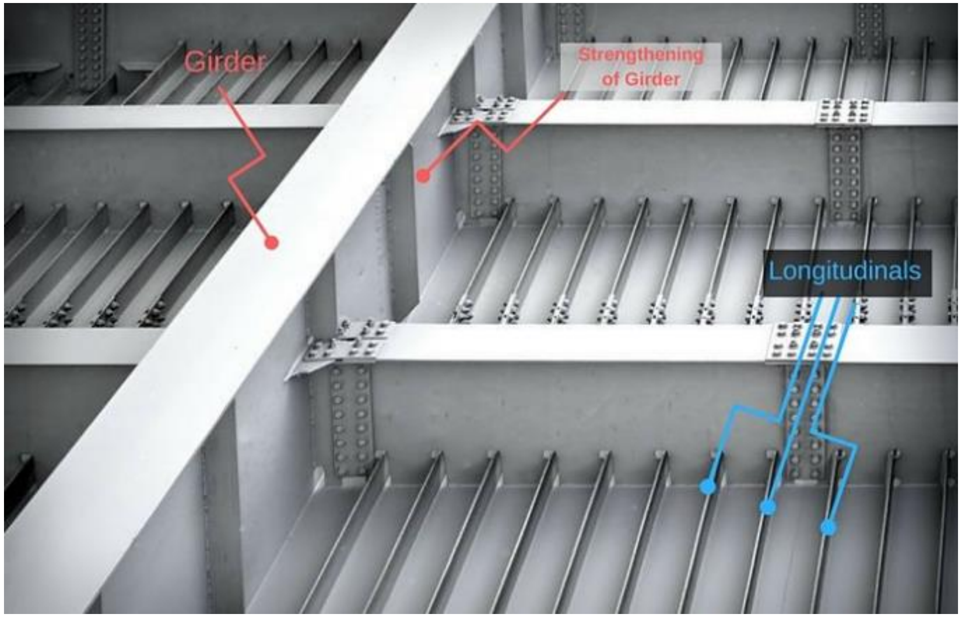
DEPARTAMENTO DE ENGEHARIA NAVAL E OCEÂNICA
ESCOLA POLITÉCNICA DA USP

Análise de Vigas : σ_x e τ_{xy}

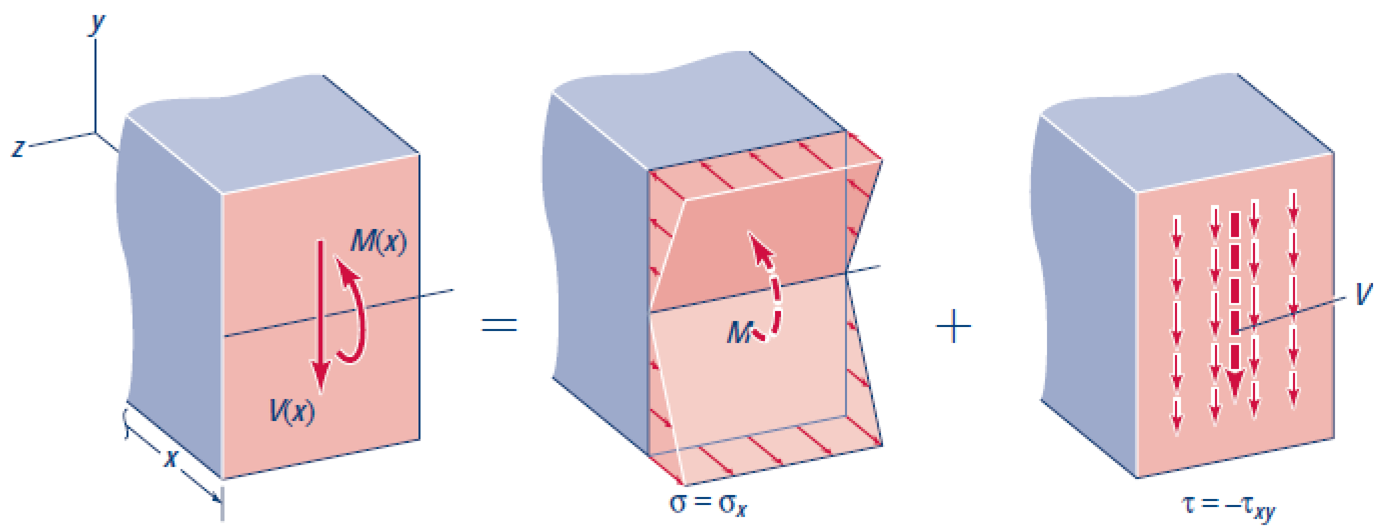


PNV 3212 – Mecânica Dos Sólidos I

2020

Agenda

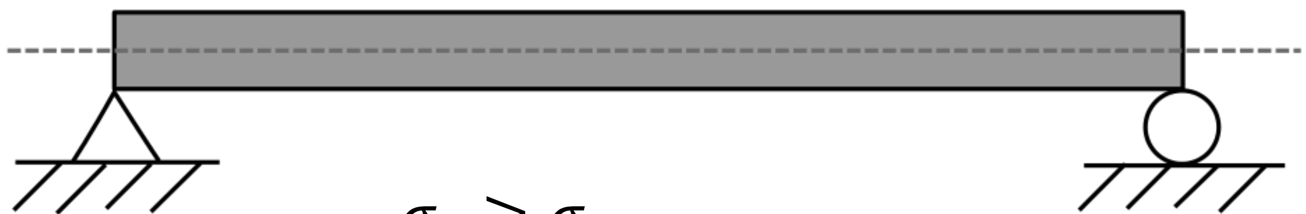
- Motivação
- Cálculo de σ_x e τ_{xy}
  - Teoria de Euler-Bernoulli



(a) Stress resultants. (b) Normal (flexural) stress. (c) Transverse shear stress.

