PEF3208
Aula 8
17 abr
PROF. NAKAO

ESFORÇOS SOLICITANTES EM ESTRUTURAS ESPACIAIS

No esboço dos gráficos dos esforços solicitantes deve-se observar que:

- Se a estrutura é uma barra poligonal engastada em um único ponto o caminhamento dos esforços deve ser da extremidade livre para o apoio, não havendo a necessidade da obtenção das reações que poderão ser obtidas a partir dos esforços solicitantes junto ao engastamento.

- Se a estrutura tem vários apoios é necessário obter as reações nos apoios, fazer o diagrama do corpo livre e acompanhar o caminhamento dos esforços entre os apoios, pois é a estrutura que leva os esforços até os apoios.

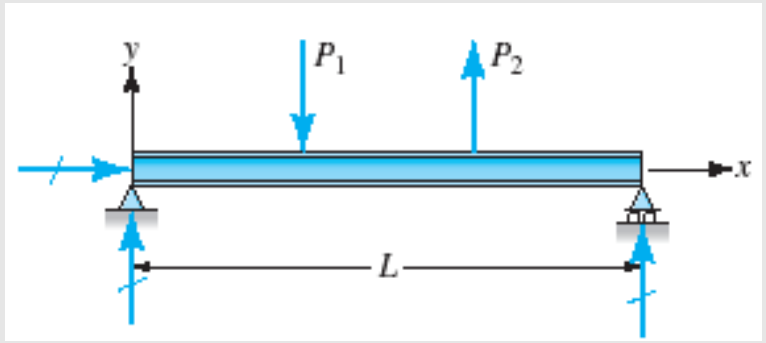
ESFORÇOS SOLICITANTES EM ESTRUTURAS ESPACIAIS

No esboço dos gráficos dos esforços solicitantes deve-se observar que:

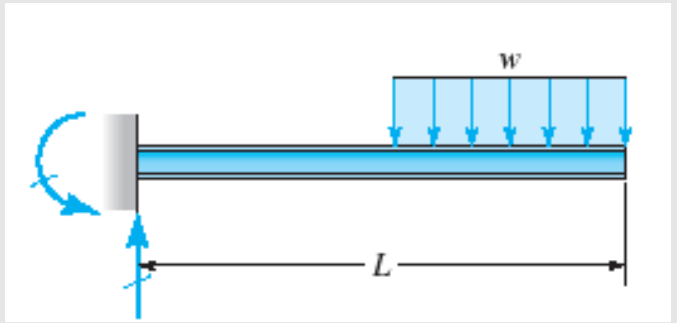
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- **O "teorema do corte" deve ser aplicado em seções próximas a esforços ativos ou quando se tem mudança de carregamento ou quando há mudanças de direção das barras da estrutura.**
- As forças normais e os momentos fletores e de torção são reduzidos/transferidos de uma seção a outra sem novos acréscimos.
- As forças cortantes são reduzidas/transferidas de uma seção para a outra acrescentando um momento fletor para que os sistemas sejam mecanicamente equivalentes.

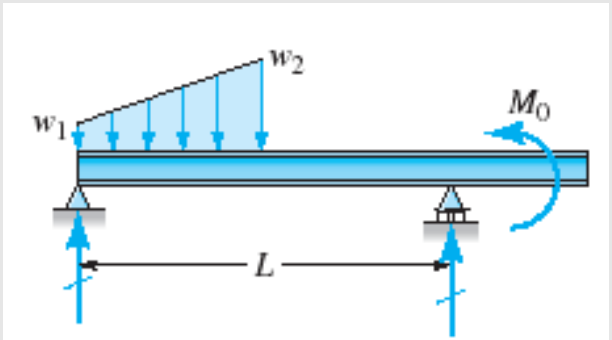
Types of beams



Simple beam
(Viga biapoiada)



Cantilever beam
(Viga em balanço/engastada)



Overhang beam
(viga biapoiada com balanço)

Example (cantilever beam)

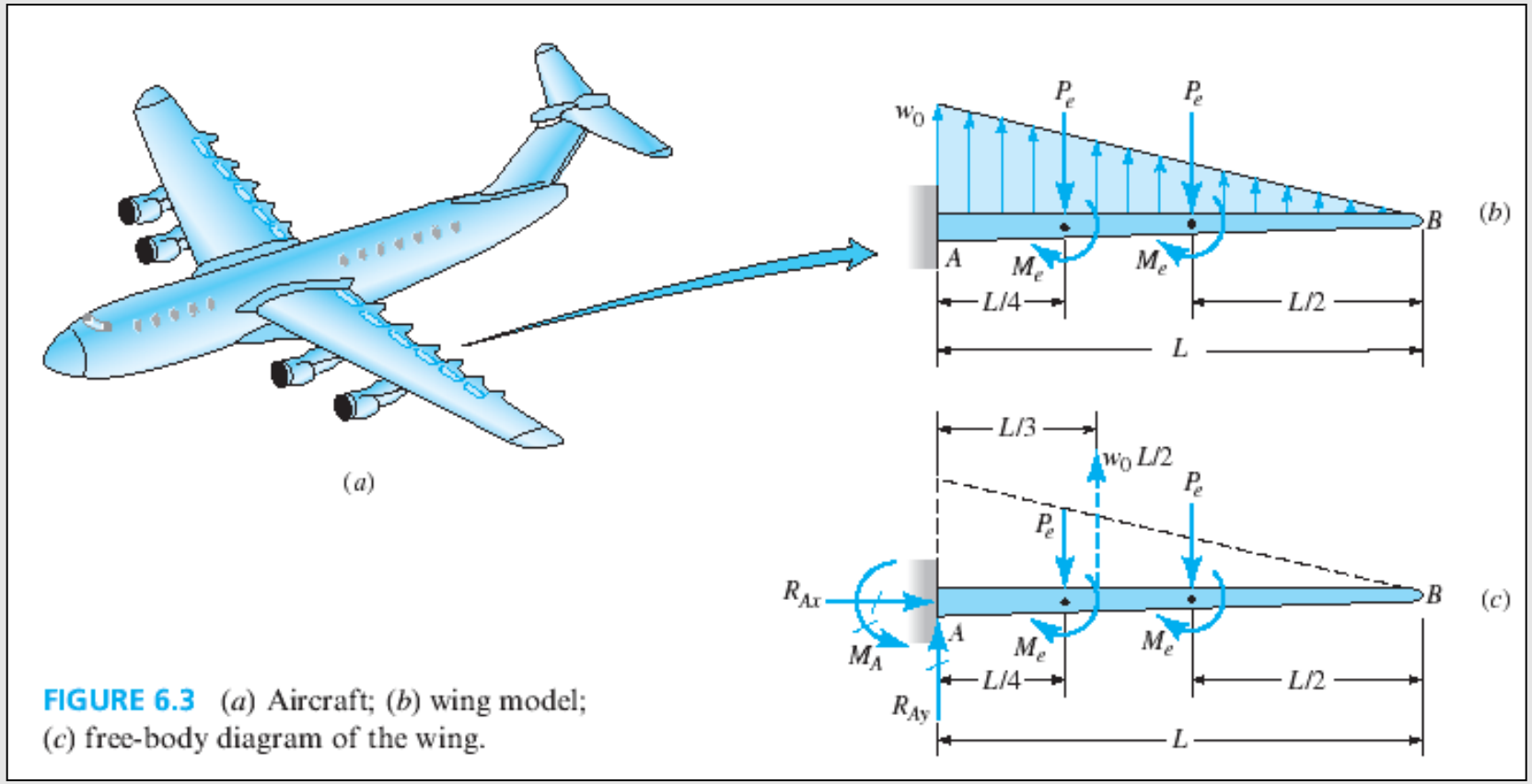


FIGURE 6.3 (*a*) Aircraft; (*b*) wing model; (*c*) free-body diagram of the wing.

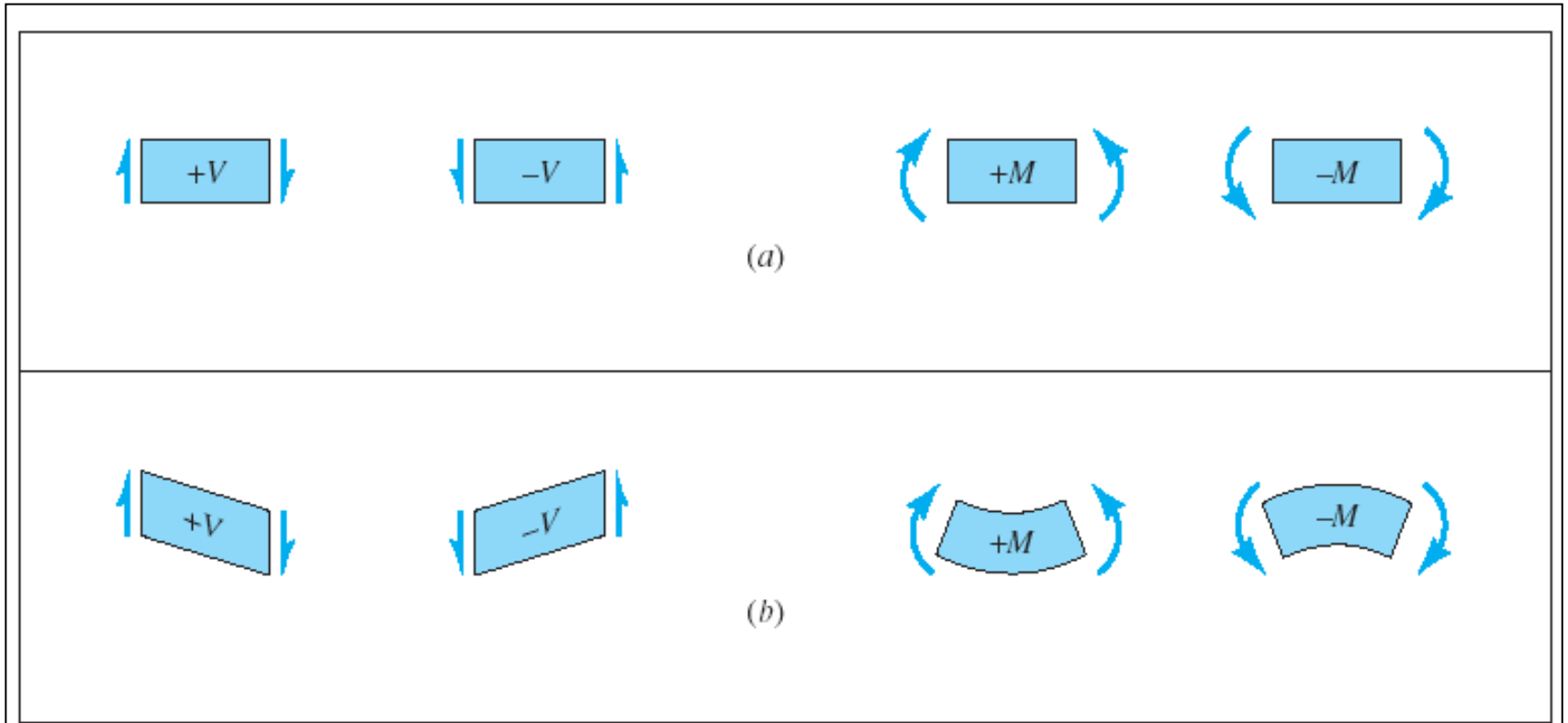
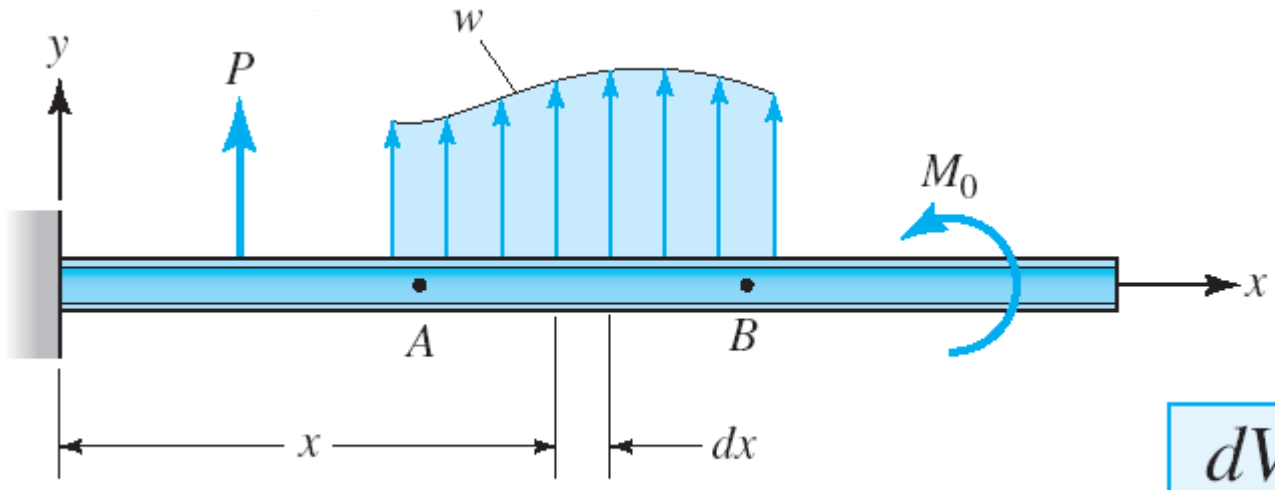


FIGURE 6.8 Sign convention for beams: (*a*) definitions of positive and negative shear force and bending moment; (*b*) deformations (highly exaggerated) of beam segments caused by shear and moment.

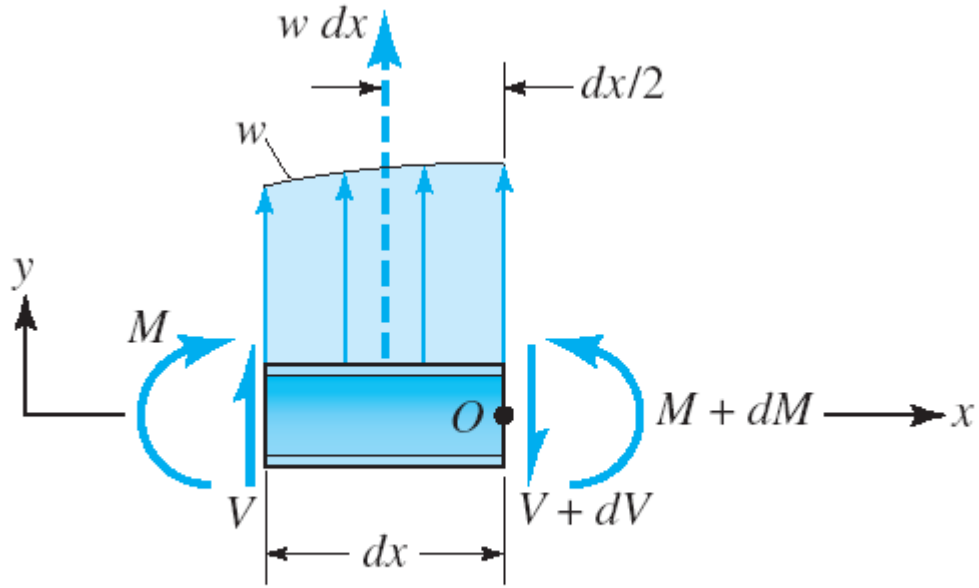
Shear force = força cortante

Bendind moment = momento fletor



(a)

$$\frac{dV}{dx} = w$$



(b)

$$\frac{dM}{dx} = V$$

REVIEW OF STATIC EQUILIBRIUM

Types of loads: External loads and Internal loads

External loads are due to surface forces and body forces

- **Surfaces forces can be for example, a concentrated load acting at a point or a distributed load both acting on the surface of a body**
- **Body forces act on a volumetric portion of the body, forexample, magnetic force or gravitational force**
- **Reaction forces caused by the supports**

Internal loads can be considered as forces of interaction between the constituent material particles of the body

CONDITIONS OF EQUILIBRIUM

When a system of forces acting upon a body has zero resultant, the body is said to be in force equilibrium.

The equations of static equlibrium require:

$$\sum F_x = 0;\ \sum F_y = 0, \text{ and } \sum F_z = 0$$

$$\sum M_x = 0;\ \sum M_y = 0, \text{ and } \sum M_z = 0$$

In other words, for a body to be in static equilibrium, the sum of all forces acting upon a body in any direction is zero and also the sum of all moments taken about any axis is also zero.

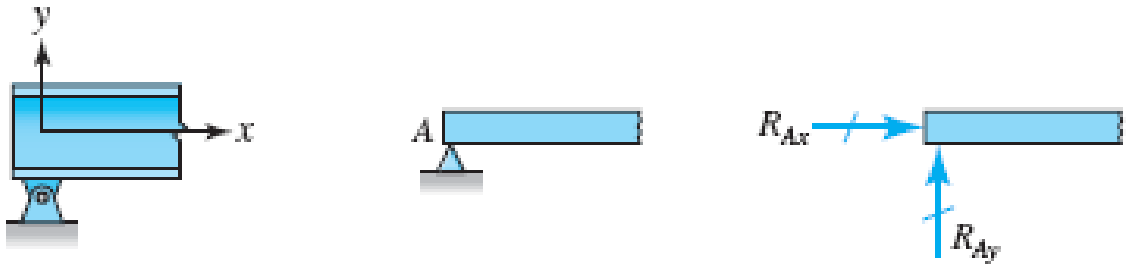
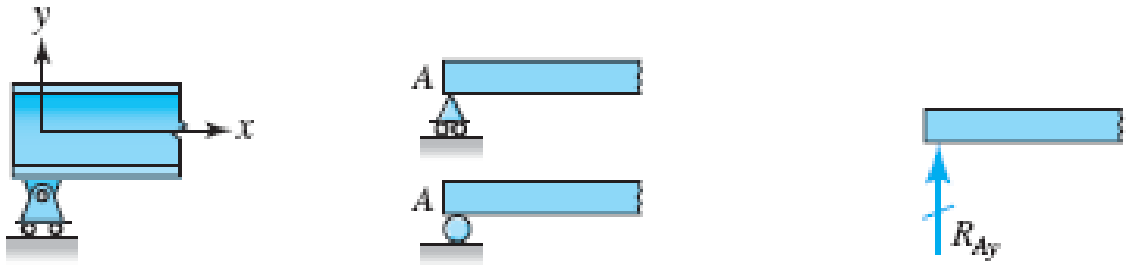
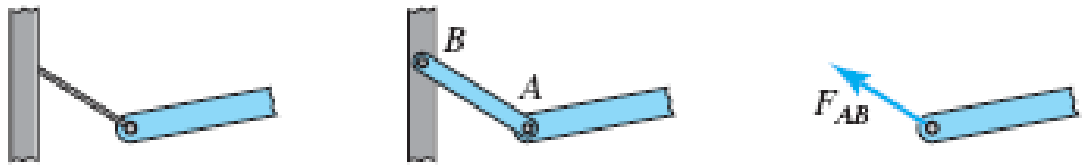
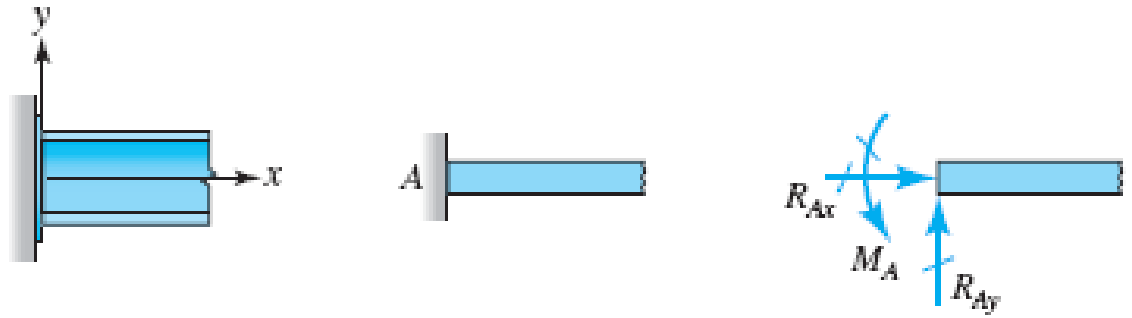
Planar Equations of Equilibrium

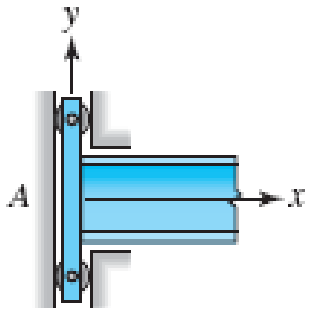
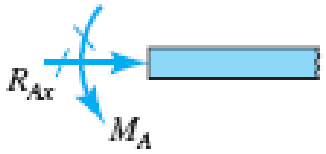
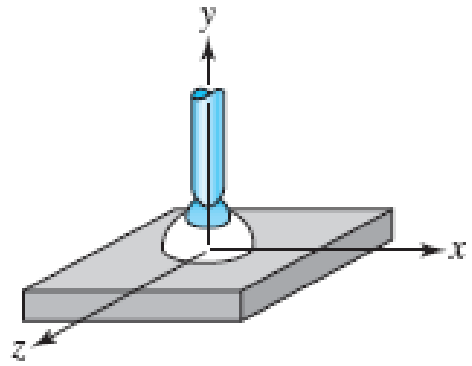
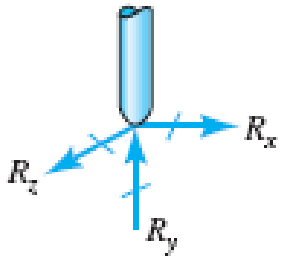
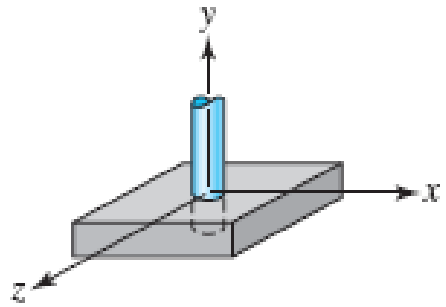
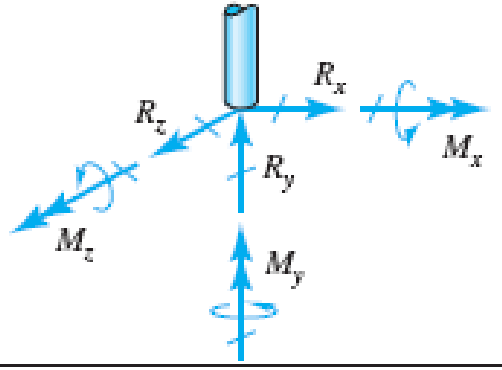
For a planar body to be in equilibrium, any one of the following sets of 3 equations may be used to solve for the unknown variables.

1. **$\sum F_x = 0$, $\sum F_y = 0$, and $\sum M_A = 0$, where the resultant moment is with respect to any axis z or any point A in the xy-plane, or**

2. **$\sum F_x = 0$, $\sum M_A = 0$, and $\sum M_B = 0$, provided that the line connecting the points A and B is not perpendicular to the x axis, or**

3. **$\sum M_A = 0$, $\sum M_B = 0$, and $\sum M_C = 0$, where points A, B, and C are not collinear**

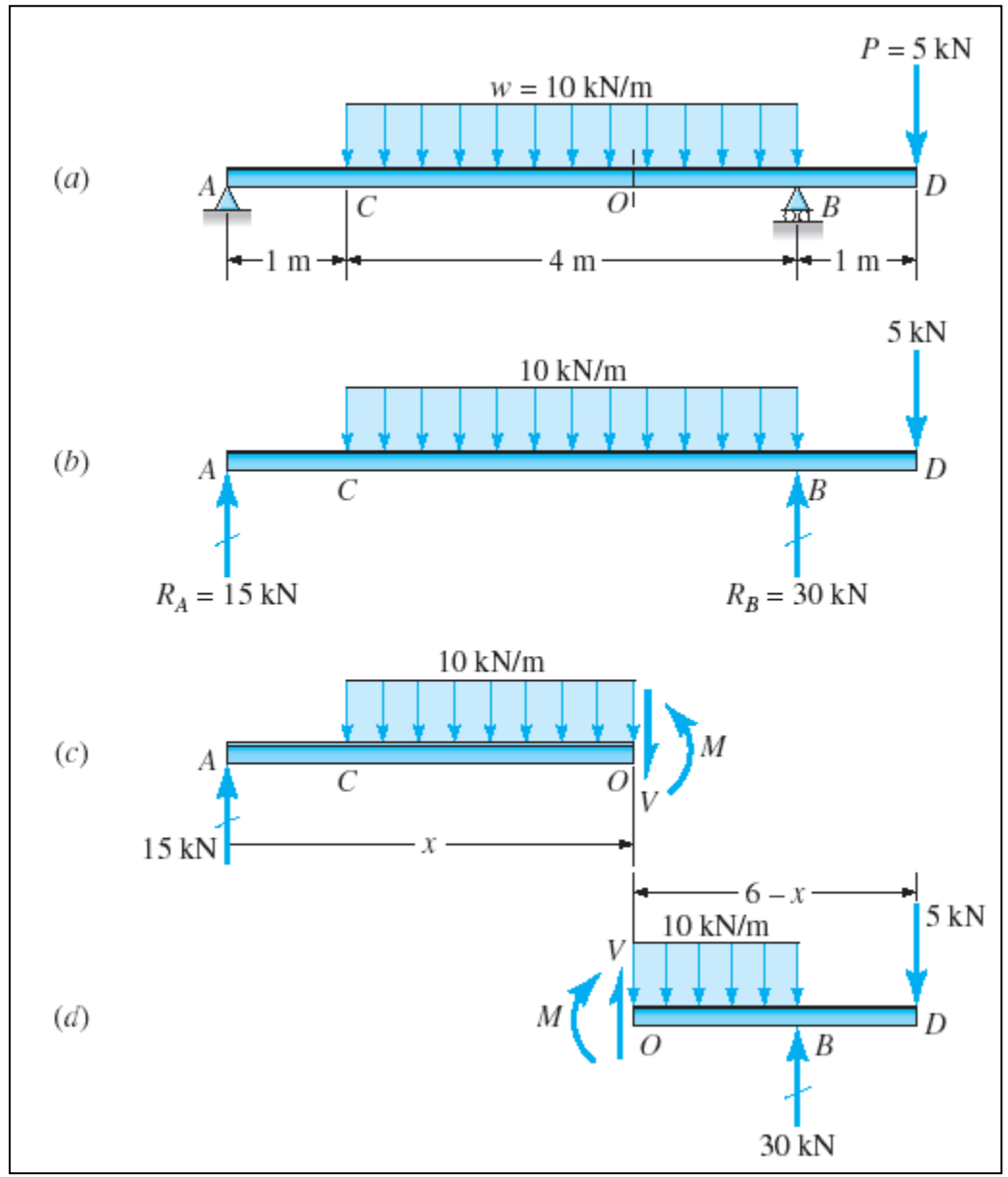
Type of Support or Connection		Reaction*
1. Hinge or pin		
2. Roller		
3. Cable or link		
4. Fixed or clamped		

5. Guided		
6. Ball and socket (nonplanar)		
7. Fixed (nonplanar)		

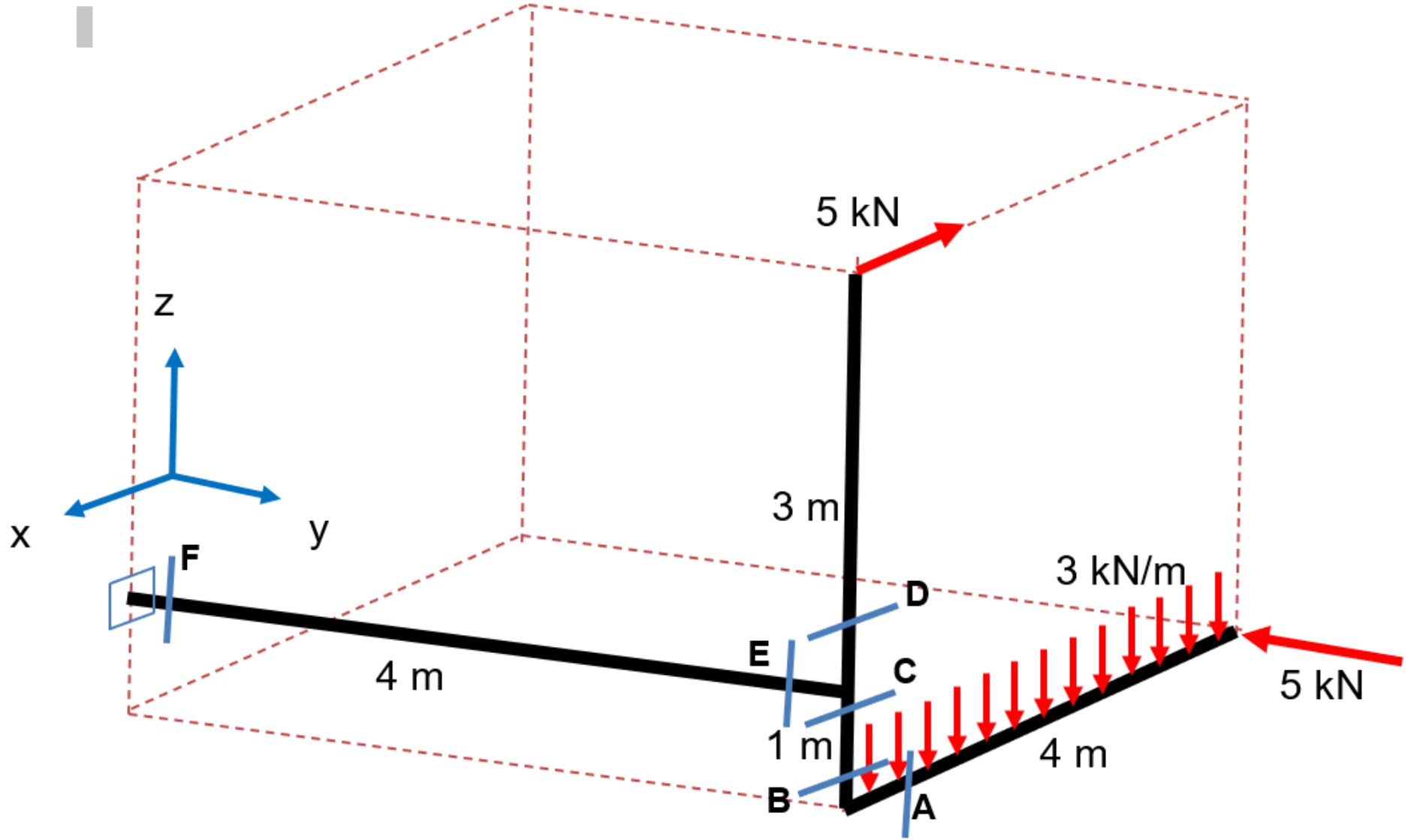
*Usually the reaction will be identified in this text with a *slash* drawn through its vector as shown.

