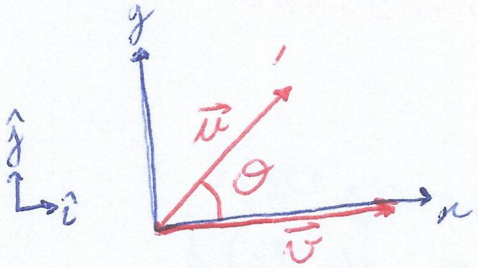
TRABALHO E ENERGIA (AULA 7 - 30/04/2020)

→ PRODUTO DE VETORES

→ PRODUTO ESCALAR (INTERNO)

$$\vec{u} \cdot \vec{v} = u_x \cdot v_x + u_y v_y + u_z v_z$$

DADOS DOIS VETORES:



$$\vec{u} = |u| \cos\theta \, \hat{i} + |u| \operatorname{sen}\theta \, \hat{j}$$

$$\vec{v} = |v| \hat{i} + 0$$

$$\therefore \boxed{\vec{u} \cdot \vec{v} = |u| \cdot |v| \cdot \cos\theta}$$

→ PRODUTO ESCALAR PODE SER CONSIDERADO COMO O PRODUTO ENTRE O MÓDULO DE UM VETOR E A COMPONENTE DE OUTRO VETOR NA DIREÇÃO DO PRIMEIRO.

$$\vec{u} \cdot \vec{u} = u_x^2 + u_y^2 + u_z^2 = |u|^2$$

$$\therefore |\vec{u}| = \sqrt{\vec{u} \cdot \vec{u}}$$

→ PROPRIEDADES:

I) $\vec{u} \cdot \vec{v} = \vec{v} \cdot \vec{u}$

II) $\vec{u} \cdot (\vec{v} + \vec{w}) = \vec{u} \cdot \vec{v} + \vec{u} \cdot \vec{w}$

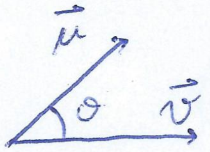
III) $m\vec{u} \cdot (\vec{v}) = m(\vec{u} \cdot \vec{v}) = \vec{u} \cdot (m\vec{v})$

IV) $\vec{u} \cdot \vec{0} = \vec{0}$

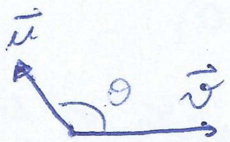
V) $|\vec{u}|^2 = \vec{u} \cdot \vec{u}$

OBSERVAÇÕES:

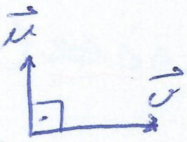
$$\vec{u}\cdot\vec{v} = |\vec{u}||\vec{v}|\cos\theta$$



$\vec{u}\cdot\vec{v} > 0$



$\vec{u}\cdot\vec{v} < 0$



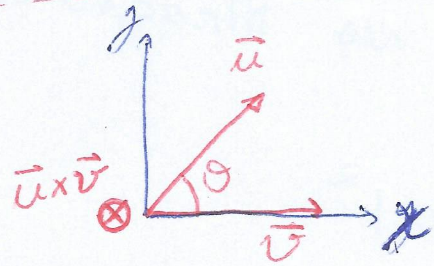
$\vec{u}\cdot\vec{v} = 0$

$\Downarrow$

CONDIÇÃO DE ORTOGONALIDADE

→ PRODUTO VETORIAL

$$\vec{u}\times\vec{v} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ u_x & u_y & u_z \\ v_x & v_y & v_z \end{vmatrix} = (u_y v_z - u_z v_y)\hat{\imath} - (u_x v_z - u_z v_x)\hat{\jmath} + (u_x v_y - u_y v_x)\hat{k}$$



$$\Rightarrow \quad \vec{u}\times\vec{v} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ u_x & u_y & 0 \\ v_x & 0 & 0 \end{vmatrix} = 0\hat{\imath} + 0\hat{\jmath} + (-u_y v_x)\hat{k}$$

