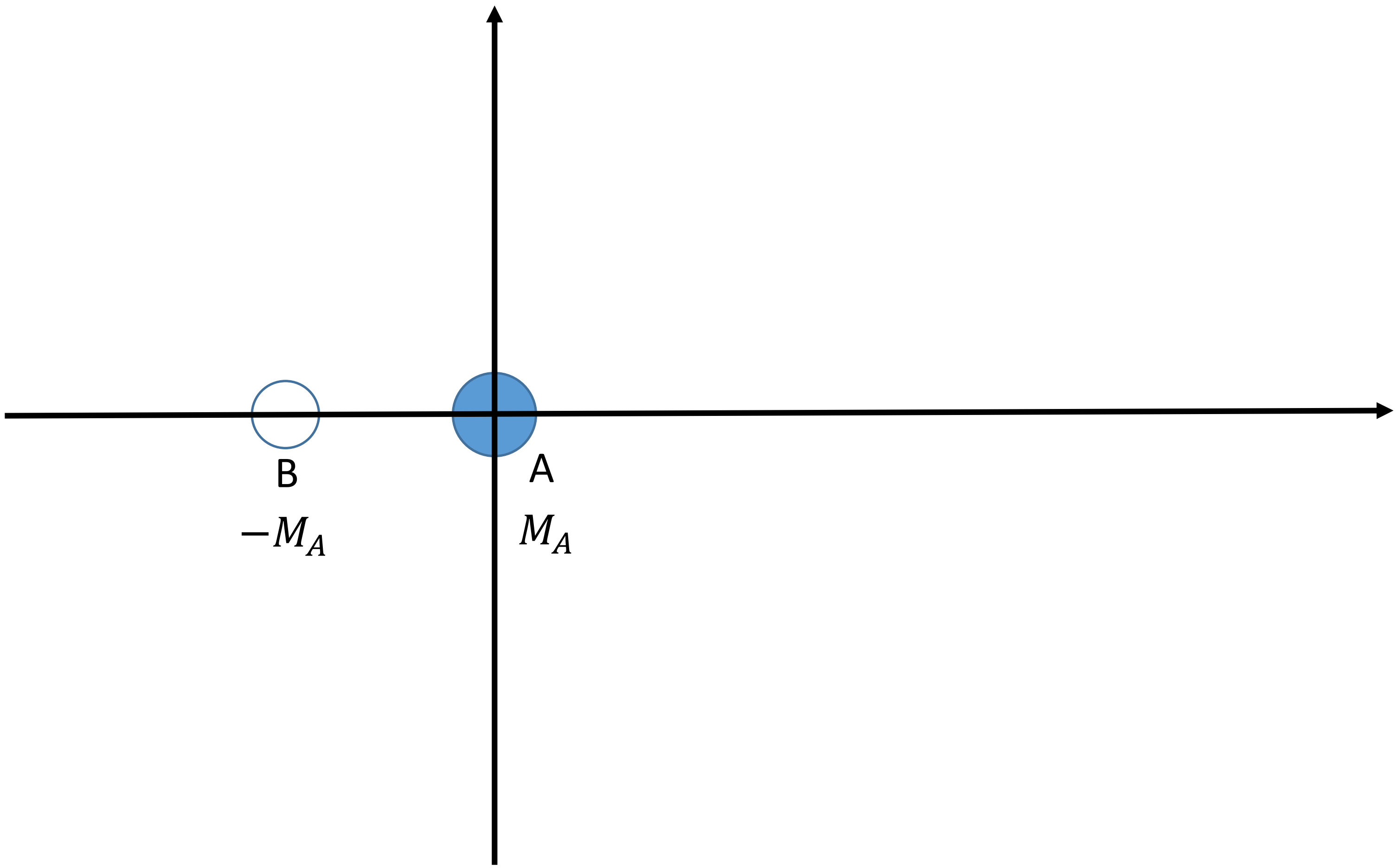
$$M_A \rightarrow \alpha = \frac{M_A}{\pi(2R)^2} \qquad M_A = 4\alpha\pi R^2$$

