**Universidade de São Paulo - Escola Politécnica**
**Departamento de Engenharia de Estruturas e Geotécnica**
**Graduate Program in Civil Engineering (Structures)**

**PEF-5737 PROGRAMME**
**NON-LINEAR DYNAMICS AND STABILITY**
**Third Period 2018**

## Lectures 9-12

# Global Nonlinear Dynamics for Engineering Design and System Safety

Giuseppe Rega

*Department of Structural and Geotechnical Engineering*
*Sapienza University of Rome, Italy*

| DAY | TIME | LECTURE |
| --- | --- | --- |
| Monday 05/11 | 14.00 -14.45 | Historical Framework - A Global Dynamics Perspective in the Nonlinear Analysis of Systems/Structures |
| | 15.00 -15.45 | Achieving Load Carrying Capacity: Theoretical and Practical Stability |
| | 16.00 -16.45 | Dynamical Integrity: Concepts and Tools_1 |
| Wednesday 07/11 | 14.00 -14.45 | Dynamical Integrity: Concepts and Tools_2 |
| | 15.00 -15.45 | Global Dynamics of Engineering Systems |
| | 16.00 -16.45 | Dynamical integrity: Interpreting/Predicting Experimental Response |
| Monday 12/11 | 14.00 -14.45 | Techniques for Control of Chaos |
| | 15.00 -15.45 | A Unified Framework for Controlling Global Dynamics |
| | 16.00 -16.45 | Response of Uncontrolled/Controlled Systems in Macro- and Micro-mechanics |
| Wednesday 14/11 | 14.00 -14.45 | A Noncontact AFM: (a) Nonlinear Dynamics and Feedback Control<br>(b) Global Effects of a Locally-tailored Control |
| | 15.00 -15.45 | Exploiting Global Dynamics to Control AFM Robustness |
| | 16.00 -16.45 | Dynamical Integrity as a Novel Paradigm for Safe/Aware Design |

# 12 Hours - Contents (1)

0. **Exploiting Global Dynamics** in a thermomechanically coupled plate
1. **Theoretical stability** of systems/structures. **Solution/attractor robustness** and **basin erosion** in **phase-space** and **control parameter space. Practical stability** for achieving load carrying capacity
2. Safe basin. Concepts and tools of **dynamical integrity**: measures, profiles, charts
3. Dynamical integrity for
   - **analysing global dynamics**,
   - **interpreting and predicting experimental behavior**,
   - **getting hints towards engineering design**

   Theoretical and practical existence of solutions. Wanted/unwanted **competing attractors**

   Escape as dynamical system representation of **failure mechanisms** of **different physical systems**

# 12 Hours - Contents (2)

4. Techniques for **control of chaos**: **local** and **global** control of nonlinear response. **Global effects** of **locally-tailored** control

5. Response of **uncontrolled** vs **controlled** systems/models in structural dynamics, by also considering the effect of **system imperfections**:
   - archetypal oscillators; discrete systems; piecewise smooth systems;
   - slender structures liable to unstable interacting buckling;
   - reduced order models in micro/nano-mechanics

6. An illustrative system in micro-mechanics: a **noncontact AFM**
   - highlighting **unsafe overall dynamics** under **feedback control**
   - **enhancing dynamical robustness** via **global control**

7. Effects of **stochasticity** in system parameters and excitation

8. Dynamical Integrity as
   - A **criterion** for **practical stability** and **load carrying capacity**
   - A **novel paradigm** for **safe/aware engineering design**

| DAY | TIME | LECTURE |
| --- | --- | --- |
| Monday 05/11 | 14.00 -14.45 | **Historical Framework - A Global Dynamics Perspective in the Nonlinear Analysis of Systems/Structures** |
| | 15.00 -15.45 | Achieving Load Carrying Capacity: Theoretical and Practical Stability |
| | 16.00 -16.45 | Dynamical Integrity: Concepts and Tools_1 |
| Wednesday 07/11 | 14.00 -14.45 | Dynamical Integrity: Concepts and Tools_2 |
| | 15.00 -15.45 | Global Dynamics of Engineering Systems |
| | 16.00 -16.45 | Dynamical integrity: Interpreting/Predicting Experimental Response |
| Monday 12/11 | 14.00 -14.45 | Techniques for Control of Chaos |
| | 15.00 -15.45 | A Unified Framework for Controlling Global Dynamics |
| | 16.00 -16.45 | Response of Uncontrolled/Controlled Systems in Macro- and Micro-mechanics |
| Wednesday 14/11 | 14.00 -14.45 | A Noncontact AFM: (a) Nonlinear Dynamics and Feedback Control<br>(b) Global Effects of a Locally-tailored Control |
| | 15.00 -15.45 | Exploiting Global Dynamics to Control AFM Robustness |
| | 16.00 -16.45 | Dynamical Integrity as a Novel Paradigm for Safe/Aware Design |

# 9.1 – Historical Framework – A Global Dynamics Perspective in the Nonlinear Analysis of Systems/Structures

Giuseppe Rega

*Department of Structural and Geotechnical Engineering*
*Sapienza University of Rome, Italy*

***Coworker:*** S. Lenci

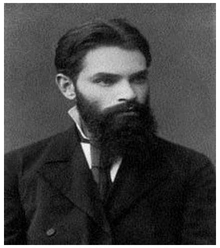
# Nonlinear Dynamics in Mathematical-Physical Sciences

*"If modern nonlinear dynamics has a father it is Henri Poincaré (1854-1912)"*


H. Poincaré

**LATE 1800** **H.POINCARE'** → Origin of **CHAOS THEORY**

Birth of **ALGEBRAIC TOPOLOGY**


A.Lyapunov

**EARLY 1900** **A. LYAPUNOV** → Theory of **MOTION STABILITY**

Following Poincaré intuitions, fundamentals of **science of nonlinear/complex systems** developed within the mathematical-physical community

Two soviet schools of thought to pursue solutions to nonlinear systems

(i) *Moscow school* (**Andronov** and followers'): via ***qualitative methods,*** with applications mostly in radiophysics and electrical engineering;


A.Andronov

(i) *Kiev school* (**Krylov-Bogoliubov-Mitropolski**): via ***analytical methods*** (quantitative), mostly dealing with problems in nonlinear mechanics


Y. Miitropolski

**Mid of XX CENTURY (60s and 70s)** → explosion of **DYNAMICAL SYSTEMS THEORY**

**Theories of CATASTROPHE, COMPLEXITY, CHAOS, FRACTALS, TURBULENCE**

. What about **Nonlinear Dynamics** in ***Mechanics*** ? **Four main hystorical stages** →

1980 1985 1990 1995 2000 2005 2010 2015

## NONLINEAR OSCILLATIONS – ANALYTICAL TECHNIQUES

**Systems/Models**:

**Archetypal** single-dof oscillators: Helmholtz, Duffing, van der Pol, pendulum, piece-wise linear, impact

- ***representative of discrete mechanical systems***, or
- ***reliable discretized single-mode models of continuous systems (structures)*** suitable to investigate fundamental aspects of ND behavior

Self-excited oscillations. Two-degree-of-freedom systems. Rotating systems

**Analysis/Solutions**:

- Problems with ***small nonlinearities* - Asymptotics**: perturbation (Lindstedt-Poincaré; multiple time scales), averaging (generalized method, Krylov-Bogoliubov-Mitropolski); Melnikov, Holmes-Marsden
- Problems with also ***large nonlinearities*** - Method of harmonic balance, Homotopy analysis method

Exact and approximate solutions

PDE ***Direct perturbation*** ***NNM*** ***Invariant manifold*** technique

**Phenomena**:

- Primary/secondary/internal/multiple ***resonances***; external/parametric excitation
- ***Weakly ND regular*** responses; ***nonlinear modal interaction***
- Nonstationary vibrations

**Tools**: characterizing ND response through:

- Backbone, frequency/force-response curves; time histories; phase portraits; Poincaré map; power spectra

1980 1985 1990 1995 2000 2005 2010 2015

**NONLINEAR OSCILLATIONS – ANALYTICAL TECHNIQUES**

**BIFURCATIONS – COMPLEX DYNAMICS – GEOMETRICAL/COMPUTATIONAL TECHNIQUES**

**Systems/Models**: Maps and Flows

- ***Smooth continuous*** systems: multimode models (2-4 dof)
- ***Non-smooth*** systems. Systems with *time delay* (machining, manufacturing). Multibody systems. Vehicle systems