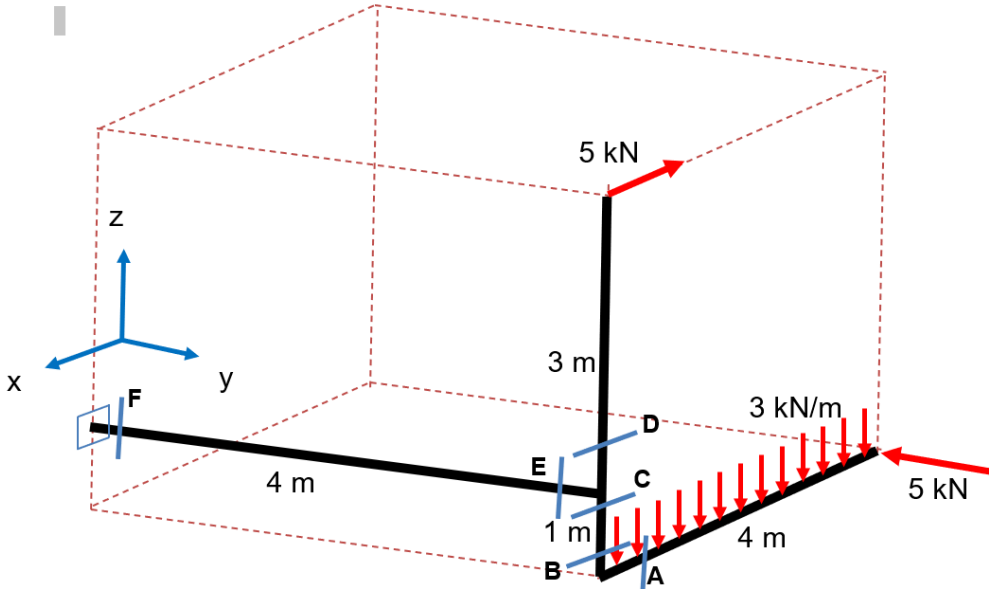
Freebody diagrams

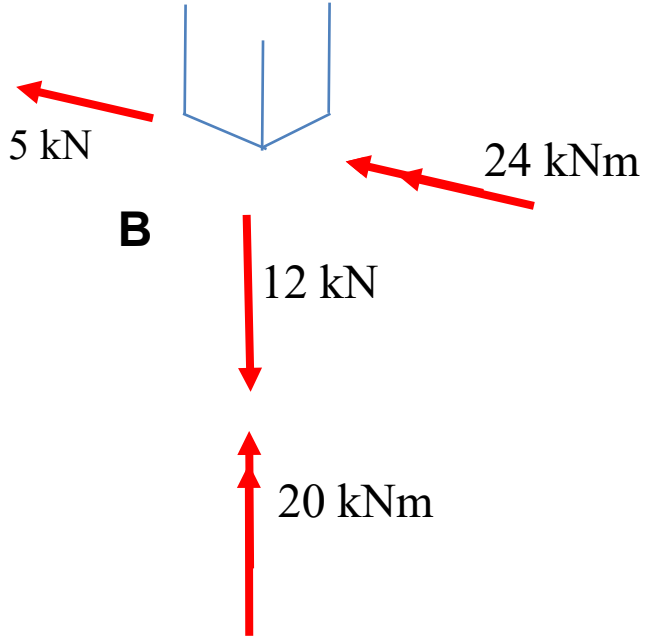
1. **Select the free body to be used.**
2. **Detach this body from its supports and separate it from any other bodies. (If internal force resultants are to be found, use the method of sections).**
3. **Show on the sketch all of the external forces acting on the body. Location, magnitude, and direction of each force should be marked on the sketch.**
4. **Label significant points and include dimensions. Any other detail, however, should be omitted.**

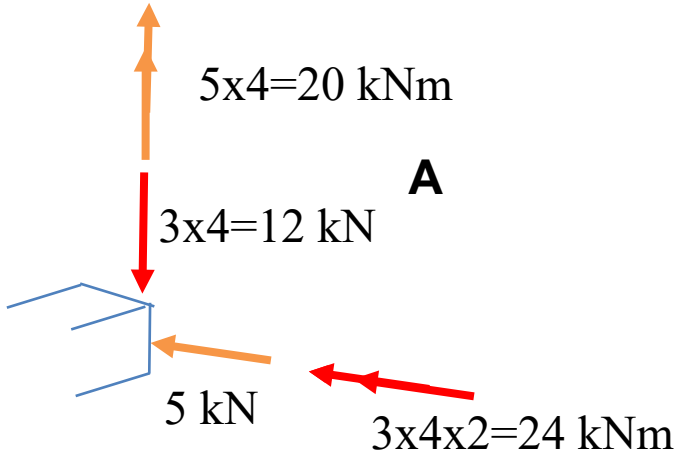

(a)
P = 5 kN
w = 10 kN/m
A
C
O
B
D
1 m
4 m
1 m
(b)
5 kN
10 kN/m
A
C
B
D
R_A = 15 kN
R_B = 30 kN
(c)
10 kN/m
M
A
C
O
V
15 kN
x
(d)
6 – x
5 kN
10 kN/m
V
M
O
B
D
30 kN

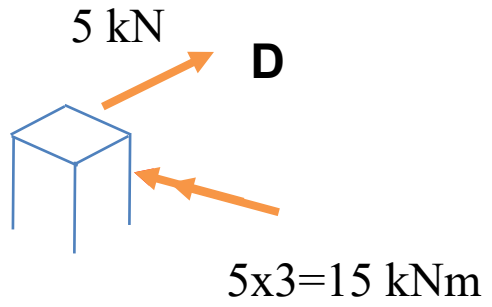
Exemplo 1. Esboçar os diagramas dos esforços solicitantes na estrutura da figura.