$$M_B \rightarrow \alpha = \frac{M_B}{\pi(R)^2} \qquad M_B = \alpha\pi R^2$$

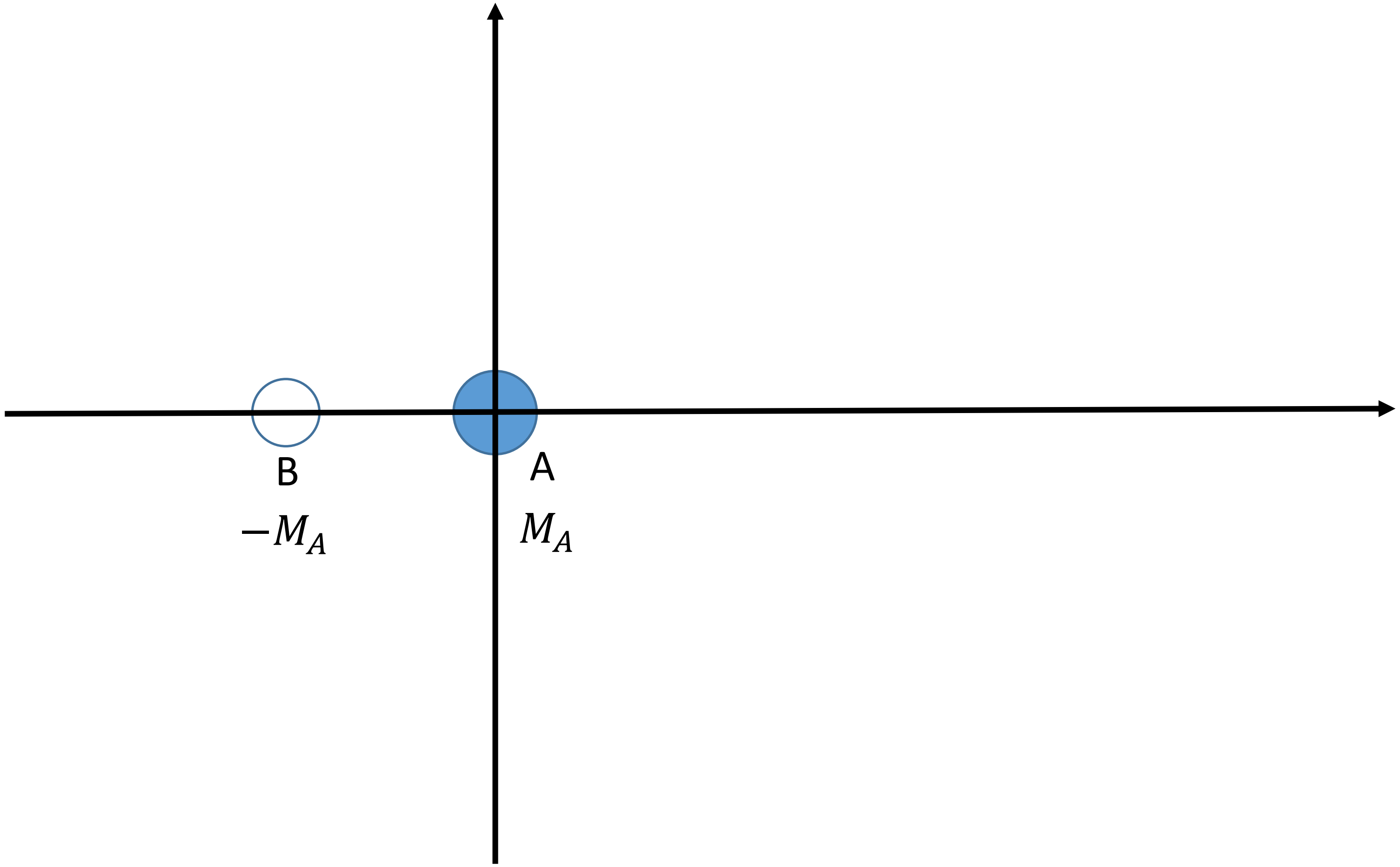
$$M = M_A - M_B = 3\alpha\pi R^2$$

Exemplo 3

$$x_{CM} = \frac{m_A x_A + m_B x_B}{M}$$



Exemplo 3



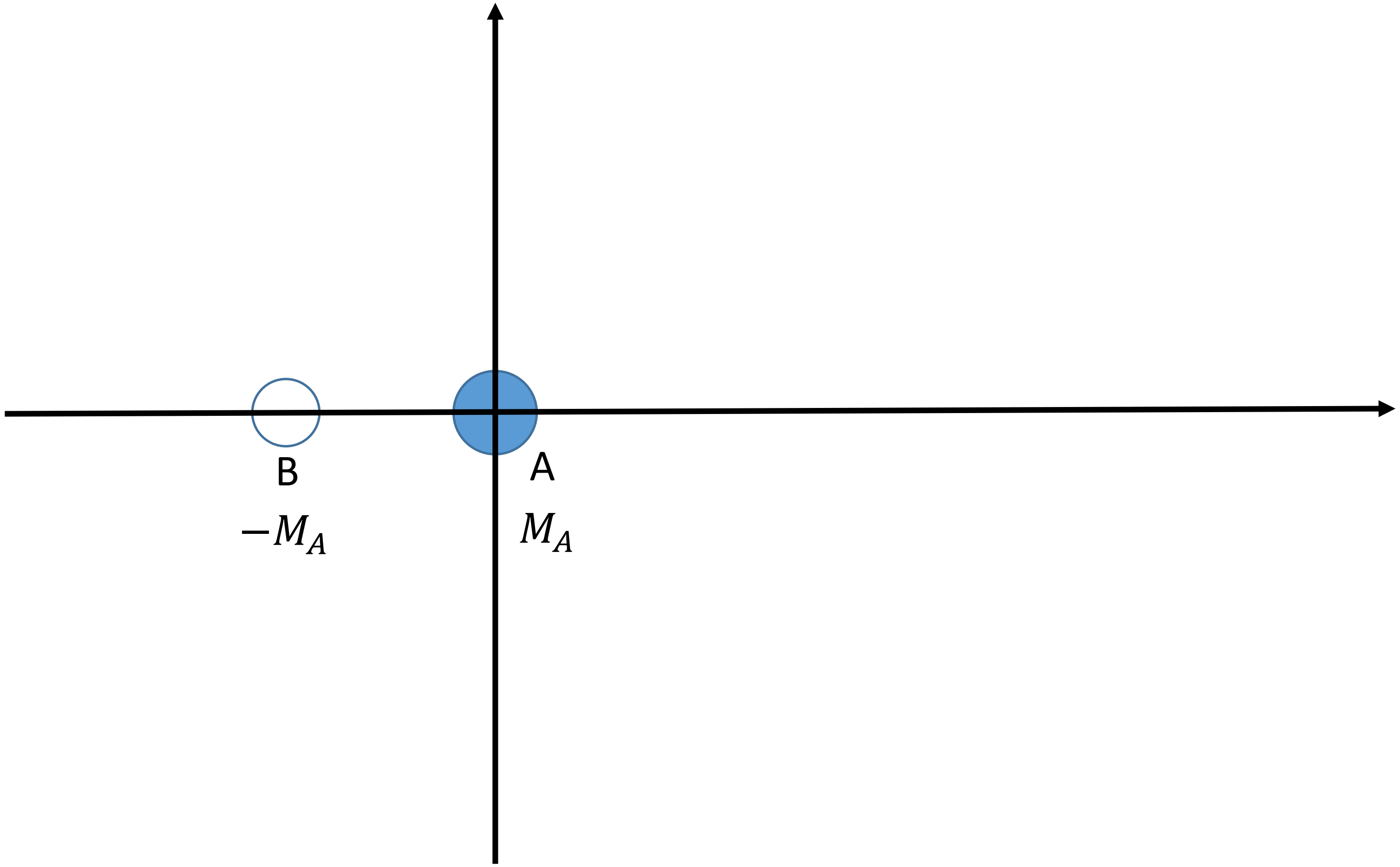
$$x_{CM} = \frac{m_A x_A + m_B x_B}{M}$$

