Dynamical Integrity in Structural and Nonlinear Dynamics

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Remind of some theoretical issues

Continuous dynamical systems (1)

a continuous dynamical system is a time-differential equation

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), t), \quad \mathbf{x}(t_0) = \mathbf{x}_0$$

and practically is defined by the function

$$\mathbf{f}(\mathbf{x}, t) : \Omega \times I \to \Omega$$

where $I \subset \mathrm{IR}$***. Under mild conditions (*** $\mathbf{f}$ ***lipschitzian) the solution exists for all*** $t \in [t_0, \infty[\subset I$

$\Omega \times I$ ***is the phase space***

$(\mathbf{x}_0, t_0) \in \Omega \times I$ ***are the initial conditions***

Continuous dynamical systems (2)

if $\mathbf{f}(\mathbf{x})$ ***does not depend on time, the system is autonomous, the phase space is*** Ω ***and the initial condition is*** $\mathbf{x}_0$

if $\mathbf{x}\in\Omega\subset\mathrm{IR}^N$ ***is an*** N-***vector, the system is*** N-***dimensional***

if $\mathbf{x}\in\Omega\subset$functional space ***is a function, the system is infinito-dimensional and the governing equation is a partial differential equation***

Discrete dynamical systems

a discrete dynamical system is a map

$$\mathbf{x}_{n+1} = \mathbf{g}(\mathbf{x}_n)$$

and practically is defined by the function

$$\mathbf{g}(\mathbf{x}) : \Omega \to \Omega$$

the solution is always defined in (discrete) time

Ω ***is the phase space***

$\mathbf{x}_0$ ***is the initial condition***

Stability

a solution is stable if solutions starting close to it remain close in time (rough definition)

if neighbouring solutions converge to it, then it is an attractor (rough definition) and it has its own basin of attraction (the set of all initial conditions leading to the attractor)

example of attractors are equilibrium, periodic, quasi-periodic and chaotic solutions

Basin (1)

basin, or 'safe basin':

a set of initial conditions sharing some common properties

it is a subset of the phase space $\Omega \times I$

what property? Whatever:

- same steady state behaviour (→ same attractor)

- never escaping from a potential well

- never reaching a given displacement threshold

- …

Basin (2)

we will frequently (but not always) consider 'basin of attraction', i.e. the common property is the attractor

sometimes the basin will be the union of different basins of attraction

if $\mathbf{f}(\mathbf{x},t)=\mathbf{f}(\mathbf{x},t+T)$***, the system is*** T***-periodic and repeats itself every*** T***; the basin is then a subset of*** Ω ***for a fixed*** t_0

(in dynamical system language we are considering a stroboscopic Poincarè section of the flux)

What dynamical integrity is?
A phenomenological introduction

Again on stability (1)

stability is a basic and very important concept

it means that under small perturbations the system does not change the response substantially

since we live in an imperfect world, in practice we experience only stable solutions (persisting after perturbations which are everywhere!)

(but unstable solution are important from a theoretical point of view, be careful)

Again on stability (2)

***first key question*: how *small* have to be perturbations?**

from a mathematical point of view the magnitude of perturbations is not important (e.g.* 10^{-50} *is ok)

but from a practical point of view it is important, since in our real world imperfections have a finite magnitude

stability is not enough for practical applications

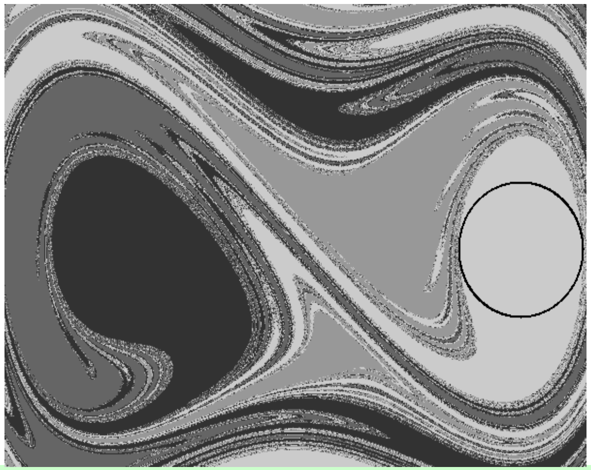
Dynamical integrity (1)

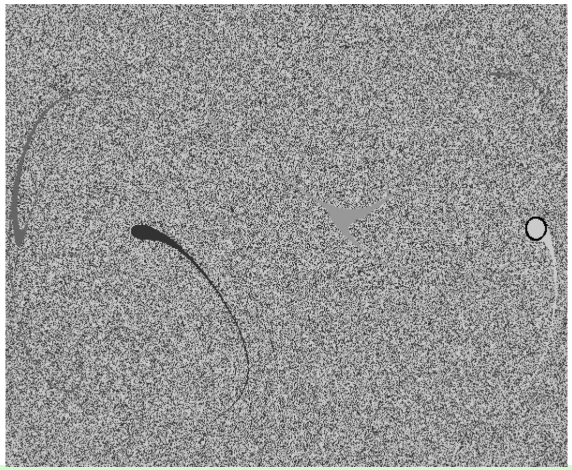
in practice we need robustness to sufficiently 'large' perturbations to use an attractor

but perturbations are different initial conditions
→ we need a 'large' basin of attraction

example: both are stable attractors, but with different basins of attraction

can be used in practice





unuseful in practice

Dynamical integrity (2)

we need to study the basin of attraction

the study of the topology of basins, of their evolution by varying parameters, etc., is the subject of dynamical integrity

extending the previous idea and motivation, dynamical integrity analyzes basins, not only basins of attraction

second key question: how to measure the dynamical integrity? (see later)

From local to global

- ***stability is a local property of the attractor***
- ***dynamical integrity is a global property***

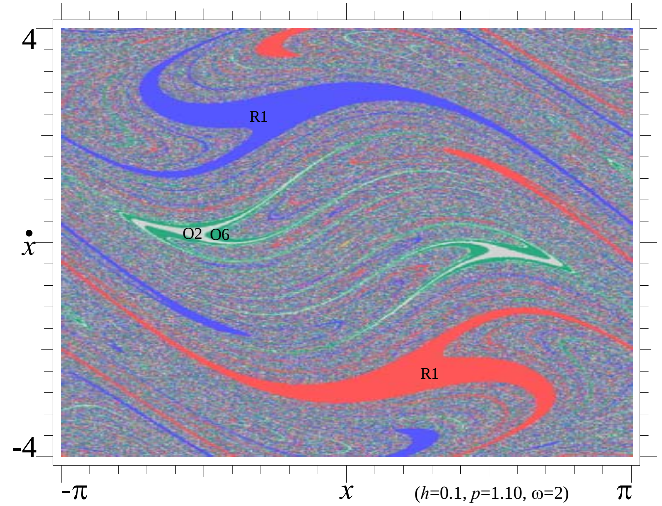
from local to global dynamics

more information, more knowledge of the system, more useful

but more difficult (e.g. heavy numerical simulations required)

Some considerations

since basins can be complex (even fractal), dynamical integrity is not the simple study of the magnitude of basins



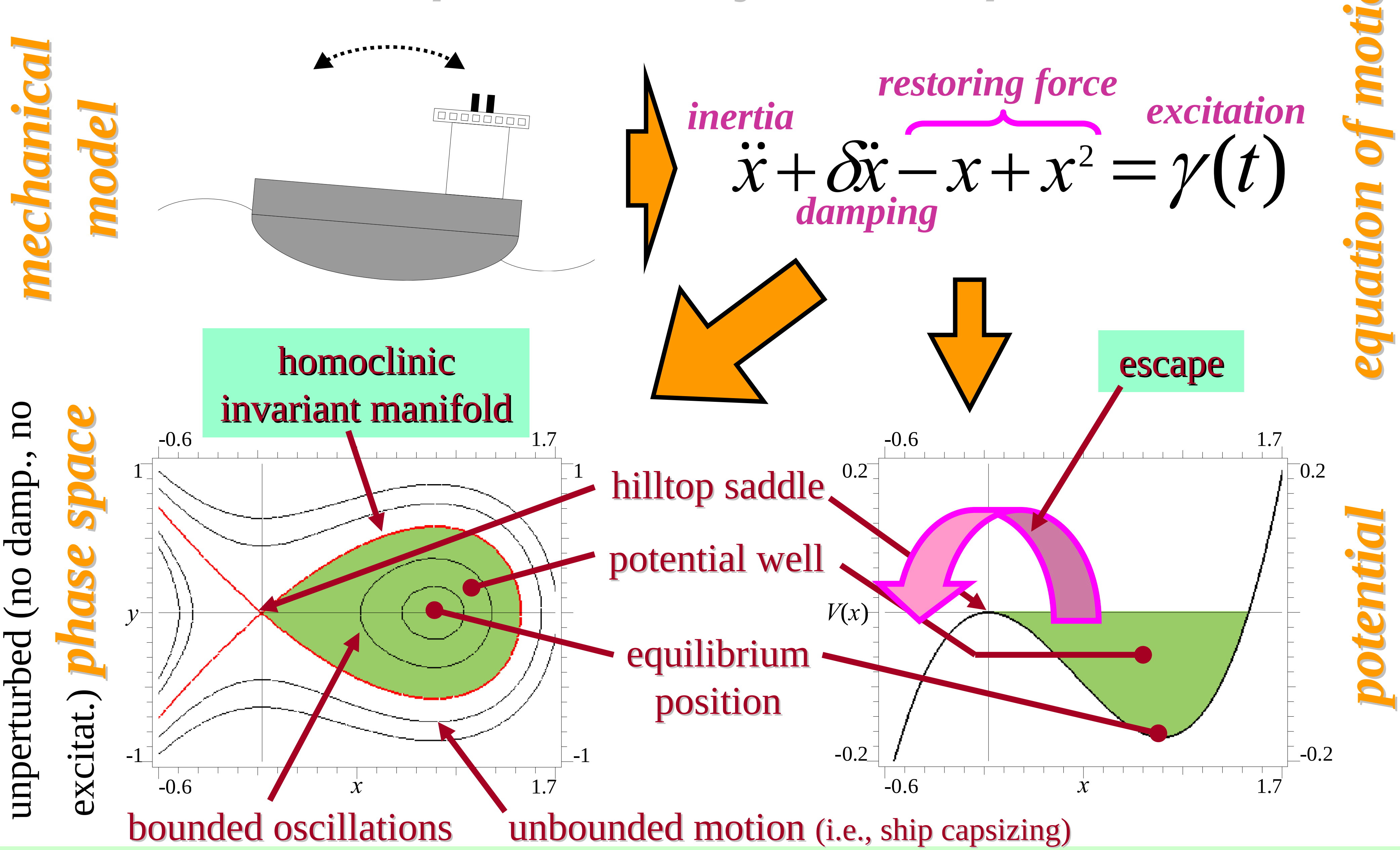
bad news for designer, good news for researchers!

let us start the study in more detail, also by introducing other useful tools

Dynamical systems background

Models (mech-math), phase space, potential

an overall picture of dynamical problems

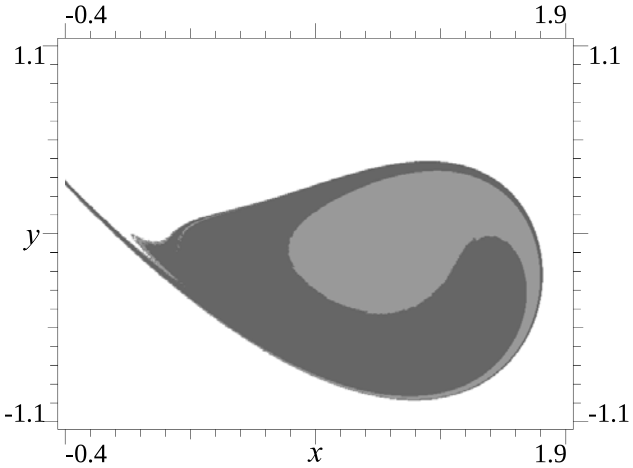


Main points for integrity

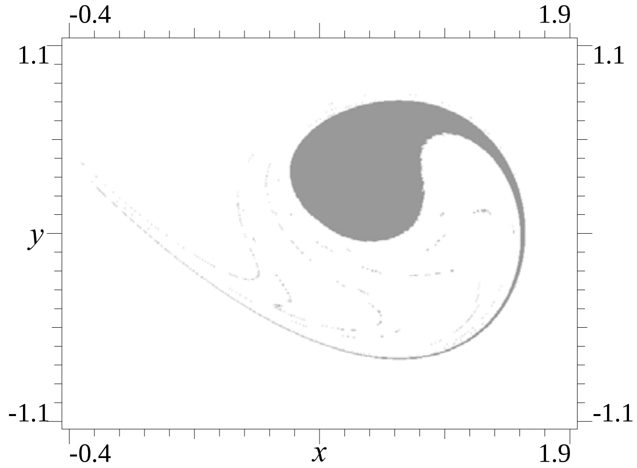
1) invariant manifolds

2) basin erosion*, *i.e., how a basin reduces (in magnitude, in shape, etc.) by varying a governing parameter

from



to



3) escape*, *or getting out of a potential well. It is inevitable at the end of the erosion

Relevance of invariant manifolds

Invariant manifolds ***"provide a useful stepping stone in the understanding of the overall system dynamics"*** [Katz & Dowell, 1994]

"…it is not an exaggeration to claim that in virtually every manifestation of chaotic behaviour known thus far, some type of homoclinic behaviour is lurking in the background…" [Kovacic & Wiggins, 1992]

- ***stable manifold (inset) are boundaries of basins of attractions (this is why they are so important for dynamical integrity)***

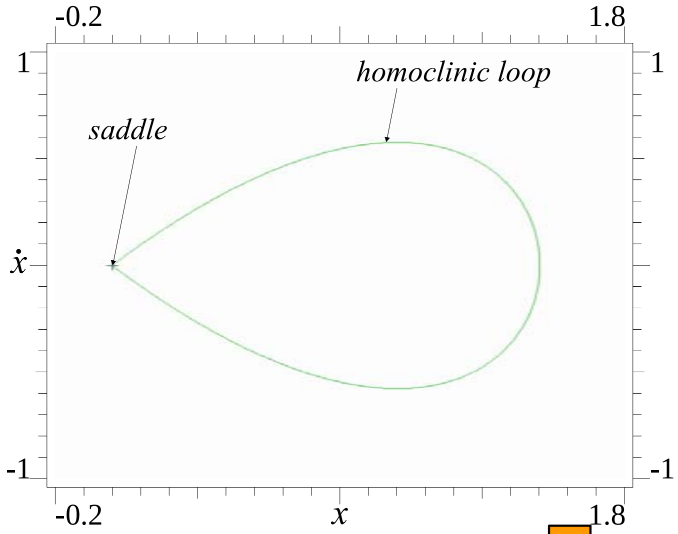
- ***skeleton of chaotic attractors***
- ***involved in many topological phenomena***

Stable and unstable manifolds (and their distance)

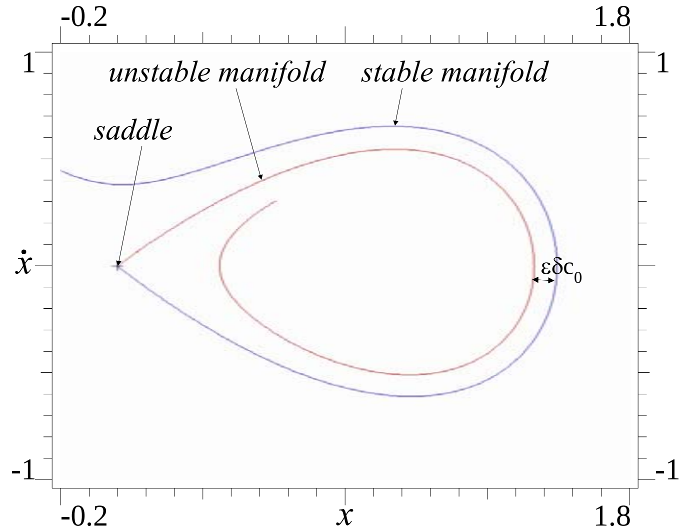
distance=(costant part) $\varepsilon\delta a_0$ +(oscillating part) $\varepsilon\gamma_1 \cos(\omega t)\, a_1(\omega)$

Helmholtz: $x'' + \varepsilon\delta x' - x + x^2 = \varepsilon\gamma_1 \underline{\sin(\omega t)}$

unperturbed

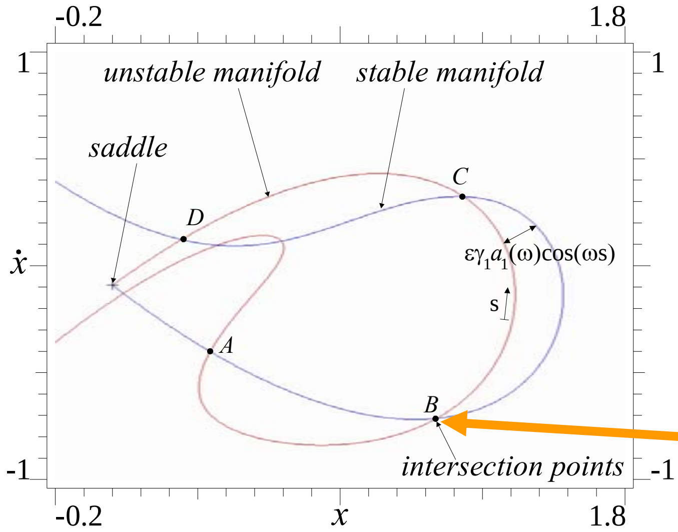


damping



Keep manifolds disjont

harmonic excitation

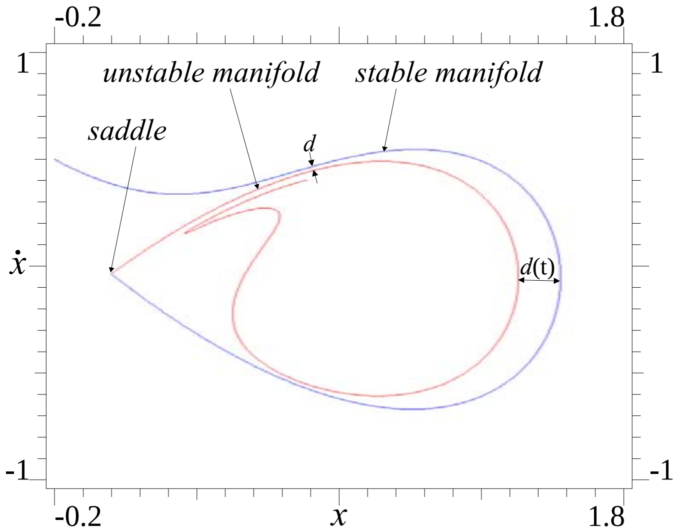


Enforce manifolds intersection

who wins?

homoclinic intersection, root of chaos and fractality

Manifold distance: harmonic excitation



$$d(t) = \varepsilon\delta a_0 + \varepsilon\gamma_1 \cos(\omega t)\, a_1(\omega)$$

The <u>structure</u> of the distance is system-independent

***The <u>coefficients</u> are system-dependent, and can be computed exactly** (piece-wise linear systems) **or approximately** (Melnikov)*

homoclinic bifurcation $\Leftrightarrow 0 = d = \min_t\{d(t)\} = \varepsilon\delta a_0 - \varepsilon\gamma_1 a_1(\omega)$

$$\gamma_{1,cr}^{h} = \delta \frac{a_0}{a_1(\omega)}$$

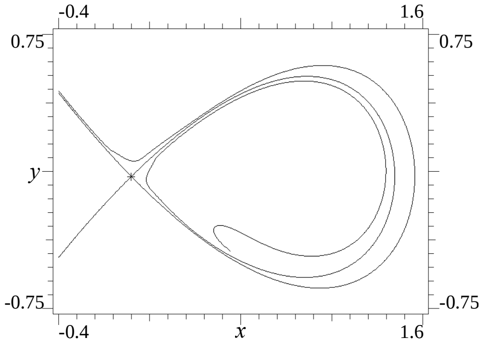
homoclinic bifurcation threshold

Relevance of homo/heteroclinic bifurcations

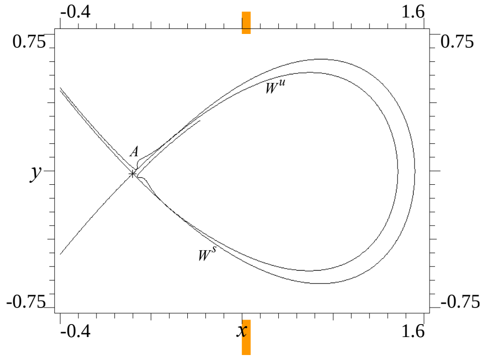
***Homo/heteroclinic bifurcations* of selected saddles are the mechanisms responsible for:**

- ***starting of fractalization of basin boundaries and sensitivity to initial conditions***
- ***appearance/disappearance of chaotic attractors or their sudden enlargement/reduction***
- ***triggering phenomena of basins erosion suddenly leading to out-of-well dynamics:***
  - ***transition from single-well to cross-well chaos in multi-well systems***
  - ***escape from potential well in single-well systems***

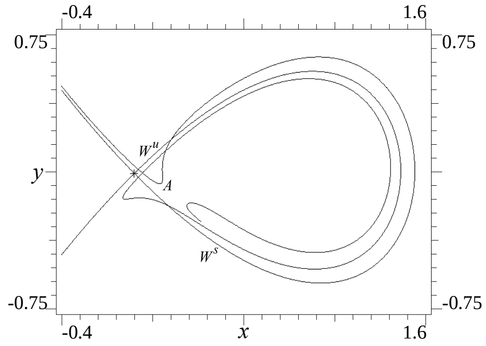
Homoclinic bifurcation, basins of attraction



detached manifolds



manifolds tangency

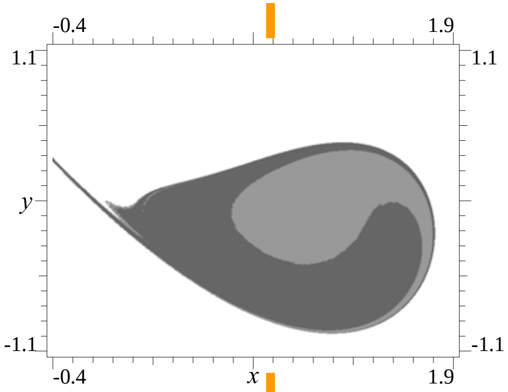


manifolds intersection (tangle)

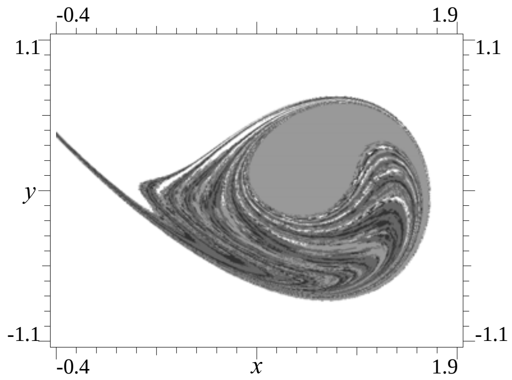
homoclinic bifurcation

***varying one parameter** (e.g., increasing excitation amplitude)*

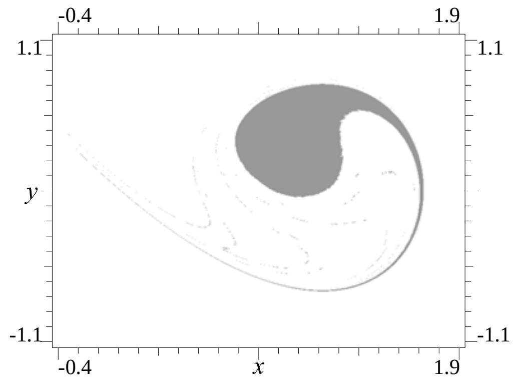
associated basins of attraction erosion



uneroded basin



erosion starts



toward complete erosion and inevitable escape

example of erosion

Escape from a potential well

- ***effects of overcoming a potential hill:***

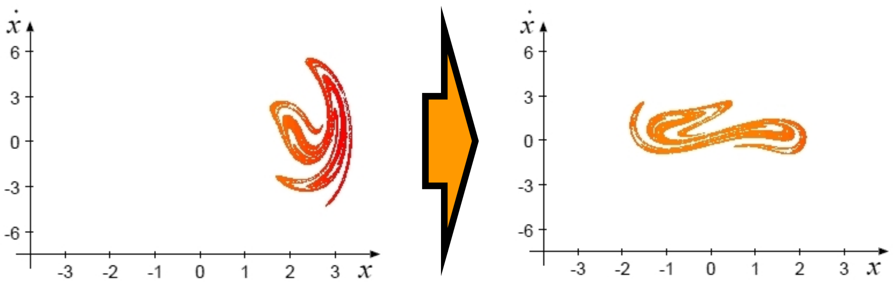
increasing dangerousness

- ***scattered periodic motions***
- ***scattered chaotic motions***
- ***unbounded motions***

- ***destroying the structure by fatigue***
- ***failure of the structure***

dynamical effects

practical effects



capsizing

overturning

Dynamical integrity

Background: attractors and basins

eroded basins is a critical state for the structure possibly corresponding to its incipient failure

using the structure only when erosion is totally prevented is too conservative if the erosion is not sharp

attractors must be paralleled by uncorrupted basins for safe practical applications

necessity of a detailed investigation of the safe basin integrity/erosion (and of its possible control)

(pioneering: J.M.T. Thompson and co-workers)

Main lines of analysis of dynamical integrity

- ***considering the definition of "safe basin"***
  (transient vs steady dynamics, fractality of the basin can be accepted or must be prevented, etc.)
- ***measuring the integrity of the safe basins***
  (crucial for quantitatively assessing structural reliability)
- ***investigating basins evolution due to variation of system parameters ⇨ "erosion profiles", which are of great practical interest***
- ***(erosion of safe basins is unwanted ⇨ necessity of its reduction: applying a control method side by side with a non-controlled, reference, case; important, but out of the scopes of this talk)***

Issues in the definition of safe basins (1)

- ***hardening vs softening systems***
  - ***different out-of-well phenomena after erosion (cross-well motion, escape, overturning, etc.)***
  - ***hardening systems:***
    - → ***erosion due to interpenetration of basins from adiacent wells***
    - → ***basins do not change in magnitude but become tangled***
    - → ***erosion does not usually entail immediate unwanted events***
  - ***softening systems:***
    - → ***erosion is owed to the penetration of the “infinity” attractor***
    - → ***basins reduce in magnitude***
    - → ***erosion is dangerous from a practical point of view, because it leads to immediate failure of the system (e.g., ship capsizing)***

Issues in the definition of safe basins (2)

- ***basin boundaries being invariant manifolds or not***
  - ***theoretical interest***
  - ***explanation of erosion in terms of global bifurcations or other "classical" dynamical events***
- ***fractality vs compactness of the safe basin***
  - ***degree of fractality is important because linked to S.I.C.***
  - ***during erosion, the basin magnitude can remain unchanged but becomes tangled, forewarning the boundary crisis triggering the out-of-well phenomenon***
  - ***for fractal basins it makes more sense to refer to the compact 'core' of the basin surrounding in-well attractors***

Issues in the definition of safe basins (3)

- ***transient vs steady dynamics***
  - ***the correct framework is suggested by the considered mechanical system** (e.g., in some cases, temporary escape from the potential well may be unessential, while it must be avoided in other situations)*
  - ***in short excitation problems** (impacts, seismic loads, etc.) **transient dynamics is important, while with stationary excitations steady state dynamics are of major interest***
  - ***minor importance if the transient is short** (e.g., highly damped systems) **while crucial when transient is long***
- ***independence of the excitation phase***
  - ***recently highlighted (overturning of rigid blocks)***
  - ***need of phase-independent arguments***

Safe basin of attraction

- ***two possible definitions:***

(1) "the set of all i.c. approaching bounded attractors belonging to a given potential well as $t \to \infty$"

→ ***∪ of classical basins of attraction of all attr.s of a given potential well***
→ ***most intuitive and simple***
→ ***ignores transient dynamics***

(2) "..precluding any i.c. which leads to…an attractor or transient which spans both wells...the set of remaining i.c. that lead to steady state motion confined to one well…is the safe basin of attraction" [Thompson]

→ ***eliminates from previous defin. the i.c. leading to transient out-of-well***

in both cases the safe basin is a property of the potential well and not of the attractors

Comparison of the two definitions

- ***it is the mechanical application which usually suggests whether the transient is relevant for the integrity or not ⇨ this automatically provides the right definition of safe basin***

- ***complementary elements of comparison:***
  - ***safe basins (1) are bounded by invariant manifolds** (→ can be studied in terms of dynamical systems theory)**, and can be computed by standard numerical techniques***
  - ***safe basins (2) require time consuming ad-hoc algorithms** (due to on-line continuous check on the state of the system)** and needs care in defining the boundary of potential well in the dynamical case***
  - ***safe basins (2)** $\subset$ **safe basins (1)***

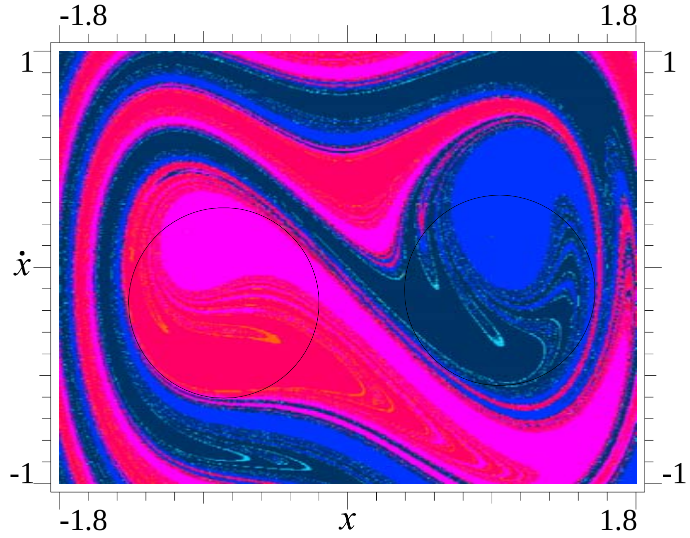
Quantitative measures of integrity (1)

***second key question*: how to <u>measure</u> the basin integrity?**

- ***Global Integrity Measure (GIM): the normalized magnitude (area in 2D) of the safe basin***
  - ***most intuitive, but does not take care of the fractality of the safe basin, and can be unuseful in practice***
- ***Local Integrity Measure (LIM): the normalized attractor-basin boundary minimum distance***
  - ***rules out the fractality of the basins and focuses on the compact part of safe basin surrounding the attractor***
  - ***property of the attractor and not of the potential well***
  - ***numerically onerous (especially with chaotic attractors)***

Quantitative measures of integrity (2)

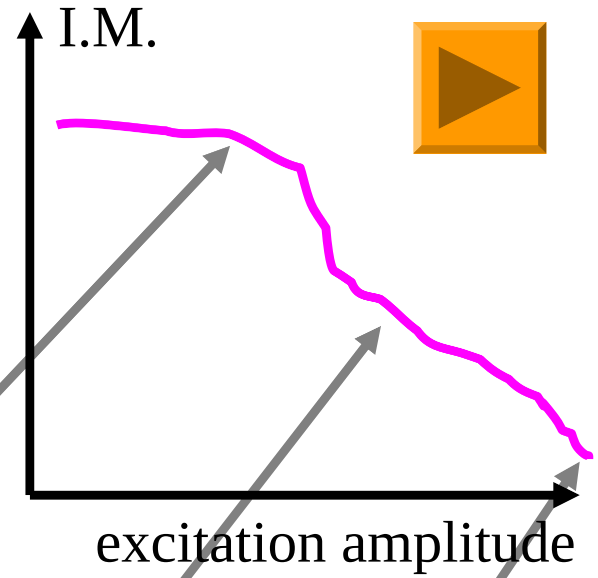
- ***Integrity Factor (IF): normalized radius of the largest sphere entirely belonging to safe basin***
  - ***computationally easy***
  - ***measure of the compact part***
  - ***elimination of fractal tongues from integrity evaluation***



The LIM and IF do not have a clear theoretical background permitting an in-depth investigation (although they are certainly somehow linked to classical dynamical phenomena). Both can be determined only numerically, with a markedly different effort in the two cases

Erosion profiles

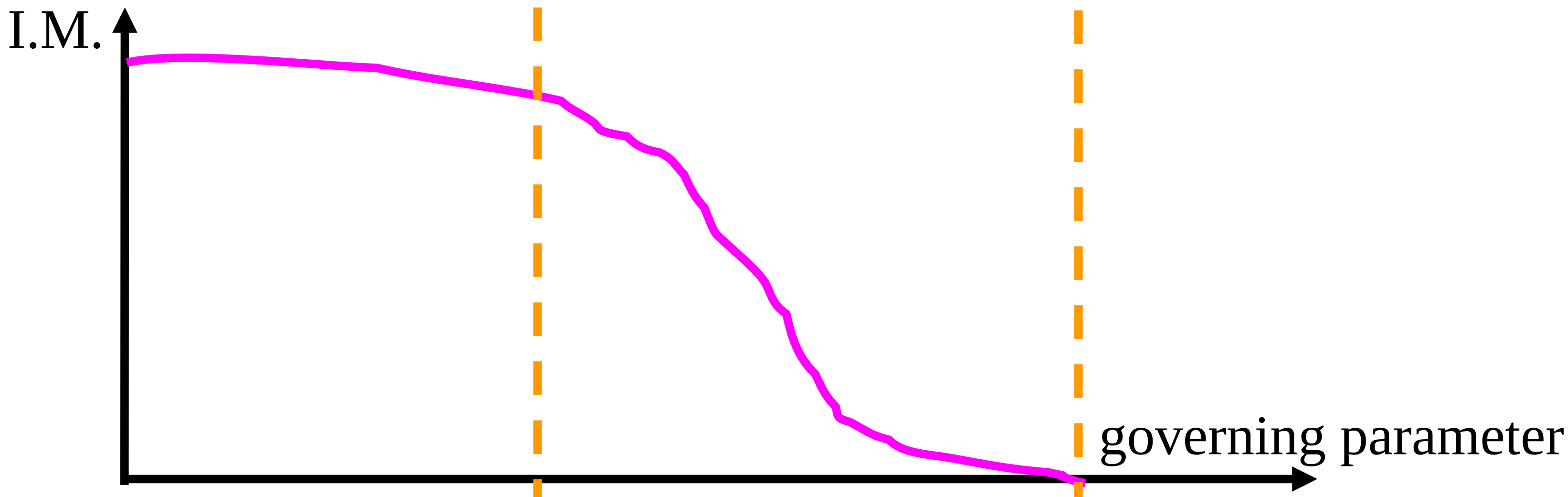
- ***integrity measures permit to study how the structure reliability changes when parameters vary***
- ***erosion profiles: integrity measure as a function of excitation amplitude***
- ***irrespective of safe basin definitions, exact/approximate information from:***



  - ***homo/heteroclinic bifurcation of the manifolds surrounding the potential well which triggers the erosion***
  - ***then erosion proceeds with complex mechanisms, which may involve secondary homo/heteroclinic bifurcations***
  - ***erosion ends with the onset of out-of-well phenomena which may represent the physical "failure"***

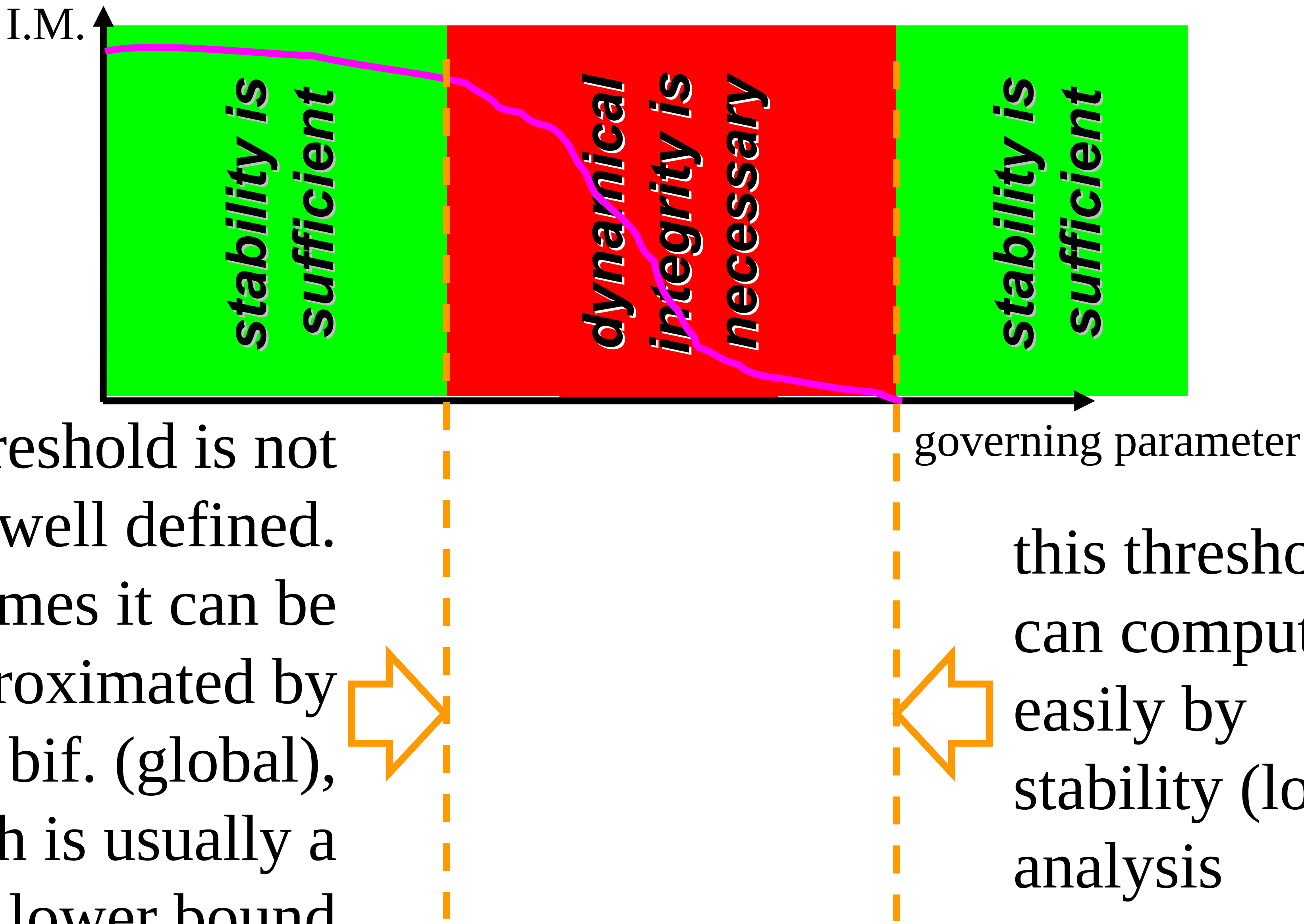
Stability vs dynamical integrity analyses (1)

- ***the end of the erosion corresponds to the disappearance of the attractor, i.e., to the loss of stability***



stable		unstable
uneroded	erosion	inevitable escape
safe:	***unsafe:***	***no attractor:***
stability detect it	stability miss it	stability detect it
dyn. int. detect it	dyn. int. detect it	dyn. int. detect it

Stability vs dynamical integrity analyses (2)



this threshold is not well defined. Sometimes it can be approximated by hom. bif. (global), which is usually a lower bound

this threshold can computed easily by stability (local) analysis

- ***more involved erosion profiles may occur***

System Integrity Scenario

system dynamical integrity

evaluated through the topological concept of

safe basin erosion

main global bifurcation event — triggers → safe basin erosion

safe basin definition / safe basin measure — prerequisites → safe basin erosion

safe basin erosion — leads to → **escape**

out-of-well phenomena

to infinity

- ship capsize
- rigid block overturning
- MEM sensors pull-in…..

to neighbouring wells

- cross-well chaos
- scattered periodic motion
- MEM switches pull-in…..

failure → avoiding escape

different dyn. → avoiding or realizing escape

Integrity of in-well dynamics

Helmholtz
Duffing
Rigid block
MEMS
Augusti's 2-d.o.f. model

Overall aims

Investigating the dynamical integrity of different nonlinear mechanical oscillators

(Helmholtz, rigid block, MEMS, pendula)

- ***showing practical examples of erosion profiles***
- ***discussing specific mechanical issues***
- ***discussing dynamical issues***

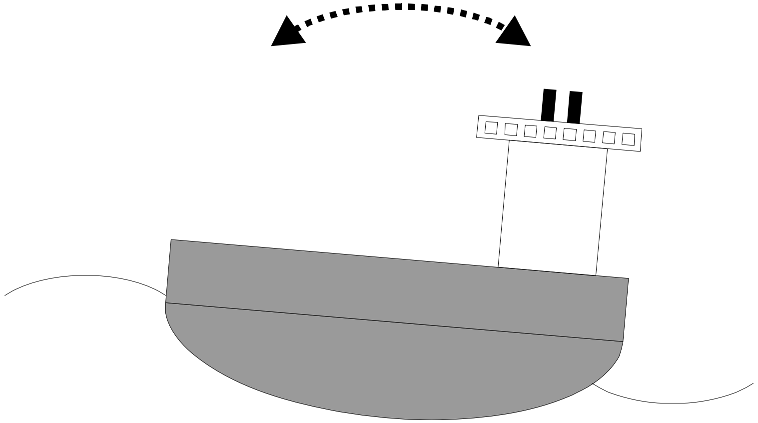
different systems ⇒ ***different dynamical phenomena***

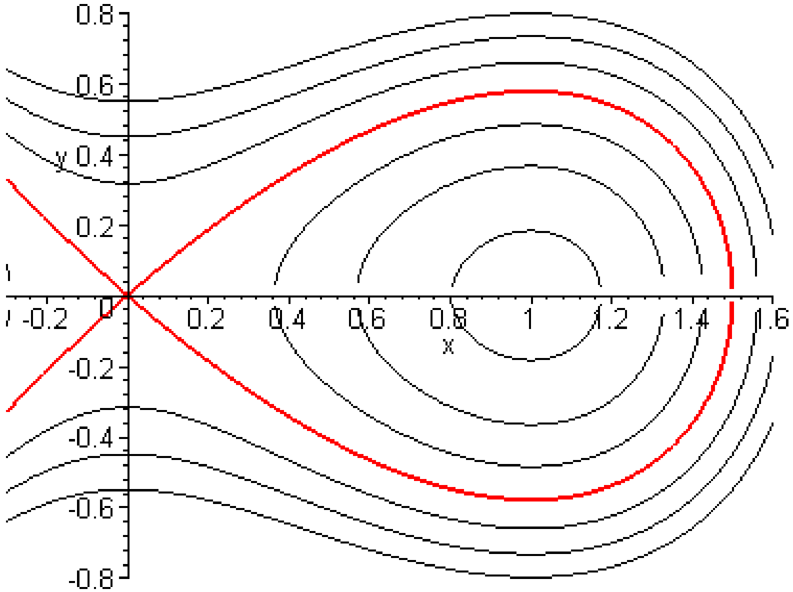
- ***safe basin (1), i.e., steady dynamics, always used***

Mechanical and dynamical issues

- ***hardening*** ***(Duffing)*** ***vs*** ***softening*** ***(Helmholtz, rigid block, MEMS)*** ***systems***
- ***symmetric*** ***(Duffing, rigid block)*** ***vs*** ***asymmetric*** ***(Helmholtz, MEMS)*** ***systems***
- ***smooth*** ***(Helmholtz, Duffing, MEMS)*** ***vs*** ***non-smooth*** ***(rigid block)*** ***systems***
- ***various "failure" phenomena: capsizing (Helmholtz), overturning (rigid block), pull-in (MEMS)***
- ***erosion of system without (rigid block) and with (Helmholtz, Duffing, MEMS) internal frequency***
- ***GIM vs IF (rigid block, MEMS)***
- ***harmonic and other excitations***

Helmholtz oscillator





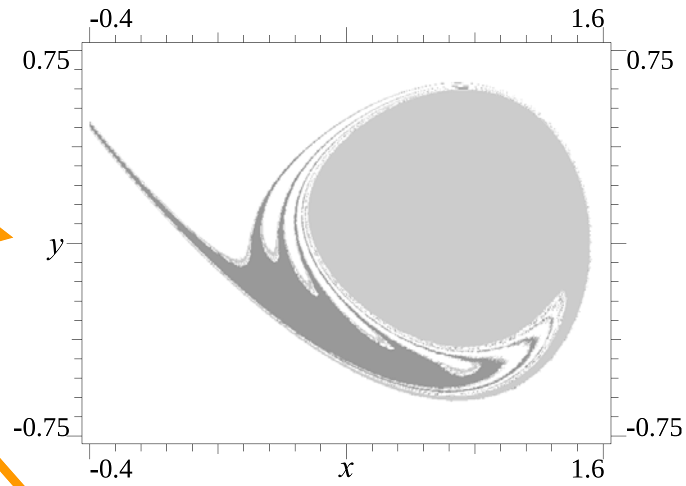
$$\ddot{x}+0.1\dot{x}-x+x^2=\gamma(\omega t)=\gamma_1\sum_{j=1}^{\infty}\frac{\gamma_j}{\gamma_1}\sin(j\omega t+\Psi_j)$$

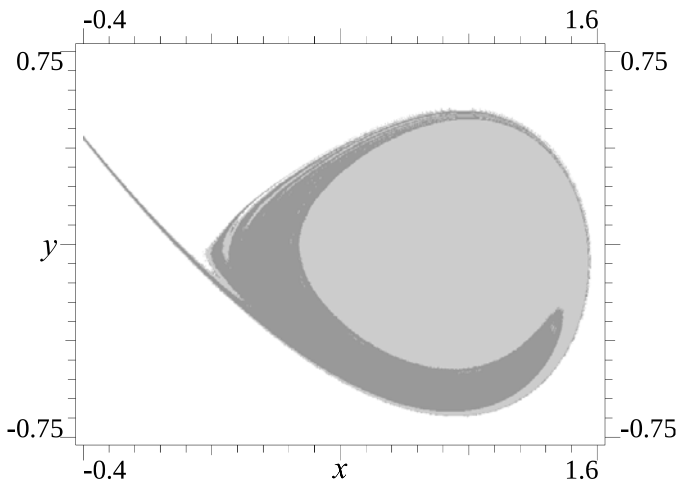
$\varepsilon\gamma_1$ = ***overall excitation amplitude***

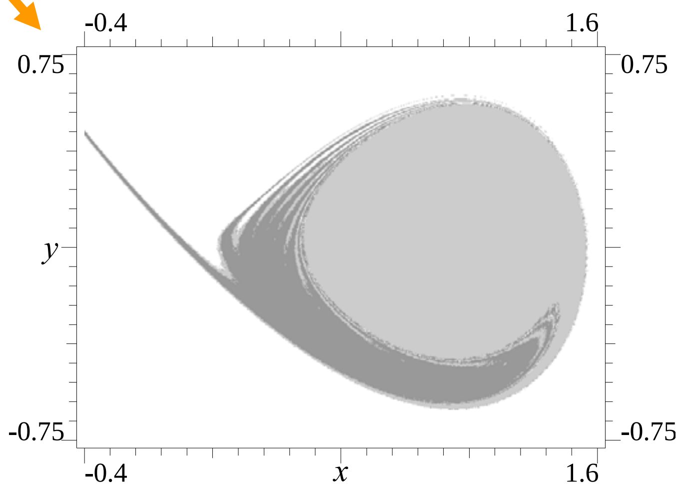
γ_j/γ_1; Ψ_j = ***parameters governing the shape of the excitation***

Helmholtz: comparison of different excitations (1)

harmonic
harmonic + 1 super.
harmonic + 2 super.







regularization by adding (clever) superharmonics

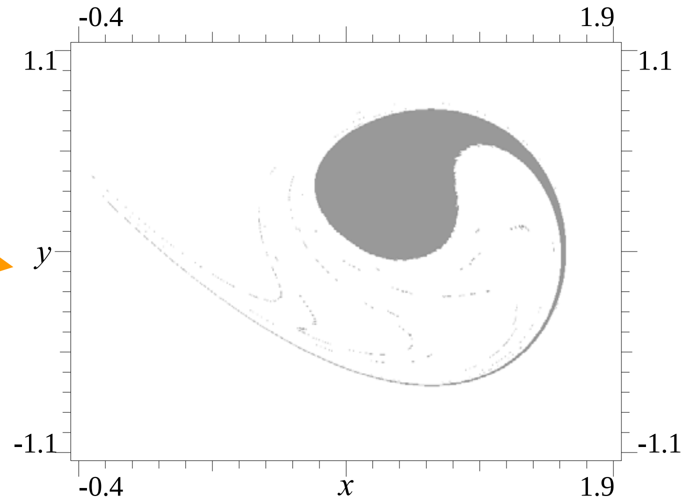
Helmholtz: comparison of different excitations (2)

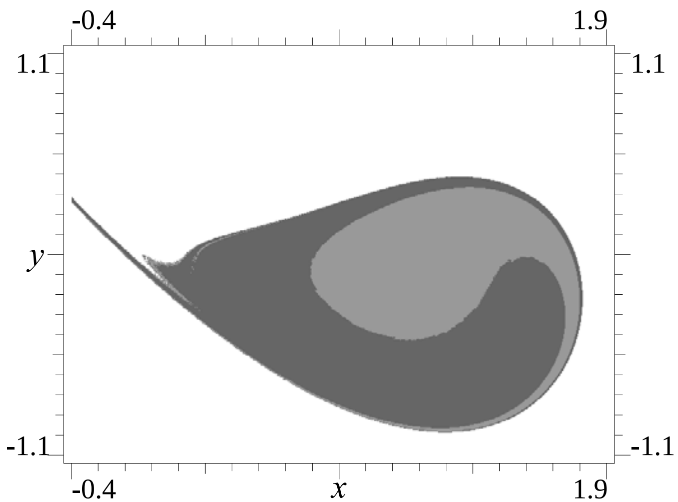
higher excitation amplitude

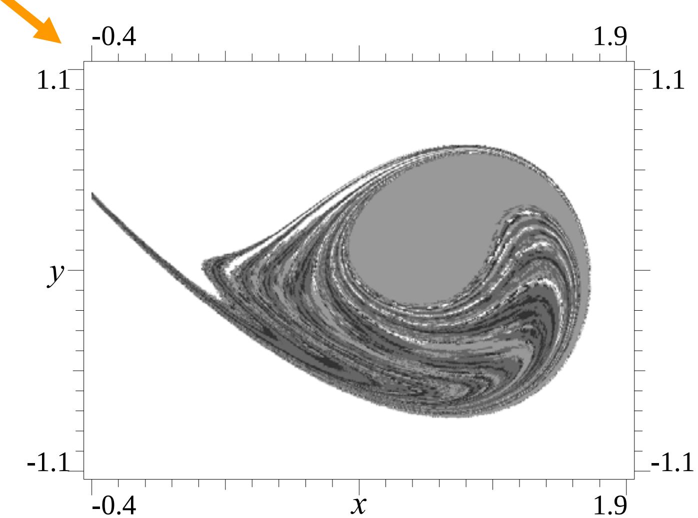
harmonic

harmonic + 1 super.

harmonic + 2 super.