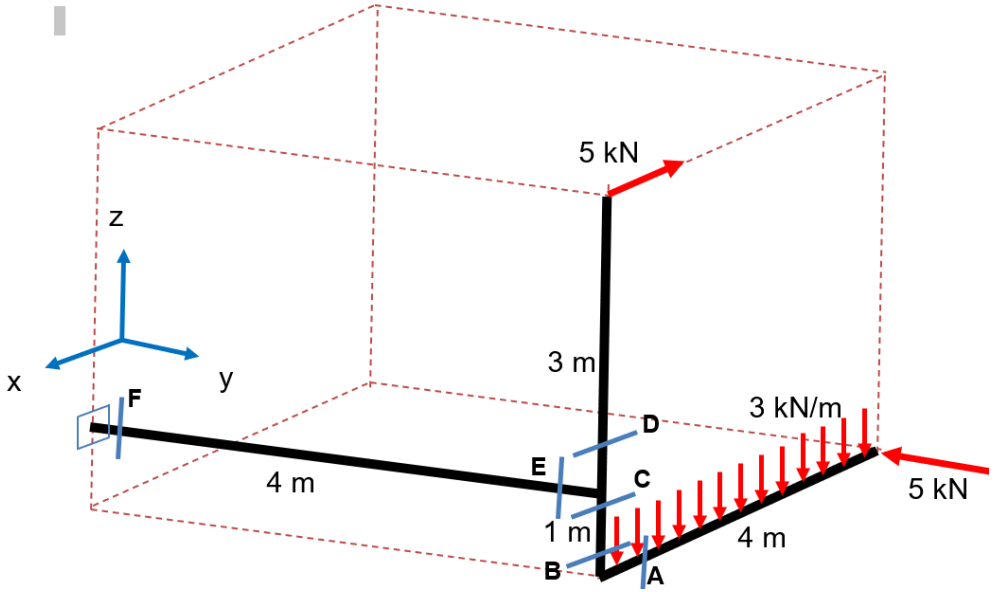


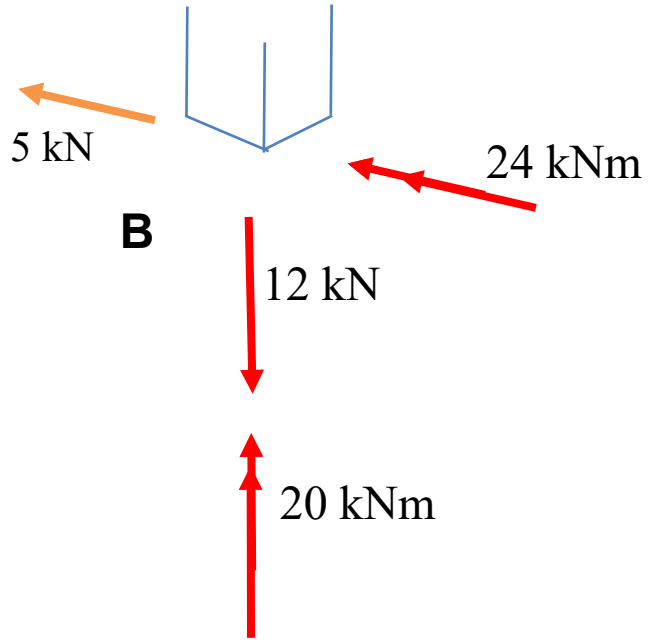

5 kN
z
x
y
3 m
F
D
3 kN/m
E
C
4 m
5 kN
1 m
4 m
B
A

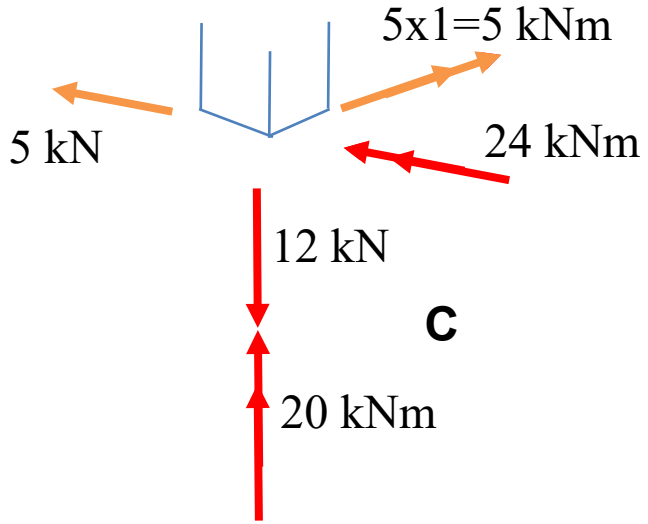

5 kN
24 kNm
B
12 kN
20 kNm

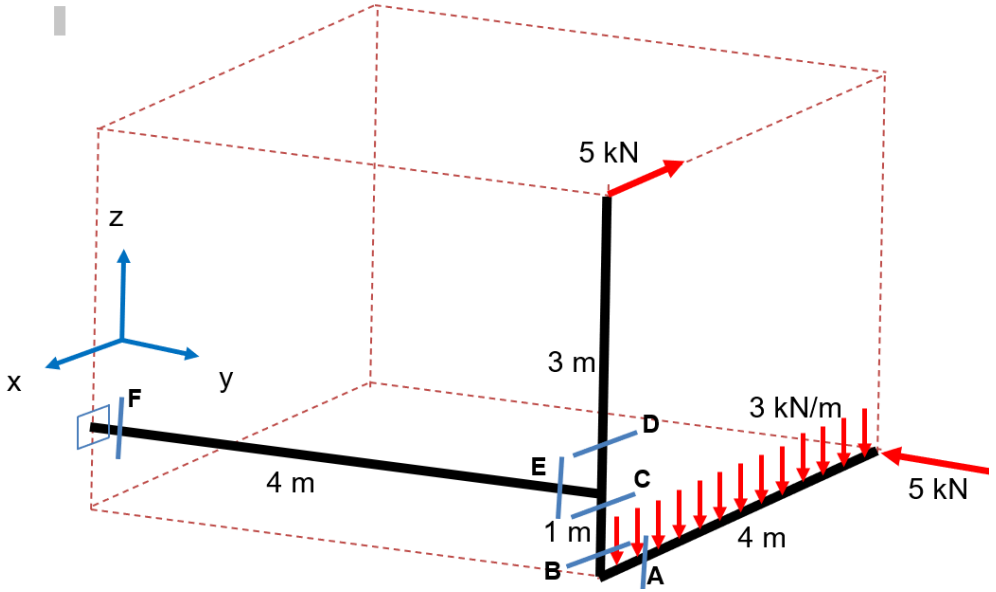

5x4=20 kNm
A
3x4=12 kN
5 kN
3x4x2=24 kNm

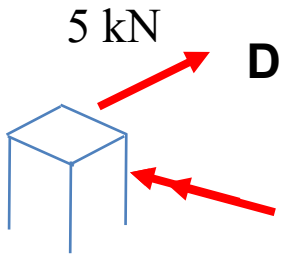

5 kN
D
5x3=15 kNm


5 kN
z
x
y
3 m
F
D
3 kN/m
E
C
4 m
5 kN
1 m
4 m
B
A

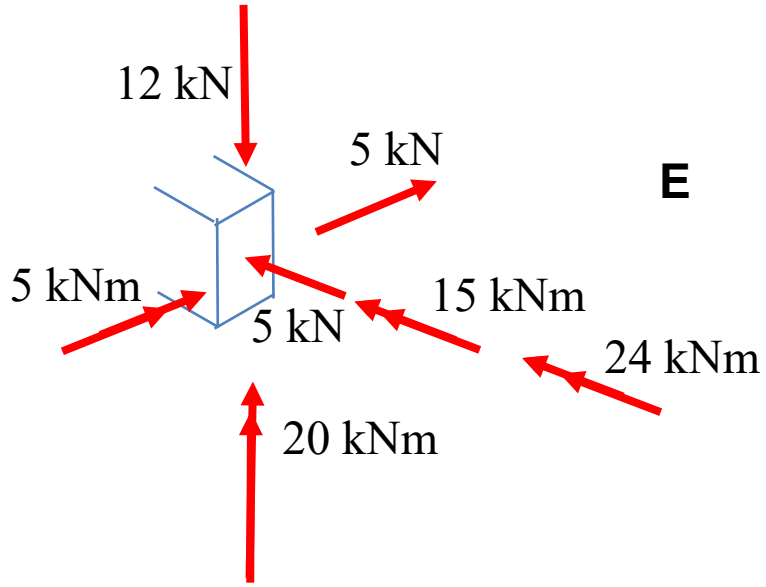

5 kN
24 kNm
B
12 kN
20 kNm

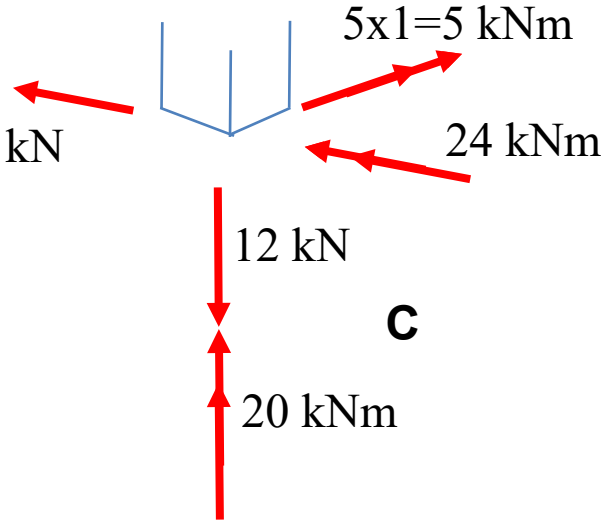

5x1=5 kNm
5 kN
24 kNm
12 kN
C
20 kNm

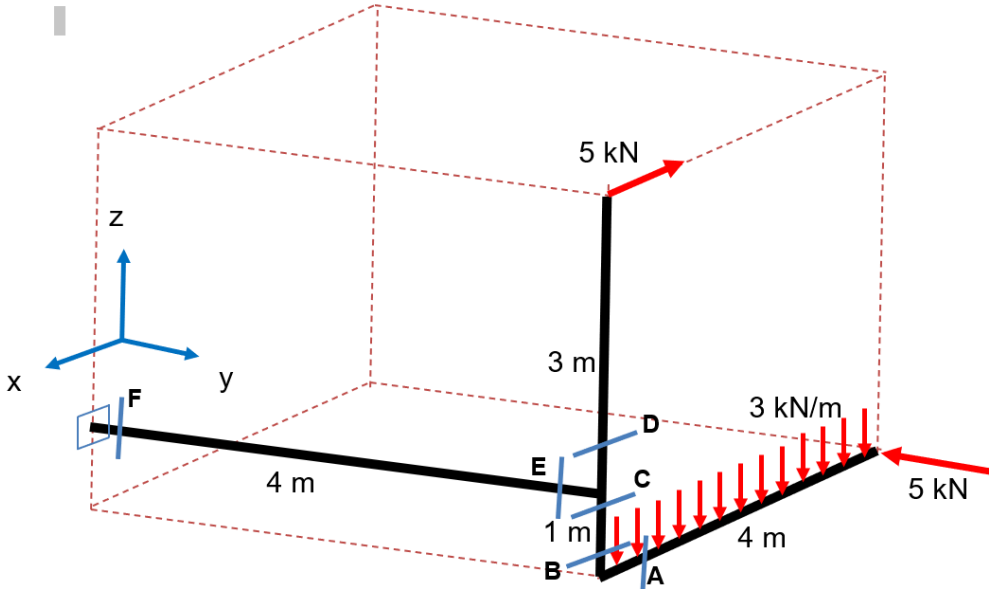

5 kN
z
x
y
3 m
F
D
3 kN/m
E
C
4 m
1 m
4 m
5 kN
B
A

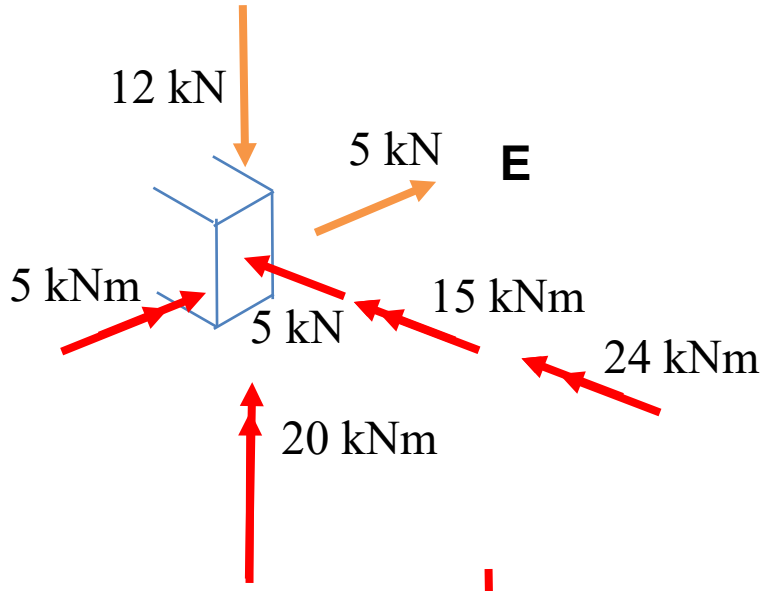

5 kN
D

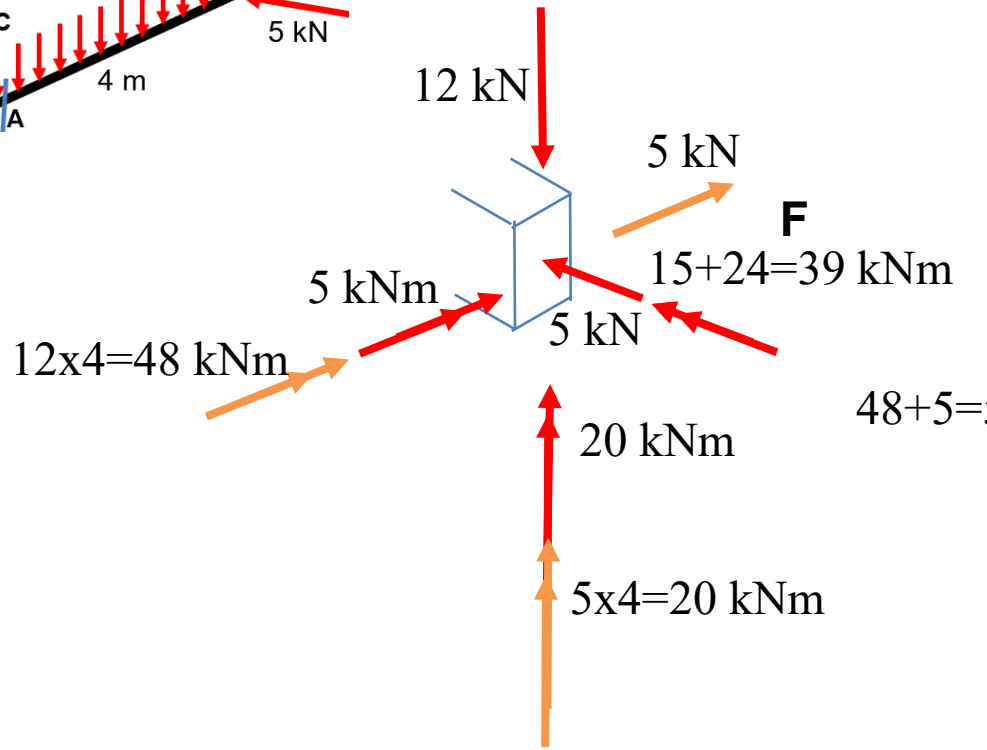
5x3=15 kNm

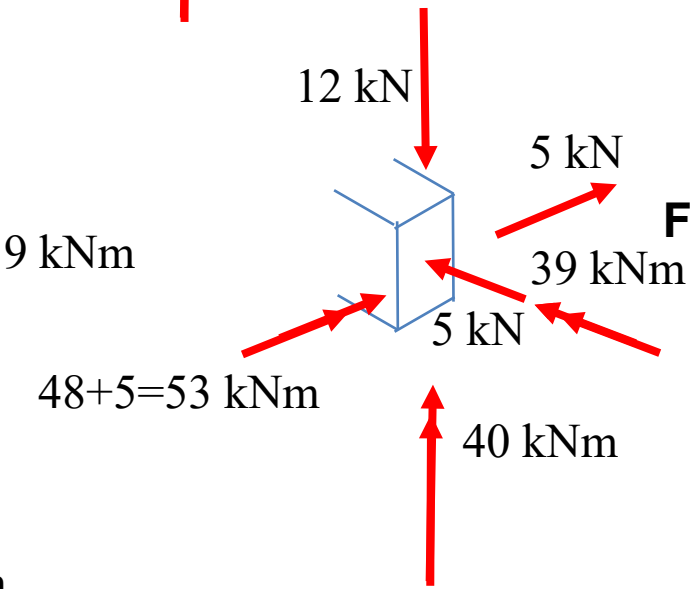

12 kN
5 kN
E
5 kNm
5 kN
15 kNm
24 kNm
20 kNm

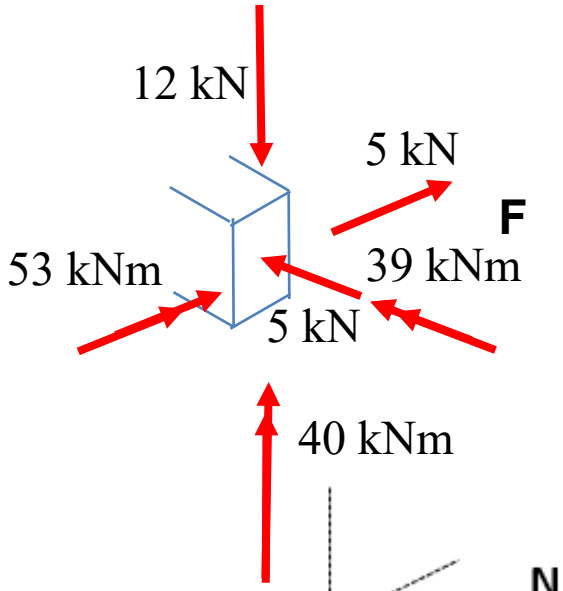

5x1=5 kNm
5 kN
24 kNm
12 kN
C
20 kNm

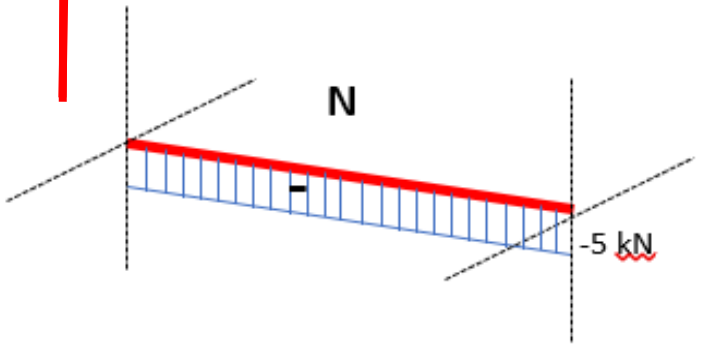

5 kN
z
x
y
3 m
F
D
3 kN/m
E
C
4 m
5 kN
1 m
4 m
B
A

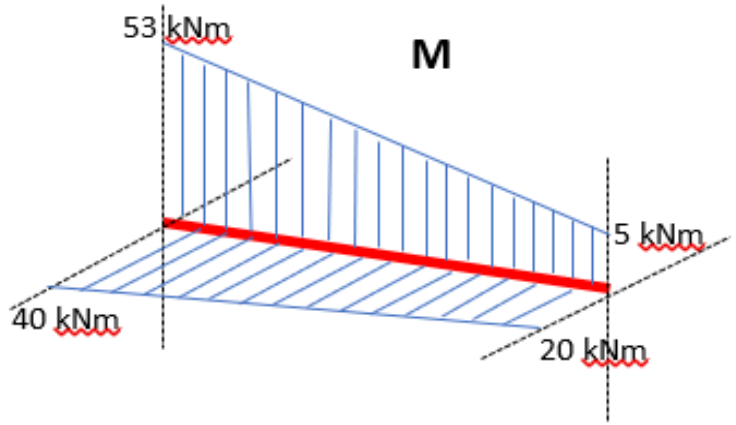

12 kN
5 kN
E
5 kNm
15 kNm
5 kN
24 kNm
20 kNm

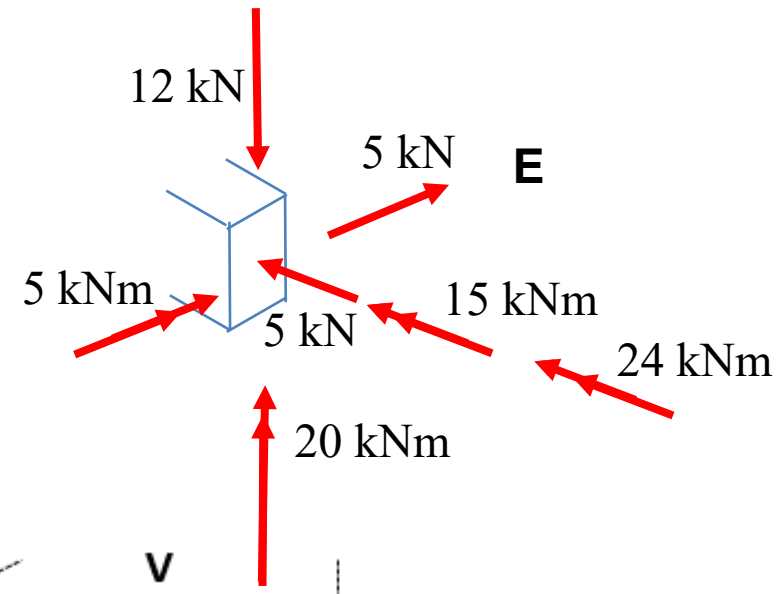

12 kN
5 kN
F
15+24=39 kNm
5 kNm
5 kN
12x4=48 kNm
20 kNm
5x4=20 kNm

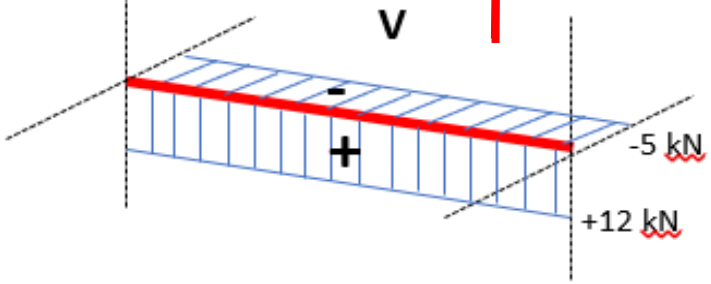

12 kN
5 kN
F
39 kNm
5 kN
48+5=53 kNm
40 kNm

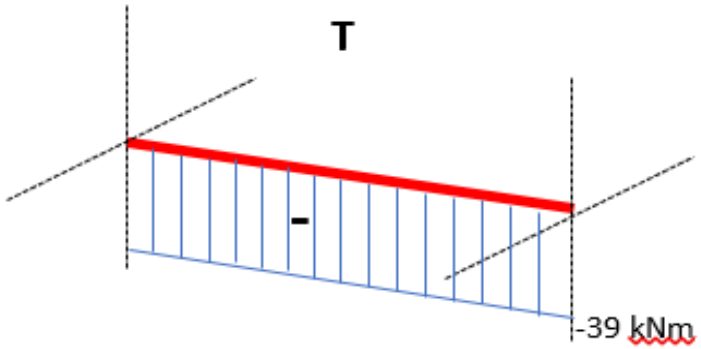

12 kN
5 kN
F
53 kNm
39 kNm
5 kN
40 kNm

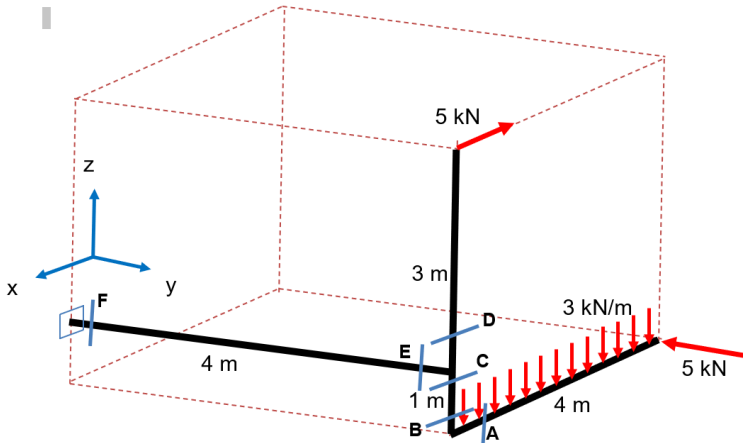

N
-
-5 kN

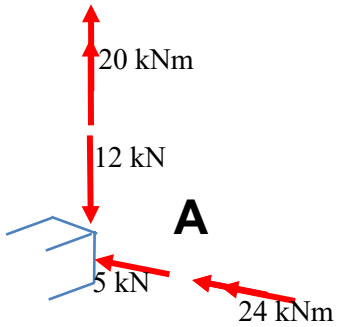

53 kNm
M
5 kNm
40 kNm
20 kNm

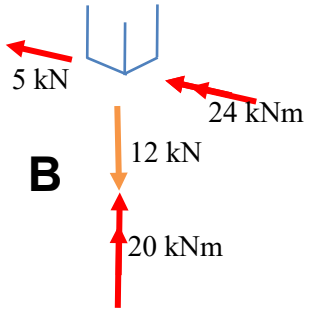

12 kN
5 kN
E
5 kNm
15 kNm
5 kN
24 kNm
20 kNm

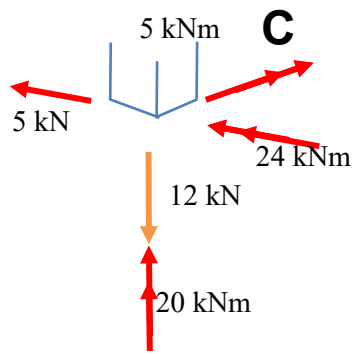

V
-
+
-5 kN
+12 kN

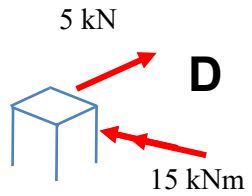

T
-
-39 kNm

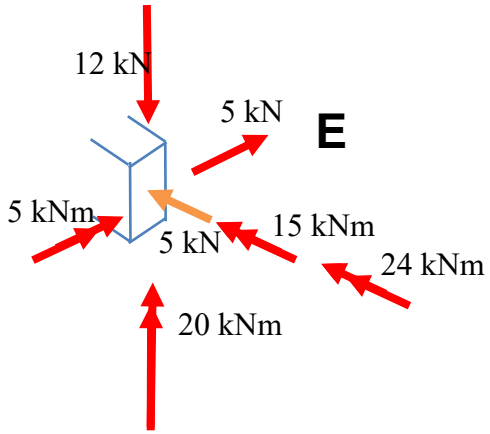

5 kN
z
x
y
F
3 m
D
3 kN/m
E
C
4 m
1 m
4 m
5 kN
B
A

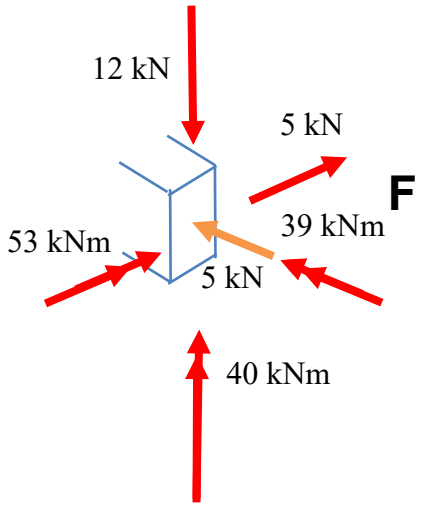

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A
5 kN
24 kNm

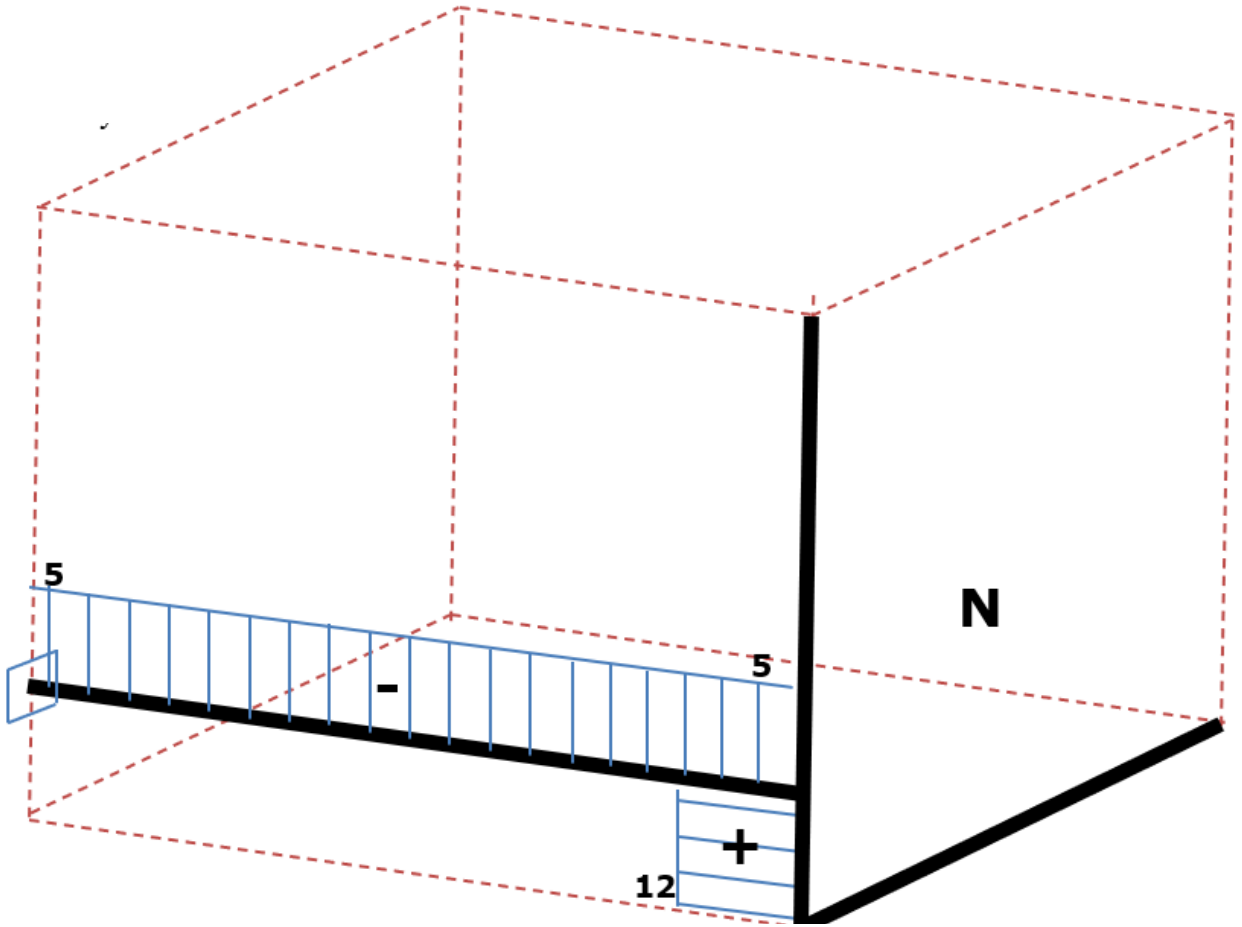

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B
12 kN
20 kNm

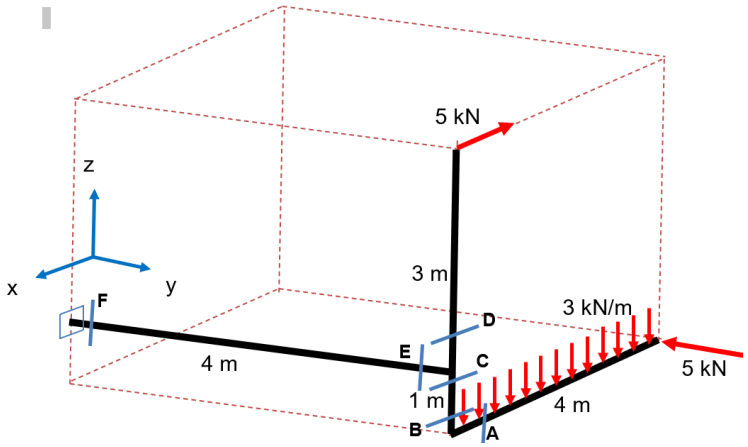

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C
5 kN
24 kNm
12 kN
20 kNm

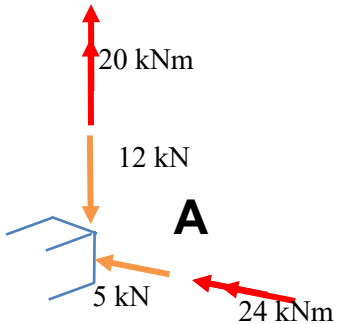

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D
15 kNm

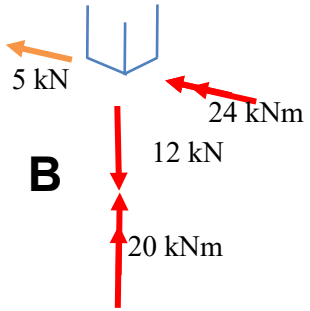

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20 kNm

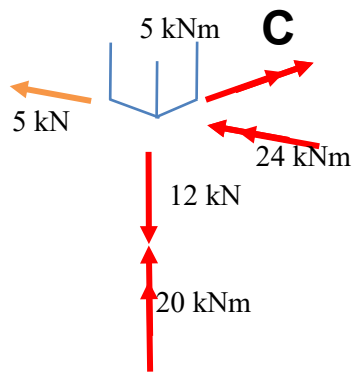

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F
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39 kNm
5 kN
40 kNm

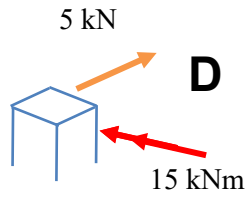

5
N
-
5
+
12

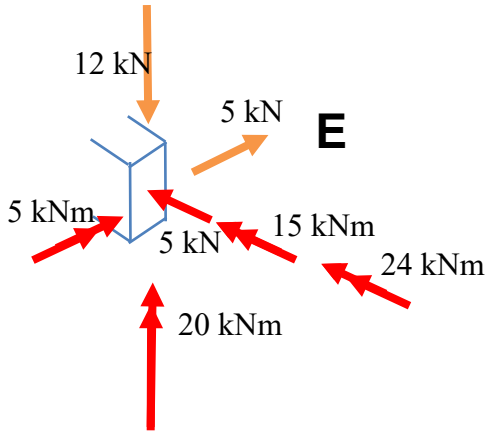

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z
x
y
F
3 m
D
3 kN/m
E
C
4 m
1 m
4 m
5 kN
B
A

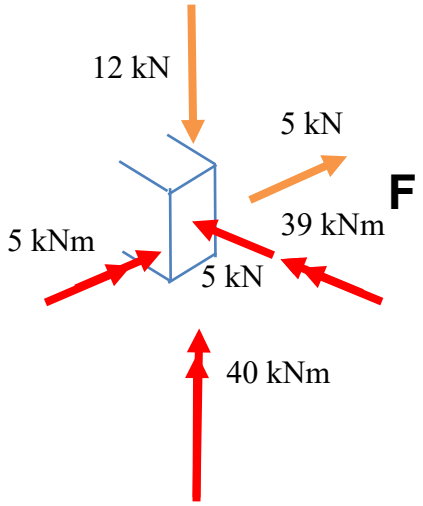

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A
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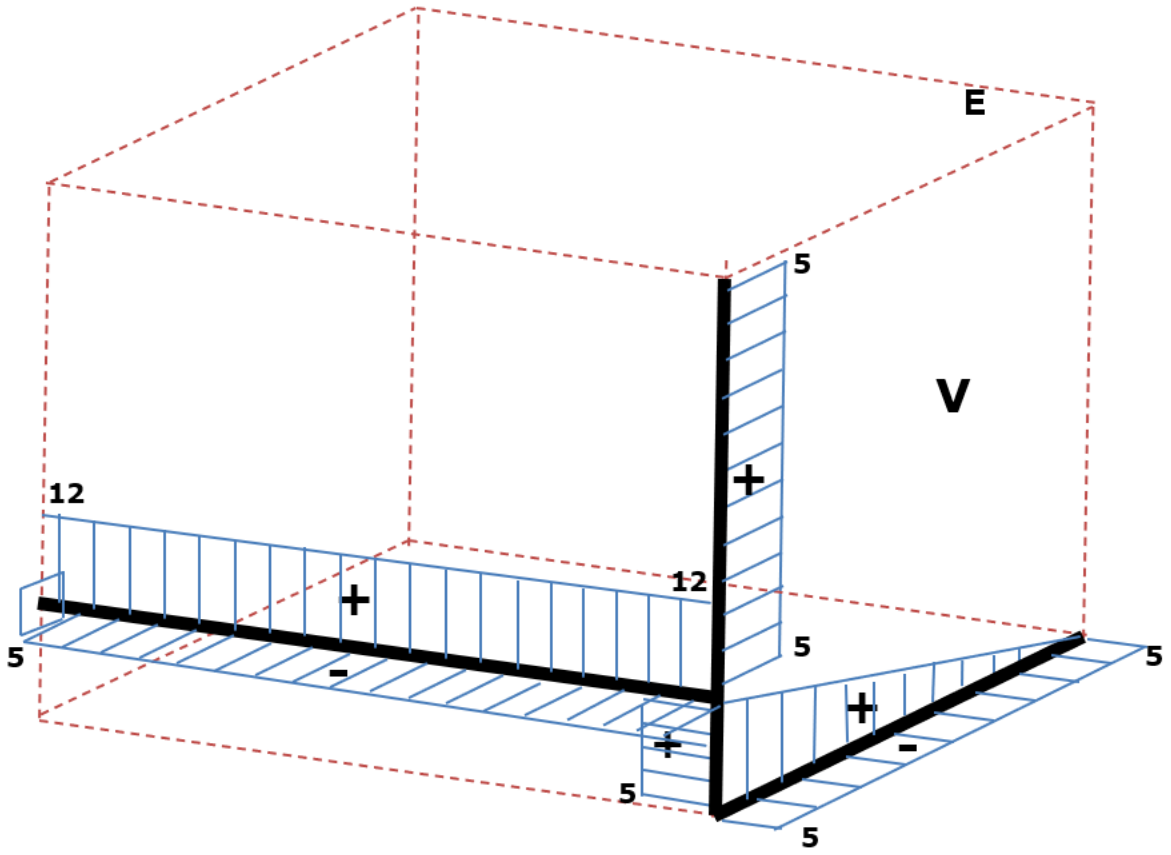

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B
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20 kNm

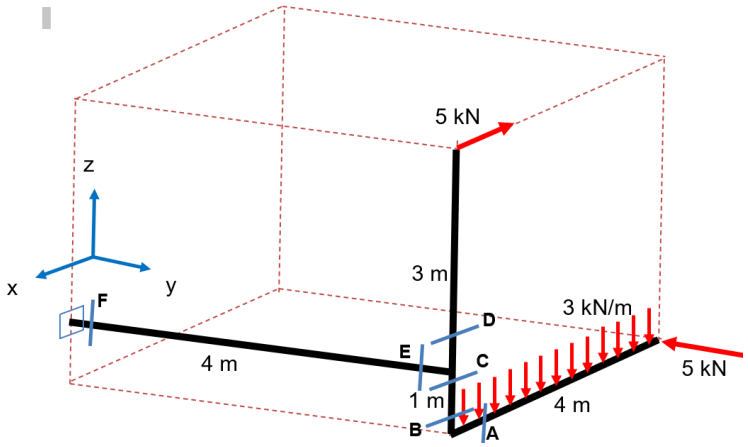

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C
5 kN
24 kNm
12 kN
20 kNm

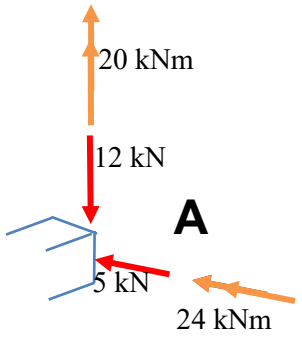

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D
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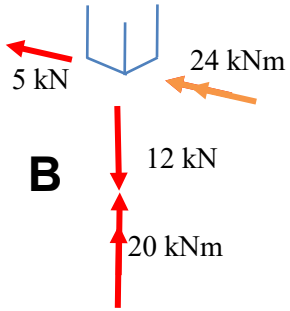

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E
5 kNm
5 kN
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20 kNm

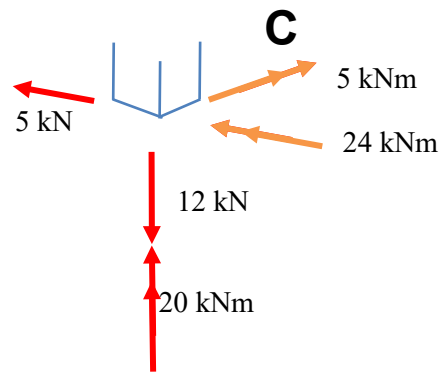

12 kN
5 kN
F
5 kNm
39 kNm
5 kN
40 kNm

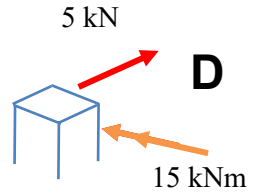

E
5
V
12
+
12
5
5
+
-
5
+
5
5

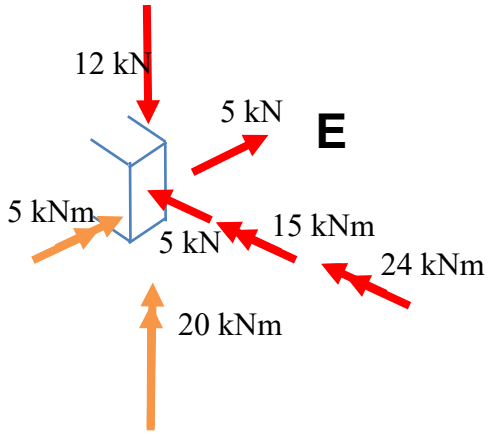

5 kN
z
x
y
3 m
F
D
E
C
B
A
4 m
1 m
4 m
3 kN/m
5 kN

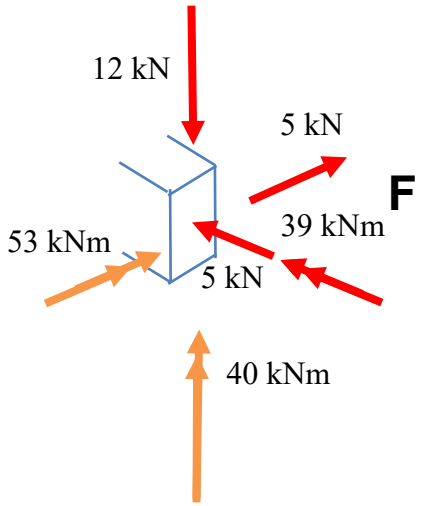

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A
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24 kNm