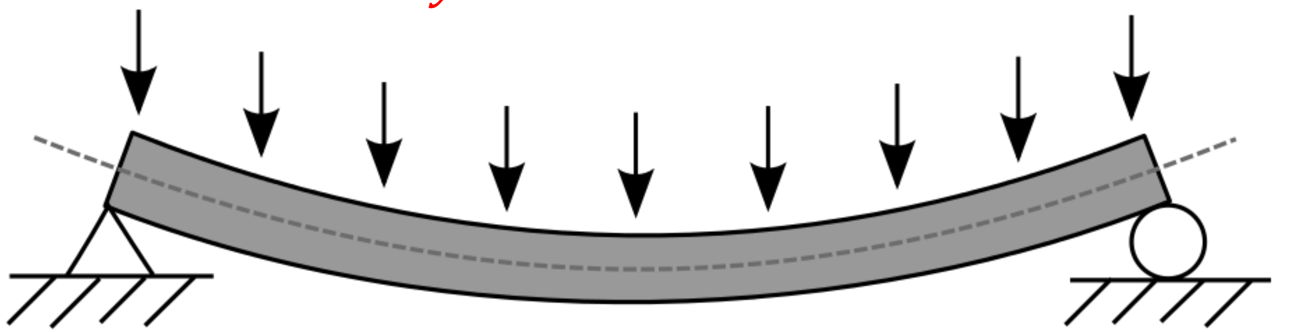
Motivação

- Projeto/Análise dos elementos estruturais (Vigas)
  - Distribuição de tensões(Normal e Cisalhamento)



$$\sigma_x \geq \sigma_{escoamento?}$$

$$\color{red}{\tau_{xy}} \geq \tau_{escoamento?}$$

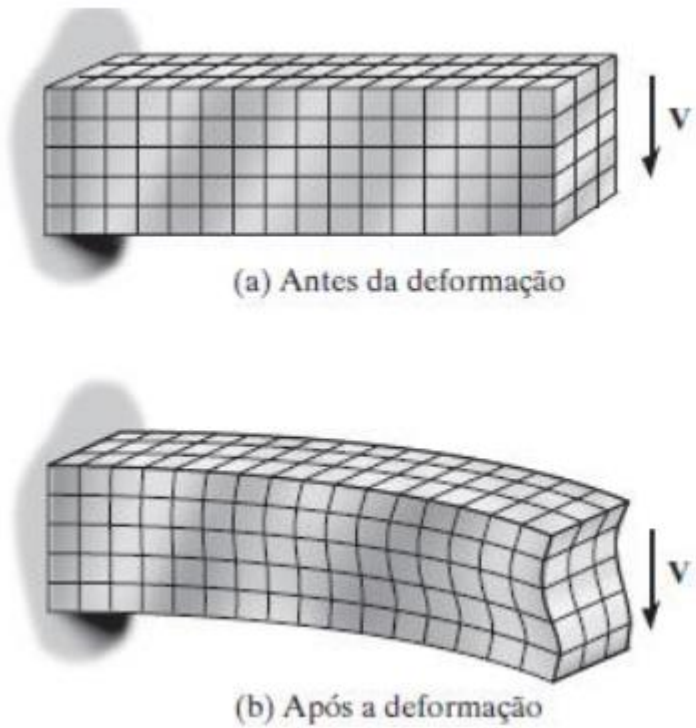


Tensões de Cisalhamento

- **Hipóteses**
  - Problema é independente do tempo.
  - O formato da viga é um prisma reto, cujo comprimento é muito maior que as outras dimensões (**Esbelta**).
  - A viga é constituída de um material **linearmente elástico**.
  - O efeito Poisson é negligenciável.
  - A seção transversal é simétrica em relação ao plano vertical.
  - Planos perpendiculares à linha neutra permanecem quase planos e perpendiculares ao eixo deformado depois da deformação (**Navier**).
  - O ângulo de rotação da seção transversal é muito pequeno.
  - O efeitos de momento de inércia da rotação é desprezado.
  - ~~**Flexão Pura**~~.
  - A viga é constituída de material homogêneo .
  - The distribution of flexural stress on a given cross section is not affected by the deformation due to shear.
  - Distorção da seção transversal é pequena o suficiente para ser desprezada!

Tensões de Cisalhamento

- **Hipóteses**
  - Distorção da seção transversal é pequena o suficiente para ser desprezada!



Tensões de Cisalhamento

- **Objetivo**
  - Relacionar

Carregamento $\left\{ \mathrm{V}(x) \right.$

$$\mathrm{V}(x) \longleftrightarrow \mathrm{A}(x), I(x)$$

$\left. \mathrm{A}(x), I(x) \right\}$ **Propriedades da seção transversal**

$$\begin{array}{ccc} \mathrm{V}(x) & & \mathrm{A}(x), I(x) \\ & \nwarrow\!\searrow \quad \nearrow\!\swarrow & \\ & \tau(x) & \\ & \updownarrow & \end{array}$$

$$\tau(x) \sim V(x) \qquad \tau(x) \sim \frac{1}{I(x)} \qquad \tau(x) \sim \int y dA$$

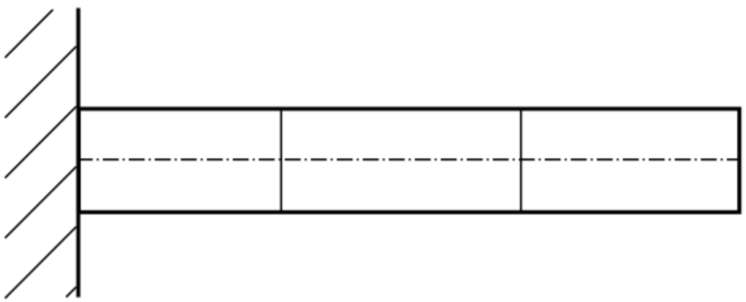
$$\tau(x) \sim \frac{V(x) \times Q}{I(x)}$$

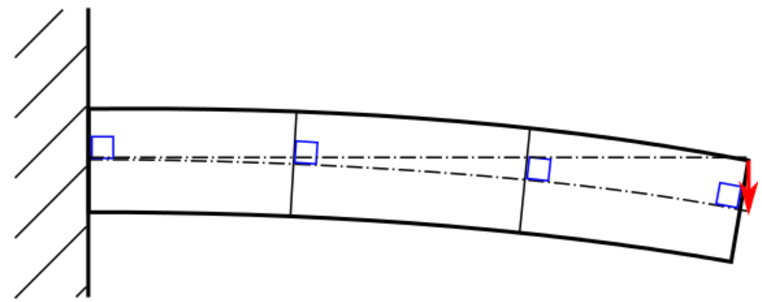
Tensões de Flexão

- **Caminho**
  1. Premissa plausível da deformação da seção transversal
     - we have no convenient way to characterize the displacement due to shear (like the “plane sections” assumption that was used to characterize displacement due to flexure) we must follow a different approach
  2. Equilíbrio da Seção (Forças)