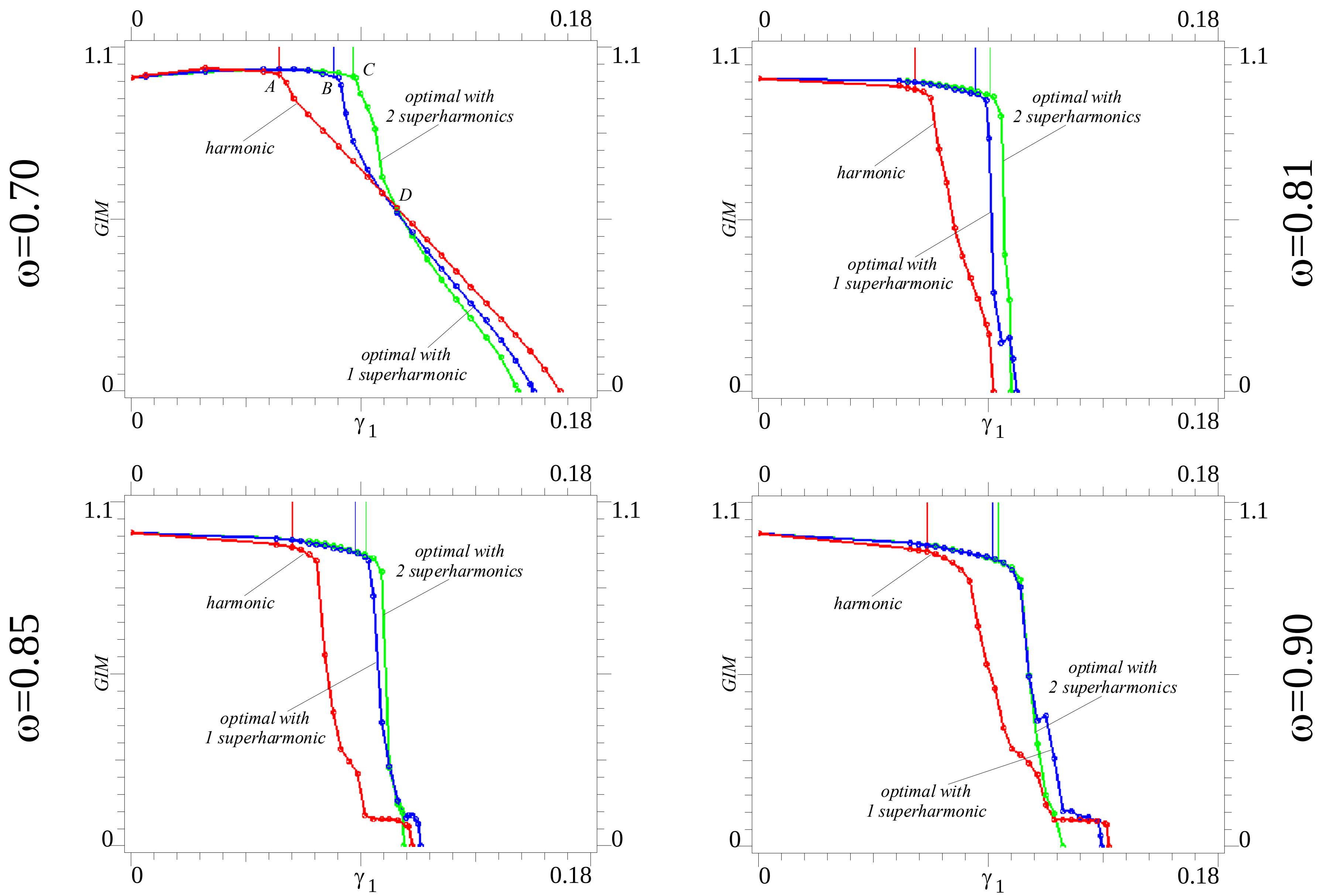






strong reduction for fixed excitation amplitude

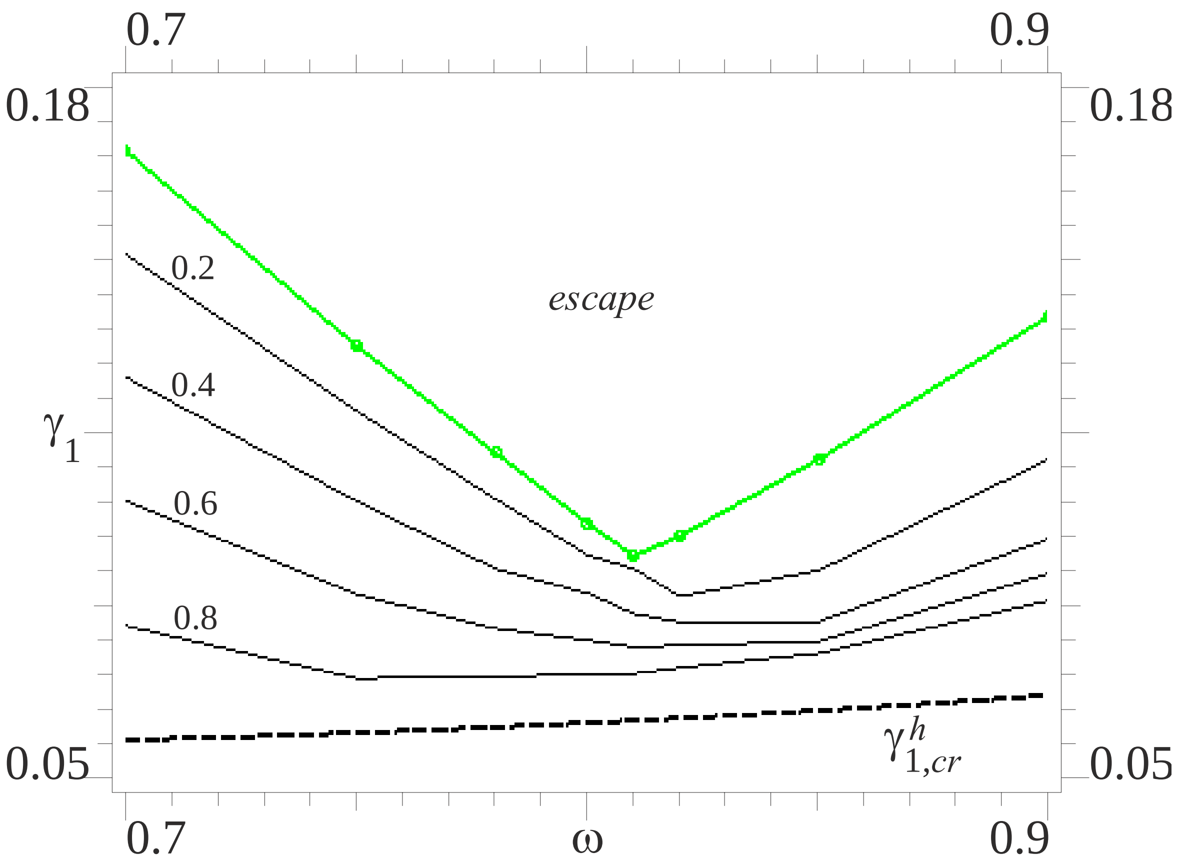
Helmholtz: erosion profiles



- ***safe basin: classical basin of attraction; integrity through GIM***
- ω=0.81 ***is the vertex of the escape V-region in parameter plane***

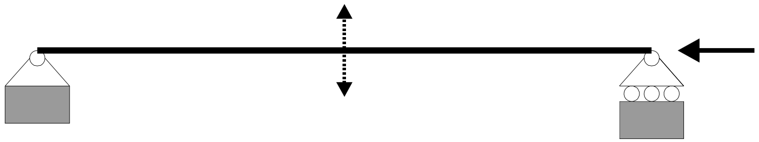
Helmholtz: excitation phase-amplitude chart

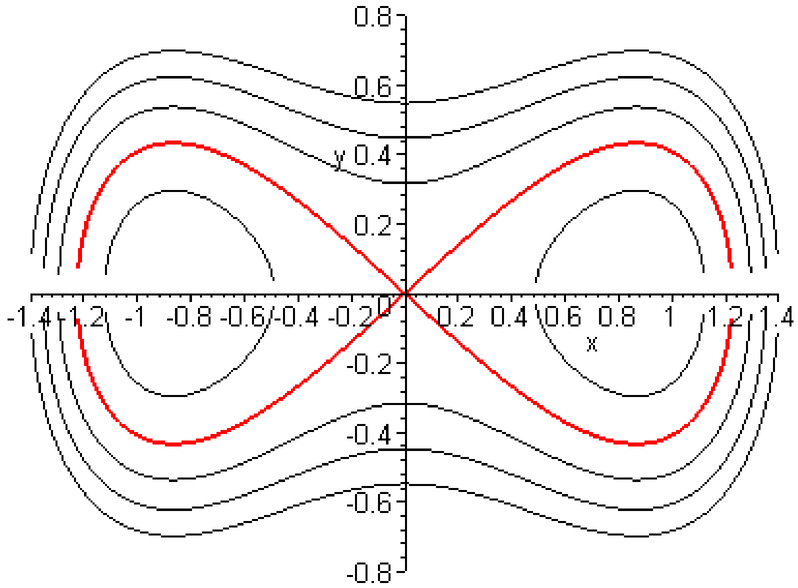
contour plot of the GIM with harmonic excitation



- ***“Dover cliff” profiles***
- ***starting points of erosion = homoclinic bifurcations (OK!)***
- ***sharpness close to the vertex, dullness elsewhere***

Duffing oscillator





$$\ddot{x}+\varepsilon\delta\ddot{x}-\frac{x}{2}+\frac{x^3}{2}=\varepsilon\gamma(\omega t)=\varepsilon\gamma_1\sum_{j=1}^{\infty}\frac{\gamma_j}{\gamma_1}\sin(j\omega t+\Psi_j)$$

$\varepsilon\gamma_1$= ***overall excitation amplitude***

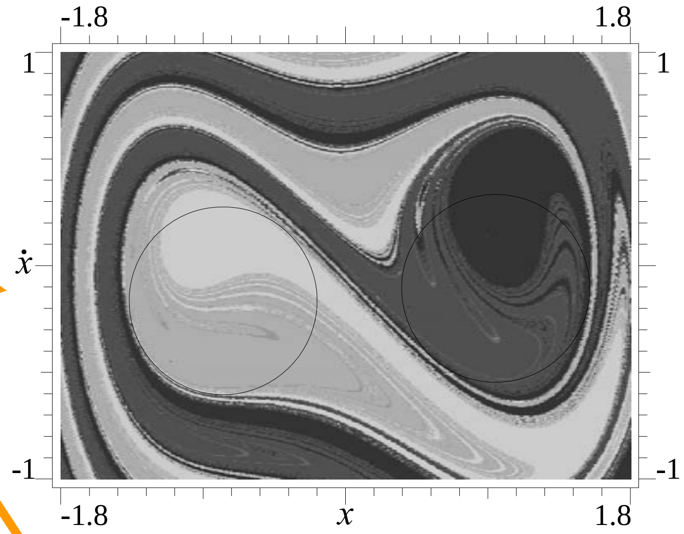
γ_j/γ_1; Ψ_j= ***parameters governing the shape of the excitation***

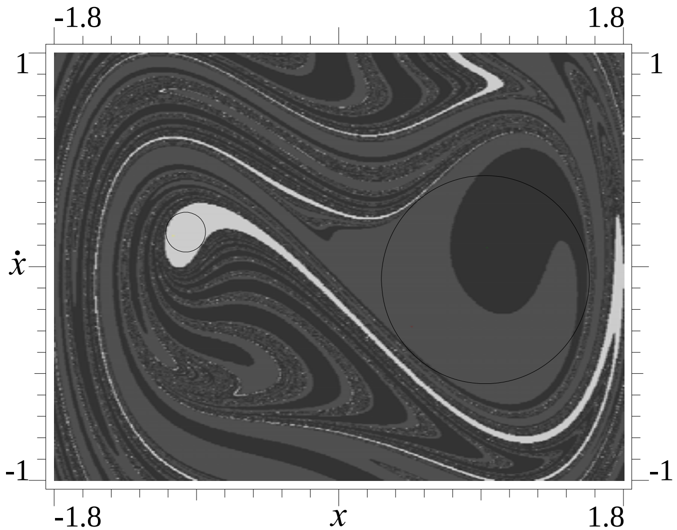
Duffing: comparison of different excitations

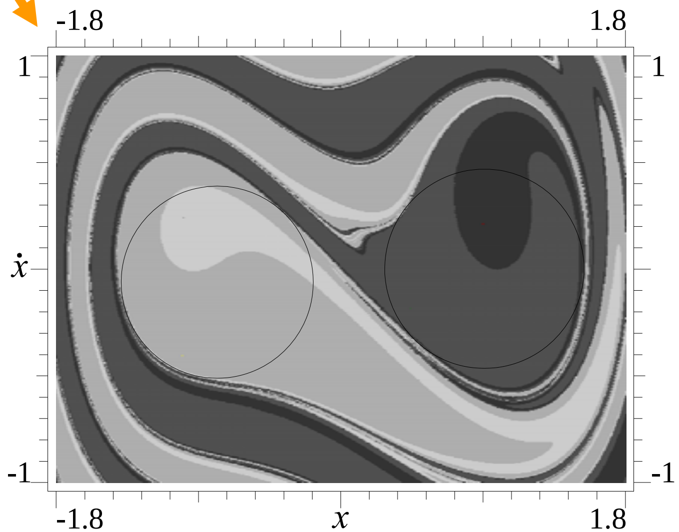
harmonic

harmonic + 1 sym. super.

harmonic + 1 unsym. super.



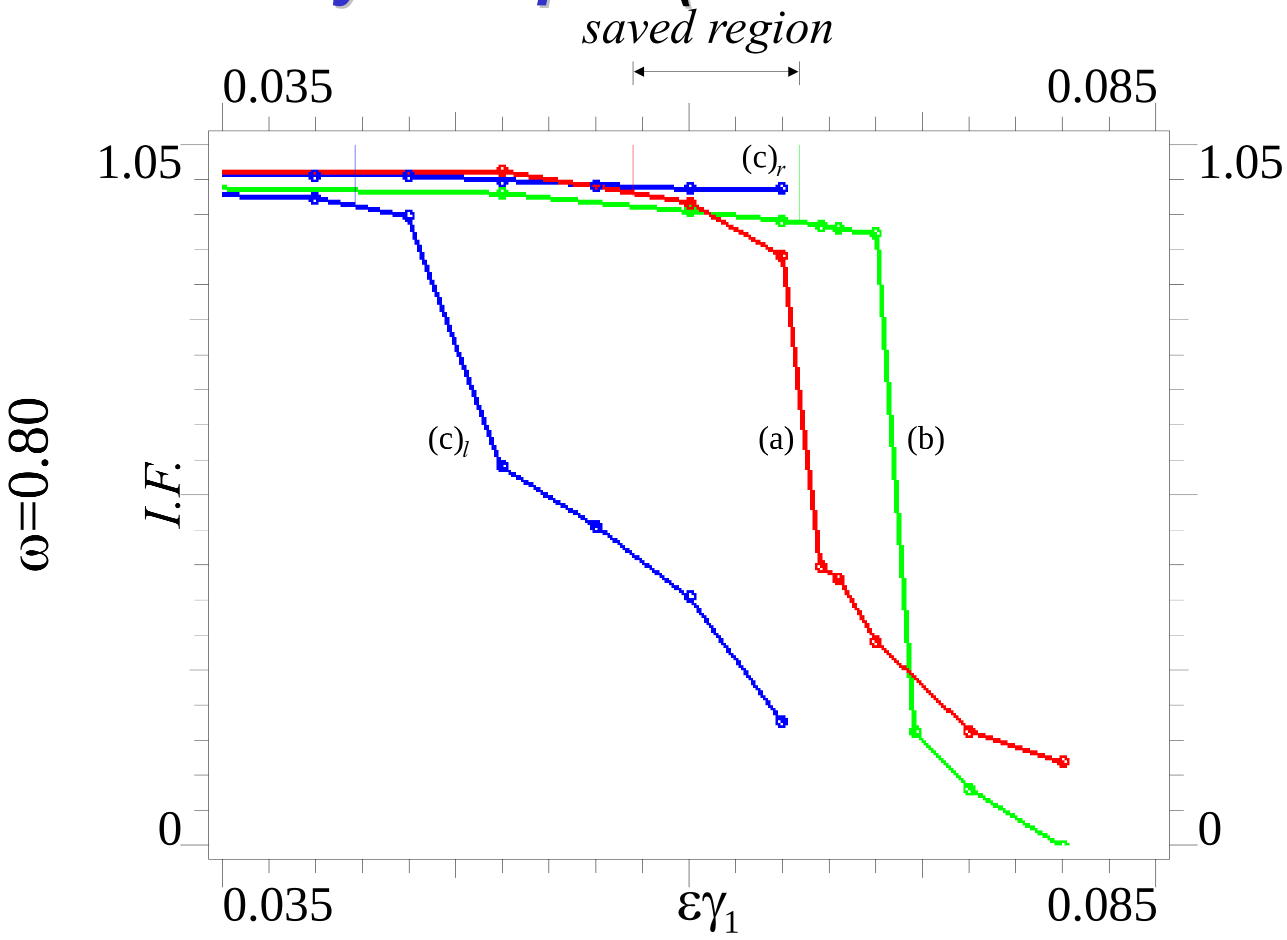




localized vs scattered reduction of fractality

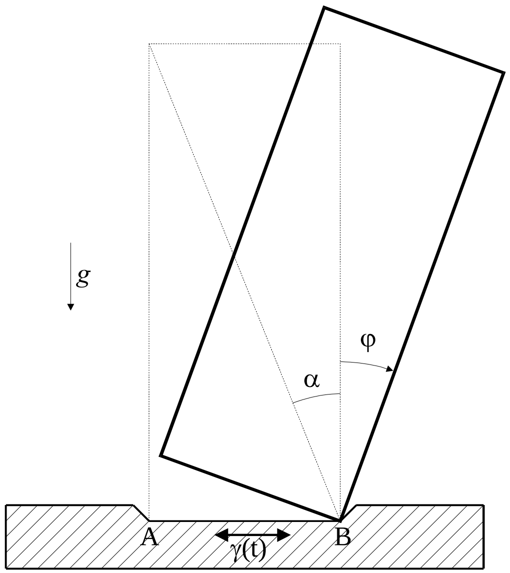
Duffing: erosion profiles

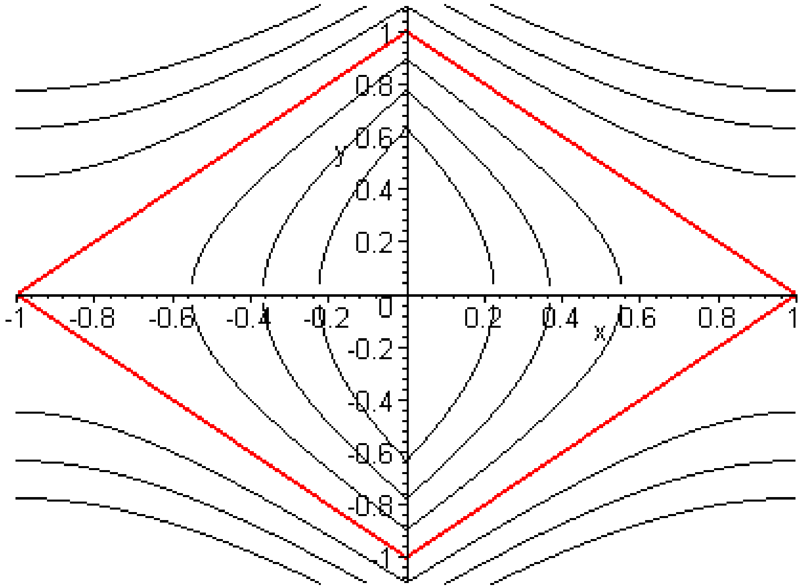
(a) harmonic, (b) harmonic + 1 sym. super., (c) harmonic + 1 unsym. super. (in the two different wells)



- ***safe basin: classical basin of attraction; integrity through IF***
- ω=0.80 ***is very close to the vertex of the escape V-region***

Rigid block





Heteroclinic bifurcation

rocking around the left corner:

$$\ddot{\varphi}+\delta\dot{\varphi}-\varphi-\alpha+\gamma(t)=0,\quad \varphi<0,$$

rocking around the right corner:

$$\ddot{\varphi}+\delta\dot{\varphi}-\varphi+\alpha+\gamma(t)=0,\quad \varphi>0,$$

impact (Newton law):

$$\dot{\varphi}(t^+)=r\,\dot{\varphi}(t^-),\quad \varphi=0,$$

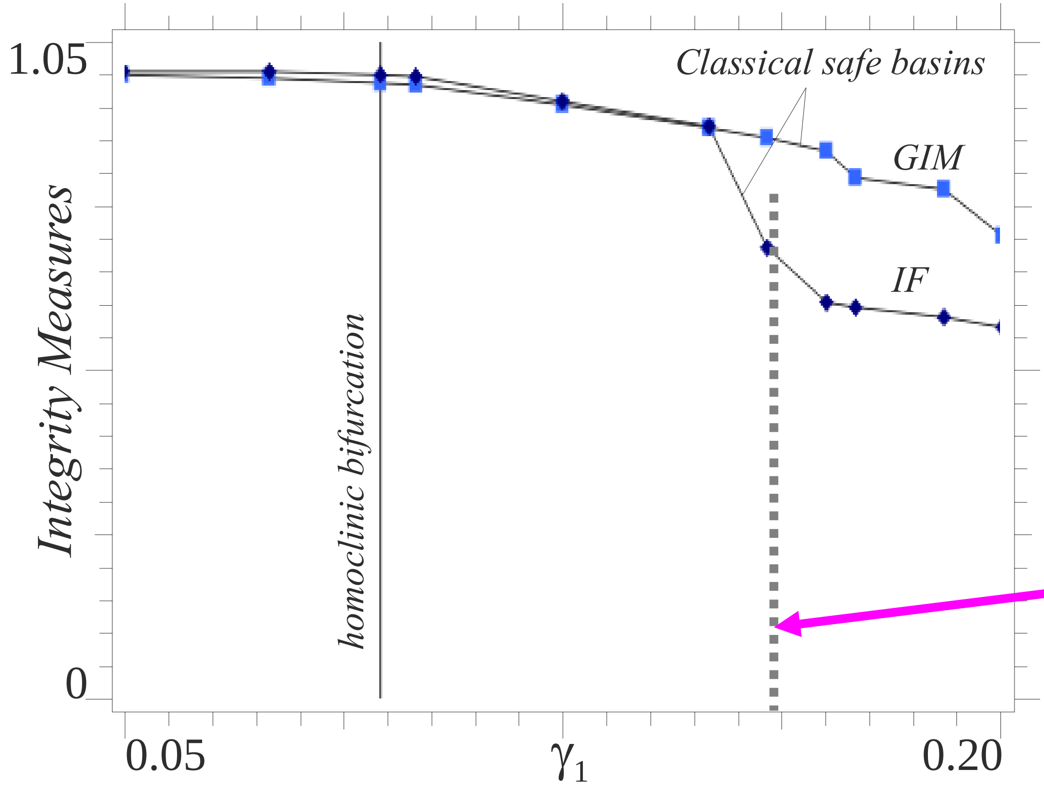
$T=2\pi/\omega$***-periodic generic excitation:***

$$\gamma(t)=\Sigma_j\,\gamma_j\cos(j\,\omega t+\psi_j)$$

overturned positions $\varphi=\pm\pi/2$

Rigid block: erosion profiles, different measures

α=0.2, δ=0.02, r=0.95, ω=3.5 (slightly damped) – harmonic excitation

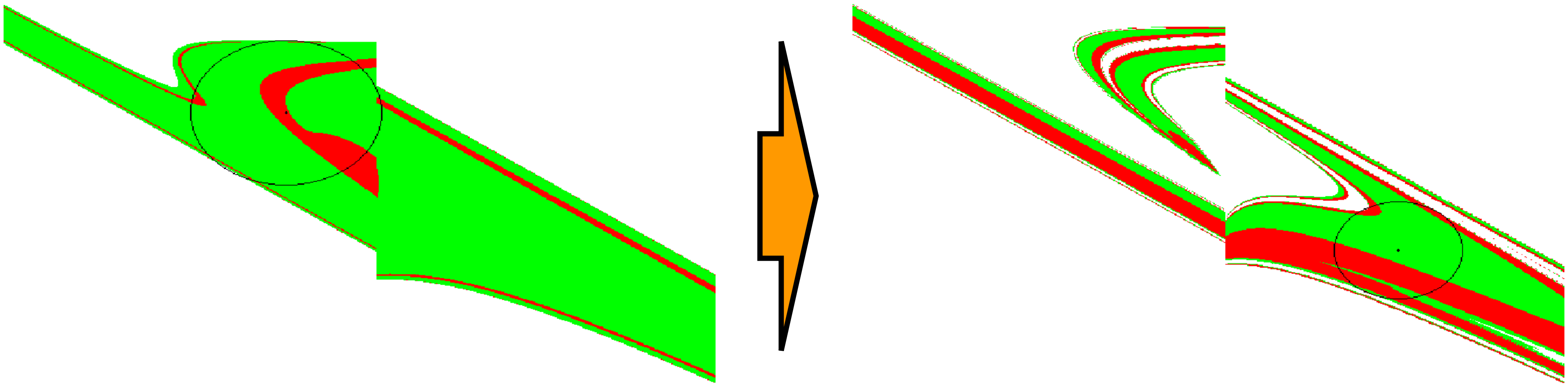


- ***no resonance frequency around which focusing numerical analyses***
- ***likely effect of a secondary global bifurcation***

- ***GIM misses sharp fall or erosion profile***
- ***high values after fall: absence of resonance?***
- ***homoclinic bifurcation slowly triggers erosion***
- ***effects of non-smoothness***

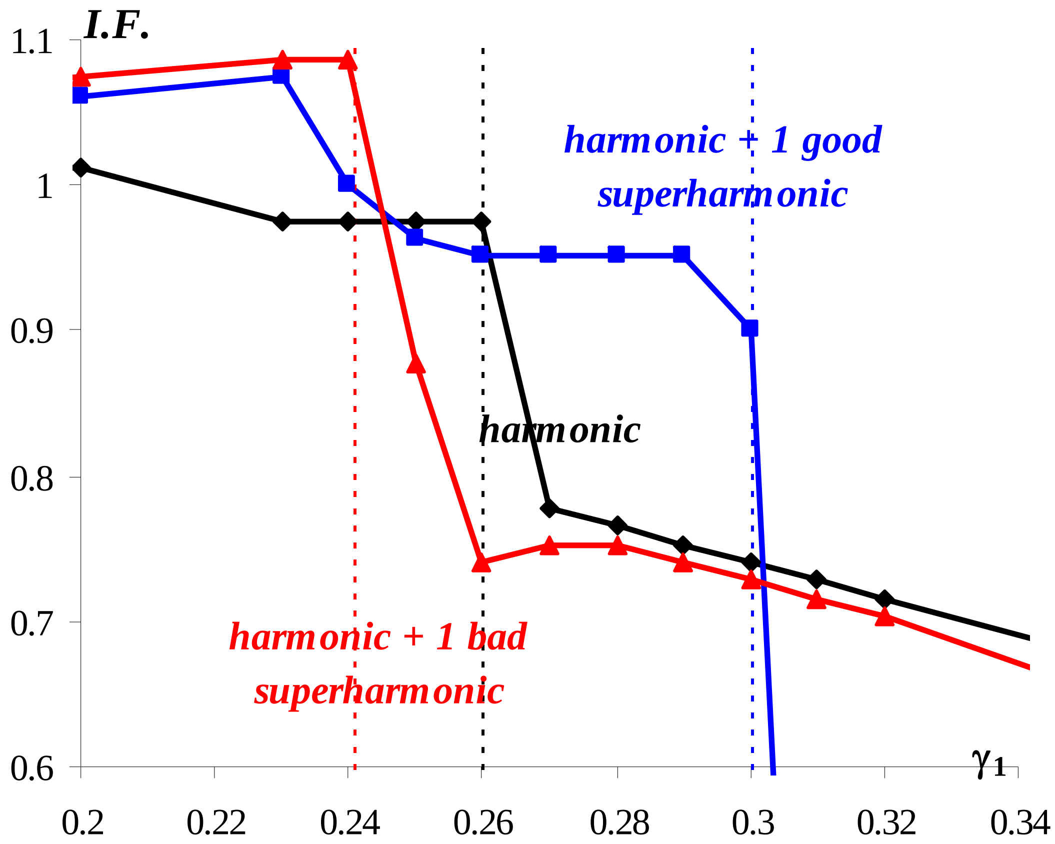
Rigid block: example of basins erosion

γ_1=0.20 α=0.2, δ=0.5, r=0.7, ω=1.5 **γ_1=0.35**



Rigid block: erosion with different excitations

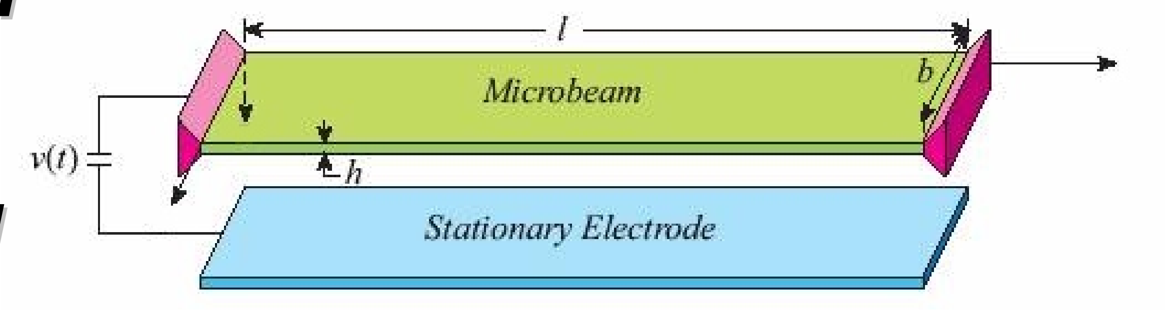
α=0.2, δ=0.5, r=0.7, ω=1.5 (strongly damped)



- ***now coincidence between the critical threshold and sudden falls of profiles, for all excitations***
  (different from what observed in other examples)

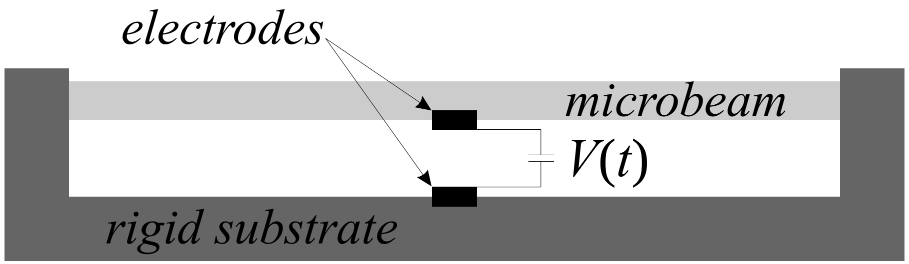
Micro-electro-mechanical systems (MEMS)

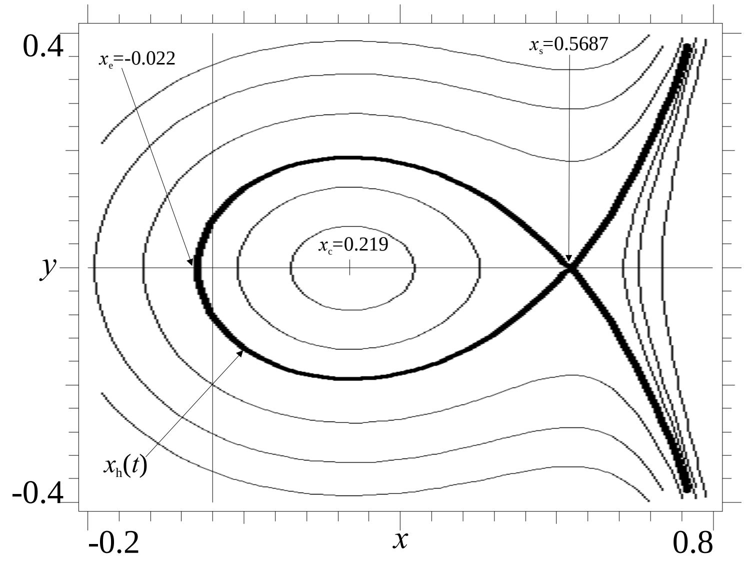
- ***nonlinear dynamics of a thermoelastic microbeam***
- ***axial load, modeling residual stresses***
- ***concentrated electrodynamic transverse force applied at mid-span (the actuation)***
- ***both ends are fixed***



- ***geometric nonlinearity due to membrane stiffness***
- ***ultra-high vacuum environment***

MEMS: single-d.o.f. model





- ***small electrodynamic force***
- ***small visco- and thermo-elastic damping***

$\rightarrow$ ***temperature condensation***

$$\ddot{x} + \alpha x + \beta x^3 - \frac{\gamma}{(1-x)^2} = \varepsilon \left\{ -\tilde{\mu}\dot{x} + \frac{\tilde{\eta}\sum_{j=1}^{N}(\eta_j/\eta_1)\sin(j\Omega t + \Psi_j)}{(1-x)^2} \right\}$$

substrate at $x=1$ ***overall excitation amplitude***

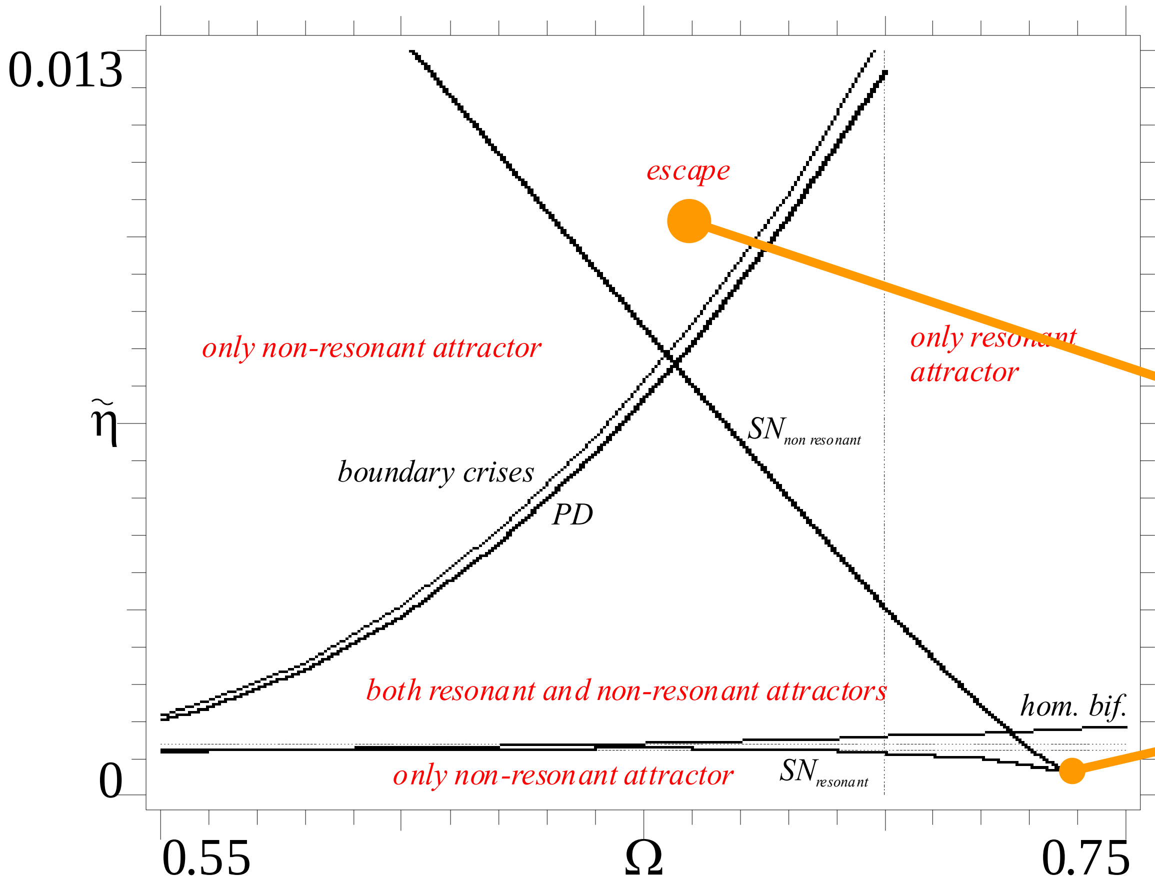
$\gamma>0$ ***magnitude of the electrostatic force, ≈ the square of the constant (DC) input voltage***

Ω ***frequency of the periodic electrodynamic force***

$\eta_j>0$ ***and*** Ψ_j : ***relative amplitudes and phases of the j-th harmonic of the electrodynamic force, i.e., of the oscillating (AC) voltage***

MEMS: reference response chart (harmonic excitation)