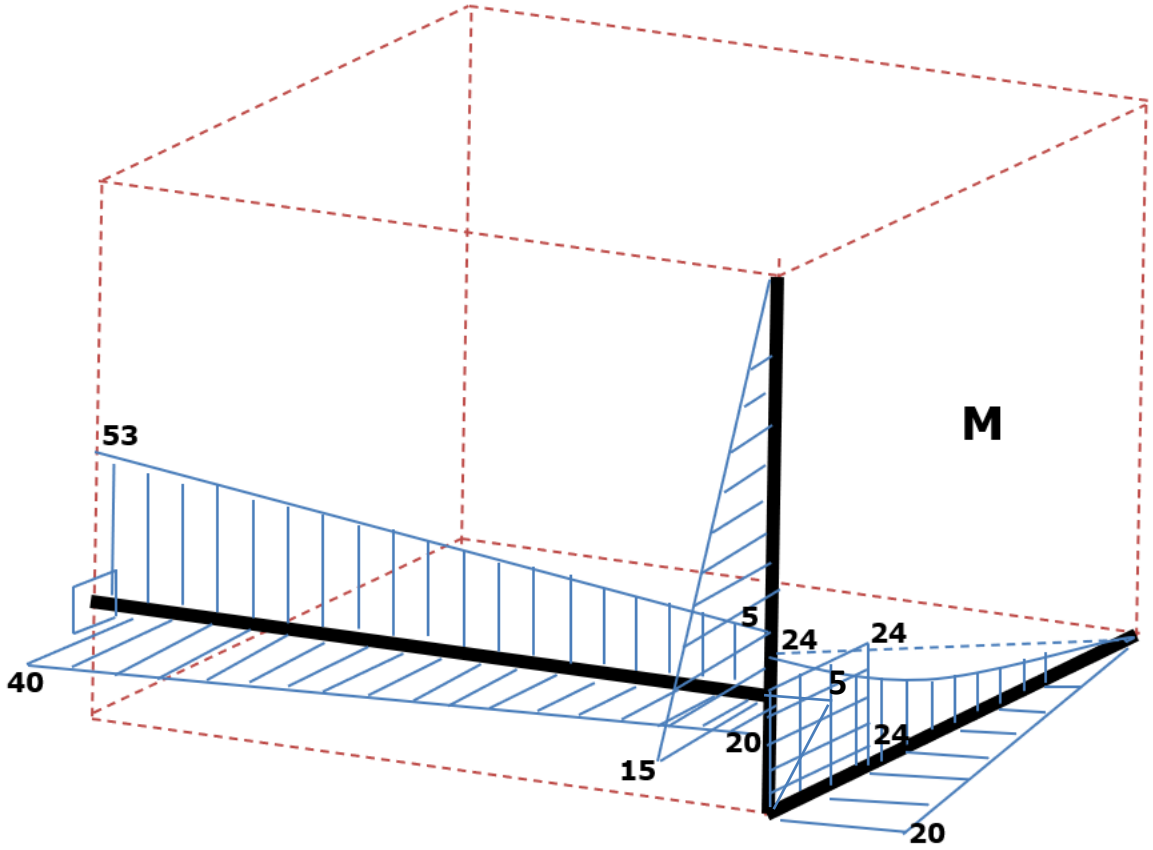

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B
12 kN
20 kNm

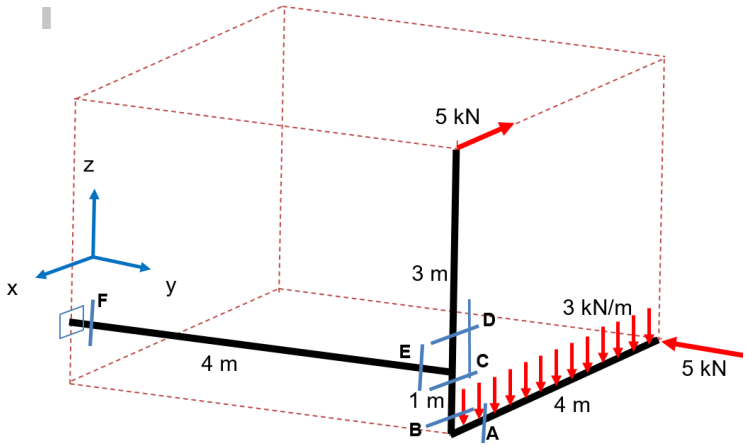

C
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5 kN
24 kNm
12 kN
20 kNm

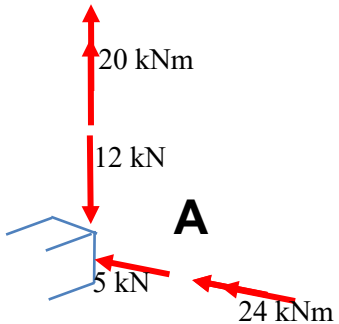

5 kN
D
15 kNm

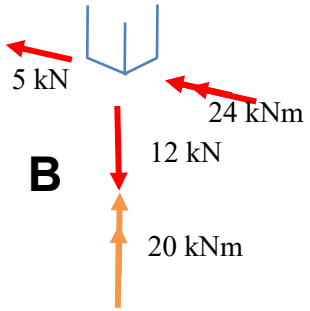

12 kN
5 kN
E
5 kNm
5 kN
15 kNm
24 kNm
20 kNm

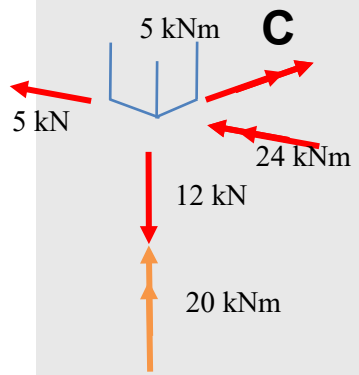

12 kN
5 kN
F
39 kNm
53 kNm
5 kN
40 kNm

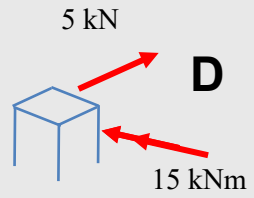

M
53
40
5
24
24
5
20
15
24
20

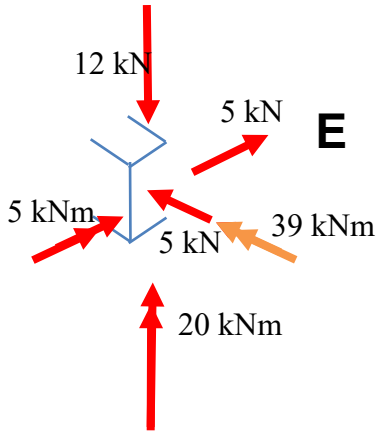

5 kN
z
x
y
F
4 m
E
D
C
3 m
1 m
B
A
3 kN/m
4 m
5 kN

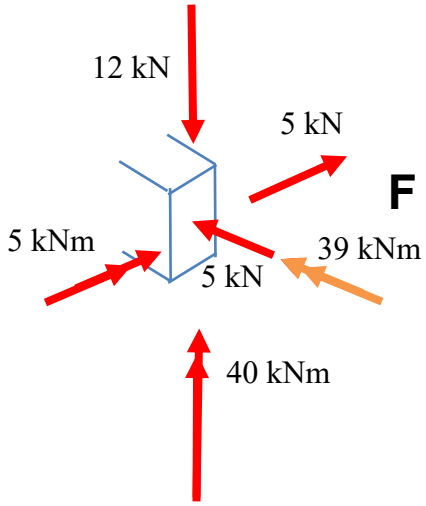

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A
5 kN
24 kNm

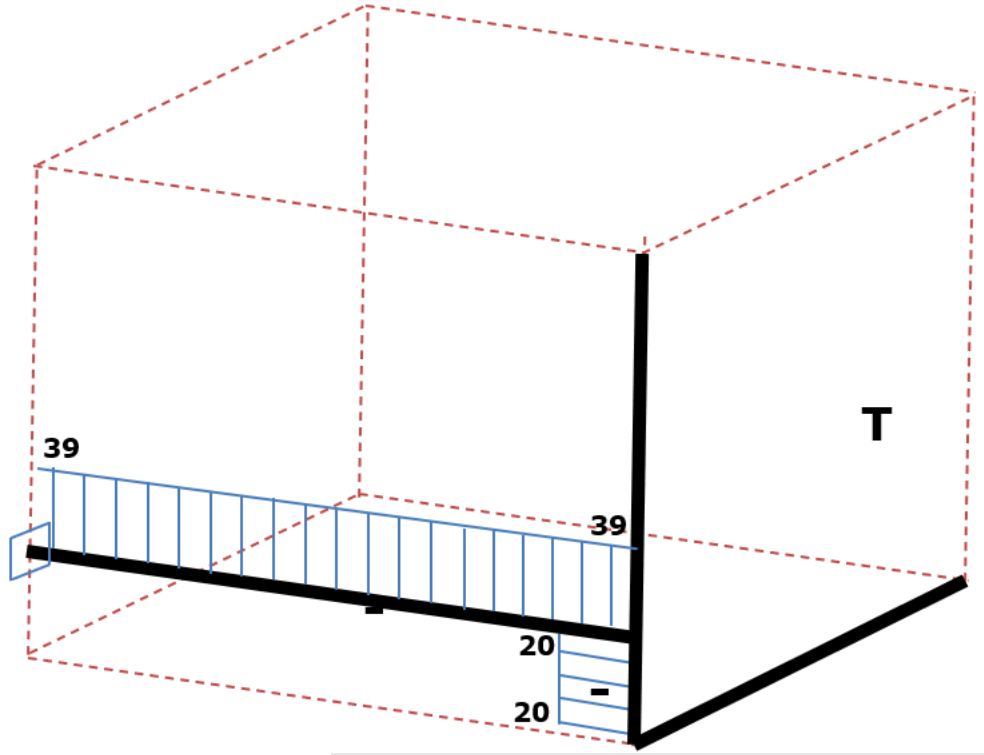

5 kN
24 kNm
B
12 kN
20 kNm


5 kNm
C
5 kN
24 kNm
12 kN
20 kNm


5 kN
D
15 kNm


12 kN
5 kN
E
5 kNm
5 kN
39 kNm
20 kNm


12 kN
5 kN
F
5 kNm
5 kN
39 kNm
40 kNm


T
39
39
20
20

PEF3208
Aula 8
17 abr
PROF. NAKAO

Ftool

E-aulas

http://eaulas.usp.br/portal/video.action?idItem=10577

https://edisciplinas.usp.br/course/view.php?id=73700

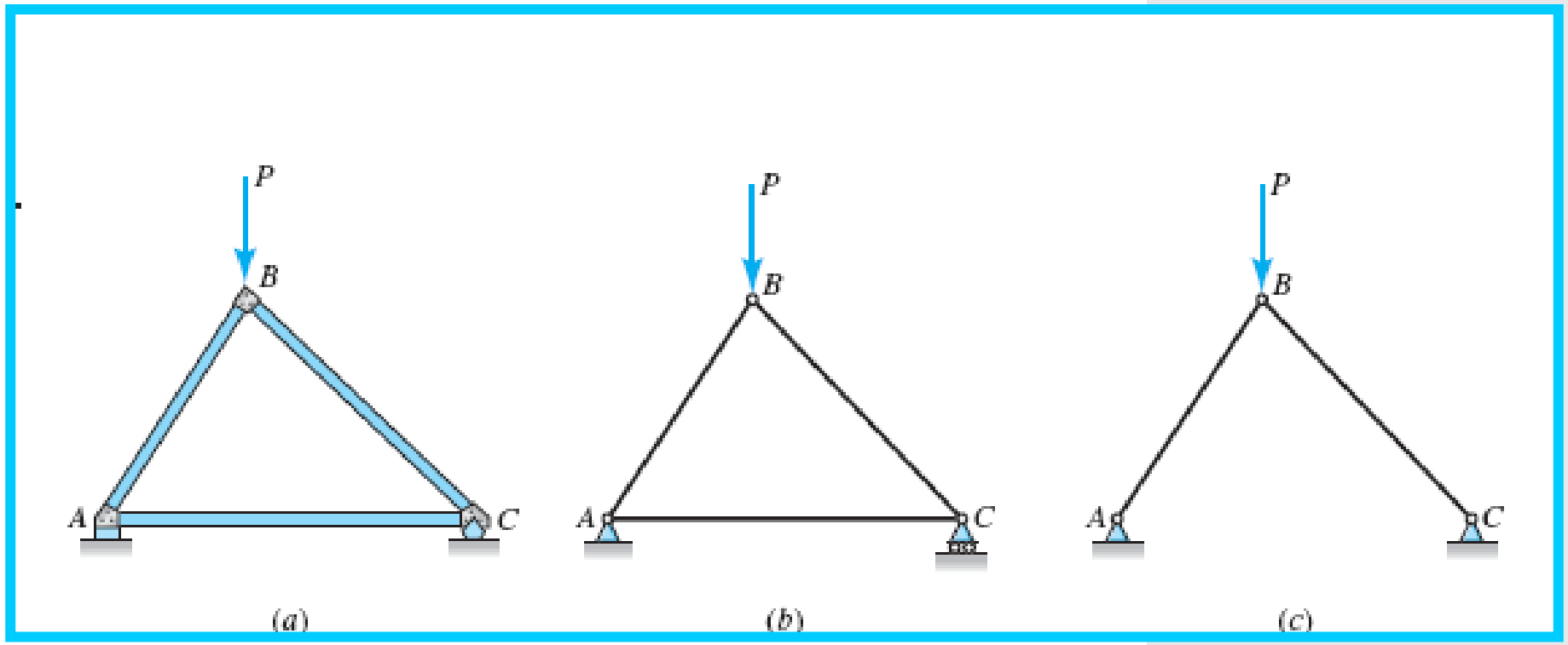


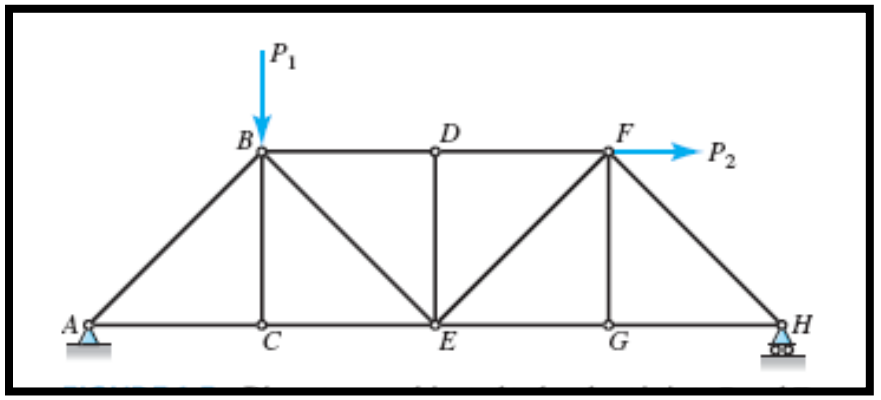
FtoolTutorial.exe

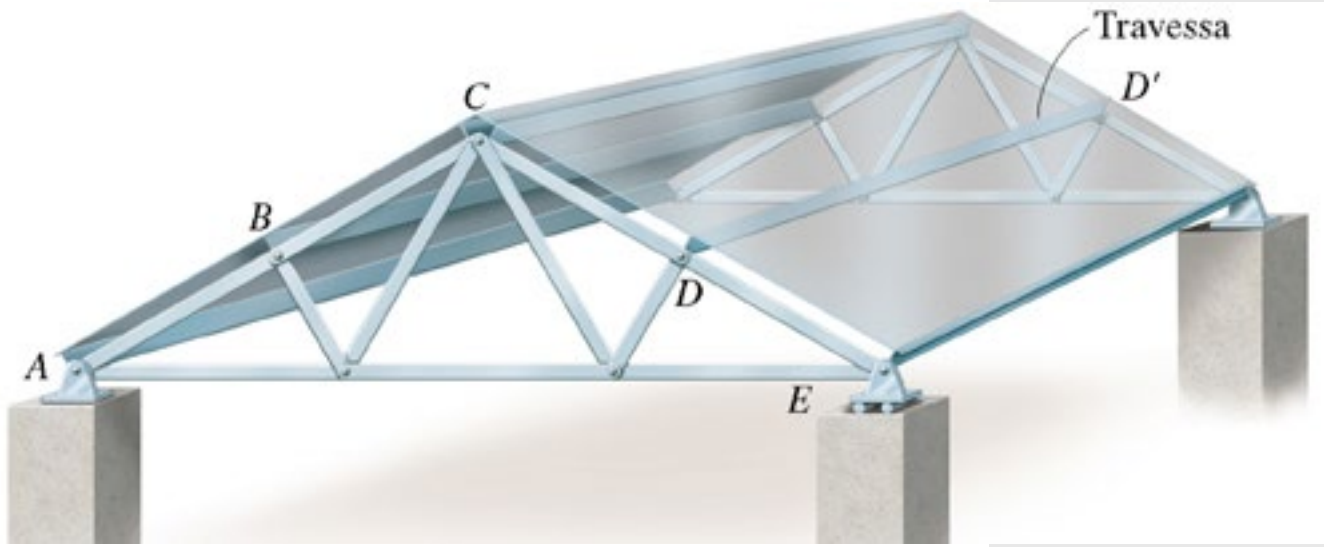
Aula FTool.mp4

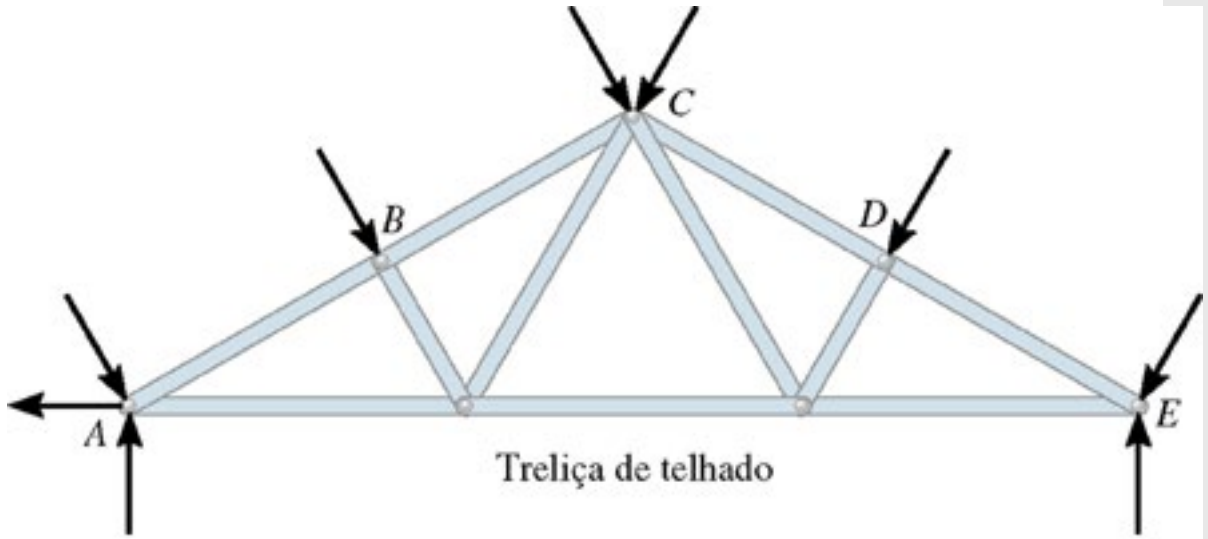
Treliças

EXEMPLOS DE TRELIÇAS

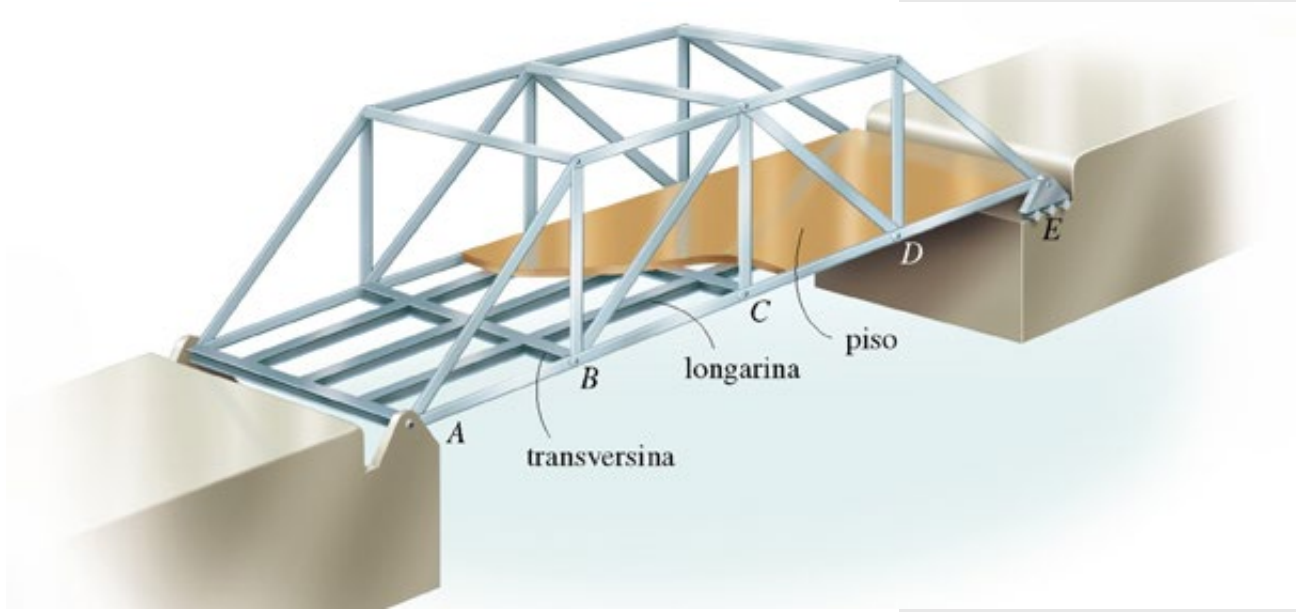


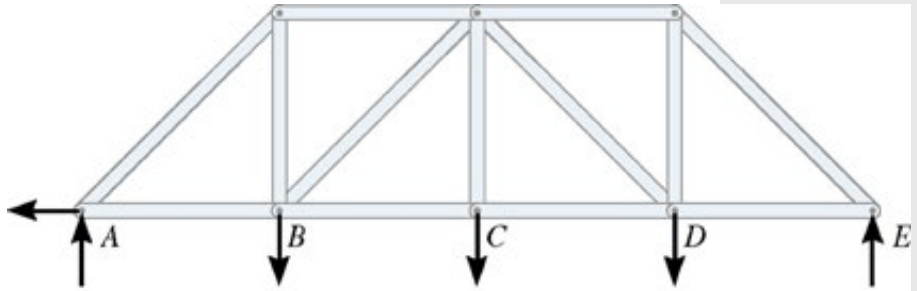




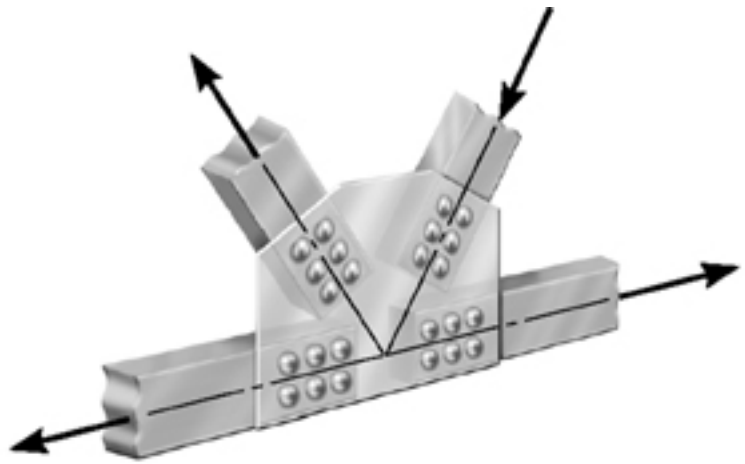


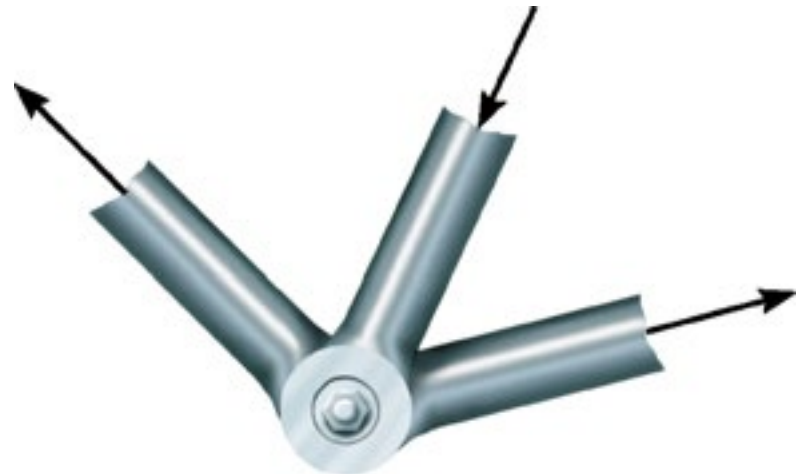
Treliça de telhado

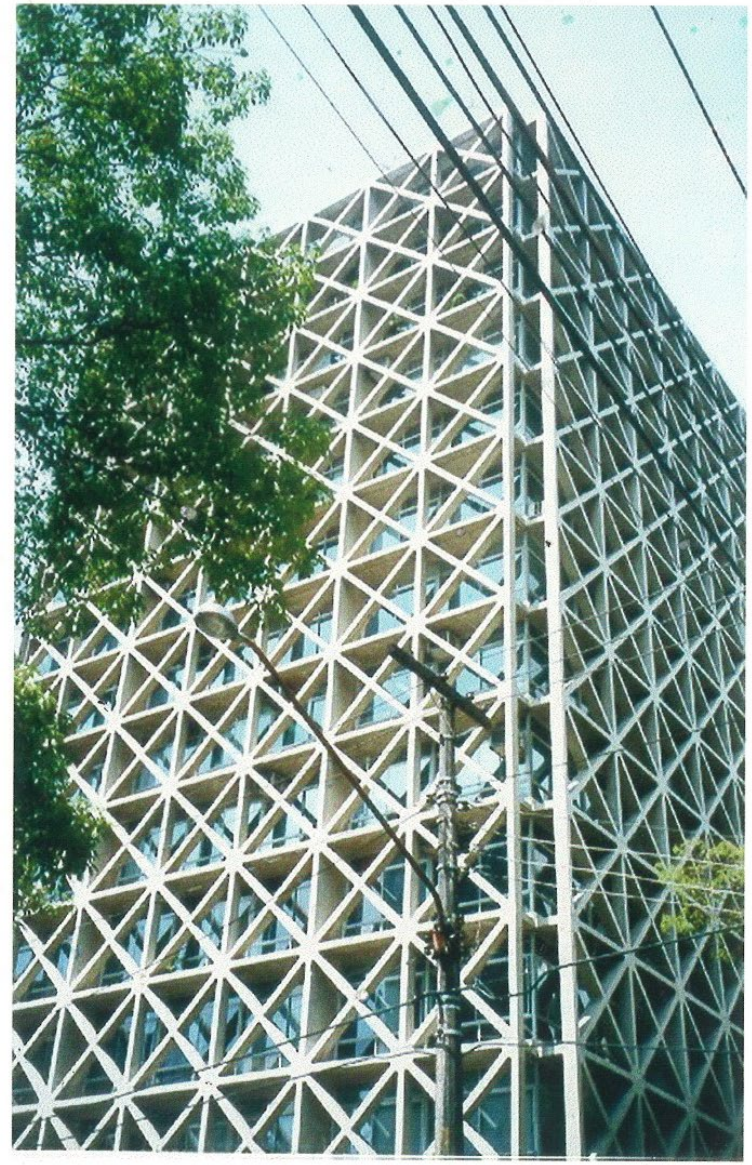




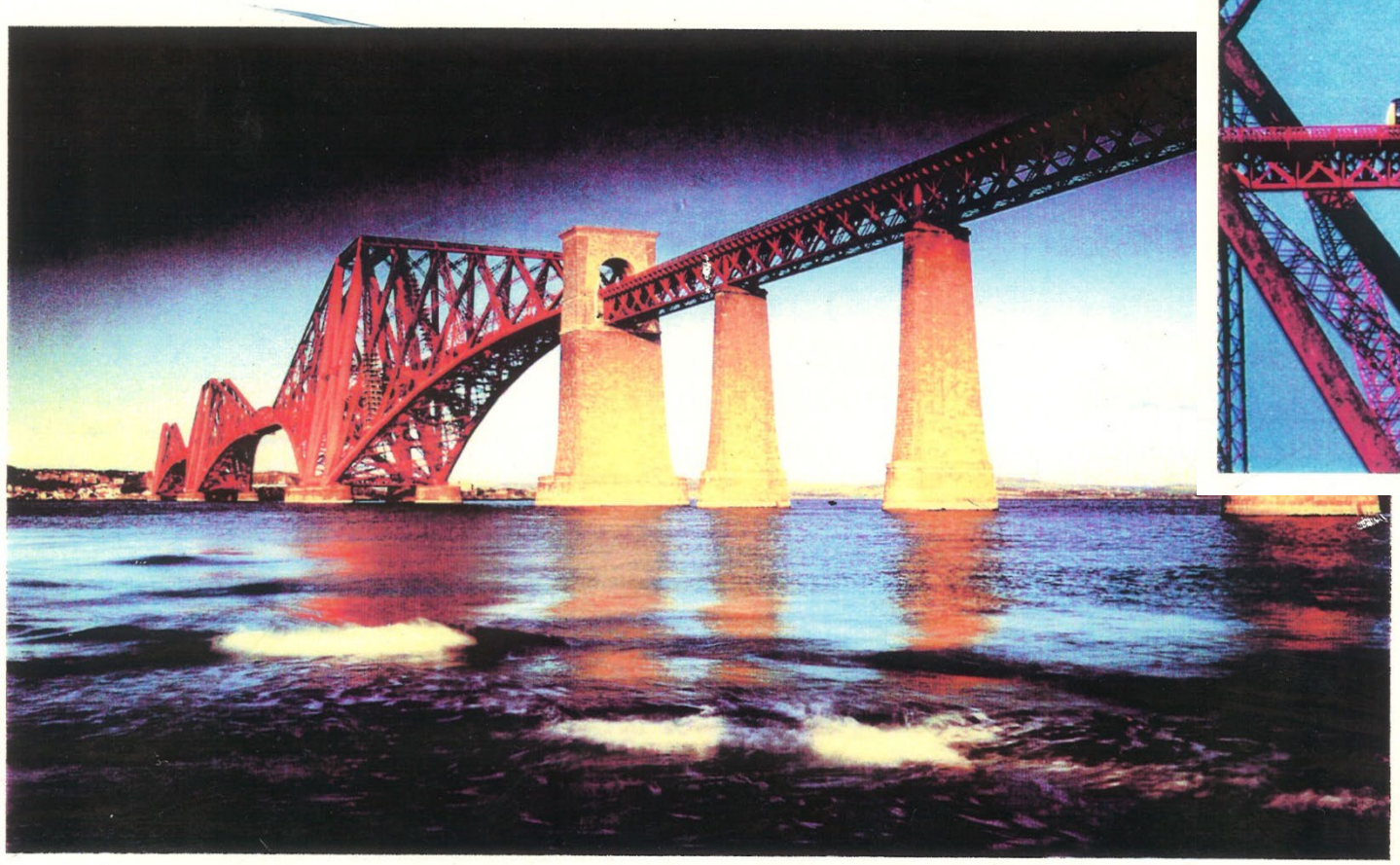
Treliça da Ponte

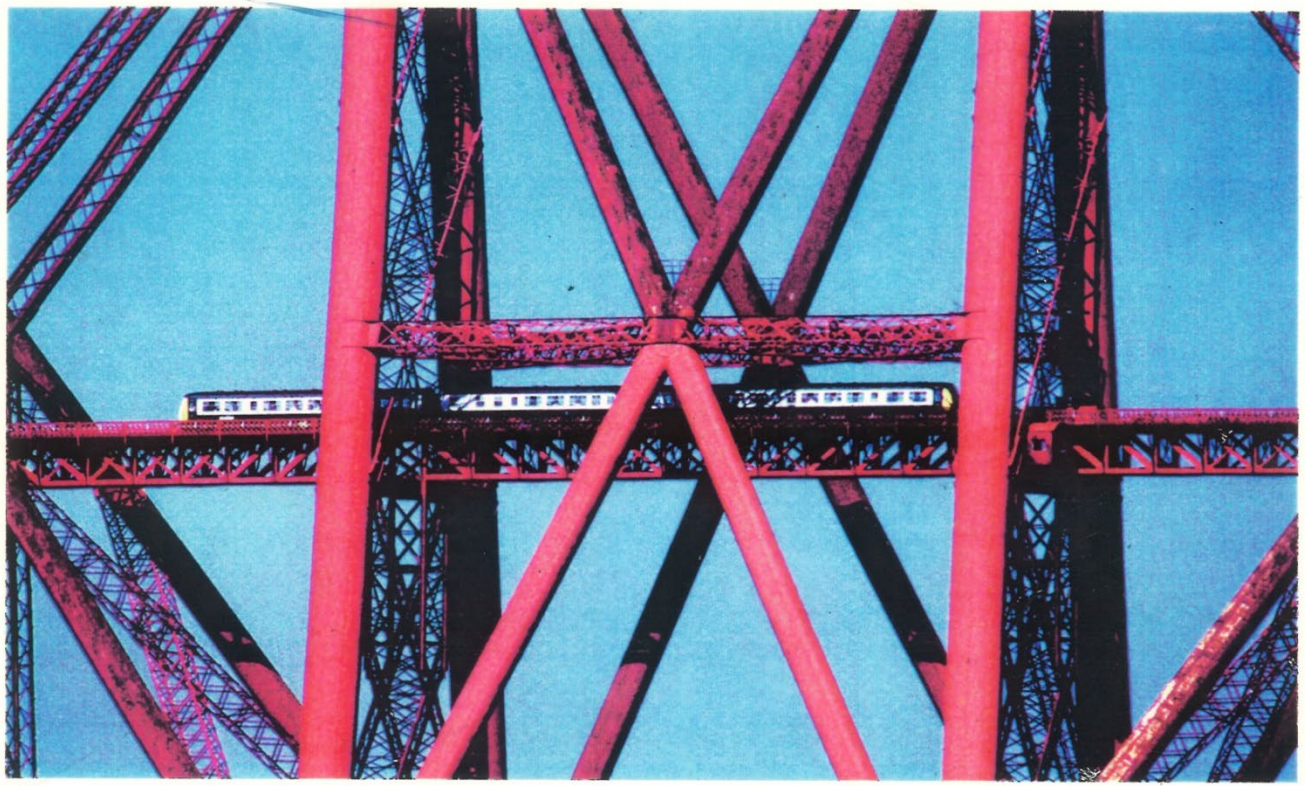






Edifício Acal
(fotografia de Daniella Pinholi Cardoso)





Firth of Forth Bridge, 1889



Ponte estaiada sobre o rio Pinheiros
(fotografia de Anderson Glauco Benite)



Treliça de escoramento dos segmentos
do tabuleiro da ponte
(fotografia de Anderson Glauco Benite)

Passarela sobre a Rodovia Raposo Tavares



Vista interior da passarela



Vista inferior da passarela

(fotografias de Anderson Glauco Benite)



Terminal Rodoviário Amador Aguiar



Cobertura
(fotografia de Anderson Glauco Benite)



Detalhe de um nó da treliça
(fotografia de Anderson Glauco Benite)

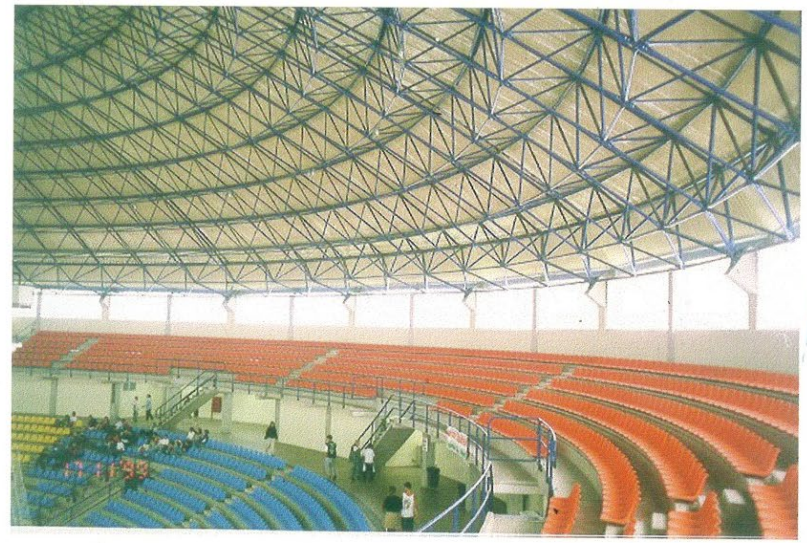


Cobertura da Estação Terminal Barra Funda do Metrô
(fotografia de André Hiroshi de Oliveira Nishina)



Detalhe de um nó da treliça
(fotografia de André Hiroshi de Oliveira Nishina)

Cobertura do Ginásio Poliesportivo de São Bernardo do Campo



Detalhe dos apoios da cobertura
(fotografia de Nayra Tais Savordelli)



Detalhe de um dos apoios da cobertura
(fotografia de Nayra Tais Savordelli)

REVIEW OF STATIC EQUILIBRIUM

Types of loads: External loads and Internal loads

External loads are due to surface forces and body forces

- **Surfaces forces can be for example, a concentrated load acting at a point or a distributed load both acting on the surface of a body**
- **Body forces act on a volumetric portion of the body, forexample, magnetic force or gravitational force**
- **Reaction forces caused by the supports**

Internal loads can be considered as forces of interaction between the constituent material particles of the body

CONDITIONS OF EQUILIBRIUM

When a system of forces acting upon a body has zero resultant, the body is said to be in force equilibrium.
The equations of static equlibrium require:

$\sum F_x = 0$; $\sum F_y = 0$, and $\sum F_z = 0$

$\sum M_x = 0$; $\sum M_y = 0$, and $\sum M_z = 0$

In other words, for a body to be in static equilibrium, the sum of all forces acting upon a body in any direction is zero and also the sum of all moments taken about any axis is also zero.

Planar Equations of Equilibrium

For a planar body to be in equilibrium, any one of the following sets of 3 equations may be used to solve for the unknown variables.

1. $\sum F_x = 0$, $\sum F_y = 0$, and $\sum M_A = 0$, where the resultant moment is with respect to any axis z or any point A in the xy-plane, or

2. $\sum F_x = 0$, $\sum M_A = 0$, and $\sum M_B = 0$, provided that the line connecting the points A and B is not perpendicular to the x axis, or

3. $\sum M_A = 0$, $\sum M_B = 0$, and $\sum M_C = 0$, where points A, B, and C are not collinear

Freebody diagrams

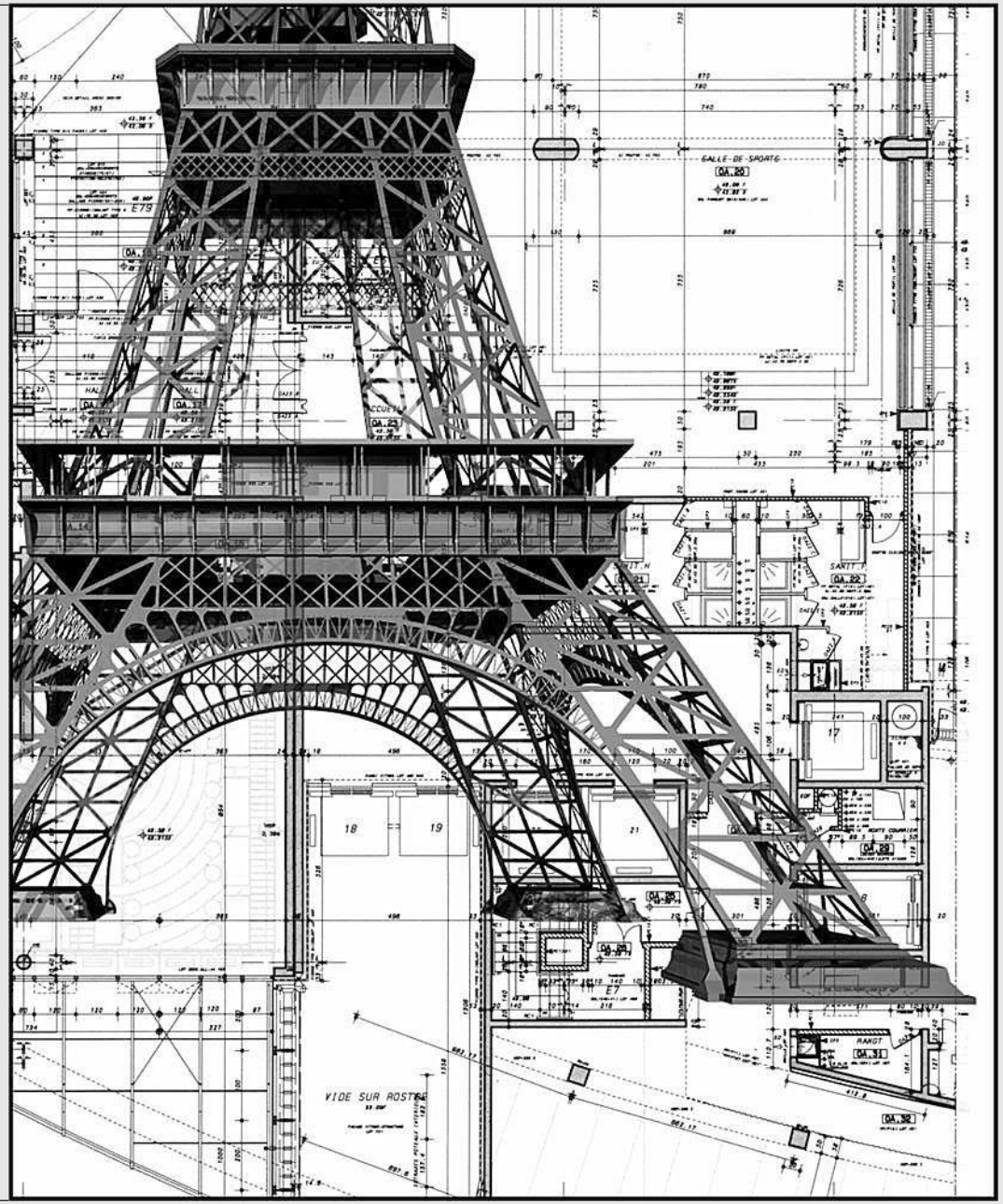
1. **Select the free body to be used.**
2. **Detach this body from its supports and separate it from any other bodies. (If internal force resultants are to be found, use the method of sections).**
3. **Show on the sketch all of the external forces acting on the body. Location, magnitude, and direction of each force should be marked on the sketch.**
4. **Label significant points and include dimensions. Any other detail, however, should be omitted.**

PEF3208 2020
AULA 8 - TRELIÇAS

Prof. Martin P. Schwark

TRELIÇAS

- São estruturas reticuladas (formadas por barras)
- Todas as barras são articuladas nas duas extremidades
- Os carregamentos são aplicados apenas nos nós da estrutura
- Desta forma, as barras são solicitadas apenas por força normal, constante ao longo do seu eixo
- Principais classificações:
  - Disposição: Planas/espaciais (neste curso abordaremos apenas treliças planas)
  - Estaticidade: hipostáticas/isostáticas/hiperestáticas (neste curso abordaremos apenas treliças isostáticas)
  - Formação: Simples/compostas/complexas (neste curso abordaremos apenas treliças simples e compostas)
- Métodos de resolução apresentados neste curso:
  - Método do Equilíbrio dos Nós
  - Método de Ritter

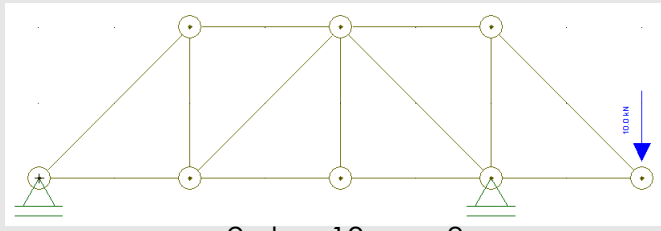


ESTATICIDADE

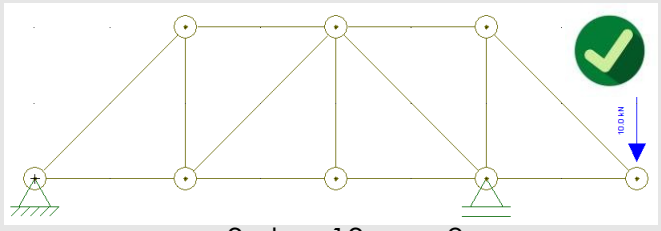
- r – número de reações externas
- b – número de barras
- n – número de nós

- r + b < 2n hipostática
  Pode apresentar movimento

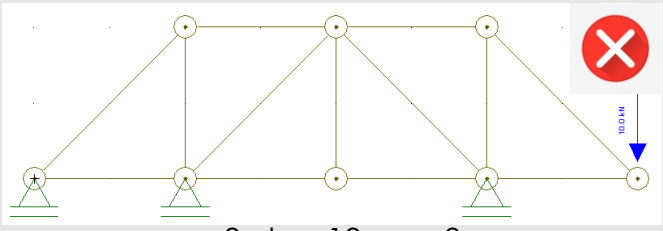
- r + b = 2n isostática (condição necessária mas não suficiente, podendo ser hipostática)
  Não pode apresentar movimento, mas se torna hipostática com a liberação de qualquer vínculo
- r + b > 2n hiperestática (condição necessária mas não suficiente, podendo ser hipostática)
  Não pode apresentar movimento, e pode ter vinculações suprimidas sem se tornar hipostática



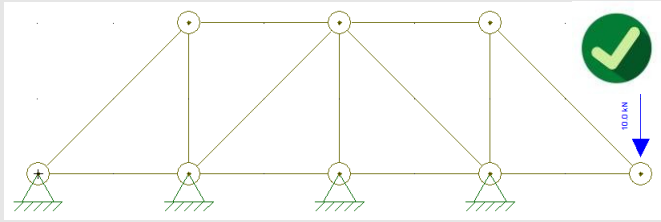
r = 2 b = 13 n = 8



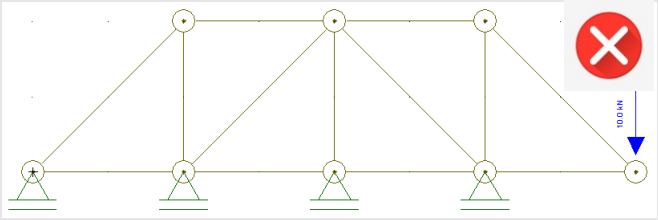
r = 3 b = 13 n = 8



r = 3 b = 13 n = 8



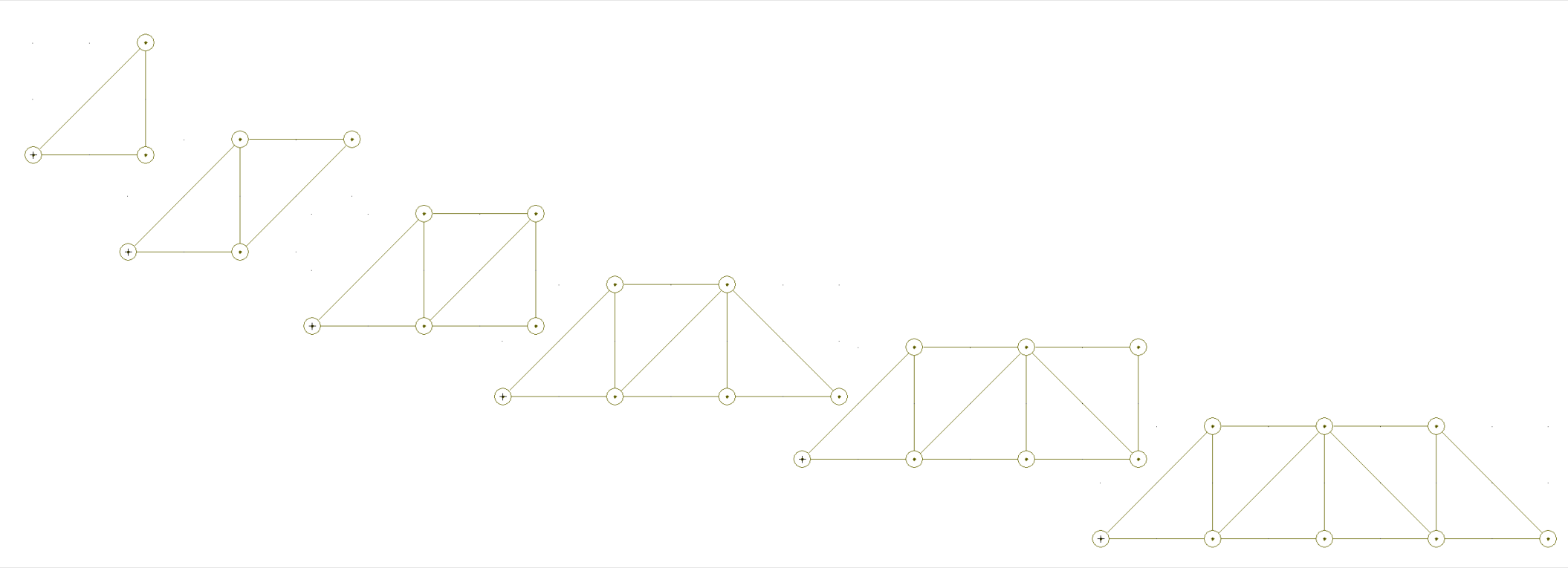
r = 8 b = 13 n = 8



r = 4 b = 13 n = 8

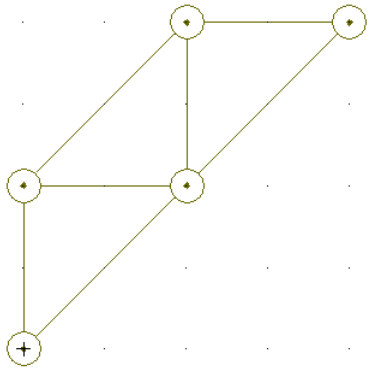
TRELIÇAS SIMPLES

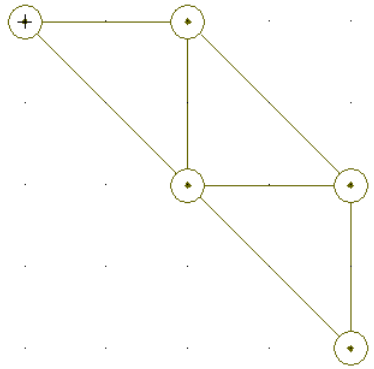
- São formadas a partir de um triângulo de barras, adicionando sucessivamente duas barras e um nó