**Analysis/Solutions**:

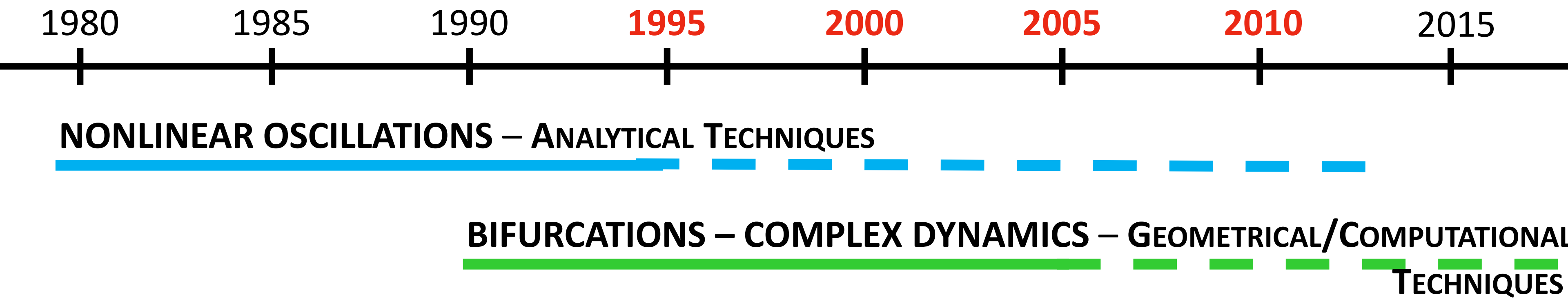
- ***Local analysis:*** *continuation* (arc-length, shooting), *numerical simulation* (brute force). Structural stability
- ***Global analysis***: invariant manifolds; homo/heteroclinic orbits; center manifold; cell mapping. System dimension calculation

**Phenomena**: ***strongly*** ND

- ***Local*** bifurcations: nonlinear multimodal interaction in regular regime. Slow/fast-scale phenomena with energy transfer between modes
- ***Global*** bifurcations: homo/heteroclinic tangles. Transition to ***complex dynamics***: quasiperiodicity, ***chaos***
- Resonance capture. Synchronization. Stick-slip oscillations.

**Tools**:

- ***Quantitative measures*** of ***nonregular*** responses: fractal dimension, Lyapunov exponents. ***Basins of attraction***: attractor-basin phase portraits




**EXPERIMENTAL NONLINEAR DYNAMICS** – SMALL-SCALE MODELS

**Systems/Models :**

- ***flexible*** models of continuous systems; also ***real structures***, in a system identification/health monitoring perspective

**Analysis/Solutions:**

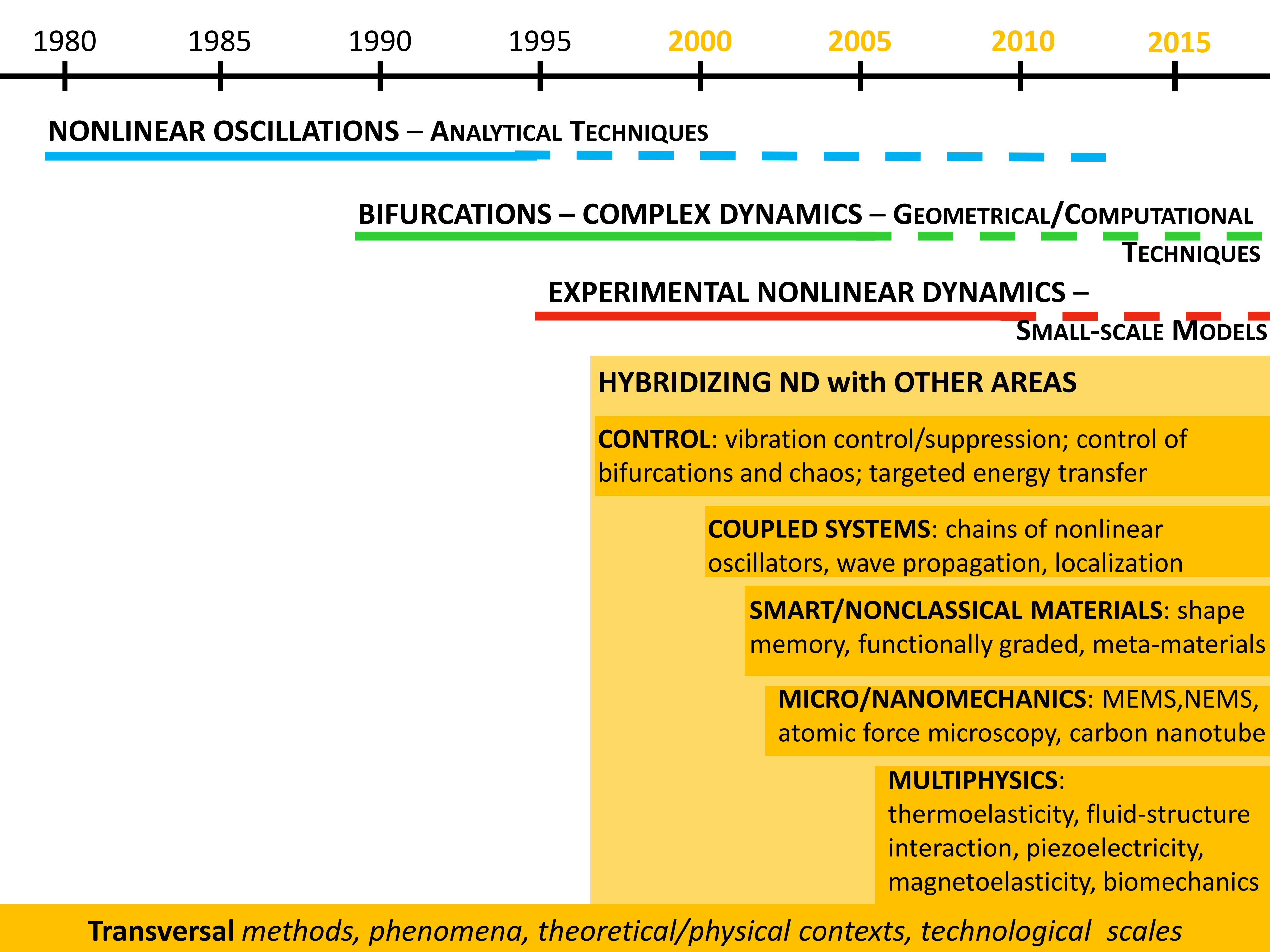
- ***experimental time series*** analysis;
- ***reconstructing dynamical properties*** from ***experimental measurements***: delay embedding, phase space reconstruction; modeling and prediction
- ***proper orthogonal decomposition***