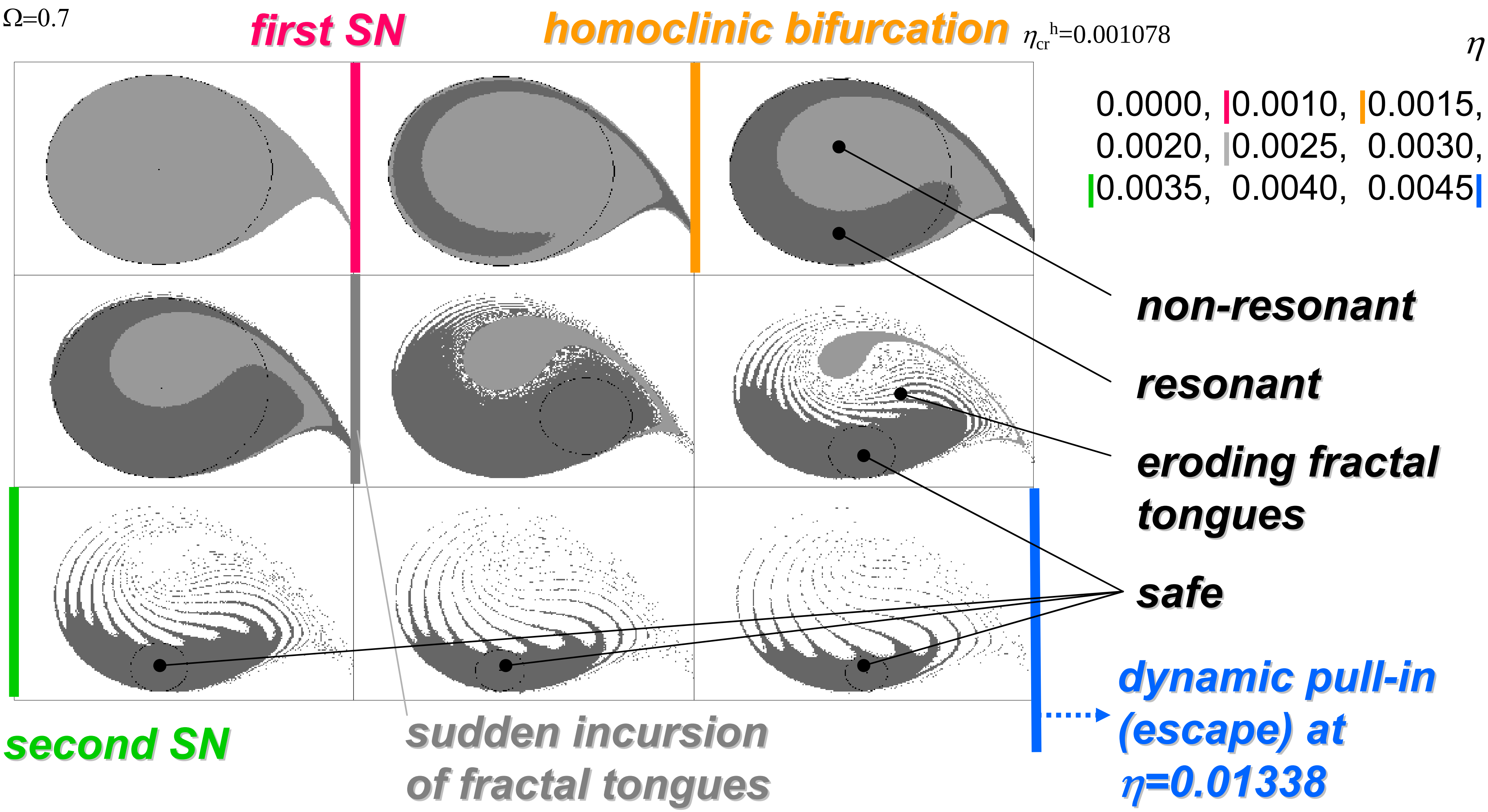
- ***many bifurcation diagrams built***



- ***same qualitative features of the Helmholtz oscillator***
- ***V-shaped region of escape (dynamic pull-in), vertex at Ω=0.655***
- ***degenerate cusp bifurcation at Ω=0.737 and η=0.000461***

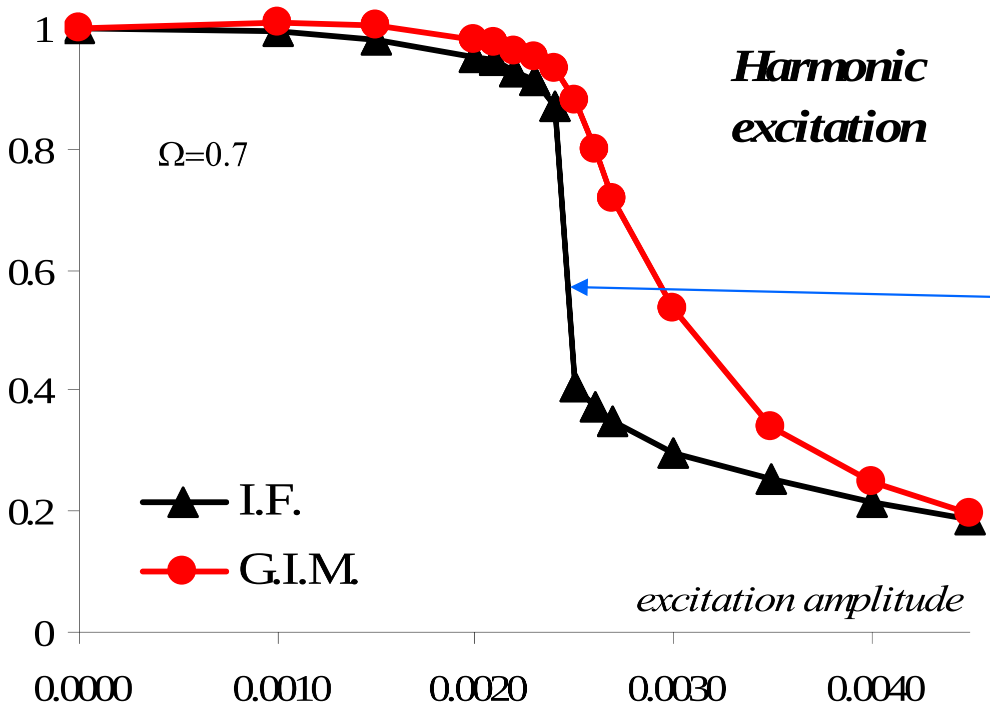
MEMS: basins erosion (harmonic excitation)

- ***classical basins of attraction (stationary regime)***



MEMS: erosion profiles (harmonic excitation)

- ***comparison of erosion profiles with Integrity Factor (I.F.) and Global Integrity Measure (G.I.M.)***

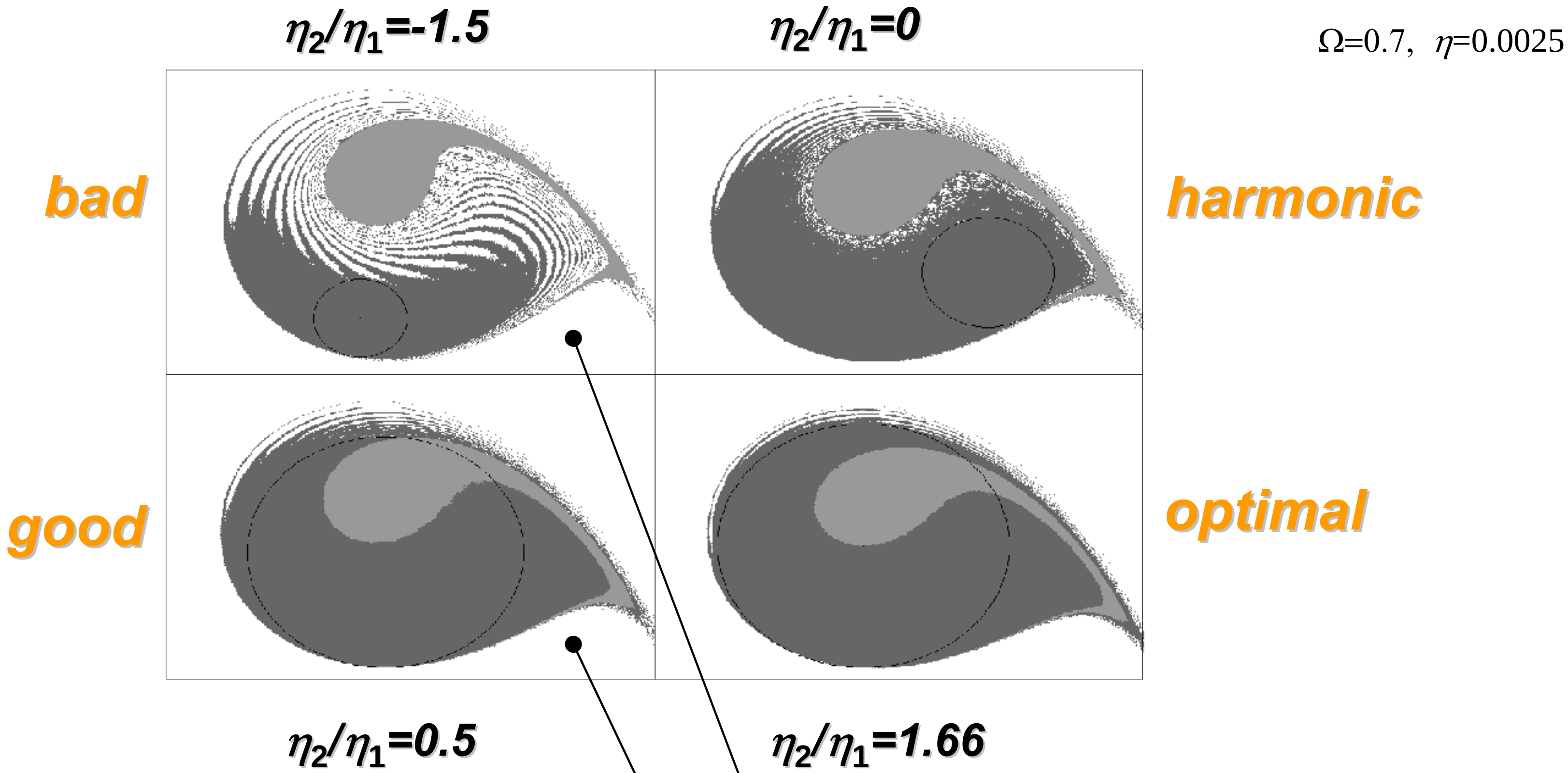


- ***I.F. better takes into account the instantaneous fractal tongues penetration***
- ***I.F.<G.I.M. → I.F. more conservative → more reliable for practical applications***

confirms rigid block results

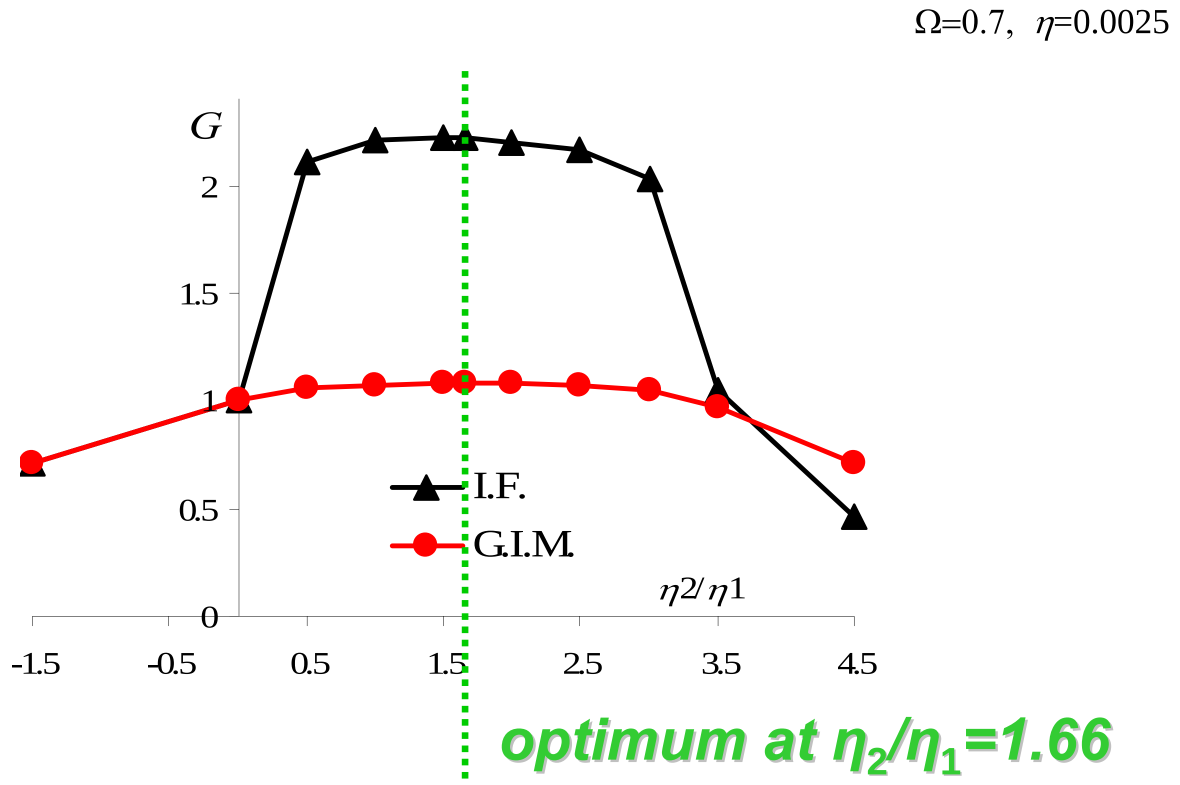
MEMS: basin erosion (1) (fixed amplitude)

- ***effects of a single added superharmonic (N=2)***



- ***the superharmonic may have dangerous effects if not properly designed***
- ***good results also for non optimal superharmonic***
- ***marginal increments around optimality***

MEMS: basin erosion (2) (fixed amplitude)

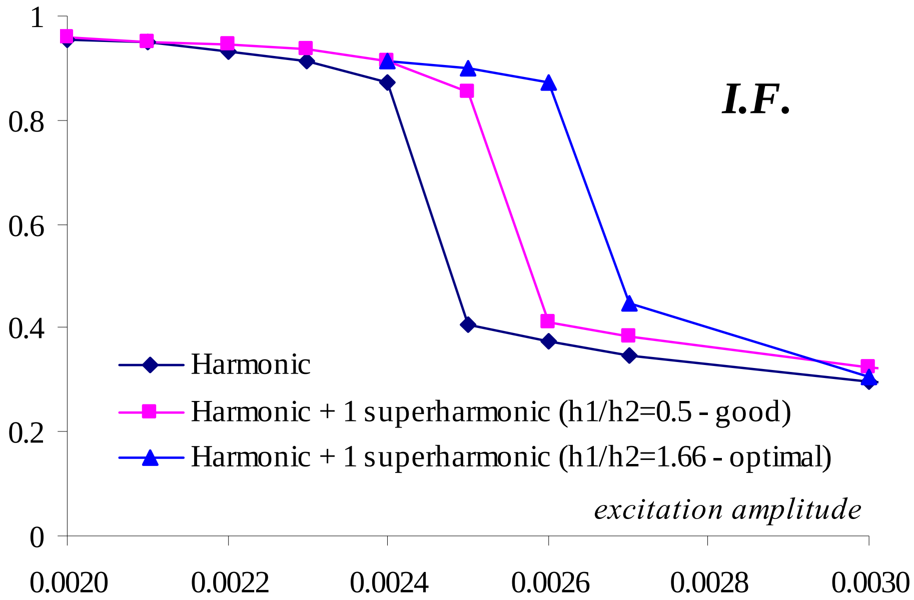


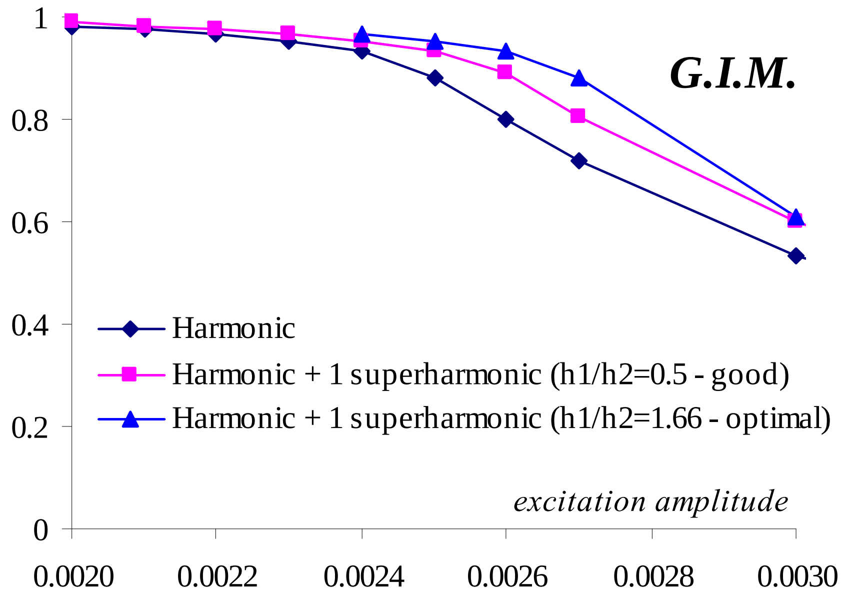
- ***almost optimal results on a broad band →***
  → practically appealing
- ***sharpness (I.F.) vs dullness (G.I.M.) due to different detection of instantaneous fractal penetration***

MEMS: basin erosion (3) (varying amplitude)

- ***effects of superharmomics on erosion profiles***

$\Omega=0.7$



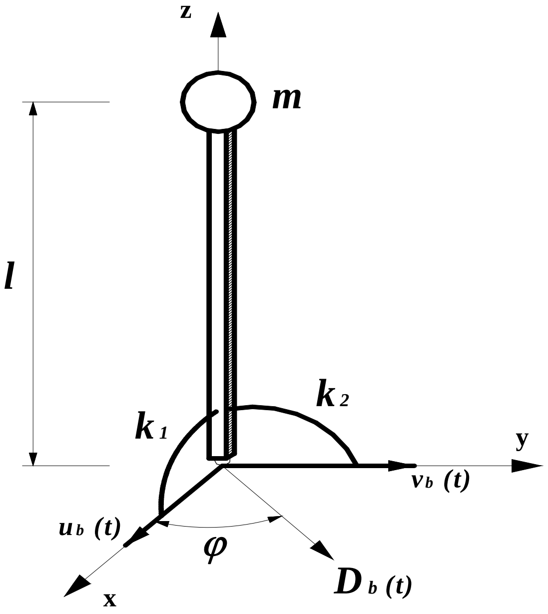


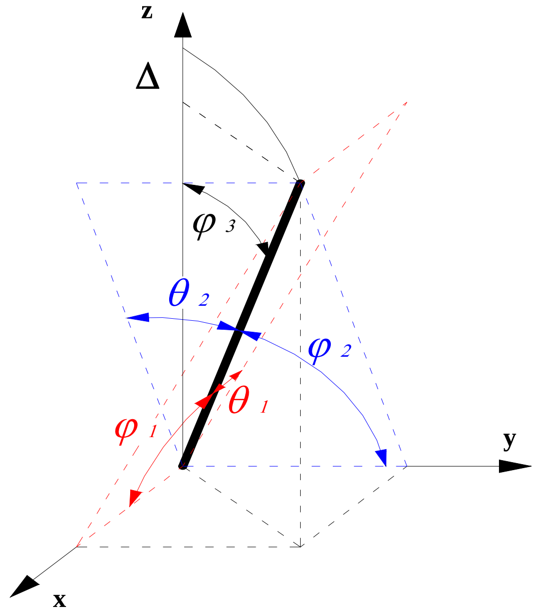
- ***shifting of erosion profiles***
- ***same horizontal shift for both measures, different vertical shift** (due to sharpness)*
- ***profiles shapes maintained by superharmonics***

Augusti's 2-dof model

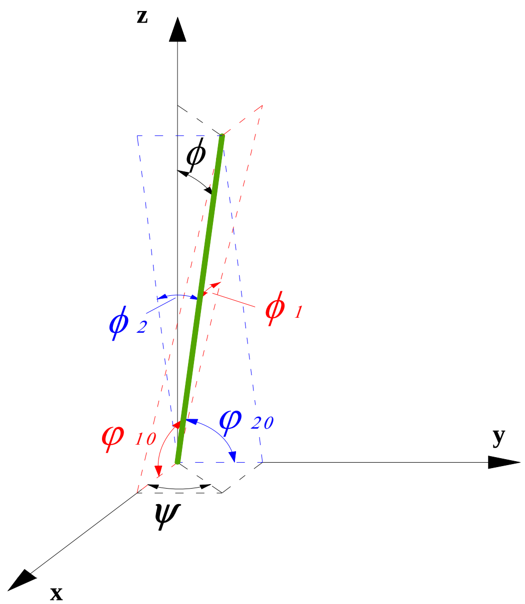
Orlando, Goncalves, Rega, Lenci, 2009

perfect system

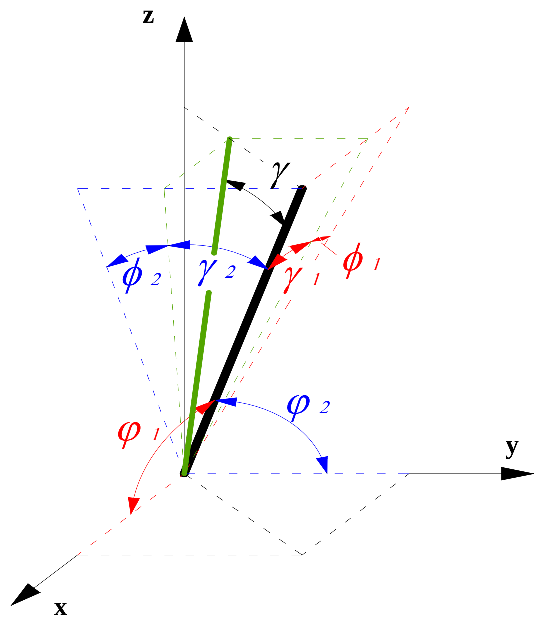




geometrically imperfect system

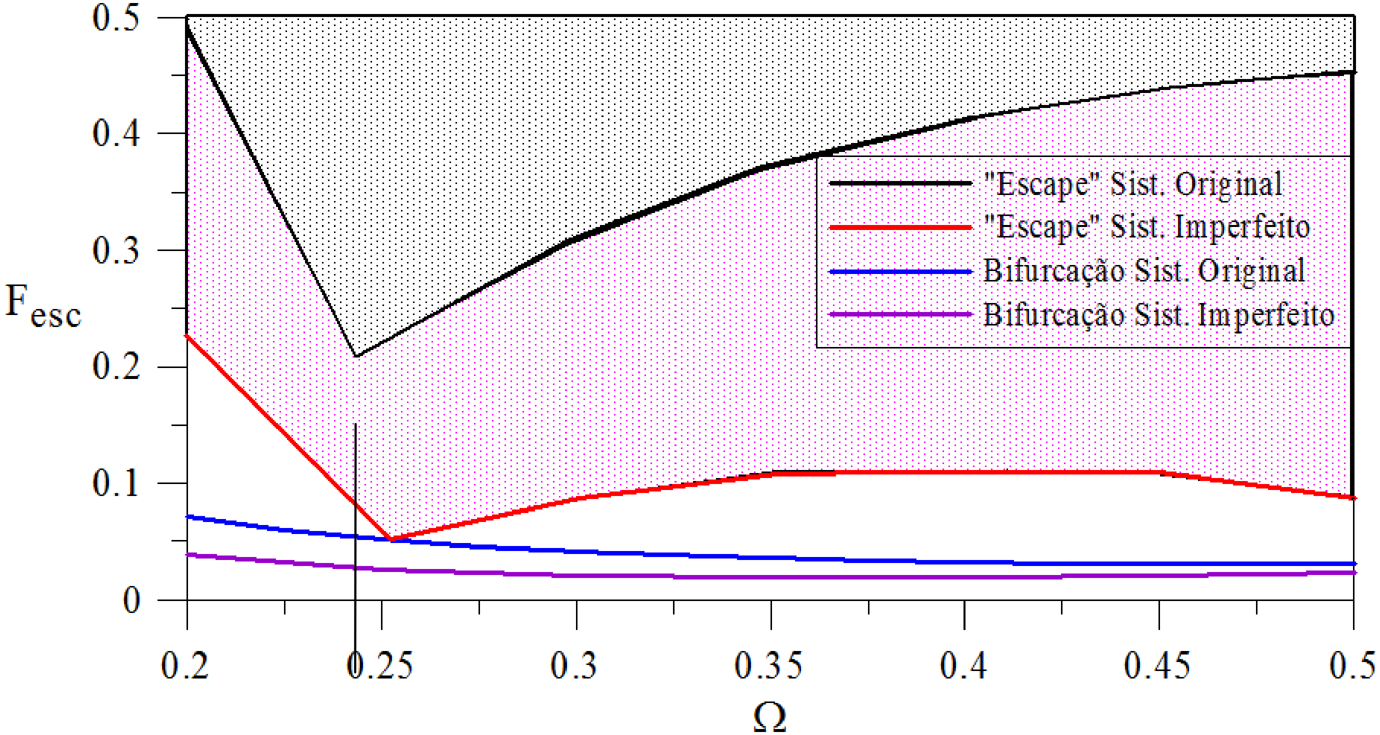


undeformed



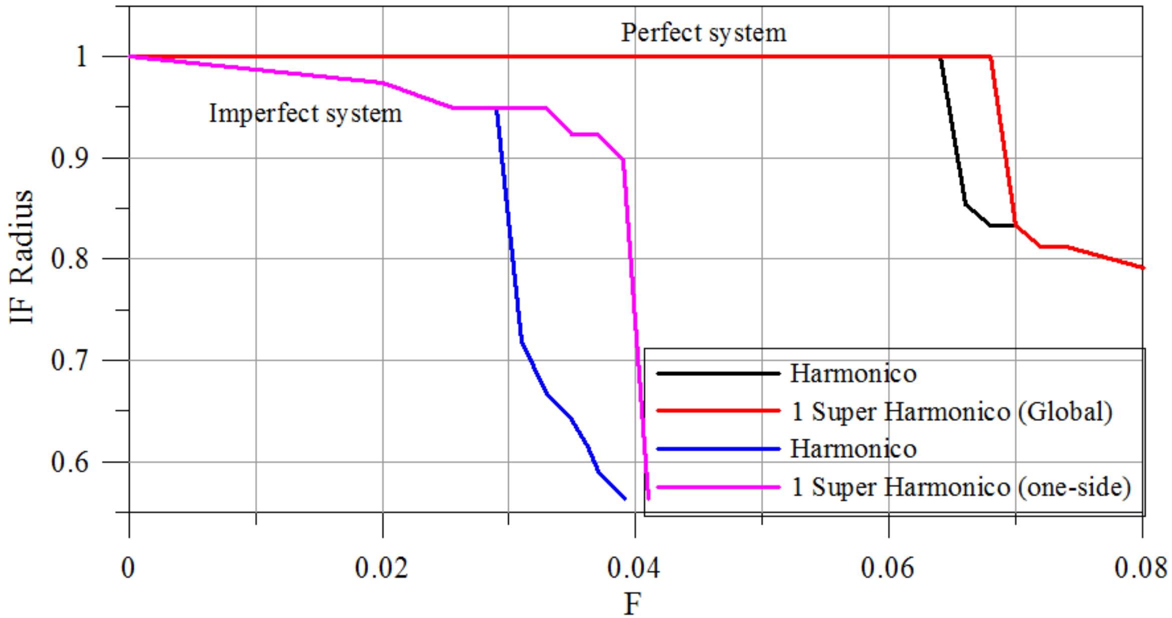
deformed

Augusti: response chart and dynamic integrity



Melnikov prediction vs escape for perfect and imperfect system

erosion profiles without and with superharmonics



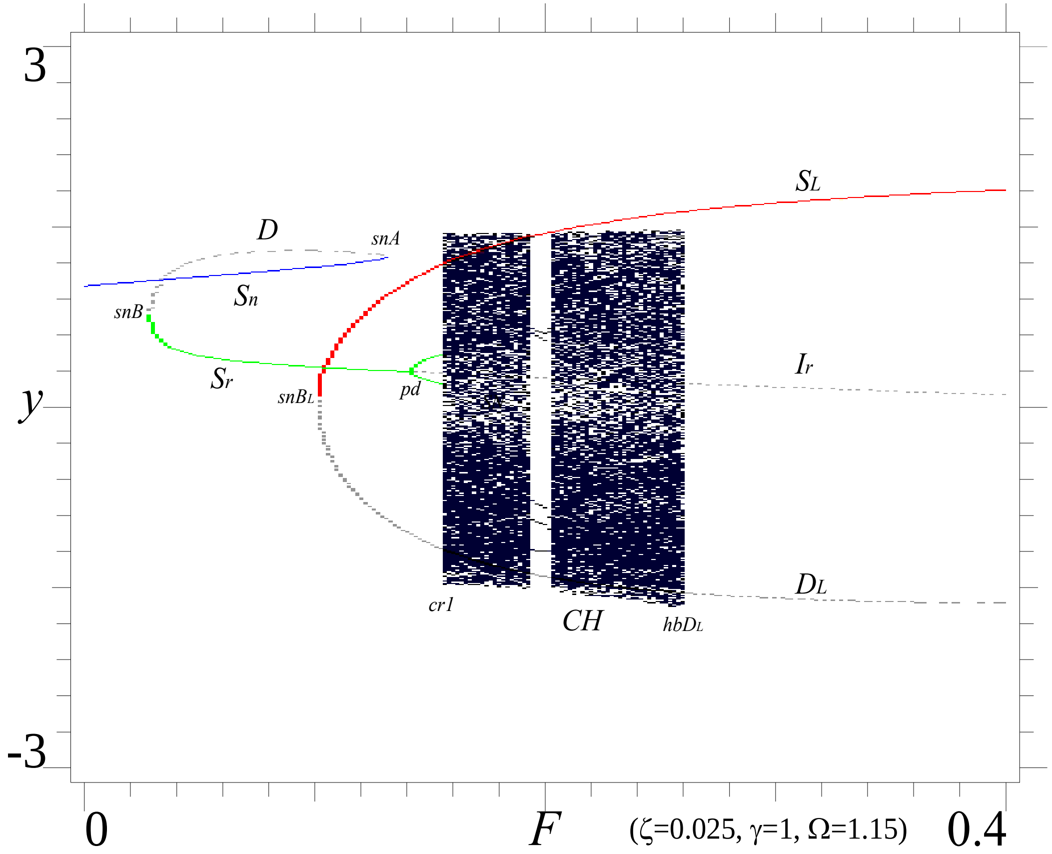
Integrity of competing (in-in/in-out) attractors

Duffing

Parametrically excited pendulum

Parametrically excited cylindrical shell

Duffing: competing non-resonant/resonant attractors (1)

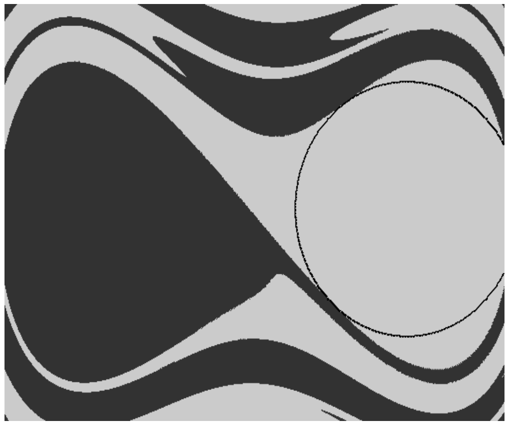


competing basins for increasing excitation amplitude:

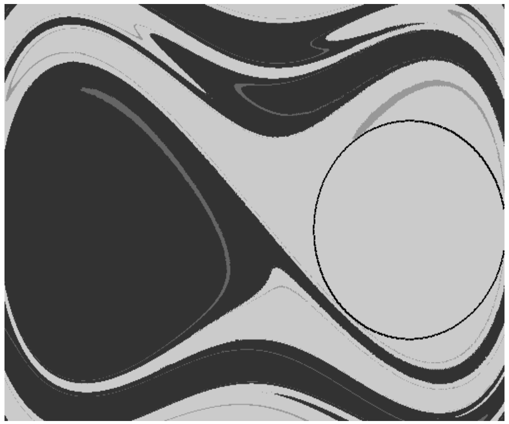
a) only non-resonant attractor

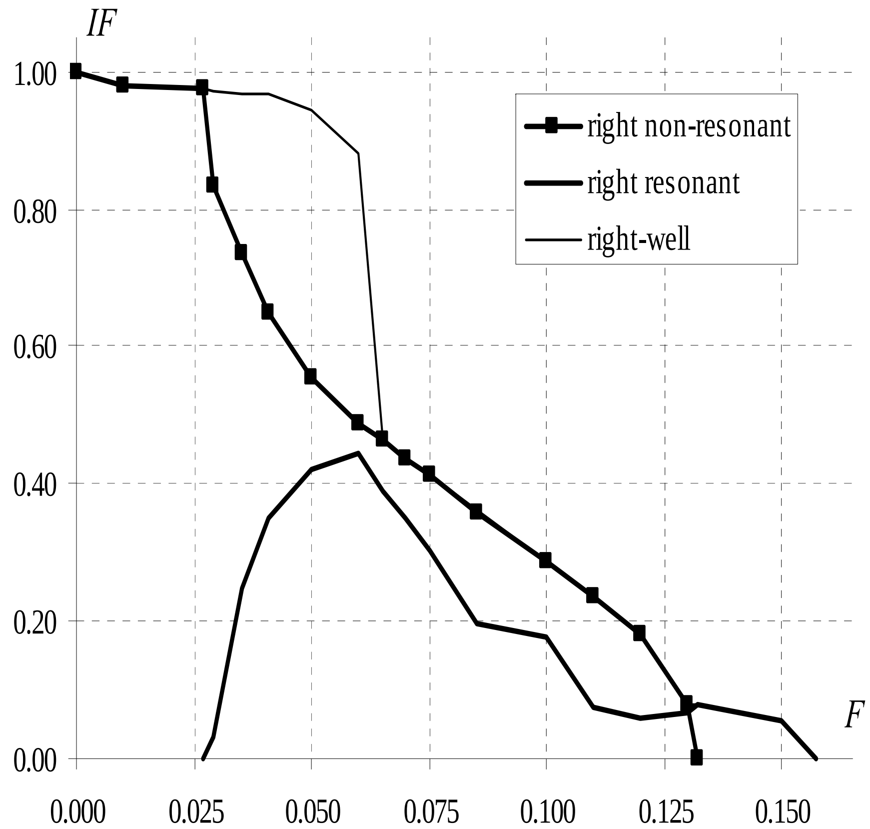
b) onset of resonant attractor (at snB): sudden fall down of S_n vs new born S_r

a) *F* = 0.027

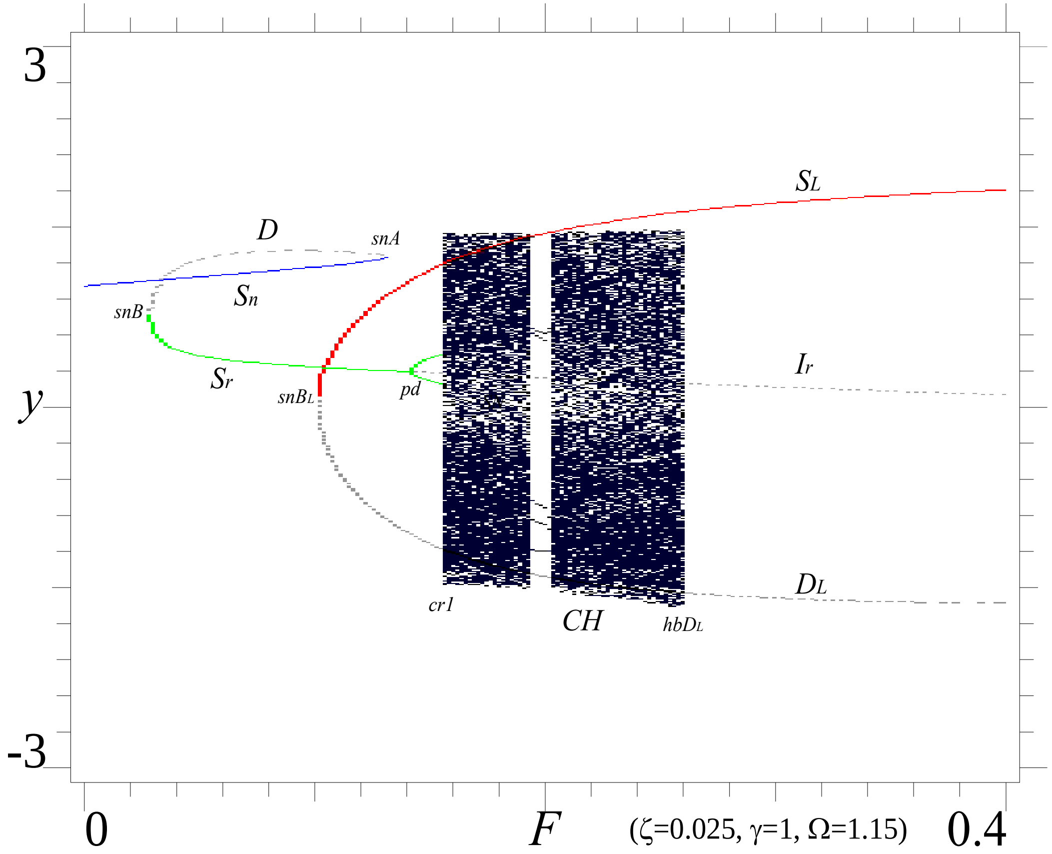


b) *F* = 0.029



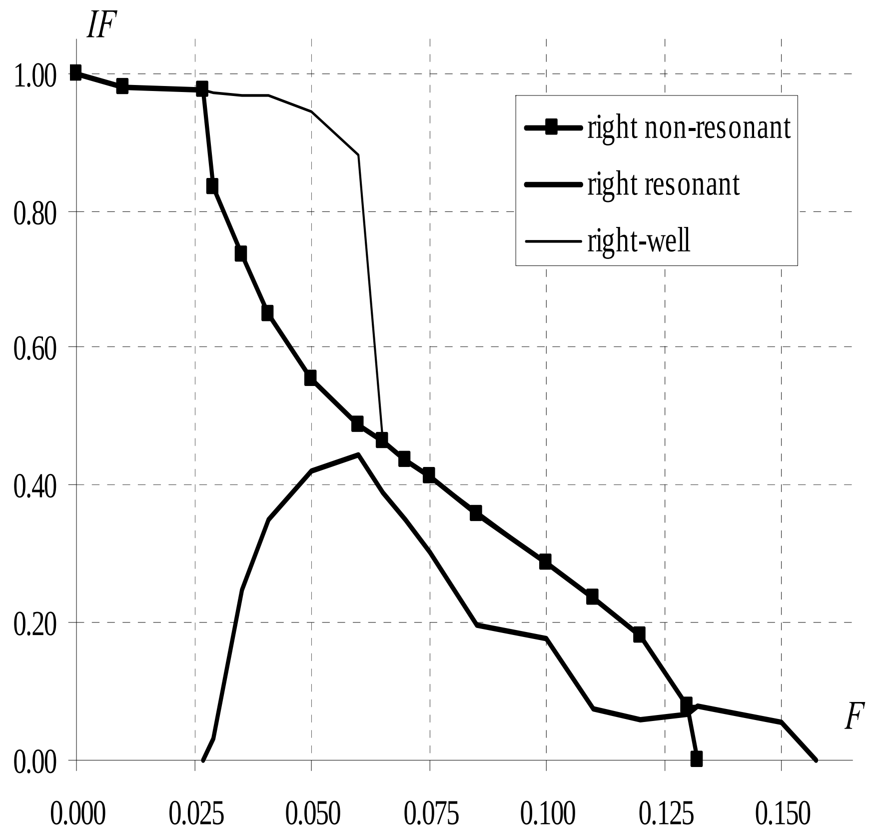


Duffing: competing non-resonant/resonant attractors (2)

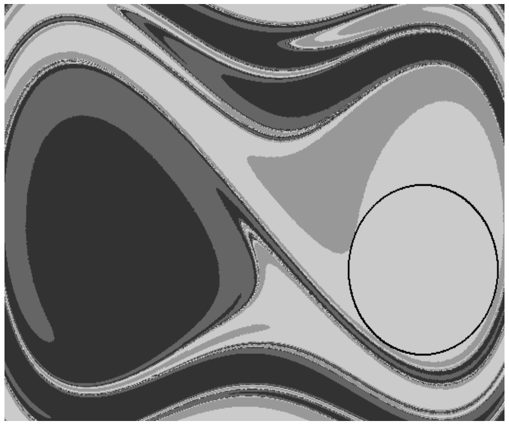


c) *fractalization of left/right well basin boundaries (hb_h): no effect on compact basins*

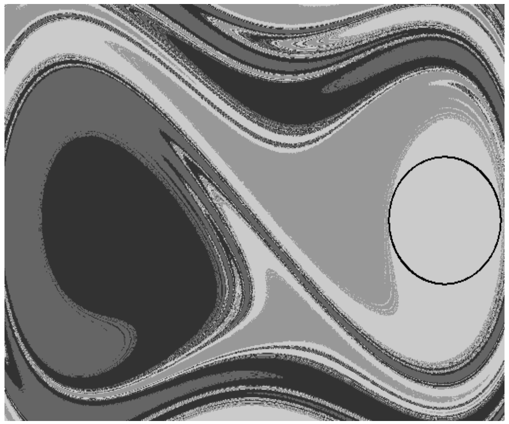
d) *maximum basin of S_r*



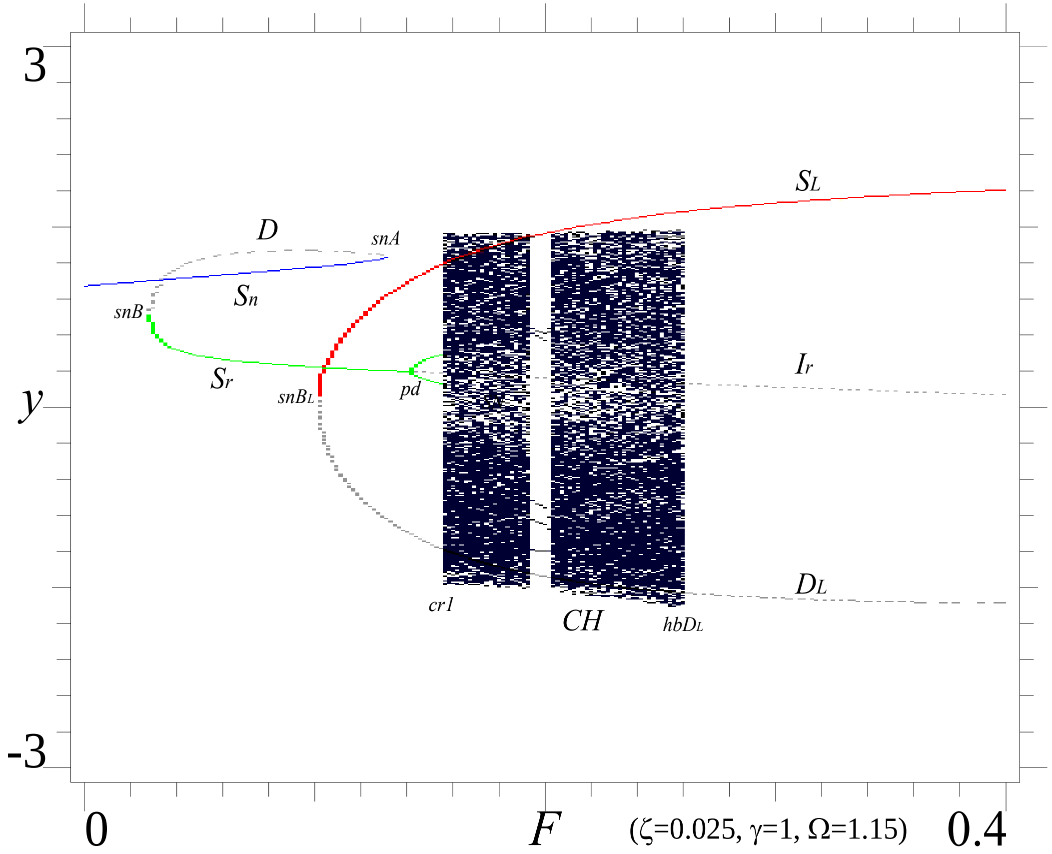
c) *F* = 0.041



d) *F* = 0.060



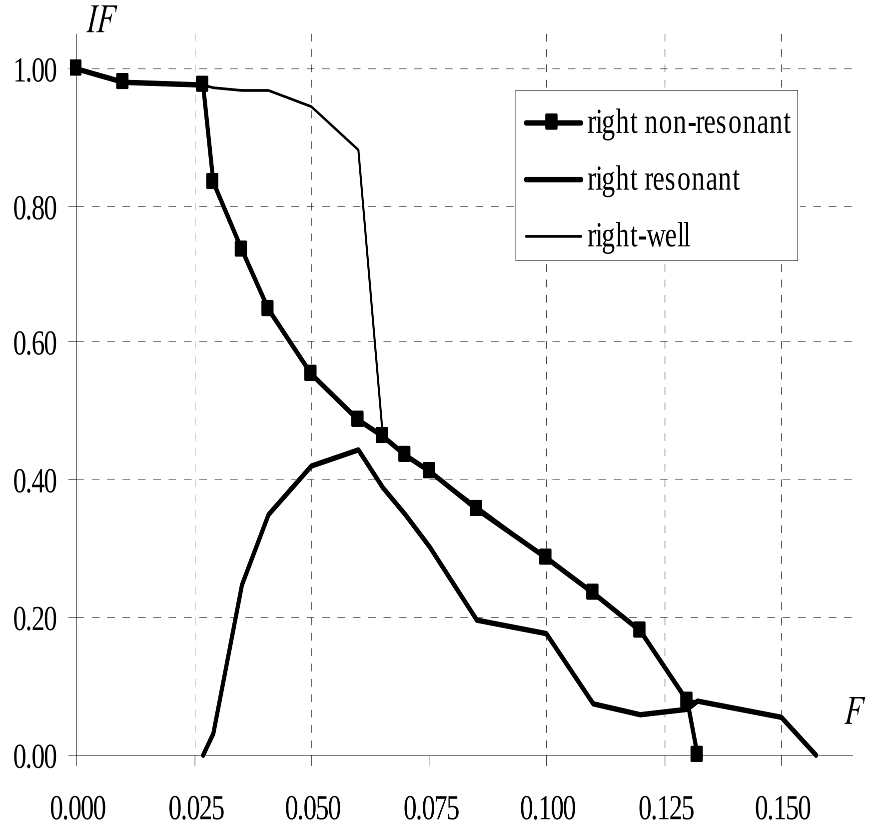
Duffing: competing non-resonant/resonant attractors (3)