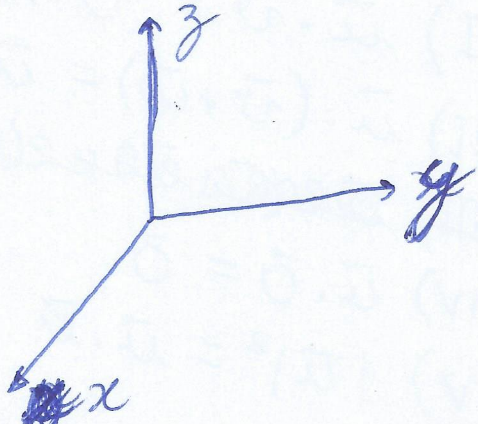
$$u_y = |\vec{u}|\,\text{sen}\,\theta$$
$$v_x = |\vec{v}|$$

$$\therefore \boxed{|\vec{u}\times\vec{v}| = |\vec{u}||\vec{v}|\,\text{sen}\,\theta}$$

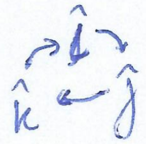
$$\vec{u}\times\vec{v} = |\vec{u}||\vec{v}|\,\text{sen}\,\theta\,(-\hat{k})$$

DEFINIÇÃO DOS VERSORES CANÔNICOS:

$$\hat{\imath}\times\hat{\jmath} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{vmatrix} = \hat{k}$$



$$\hat{\jmath}\times\hat{k} = \hat{\imath}$$



$$\hat{k}\times\hat{\jmath} = \hat{\imath}$$

→ PROPRIEDADES

I) $\vec{u} \times \vec{u} = 0$

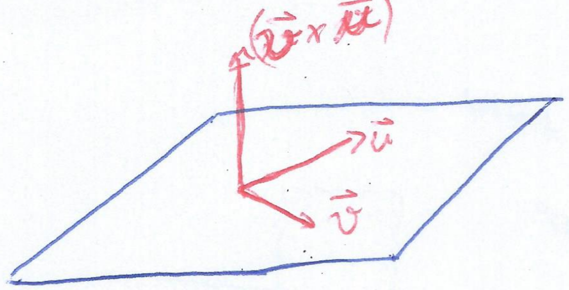
II) $\vec{u} \times \vec{v} = -\vec{v} \times \vec{u}$

III) $\vec{u} \times (\vec{v} + \vec{w}) = \vec{u} \times \vec{v} + \vec{u} \times \vec{w}$

IV) $((m\vec{u}) \times \vec{v}) = m(\vec{u} \times \vec{v}) = \vec{u} \times (m\vec{v})$

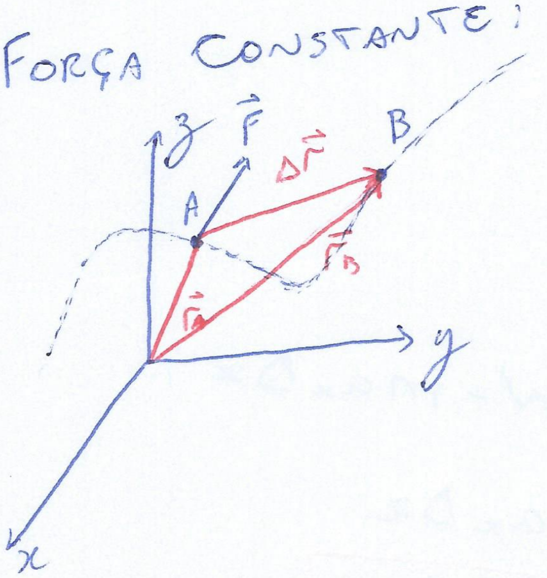
V) $\vec{u} \times \vec{v} = 0$ SE $\vec{u}$ E $\vec{v}$ SÃO COLINEARES.