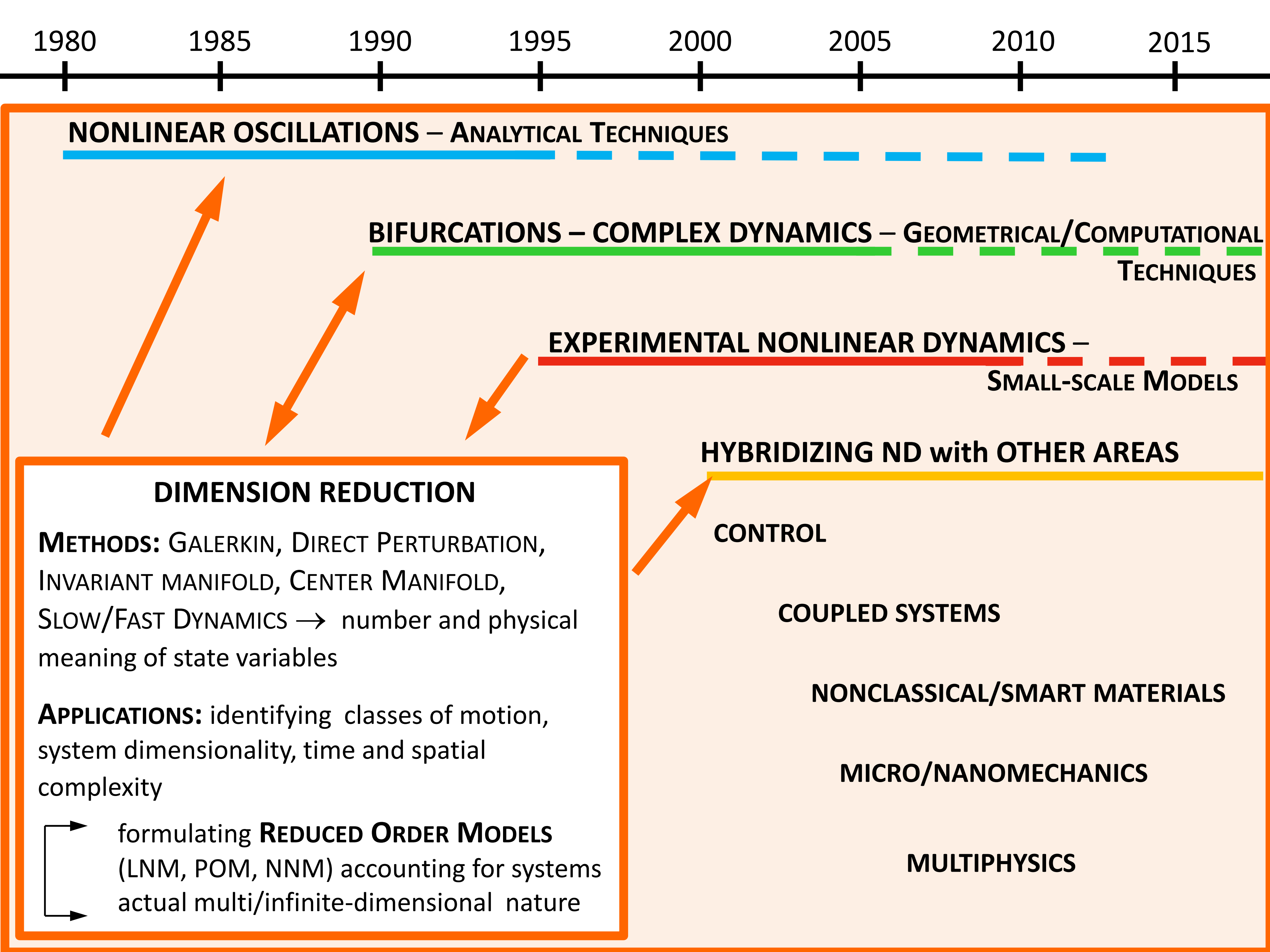
**Phenomena:**

- ***experimental vs canonical scenarios of transition*** to chaos
- ***spatial properties*** of nonlinear response: ***response dimensionality***

**Tools:**

***dimensions***: correlation (attractor), embedding (invariant saturation), topological (manifold of motion), phase-space (of system dynamics)


1980
1985
1990
1995
2000
2005
2010
2015
NONLINEAR OSCILLATIONS – ANALYTICAL TECHNIQUES
BIFURCATIONS – COMPLEX DYNAMICS – GEOMETRICAL/COMPUTATIONAL TECHNIQUES
EXPERIMENTAL NONLINEAR DYNAMICS – SMALL-SCALE MODELS
HYBRIDIZING ND with OTHER AREAS
CONTROL: vibration control/suppression; control of bifurcations and chaos; targeted energy transfer
COUPLED SYSTEMS: chains of nonlinear oscillators, wave propagation, localization
SMART/NONCLASSICAL MATERIALS: shape memory, functionally graded, meta-materials
MICRO/NANOMECHANICS: MEMS,NEMS, atomic force microscopy, carbon nanotube
MULTIPHYSICS: thermoelasticity, fluid-structure interaction, piezoelectricity, magnetoelasticity, biomechanics
Transversal methods, phenomena, theoretical/physical contexts, technological scales


1980
1985
1990
1995
2000
2005
2010
2015
NONLINEAR OSCILLATIONS – ANALYTICAL TECHNIQUES
BIFURCATIONS – COMPLEX DYNAMICS – GEOMETRICAL/COMPUTATIONAL TECHNIQUES
EXPERIMENTAL NONLINEAR DYNAMICS – SMALL-SCALE MODELS
HYBRIDIZING ND with OTHER AREAS
DIMENSION REDUCTION
METHODS: GALERKIN, DIRECT PERTURBATION, INVARIANT MANIFOLD, CENTER MANIFOLD, SLOW/FAST DYNAMICS → number and physical meaning of state variables
APPLICATIONS: identifying classes of motion, system dimensionality, time and spatial complexity
formulating REDUCED ORDER MODELS (LNM, POM, NNM) accounting for systems actual multi/infinite-dimensional nature
CONTROL
COUPLED SYSTEMS
NONCLASSICAL/SMART MATERIALS
MICRO/NANOMECHANICS
MULTIPHYSICS

# Global Nonlinear Dynamics (1)

**Global nonlinear dynamics** in **applied mechanics** dates back to the 80s: from mathematics/physics to engineering communities

Since then:

- powerful concepts and tools of **dynamical systems, bifurcation and chaos** theory
- sophisticated **analytical, geometrical** and **computational** techniques
- meaningful **experimental verifications**

→ importance of **nonlinear phenomena** in **technical applications**

# Global Nonlinear Dynamics (2)

- Achievements of last 30 years
  - entail a substantial **change of perspective**,
  - are ready to meaningfully **affect**

**analysis, control** and **design**

of mechanical/structural systems

- Highlighting the important, yet still overlooked, **role** that **GND concepts/tools** may play as regards **load carrying capacity** and **safety** of engineering systems
- Updating the classical **stability** concept via consideration of **global dynamic effects**

*Applied Mechanics Reviews*, 67(5), 050802, 2015

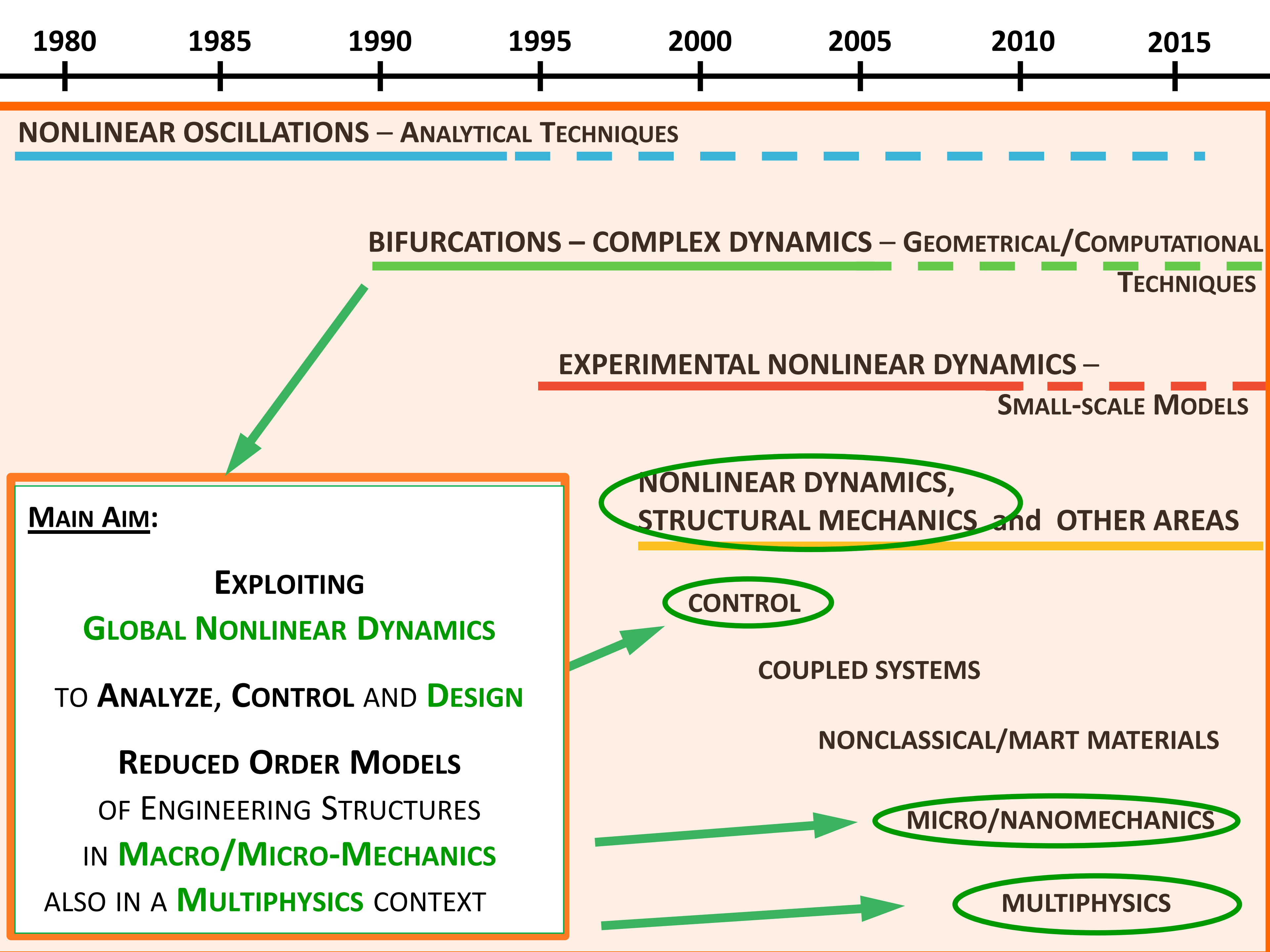
# A Global Dynamics Perspective for System Safety From Macro- to Nanomechanics: Analysis, Control, and Design Engineering