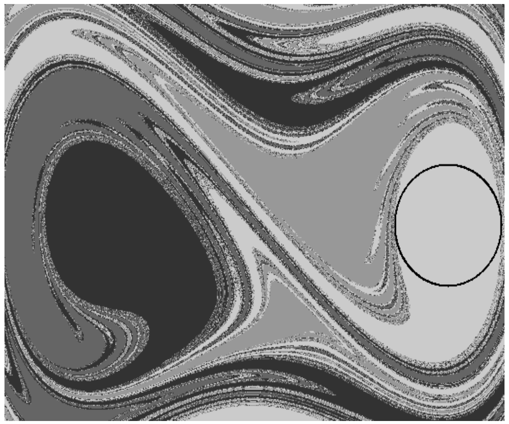
e) penetration of fractal tongues inside S_r basin

smoothly decreasing profiles till

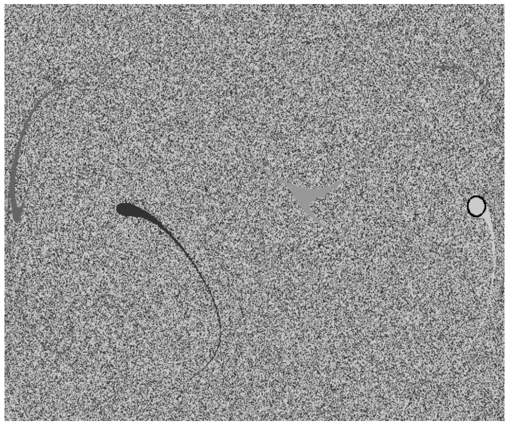
f) near disappearance of S_n (at snA) and residual integrity of S_r



e) *F* = 0.065

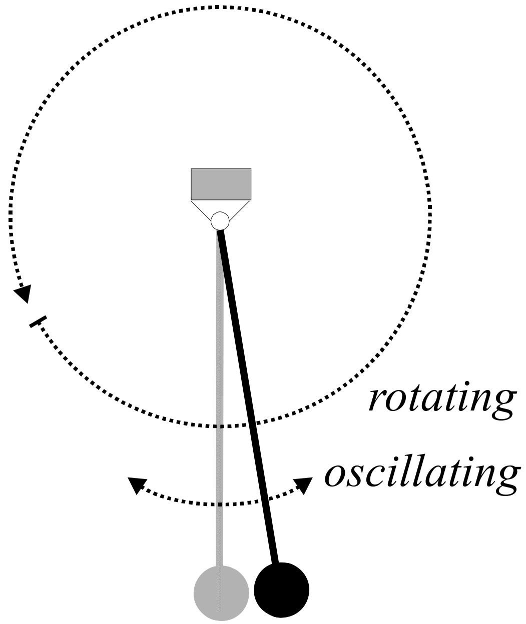


f) *F* = 0.130



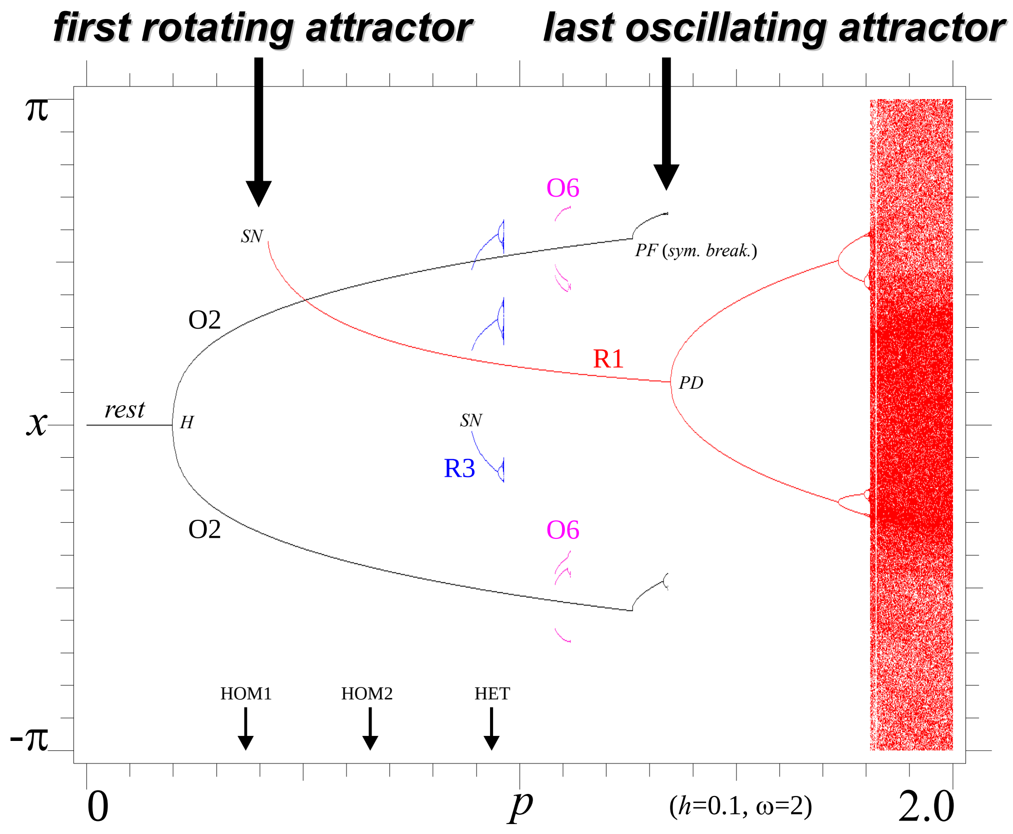
The parametrically excited mathematical pendulum

$$\ddot{x} + 0.1\dot{x} + [1 + p\cos(2t)]\sin(x) = 0$$



- ***"an antique but evergreen physical model"*** [Butikov]
- ***benchmark for main features of robustness and dynamical integrity of competing attractors***
- ***permits a cross-study of in-well attractors (oscillations) and out-of-well attractors (rotations)***
- ***has been recently shown to be of interest for practical applications*** [Xu et al., 2007]

Pendulum: bifurcation diagram and main events



attractors

O2	main oscillating solution of period 2
R1	main rotating solutions of period 1
R3	secondary rotating solutions of period 3
O6	secondary oscillating solution of period 6

main saddles

HS	hilltop saddles
DR1	direct saddles born at the SN bifurcation where R1 appear
IR1	inverse saddles after the PD bifurcation of R1
Ir	inverse saddle replacing the rest position at the H bifurcation

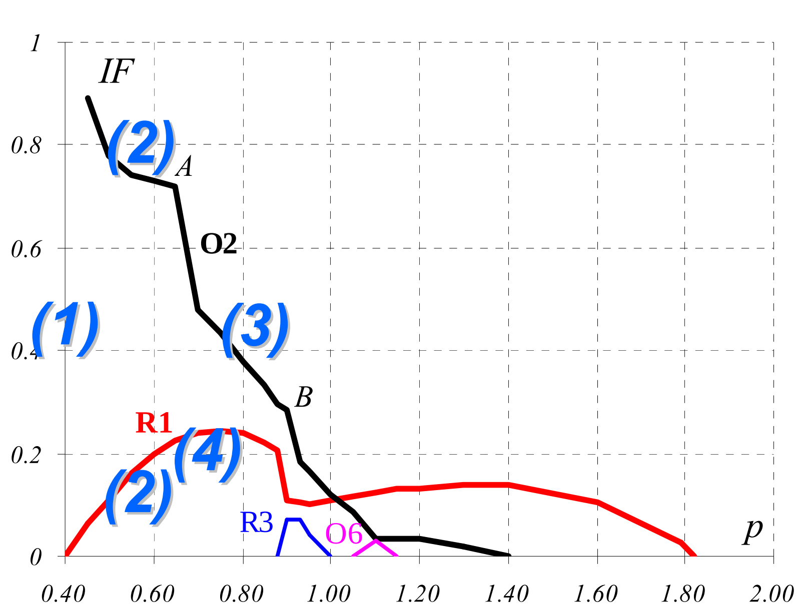
bifurcations

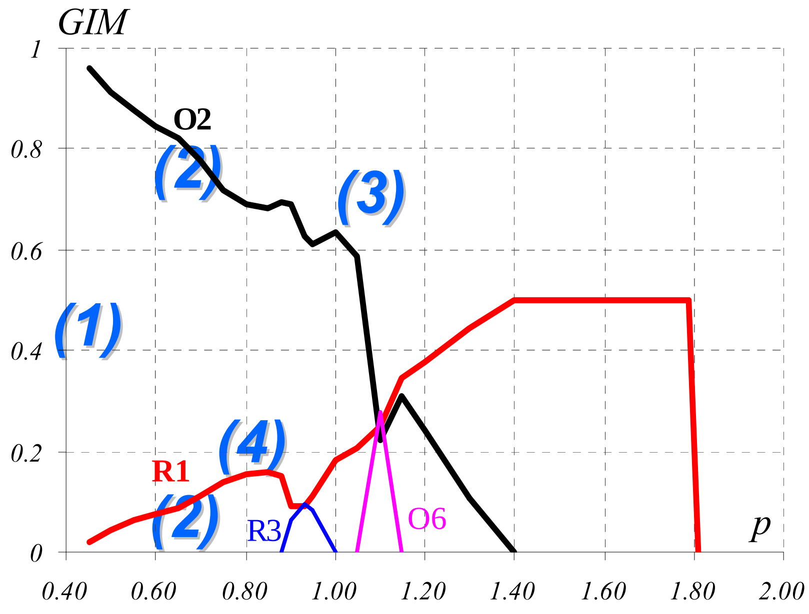
SN, PD	saddle-node, period-doubling
PF, H	pitchfork (or symmetry breaking), Hopf
CR	crisis
HOM/HET	homoclinic/heteroclinic

p	event	comment
0.196	H	the rest position loses stability. O2 appears
0.367	HOM1	homoclinic bifurcation of HS
0.418	SN	R1 appear through a SN bifurcation
0.655	HOM2	homoclinic bifurcation of DR1
0.888	SN	R3 appear through a SN bifurcation
0.935	HET	heteroclinic bifurcation of DR1 and Ir
0.948	PD	R3 undergo a PD bifurcation followed by a PD cascade
0.961	CR	the PD cascade of R3 ends by a CR. R3 disappear
1.082	SN	O6 appears through a SN bifurcation
1.111	PF	O6 undergoes a PF bifurcation, and two oscillating solutions of period 6, still named O6, appear
1.116	PD	O6 undergo a PD bifurcation followed by a PD cascade
1.118	CR	the PD cascade of O6 ends by a CR. O6 disappear
1.260	PF	O2 undergoes a PF bifurcation, and two oscillating solutions of period 2, still named O2, appear
1.332	PD	O2 undergo a PD bifurcation followed by a PD cascade
1.342	CR	the PD cascade of O2 ends by a CR. O2 disappear
1.349	PD	R1 undergo a PD bifurcation followed by a PD cascade
1.809	CR	the PD cascade of R1 ends by a CR. R1 disappear, and tumbling chaos becomes the unique attractor

- ***four main competing attractors (O2, R1, O6, R3)***
- ***ω=2 (parametric resonance)***

Pendulum: integrity profiles at parametric res., ω=2





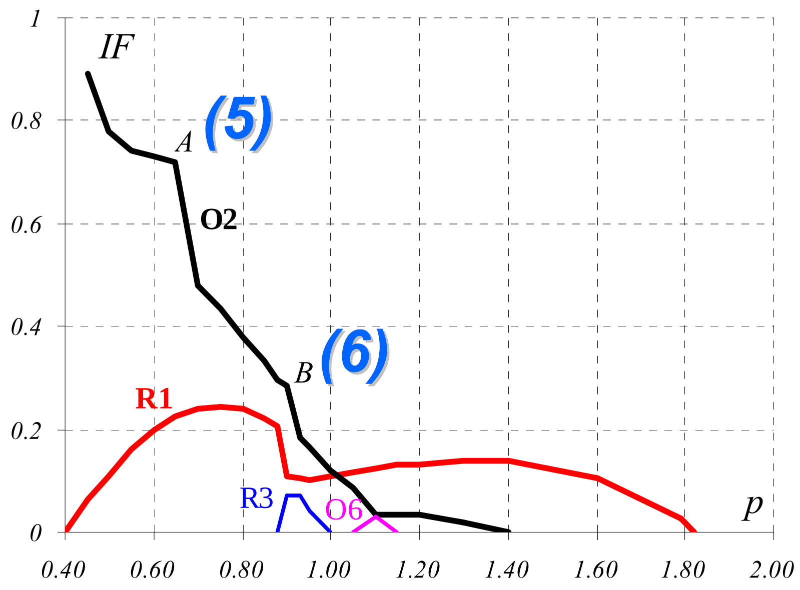
(1) starts when R1 are born by a SN

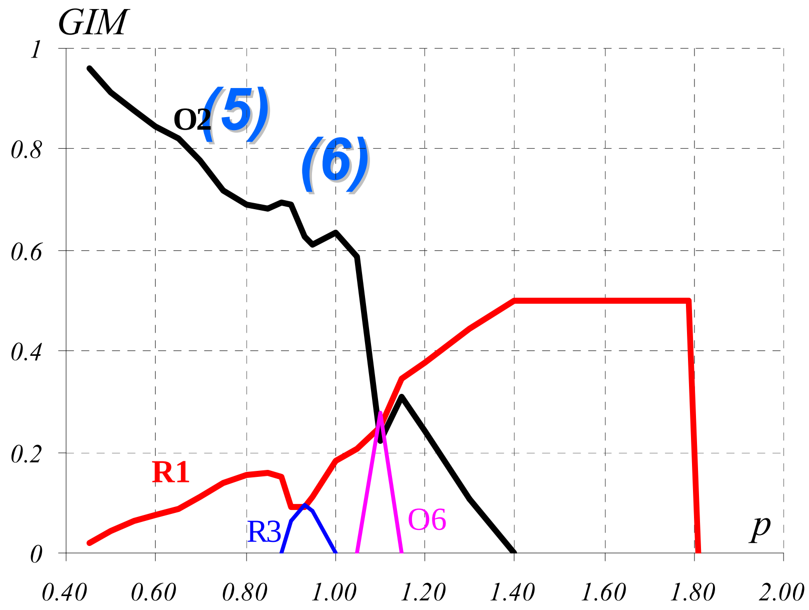
(2) R1 basins grow up against the O2 basin. This is described by IF and GIM, to a different extent

(3) both integrity curves of O2 have the classical "Dover cliff" behaviour

(4) IF and GIM integrity curves of R1 have a different qualitative behaviour

Pendulum: sudden falls

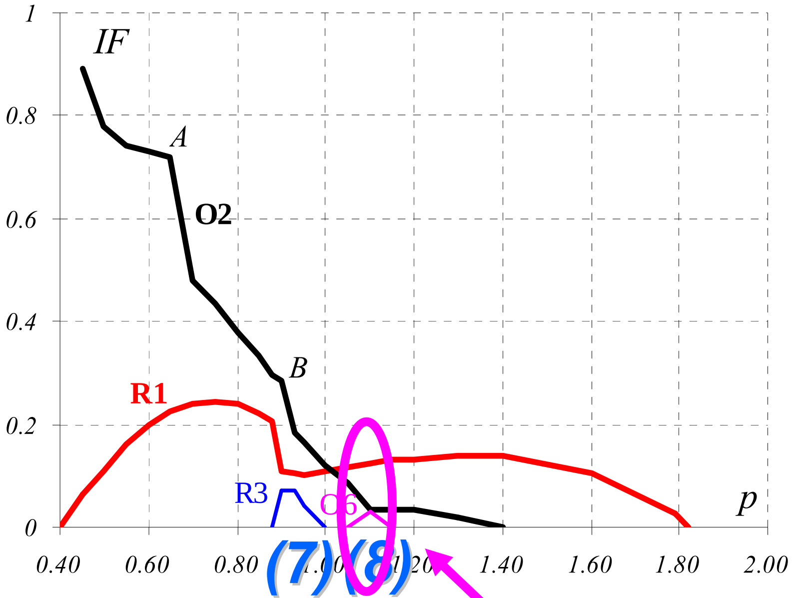


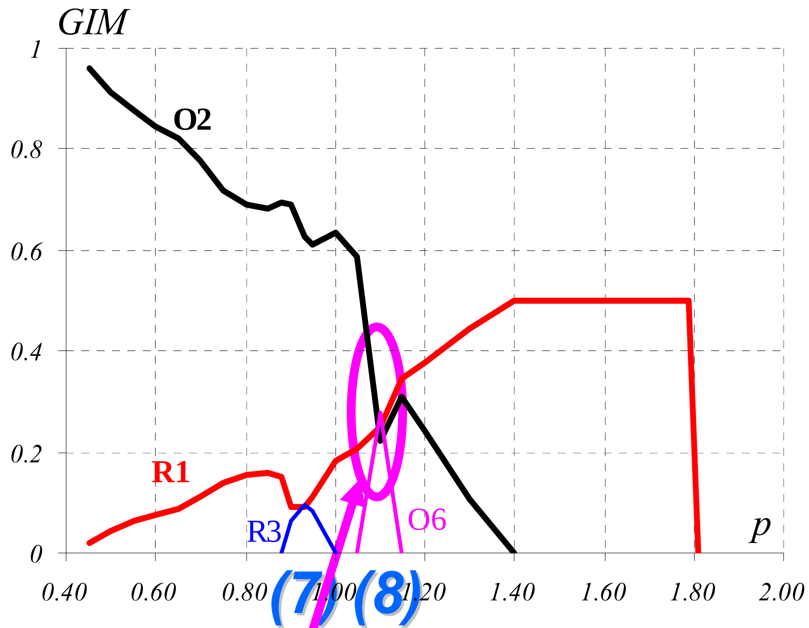


(5) sharp fall due to the homoclinic bifurcation of DR1: evidenced by IF but not by GIM

(6) sharp fall due to the het. bif. of DR1 and Ir: drastic reduction of the compact core of O2 basin clearly revealed by IF. With GIM this event is hardly recognizable (somehow hidden by the almost simultaneous appearance of R3)

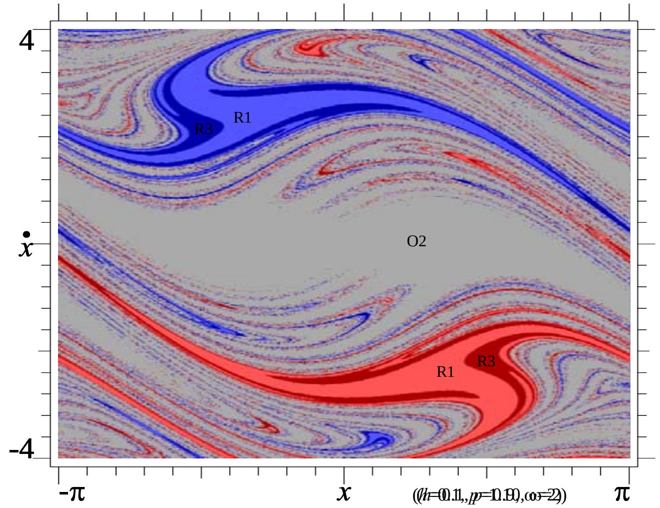
Pendulum: secondary attractors



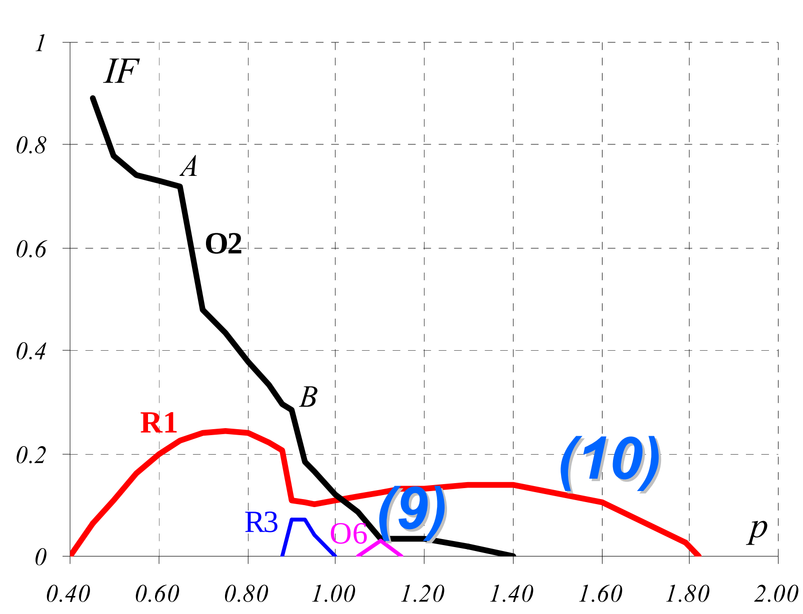


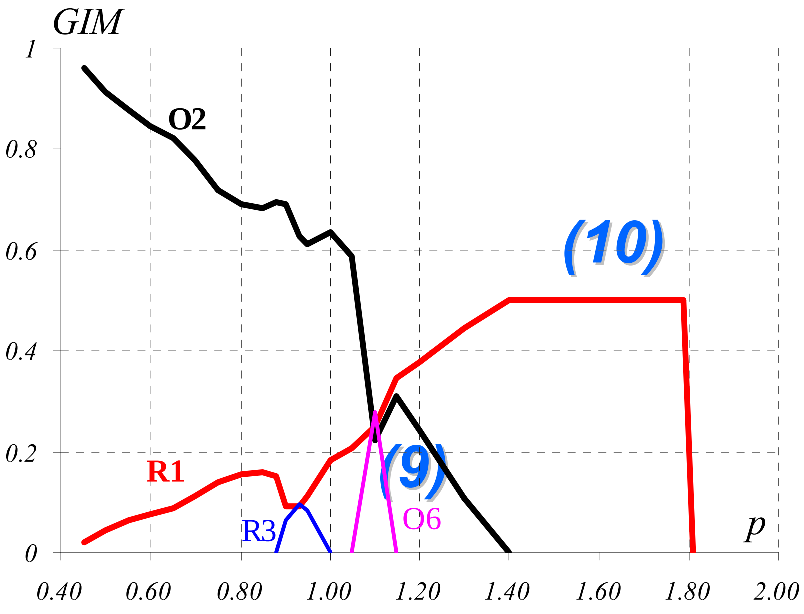
(7) *R3 appears inside the basins of R1, and thus have minor effects on O2*

(8) *O6 grows inside the O2 basin, thus having almost no effects on R1.* ***This is a case in which the GIM, which is commonly less performant, provides more information than the IF***

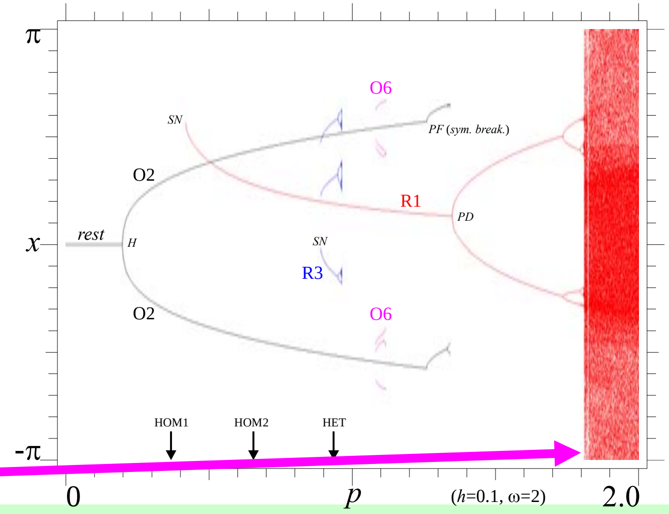


Pendulum: final part of the erosion



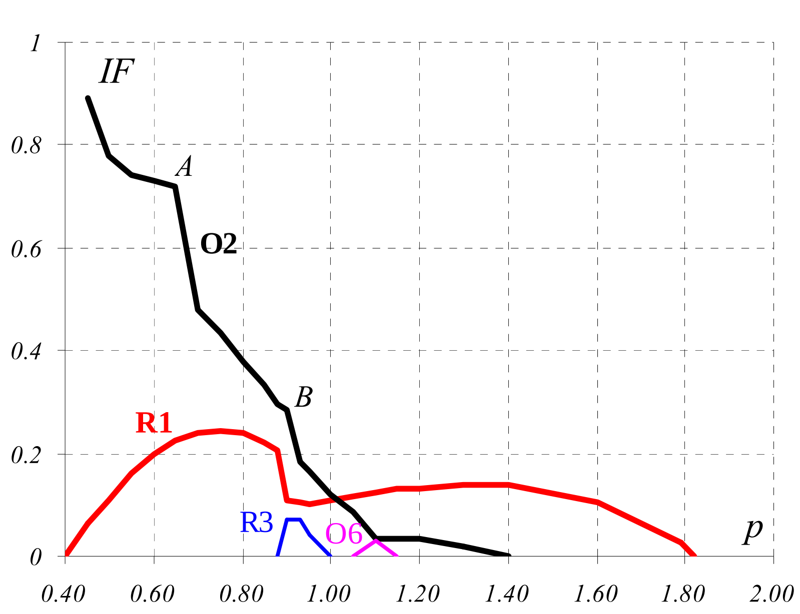


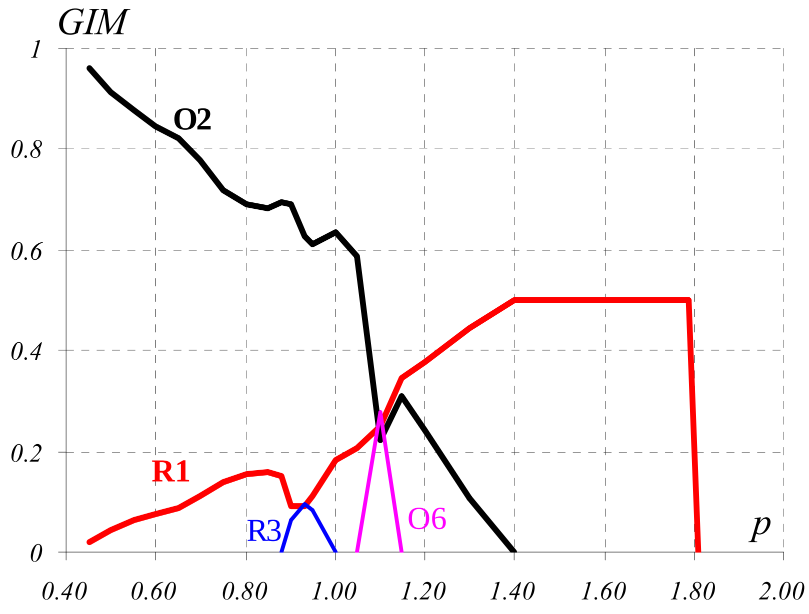
(9) O6 suddenly disappears, and O2 recovers a residual integrity by increasing the GIM and by keeping the IF constant



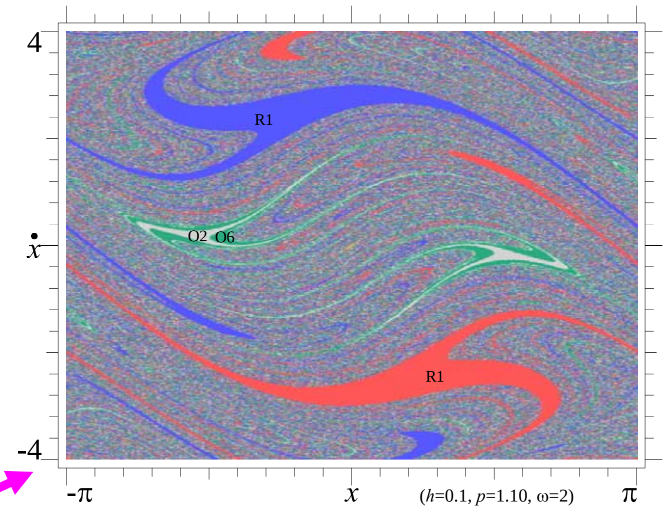
***(10) no further special events up to the end of the integrity profiles** (by the BC of the respective attractors)*

Pendulum: oscillating solutions

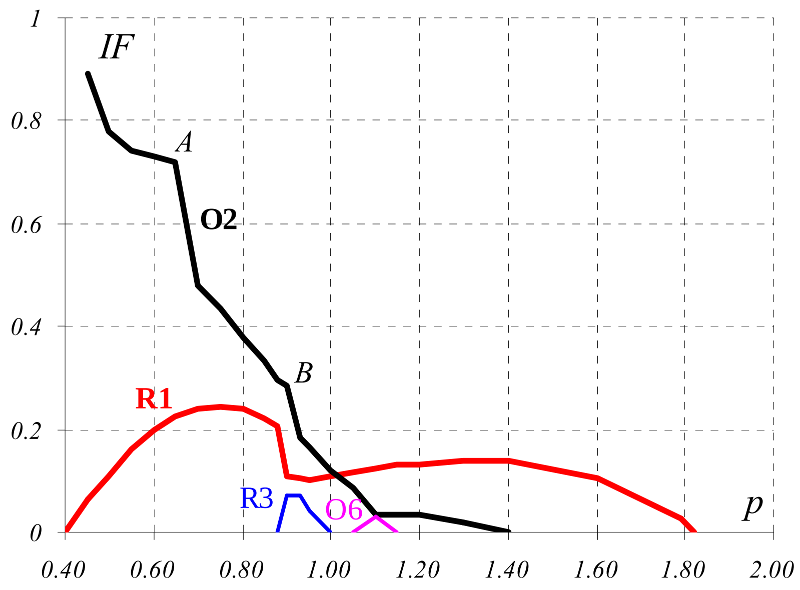


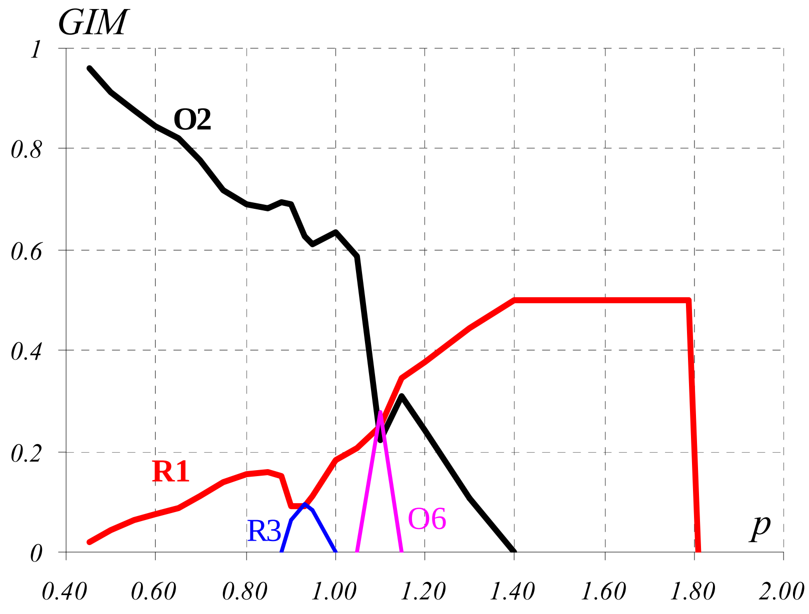


- ***IF and GIM erosion profiles of O2 are qualitatively similar. Differences in the final part: GIM→0 rapidly, IF →0 slowly***
- ***GIM>>IF in the final part, thus GIM overestimates integrity of O2, which is residual***

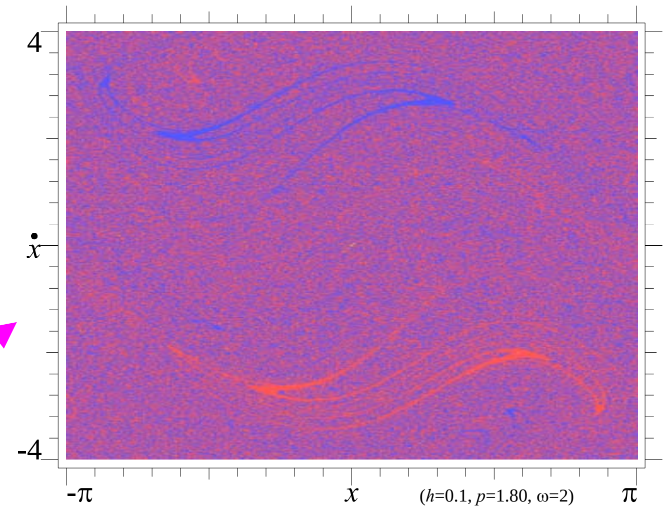


Pendulum: rotating solutions (1)

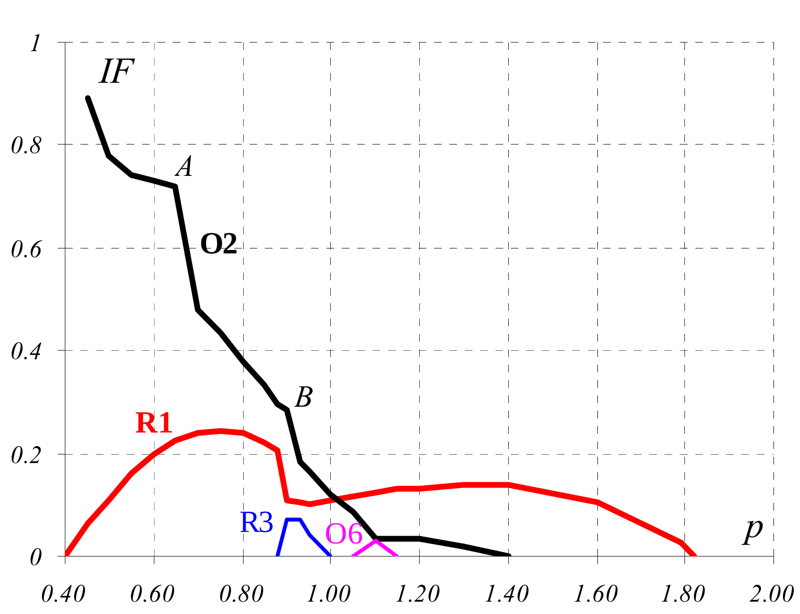


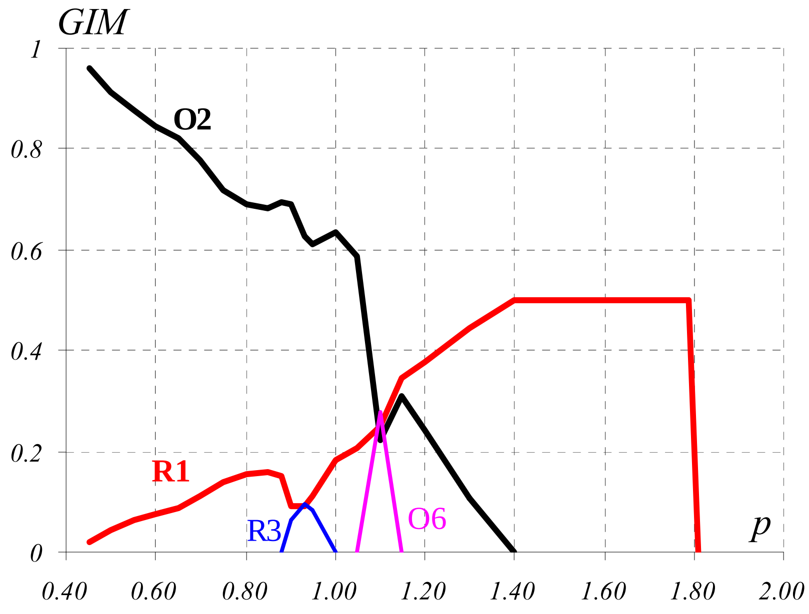


- ***R1 change 'status' for growing p. Initially they erode other (passive) attractors. Then, they are eroded by the secondary attractors, and finally they disappear by a reciprocal (self-)erosion***



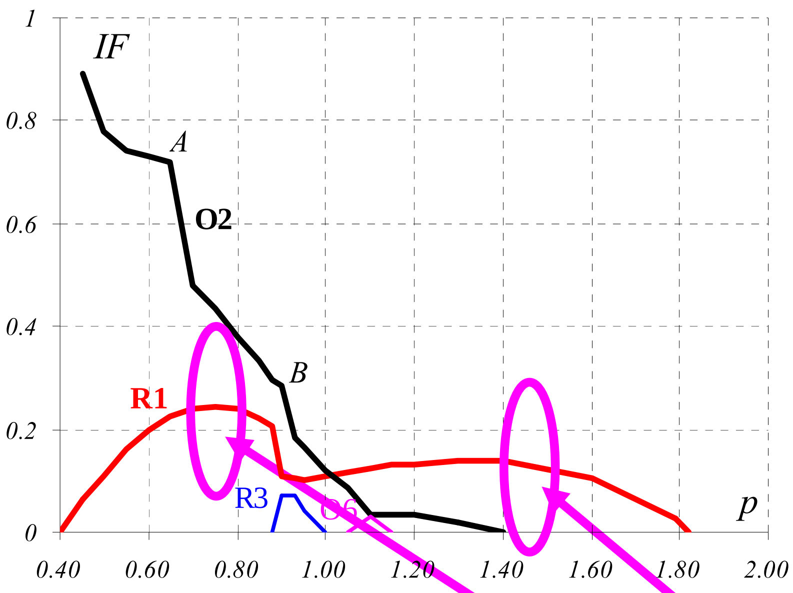
Pendulum: rotating solutions (2)

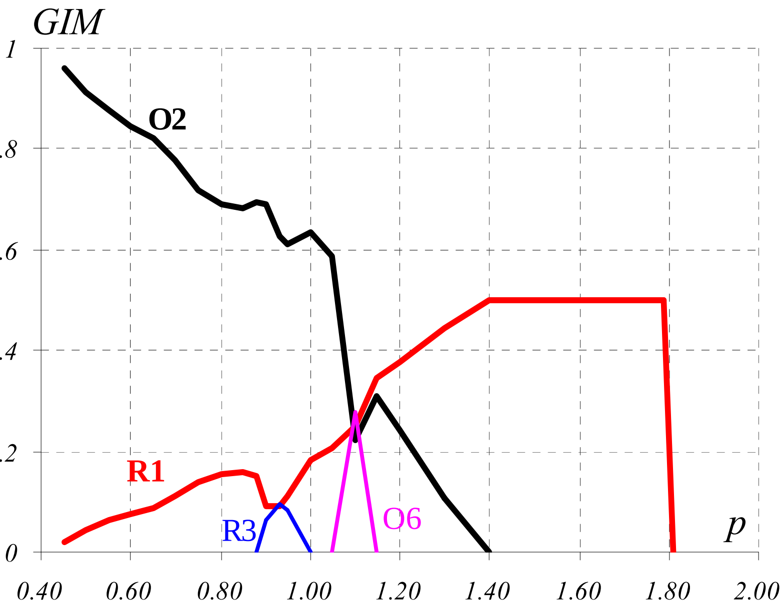




- ***differences between the IF and the GIM of R1 are much more marked than those of O2***
- ***GIM is** (almost monotonically) **increasing up to 0.5***
- ***IF initially increases, reaches a maximum, starts a dull decrement, undergoes a sudden falls due to R3, slightly increases and then slowly decreases again up to the end***

Pendulum: attractor robustness and basin integrity





- ***qualitative difference of IF and GIM: GIM is basically also a measure of attractor robustness, whereas IF is a measure of basin integrity, of major interest for safe design***
- ***sharp (O2) vs flat (R1) IF profiles***
- ***optimal operating conditions for R1***

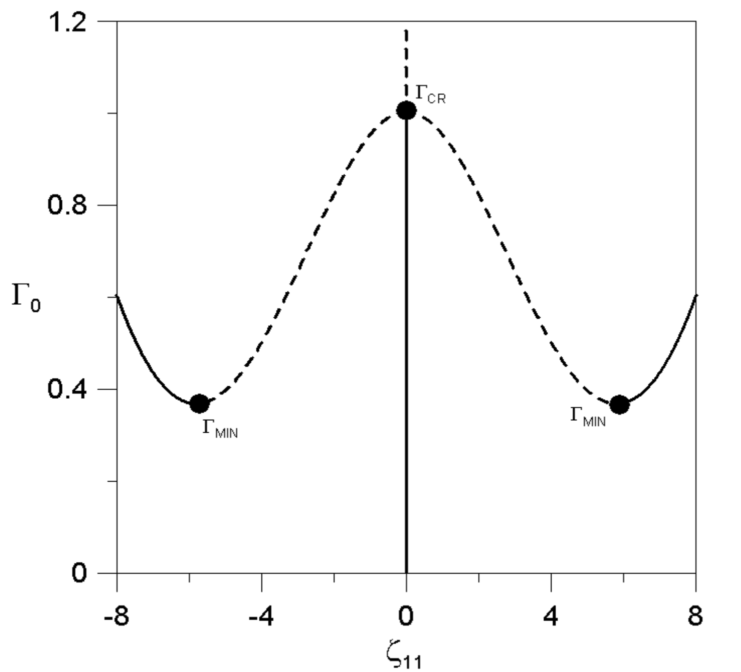
Parametrically excited cylindrical shell (2-dof model)

$$\ddot{\zeta}_{11} + 0.150761\dot{\zeta}_{11} + 1.043914\zeta_{11} + 9.274215\zeta_{11}\zeta_{02} - 1.040775\Gamma_1\cos(\Omega\tau)\zeta_{11}$$
$$-1.043914\Gamma_0\zeta_{11} + 0.274896\zeta_{11}^3 + 0.188199\zeta_{11}\zeta_{02}^2 = 0$$

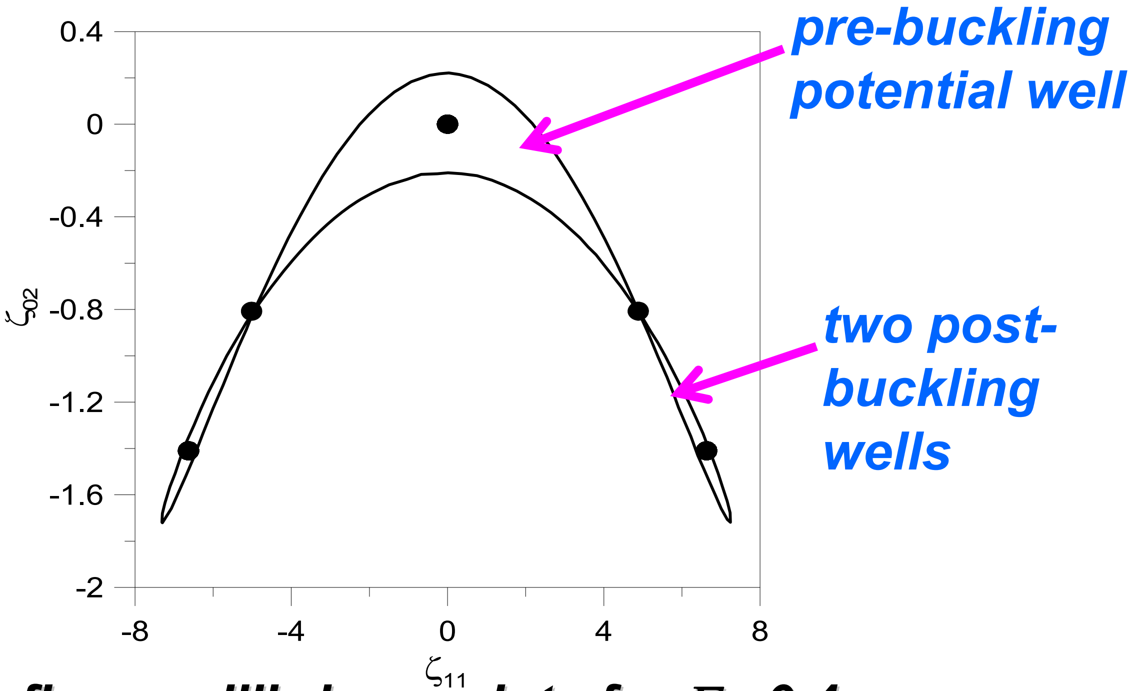
$$\ddot{\zeta}_{02} + 0.02086\dot{\zeta}_{02} - 4.16310\Gamma_0\zeta_{02} - 4.16310\Gamma_1\cos(\Omega\tau)\zeta_{02} + 69.756712\zeta_{02}$$
$$+ 2.318554\zeta_{02}^2 + 0.094099\zeta_{11}^2\zeta_{02} = 0$$