$M_A = 4\alpha\pi R^2$ $\quad x_A = 0$

$M_B = \alpha\pi R^2$ $\quad x_B = -R$

$M = 3\alpha\pi R^2$

Exemplo 3



$$x_{CM} = \frac{m_A x_A + m_B x_B}{M}$$

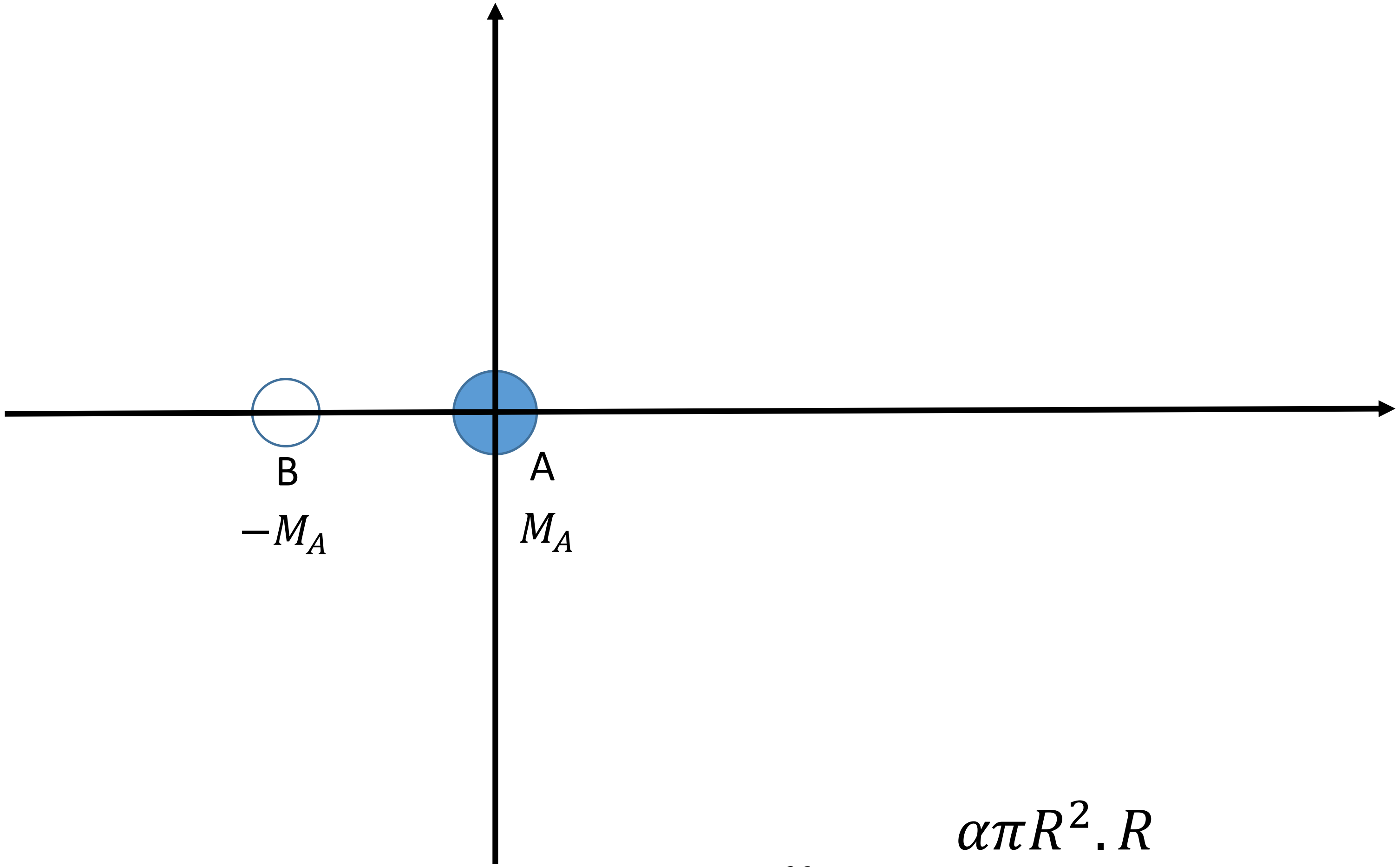
$M_A = 4\alpha\pi R^2$ $\quad x_A = 0$

$M_B = -\alpha\pi R^2$ $\quad x_B = -R$

$M = 3\alpha\pi R^2$

$$x_{CM} = \frac{4\alpha\pi R^2 . 0 + (-\alpha\pi R^2).(-R)}{3\alpha\pi R^2}$$

Exemplo 3



$$x_{CM} = \frac{m_A x_A + m_B x_B}{M}$$