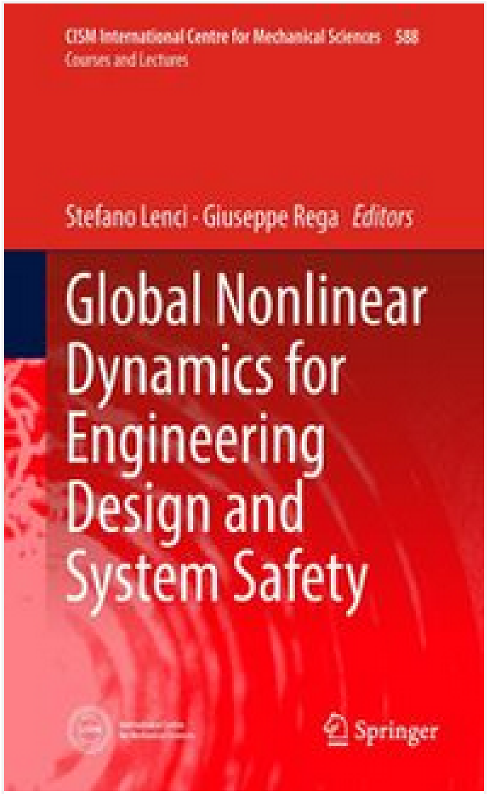
**Giuseppe Rega**[1]
Department of Structural and Geotechnical Engineering,
Sapienza University of Rome,
Via Antonio Gramsci 53,
Rome 00197, Italy
e-mail: giuseppe.rega@uniroma1.it

**Stefano Lenci**
Department of Civil and Building Engineering, and Architecture,
Polytechnic University of Marche,
via Brecce Bianche,
Ancona 60131, Italy
e-mail: lenci@univpm.it

*The achievements occurred in nonlinear dynamics over the last 30 years entail a substantial change of perspective when dealing with vibration problems, since they are now deemed ready to meaningfully affect the analysis, control, and design of mechanical and structural systems. This paper aims at overviewing the matter, by highlighting and discussing the important, yet still overlooked, role that some relevant concepts and tools may play in engineering applications. Upon dwelling on such topical concepts as local and global dynamics, bifurcation and complexity, theoretical and practical stability, attractor robustness, basin erosion, and dynamical integrity, recent results obtained for a variety of systems and models of interest in applied mechanics and structural dynamics are overviewed in terms of analysis of nonlinear phenomena and their control. The global dynamics perspective permits to explain partial discrepancies between experimental and theoretical/numerical results based on merely local analyses and to implement effective dedicated control procedures. This is discussed for discrete systems and reduced order models of continuous systems, for applications ranging from macro- to micro/nanomechanics. Understanding of basic phenomena in nonlinear dynamics has now reached such a critical mass that it is time to exploit their potential to enhance the effectiveness and safety of systems in technological applications and to develop novel design criteria.*
[DOI: 10.1115/1.4031705]

*Keywords: nonlinear dynamics, mechanical/structural systems, local versus global behavior, dynamical integrity, experiments in macro/micromechanics, control, engineering design*


1980
1985
1990
1995
2000
2005
2010
2015
NONLINEAR OSCILLATIONS – ANALYTICAL TECHNIQUES
BIFURCATIONS – COMPLEX DYNAMICS – GEOMETRICAL/COMPUTATIONAL TECHNIQUES
EXPERIMENTAL NONLINEAR DYNAMICS – SMALL-SCALE MODELS
NONLINEAR DYNAMICS, STRUCTURAL MECHANICS and OTHER AREAS
CONTROL
COUPLED SYSTEMS
NONCLASSICAL/MART MATERIALS
MICRO/NANOMECHANICS
MULTIPHYSICS
MAIN AIM:
EXPLOITING
GLOBAL NONLINEAR DYNAMICS
TO ANALYZE, CONTROL AND DESIGN
REDUCED ORDER MODELS
OF ENGINEERING STRUCTURES
IN MACRO/MICRO-MECHANICS
ALSO IN A MULTIPHYSICS CONTEXT




BOOK

# Global Nonlinear Dynamics for Engineering Design and System Safety - 2019

**Editors:** Prof. Stefano Lenci · Prof. Giuseppe Rega

This is the first book which exploits concepts and tools of global nonlinear dynamics for bridging the gap between theoretical and practical stability of systems/structures, and for possibly enhancing the engineering design in ma

# BASINS OF ATTRACTION

- **Basin of attraction**: The set of initial conditions in phase space leading to a given (generally bounded) attractor as t $\rightarrow \infty$, i.e. at steady nonlinear dynamics

***Nonlinear systems*** ⟶ ***Multistability*** *(coexisting attractors)* ⟶ ***Competing basins***

- **1-d.o.f.** mechanical system: **2D** phase space (displacement and velocity)
- **n-d.o.f.** mechanical system: **2nD** phase space (displacement and velocity of each dof)

- Global analyses to be pursued in actual ***multidimensional phase space***, which is only possible via systematic implementation of effective ***parallel computing*** technique (research going on)

- Yet, analysis of ***multidimensional basins of attraction*** via a ***variety of properly selected 2D cross-sections*** can already allow to get a ***reliable representation/ description*** of ***some main features*** of ***nonlinear dynamic response*** of ***MDOF systems***

**Let's start** with a ***coupled multiphysics*** context characterized by ***slow-fast dynamics:***

**Exploiting Global Dynamics**

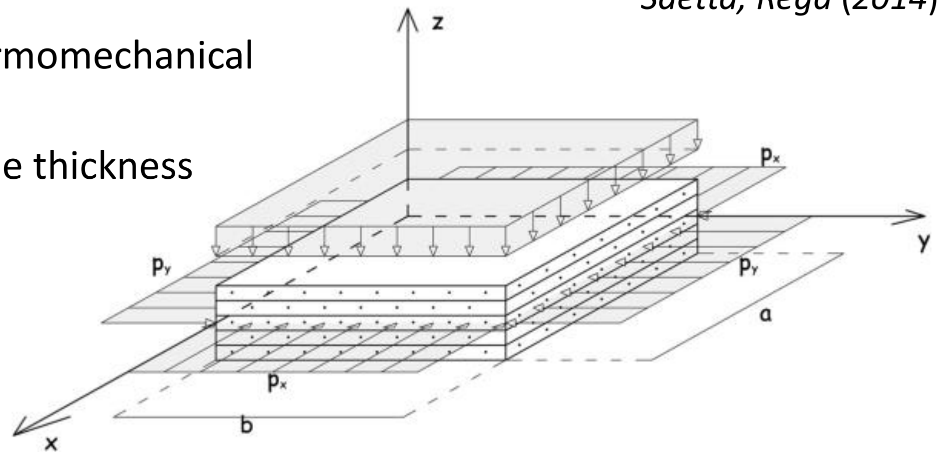
to unveil **Transient Thermal Effects** in the ***Steady Response***

# MACROMECHANICS: A THERMOMECHANICALLY COUPLED LAMINATED PLATE

**2D unified continuum formulation**

*Saetta, Rega (2014)*

- Classical ***von Kármán*** laminated plate with Thermomechanical Coupling (**CTC**)
- Prescribed ***linear temperature variation*** along the thickness
- 5 generalized variables $\{u, v, w, T_0, T_1\}$




For symmetric cross-ply laminates:

- Static condensation of in-plane displacements
- Single-mode mechanical approximation