Gonçalves, Silva, Rega, Lenci, 2009

ζ_{11}, ζ_{02} *basic, axisymm. mode with twice number of half waves in axial direction*



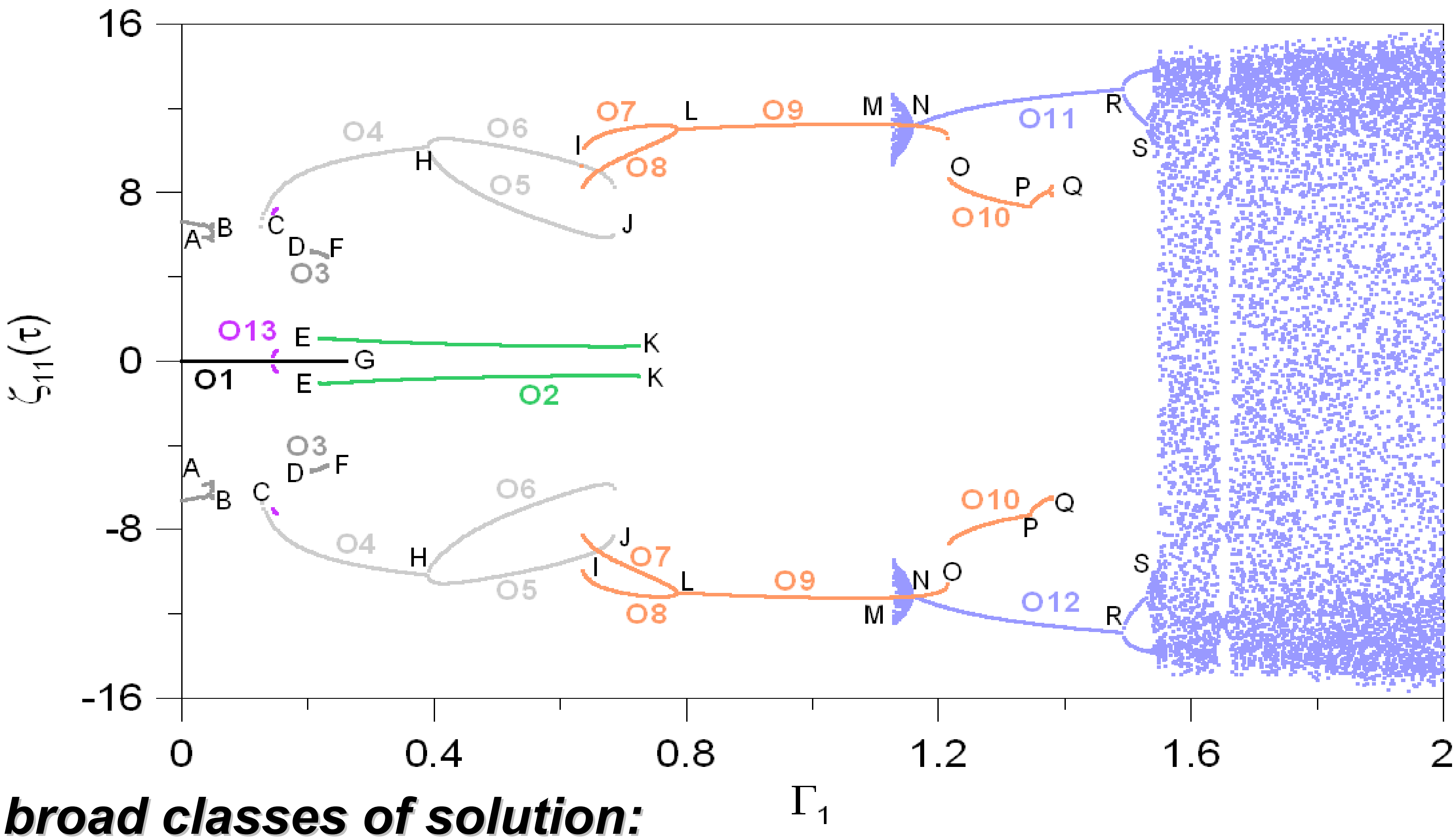
post-buckling response path (Γ$_0$, static load)



five equilibrium points for Γ$_0$=0.4 (two heteroclinic and two homoclinic orbits)

Shell, sub-critical scenario: bifurcation diagram

increasing axial load amplitude Γ_1 in the main parametric instability region



five different broad classes of solution:

(1) trivial pre-buckling,

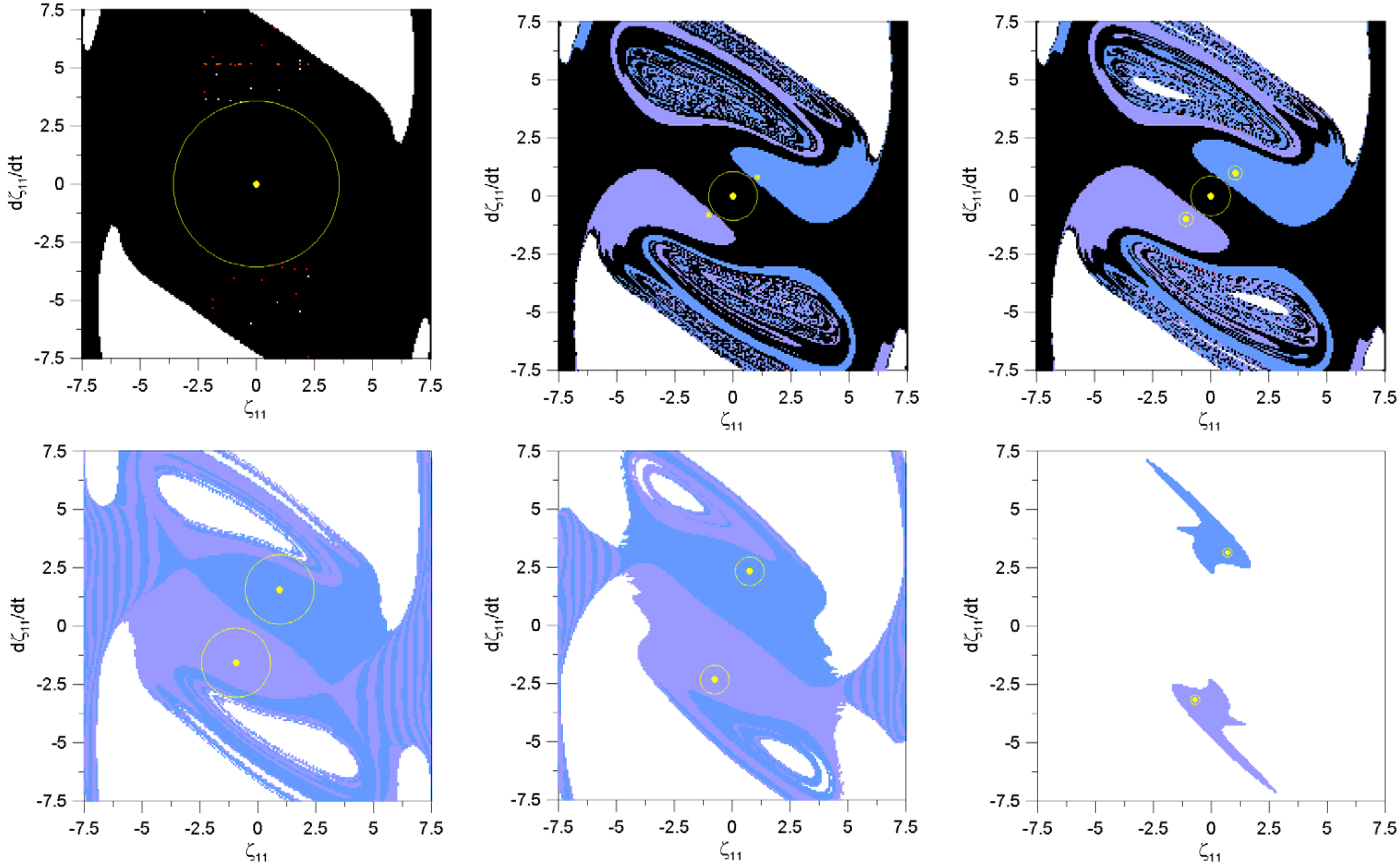
(2) non-trivial 2T within the pre-buckling well,

(3) small amplitude vibrations within each of the post-buckling wells,

(4) medium amplitude cross-well,

(5) very large-amplitude cross-well period three, robust in the range

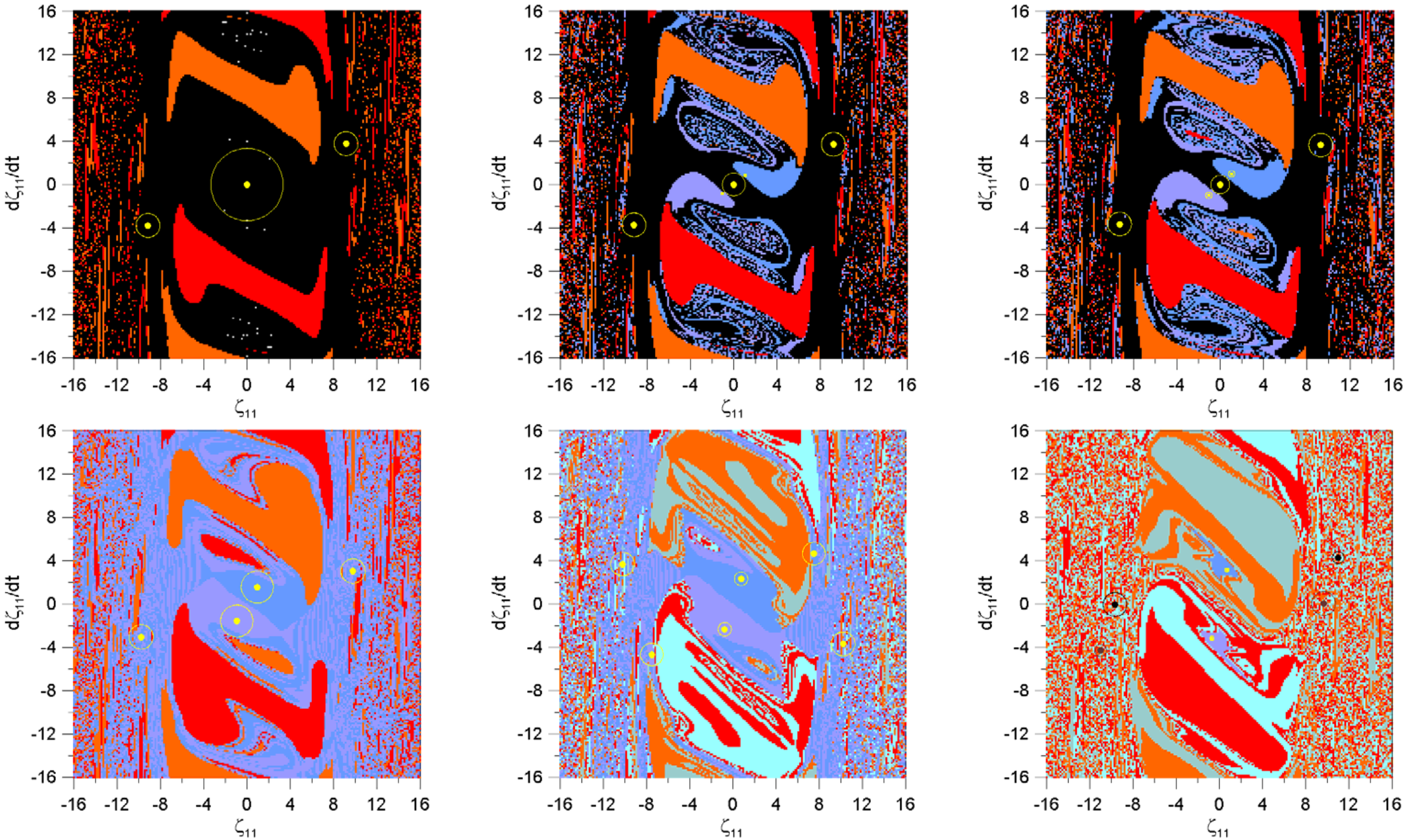
Shell, sub-critical: attractor-basin portrait (1)



cross-sections of 4D basins of attraction: in-well pre-buckling attractors

Black: trivial. Light and dark blue: period two. White: escape

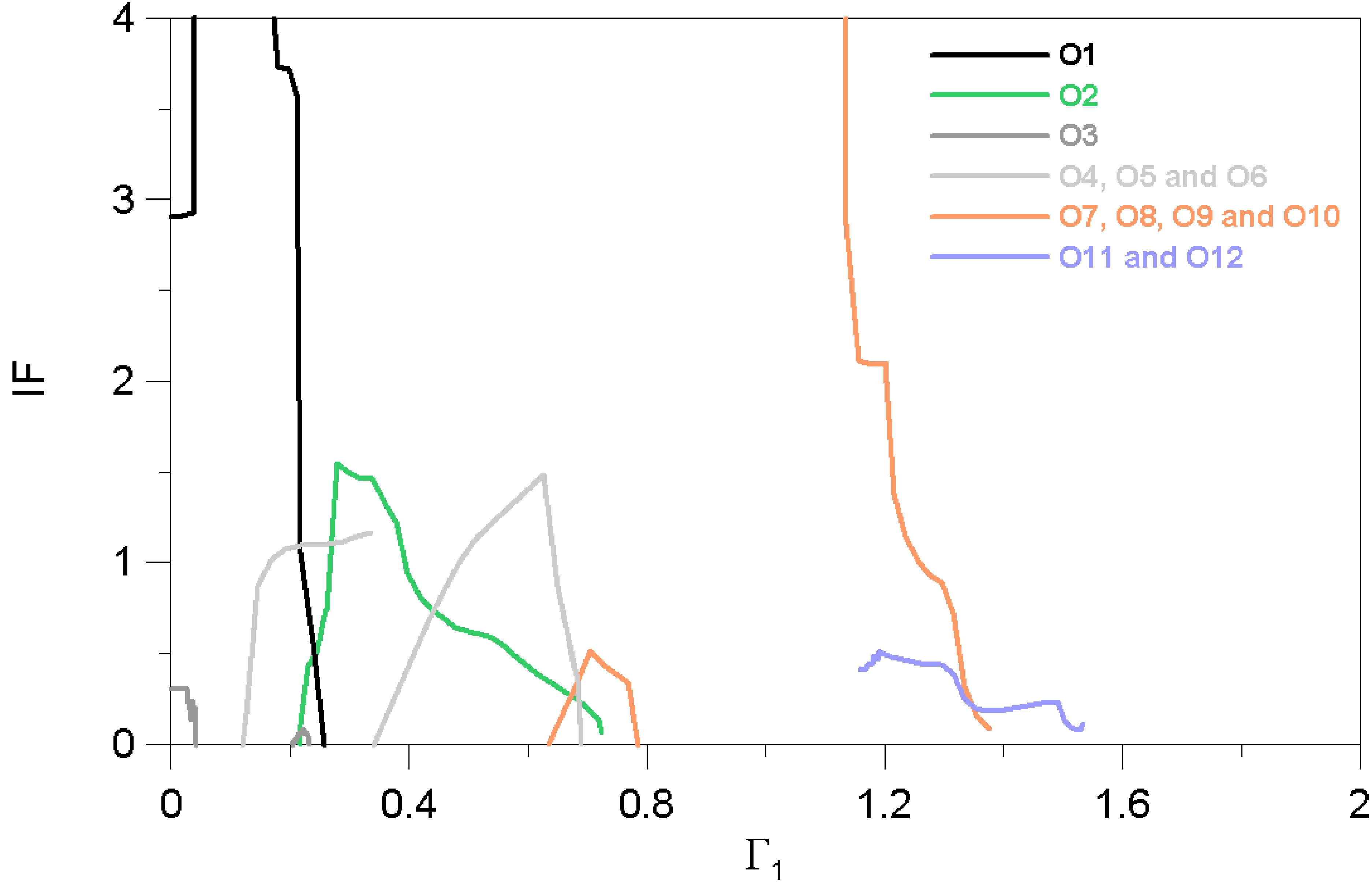
Shell, sub-critical: attractor-basin portrait (2)



Topological complexity of in-well and out-of-well attractors.

***Remark:* Being basins of attraction in a 4D hyper-volume, it is not easy to detect touching of the hypersphere with the nearest competing basin**

Shell, sub-critical scenario: dynamic integrity



erosion profiles of competing attractors

Dynamical integrity and experiments

MEMS

Parametrically excited pendulum

Motivations

☑ ***theoretical work***
- ***concept, definitions, safe basins, integrity measures, etc.***

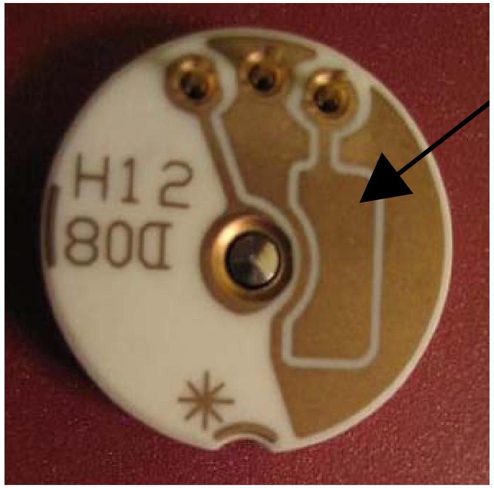
☑ ***numerical work***
- ***analyses of the dynamics of various mechanical systems and model by extensive numerical simulations***

☐ ***experimental work?***
- ***is dynamical integrity also useful in experiments?***
- ***can it help in explaining some 'strange' behaviour?***

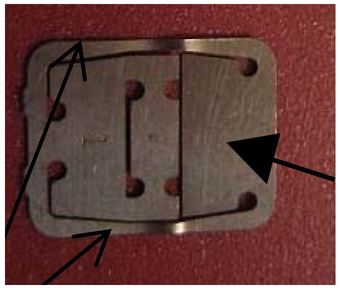
MEMS: the mechanical system

Al Saleem, Younis, JMM, 2008; Ruzziconi, Younis, Lenci, 2009

lower electrode (substrate)



upper electrode (proof mass)

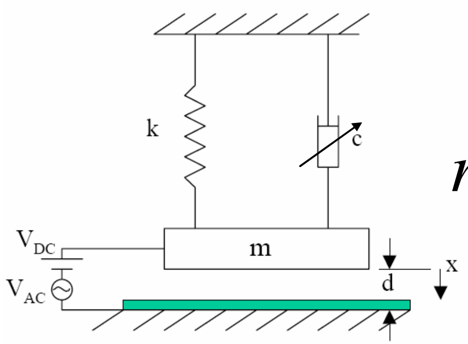


cantilever beams

capacitive accelerometer

the proof mass is suspended over the substrate by the two cantilever beams

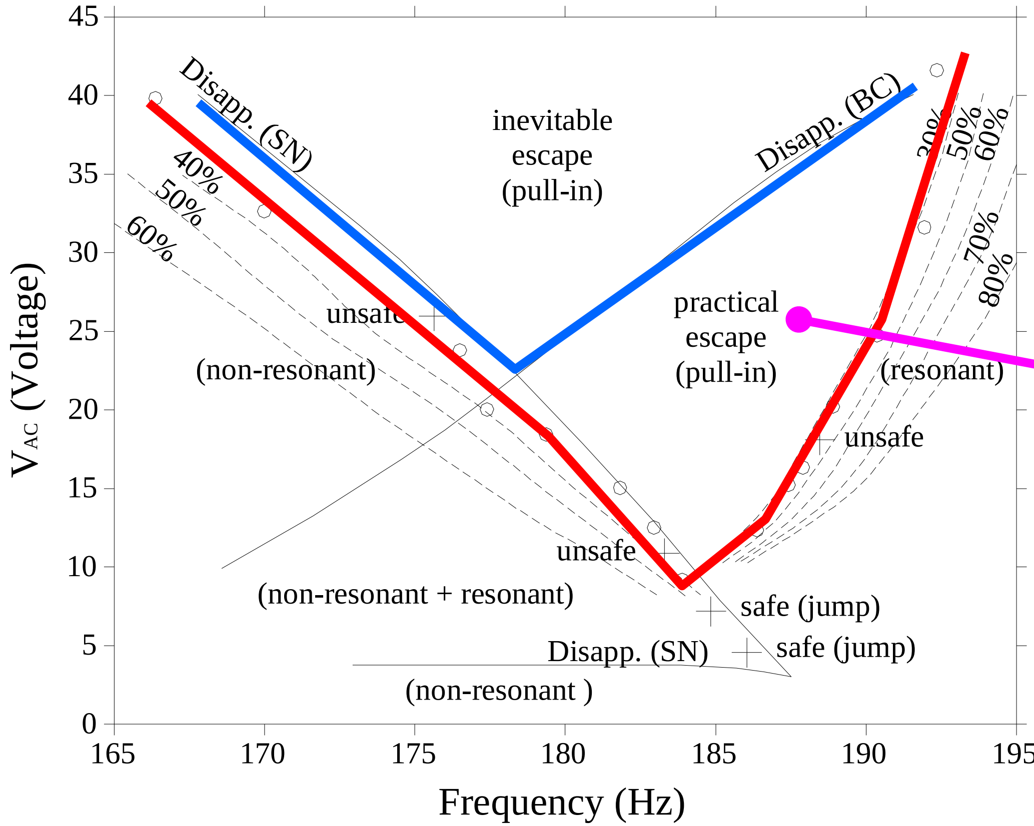
s.d.o.f. model



$$m\ddot{x} + c(x)\dot{x} + kx = \frac{\varepsilon A[V_{DC} + V_{AC}\cos(\Omega t)]^2}{2(d-x)^2}$$

MEMS: escape (pull-in) threshold (1)

dot and crosses = experimental escape threshold



theoretical threshold

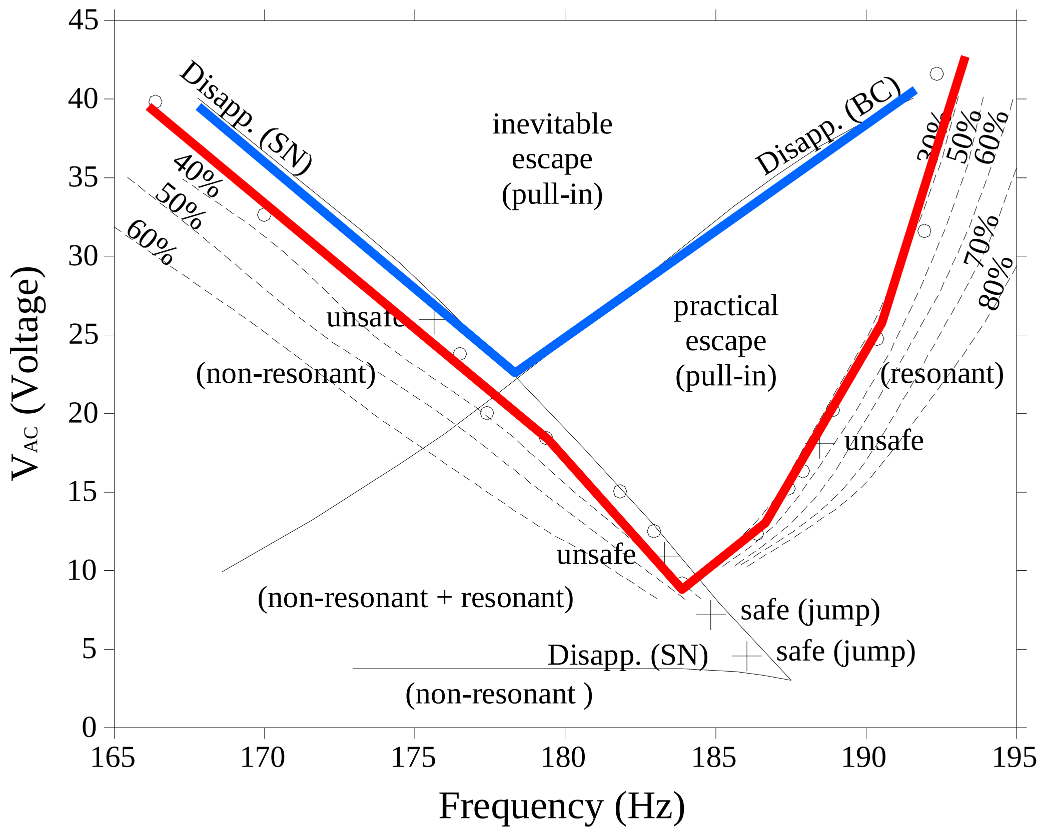
experimental threshold

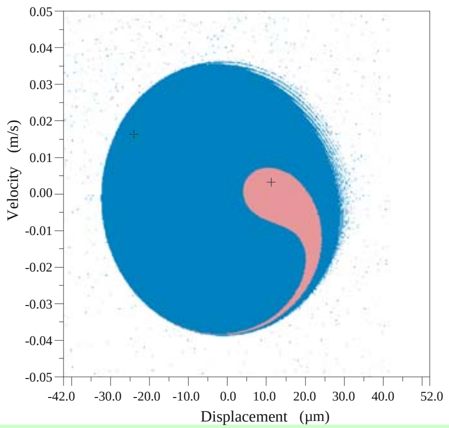
discrepancy between theoretical and experimental threshold

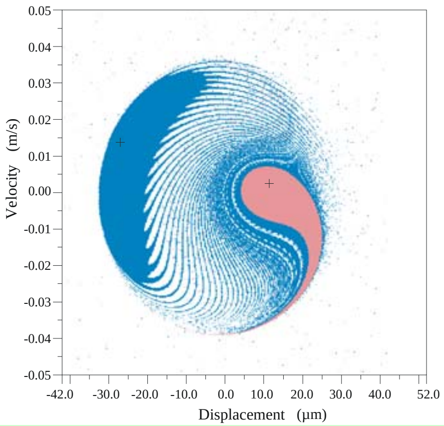
- ***escape (pull-in) occurs when IF is 40%: remarkable coincidence of experimental pull-in and contour level of IF***

MEMS: escape (pull-in) threshold (2)

dot=experimental escape threshold







the discrepancy can be explained by noting that the basins are eroded, so that, while theoretically present, they are practically unaccesible, not even by slow sweeping (to much noise)

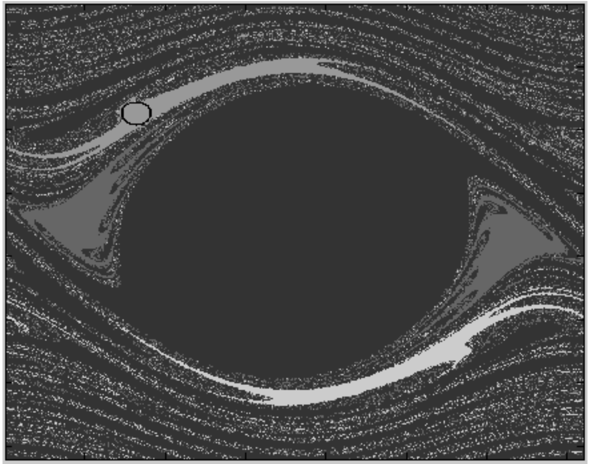
Parametrically excited pendulum

$$\ddot{\varphi}+0.015\dot{\varphi}+[1+p\cos(\omega t)]\sin(\varphi)=0$$

Luzi, Venturi, Lenci, 2008

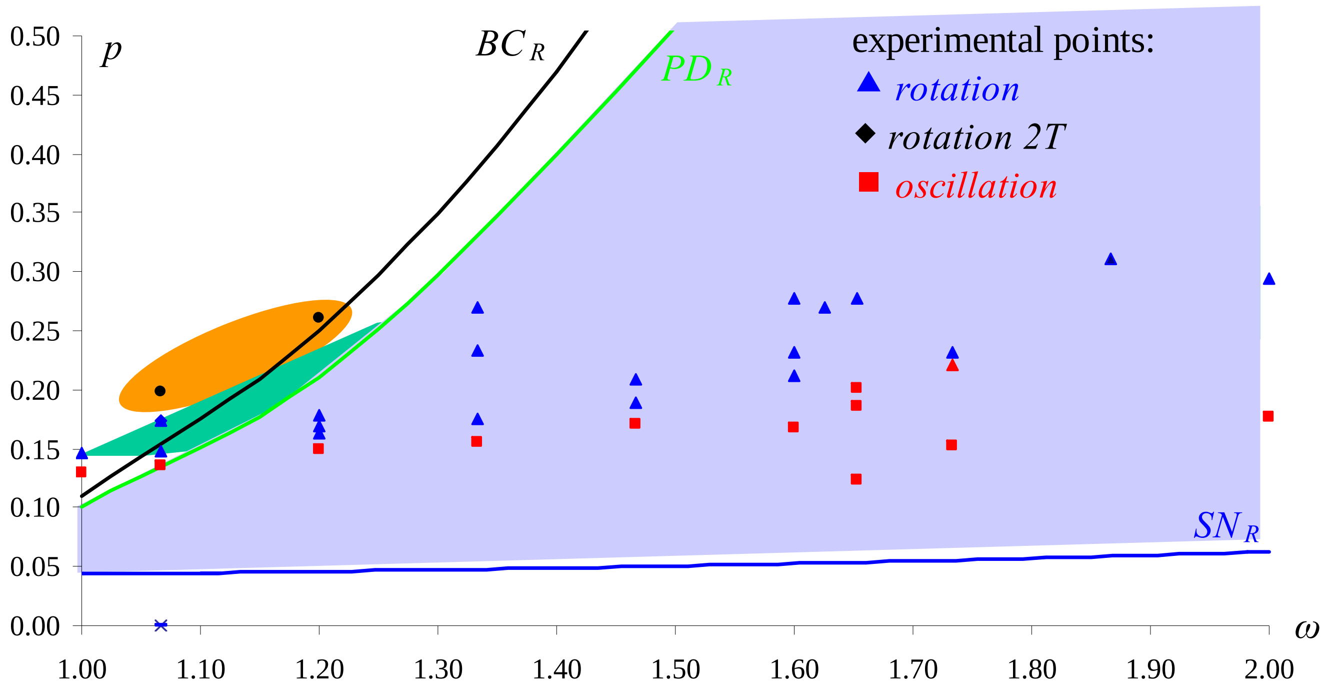
- ***damping measured experimentally from free damped oscillations***
- ***main interest in rotations***
- ***rotations have small basins with respect to (competing) oscillations, so they are difficult to be detected experimentally***
- ***theoretical analysis developed before***

excitation



Pendulum: existence of rotations

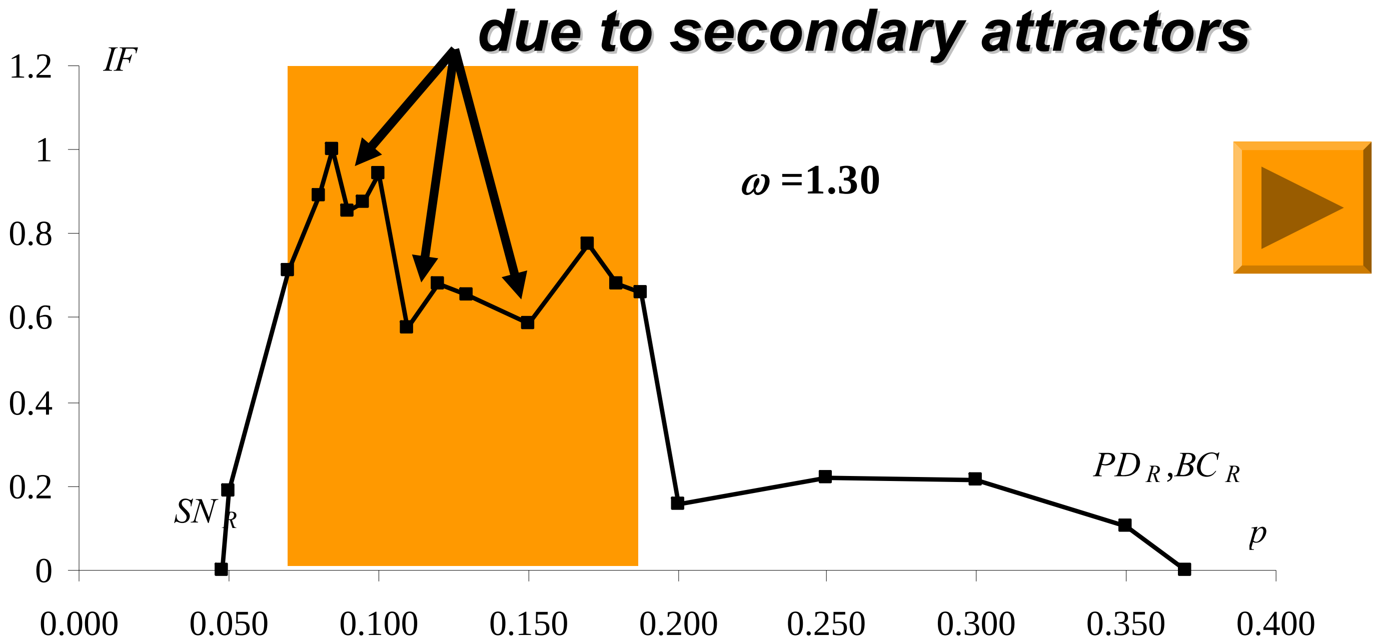
- ***analytical (numerical) vs experimental results***



- ***PD captured experimentally***
- ***theory: rotations exist in a large region***
- ***experiments: rotations exist in a narrow strip***

Pendulum: erosion profile

- ***the difference can be explained by looking at the erosion profile of rotation:***



- ***only in the central strip IF is high enough to 'overcome' imperfections***

Conclusions on experiments

- ***these examples agree in showing that when the dynamical integrity is residual the attractor cannot be detected***
- ***loss of dynamical integrity corresponds to practical instability***
- ***IF is a good measure to assess practical existence of attractors***
- ***regions of large IF facilitate the application of control methods***