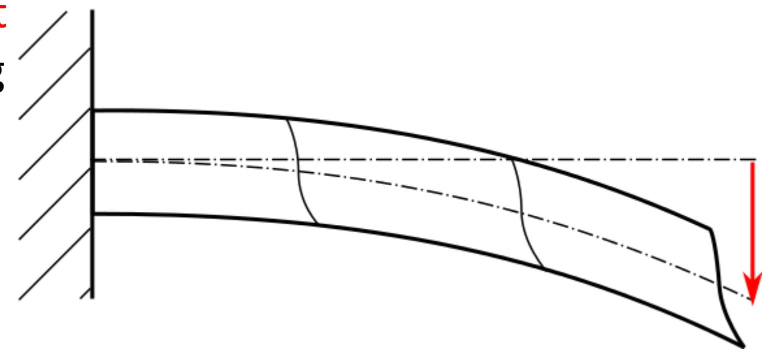
Tensões de Cisalhamento

1. Shear strain distribution



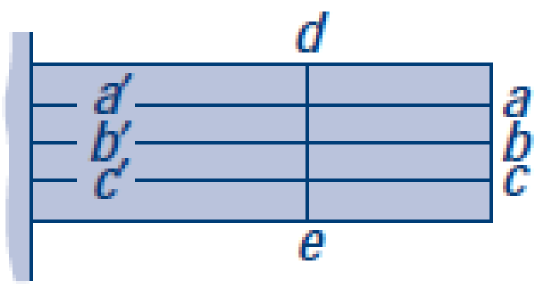


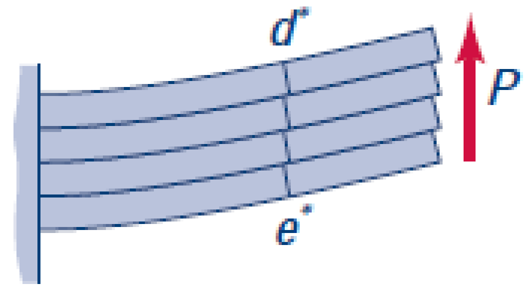
Because of shear deformation, plane sections do not remain plane, as they do in the case of pure bending



Tensões de Cisalhamento

Tábuas sem acoplamento → free to slip along the surfaces of contact



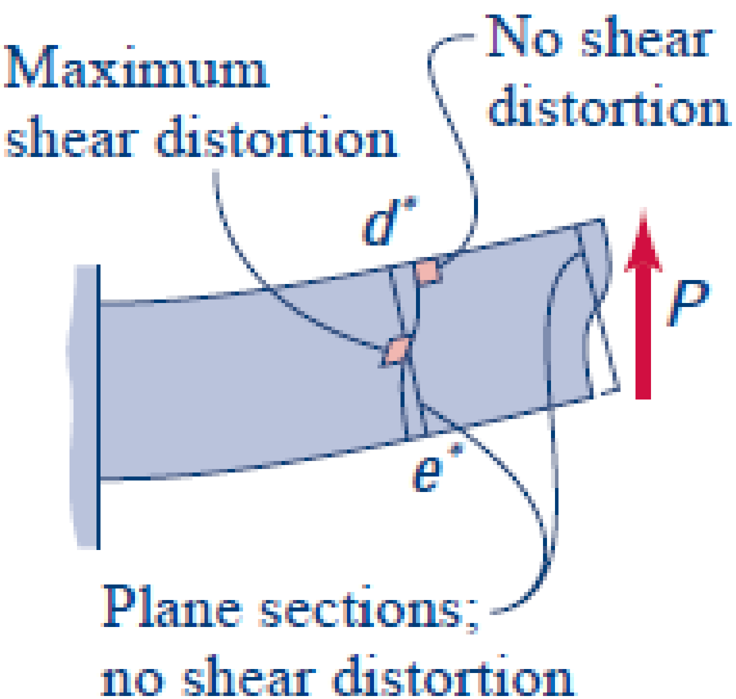


Slip between non-bonded

- Planes parallel to the neutral surface (i.e., horizontal planes) must be able to transmit shear!

Tensões de Cisalhamento

Tábuas com acoplamento



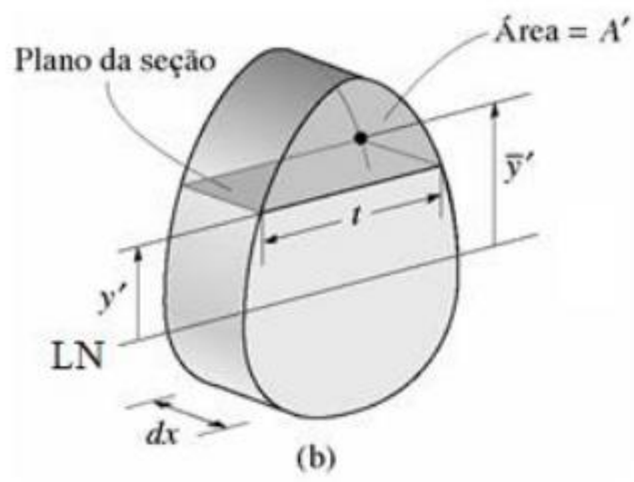
- the beam will undergo shear deformation
- Because of shear deformation, plane sections do not remain plane
- **Shear deformation has little effect on the distribution of flexural stress as long as the beam is slender**

Tensões de Cisalhamento

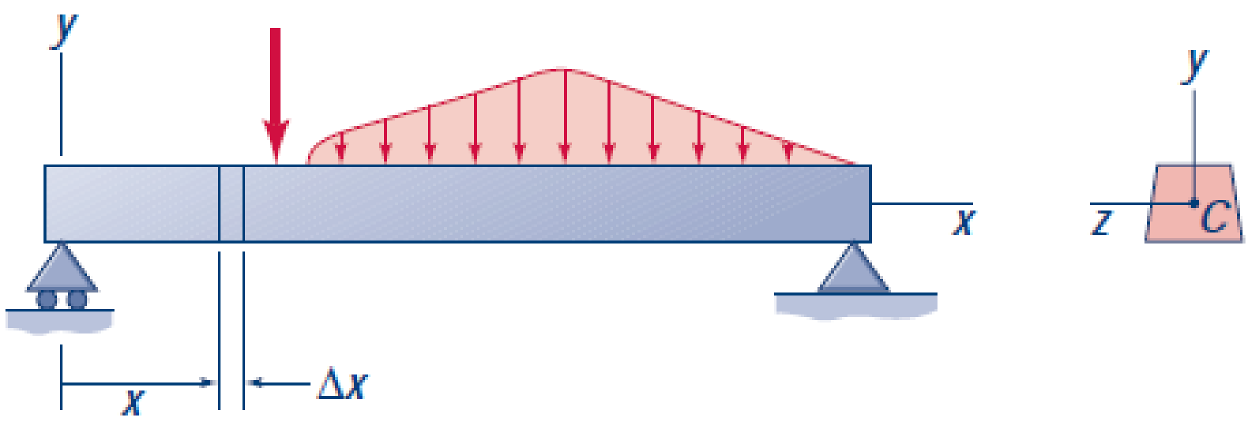
- Fórmula

$$\tau(x) = \frac{V(x) \times Q}{I(x) \times t}$$

$$Q = \int_{A'} y\,dA'$$



Tensões de Cisalhamento



(*a*) Profile view.

