Exercises - Date: 29/04/20

1) Consider the individual observations of ~~x~~ the random variable X, taken from a process with mean $\mu_0 = 100$. and s.d $\sigma = 1$.

i	X_i	i		i		i	
1	100.23	6	99.86	11	99.73	16	100.86
2	100.19	7	99.6	12	98.00	17	100.21
3	102.02	8	100.35	13	101.39	18	101.85
4	99.59	9	99.38	14	101.64	19	101.43
5	99.81	10	100.83	15	101.41	20	100.82

a) Build a Shewhart control chart to monitor these individual observations. Is the process in-control? Use $\alpha = 0.0027$

b) Build the cusum chart ~~and~~ for these individual observation. Use as a decision rule: if $S_i > 5$, the process is out-of-control, with $S_i = \sum_{j=1}^{i} (X_j - \mu_0)$. Is the process in-control?

c) Build an EWMA chart for this data set. Use the constant $\lambda = 0.2$, ~~and~~ $k = 3$ and

$$Var(Y_i) \approx \sigma^2 \left(\frac{\lambda}{2-\lambda}\right)\left[1 - (1-\lambda)^{2i}\right], \text{ being}$$

$Y_i = \lambda X_i + (1-\lambda) Y_{i-1}$, starting $Y_0 = \mu_0$ to set the control limits. That is:

$\mu_0 \pm 3\sqrt{Var(Y_i)}$. Is the process in control.

d) Compare the 3 control charts and write some comments.

2) Consider that when the process is in-control, their parameters are: $\mu_0 = 5.0$ and $\sigma_0 = 5$.

a) Determine the control limits for $\bar{X}$ chart considering sample sizes of 5 units. Use $\alpha = 0.0027$.

b) If the process mean shifts to $\mu_1 = 7.5$, what is the probability to signal such change at the first sample after the shift? And to signal before the fourth sample?

c) Obtain the control limits for S^2 control chart. Use $\alpha = 0.0027$.

d) If σ^2 shifts to $\sigma_1 = 6$, what is the probability of S^2 chart signals at the first sample after the shift.

e) If σ shifts to $\sigma_1 = 6$, what is the probability the $\bar{X}$ chart triggers a signal at the first sample after the change?

f) ~~B~~ In item e) if σ shifts to $\sigma_1 = 6$ and the mean to $\mu_1 = 6.00$, what is the probability the $\bar{X}$ charts signals at the first sample after the change.

g) Now consider the two control charts to monitor jointly mean and s.d. Please re-state the control limits to attend an overall type I error $\alpha_{all} = 0.0027$.

h) What is the probability to detect the change $\sigma_1 = 6$ using both control charts at the first sample after the shift.

i) If μ shifts to $\mu_1 = 6$ and $\sigma_1 = 6$, using both charts, what is the probability to signal these shifts at the first sample after the shifts.

3) Consider the capability index expressed as $C_p = \dfrac{USL - LSL}{6\sigma}$, USL and LSL are respectively the upper and lower specification limits. Thus if $C_p = 1 \Rightarrow USL - LSL = 6\sigma$. This means that the range of the specification limits lays in a range of six standard deviation. Using what you learned from our classes, propose a control chart to monitor this index meets the target value of one? Use $\alpha = 0.0027$. What you suggest?

4) Conside o processo de enchimento de leite de longa vida, com $\mu_0 = 1000$ ml e $\sigma_0 = 10$ ml. Os engenheiros definiram como limites de especificação: [985; 1015] ml.

Determinem as % de não conformidades e os poderes do gráfico de $\bar{X}$ construído com uma amostra de tamanho $n = 5$, quando μ_0 muda para $\mu_1 = 990;\ 995;\ 1005,\ 1010$.

5) Um pesquisador quer avaliar o desempenho do gráfico de controle X considerando várias intervalos de inspeção. Deseja que o tempo médio entre os falsos alarmes não seja inferior a 600 horas e dispõem recursos de inspecionar no máximo 24 unidades por hora. Obter o tempo esperado até o sinal considerando as seguintes alternativas:

h = 2 horas; 1 hora; 30 min, 15 min; 20 min, 40 min

e p/ $\mu_1 = \mu_0 + \delta\sigma$ com $\delta = 0{,}25;\ 0{,}5;\ 1;\ 1{,}25;\ 1.5$ e 2.

Discutem qual(is) melhores políticas para este caso.