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- …

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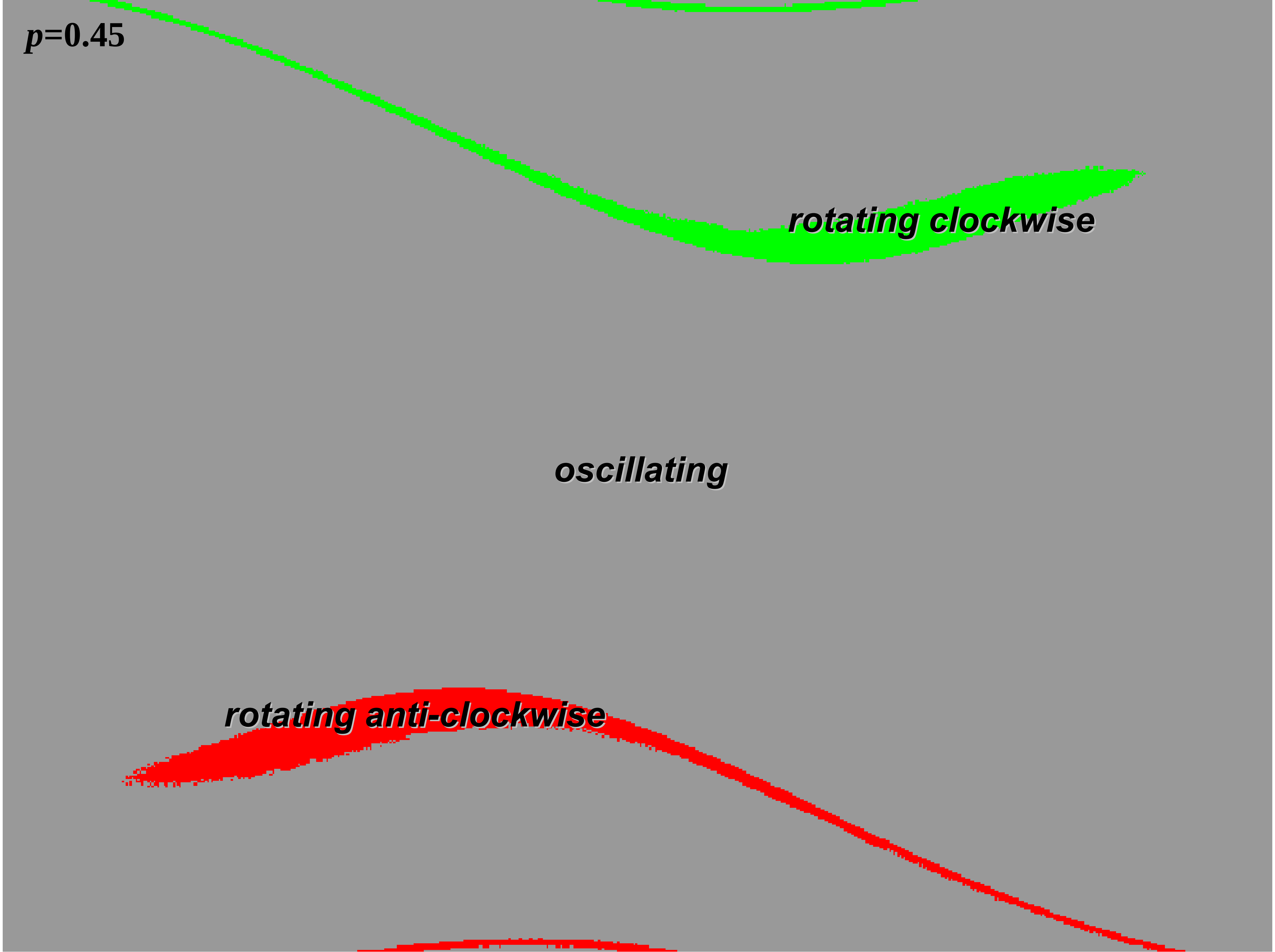
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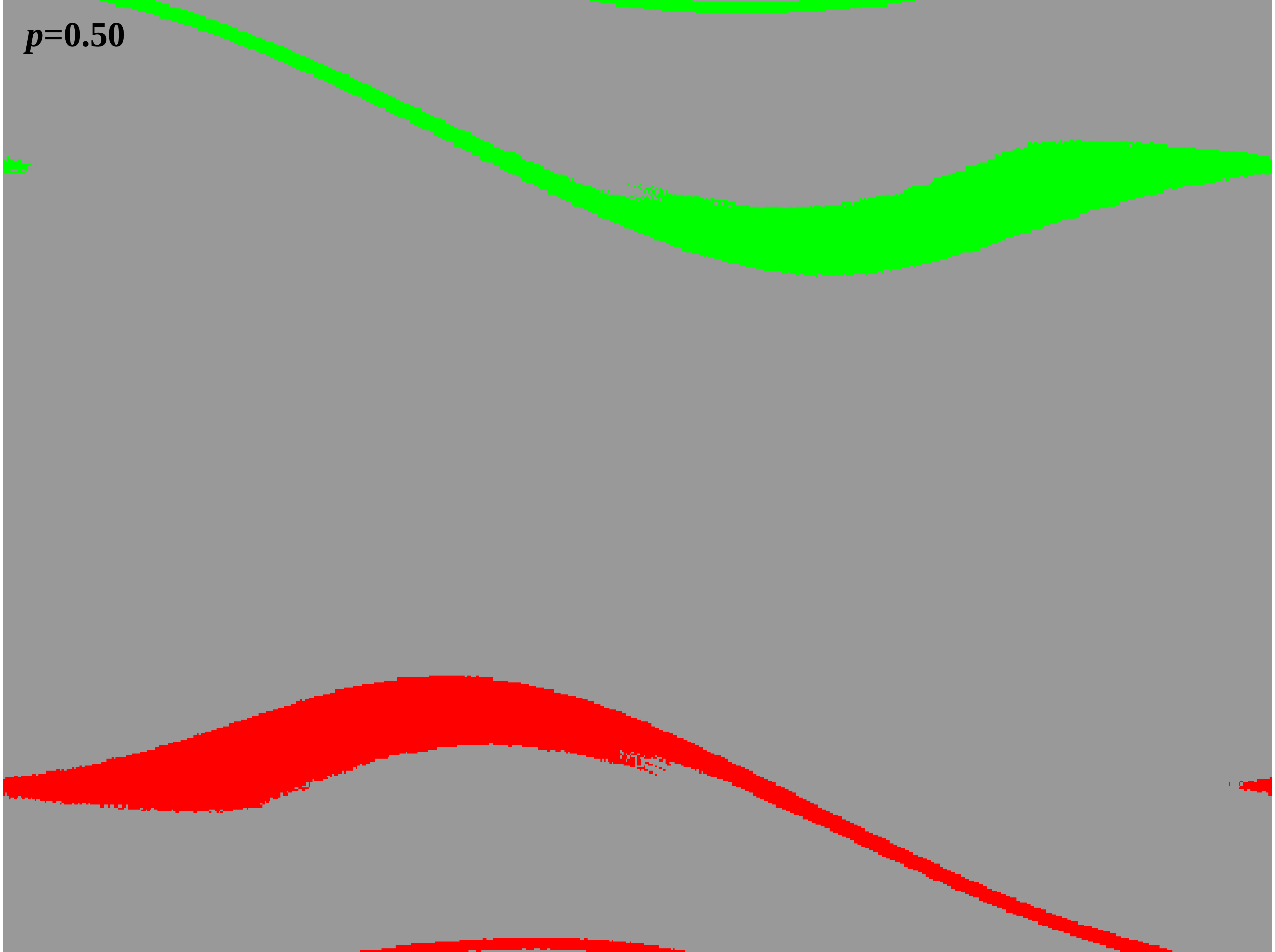
rotating clockwise
oscillating
rotating anti-clockwise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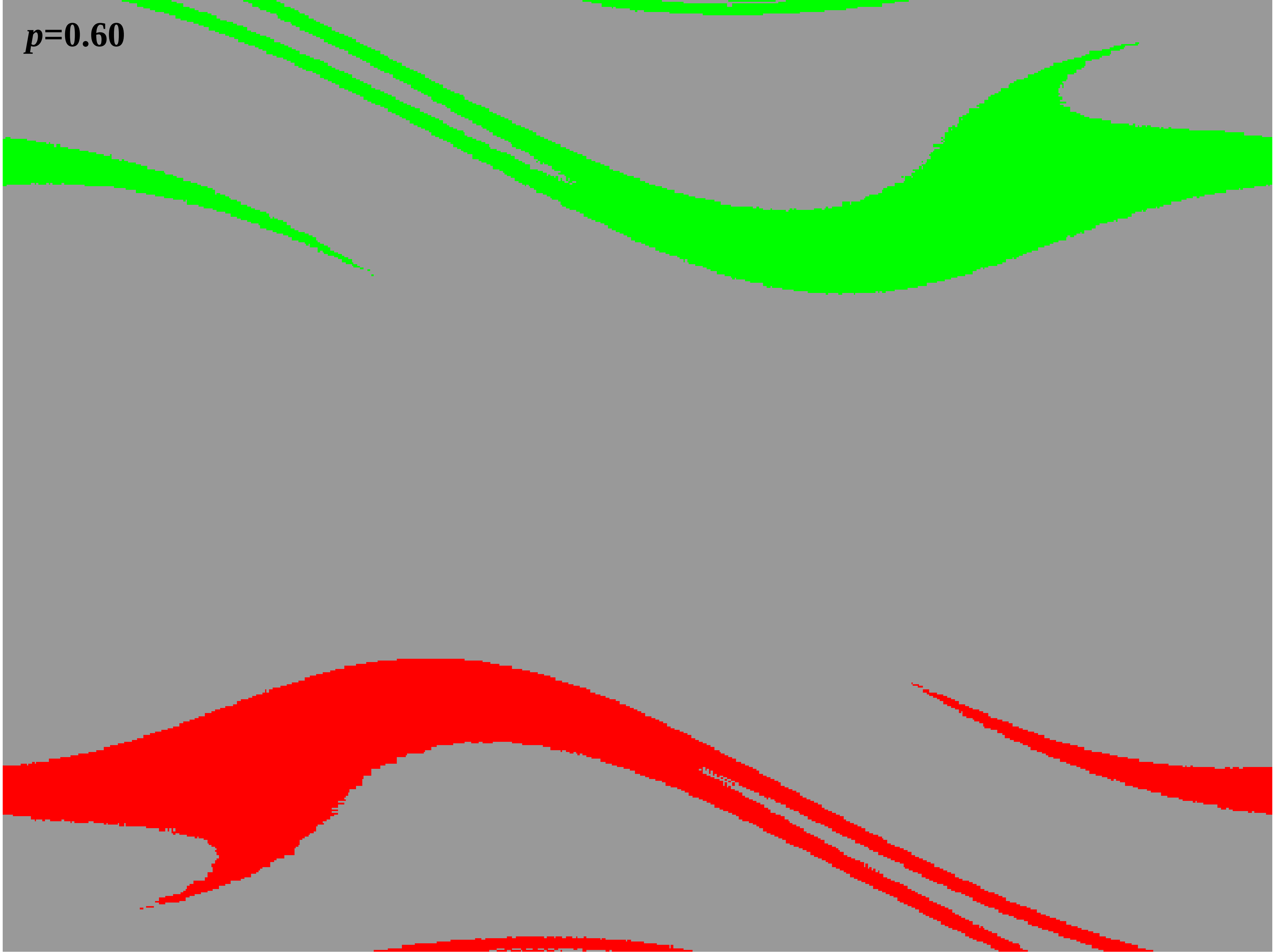
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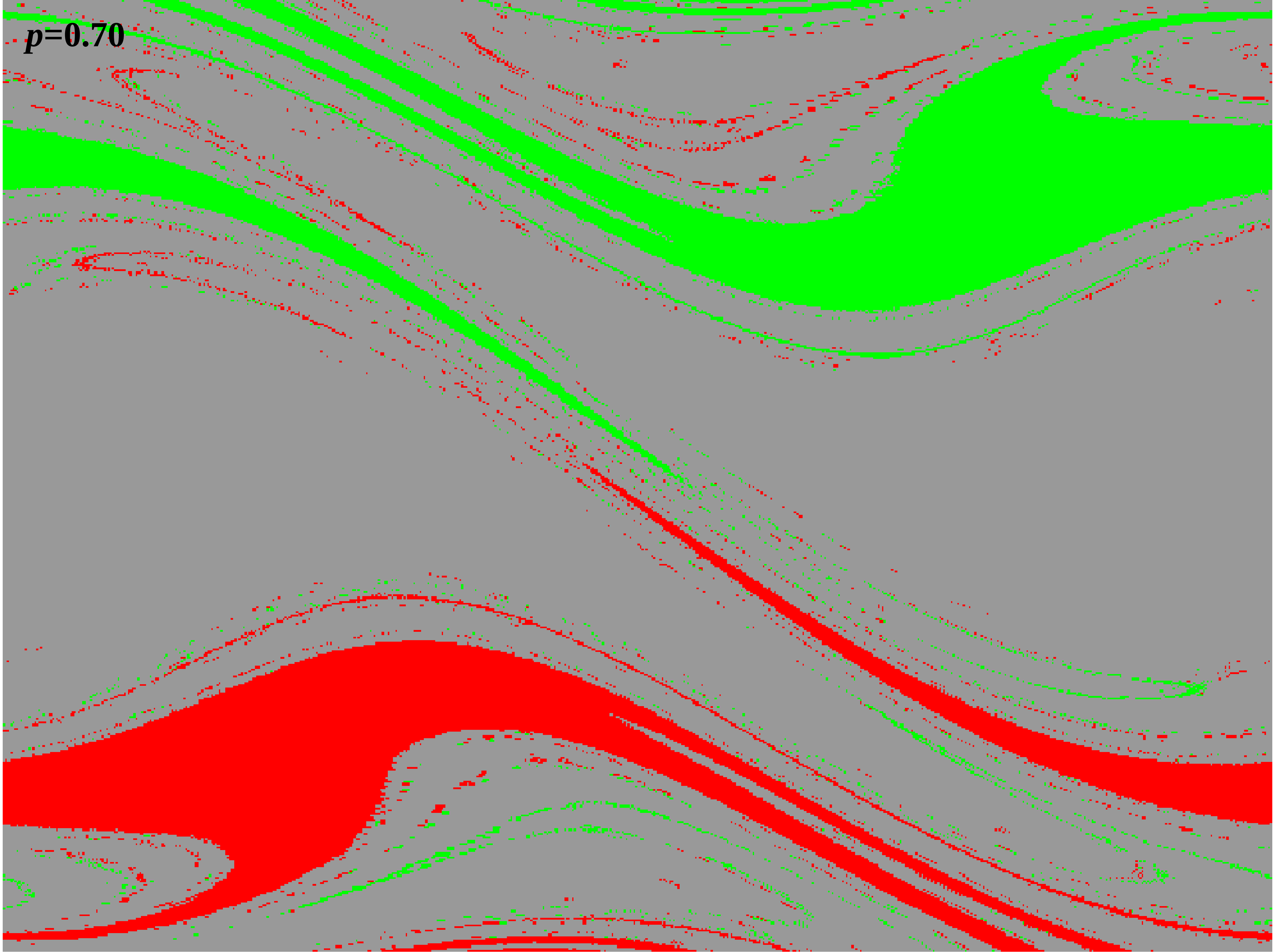
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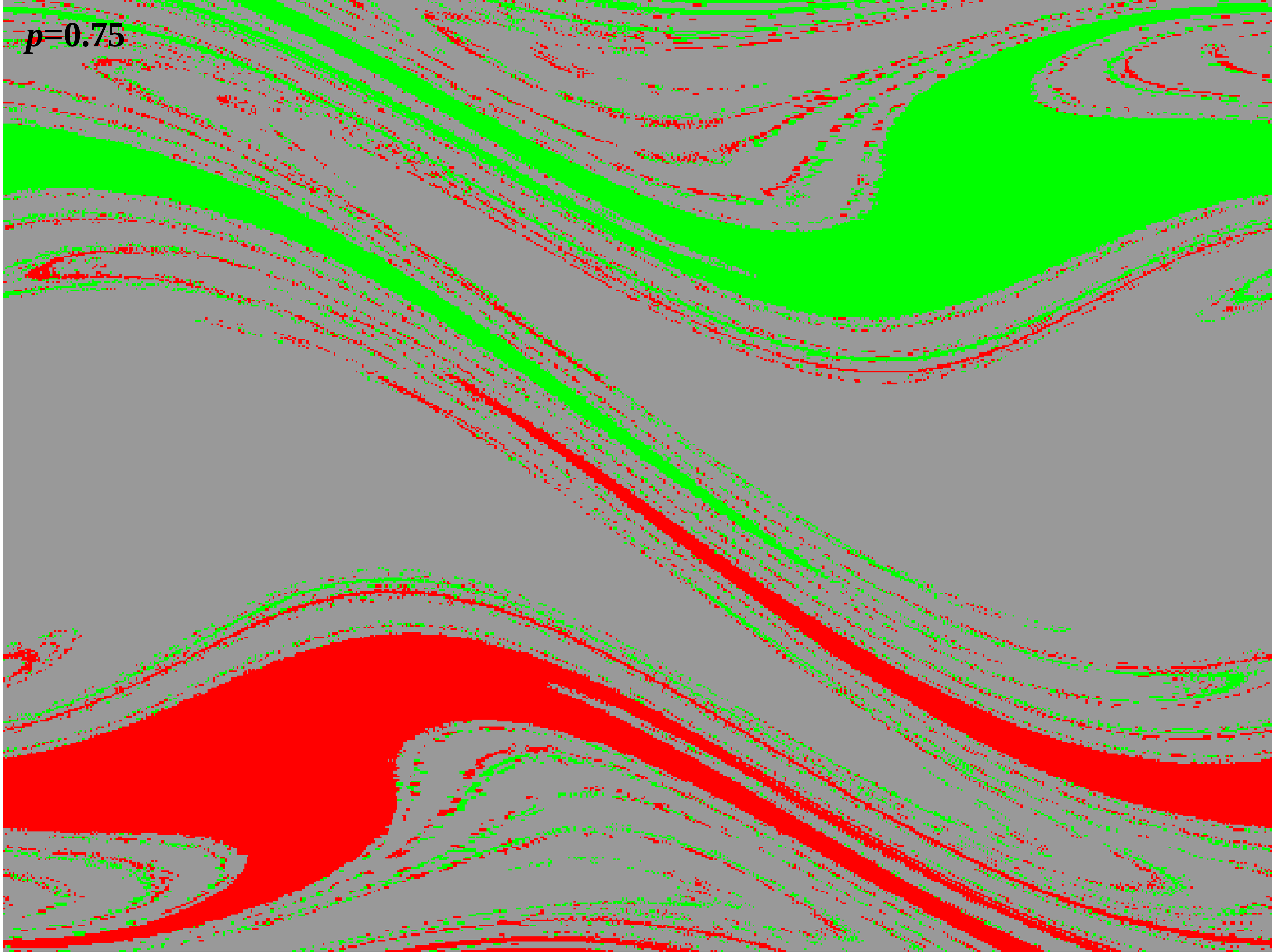
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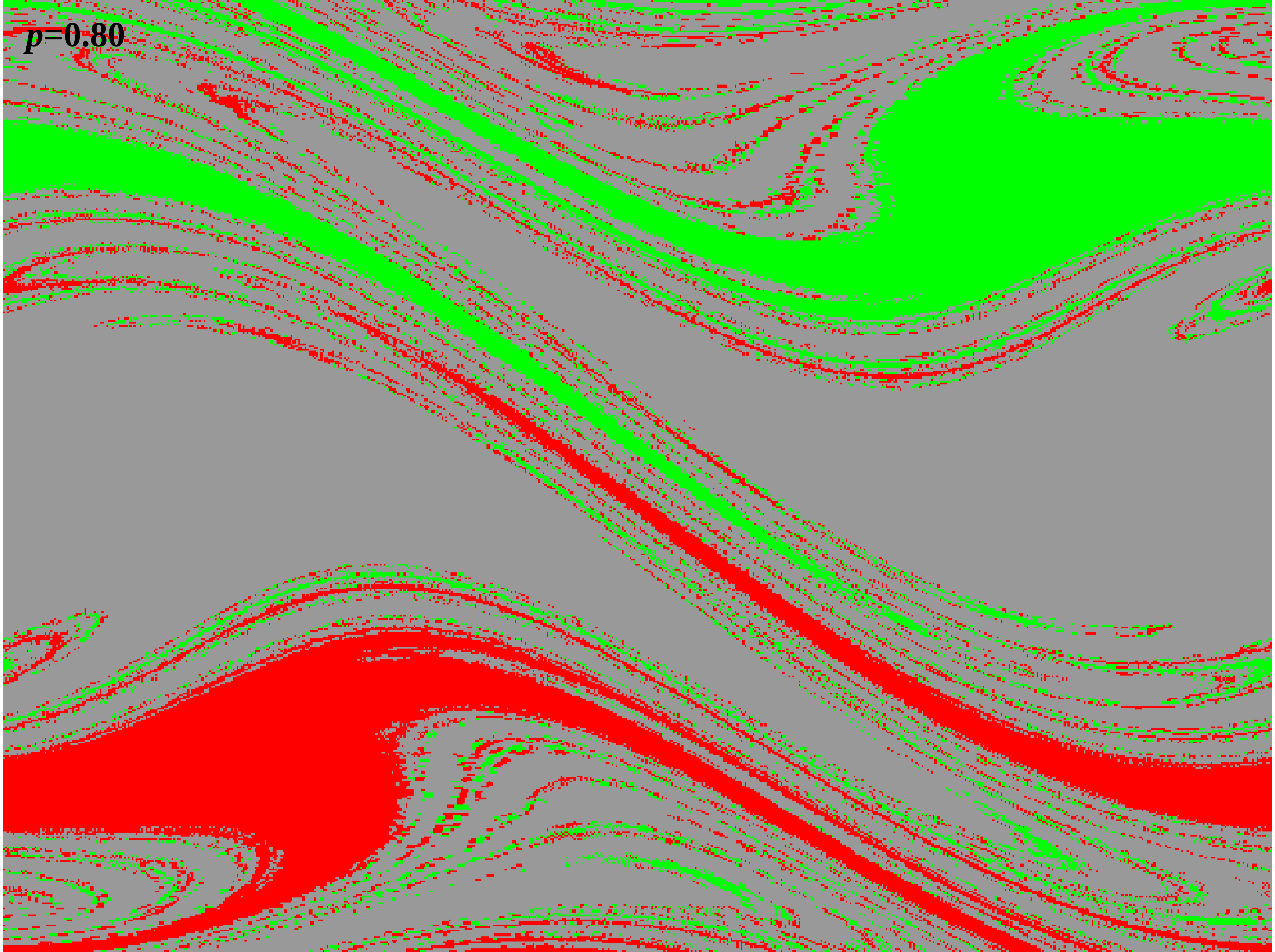
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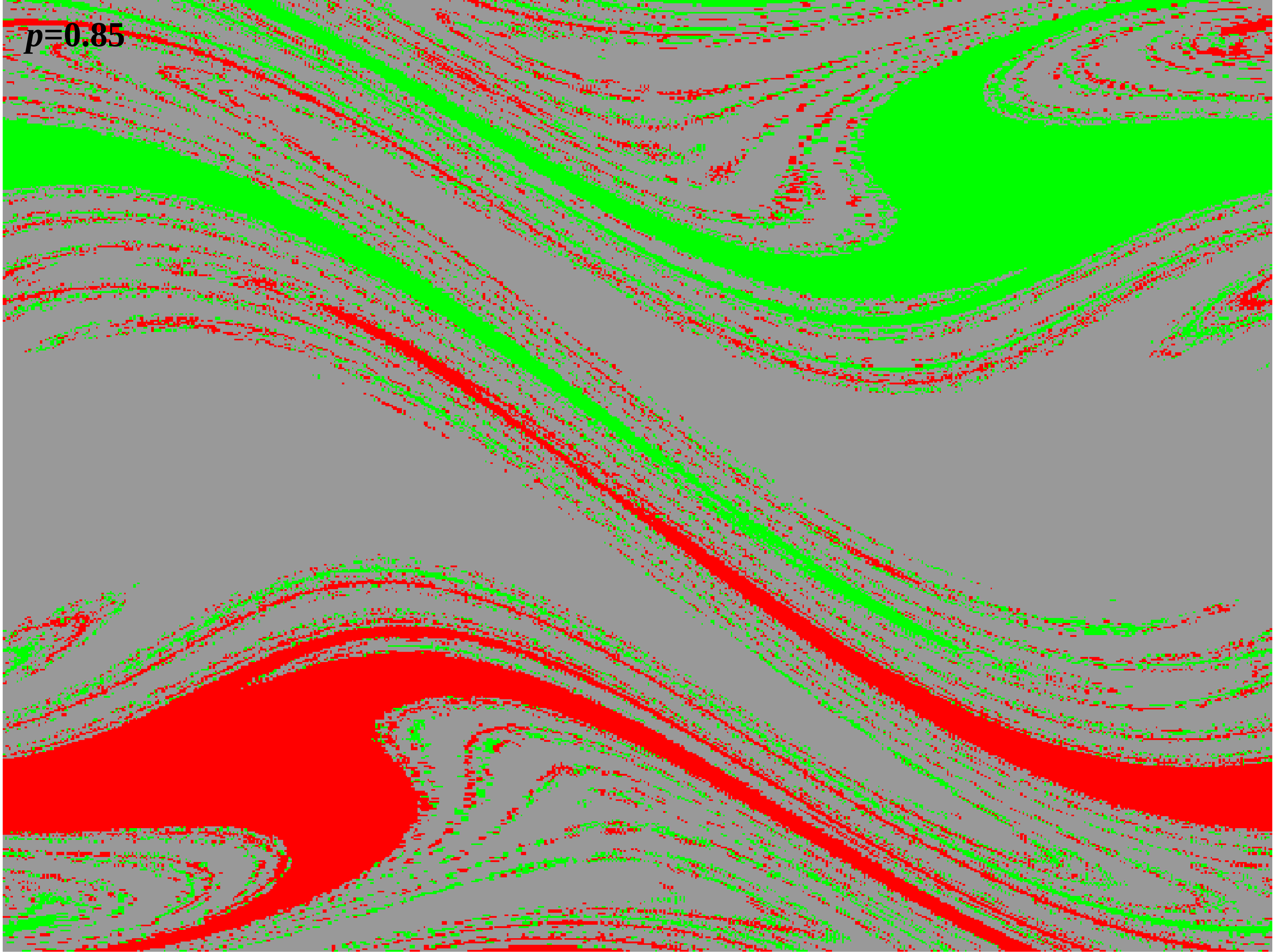

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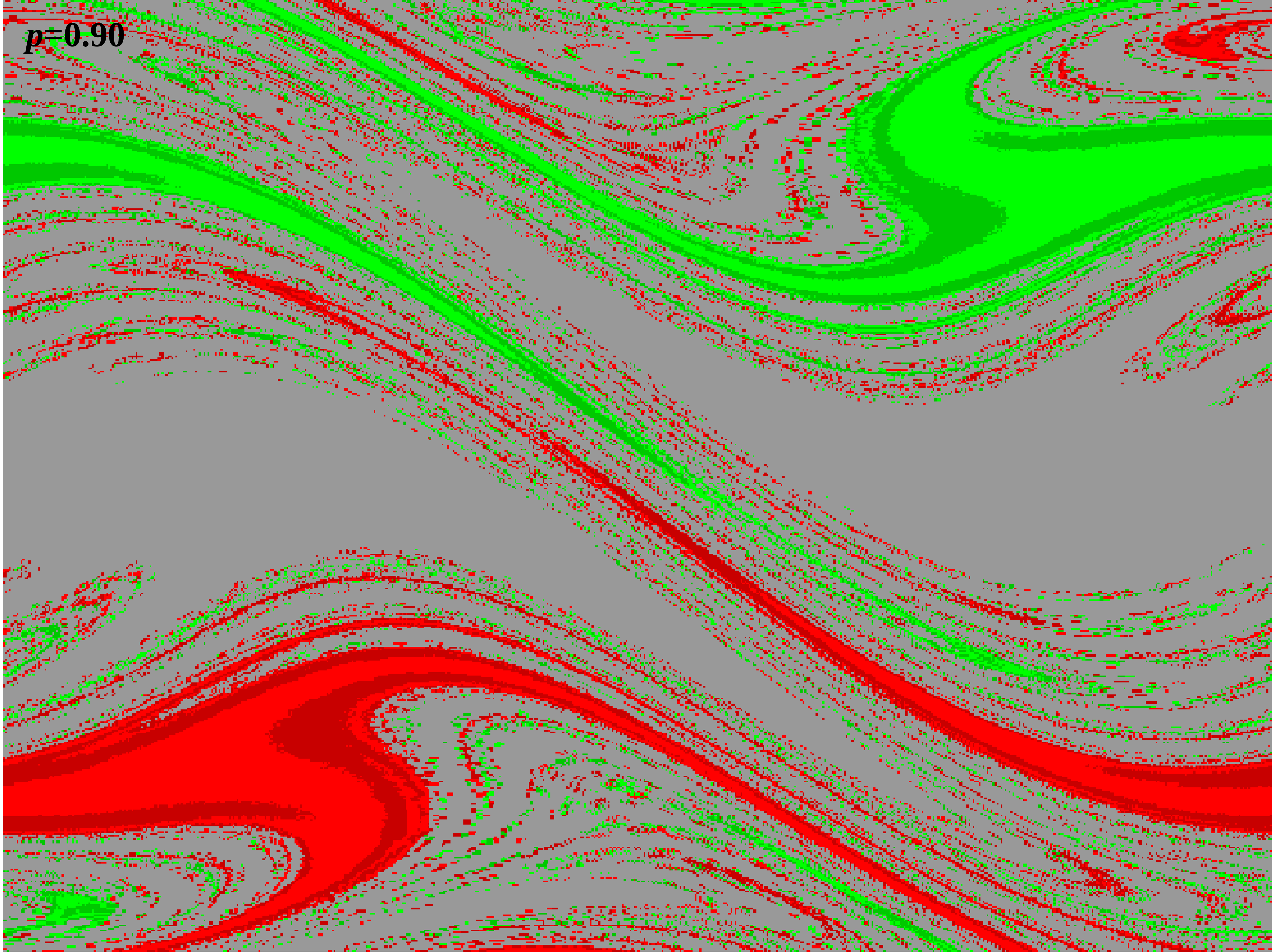
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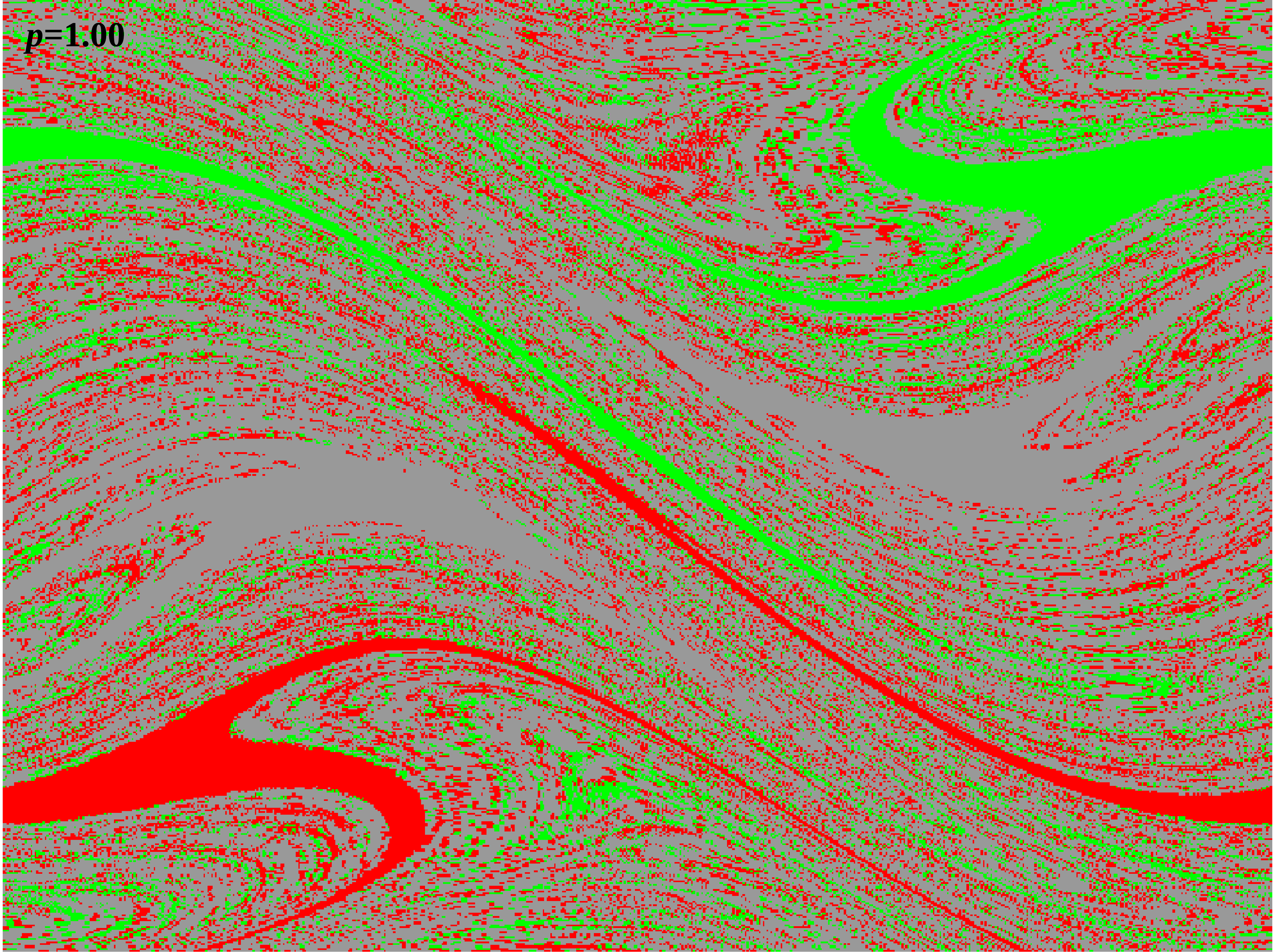
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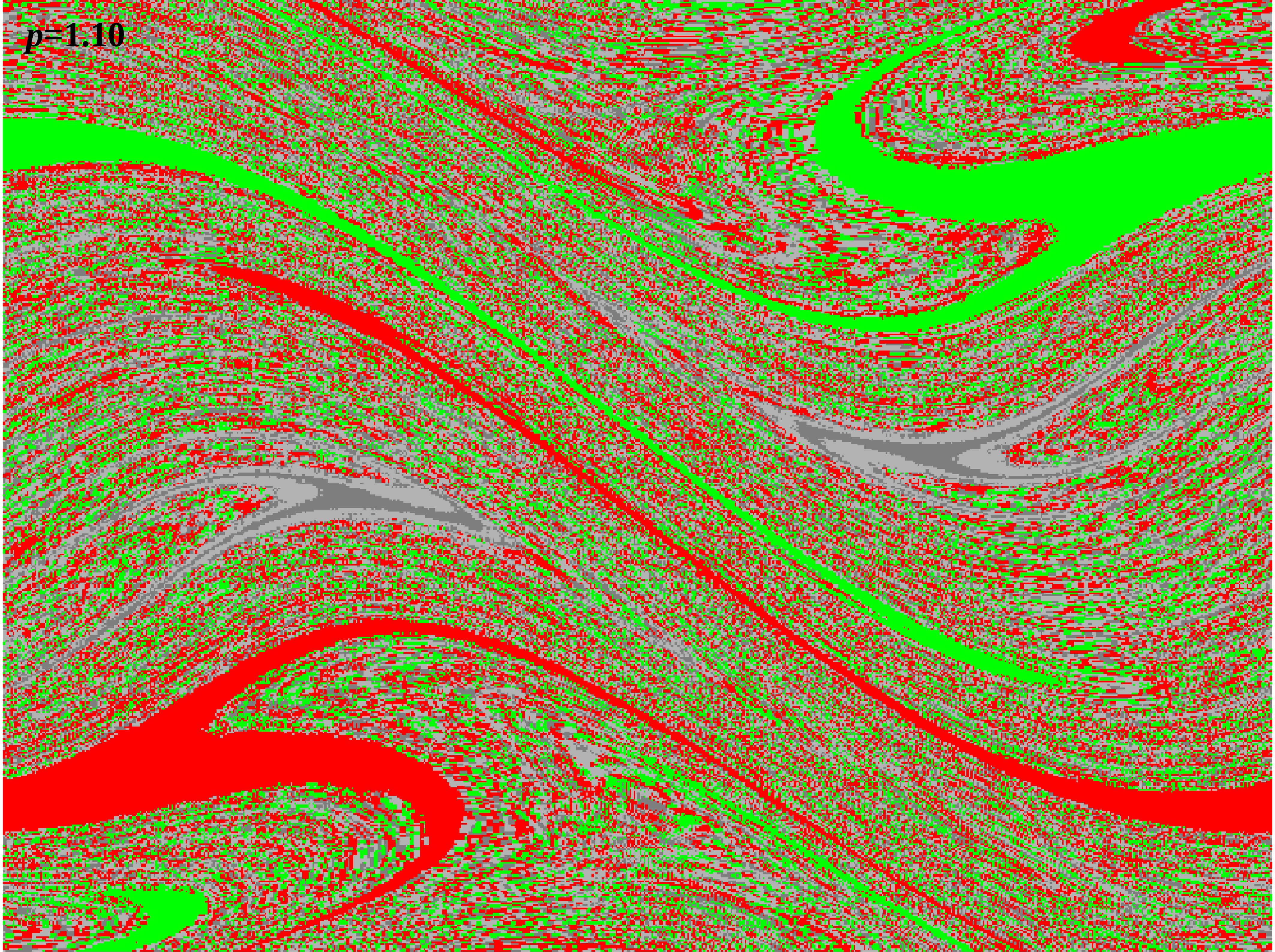
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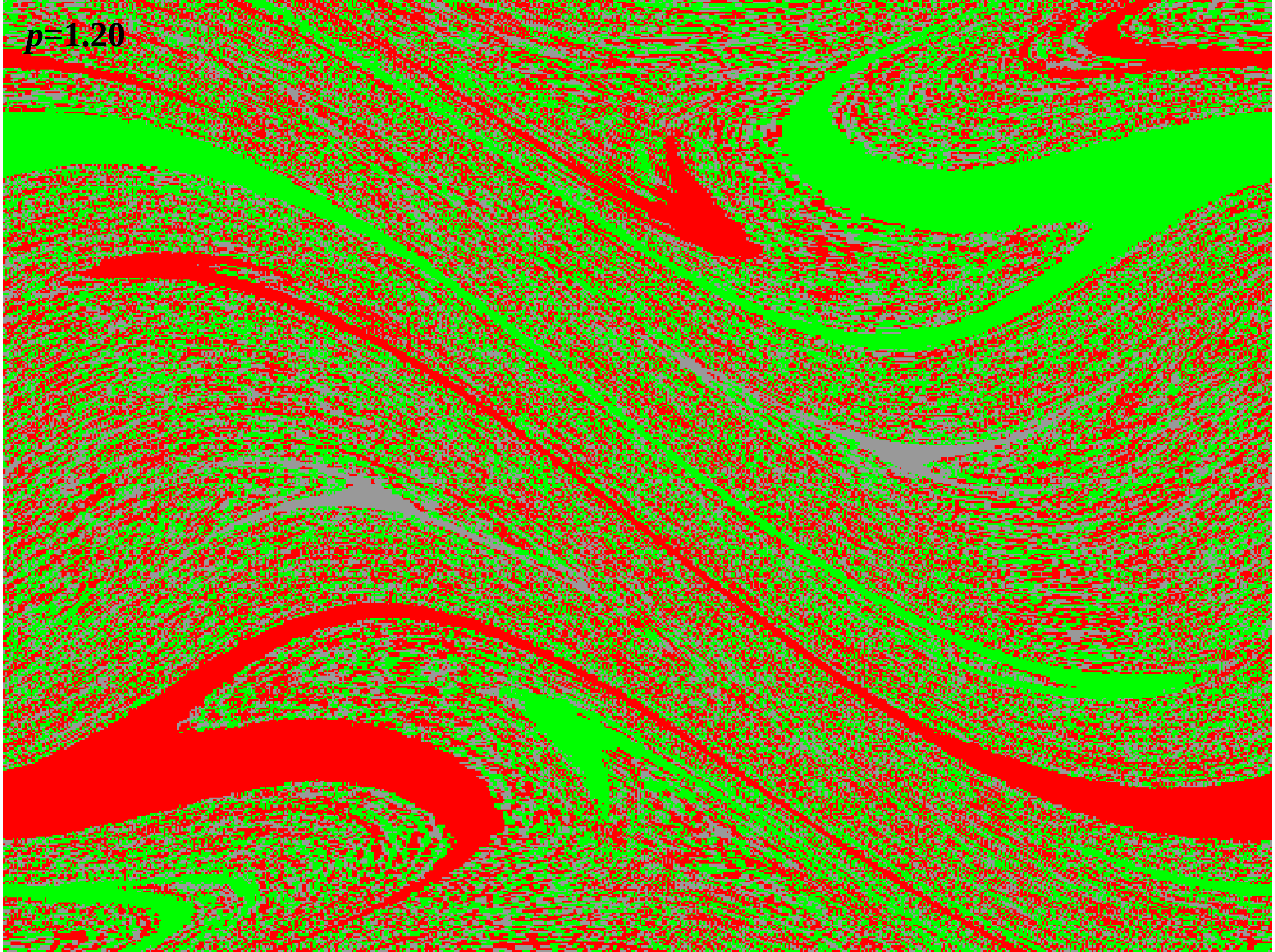
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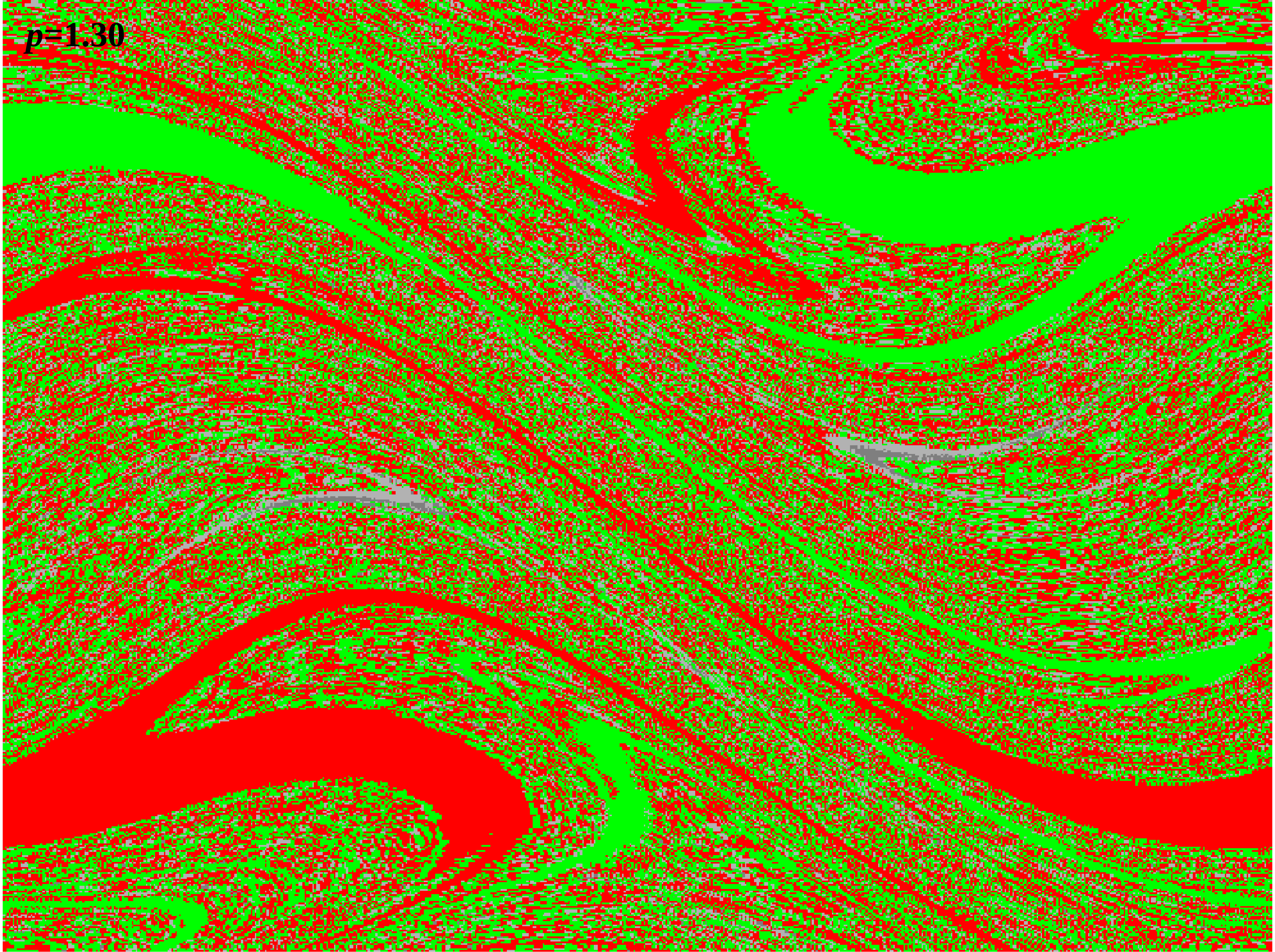
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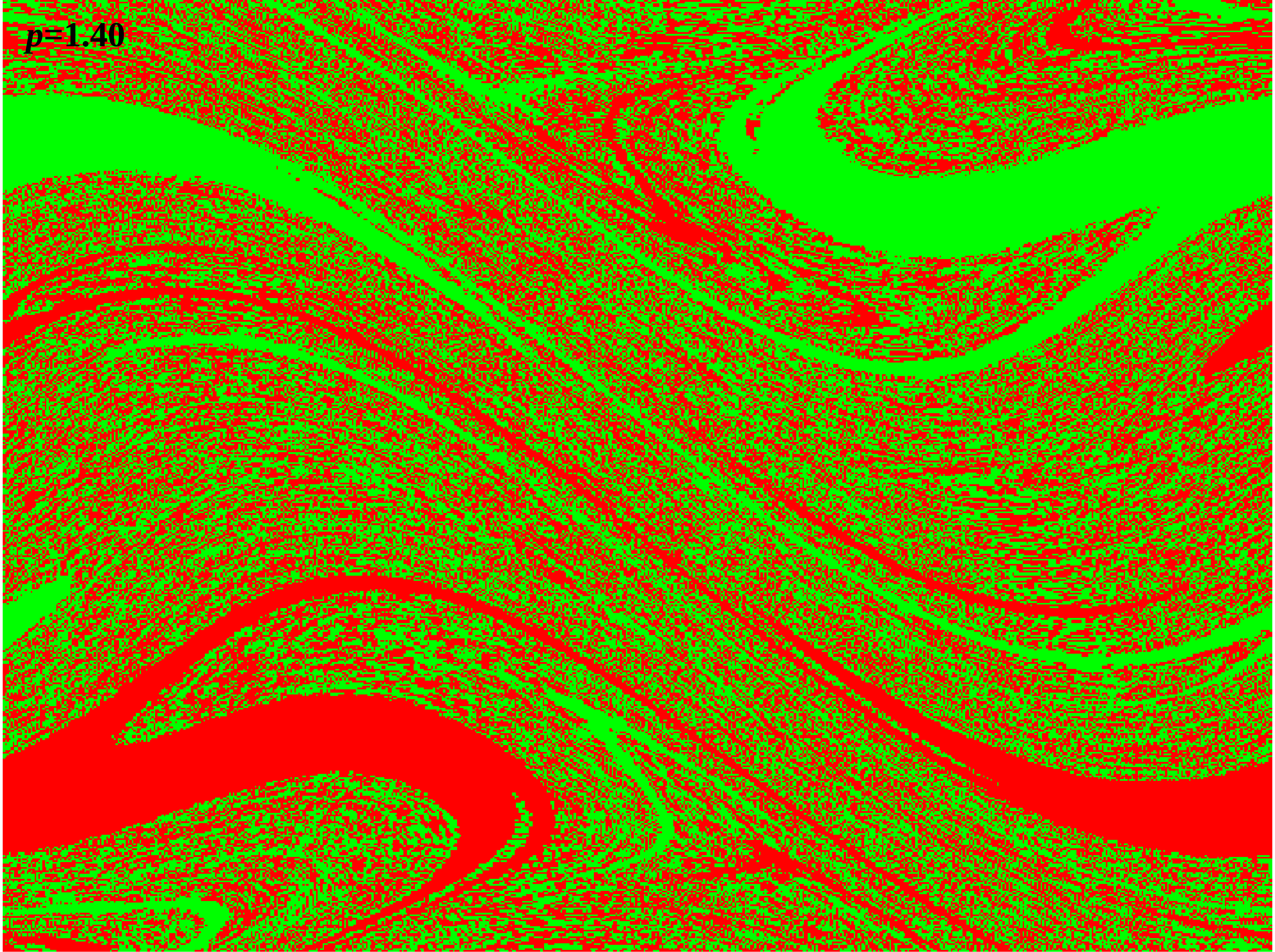
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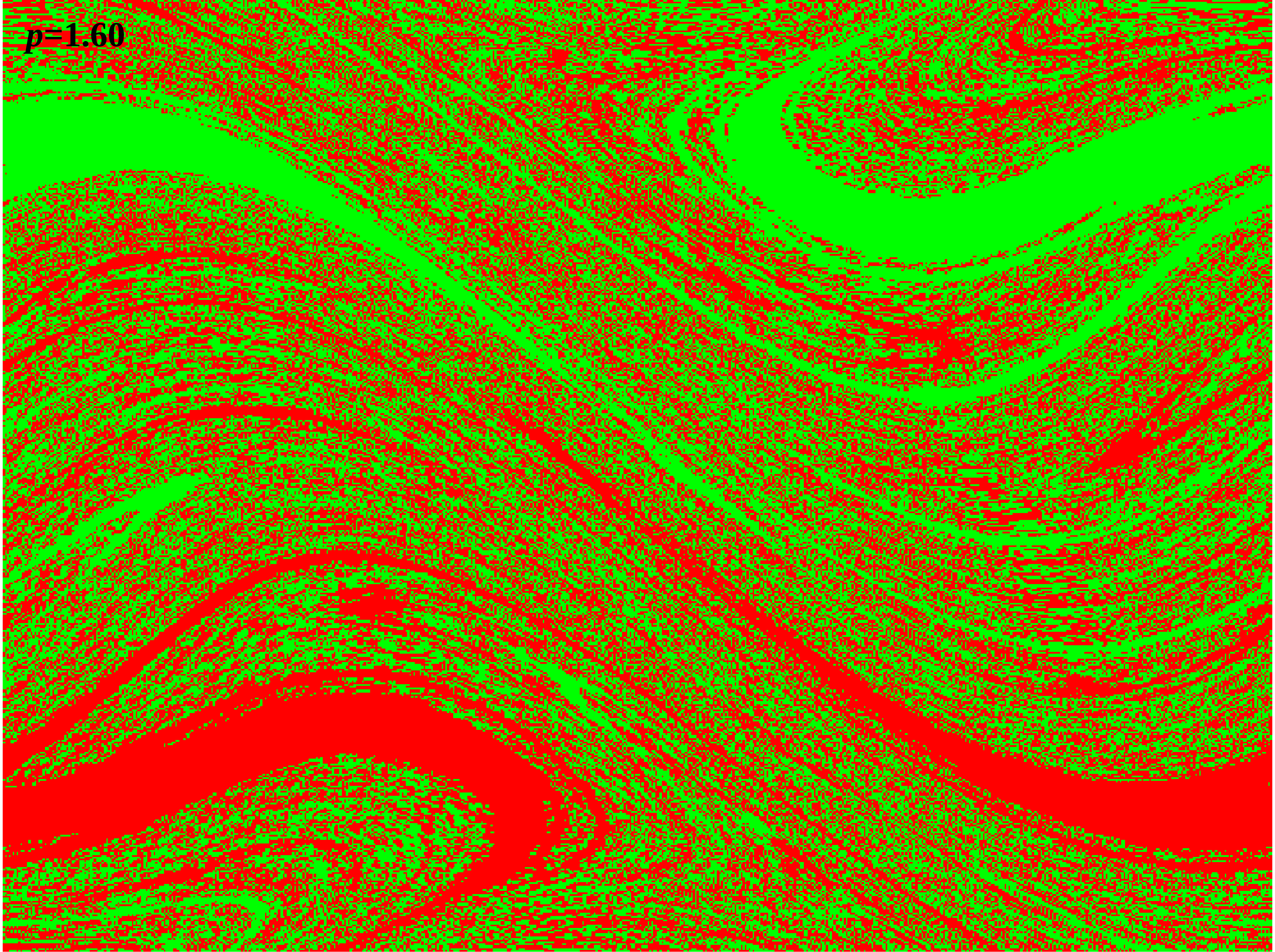
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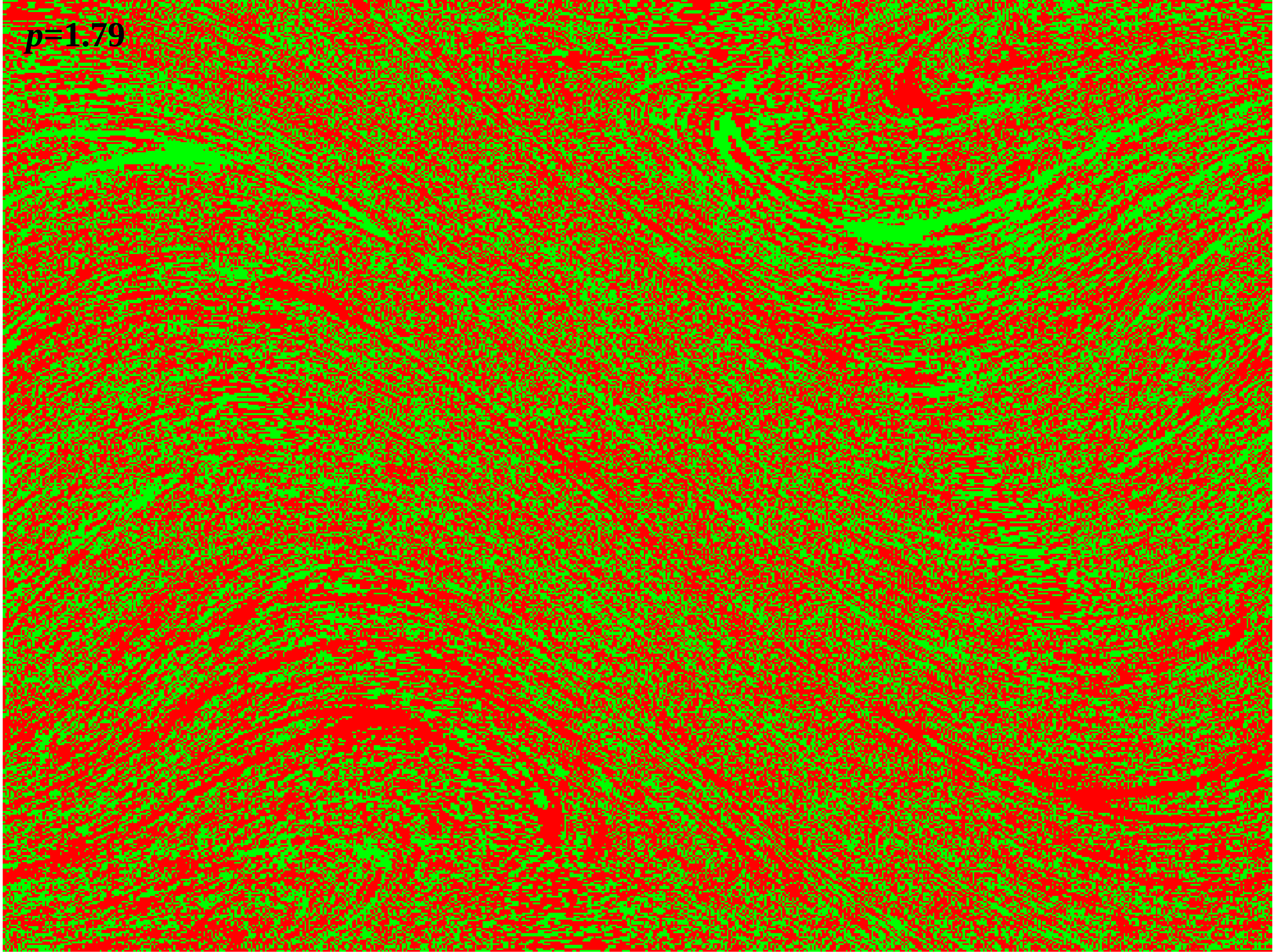
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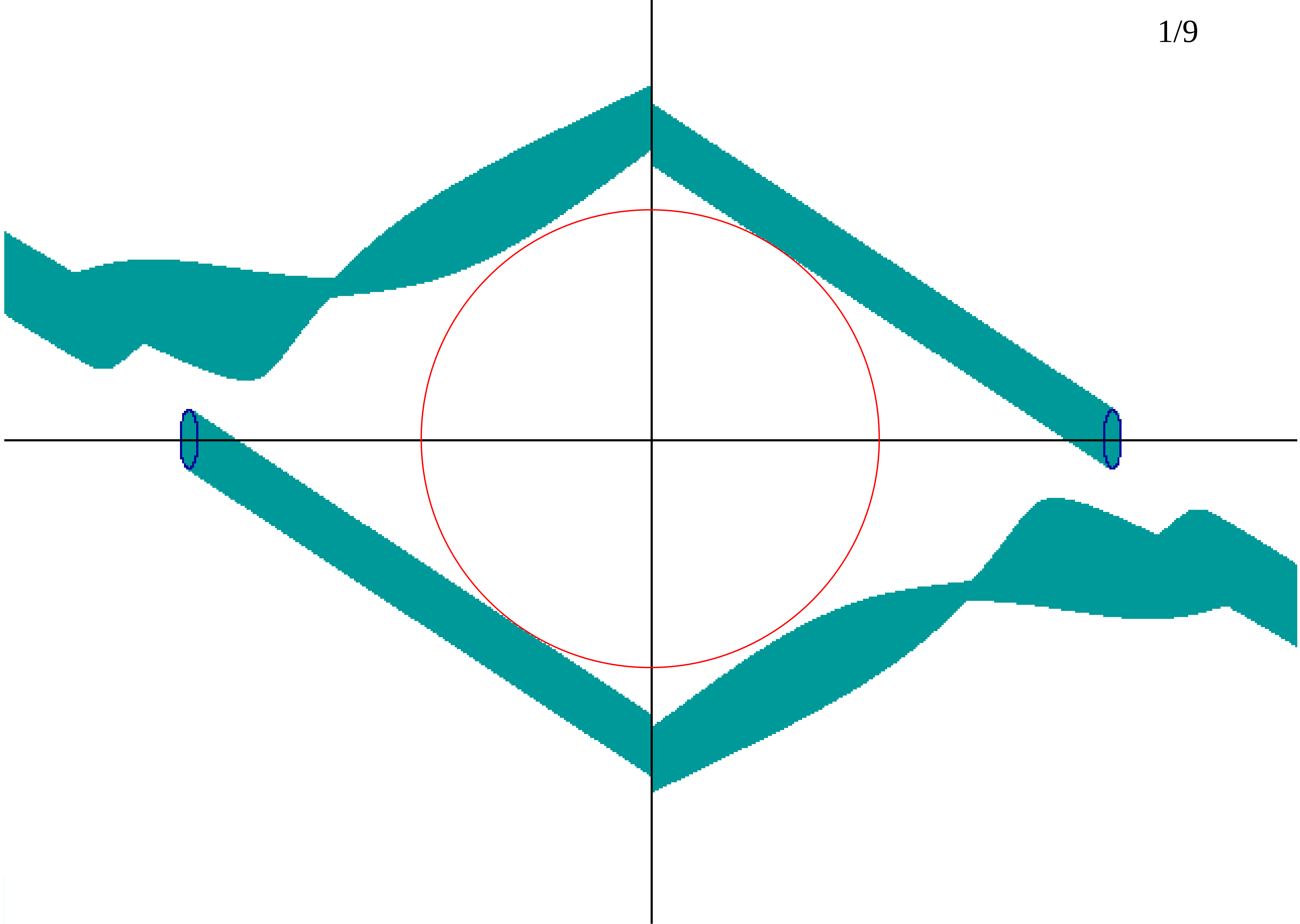
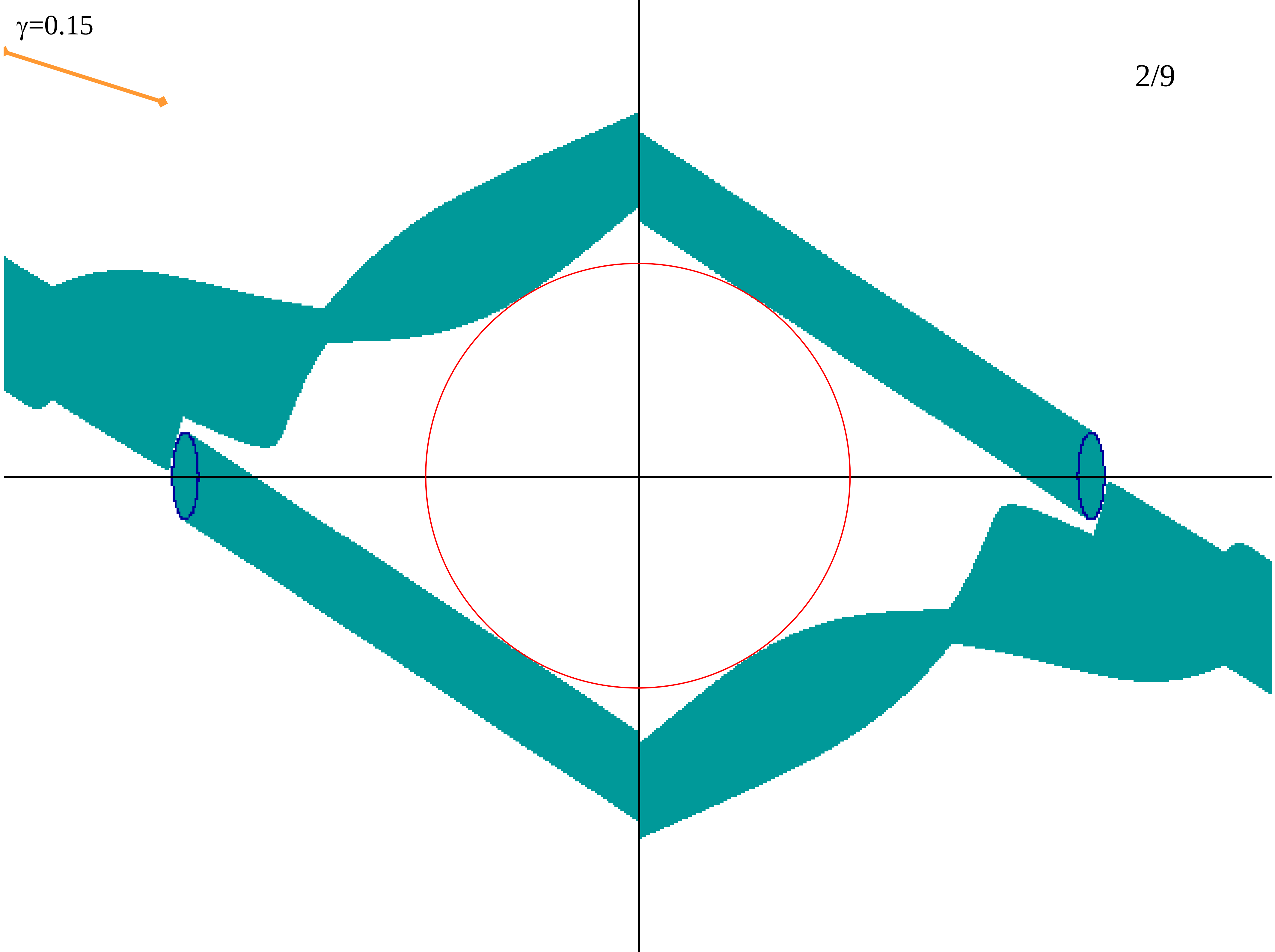
γ=0.10; ω=5, δ=0.02, α=0.2, r=0.95; $x_{min/max}$=±0.28, $y_{min/max}$=±0.32,