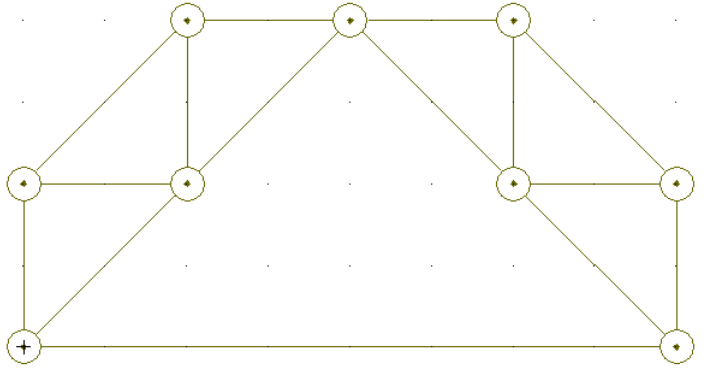


TRELIÇAS COMPOSTAS

- São compostas por treliças simples, ligadas entre si por nós ou barras

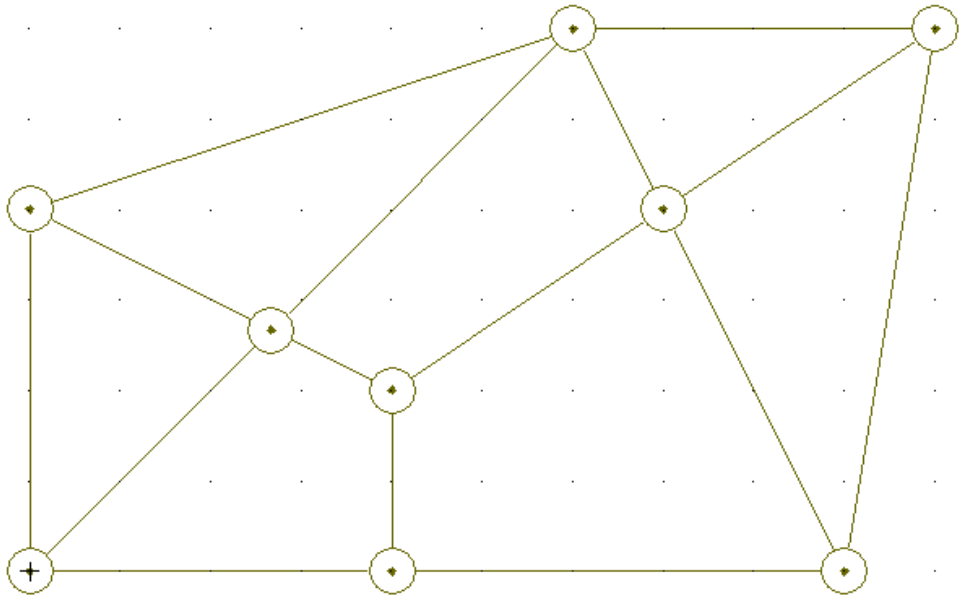






TRELIÇAS COMPLEXAS

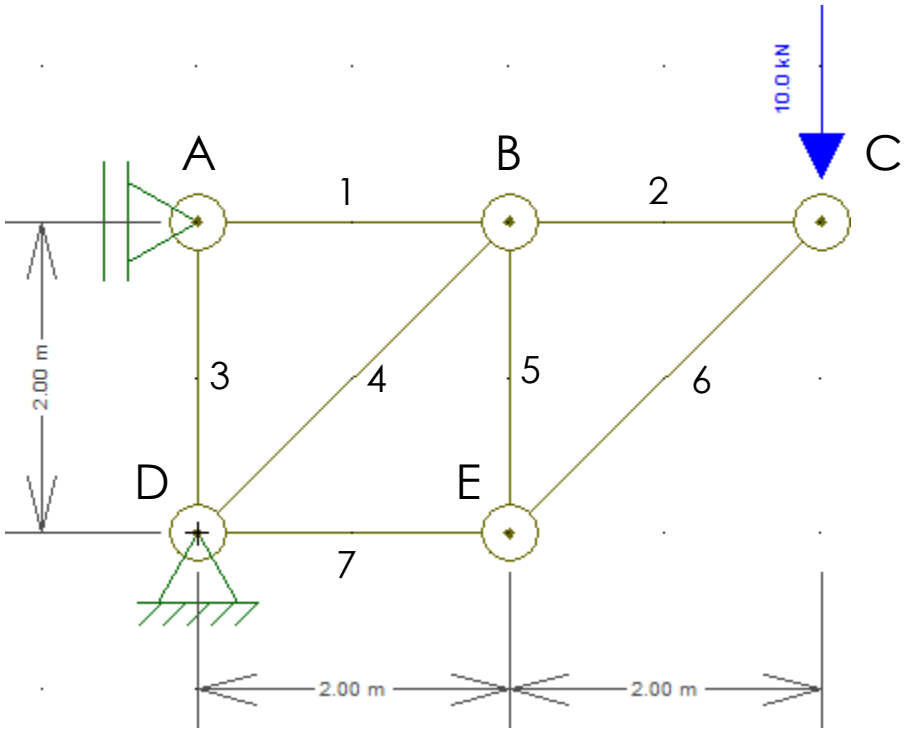
- Por exclusão



MÉTODO DO EQUILÍBRIO DOS NÓS

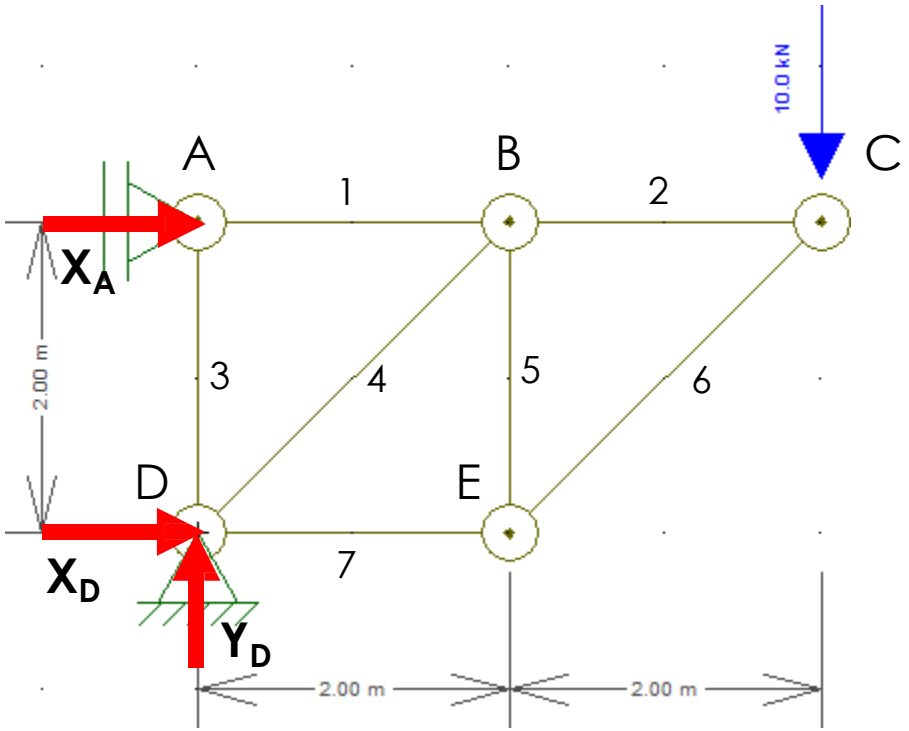
- Vamos considerar como incógnitas os esforços normais nas barras
- Se a estrutura está em equilíbrio, então seus nós também estão em equilíbrio
- Apenas duas equações de equilíbrio podem ser aplicadas para cada nó, por ser articulado nas barras
- Em treliças simples isostáticas, é possível explicitar as incógnitas uma a uma pelo equilíbrio dos nós
- Procedimento:
  - Cálculo das reações de apoio, utilizando as três equações de equilíbrio, considerando a treliça como corpo rígido
  - Cálculo sucessivo dos esforços nas barras, através do equilíbrio dos nós em que houver apenas duas incógnitas
  - No final da resolução, surgem três equações de verificação

Para os nós: $\Sigma F_x = 0$ $\Sigma F_y = 0$



Exemplo

Calcule os esforços nas barras da treliça ilustrada.



Reações de apoio

$\Sigma F_y = 0$

$Y_D - 10 = 0$

$Y_D = 10$ kN

$\Sigma M_D = 0$

$- X_A * 2 - 10 * 4 = 0$

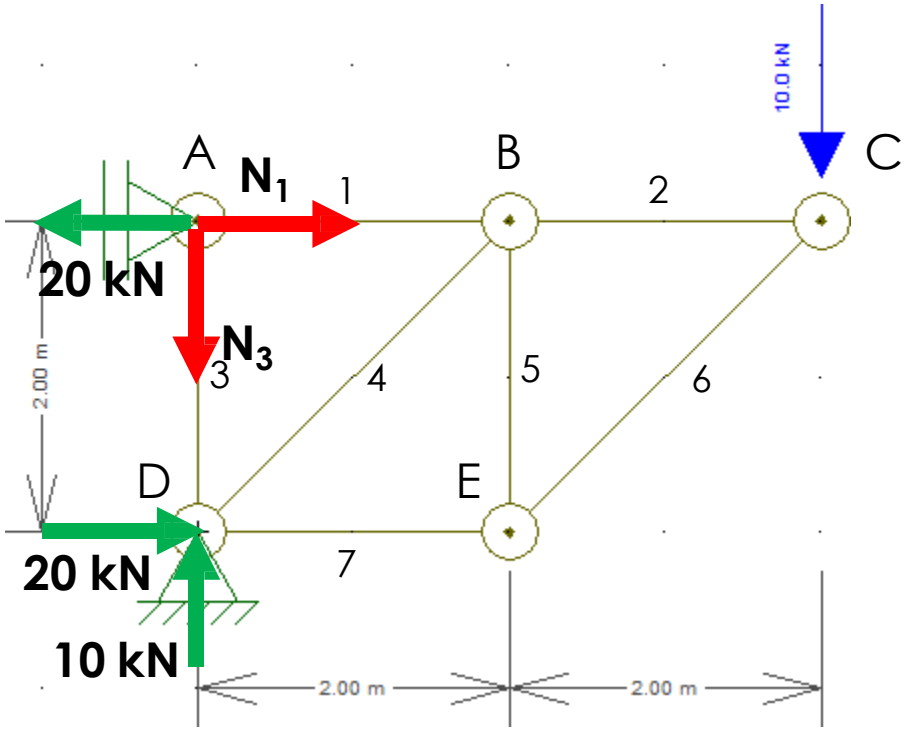
$X_A = -20$ kN

$\Sigma F_x = 0$

$X_A + X_D = 0$

$- 20 + X_D = 0$

$X_D = 20$ kN



Equilíbrio do nó A

$\Sigma F_x = 0$

$N_1 - 20 = 0$

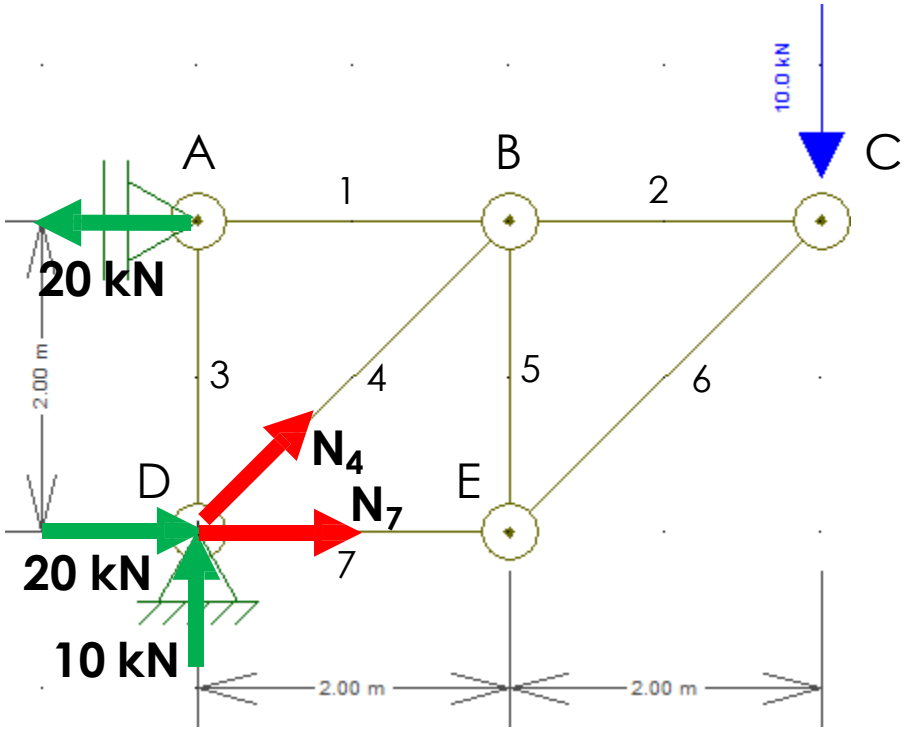
$N_1 = 20$ kN

$\Sigma F_y = 0$

$- N_3 = 0$

$N_3 = 0$

GRINTER



Equilíbrio do nó D

$\Sigma F_y = 0$

$N_4 * sen45 + 10 = 0$

$N_4 = -14{,}1$ kN

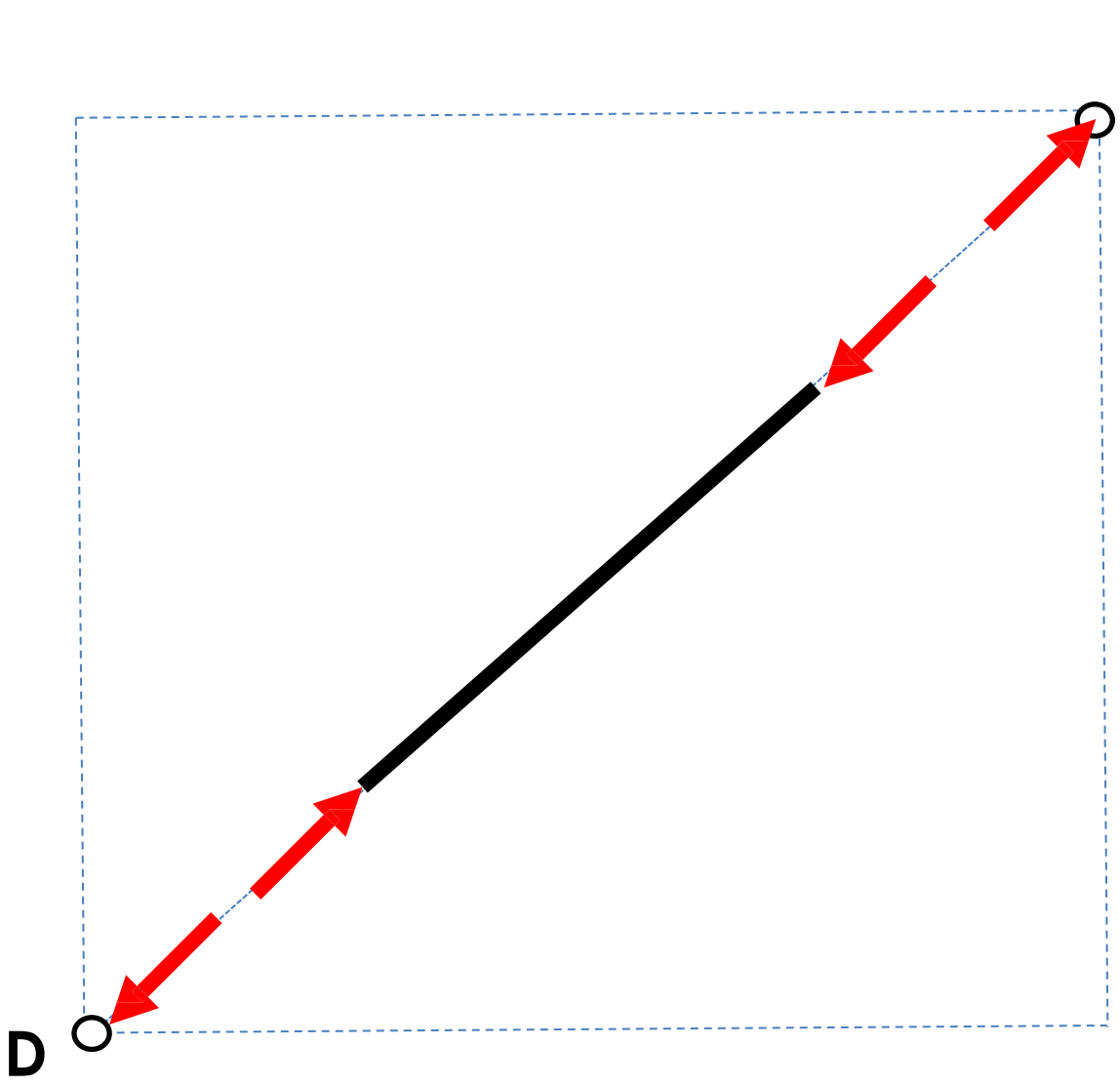
$\Sigma F_x = 0$

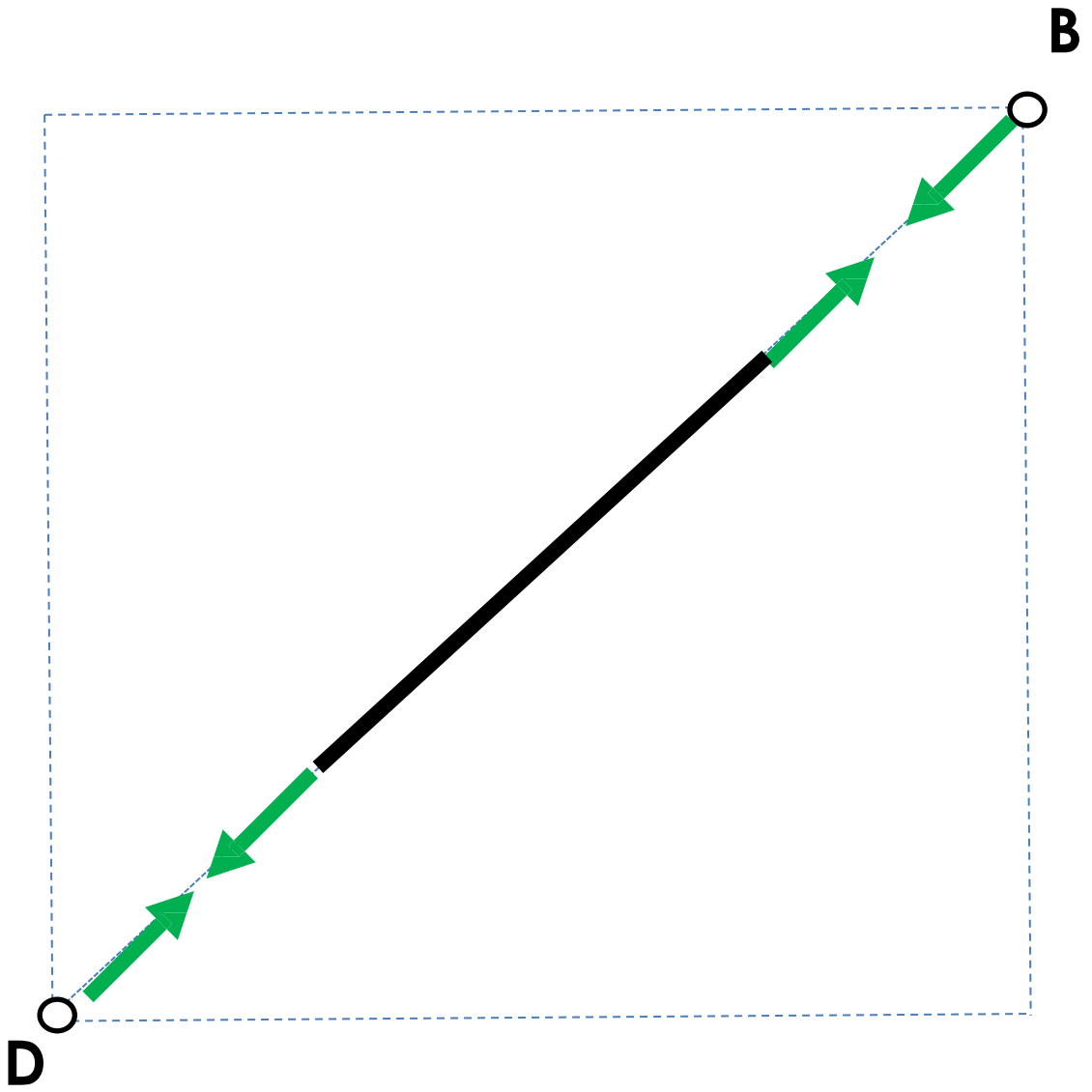
$N_7 + 20 + N_4 * cos45 = 0$

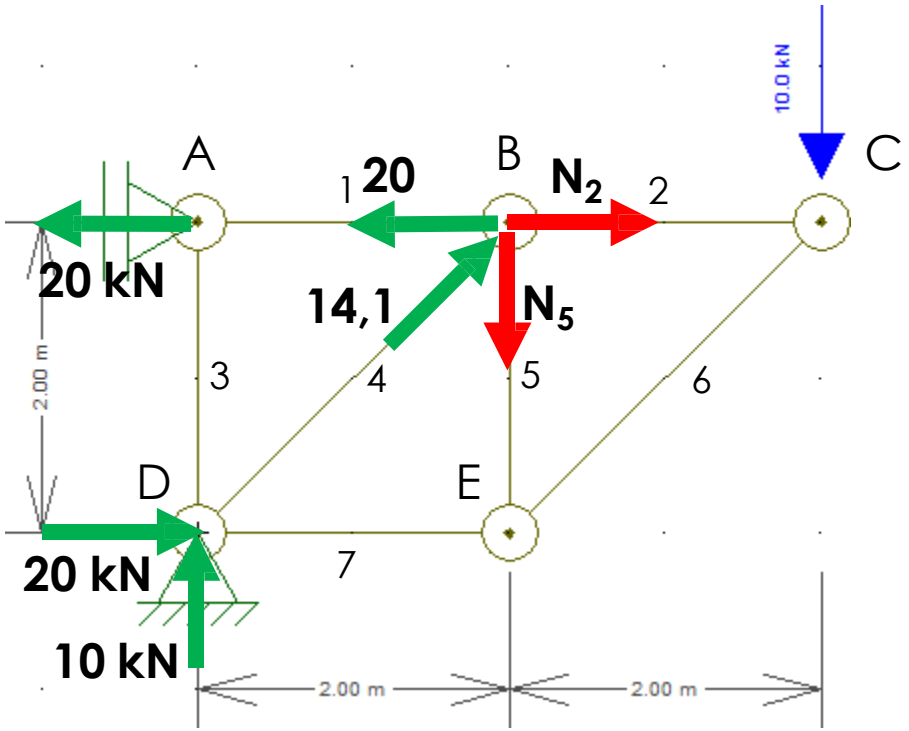
$N_7 + 20 - 10 = 0$

$N_7 = -10$ kN

GRINTER


B
D


B
D



Equilíbrio do nó B

$\Sigma F_y = 0$

$- N_5 + 14,1 * sen45 = 0$

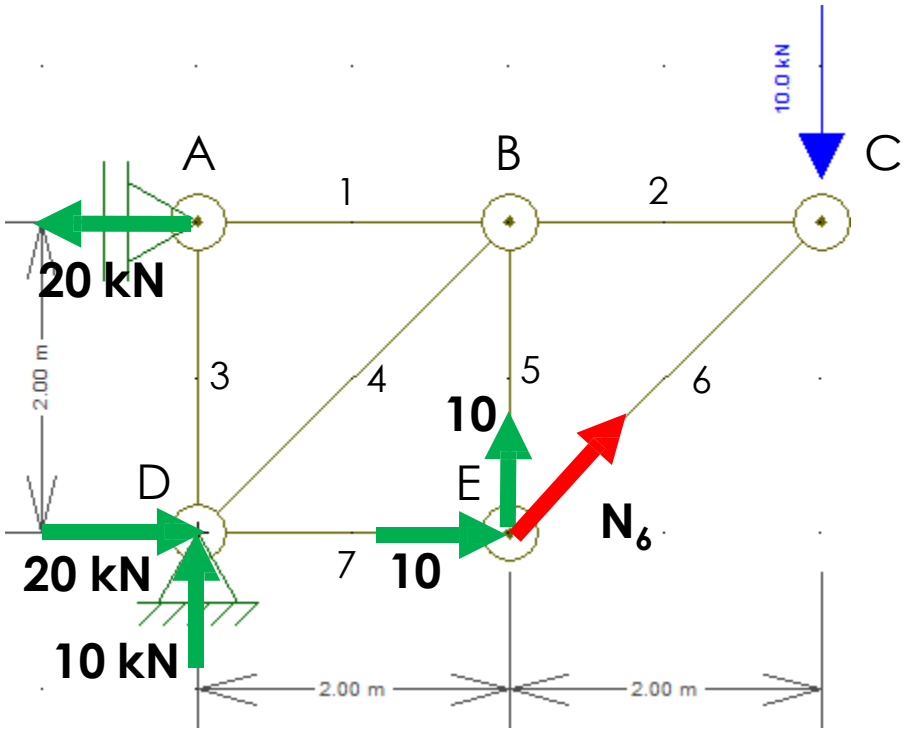
$N_5 = 10$ kN

$\Sigma F_x = 0$

$N_2 - 20 + 14,1 * cos45 = 0$

$N_2 - 10 = 0$

$N_2 = 10$ kN



Equilíbrio do nó E

$\Sigma F_y = 0$

$N_6 * \text{sen}45 + 10 = 0$

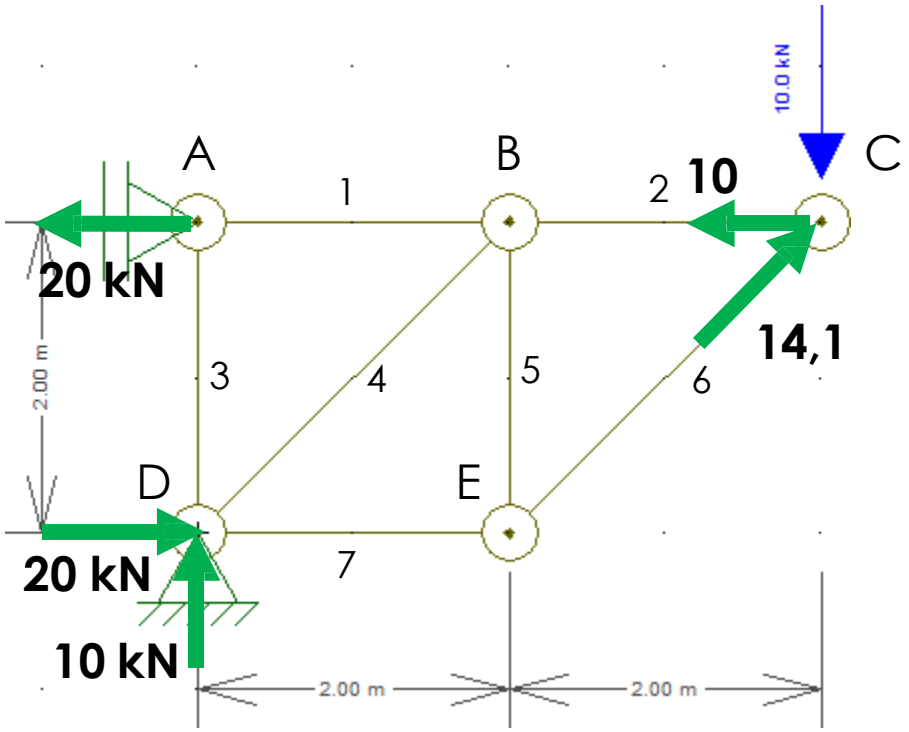
$N_6 = -14{,}1$ kN

Verificação 1

$\Sigma F_x = 0$

$N_6 * \cos 45 + 10 = 0$

$-10 + 10 = 0$ **OK**



Equilíbrio do nó C

Verificação 2

$\Sigma F_y = 0$

14,1 * sen45 - 10 = 0

10 – 10 = 0 **OK**

Verificação 3

$\Sigma F_x = 0$

14,1 * cos45 - 10 = 0

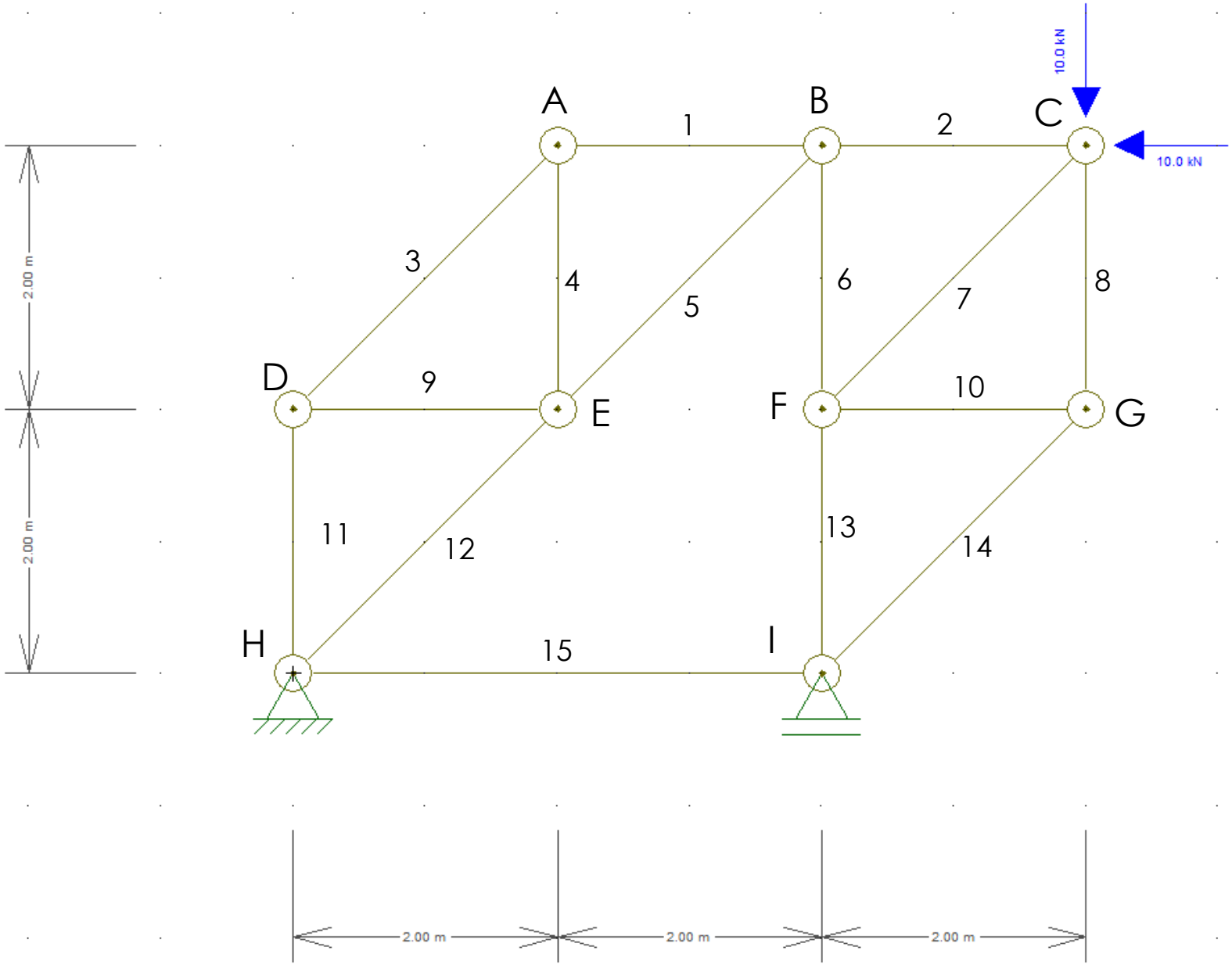
10 - 10 = 0 **OK**

MÉTODO DE RITTER

- Vamos considerar como incógnitas os esforços normais nas barras
- Se a estrutura está em equilíbrio, então qualquer parte desta estrutura, separada por um corte imaginário, também está em equilíbrio
- Para uma parte da estrutura que contenha pelo menos dois nós, as três equações de equilíbrio no plano podem ser aplicadas
- Em treliças simples ou compostas, é comum encontrar uma linha de corte ("corte de Ritter") que explicite três incógnitas, que podem ser obtidas através do equacionamento do equilíbrio de uma das partes da treliça, destacada pelo corte
- É comum mesclar o Método de Ritter com o Método do Equilíbrio dos Nós, explicitando as incógnitas de forma conveniente
- Procedimento:
  - Cálculo das reações de apoio, utilizando as três equações de equilíbrio, considerando a treliça como corpo rígido
  - Corte da treliça em duas partes contendo pelo menos dois nós cada uma, com uma linha de corte que atravesse três barras
  - Cálculo dos esforços nas três barras onde houve o corte, através do equacionamento do equilíbrio de uma das partes cortadas
  - Em seguida, pelo Método dos Nós, cálculo sucessivo dos esforços das barras, através do equilíbrio dos nós em que houver apenas duas incógnitas
  - No final da resolução, surgem três equações de verificação

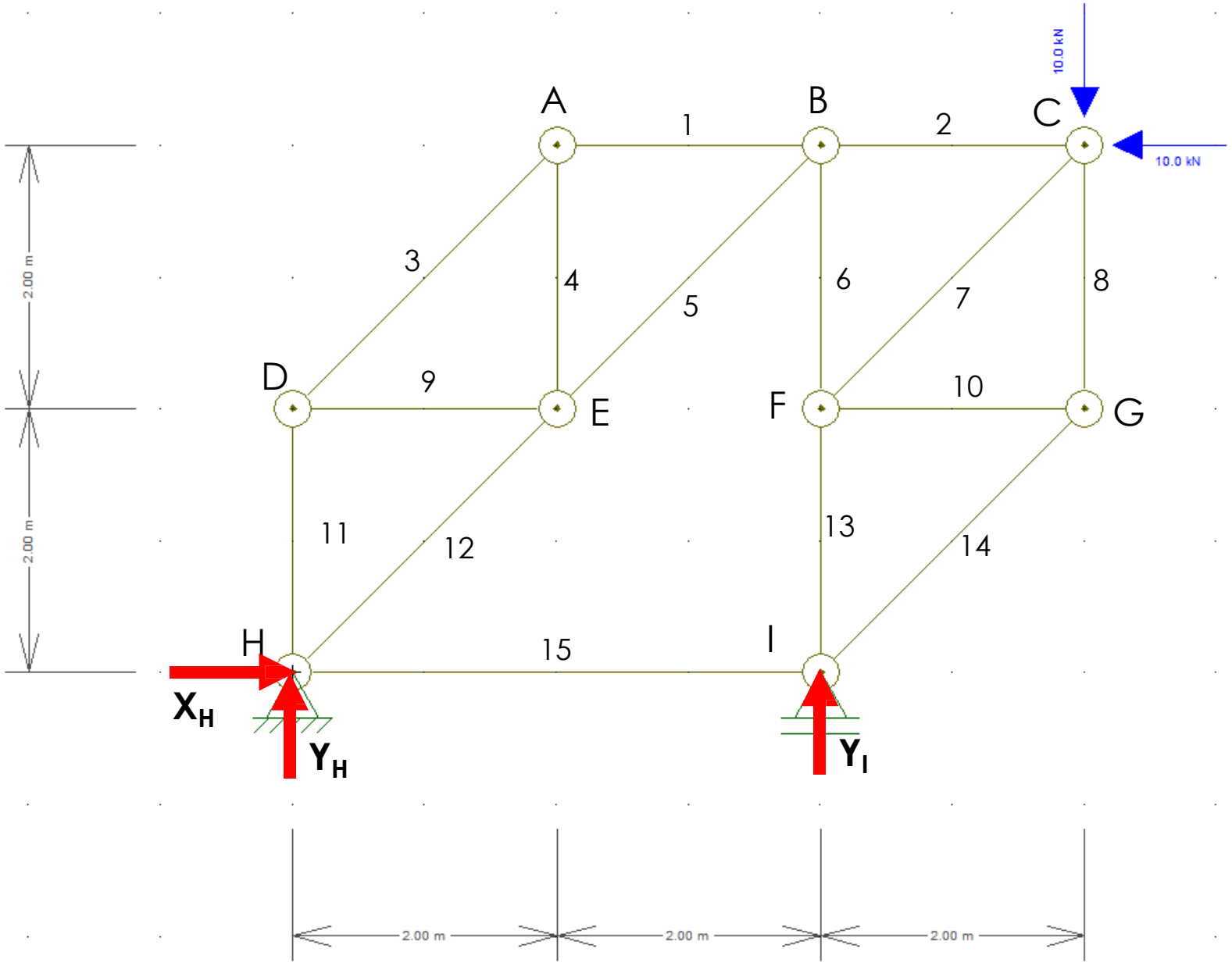
Para partes da estrutura:

$$\Sigma F_x = 0$$
$$\Sigma F_y = 0$$
$$\Sigma M_O = 0$$



Exemplo

Calcule os esforços nas barras 1, 5 e 15 da treliça ilustrada.