$M_A = 4\alpha\pi R^2$ $\quad$ $x_A = 0$

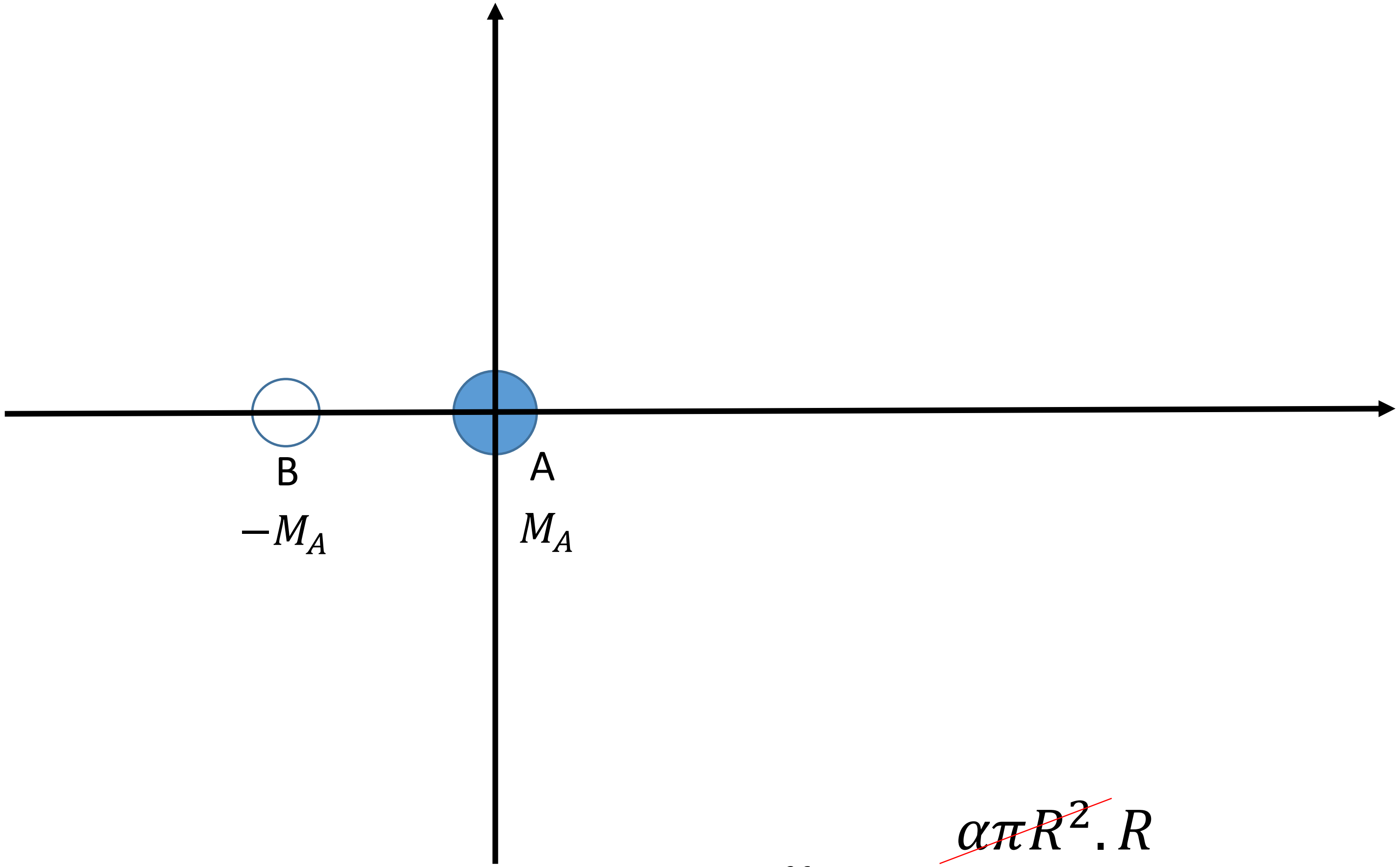
$M_B = -\alpha\pi R^2$ $\quad$ $x_B = -R$

$M = 3\alpha\pi R^2$

$$x_{CM} = \frac{4\alpha\pi R^2.0 + (-\alpha\pi R^2).(-R)}{3\alpha\pi R^2}$$

$$x_{CM} = \frac{\alpha\pi R^2.R}{3\alpha\pi R^2}$$

Exemplo 3



$$x_{CM} = \frac{m_A x_A + m_B x_B}{M}$$

$M_A = 4\alpha\pi R^2$ $\qquad$ $x_A = 0$

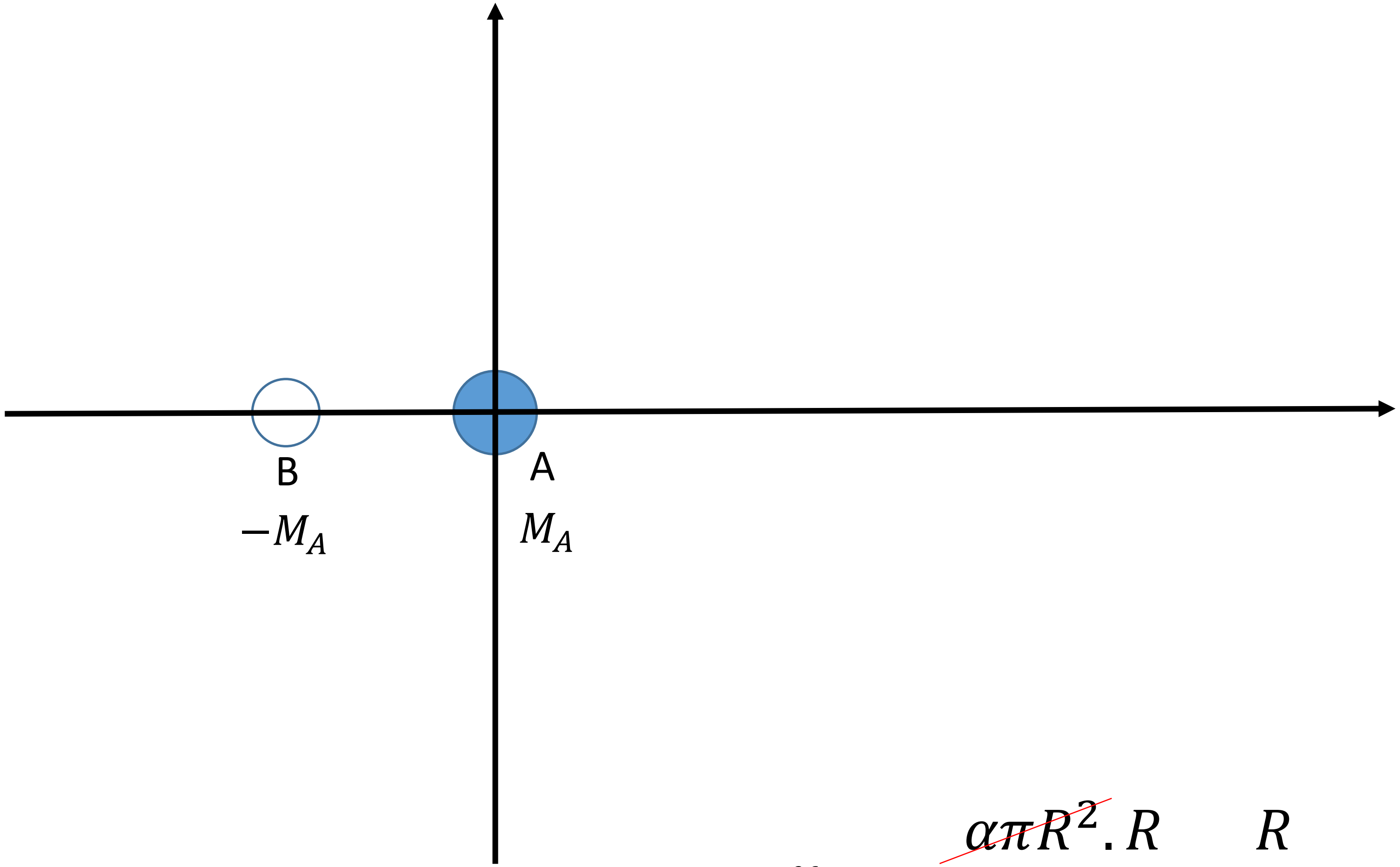
$M_B = -\alpha\pi R^2$ $\qquad$ $x_B = -R$