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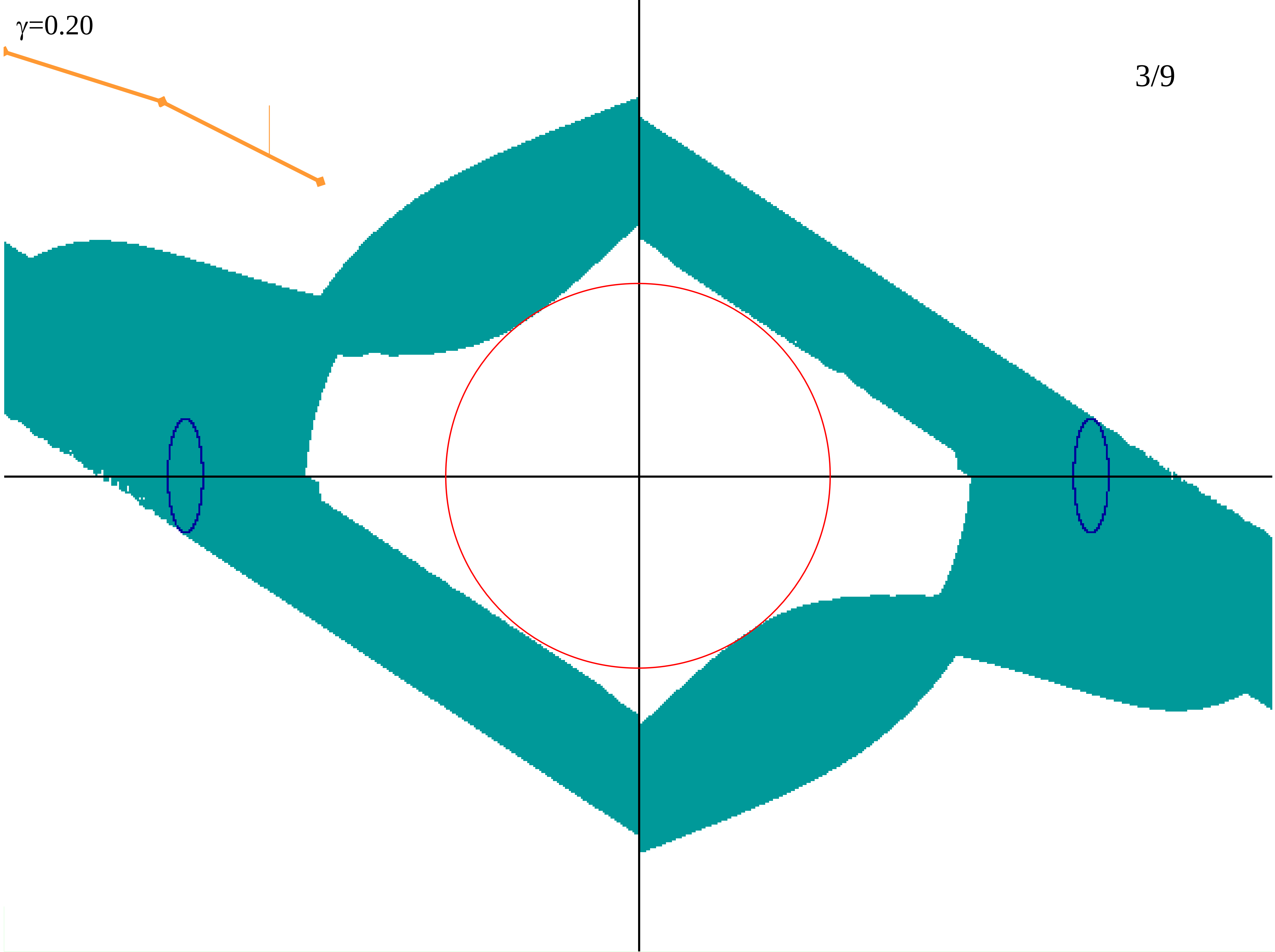
2/9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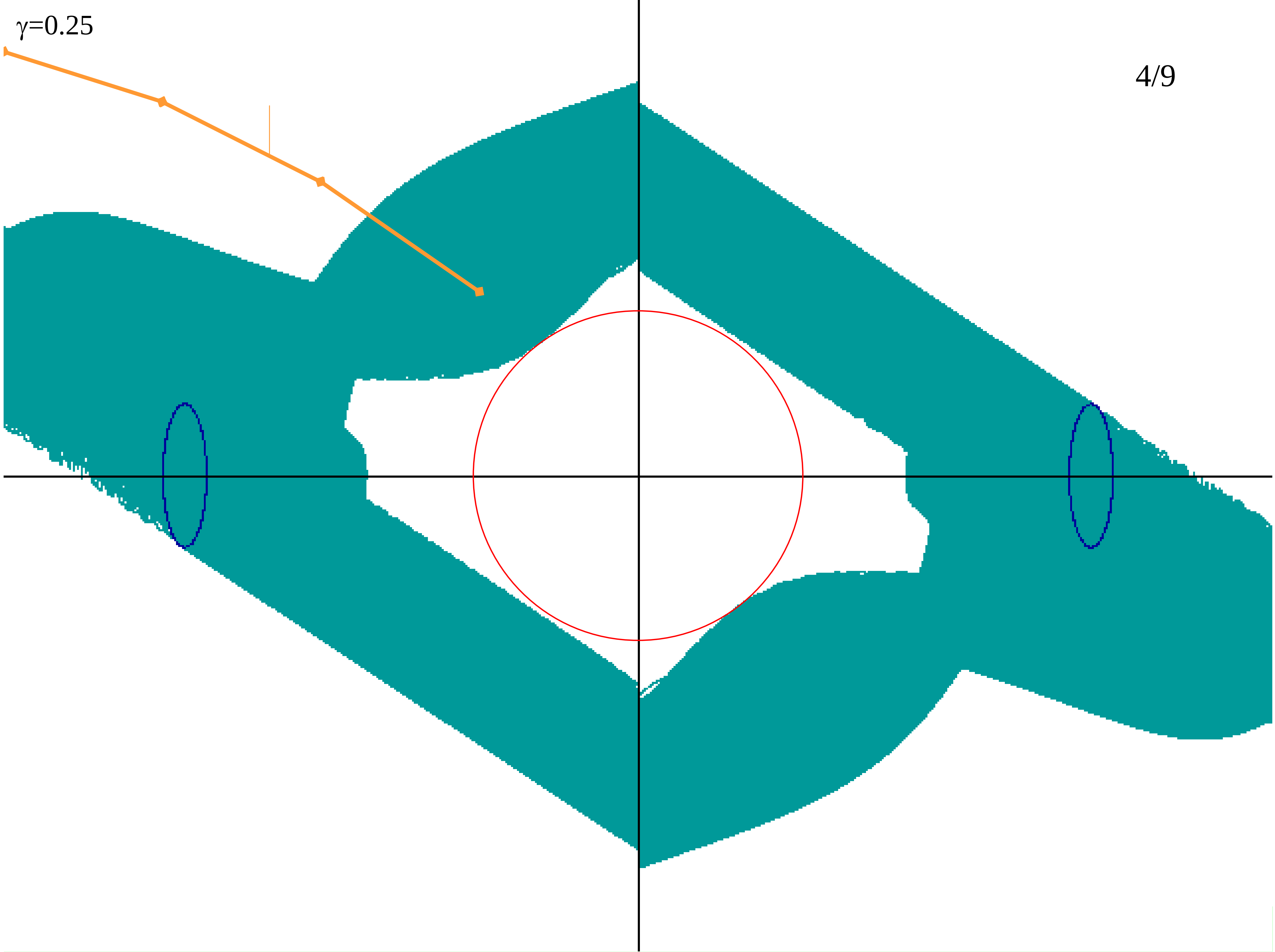
3/9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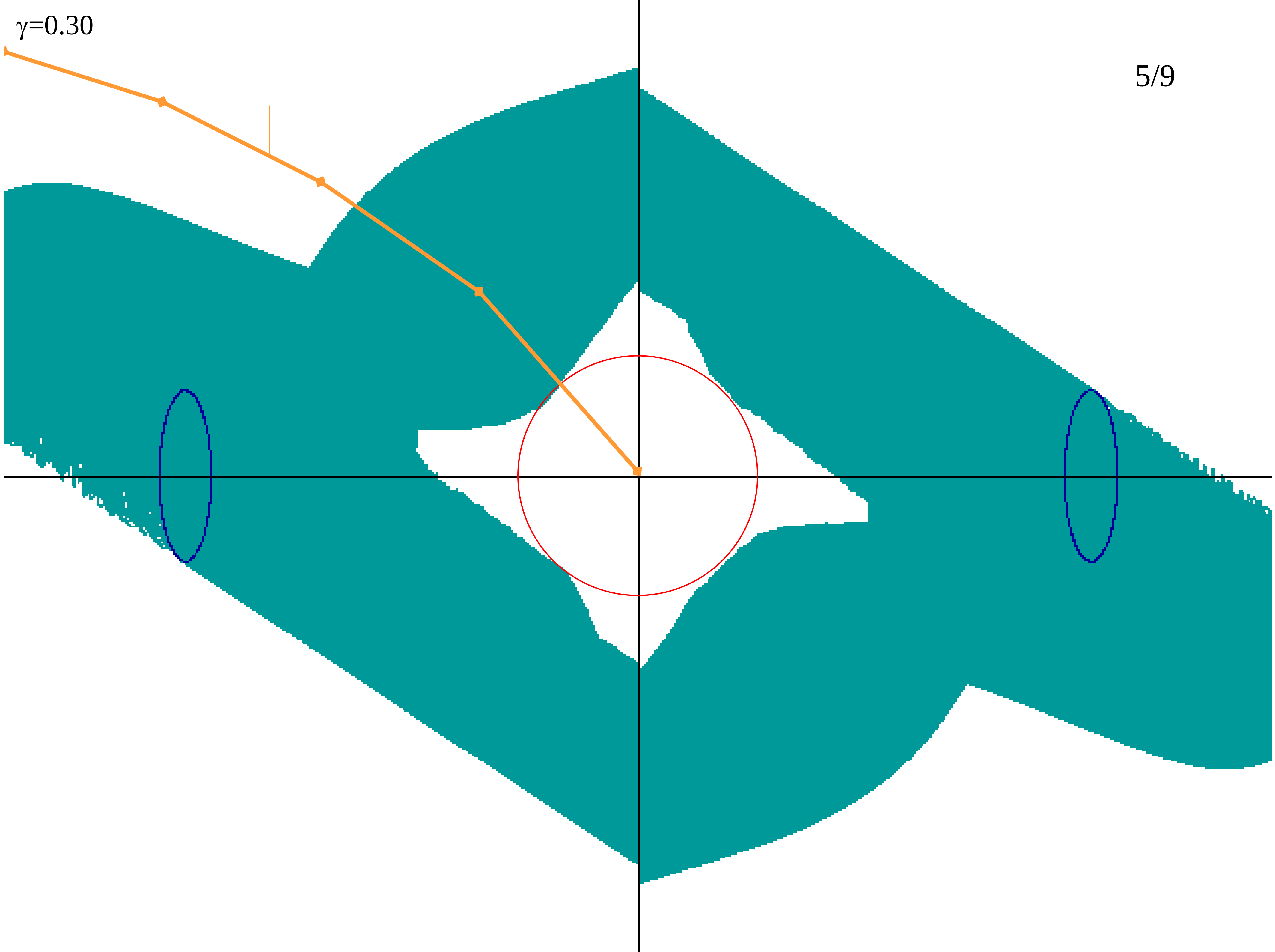
4/9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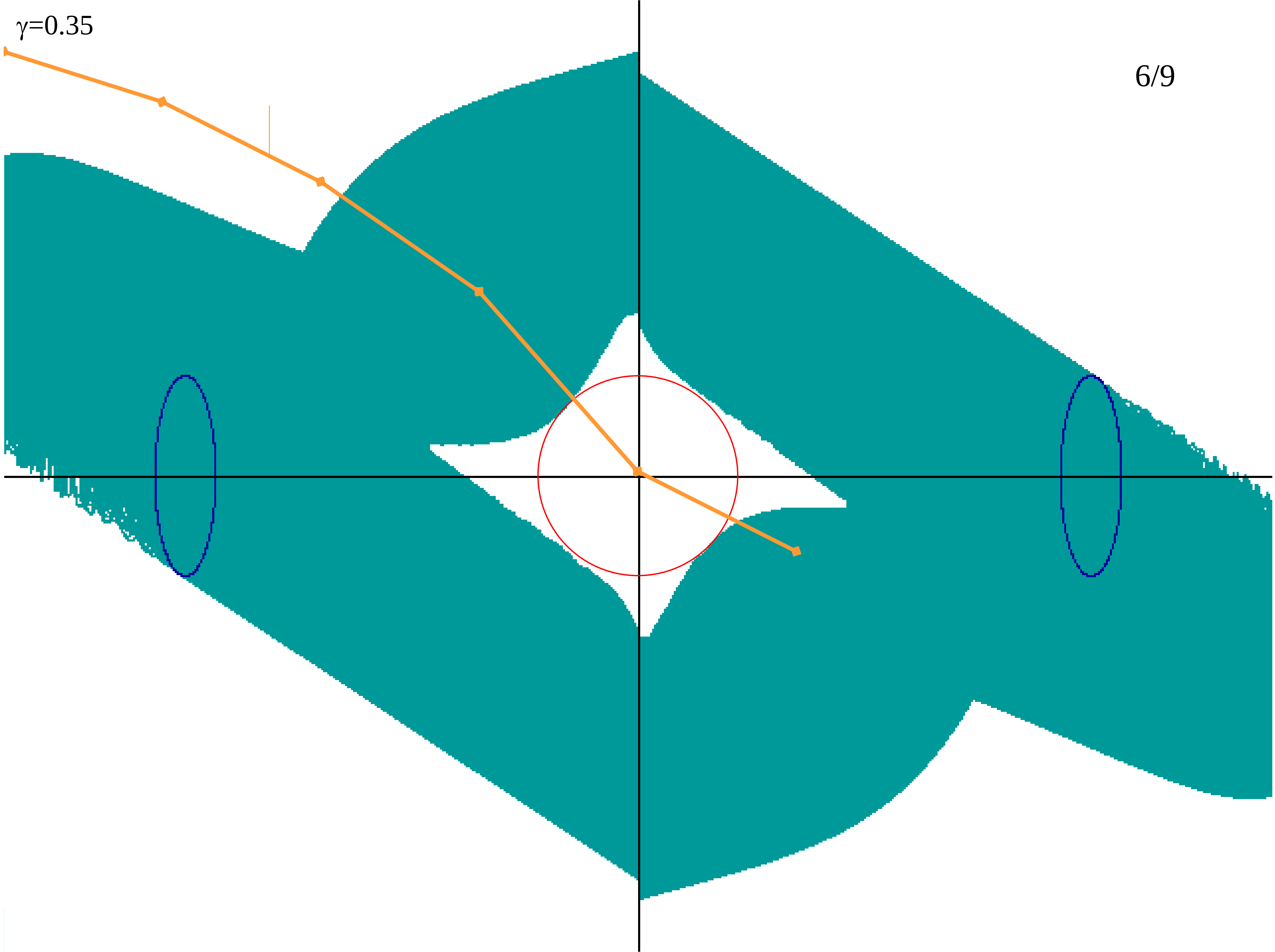
5/9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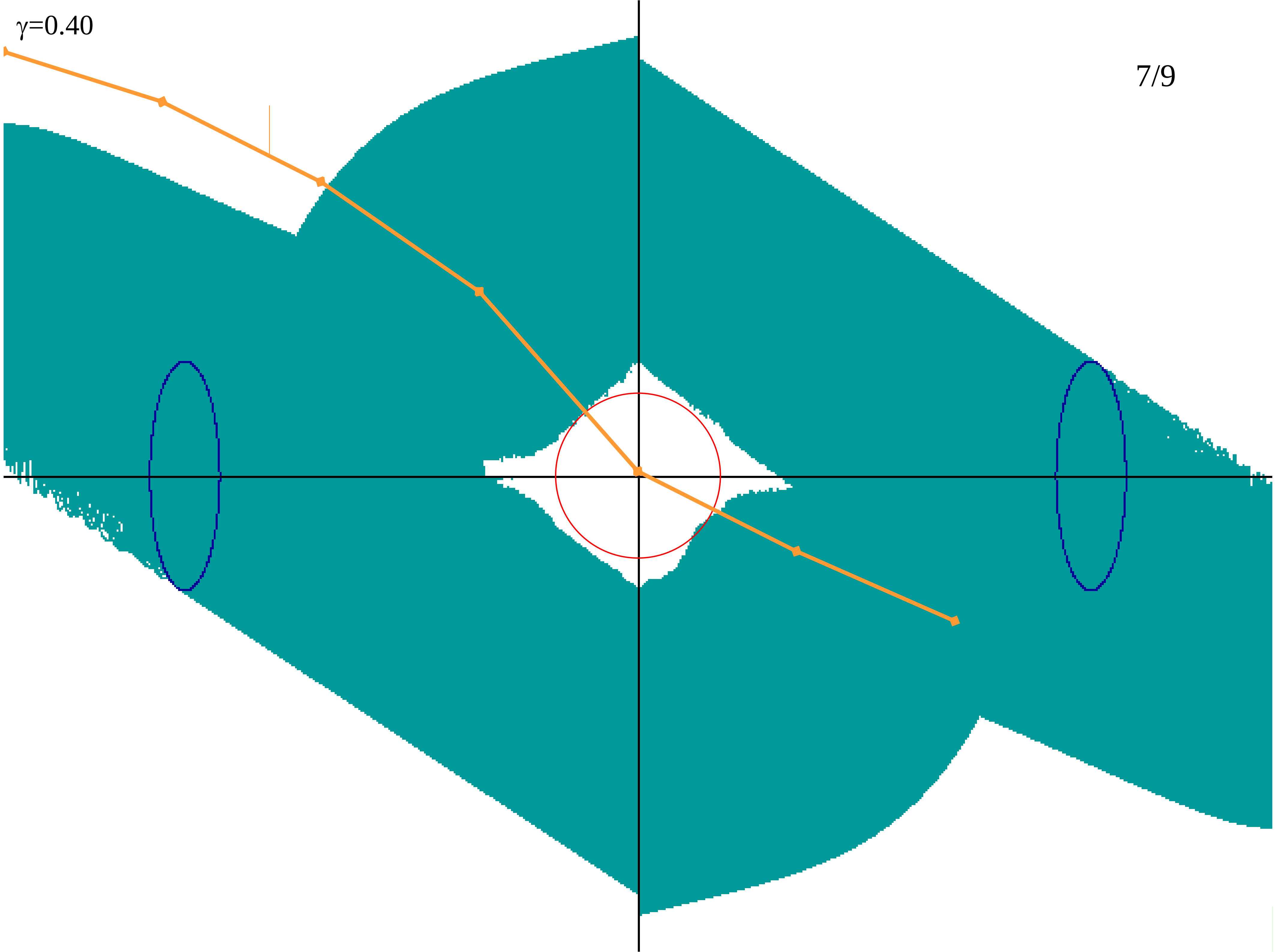
6/9



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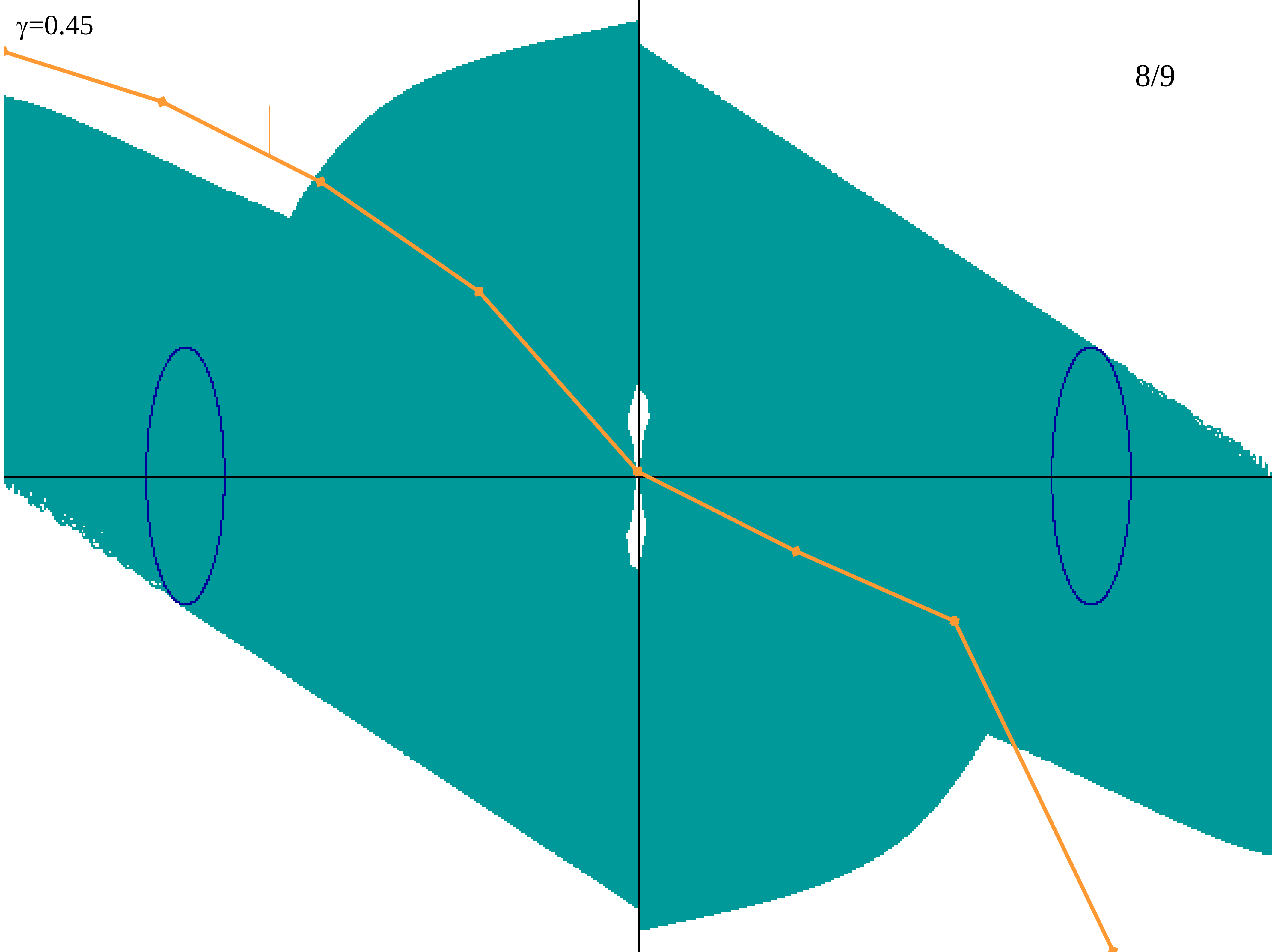
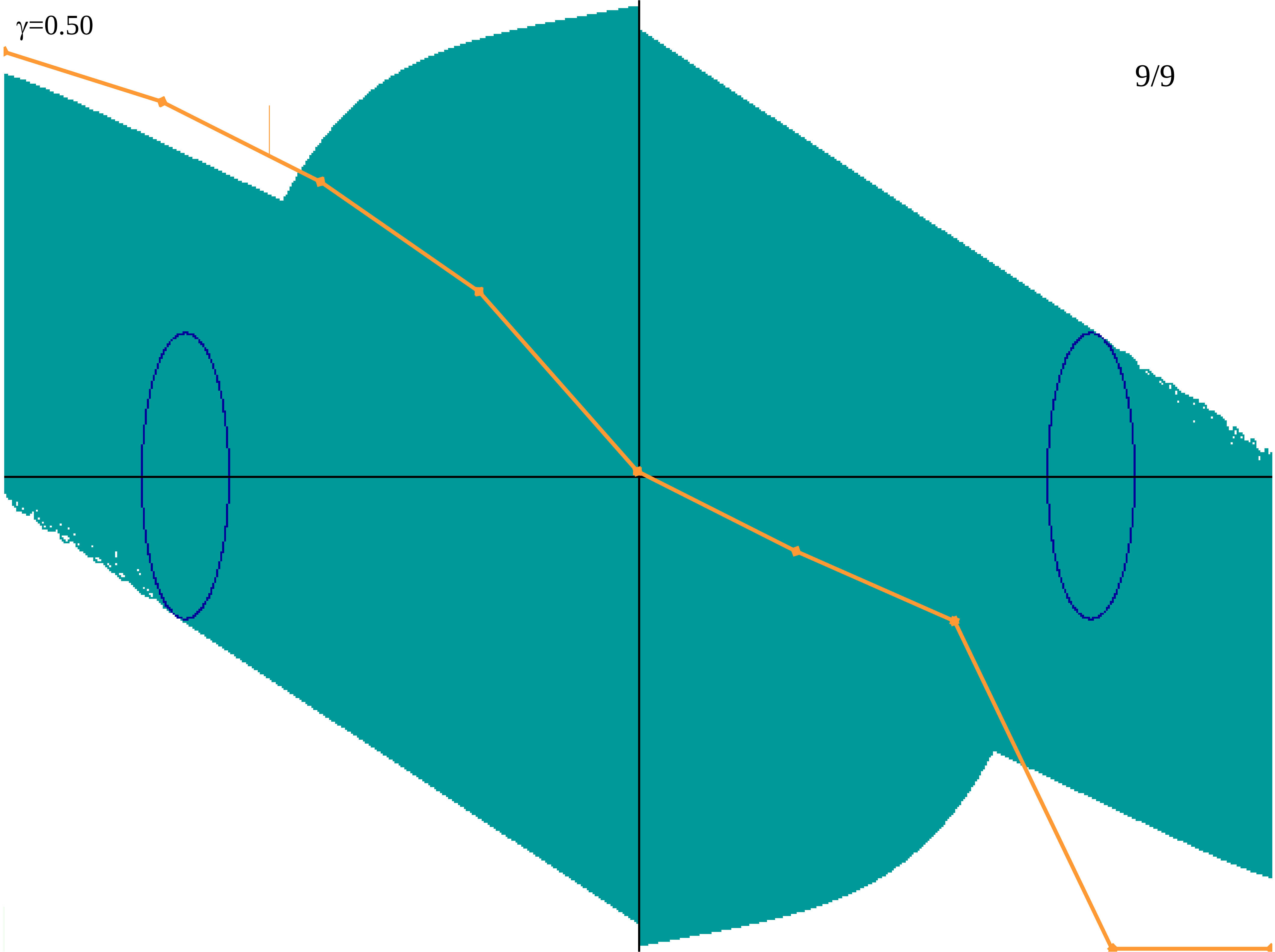
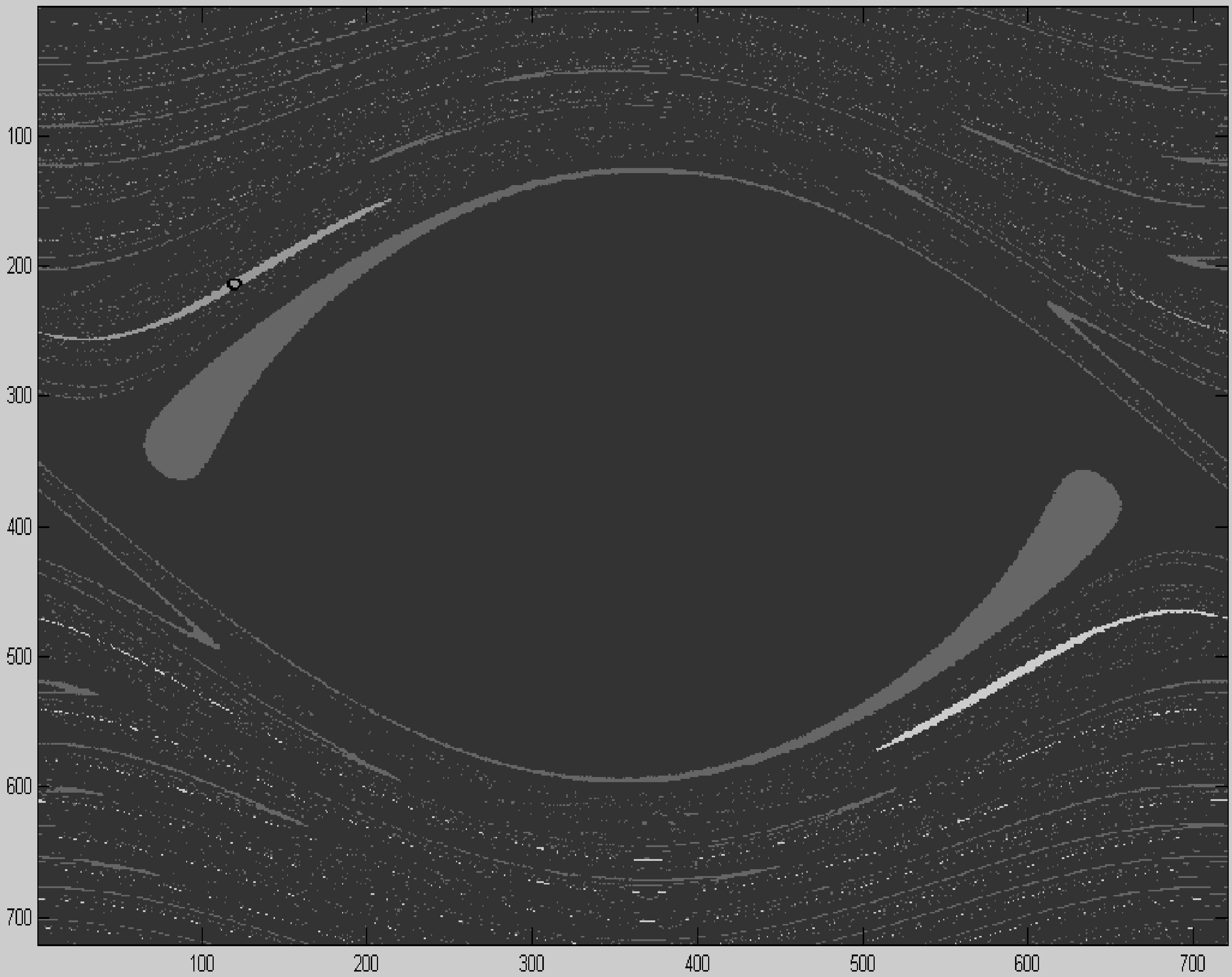
7/9