Reações de apoio

$\Sigma F_x = 0$

$X_H - 10 = 0$

$X_H = 10$ kN

$\Sigma M_H = 0$

$Y_I * 4 + 10 * 4 - 10 * 6 = 0$

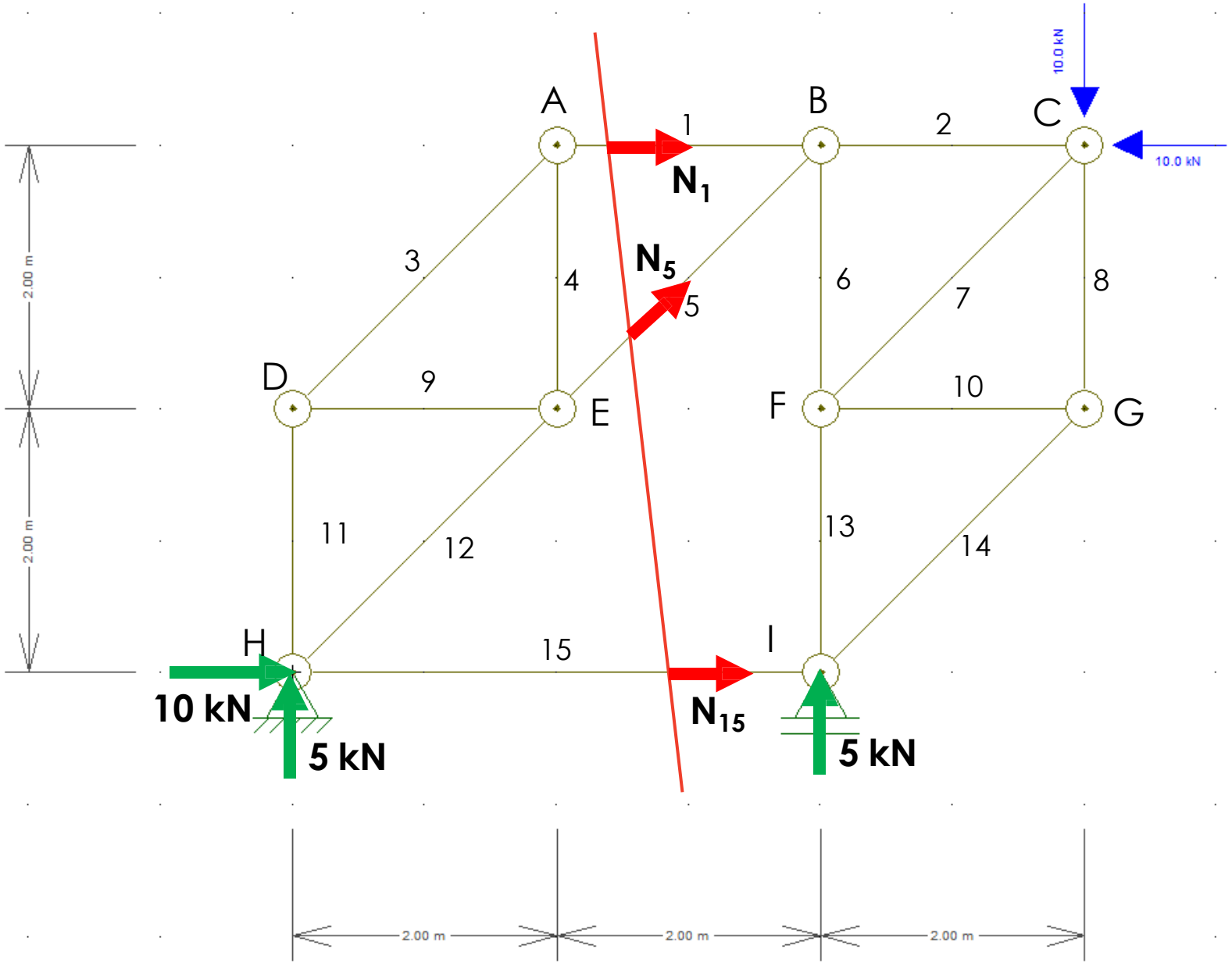
$Y_I = 5$ kN

$\Sigma F_y = 0$

$Y_H - 10 + Y_I = 0$

$Y_H - 10 + 5 = 0$

$Y_H = 5$ kN



Corte de Ritter

Equilíbrio da parte esquerda da treliça:

$\Sigma F_y = 0$

$N_5 * \text{sen}45 + 5 = 0$

$N_5 = -7{,}1$ kN

$\Sigma M_H = 0$

$-N_1 * 4 = 0$

$N_1 = 0$

GRINTER

$\Sigma F_x = 0$

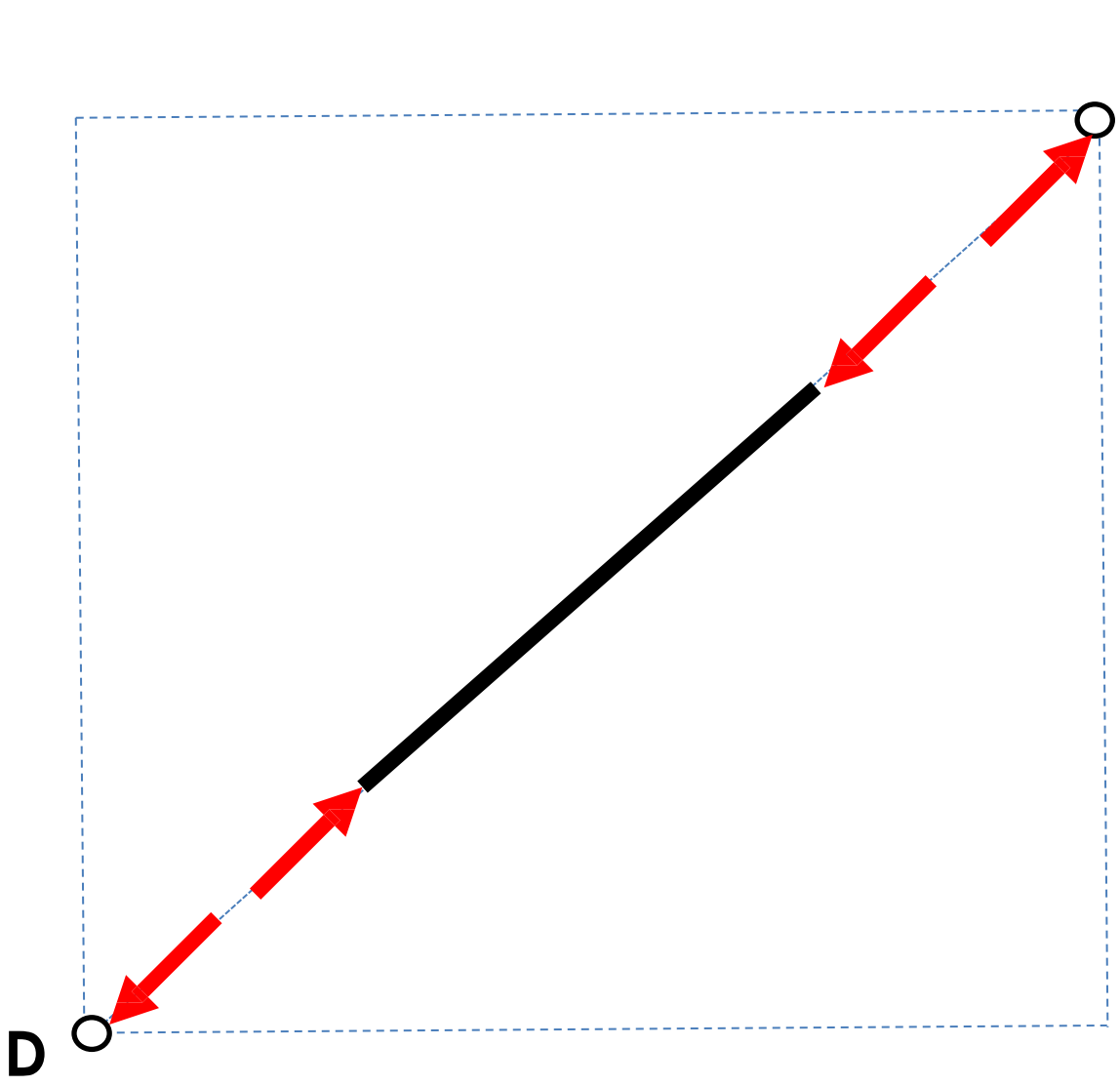
$N_1 + N_5 * \cos 45 + N_{15} + 10 = 0$

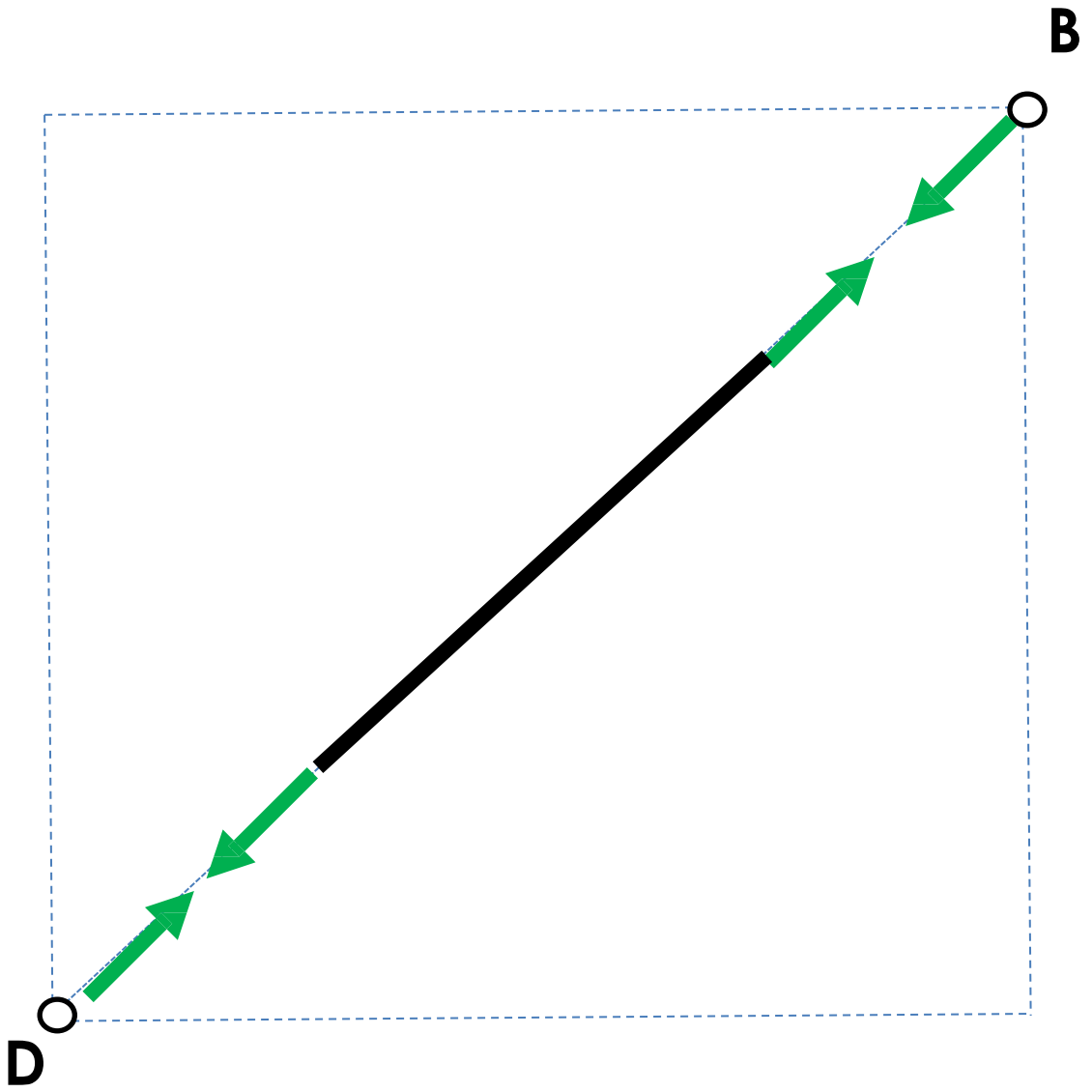
$0 - 7{,}1 * \cos 45 + N_{15} + 10 = 0$

$N_{15} = -5$ kN

REPRESENTAÇÃO DE RESULTADOS

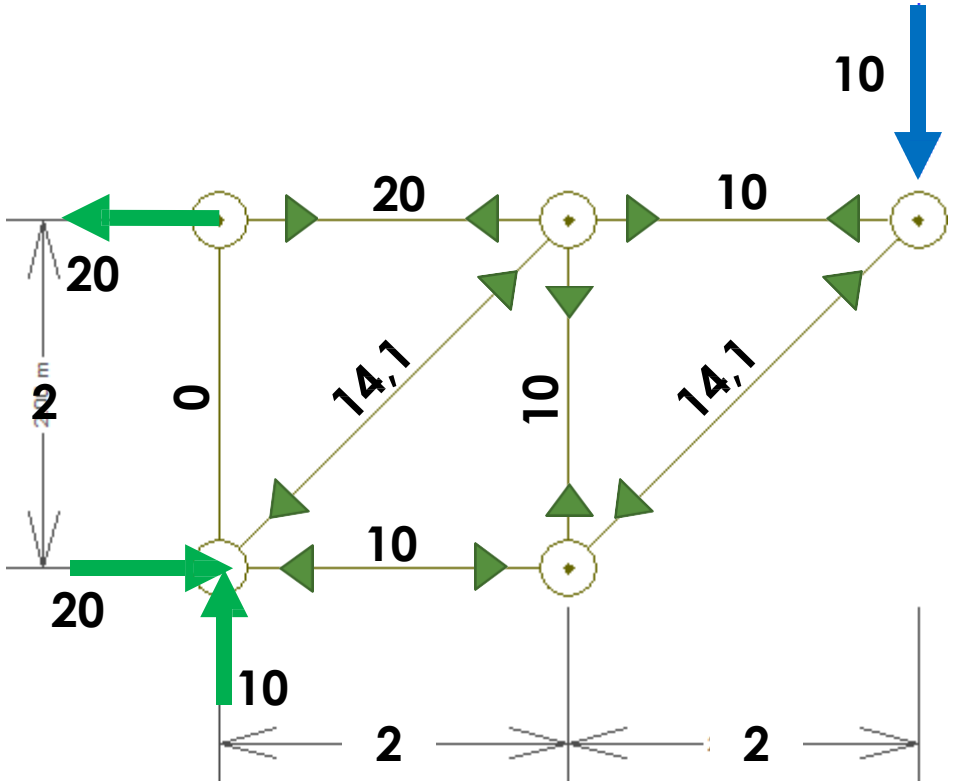
- Existem diversas formas de representar os resultados da resolução de treliças, gráficas ou tabeladas
- Quando tabeladas, deve-se respeitar a convenção de sinal (N positivo = tração)
- Uma forma de representação que auxilia no entendimento intuitivo, na verificação visual e até na resolução, é a que segue:
  - Desenhe a treliça respeitando as proporções, como corpo livre, indicando apenas seus nós, barras e cotas
  - Indique ao lado da figura a unidade de força que será utilizada
  - Indique as forças externas ativas e reativas nos nós onde ocorrerem, com seu sentido físico e valor
  - Nas duas extremidades de cada barra, junto aos nós, indique pontas de setas que mostrem o sentido físico da força aplicada no nó (atenção: é comum ocorrer confusão neste ponto, pois o esforço no nó é inverso ao esforço na barra)
  - Junto à barra, indique o valor que corresponde à força normal com que está solicitada
  - Desta forma, é rápido e intuitivo ver e verificar o equilíbrio nó por nó
- Em casos simples, é possível acelerar a resolução do equilíbrio dos nós de forma segura através desta representação gráfica, com contas rápidas, apoiadas pelo entendimento físico dos sentidos e projeções das forças


B
D


B
D

Exemplo

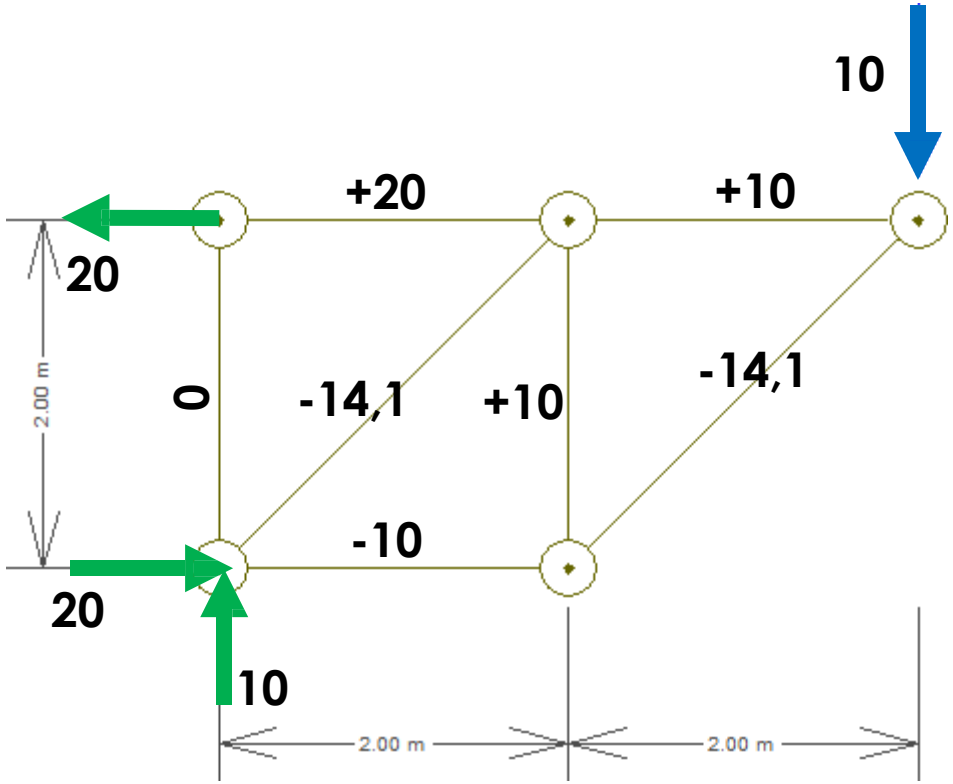
Representação dos resultados do exemplo do Método do Equilíbrio dos Nós.



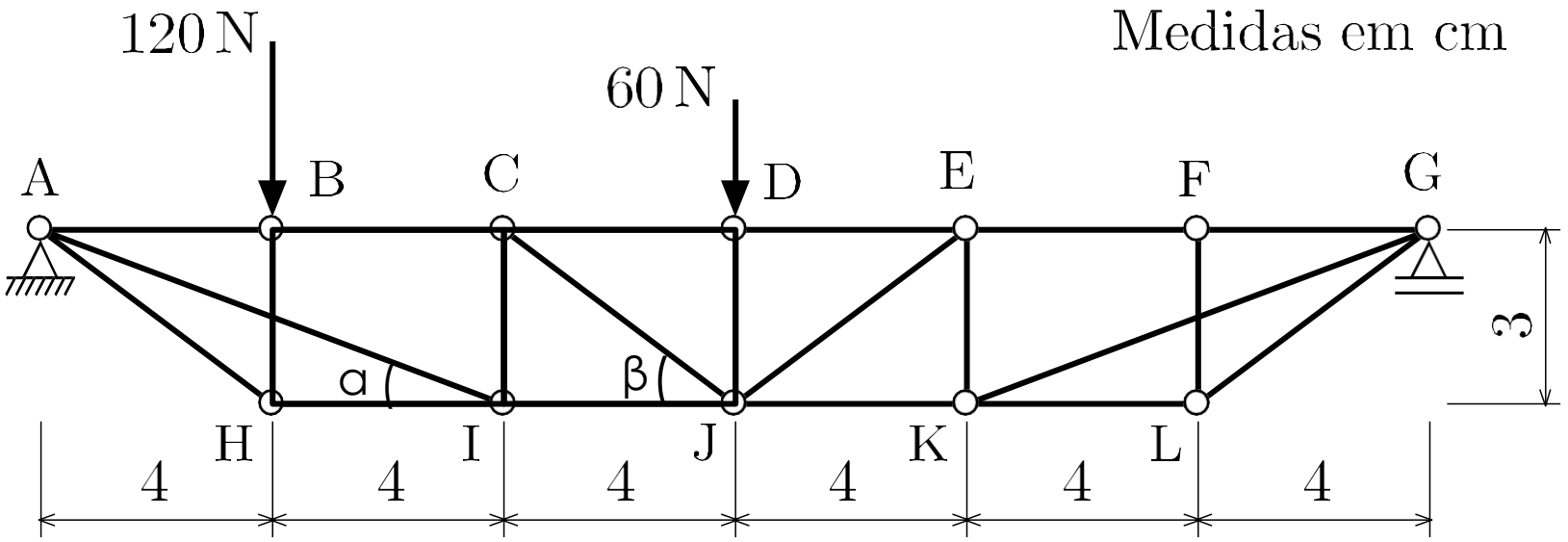
(kN, m)

Exemplo

Representação dos resultados do exemplo do Método do Equilíbrio dos Nós.



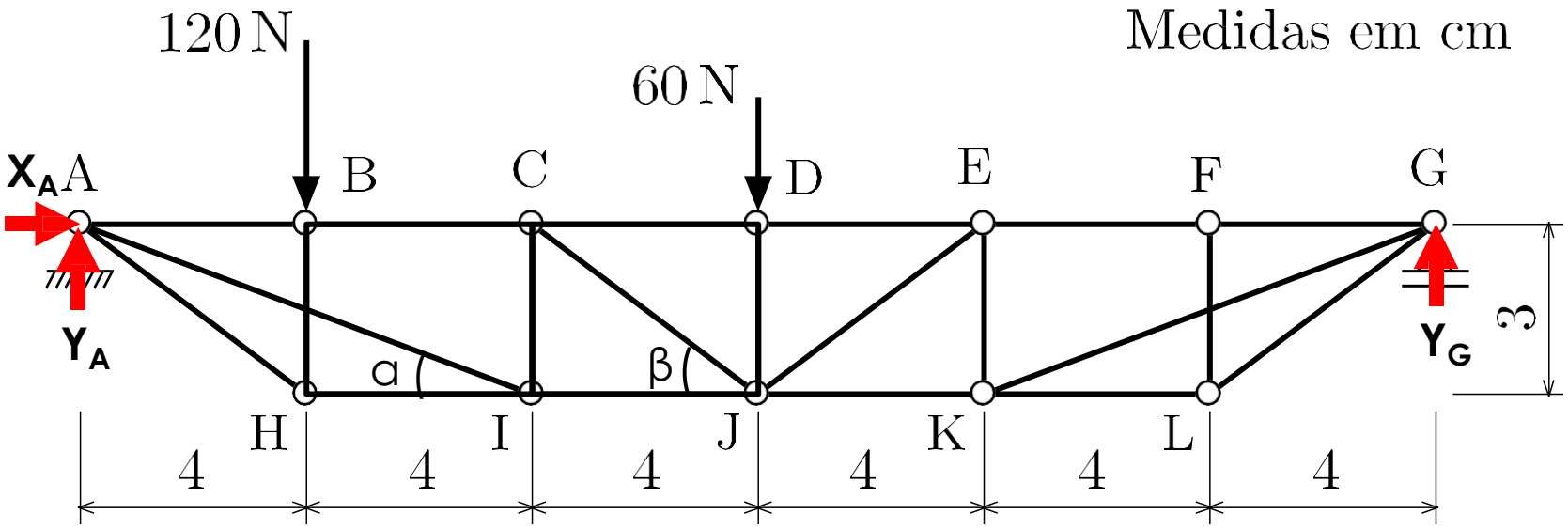
(kN)



Exemplo (P2 2019)

3ª Questão (3,0)

Na treliça plana da figura, o carregamento é formado por duas forças verticais aplicadas em B e D. Adotando sena = 0,4; cosa = 0,9; senβ = 0,6; cosβ = 0,8, determine a reação no apoio A e as forças normais nas barras AI e HI.