$M = 3\alpha\pi R^2$

$$x_{CM} = \frac{4\alpha\pi R^2 . 0 + (-\alpha\pi R^2).(-R)}{3\alpha\pi R^2}$$

$$x_{CM} = \frac{\cancel{\alpha\pi R^2}.R}{3\cancel{\alpha\pi R^2}}$$

Exemplo 3



$$x_{CM} = \frac{m_A x_A + m_B x_B}{M}$$

$M_A = 4\alpha\pi R^2$ $\quad$ $x_A = 0$

$M_B = -\alpha\pi R^2$ $\quad$ $x_B = -R$

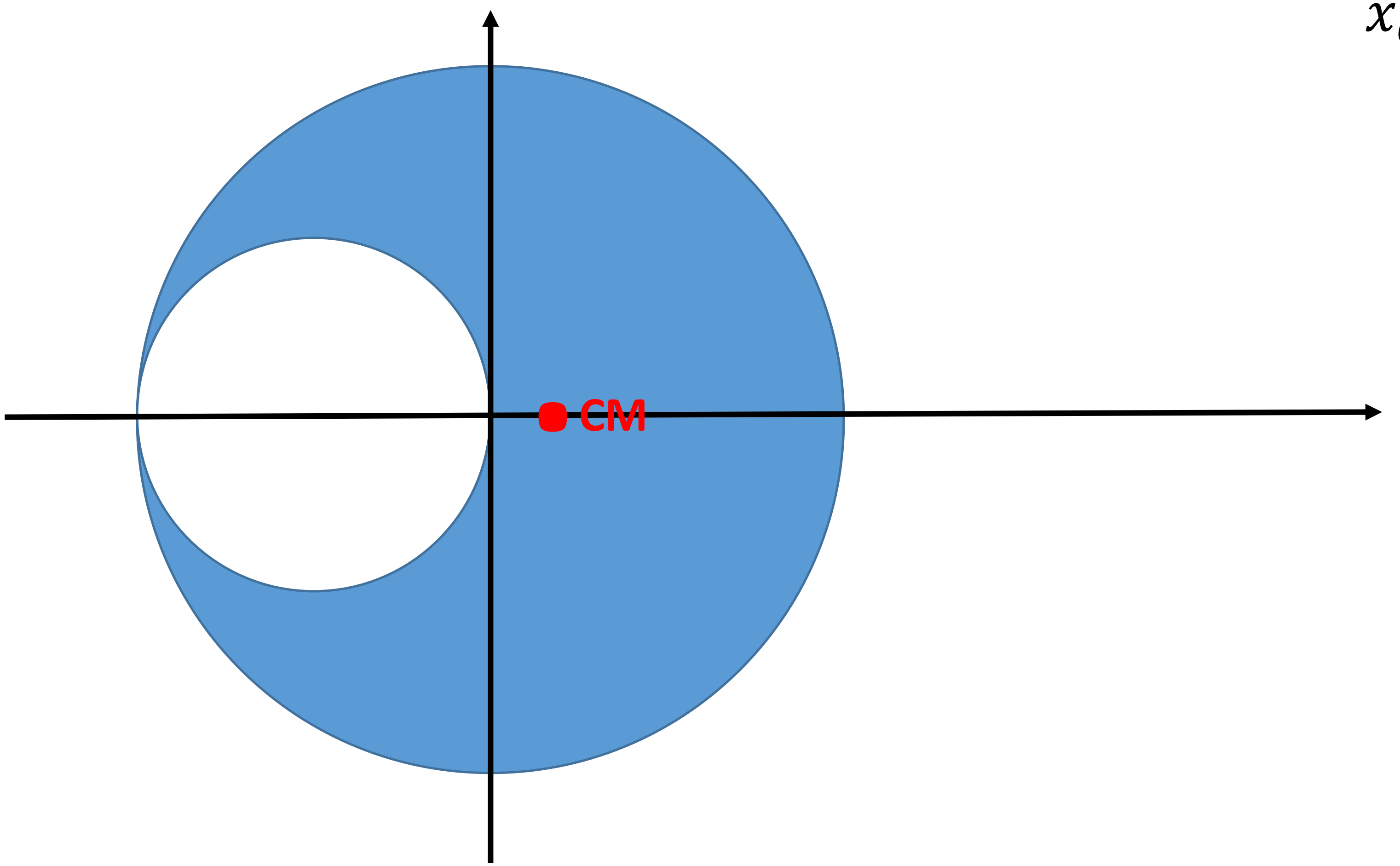
$M = 3\alpha\pi R^2$

$$x_{CM} = \frac{4\alpha\pi R^2.0 + (-\alpha\pi R^2).(-R)}{3\alpha\pi R^2}$$

$$x_{CM} = \frac{\cancel{\alpha\pi R^2}.R}{3\cancel{\alpha\pi R^2}} = \frac{R}{3}$$

Exemplo 3

$$x_{CM} = \frac{R}{3} \qquad y_{CM} = 0$$



Movimento de um Sistema de Partículas

$$x_{CM} = \frac{m_A x_A + m_B x_B}{M}$$

$$M x_{CM} = m_A x_A + m_B x_B$$

Movimento de um Sistema de Partículas

$$x_{CM} = \frac{m_A x_A + m_B x_B}{M}$$

$$M x_{CM} = m_A x_A + m_B x_B$$

$$\frac{dMx_{CM}}{dt} = \frac{d}{dt}(m_A x_A + m_B x_B)$$

Movimento de um Sistema de Partículas

$$x_{CM} = \frac{m_A x_A + m_B x_B}{M}$$

$$M x_{CM} = m_A x_A + m_B x_B$$

$$\frac{dM x_{CM}}{dt} = \frac{d}{dt}(m_A x_A + m_B x_B)$$

$$M \frac{dx_{CM}}{dt} = m_A \frac{dx_A}{dt} + m_B \frac{dx_B}{dt}$$

Movimento de um Sistema de Partículas

$$x_{CM} = \frac{m_A x_A + m_B x_B}{M}$$

$$M x_{CM} = m_A x_A + m_B x_B$$

$$\frac{dM x_{CM}}{dt} = \frac{d}{dt}(m_A x_A + m_B x_B)$$

$$M\frac{dx_{CM}}{dt} = m_A\frac{dx_A}{dt} + m_B\frac{dx_B}{dt}$$