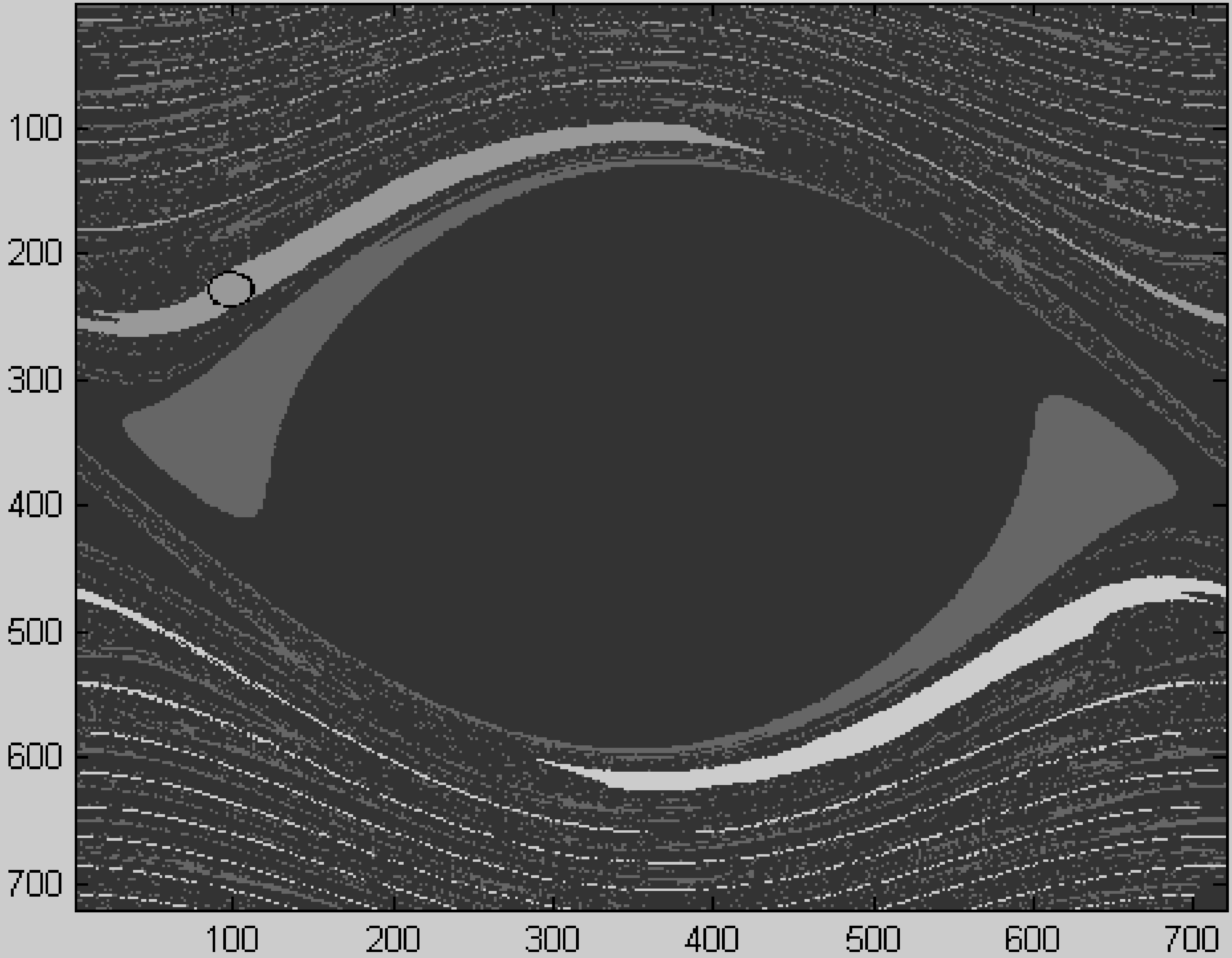

γ=0.45
8/9


γ=0.50
9/9

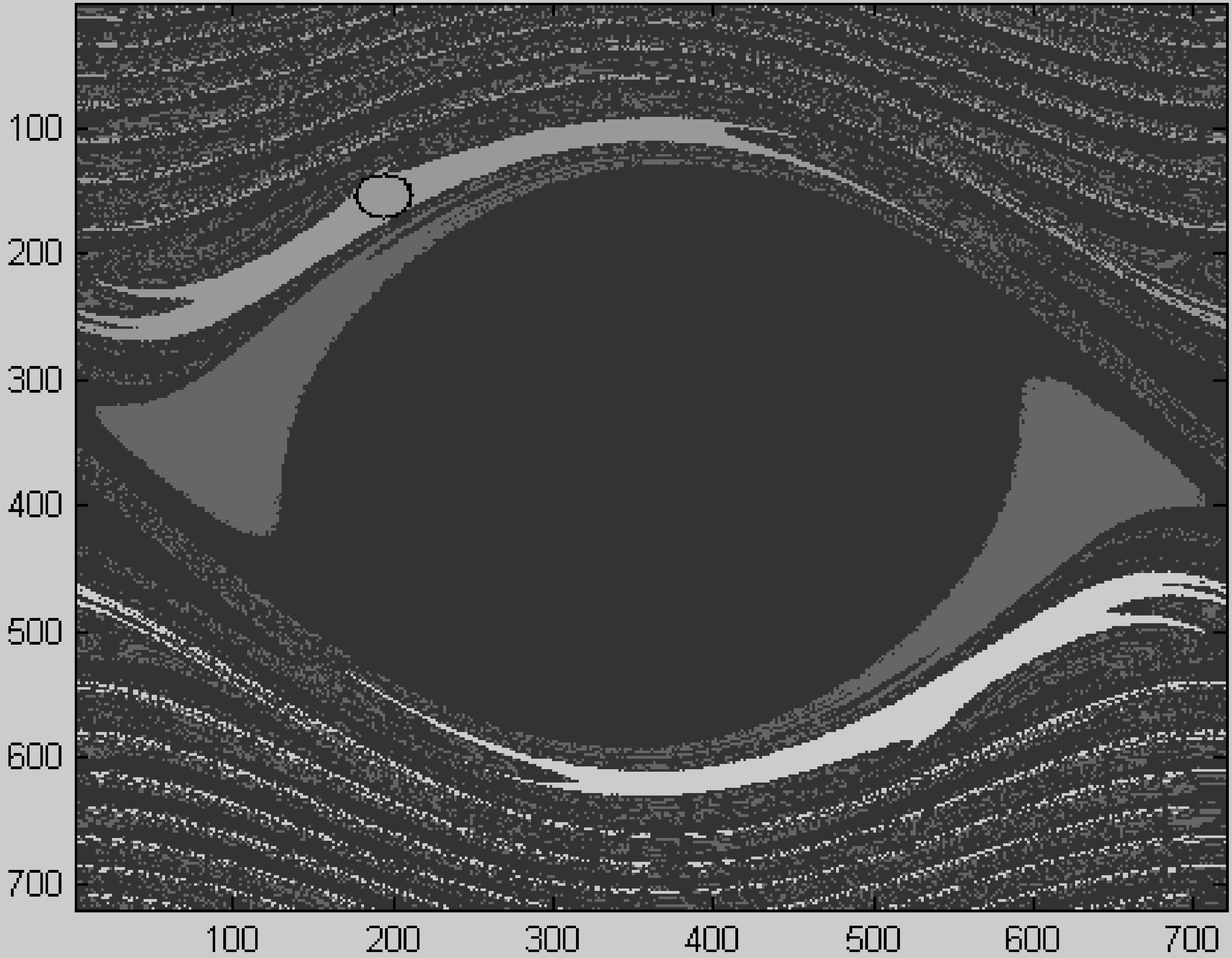
pendulum, p=0.050, ω=1.30, *c*=0.015



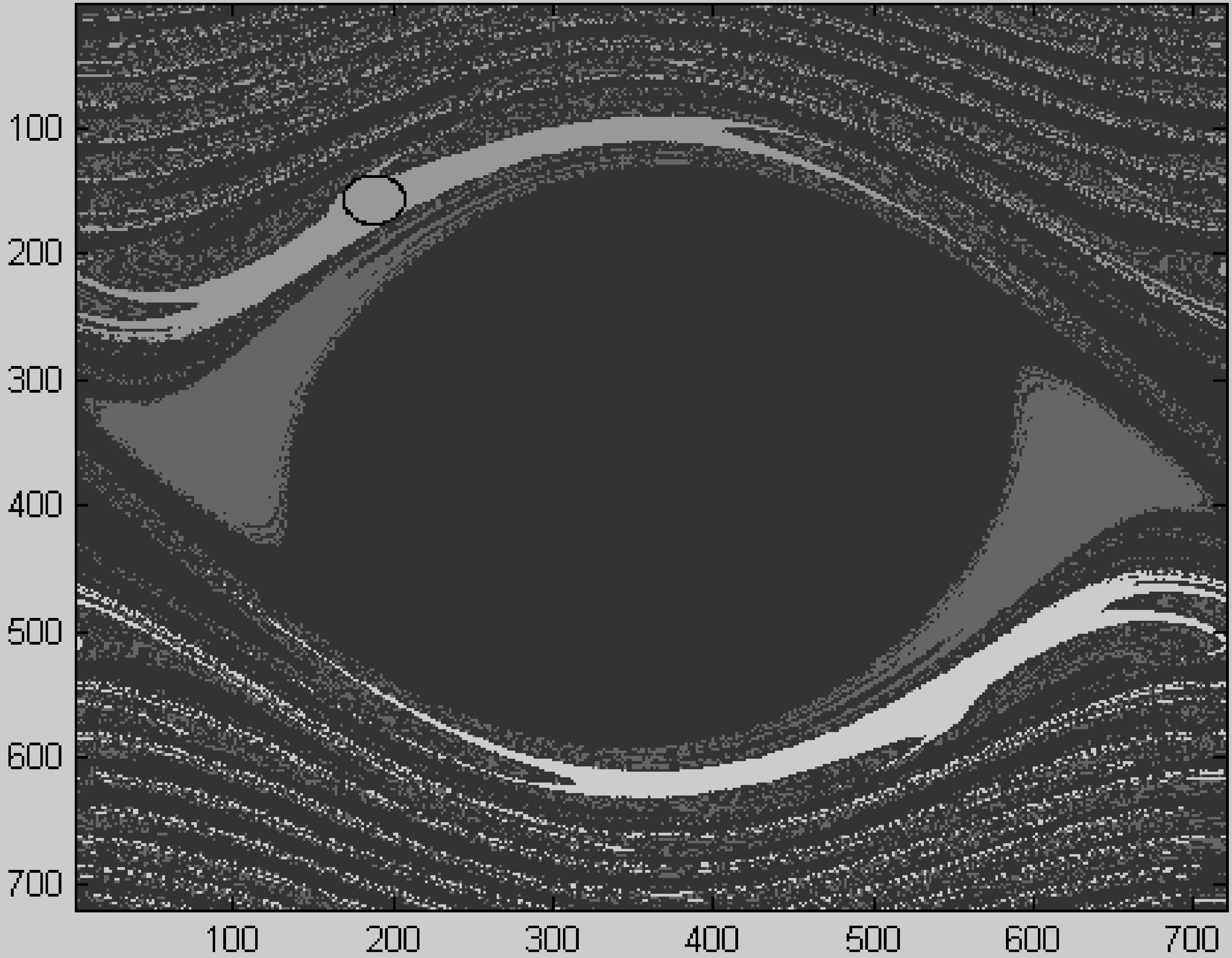
pendulum, p=0.070, ω=1.30, c=0.015



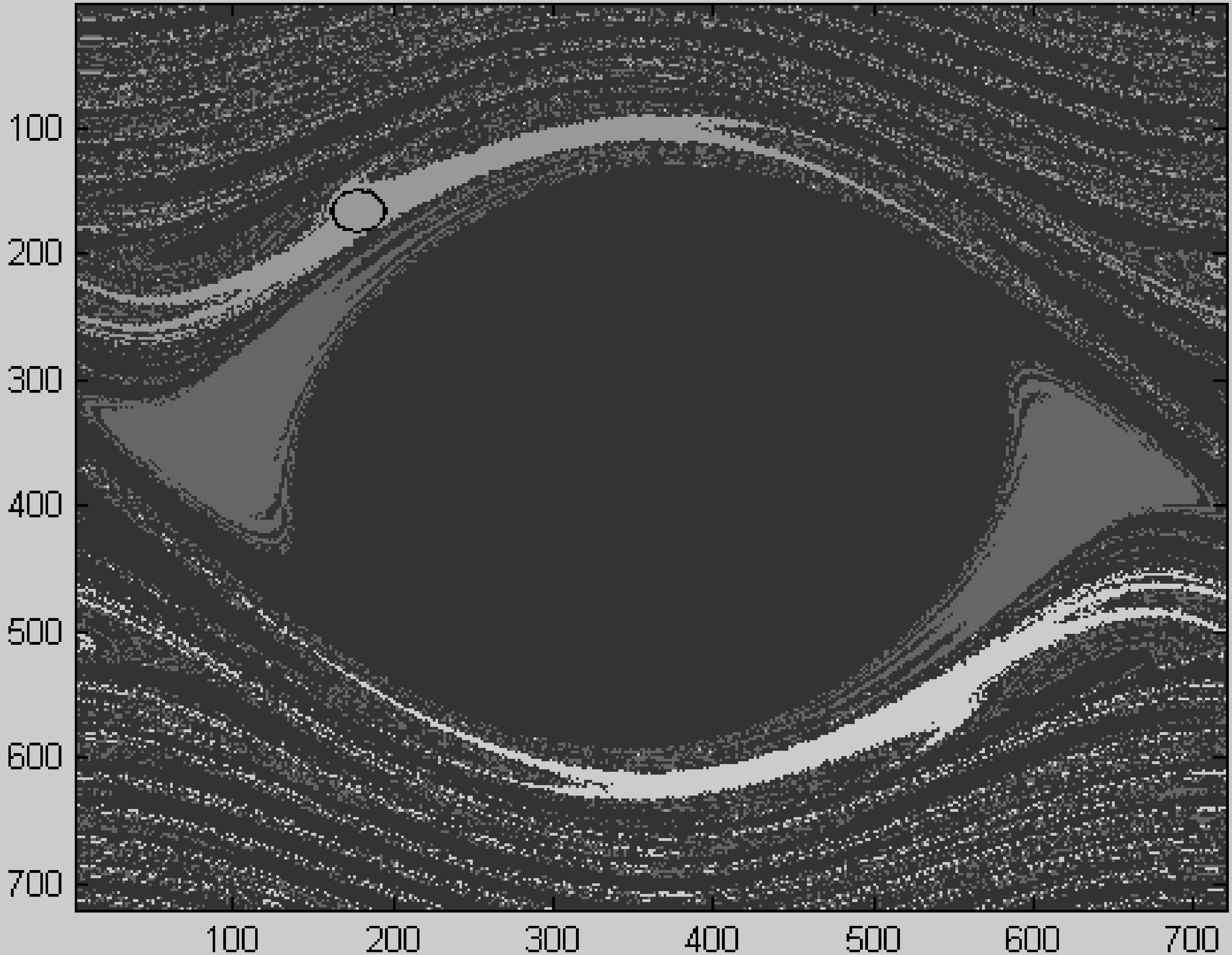
pendulum, p=0.080, ω=1.30, *c*=0.015



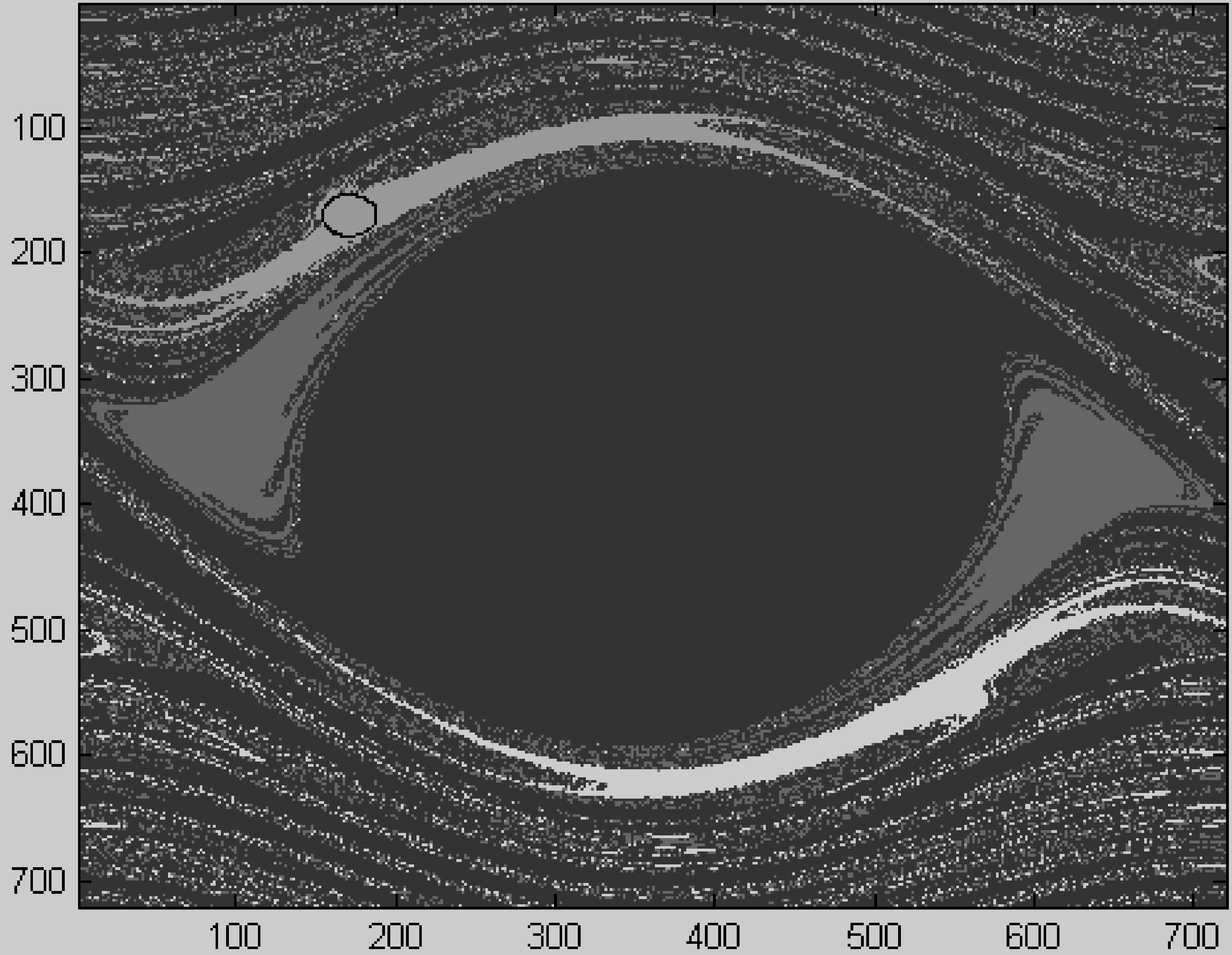
pendulum, p=0.085, ω=1.30, *c*=0.015



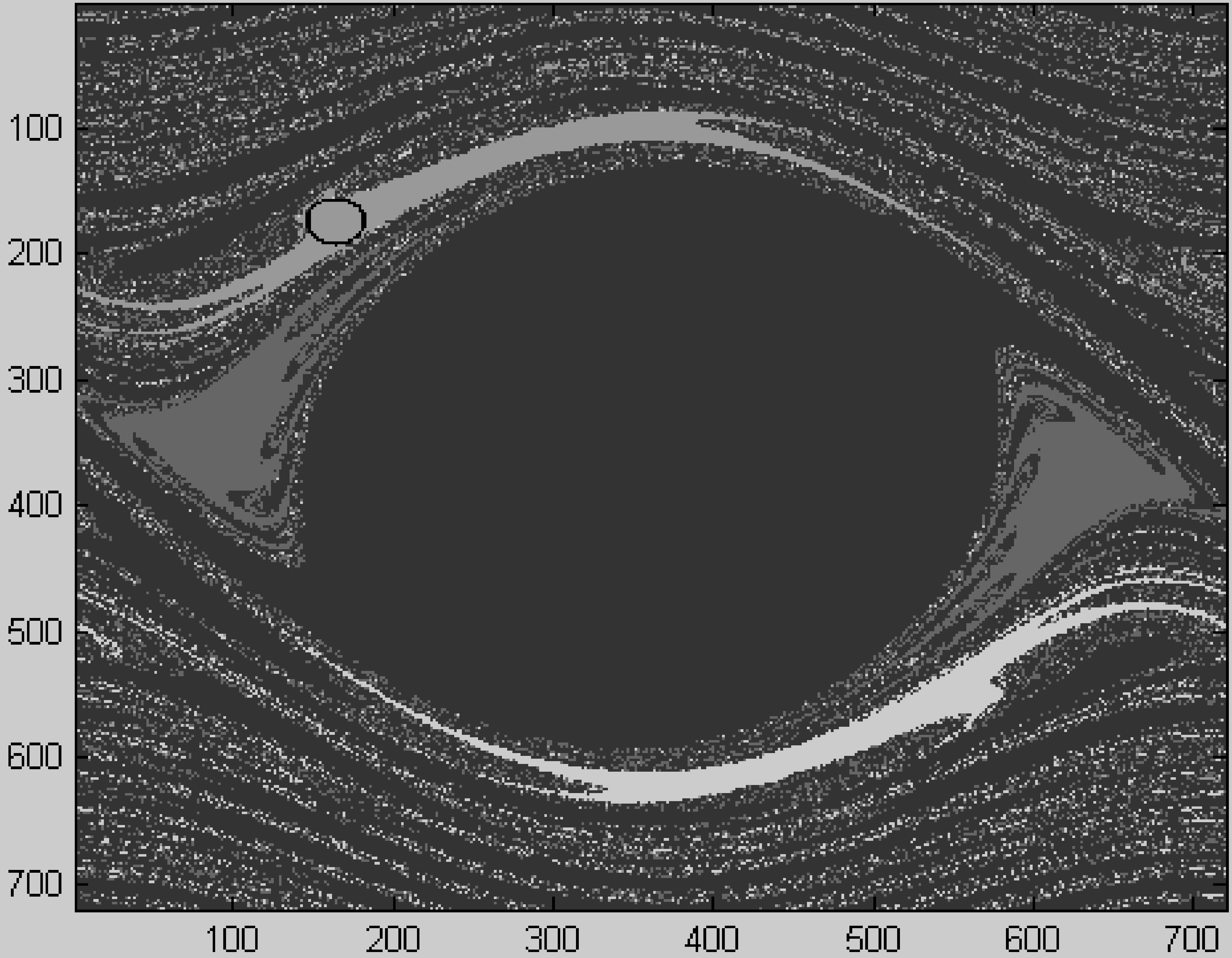
pendulum, p=0.090, ω=1.30, *c*=0.015



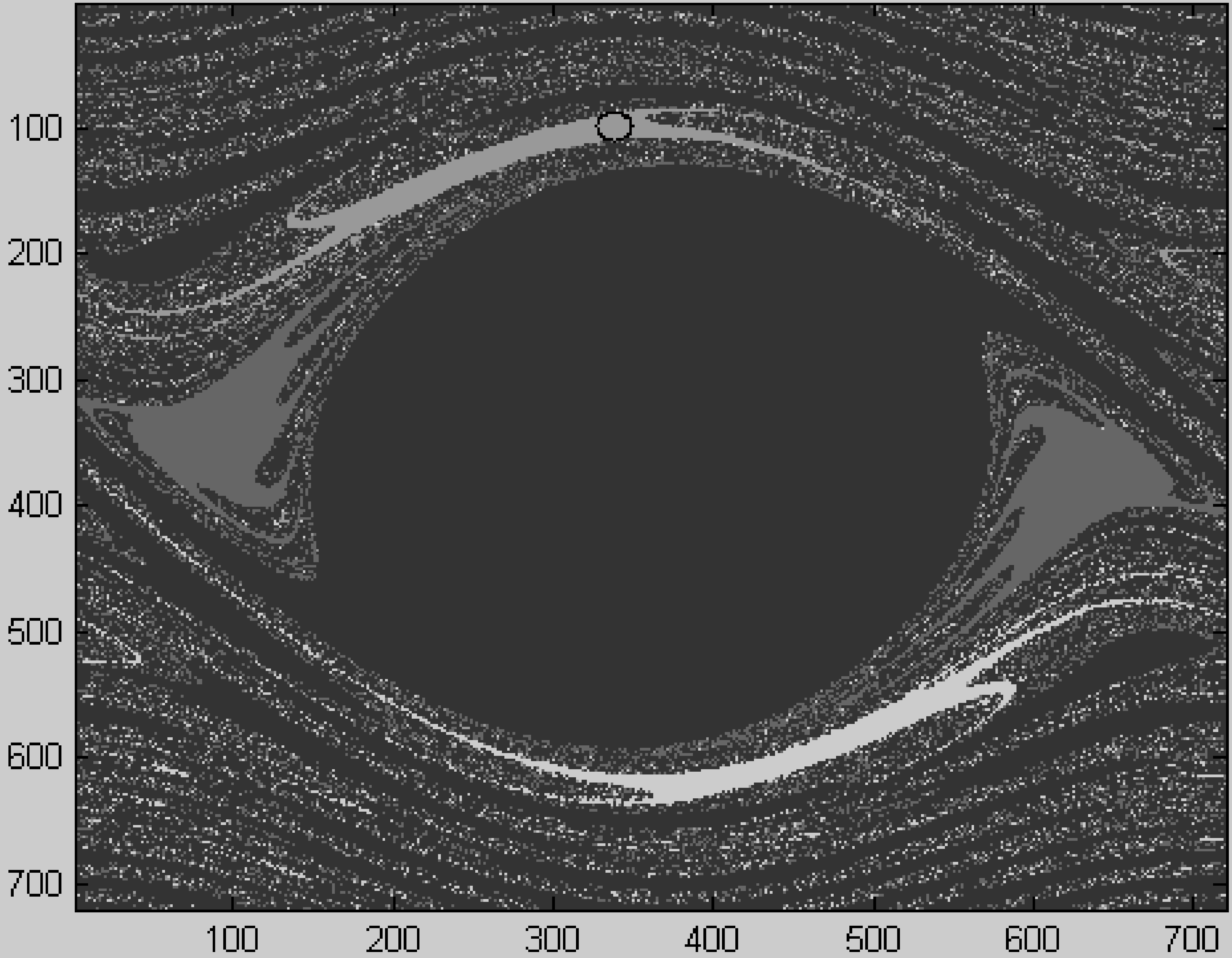
pendulum, p=0.095, ω=1.30, *c*=0.015



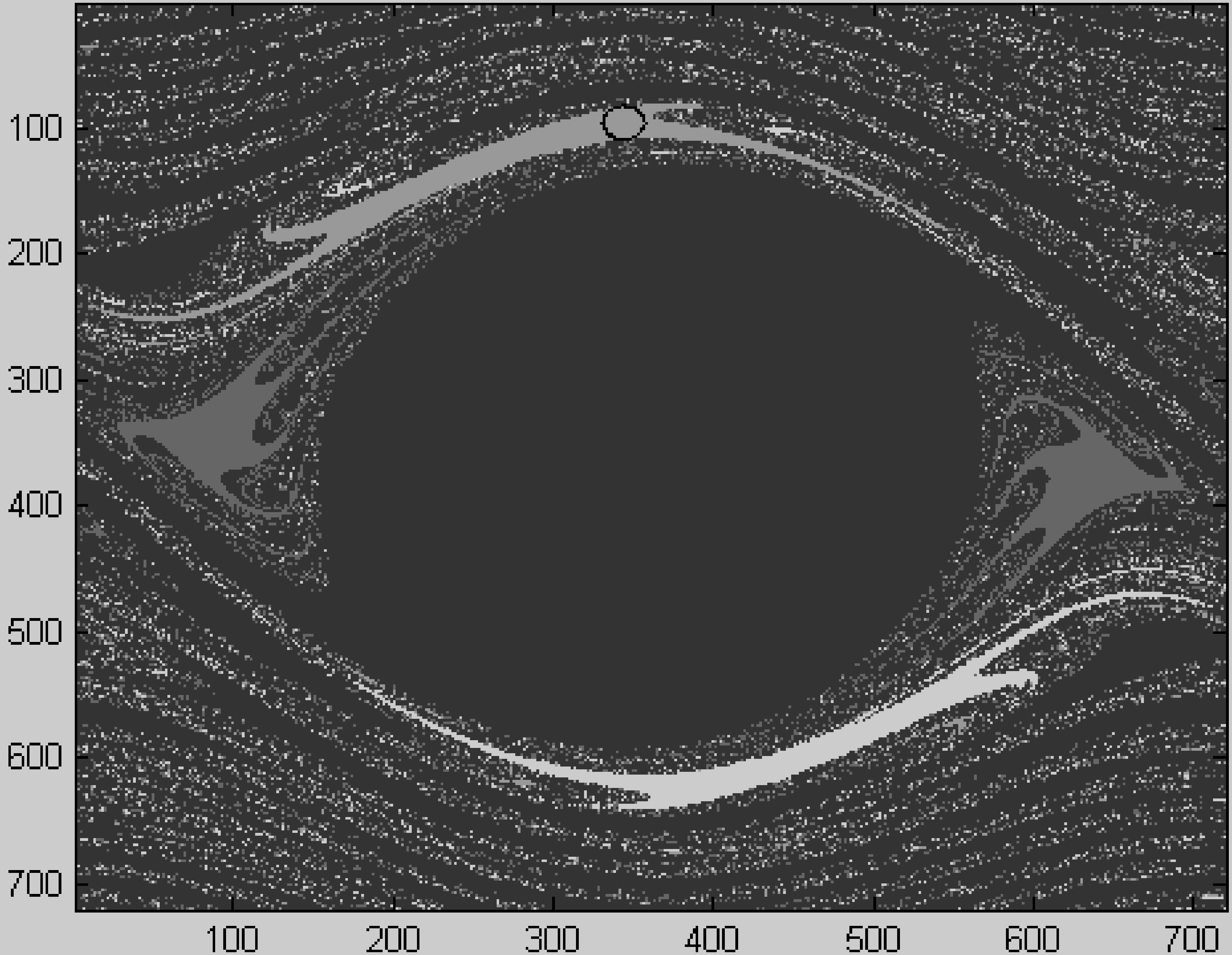
pendulum, p=0.100, ω=1.30, *c*=0.015



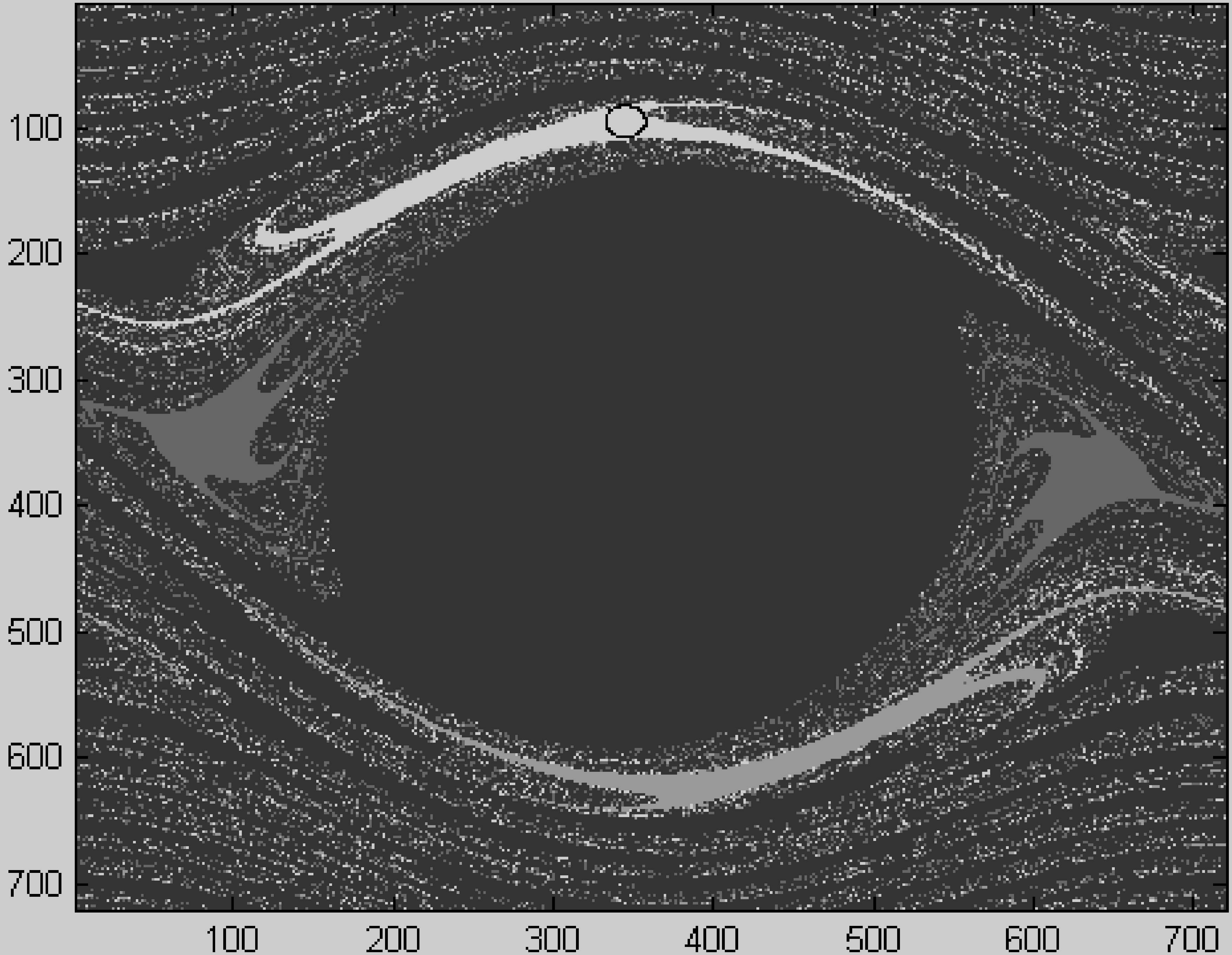
pendulum, p=0.110, ω=1.30, *c*=0.015



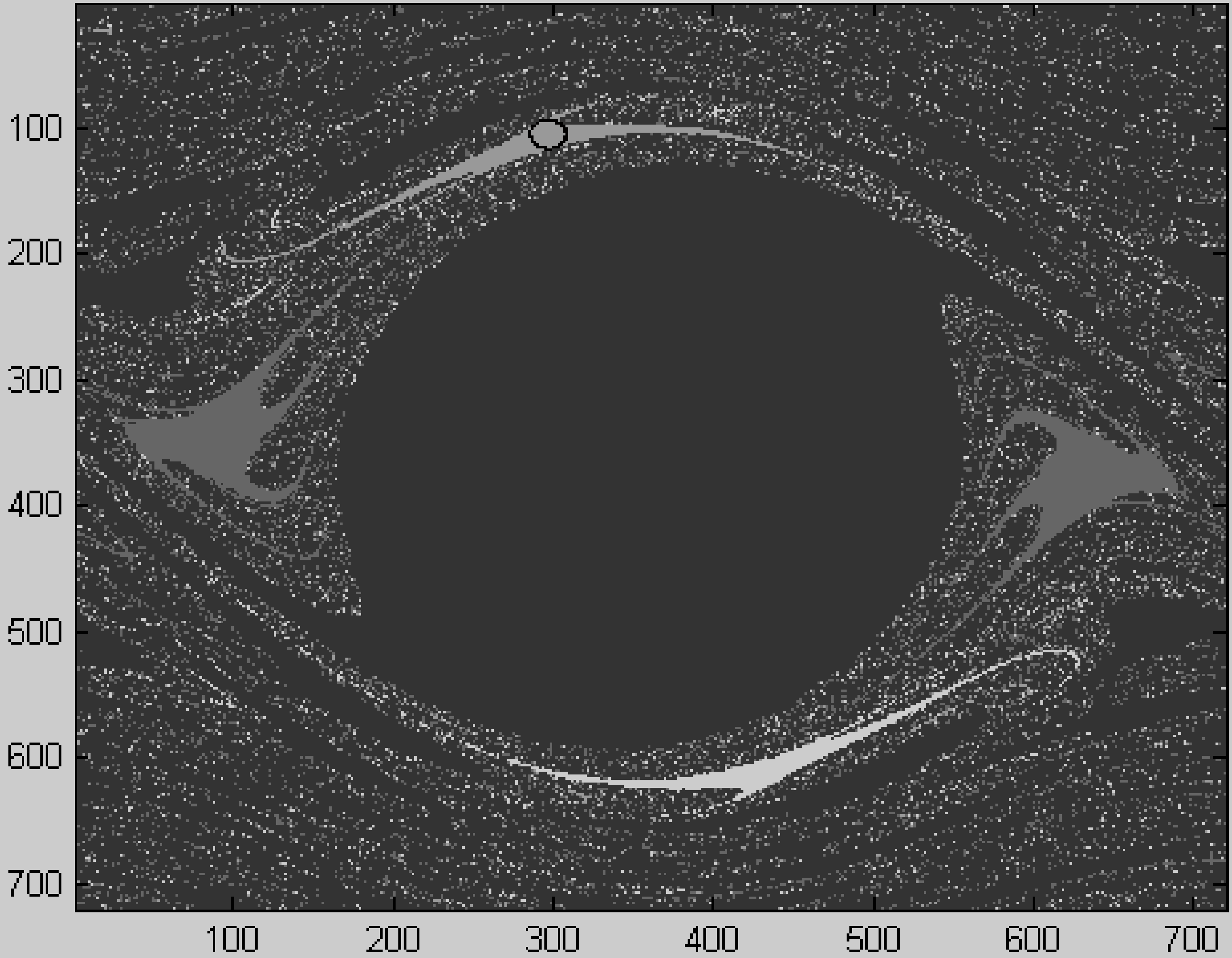
pendulum, p=0.120, ω=1.30, *c*=0.015



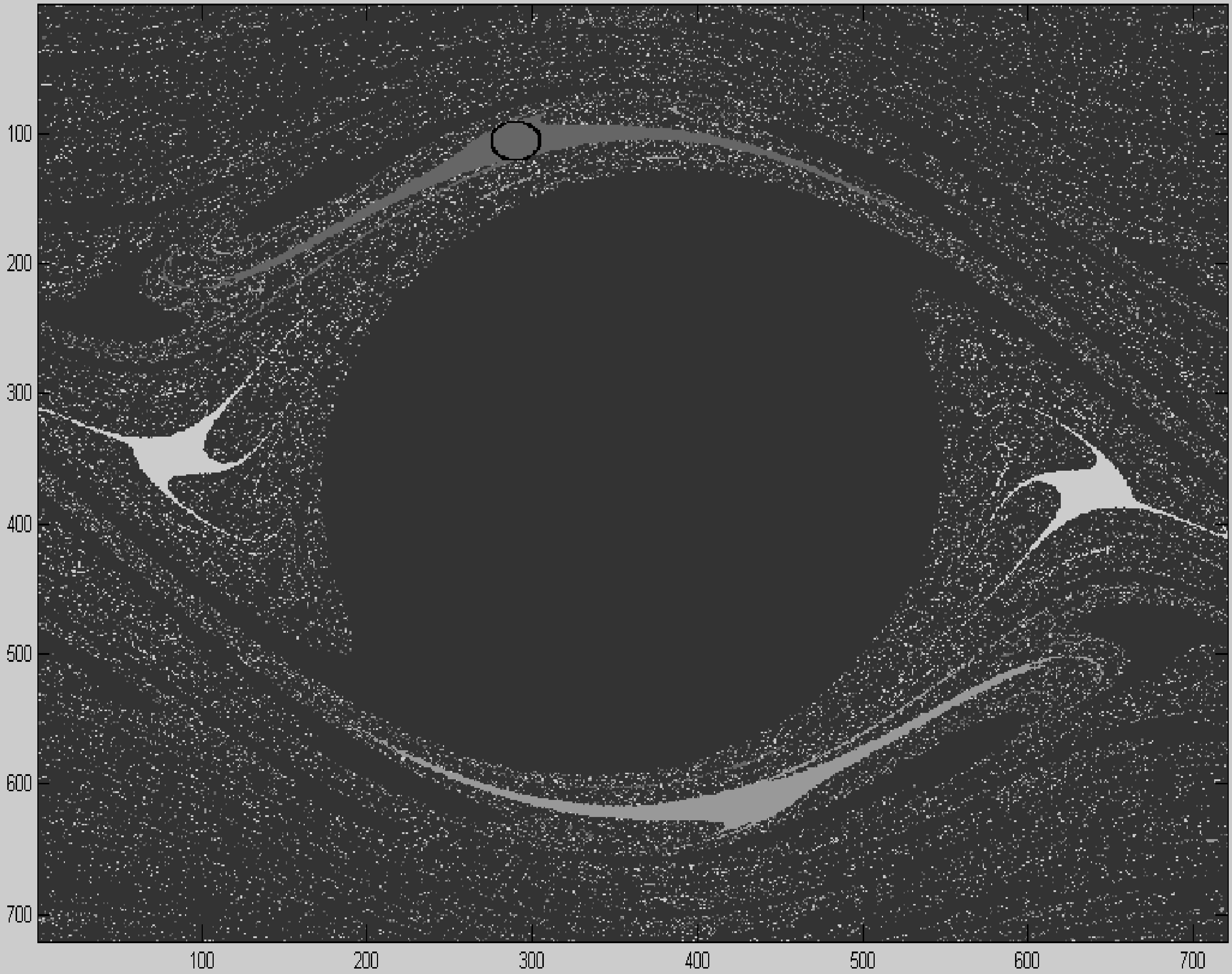
pendulum, p=0.130, ω=1.30, *c*=0.015



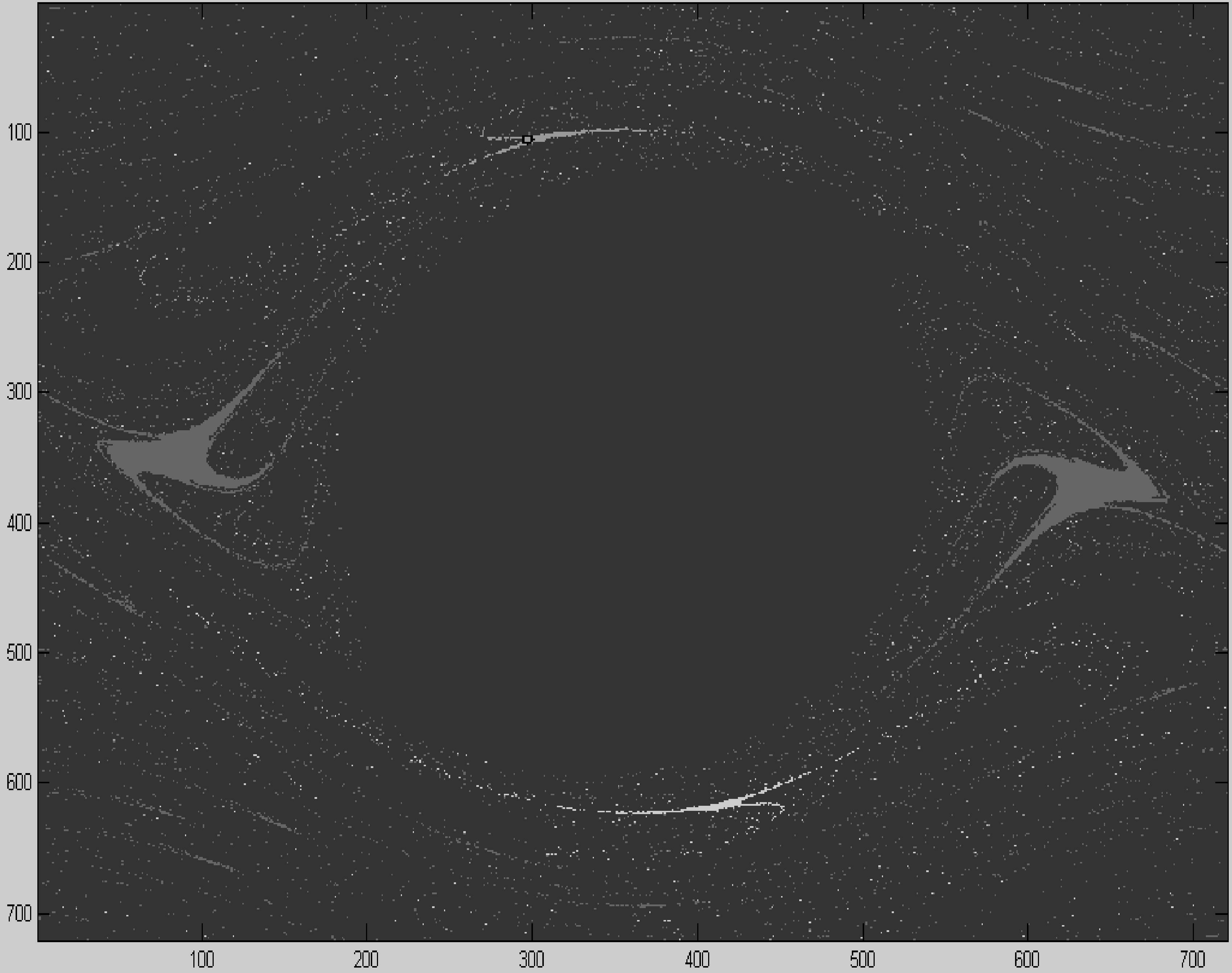
pendulum, p=0.150, ω=1.30, *c*=0.015



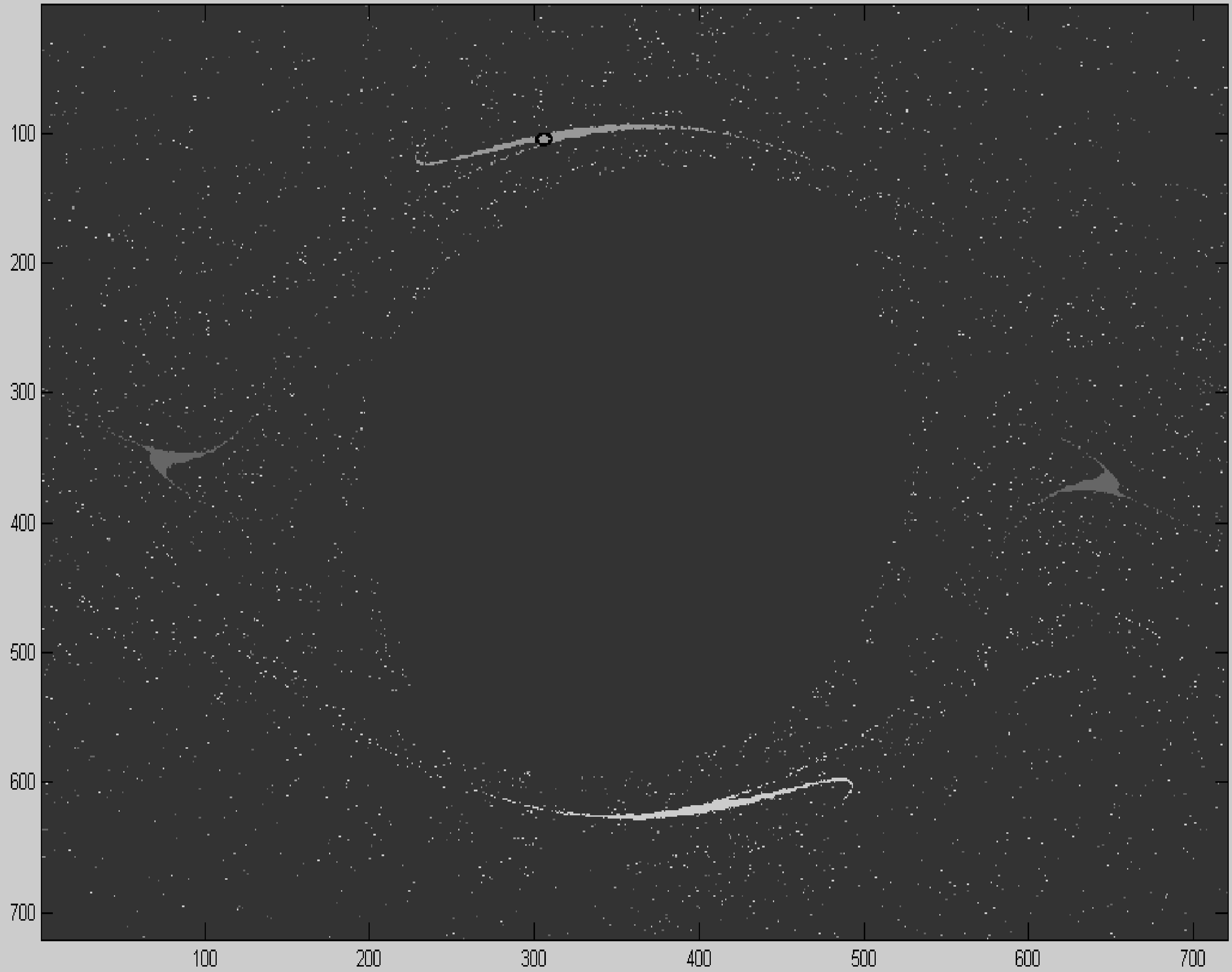
pendulum, p=0.170, ω=1.30, *c*=0.015



pendulum, p=0.200, ω=1.30, c=0.015



pendulum, p=0.250, ω=1.30, *c*=0.015



pendulum, p=0.300, ω=1.30, *c*=0.015