IV) $\vec{u} \times \vec{v} \perp \vec{u}, \vec{v}$



→ TRABALHO:

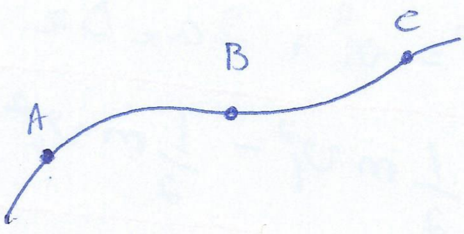
- FORÇA CONSTANTE:



$$\boxed{W_{A\to B} = \vec{F} \cdot \Delta\vec{r}}$$

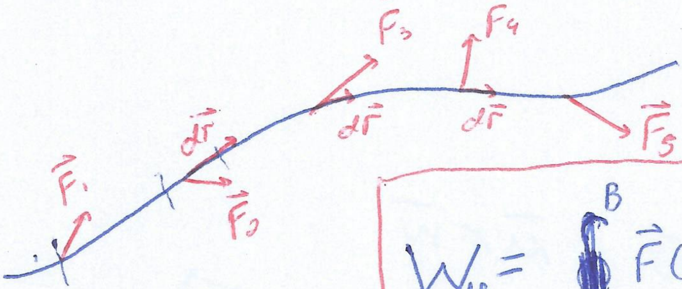
DEFINIÇÃO DE TRABALHO PARA $\vec{F}$ CONSTANTE.

- TRABALHO É CUMULATIVO

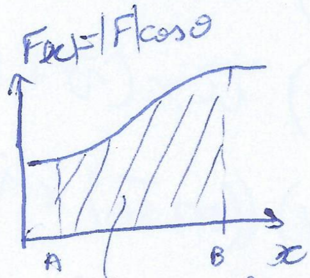


$$\boxed{W_{A\to C} = W_{A\to B} + W_{B\to C}}$$

→ FORÇA VARIANDO NO TEMPO:



$$\boxed{W_{AB} = \int_A^B \vec{F}(\vec{r}) \cdot d\vec{r}}$$ → CASO GERAL

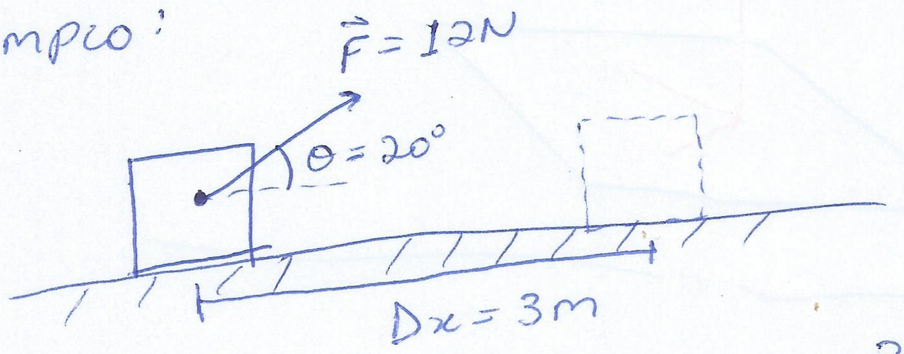


$\int_A^B$

ÁREA $= \int_A^B F_x \, dx$.

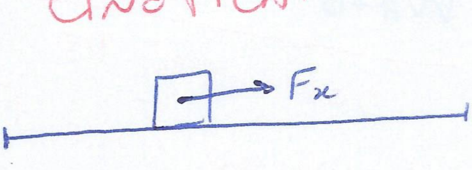
UNIDADE: N.m = J (JOULE)

EXEMPLO:



$\therefore W = \vec{F} \cdot \Delta\vec{x} = F \Delta x \cos\theta = \boxed{33,8 \, J}$

→ ENERGIA CINÉTICA:



$W = F_x . \Delta x$

MAS $F_x = m a_x \Rightarrow W = m a_x \Delta x$

DE TORRICELLI: $v_f^2 = v_i^2 + 2 a_x \Delta x$

$$\therefore \boxed{W_{TOTAL} = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2}$$

$$\boxed{K = \frac{1}{2} m v^2} \Rightarrow \text{ENERGIA CINÉTICA}$$