(*b*) Cross section.

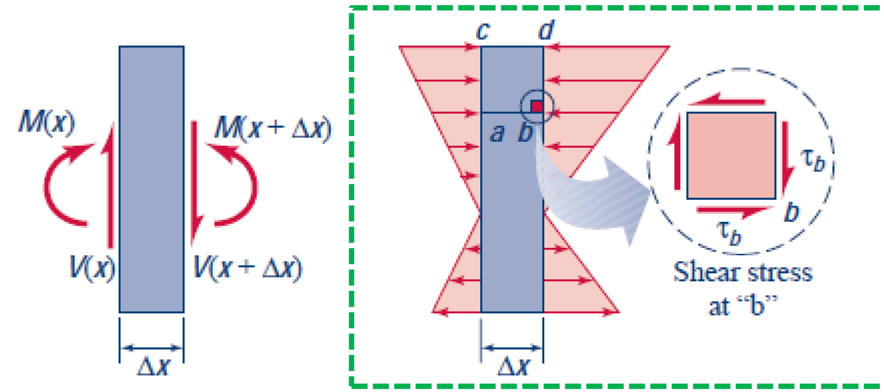
Consider the segment from x to $(x+\Delta x)$

Tensões de Cisalhamento

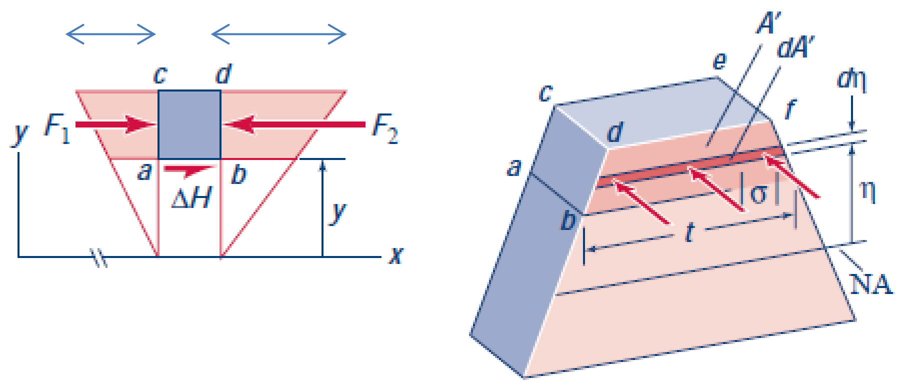
- **the transverse shear stress is equal to the longitudinal shear stress at the same point.**

$$F_i = \int_y^{h/2} \sigma dA$$

$$F_i = \int_y^{h/2} \frac{M}{I} y dA$$



(a) An element of length Δx. (b) The distribution of flexural stress.



(c) A free body diagram (minus vertical shear on ac and bd). (d) The flexural stress contributing to F_2.

Tensões de Cisalhamento

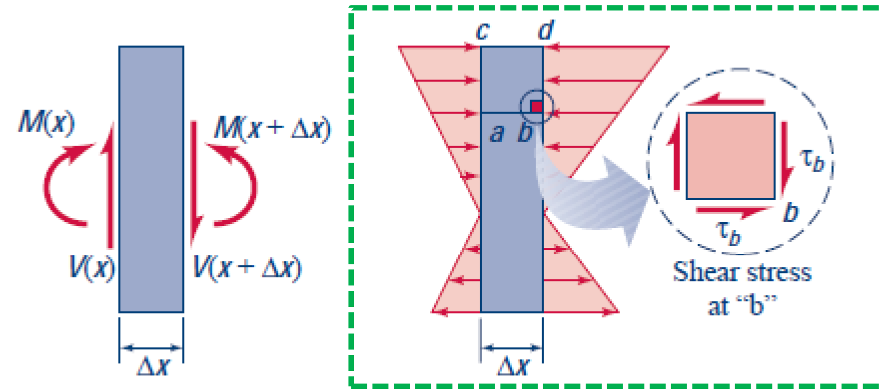
$$F_i = \int_y^{h/2} \frac{M_i}{I} y dA$$

$$\downarrow$$

$$F_2 - F_1 = \Delta H$$

$$\downarrow$$

$$\Delta H = -\frac{\Delta M}{I} \int_y^{h/2} y dA$$



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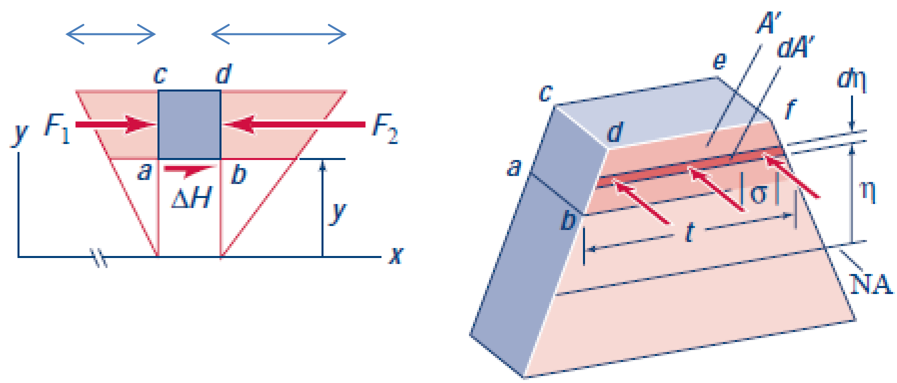


(c) A free body diagram (minus vertical shear on *ac* and *bd*). (d) The flexural stress contributing to F_2.