Reações em A

$\Sigma F_x = 0$

$X_A = 0$

$\Sigma M_G = 0$

$- Y_A * 24 + 120 * 20 + 60 * 12 = 0$

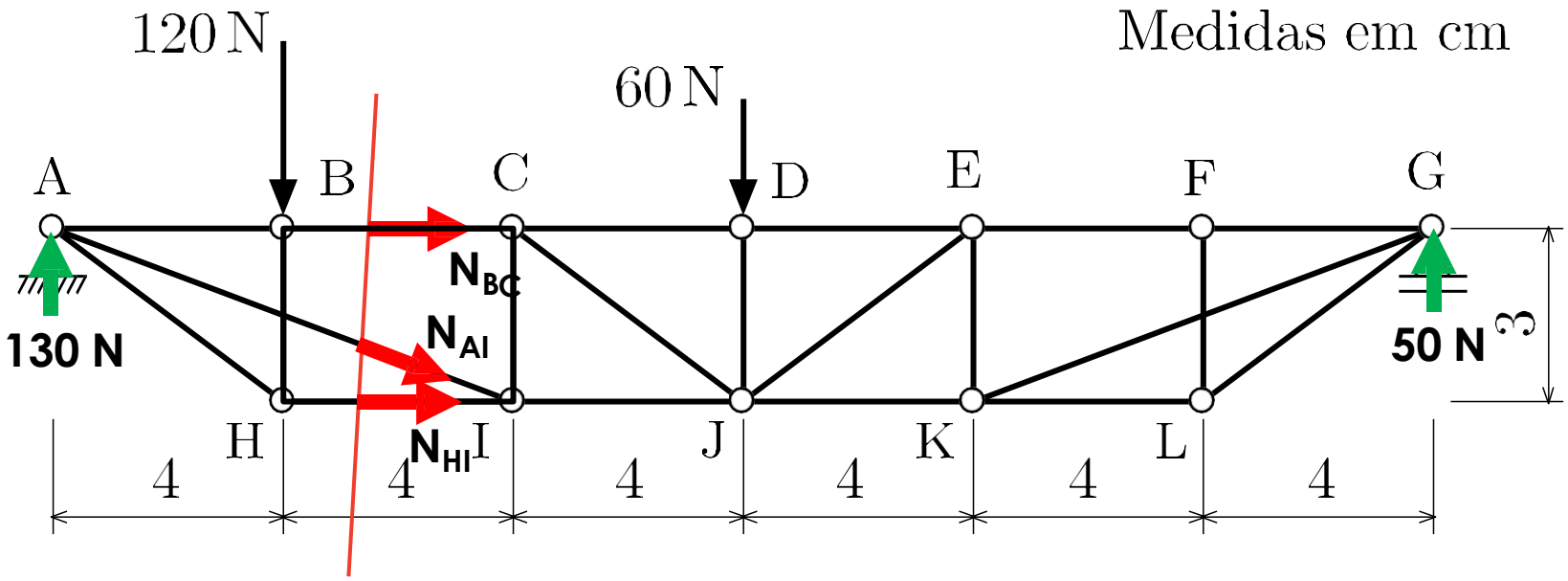
$Y_A = 130\,N$

$\Sigma F_Y = 0$

$Y_G + Y_A - 120 - 60 = 0$

$Y_G + 130 - 120 - 60 = 0$

$Y_G = 50\,N$



Corte de Ritter

Equilíbrio da parte esquerda da treliça:

$\Sigma F_y = 0$

$130 - 120 - N_{AI} * \text{sen}a = 0$

$10 - N_{AI} * 0{,}4 = 0$

$N_{AI} = 25$ N

$\Sigma M_A = 0$

$N_{HI} * 3 - 120 * 4 = 0$

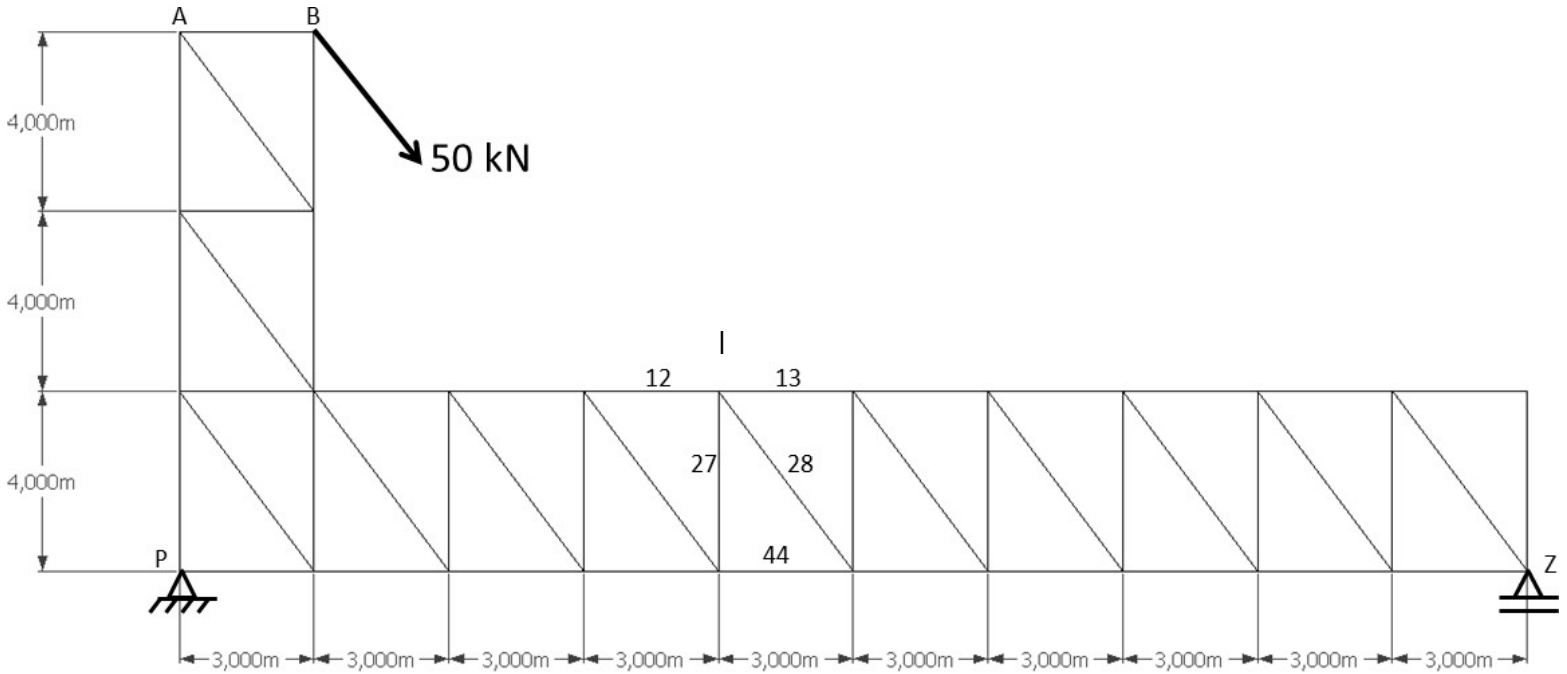
$N_{HI} = 160$ N

$\Sigma F_x = 0$

$N_{BC} + N_{AI} * \cos a + N_{HI} = 0$

$N_{BC} + 25 * 0{,}9 + 160 = 0$

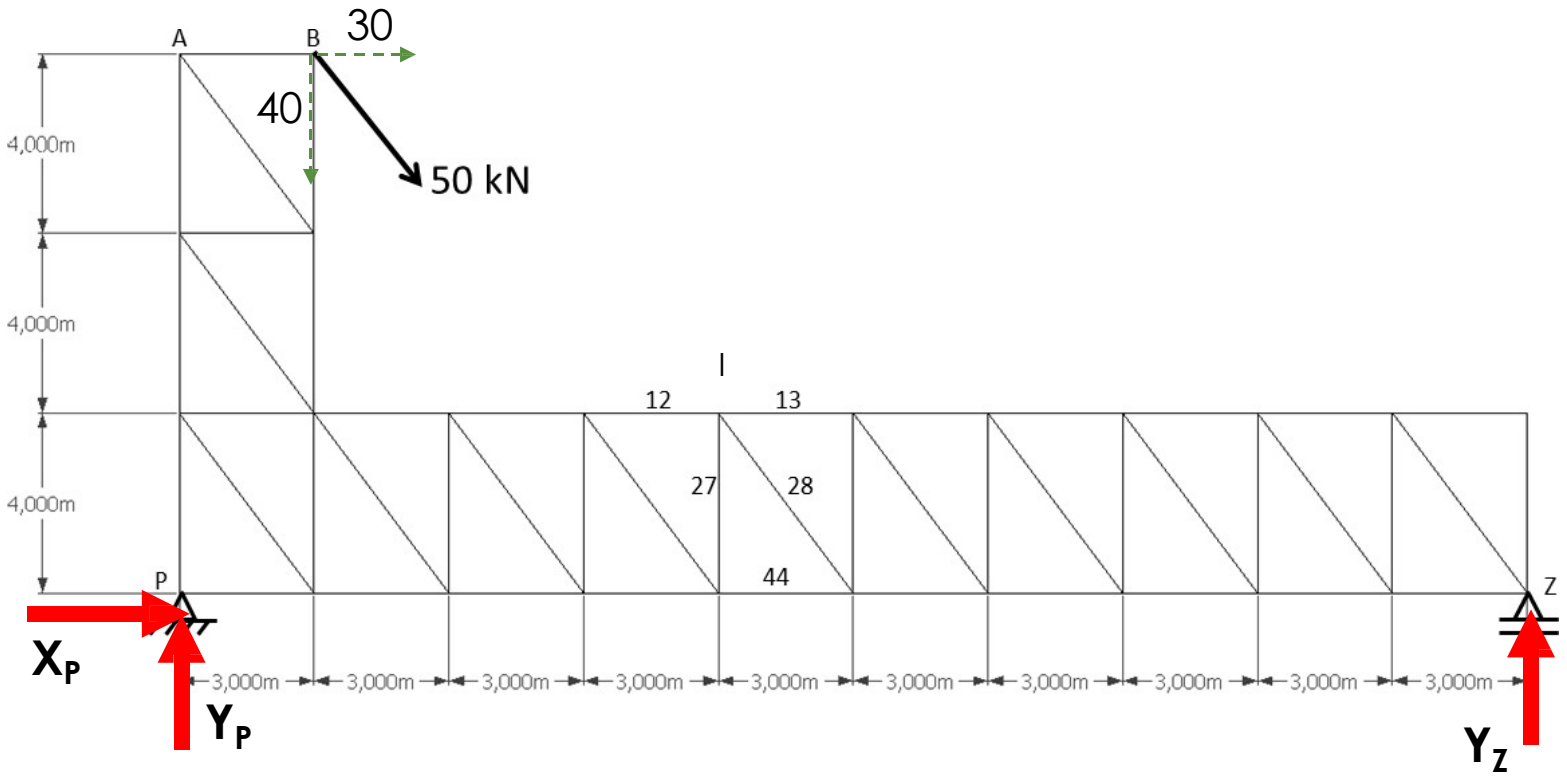
$N_{BC} = -182{,}5$ kN



Exemplo (P2 2011)

2ª Questão (3,0 pontos)

Para a treliça da figura, calcule as forças normais nas barras 12, 13, 27, 28 e 44. A força de 50kN ilustrada está aplicada no ponto B e é paralela à barra 28.



Reações de apoio

$\Sigma M_P = 0$

$Y_Z * 30 - 30 * 12 - 40 * 3 = 0$

$Y_Z = 16$ kN

$\Sigma F_y = 0$

$Y_P - 40 + Y_Z = 0$

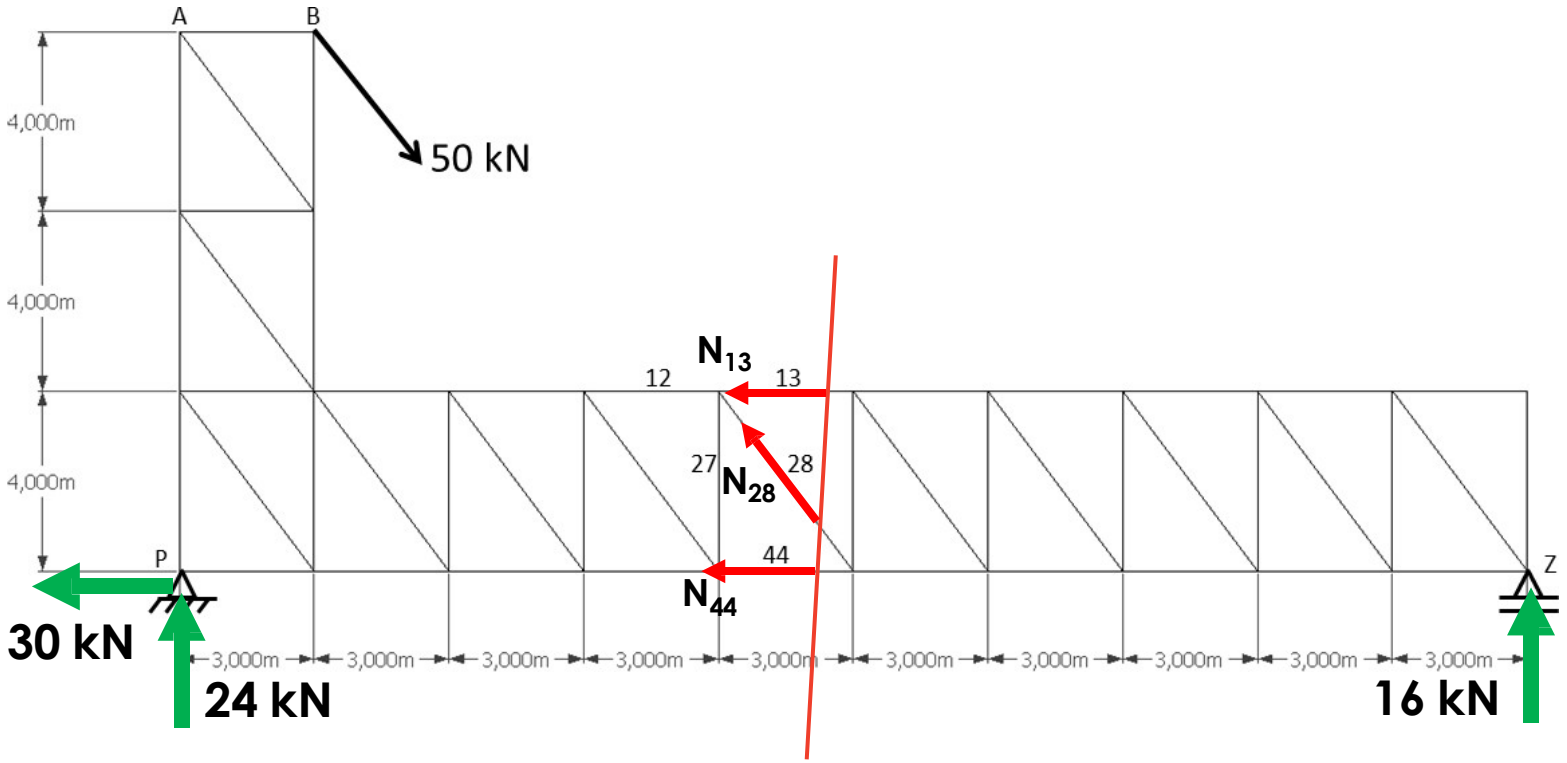
$Y_P - 40 + 16 = 0$

$Y_P = 24$ kN

$\Sigma F_x = 0$

$X_P + 30 = 0$

$X_P = -30$ kN



Corte de Ritter

Equilíbrio da parte direita da treliça:

$\Sigma F_y = 0$

$N_{28} * \text{sena} + 16 = 0$

$N_{28} * 4/5 + 16 = 0$

$N_{28} = -20$ kN

$\Sigma M_I = 0$

$- N_{44} * 4 + 16 * 18 = 0$

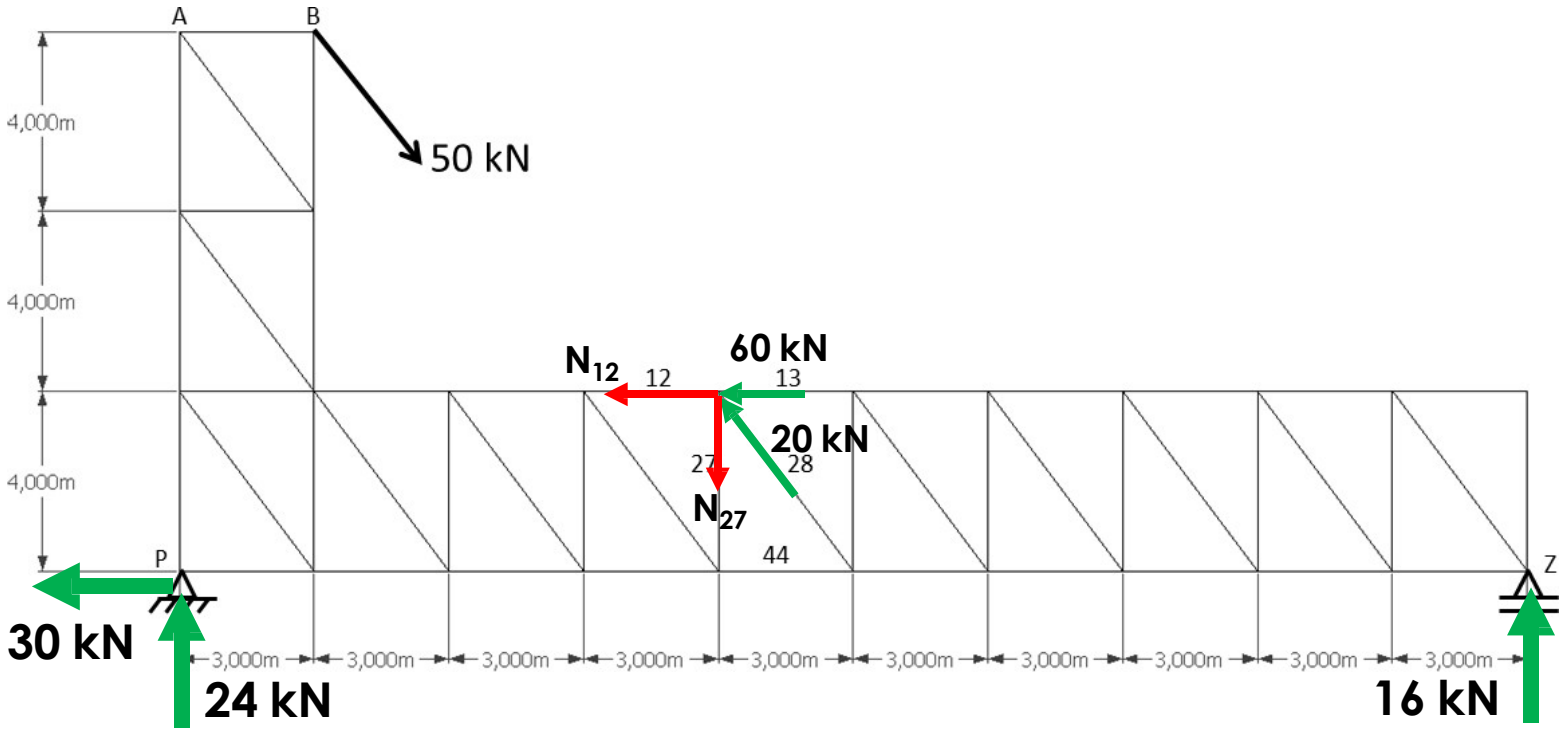
$N_{44} = 72$ kN

$\Sigma F_x = 0$

$- N_{13} - N_{28} * \text{cosa} - N_{44} = 0$

$- N_{13} + 20 * 3/5 - 72 = 0$

$N_{13} = -60$ kN



Equilíbrio do nó I

$\Sigma F_y = 0$

$- N_{27} + 20 * \text{sena} = 0$

$- N_{27} + 20 * 4/5 = 0$

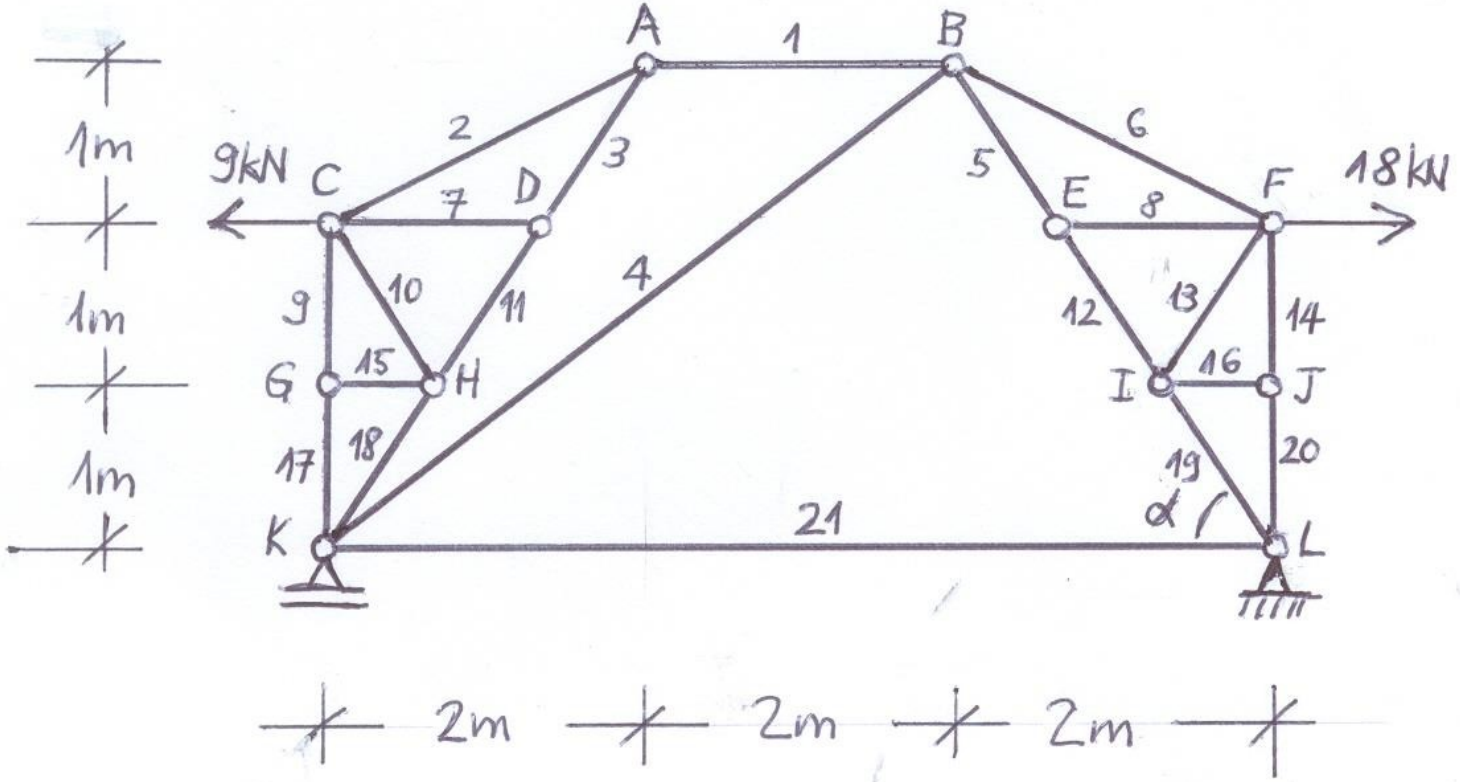
$N_{27} = 16$ kN

$\Sigma F_x = 0$

$- N_{12} - 20 * \text{cosa} - 60 = 0$

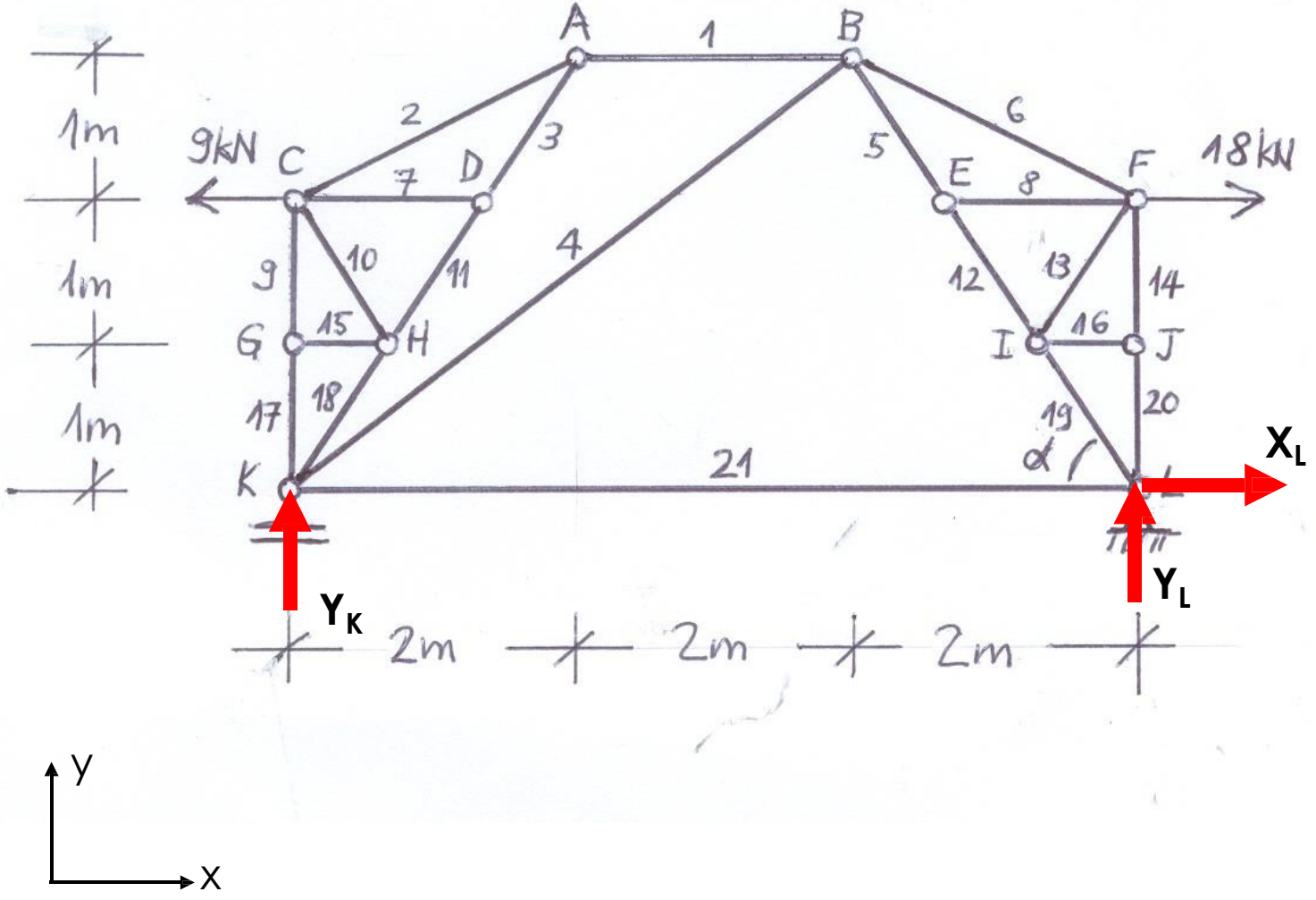
$- N_{12} - 20 * 3/5 - 60 = 0$

$N_{12} = -72$ kN



Exemplo (P2 2006)

Questão 3 (3,0) Calcule os esforços nas barras 1, 19, 20 e 21 da treliça ilustrada. Dados: a=56,31˚, cosa=0,555; sena=0,832



14/04/2020

Reações de apoio

$\Sigma F_x = 0$

$X_L + 18 - 9 = 0$

$X_L = -9$ kN

$\Sigma M_K = 0$

$Y_L * 6 + 9 * 2 - 18 * 2 = 0$

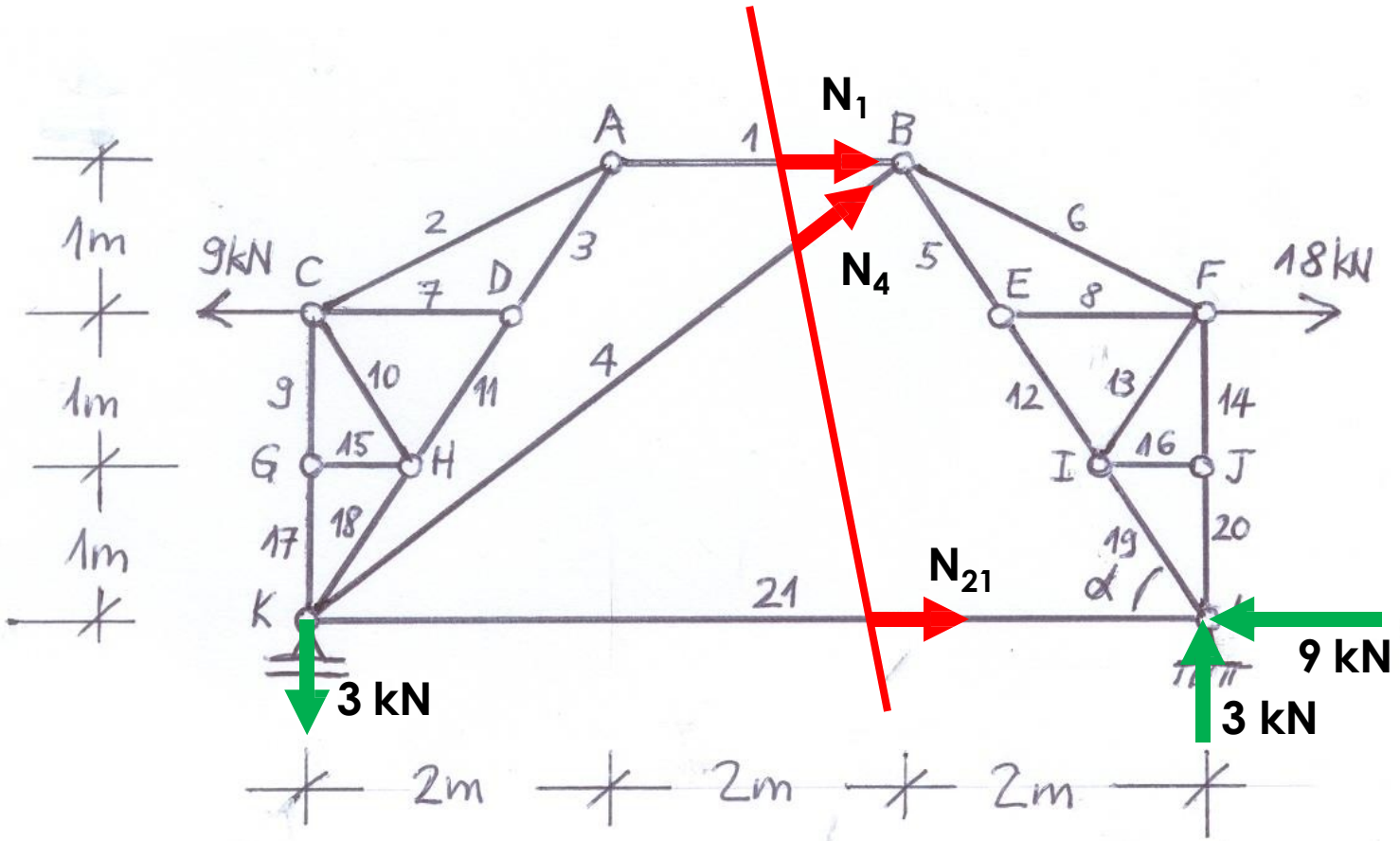
$Y_L = 3$ kN

$\Sigma F_y = 0$

$Y_K + Y_L = 0$

$Y_K + 3 = 0$

$Y_K = -3$ kN



Corte de Ritter

Equilíbrio da parte esquerda da treliça:

$\Sigma F_y = 0$

$N_4 * 3/5 - 3 = 0$

$N_4 = 5$ kN

$\Sigma M_K = 0$

$- N_1 * 3 + 9 * 2 = 0$

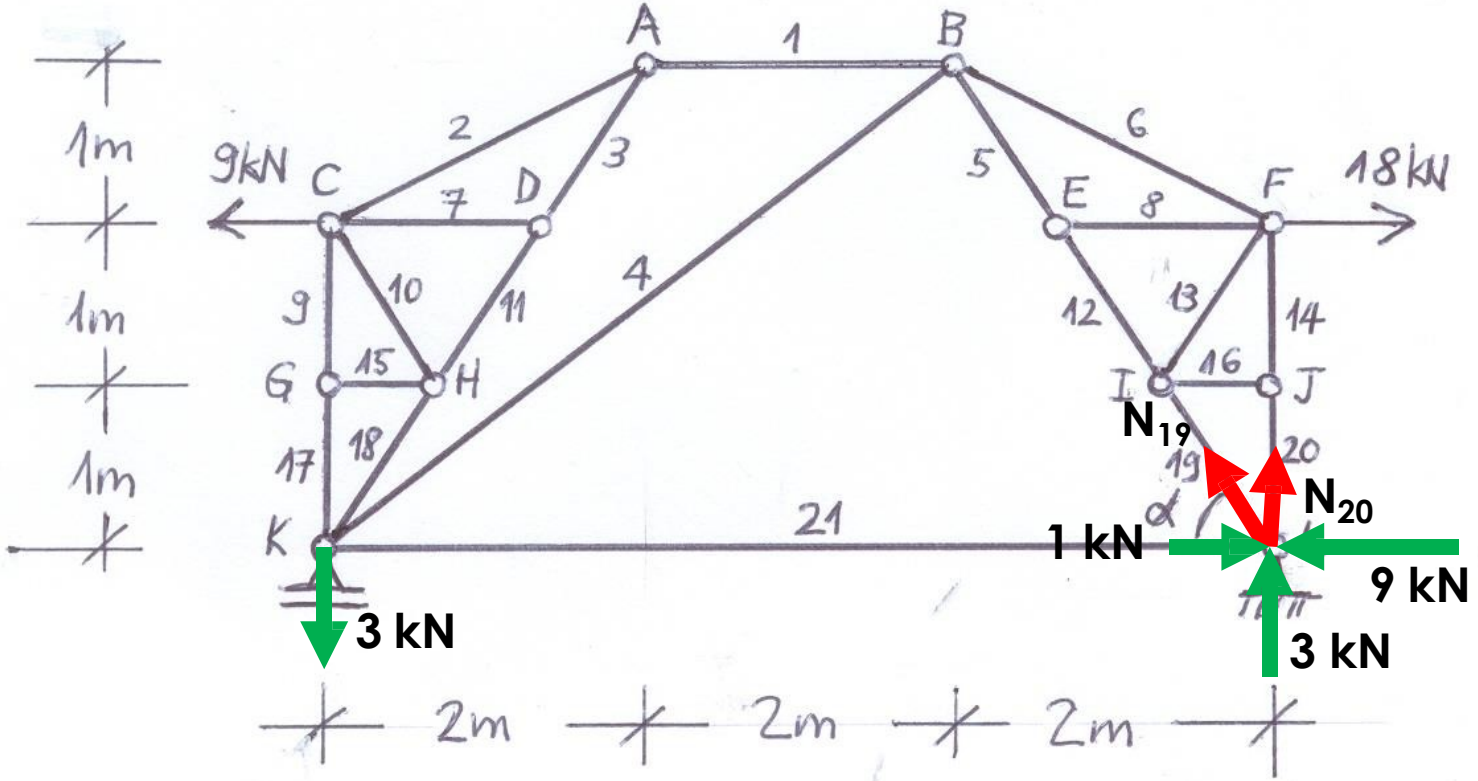
$N_1 = 6$ kN

$\Sigma F_X = 0$

$- 9 + N_1 + N_4 * 4/5 + N_{21} = 0$

$- 9 + 6 + 5 * 4/5 + N_{21} = 0$

$N_{21} = -1$ kN



Equilíbrio do nó L

$\Sigma F_x = 0$

$- N_{19} * \cos a + 1 - 9 = 0$

$- N_{19} * 0{,}555 - 8 = 0$

$N_{19} = -14{,}4$ kN

$\Sigma F_y = 0$

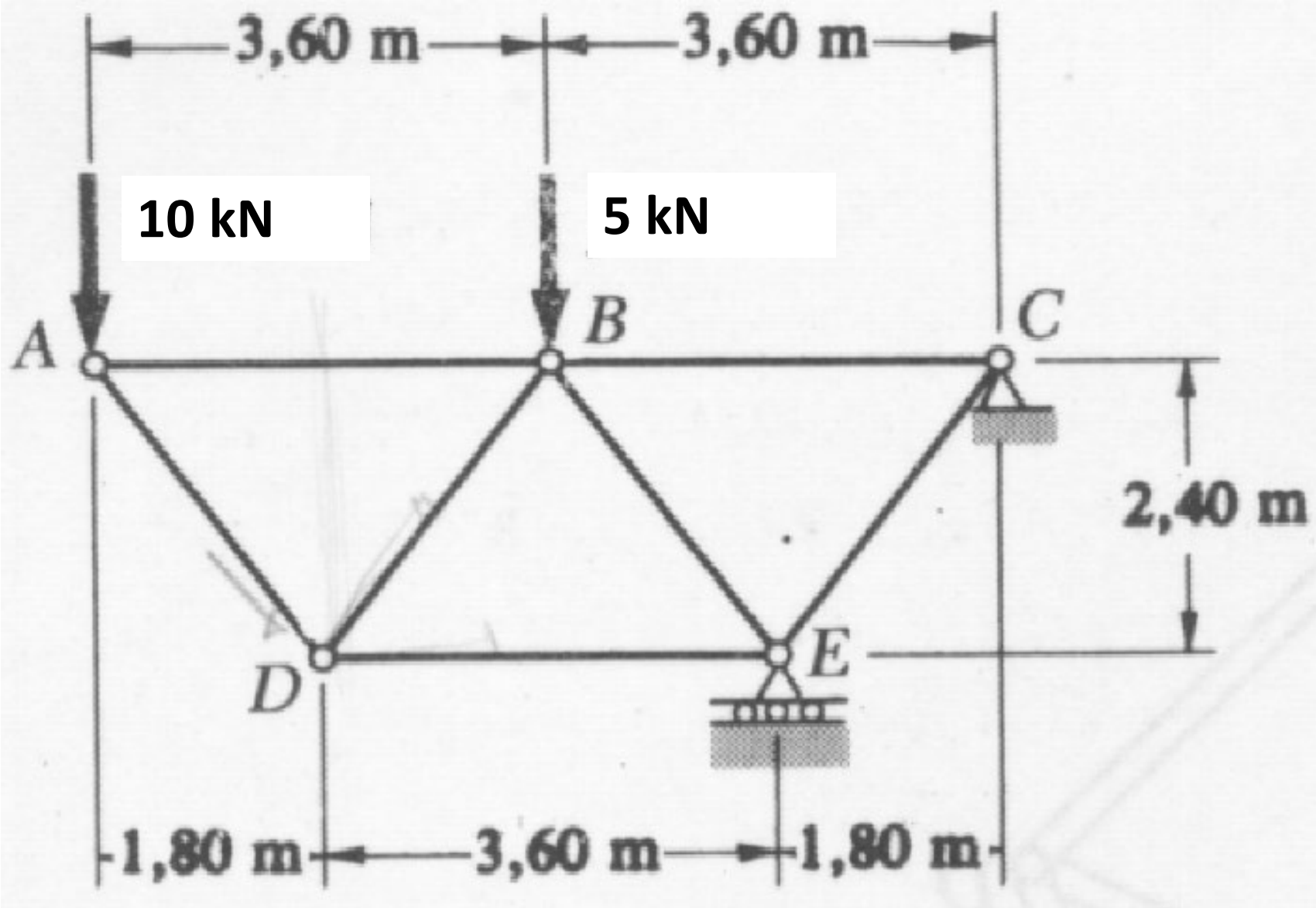
$3 + N_{19} * \text{sen}\, a + N_{20} = 0$