**FORCING TERMS: variety of MECHANICAL and THERMAL excitations**

$\boldsymbol{p}$: constant precompression

$T_\infty$: constant **temperature difference** between plate and environment

$$\ddot{W}(t) + a_{12}\dot{W}(t) + a_{13}W(t) + a_{14}W(t)^3 + a_{15}T_{R1}(t) + a_{16}T_{R0}(t)W(t) + a_{17}Cos(t) = 0$$

$$\dot{T}_{R0}(t) + a_{22}T_{R0}(t) + a_{23}\alpha_1 T_\infty + a_{24}W(t)\dot{W}(t) + a_{25}e_0(t) = 0$$

$$\dot{T}_{R1}(t) + a_{32}T_{R1}(t) + a_{33}\dot{W}(t) + a_{34}e_1(t) = 0$$

harmonic mechanical excitation

**bending-membrane** thermal excitations

**COUPLING TERMS:**
**two-way** thermomechanical interaction

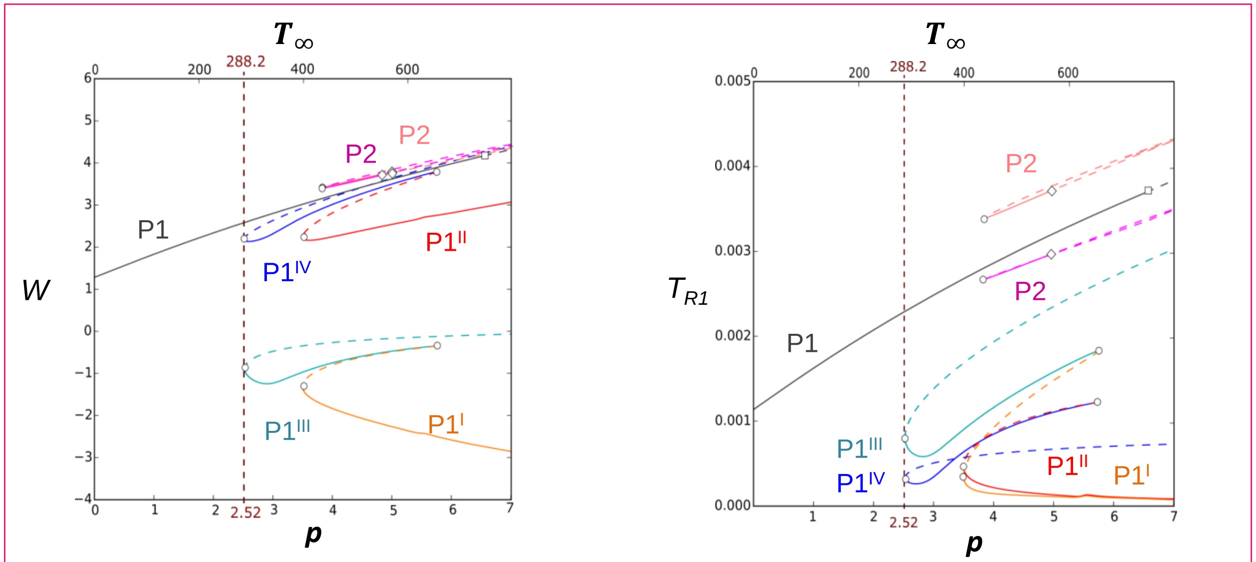
**0D formulation**: 3 coupled ODEs as a function of
reduced transversal displacement $\boldsymbol{W}$
reduced membrane temperature $\boldsymbol{T_{R0}}$
reduced bending temperature $\boldsymbol{T_{R1}}$

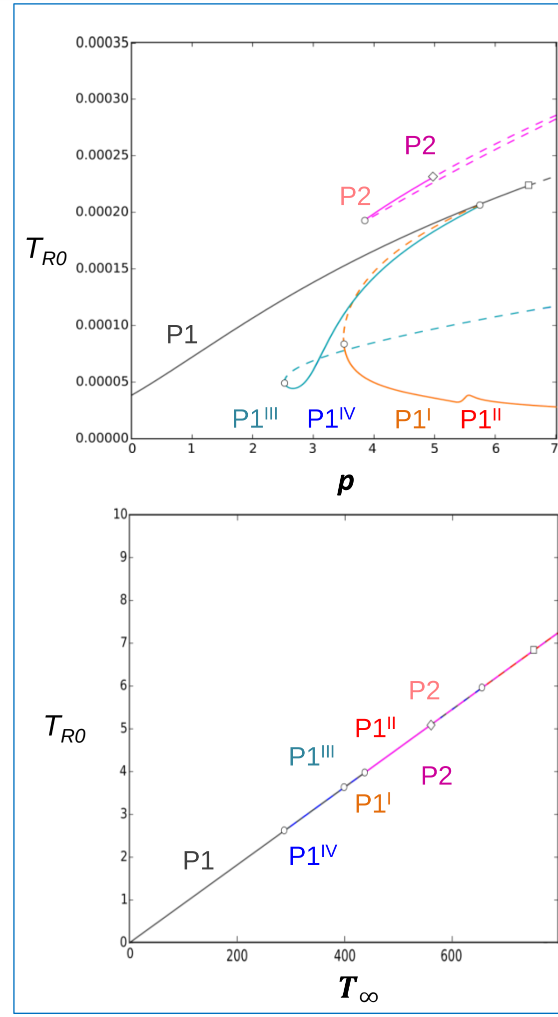
**LOCAL and GLOBAL DYNAMICS of CTC plate in ACTIVE thermal regime**

# ND IN ACTIVE THERMAL REGIME

## CONSTANT THERMAL EXCITATION $T_\infty$

- Pure thermal convection on the external surfaces
  Pure internal thermal conduction
- Directly activates the membrane temperature variable $T_{R0}$
- Modifies the mechanical linear stiffness, as precompression $p$




$$\ddot{W}(t) + a_{12}\dot{W}(t) + a_{13}W(t) + a_{14}W(t)^3 + a_{15}T_{R1}(t) + a_{16}T_{R0}(t)W(t) + a_{17}Cos(t) = 0$$

$$\dot{T}_{R0}(t) + a_{22}T_{R0}(t) + a_{23}\alpha_1 T_\infty + a_{24}W(t)\dot{W}(t) = 0$$

$$\dot{T}_{R1}(t) + a_{32}T_{R1}(t) + a_{33}\dot{W}(t) = 0$$

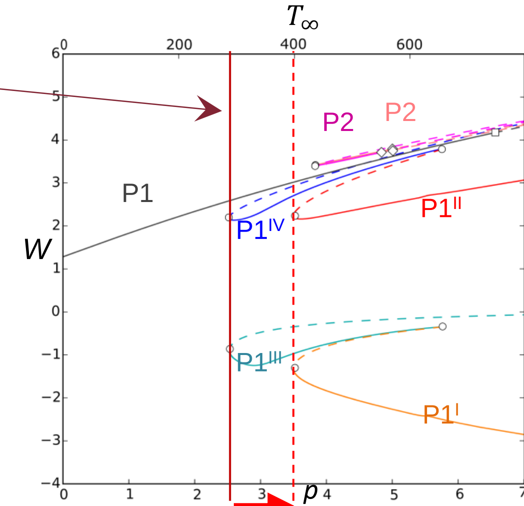
$$T_\infty = -a_{22}\,(p-1)/(a_{16}\,a_{23}\,\alpha_1)$$

# USING CONSTANT THERMAL EXCITATION: INDUCING BUCKLED RESPONSES (1)

**PLATE WARMING** due to a HOTTER ENVIRONMENT

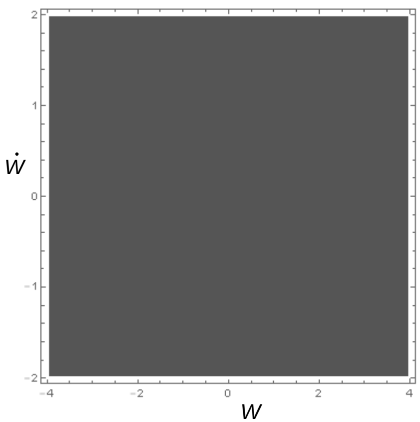
- ❑ STARTING **from a PRE-BUCKLING configuration**

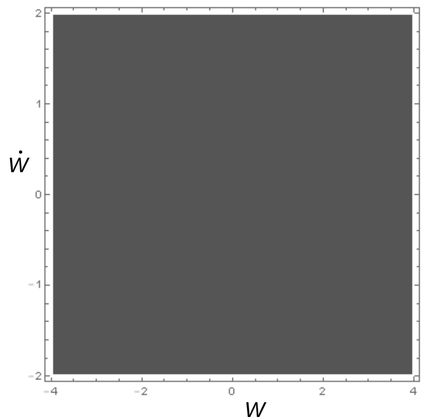
$T_{R0}(0)=T_{R1}(0) = 0.0$  $f = 1$, $p = 2.51$