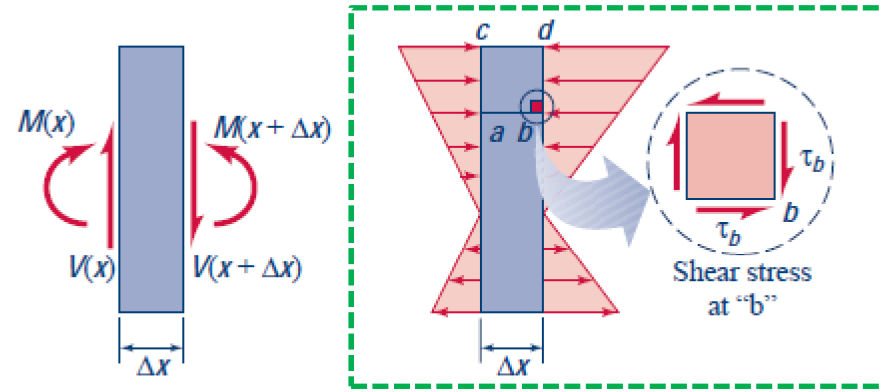
Tensões de Cisalhamento

$$\Delta H = -\frac{\Delta M}{I}\boxed{\int_{y}^{h/2} y\,dA}$$

$$Q = \int_{y}^{h/2} y\,dA$$

$$\downarrow$$

$$\Delta H = -\frac{\Delta M}{I} Q$$



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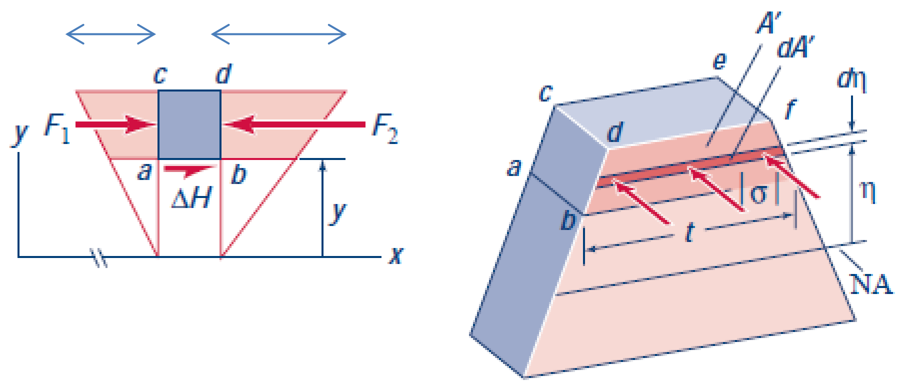


(c) A free body diagram (minus vertical shear on ac and bd). (d) The flexural stress contributing to F_2.

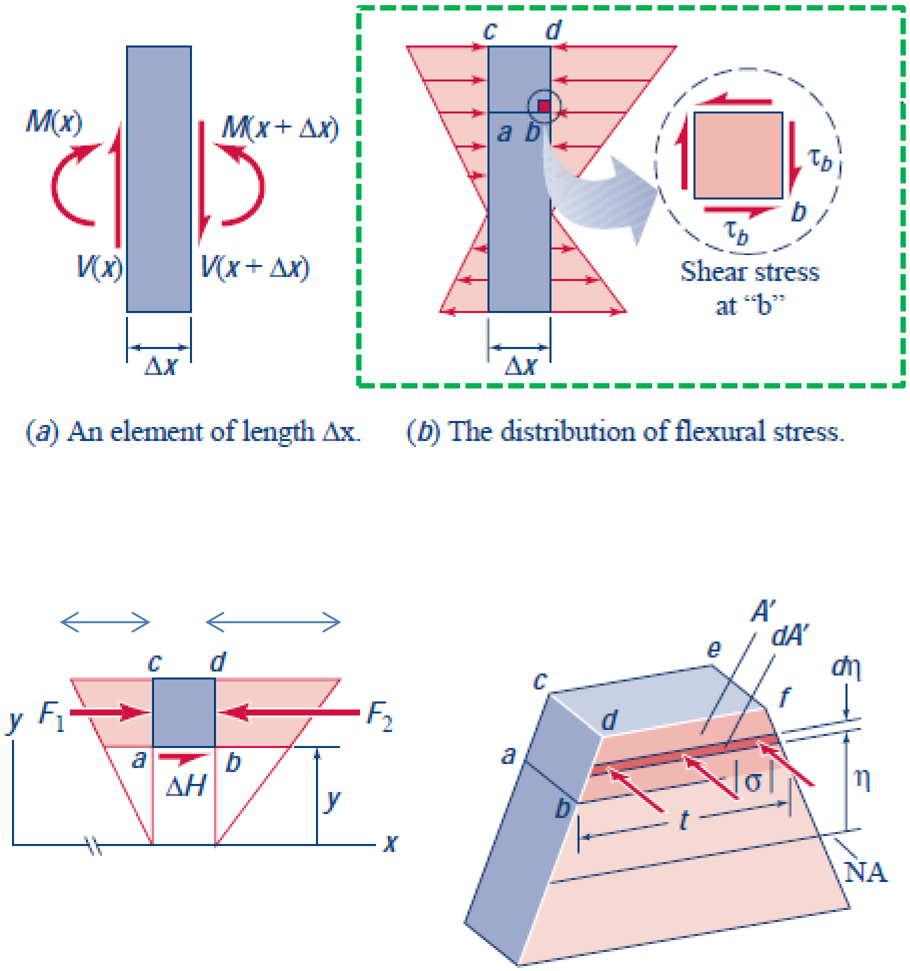
Tensões de Cisalhamento

$$\Delta H = -\frac{\Delta M}{I} Q$$

- Fluxo de Cisalhamento (shear flow)

$$q \equiv \lim_{\Delta x \to 0} \frac{\Delta H}{\Delta x}$$

$$\frac{\Delta H}{\Delta x} = q = -\frac{\Delta M}{\Delta x} \frac{Q}{I}$$



(a) An element of length Δx.
(b) The distribution of flexural stress.
(c) A free body diagram (minus vertical shear on ac and bd).
(d) The flexural stress contributing to F_2.