$$M\frac{d^2x_{CM}}{dt^2} = m_A\frac{d^2x_A}{dt^2} + m_B\frac{d^2x_B}{dt^2}$$

Movimento de um Sistema de Partículas

$$x_{CM} = \frac{m_A x_A + m_B x_B}{M}$$

$$Mx_{CM} = m_A x_A + m_B x_B$$

$$\frac{dMx_{CM}}{dt} = \frac{d}{dt}(m_A x_A + m_B x_B)$$

$$M\frac{dx_{CM}}{dt} = m_A\frac{dx_A}{dt} + m_B\frac{dx_B}{dt}$$

$$M\frac{d^2x_{CM}}{dt^2} = m_A\frac{d^2x_A}{dt^2} + m_B\frac{d^2x_B}{dt^2}$$

$$Ma_{CM} = m_A a_A + m_B a_B$$

Movimento de um Sistema de Partículas

$$x_{CM} = \frac{m_A x_A + m_B x_B}{M}$$

$$M x_{CM} = m_A x_A + m_B x_B$$

$$\frac{dM x_{CM}}{dt} = \frac{d}{dt}(m_A x_A + m_B x_B)$$

$$M\frac{dx_{CM}}{dt} = m_A\frac{dx_A}{dt} + m_B\frac{dx_B}{dt}$$

$$M\frac{d^2x_{CM}}{dt^2} = m_A\frac{d^2x_A}{dt^2} + m_B\frac{d^2x_B}{dt^2}$$

$$M a_{CM} = m_A a_A + m_B a_B = \overrightarrow{F_{res}}$$

Movimento de um Sistema de Partículas

$$x_{CM} = \frac{m_A x_A + m_B x_B}{M}$$

$$M x_{CM} = m_A x_A + m_B x_B$$

$$\frac{dMx_{CM}}{dt} = \frac{d}{dt}(m_A x_A + m_B x_B)$$

$$M\frac{dx_{CM}}{dt} = m_A\frac{dx_A}{dt} + m_B\frac{dx_B}{dt}$$

$$M\frac{d^2x_{CM}}{dt^2} = m_A\frac{d^2x_A}{dt^2} + m_B\frac{d^2x_B}{dt^2}$$

$$Ma_{CM} = m_A a_A + m_B a_B = \overrightarrow{F_{res}}$$

$$\overrightarrow{\boldsymbol{F_{res}}} = \boldsymbol{M}\overrightarrow{\boldsymbol{a_{CM}}}$$ *Segunda lei de Newton para o CM de um sistema!*