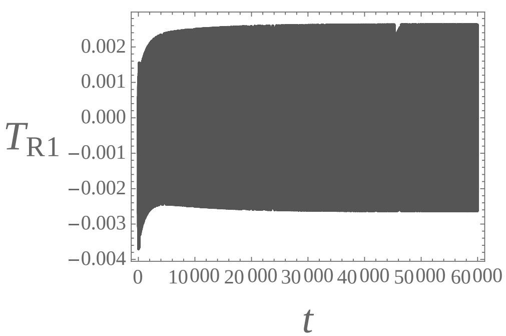

$T_\infty$ = +100 (p=+0.88)

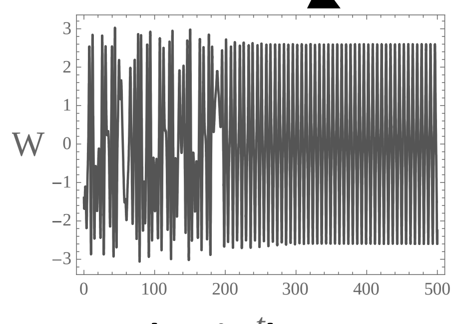
| **Before** | **With $T_\infty = +100$** |
|---|---|
| only pre-buckling $P1$ | only pre-buckling $P1$ |

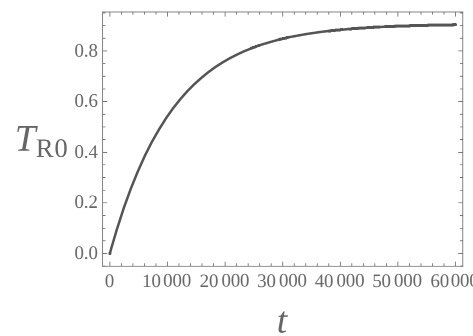



$T_{R0}$ slowly settles to $+0.9$




**Mechanical response:** quickly falls back and remains on stable pre-buckling solution

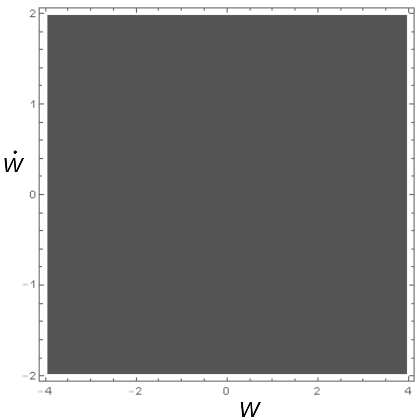
$T_{R0}$ **long transient:** slow contribution to mechanical stiffness

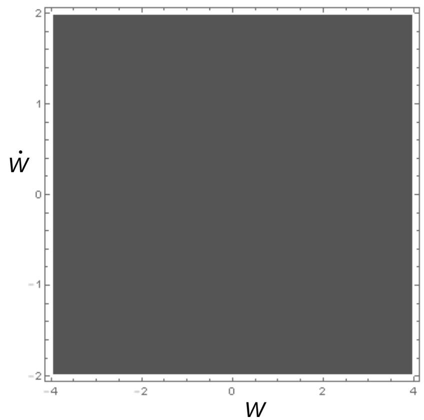
## USING CONSTANT THERMAL EXCITATION : INDUCING BUCKLED RESPONSES (2)

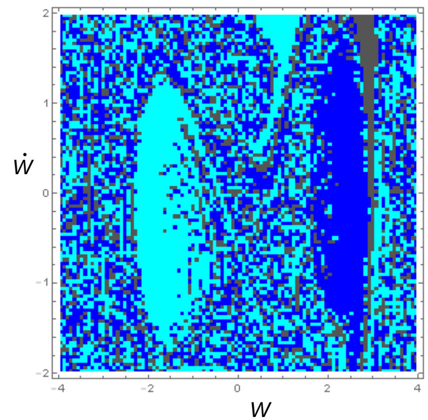
**PLATE WARMING** due to a HOTTER ENVIRONMENT

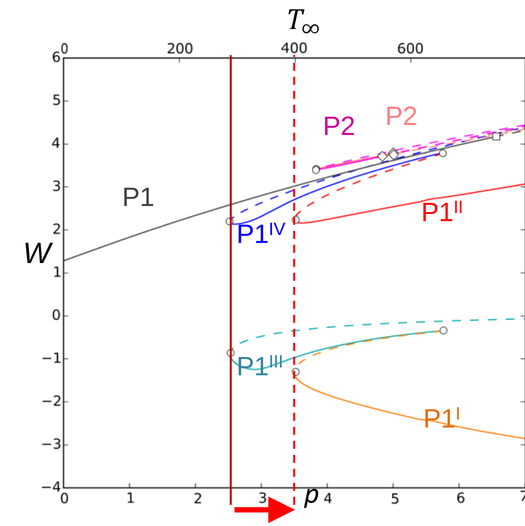
$T_{R0}(0) = T_{R1}(0) = 0.0$ $f = 1, p = 2.51$

| **BEFORE** | **WITH $T_\infty = +100$** | **UNCOUPLED SYSTEM** |
|---|---|---|
| only pre-buckling $P1$ | only pre-buckling $P1$ | pre-buckling $P1$ + buckled $P1^{III}/P1^{IV}$ |



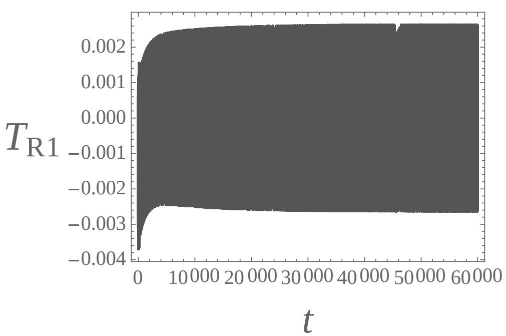



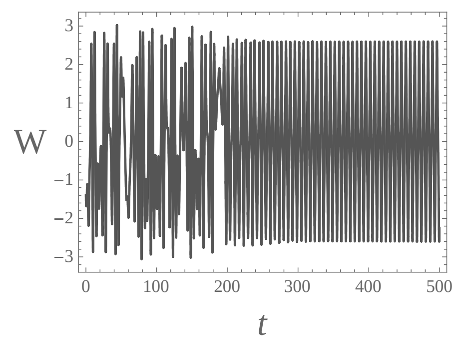



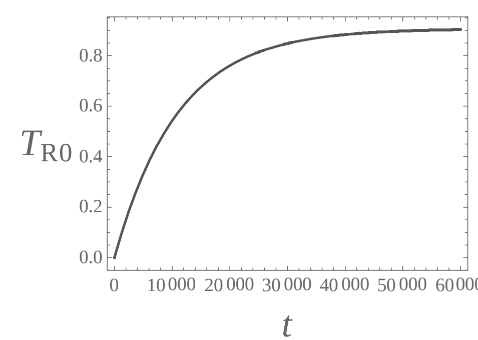

$T_\infty = +100$ (p=+0.88)

<u>usually considered</u>, sometimes unreliably

$T_{R0}$ slowly settles to $+0.9$




$T_{R0}$ LONG **TRANSIENT** **STRONGLY MODIFIES** **STEADY** DYNAMICS OF COUPLED SYSTEM

*Settimi, Saetta, Rega (2017)*

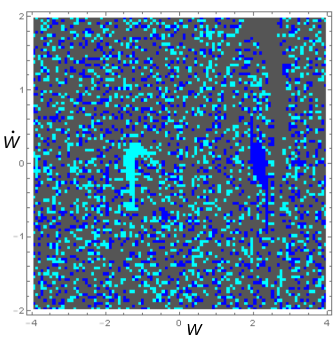
**PLATE WARMING** due to a HOTTER ENVIRONMENT

$f = 1$, $p = 2.51$, $T_{\infty} = +100$

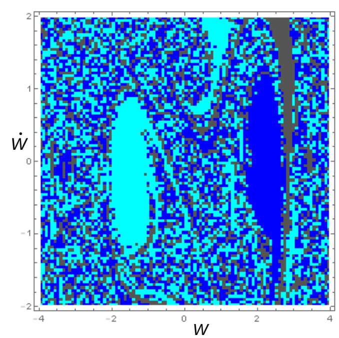
**Changing $T_{R0}$ initial conditions → Reducing thermal transient**