Tensões de Cisalhamento

- Fluxo de Cisalhamento (shear flow)

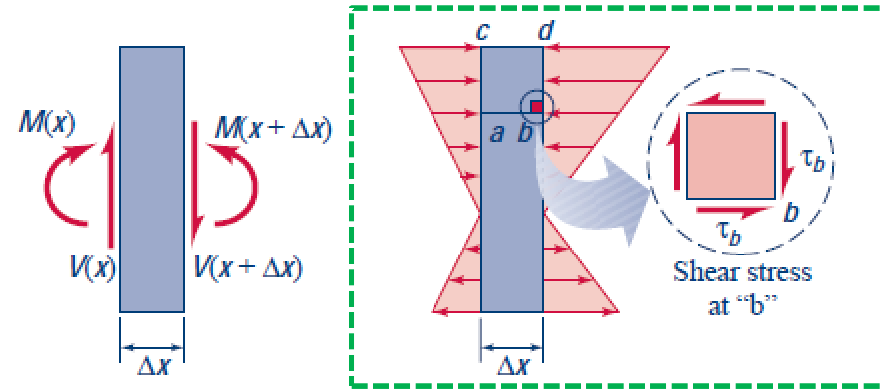
$$q = -\frac{\Delta M}{\Delta x}\frac{Q}{I}$$

↓

$$\boxed{\frac{dM}{dx} = -V(x)}$$

↓

$$q = \frac{VQ}{I} \qquad [\text{ton/m}]$$



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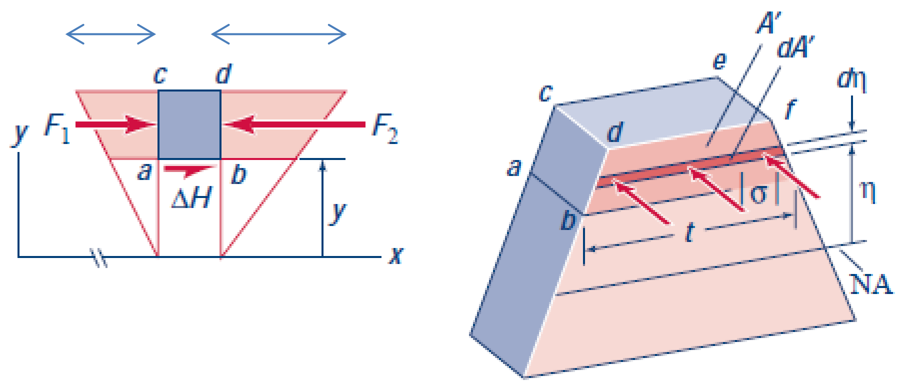


(c) A free body diagram (minus vertical shear on ac and bd). (d) The flexural stress contributing to F_2.

Tensões de Cisalhamento

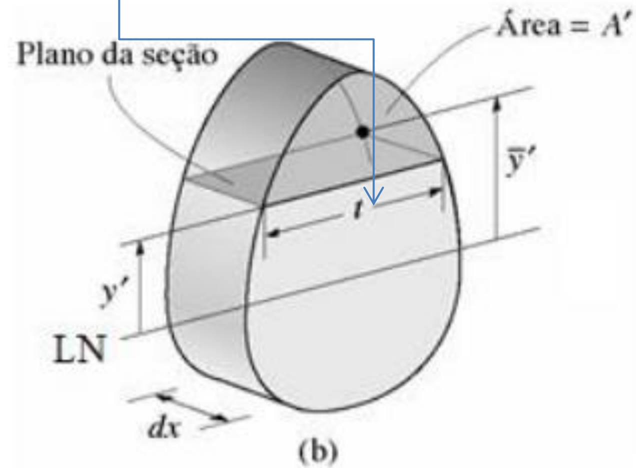
We divide ΔH by the area over which it acts, we get an *average* shear stress on the longitudinal plane at level ***y***

$$\tau_{avg}(x, y) = \lim_{\Delta x \to 0} \frac{\Delta H}{t \Delta x} = \frac{V(x) Q(x, y)}{I(x) \boxed{t(x, y)}}$$

where

τ = the *average* transverse shear stress at level y in section x,
$Q = A'\overline{y}'$ = the first moment, with respect to the neutral axis, of the cross sectional area *above* level y,[18]
I = the moment of inertia *of the entire cross section,* taken with respect to the neutral axis, and
t = the width of the cross section at level y.

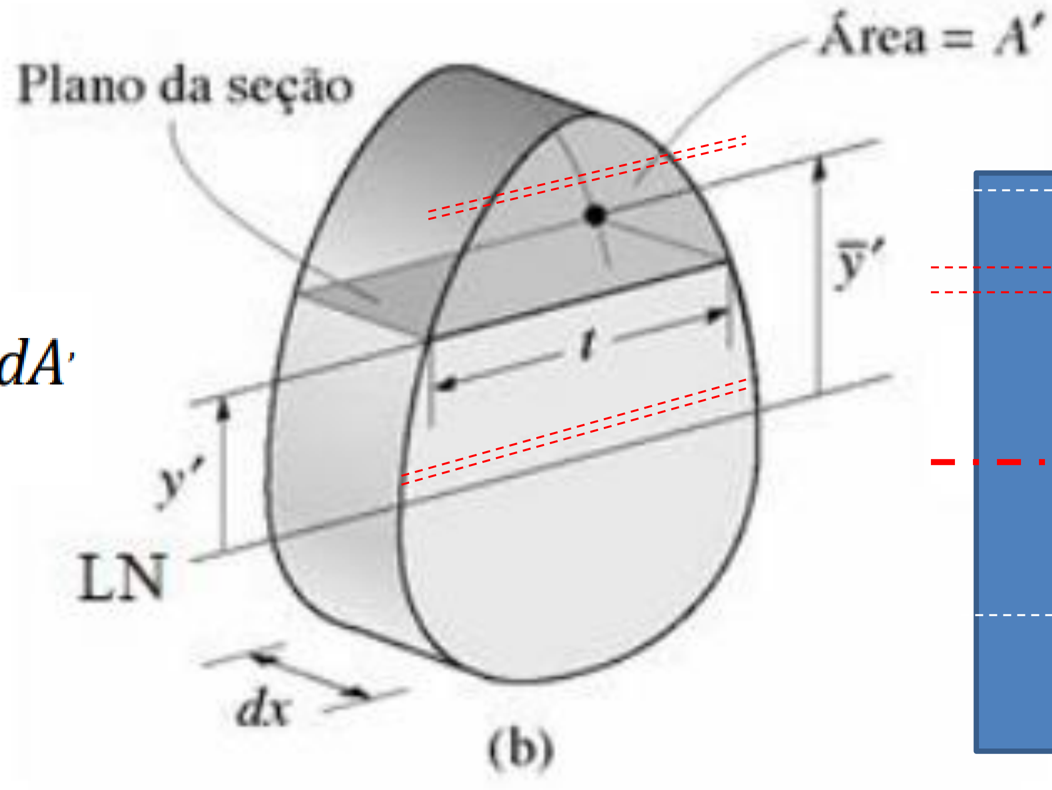
- The *sign convention* is that the shear stress acts in the same direction as the resultant shear force *V*.