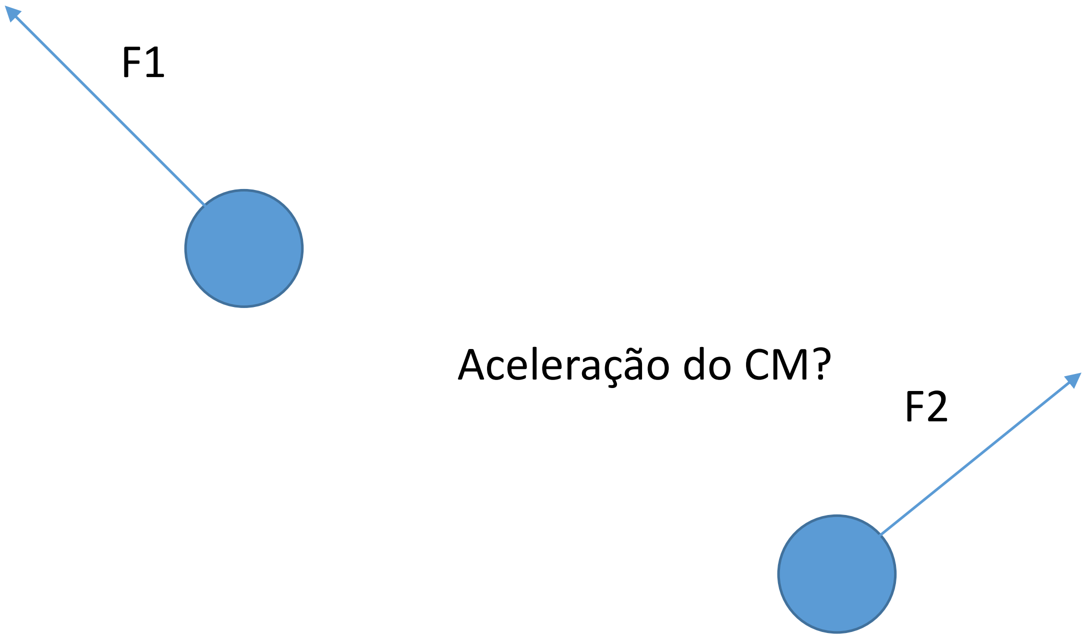
Movimento de um Sistema de Partículas

$$\overrightarrow{\boldsymbol{F_{res}}} = \boldsymbol{M}\overrightarrow{\boldsymbol{a_{CM}}}$$

- Força resultante das forças EXTERNAS que agem sobre o sistema. A força que um corpo faz no outro não é considerada;
- M é a massa total do sistema;
- A aceleração é do CM e não diz sobre a aceleração individual.

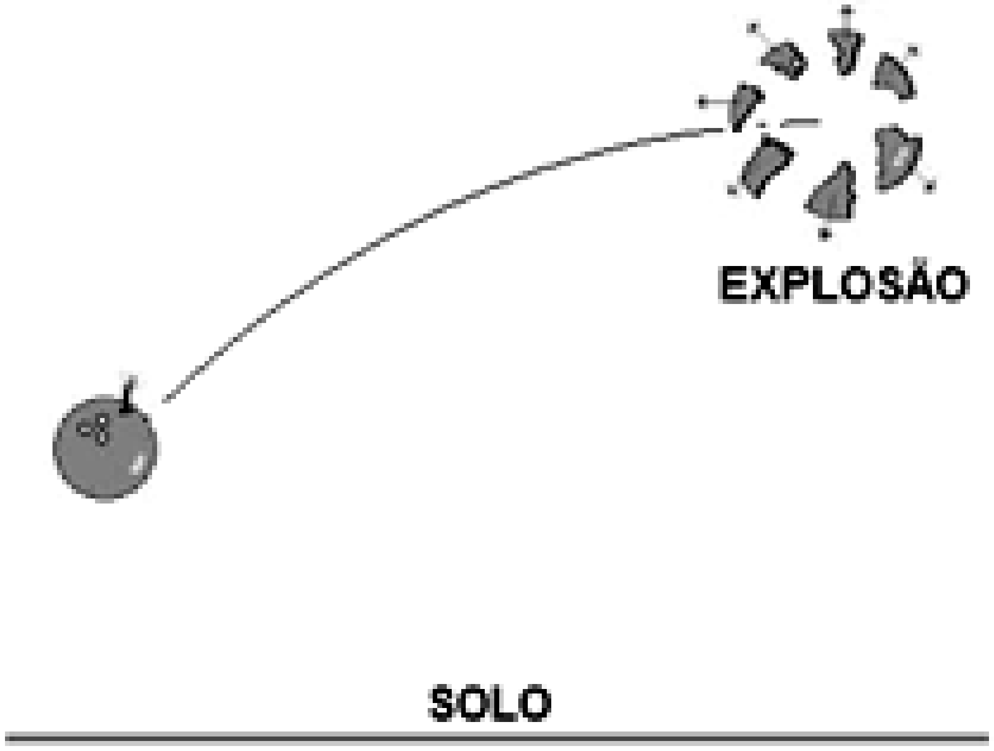
Movimento de um Sistema de Partículas

$$\overrightarrow{\boldsymbol{F_{res}}} = \boldsymbol{M}\overrightarrow{\boldsymbol{a_{CM}}}$$