$T_{R1}(0) = 0.0$

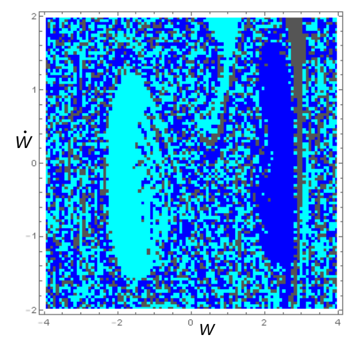
$T_{R0}(0) = 0.2$



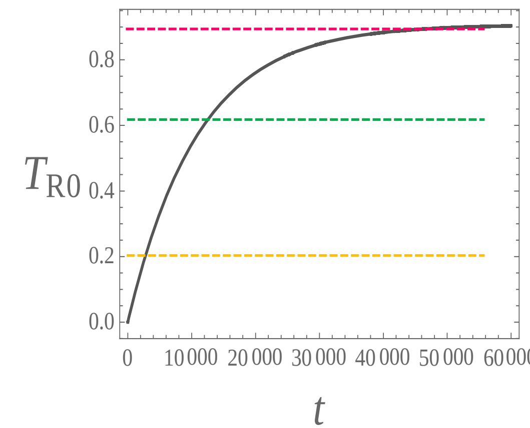

$T_{R0}(0) = 0.6$




$T_{R0}(0) = 0.9$

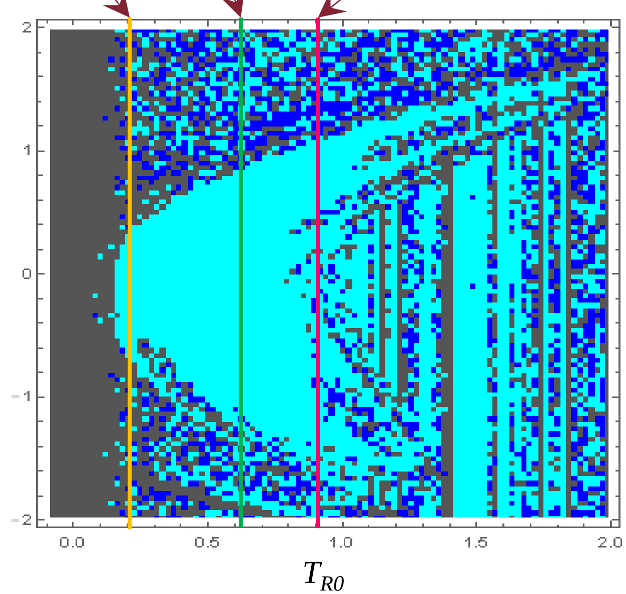



as with uncoupled system




**Cross section of 4D basins in $(T_{R0}, \dot{W})$ plane**

$T_{R1}(0) = 0$
$W(0) = -1.3$
(core of buckled $P1^{III}$ solution)

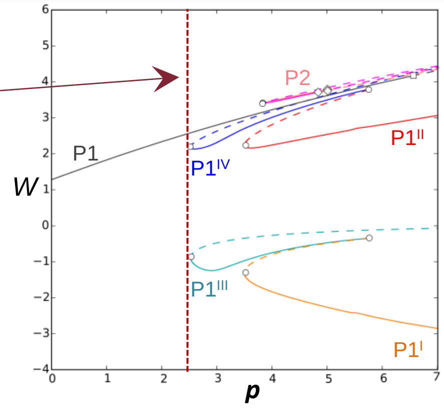



- $T_{R0}(0) < 0.2$ : monostable response
- $0.2 < T_{R0}(0) < 0.9$ : buckled basins appear and enlarge their compact part
- $T_{R0}(0) > 0.9$: strong fractalization significantly reduces basins volume making them unsafe

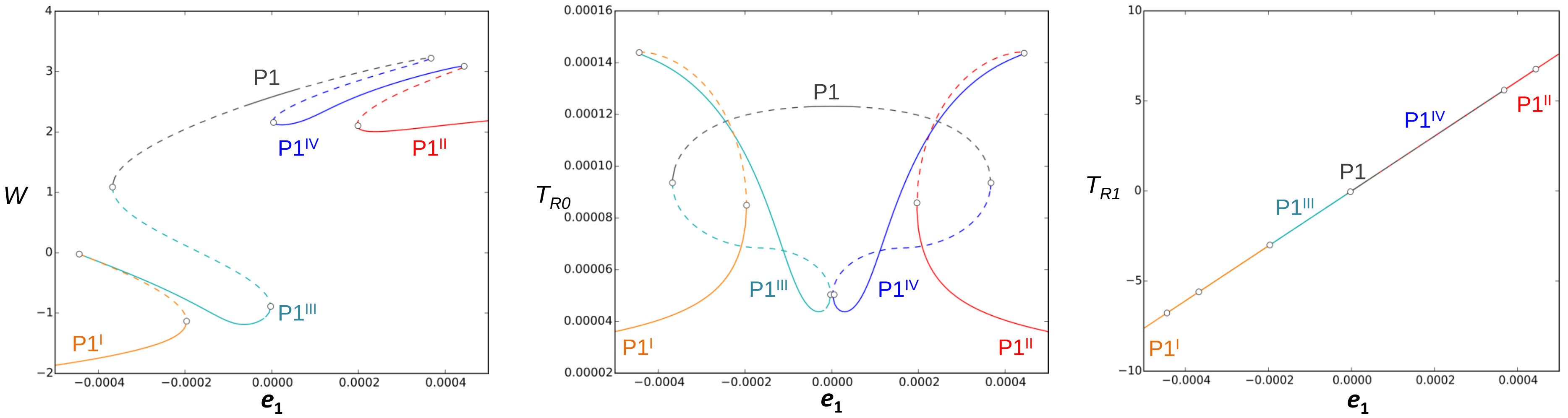
# USING BENDING THERMAL EXCITATION: INDUCING A SELECTED BUCKLING (1)

## BENDING THERMAL EXCITATION $e_1$ (linear variation along thickness)

- ❑ STARTING **from a PRE-BUCKLING configuration**




- **BUCKLED** responses ONLY around **ONE** selected varied **CONFIGURATION** (depending on the sign of excitation)



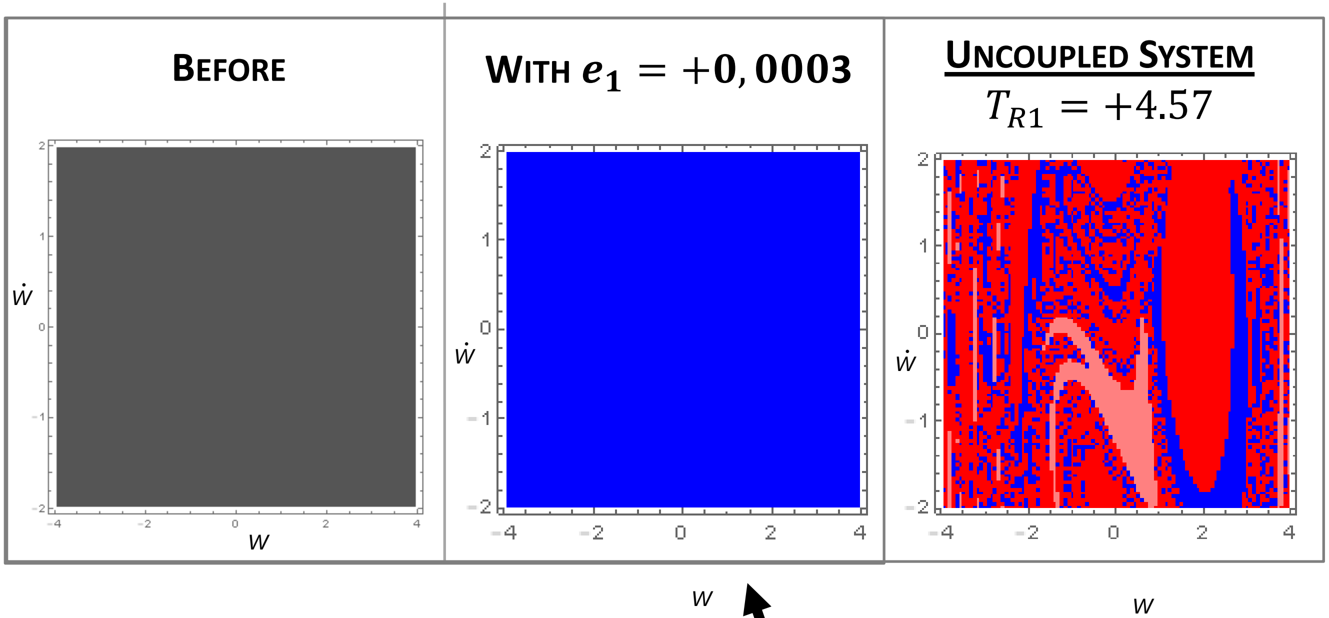

$$\ddot{W}(t) + a_{12}\dot{W}(t) + a_{13}W(t) + a_{14}W(t)^3 + a_{15}T_{R1}(t) + a_{16}T_{R0}(t)W(t) + a_{17}Cos(t) = 0$$