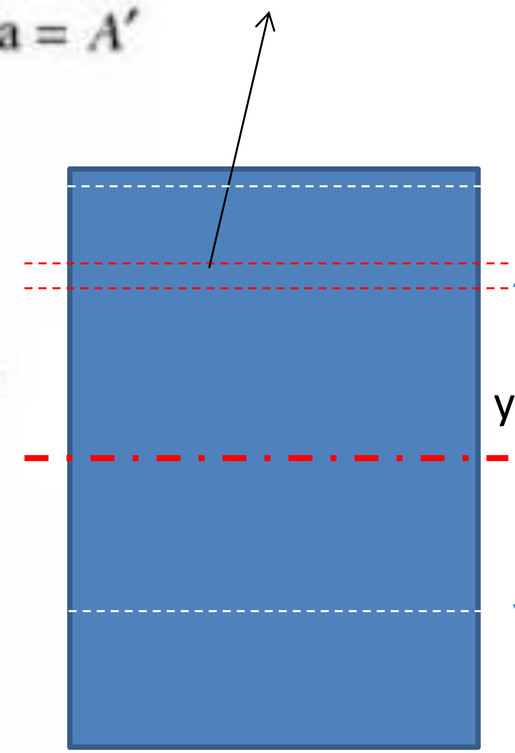
Tensões de Cisalhamento

- Fórmula

$$Q = \int_{A'} y\,dA'$$



ΔA = bdy



$\Sigma(\Delta A \; {*}y) = bdy\,(y1+y2+y3+\ldots)$

Exemplo

The rectangular beam of width b and height h (Fig. 1) is subjected to a transverse shear force V. (a) Determine the average shear stress as a function of y, (b) sketch the shear-stress distribution, and (c) determine the maximum shear stress on the cross section.

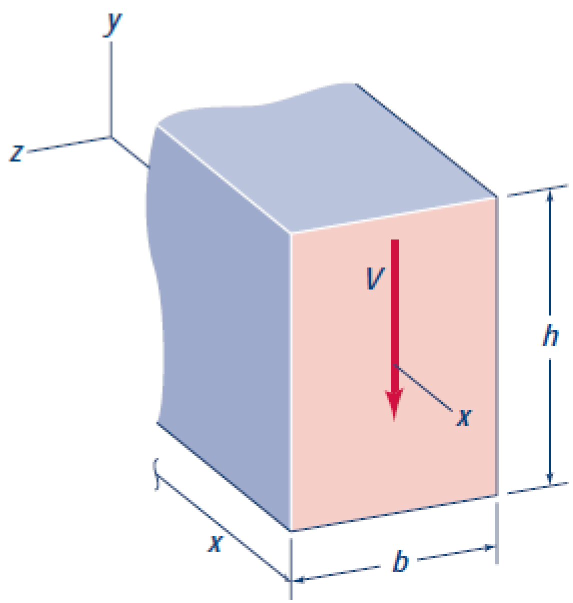
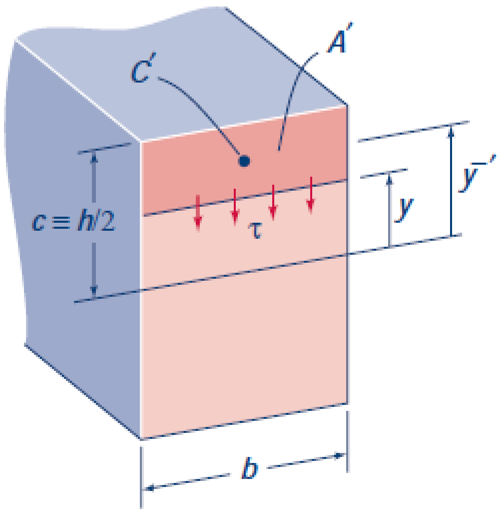


Fig. 1

Exemplo

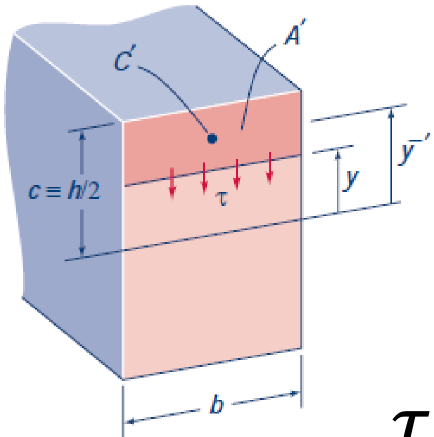


$$\tau(x) = \frac{V(x) \times Q}{I(x) \times t}$$

$$Q = \bar{A}\bar{y}$$

$$Q = \left[b\left(\frac{h}{2} - y\right)\right] \times \left[\frac{\left(\frac{h}{2} - y\right)}{2} + y\right]$$

Exemplo

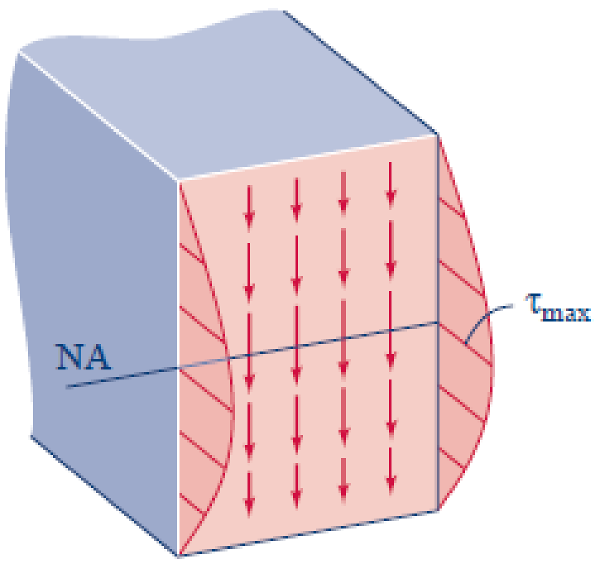


$$\tau(x) = \frac{V(x)}{\frac{bh^3}{12}b}\left[b\left(\frac{h}{2} - y\right)\right]\left[\frac{1}{2}\left(\frac{h}{2} + y\right)\right]$$