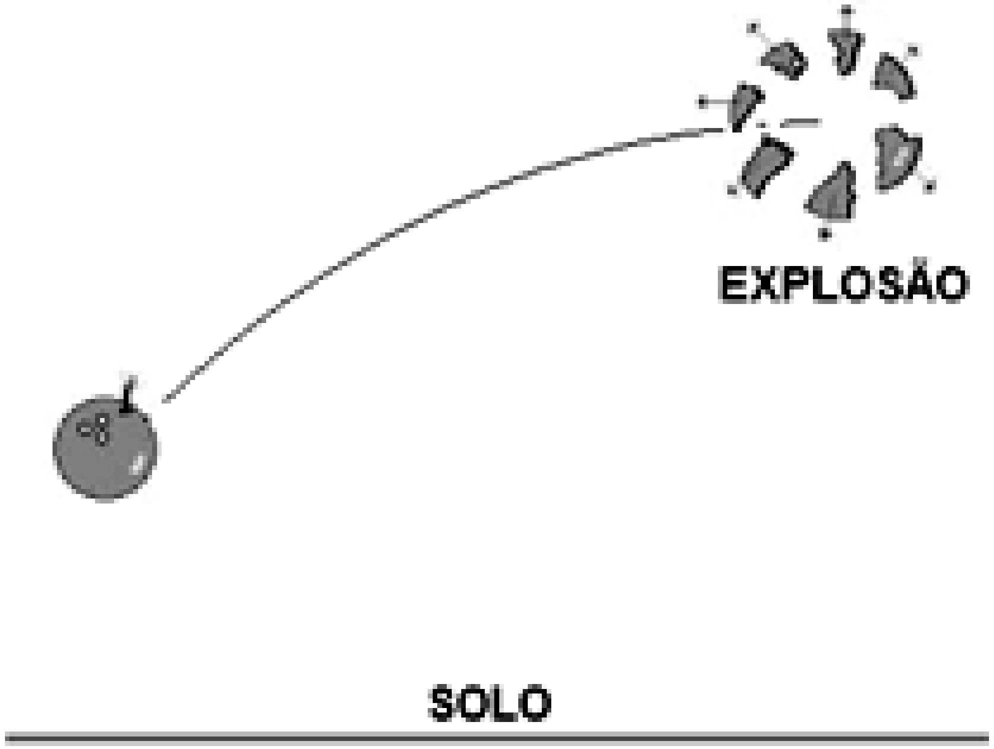


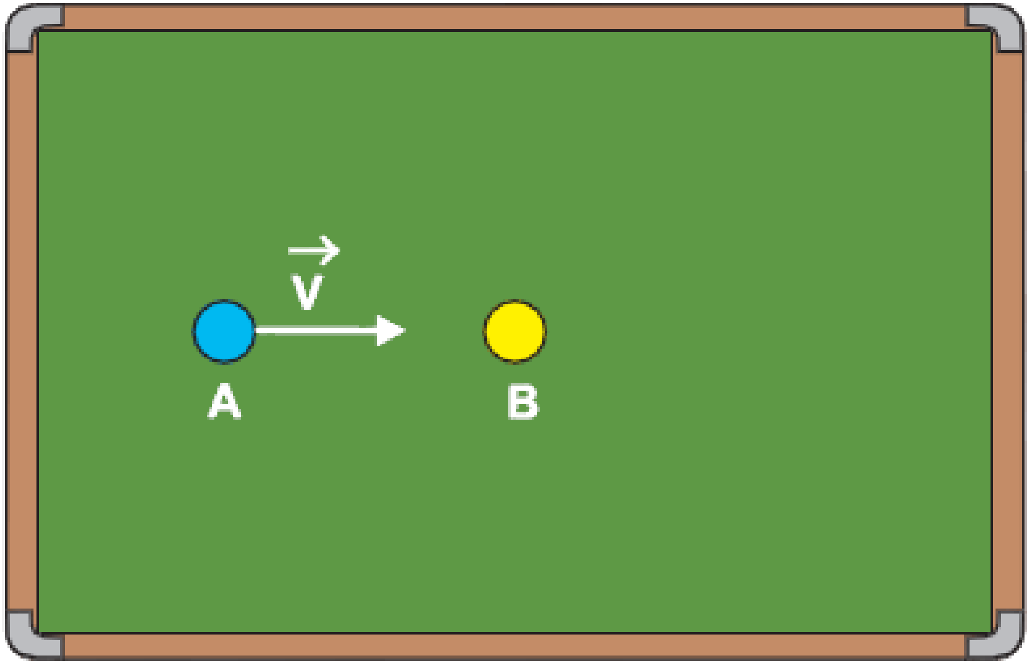
Aceleração do CM?

Movimento de um Sistema de Partículas



Movimento de um Sistema de Partículas





Momento Linear

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Neste momento a
Branca de Neve só
está com 6 anões.
O Atchim acabou de
ser posto em
quarentena

Momento Linear

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$$\vec{\boldsymbol{p}} = \boldsymbol{m}\vec{\boldsymbol{v}}$$

Mesma orientação

Momento Linear

$$\vec{\boldsymbol{p}} = \boldsymbol{m}\vec{\boldsymbol{v}}$$

$$[\vec{\boldsymbol{p}}] = [\boldsymbol{m}].[\vec{\boldsymbol{v}}] = \boldsymbol{kg}.\boldsymbol{m}/\boldsymbol{s}$$

Momento Linear – Segunda Lei de Newton

A taxa de variação com o tempo do momento angular de uma partícula = força resultante

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$$\vec{F} = \frac{d\vec{p}}{dt}$$

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$$\vec{F} = \frac{d\vec{p}}{dt} \rightarrow \vec{F} = \frac{d(m\vec{v})}{dt}$$

Para massa constante →

Momento Linear – Segunda Lei de Newton

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$$\boldsymbol{\vec{F}} = \frac{\boldsymbol{d\vec{p}}}{\boldsymbol{dt}} \rightarrow \boldsymbol{\vec{F}} = \frac{\boldsymbol{d(m\vec{v})}}{\boldsymbol{dt}}$$

Para massa constante →

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Para massa constante →

$$\vec{F} = m\frac{d\vec{v}}{dt}$$

$$\vec{F} = m\vec{a}$$

Momento Linear – Segunda Lei de Newton

De modo geral:

$$\vec{F} = \frac{d\vec{p}}{dt} \rightarrow \vec{F} = \frac{d(m\vec{v})}{dt}$$

Momento Linear - Sistema de Partículas

$$\vec{\boldsymbol{P}} = \overrightarrow{\boldsymbol{p_1}} + \overrightarrow{\boldsymbol{p_2}} + \cdots + \overrightarrow{\boldsymbol{p_n}}$$

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Relembrando...

$$Mx_{CM} = m_1x_1 + m_2x_2$$

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$$\overrightarrow{F_{Res}} = \frac{d\vec{P}}{dt}$$

Exemplo A

Um projétil é disparado sobre um campo horizontal, com velocidade inicial de 24,5 m/s, sob um ângulo de 36,9°. No ponto mais elevado da trajetória, o projétil explode e se divide em **dois fragmentos de massas iguais**. Um deles cai na vertical até o solo. Em que ponto o outro fragmento atinge o solo?