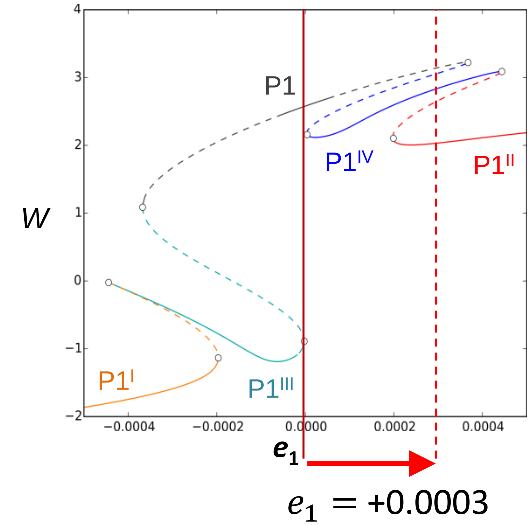
$$\dot{T}_{R0}(t) + a_{22}T_{R0}(t) + a_{23}\alpha_1 T_\infty + a_{24}W(t)\dot{W}(t) = 0$$

$$\dot{T}_{R1}(t) + a_{32}T_{R1}(t) + a_{33}\dot{W}(t) + e_1 = 0$$

# USING BENDING THERMAL EXCITATION: INDUCING A SELECTED BUCKLING (2)

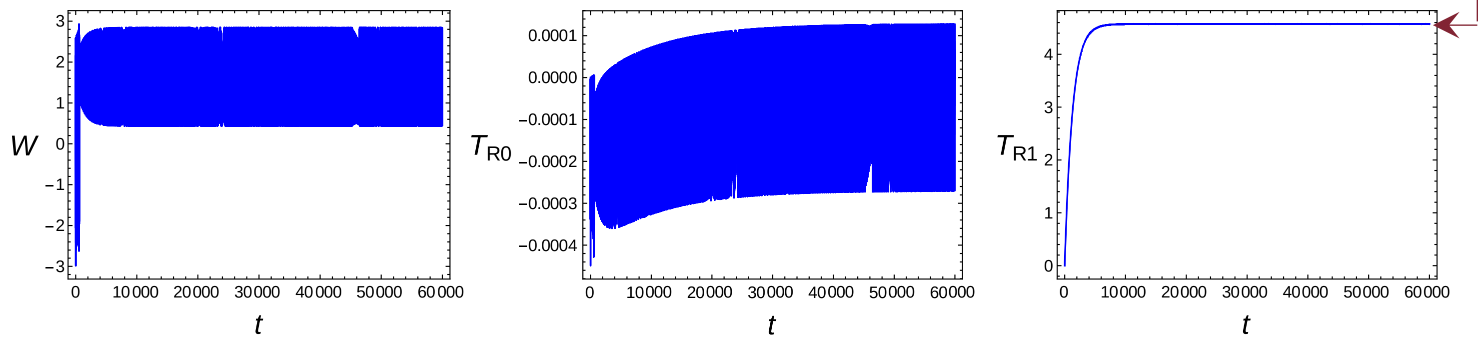
*Settimi, Rega (2018)*




- **DIFFERENT SOLUTIONS** detected
- Meaningful **DIFFERENCES** in the **BASINS** organization

$T_{R1}$ settles to $+4.57$

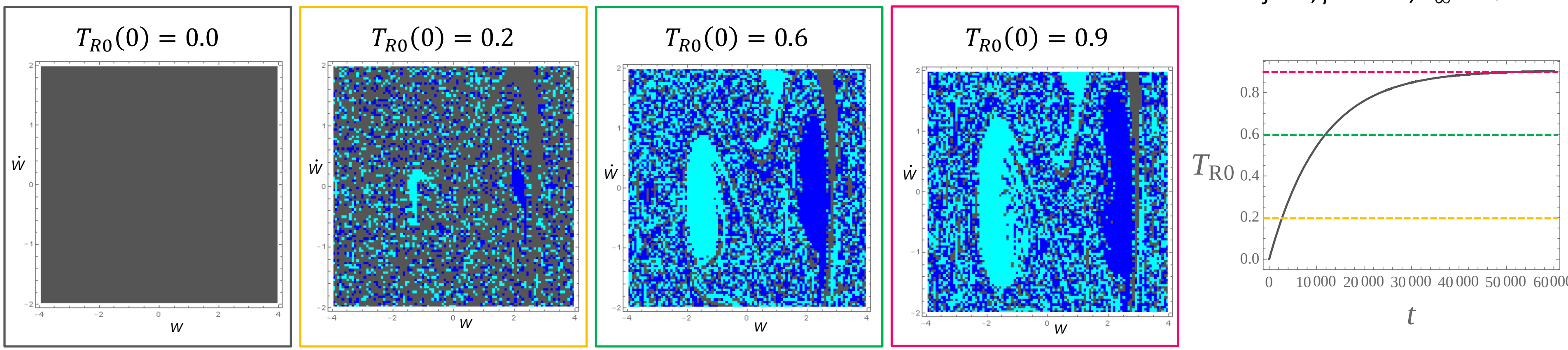



ALSO $T_{R1}$ SHORTER **TRANSIENT** **STRONGLY MODIFIES** **STEADY** DYNAMICS OF COUPLED SYSTEM

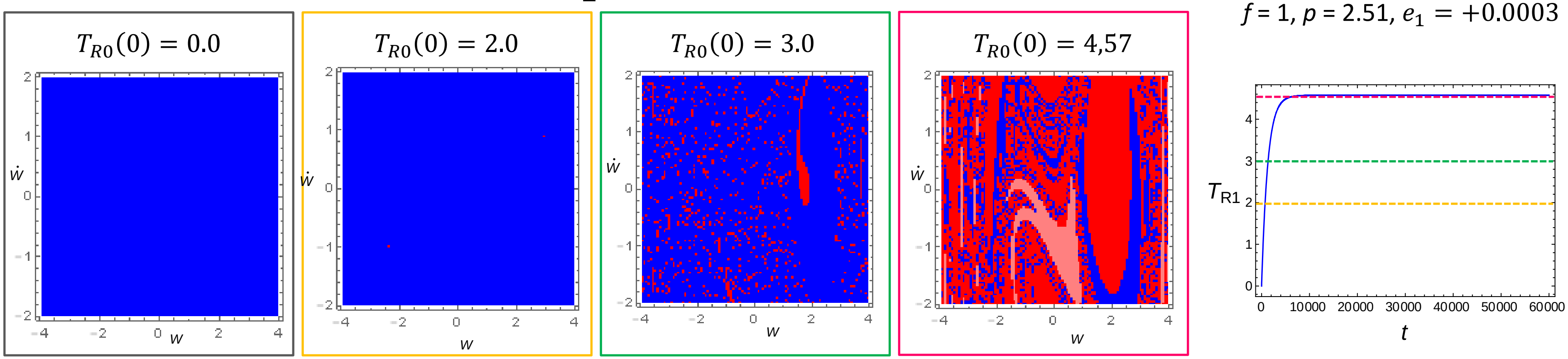
# CHANGING THERMAL INITIAL CONDITIONS

**Changing thermal initial conditions → Reducing thermal transient**

## MEMBRANE THERMAL EXCITATION $T_\infty$




## BENDING THERMAL EXCITATION $e_1$



## CONCLUSIONS

- **LOCAL** and **GLOBAL** nonlinear **DYNAMICS** of a **REDUCED MODEL of** Classical von Kármán shear indeformable single-layer orthotropic **PLATE with THERMOMECHANICAL COUPLING** (**CTC**)
- Understanding and using complex **INTERACTION** phenomena in a **MULTIPHYSICS** context characterized by **SLOW-FAST DYNAMICS**
- In **ACTIVE** thermal regime:
  - **THE TRANSIENT EVOLUTION** of the thermomechanically coupled response plays a **meaningful role in steadily modifying the system dynamics** with respect to that of the uncoupled oscillator
  - **Wrong global information provided by the uncoupled mechanical oscillator** highlights the need to consider the **ACTUAL THERMOMECHANICALLY COUPLED MODEL** in the nonlinear dynamic analysis
  - **Fundamental role** played by **GLOBAL DYNAMICS** in **unveiling transient effects on steady response**