$$\tau(x,y) = \frac{6V(x)}{bh^3}\left[\left(\frac{h^2}{4} - y^2\right)\right]$$

Exemplo

$$\tau(x,y) = \frac{6V(x)}{bh^3}\left[\left(\frac{h^2}{4} - y^2\right)\right]$$



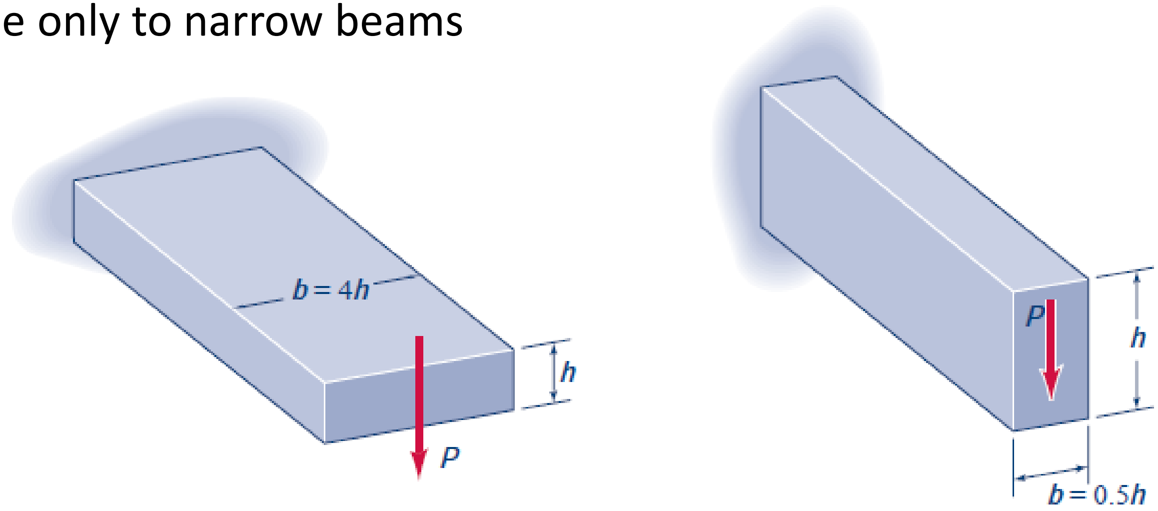
$$y = 0$$

$$\tau(x,0) = \frac{6V(x)}{bh^3}\left[\left(\frac{h^2}{4}\right)\right]$$

$$\tau(x,0) = \frac{3V(x)}{2bh} \longrightarrow \boxed{\tau_{max} = \frac{3V(x)}{2A}}$$

LIMITATIONS ON THE SHEAR-STRESS FORMULA

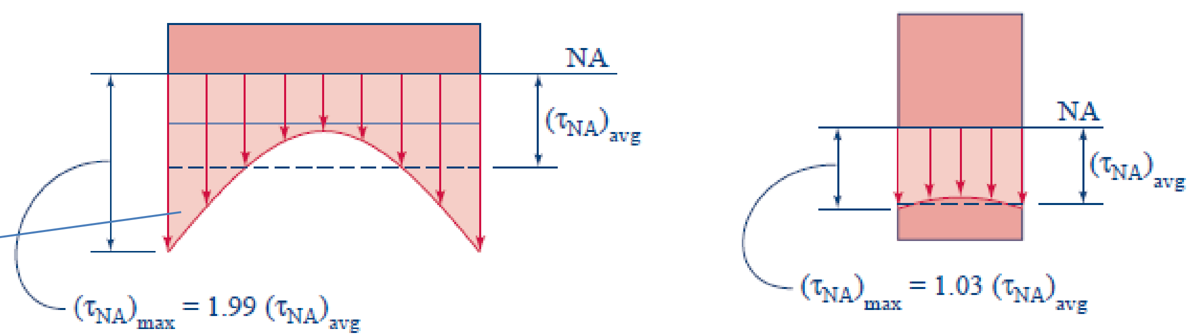
- slender beam, linearly elastic behavior, etc.
- It is applicable only to narrow beams



(a) A "wide beam," or plate.

(b) A "narrow beam."

theory of elasticity

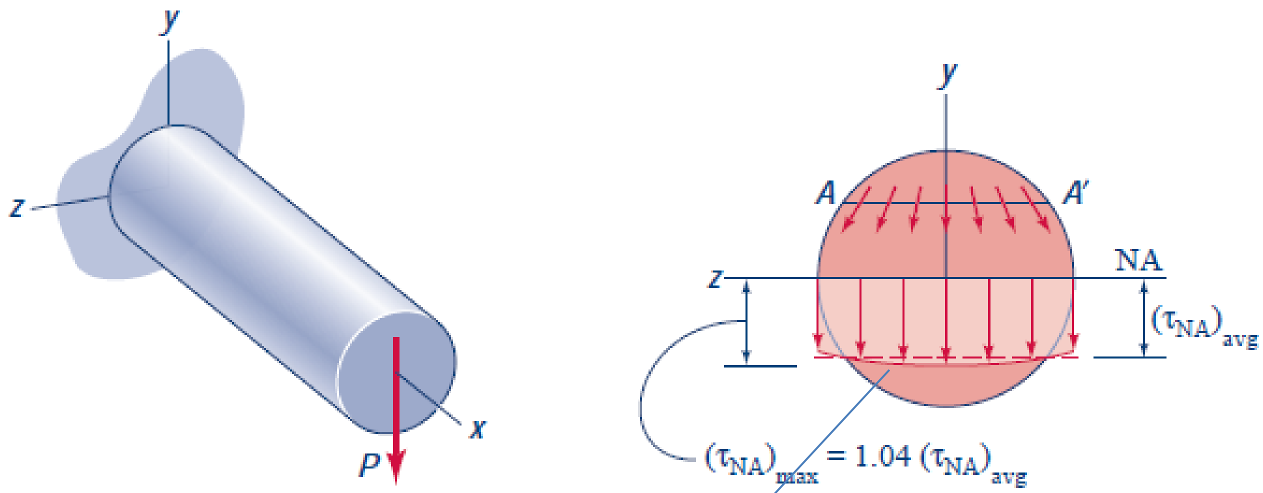


(c) Shear-stress distribution in the "wide beam" of Fig. 6.44a.

(d) Shear-stress distribution in the "narrow beam" of Fig. 6.44b.

LIMITATIONS ON THE SHEAR-STRESS FORMULA

- slender beam, linearly elastic behavior, etc.
- It is applicable only to narrow beams
- the shear stress formula does not apply where the width, $t(x, y)$, varies rapidly



(*a*) A beam with circular cross section.

(*b*) The shear-stress distribution on a circular cross section.

theory of elasticity