$3 - 14{,}4 * 0{,}832 + N_{20} = 0$

$N_{21} = 9$ kN

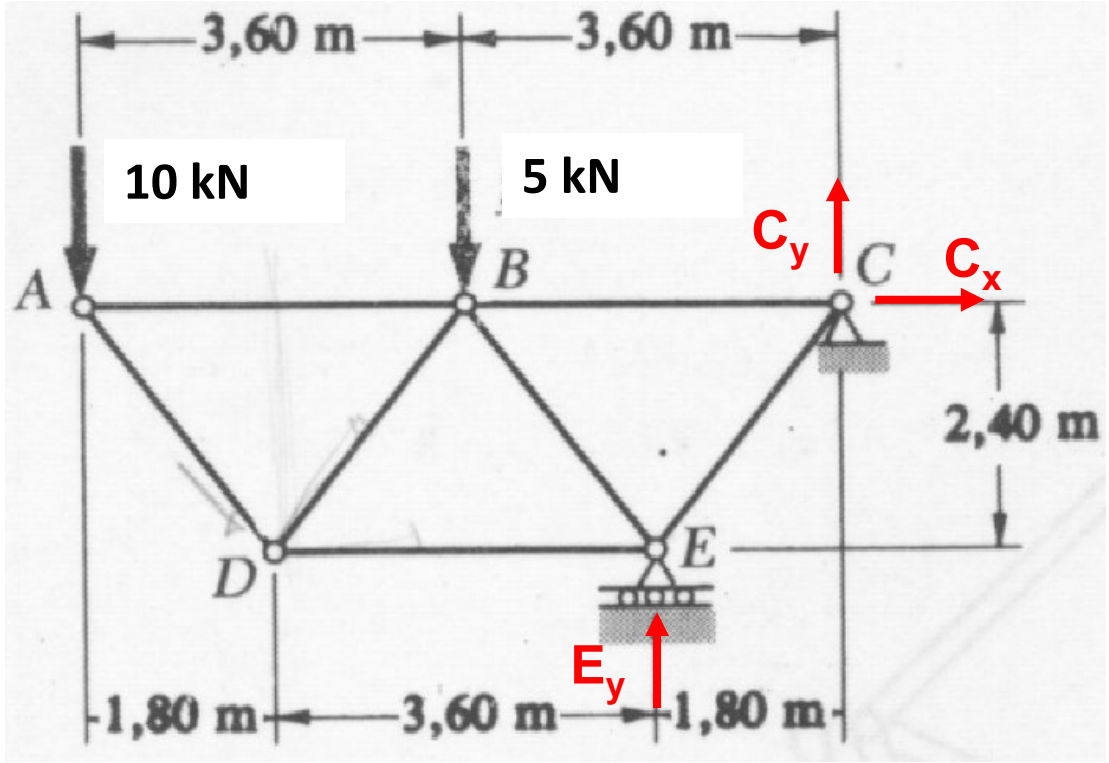
EXERCÍCIO PROF. NAKAO

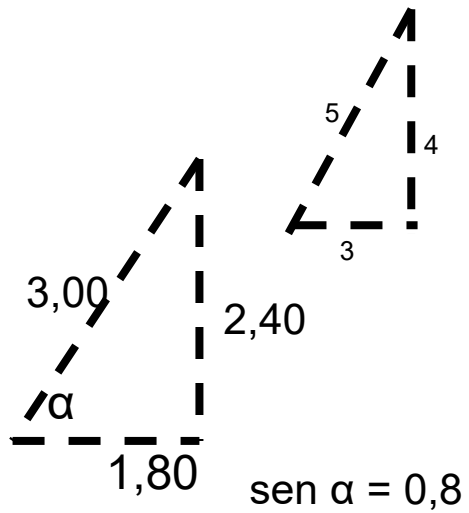
Determine os esforços normais nas barras da treliça:



1. Reações nos apoios

GRINTER





sen α = 0,8

cos α = 0,6

$\Sigma M_C = 0 = +10kN \cdot 7{,}20m + 5kN \cdot 3{,}60m - E_y \cdot 1{,}80m$

$\Longrightarrow E_y = +50\ kN$

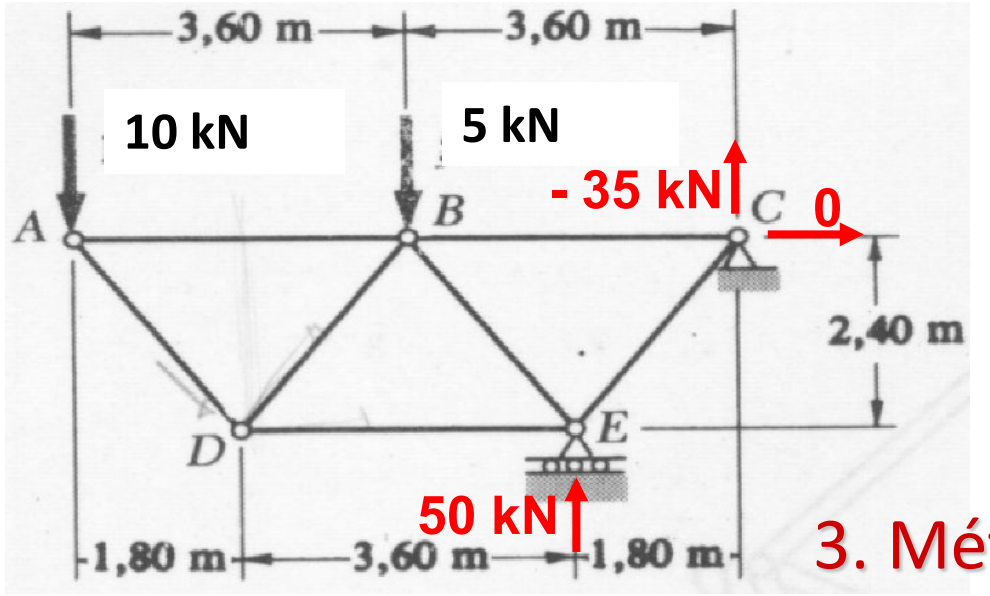
$\Sigma F_X = 0 = C_x \Longrightarrow C_x = 0$

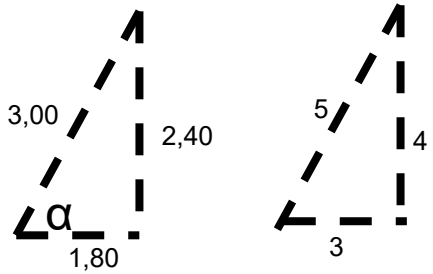
$\Sigma F_Y = 0 = -10kN - 5kN + C_y + E_y$

$\Longrightarrow C_Y = -35\ kN$

2. Diagrama de corpo livre

GRINTER





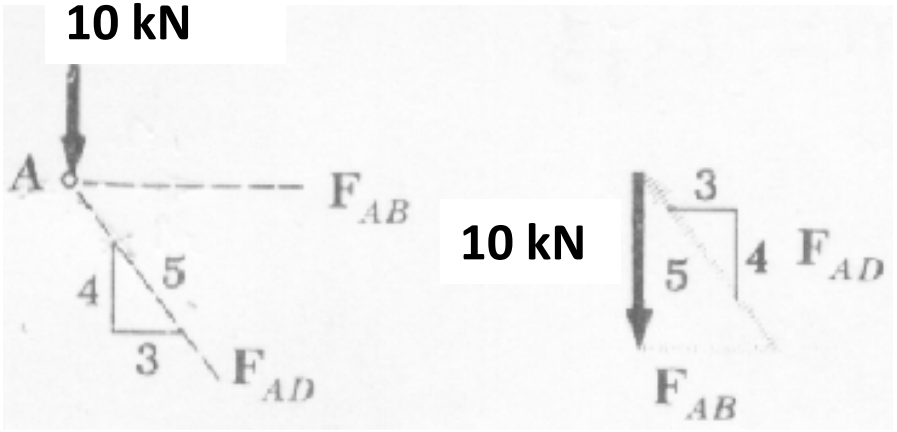
sen α = 0,8
cos α = 0,6

3. Método do equilíbrio dos nós

Nó A:

$$\Sigma F_y = 0 = -10kN - 0{,}8.F_{AD}$$

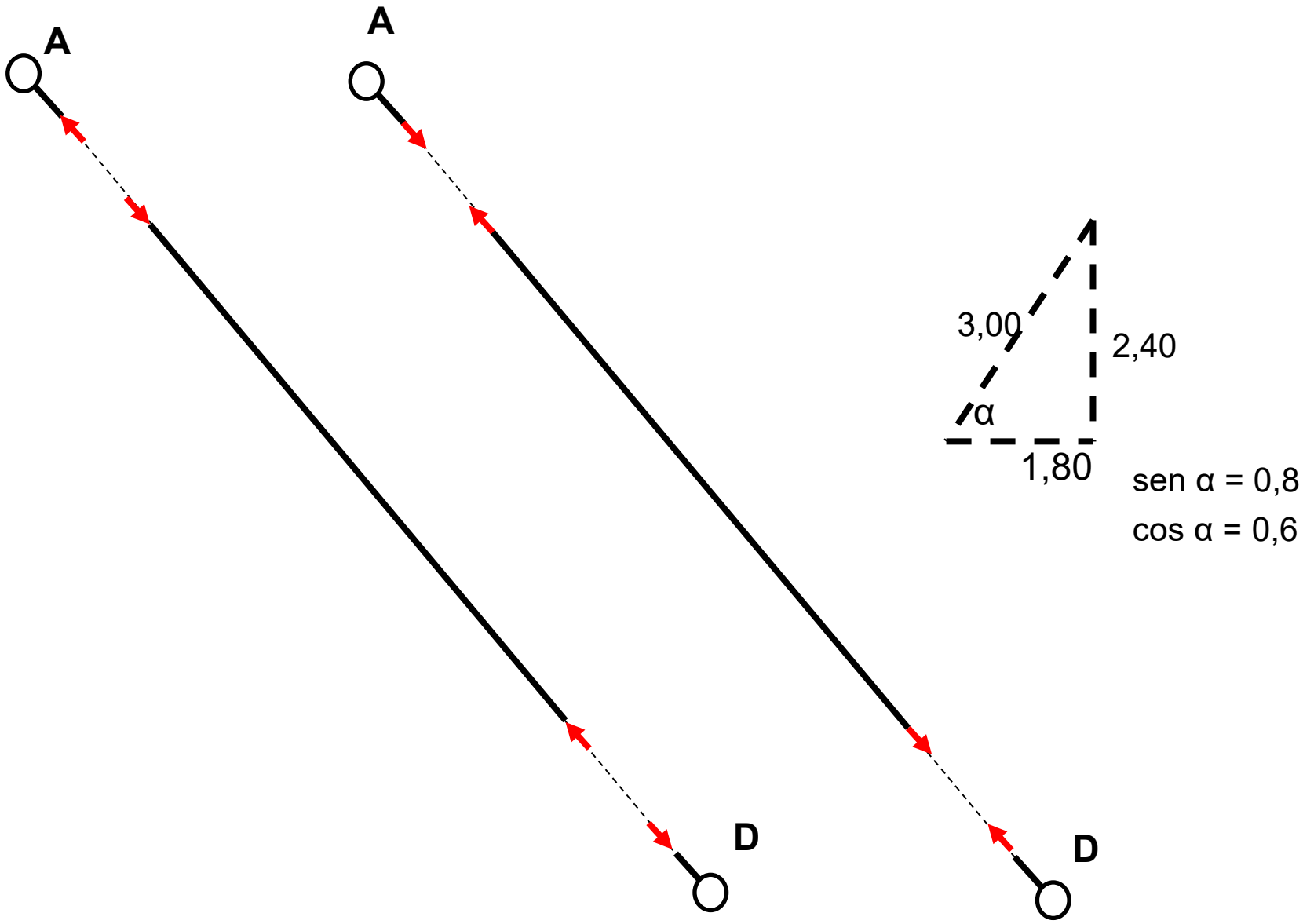
$$\Sigma F_x = 0 = F_{AD}.0{,}6 + F_{AB}$$



F_{AD} = -12,5 kN $\Longrightarrow F_{DA}$ =12,5 kN
(Compressão no nó e na barra)

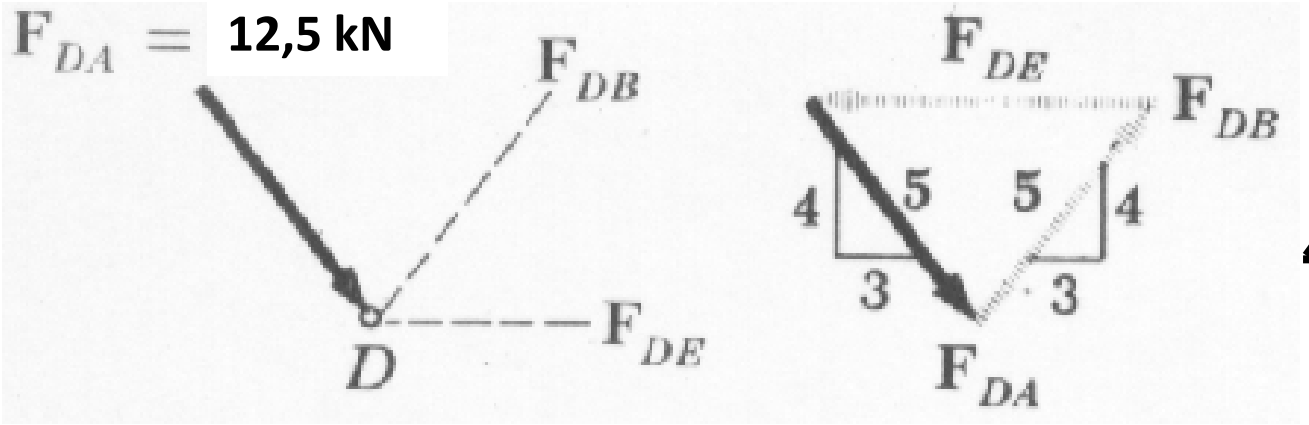
F_{AB} = +7,5 kN
(Tração no nó e na barra)

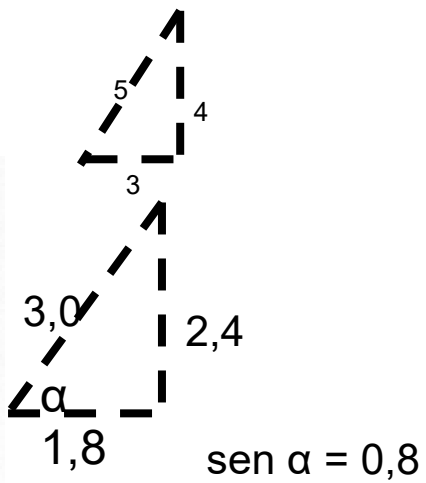
Nós A e D:



GRINTER

Nó D:





sen α = 0,8
cos α = 0,6

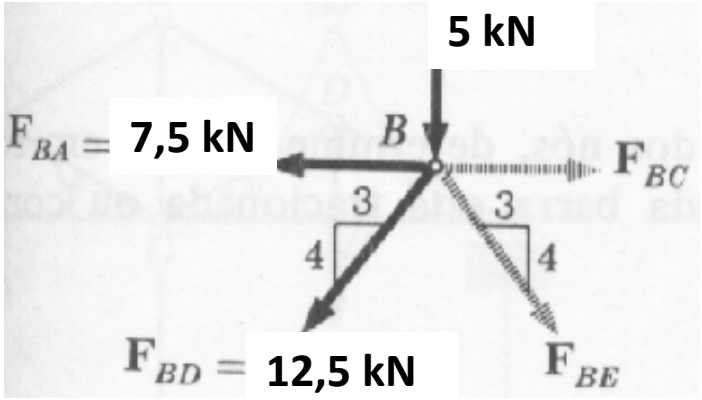
$$\Sigma F_y = 0 = -12,5 \,.\, 0,8 \; + 0,8.\, F_{DB}$$

$$\Sigma F_x = 0 = F_{DE} + 0,6 \,.\, F_{DB} + 0,6 \,.\, 12,5$$

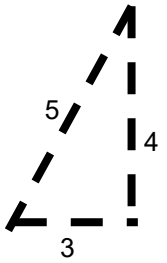
F_{DB} = +12,5 kN (Tração no nó e na barra)

F_{DE} = - 15 kN (Compressão no nó e na barra)

Nó B:



GRINTER



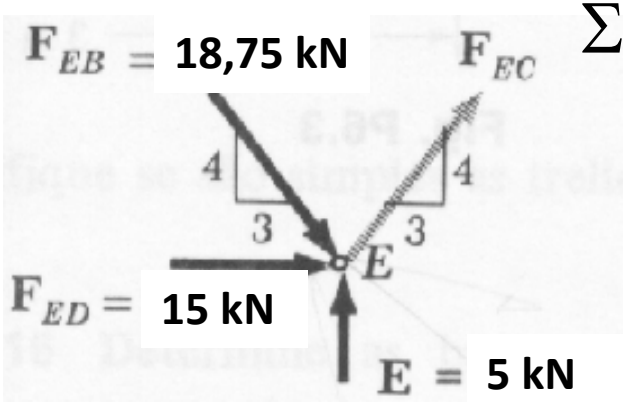
$$\Sigma F_Y = 0 = -5 - 0{,}8 \,.\, 12{,}5 - 0{,}8 \,.\, F_{BE}$$

$$\Longrightarrow F_{BE} = -18{,}75 \text{ kN (Compressão)}$$

$$\Sigma F_X = 0 = F_{BC} - 7{,}5 - 0{,}6 \,.\, 12{,}5 - 0{,}6 \,.\, 18{,}75$$

$$\Longrightarrow F_{BC} = 26{,}25 \text{ kN (Tração)}$$

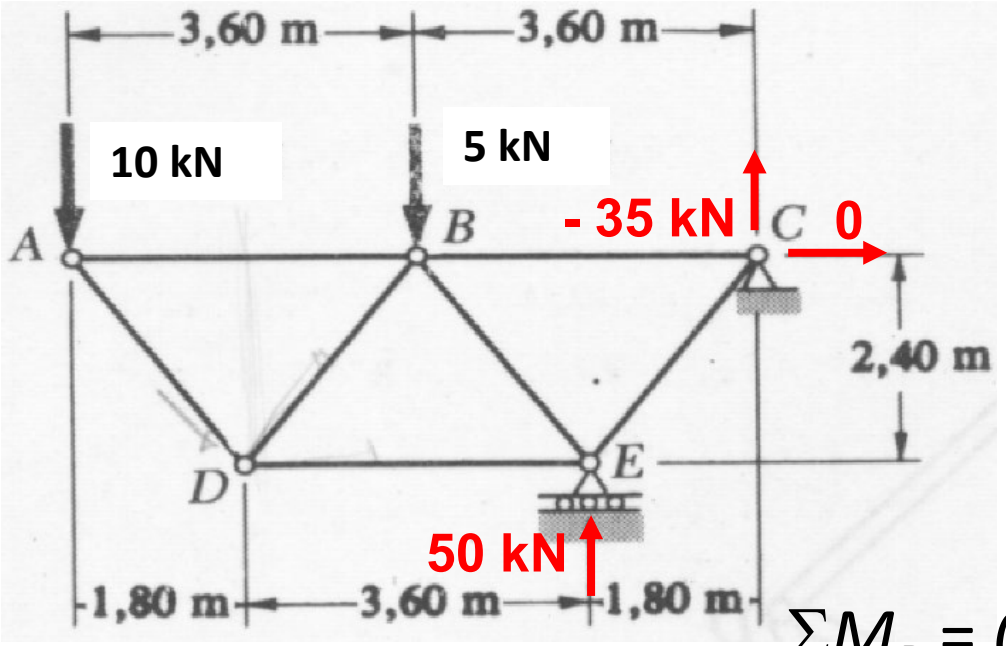
Nó E:



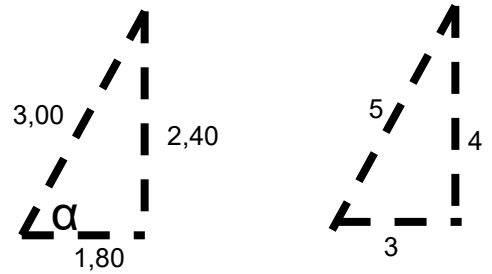
$$\Sigma F_X = 0 = 0{,}6 \,.\, F_{EC} + 15 + 0{,}6 \,.\, 18{,}75$$

$$\Longrightarrow F_{EC} = -43{,}75 \text{ kN (Compressão)}$$

4. Método de Ritter



GRINTER

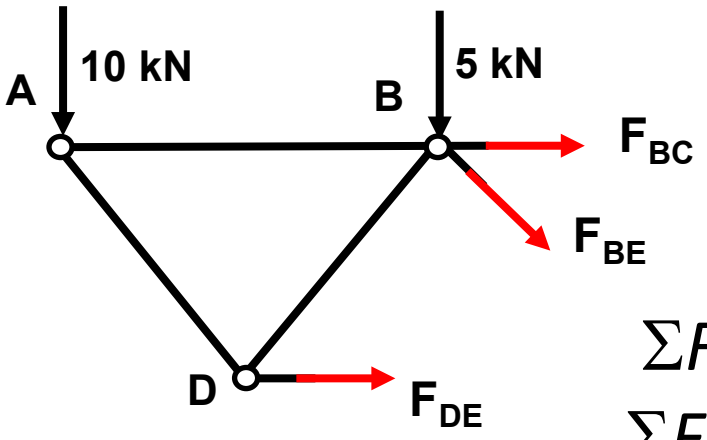


sen α = 0,8

cos α = 0,6

$\Sigma M_B = 0 = 10kN.3,60m + F_{DE}.2,40m$

$\Longrightarrow F_{DE} = -15\ kN$



$\Sigma F_y = 0 = -10kN - 5kN - F_{BE}.0,8$

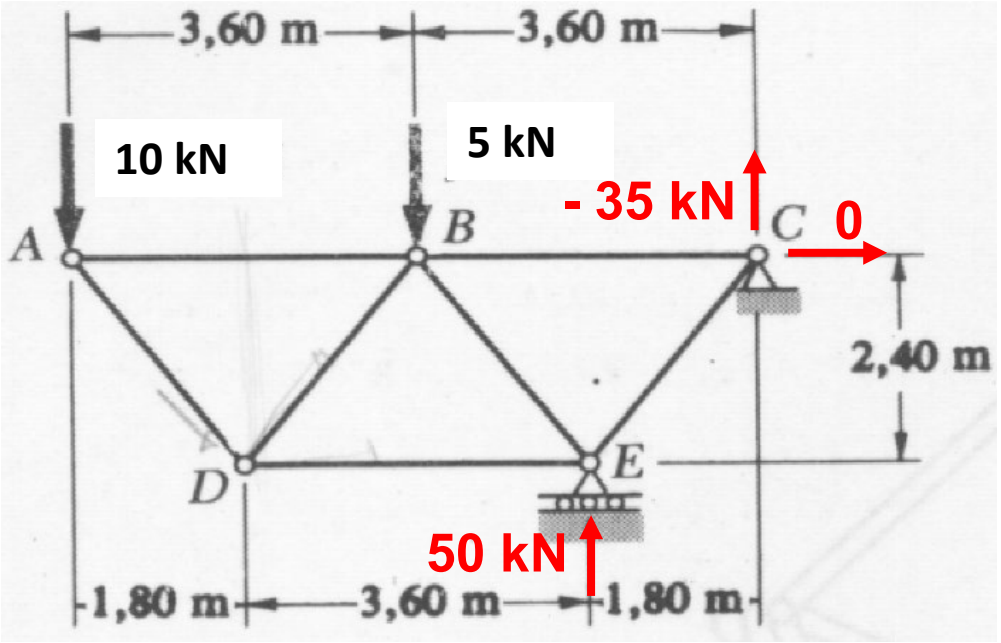
$\Longrightarrow F_{BE} = -18,75\ kN$

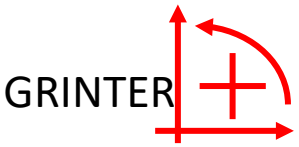
$\Sigma F_x = 0 = F_{DE} + F_{BC} + 0,6\ .\ F_{BE}$

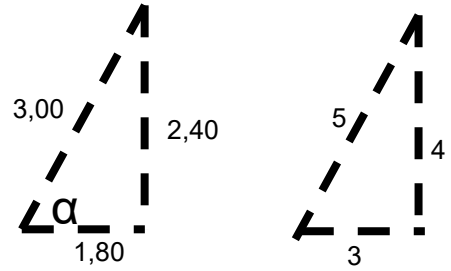
$\Sigma F_x = 0 = -15\ kN + F_{BC} + 0,6.\ (-18,75\ kN)$

$\Longrightarrow F_{BC} = 26,25\ kN$

4. Método de Ritter

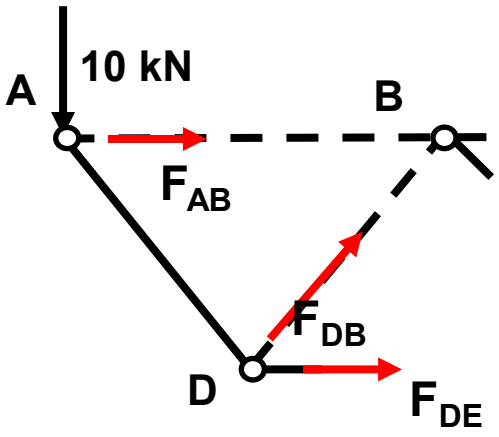






sen α = 0,8

cos α = 0,6



$$\Sigma M_D = 0 = 10kN.1,80m - F_{AB}.2,40m$$

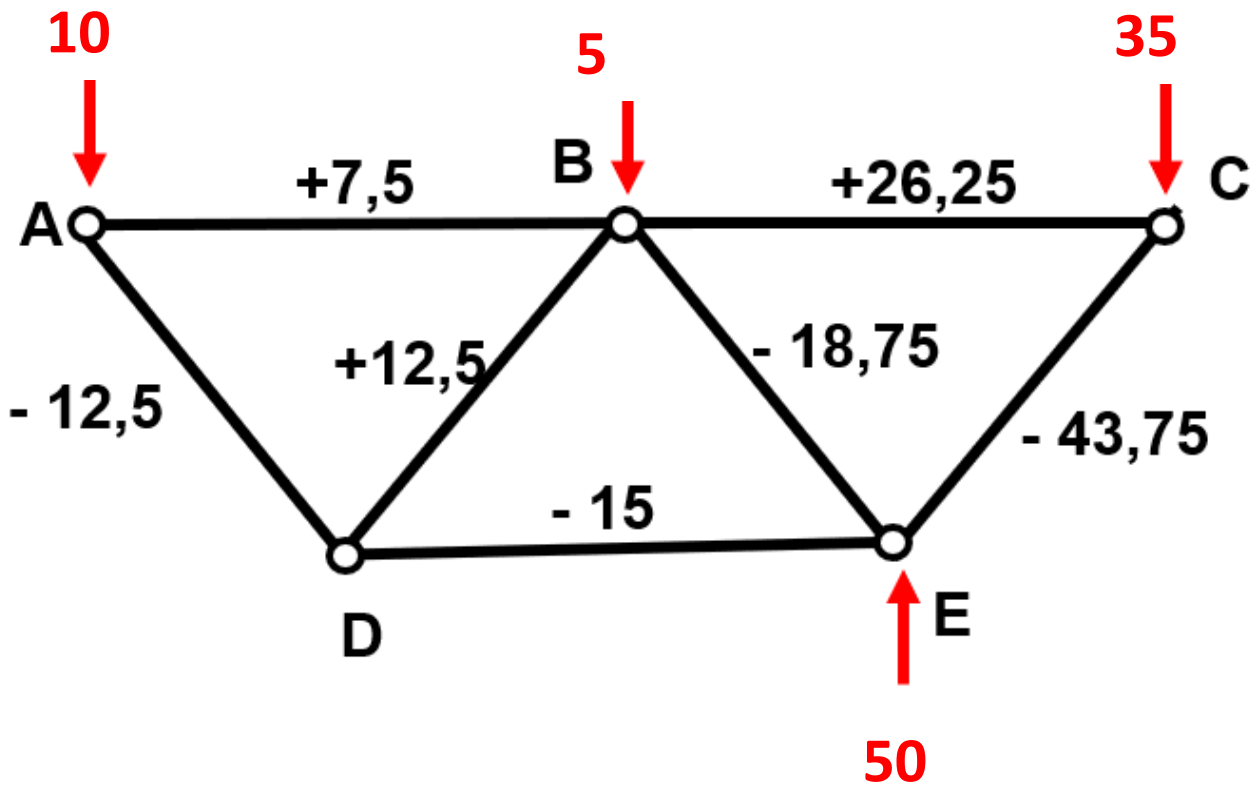
$$\Longrightarrow F_{AB} = 7,5\ kN$$

$$\Sigma F_y = 0 = -10kN + F_{DB}.0,8$$

$$\Longrightarrow F_{DB} = 12,5\ kN$$

$$\Sigma M_B = 0 = 10kN.3,60m + F_{DE}.2,40m$$

$$\Longrightarrow F_{DE} = -15\ kN$$


10
5
35
A
B
C
+7,5
+26,25
- 12,5
+12,5
- 18,75
- 43,75
- 15
D
E
50

Example (cantilever beam)

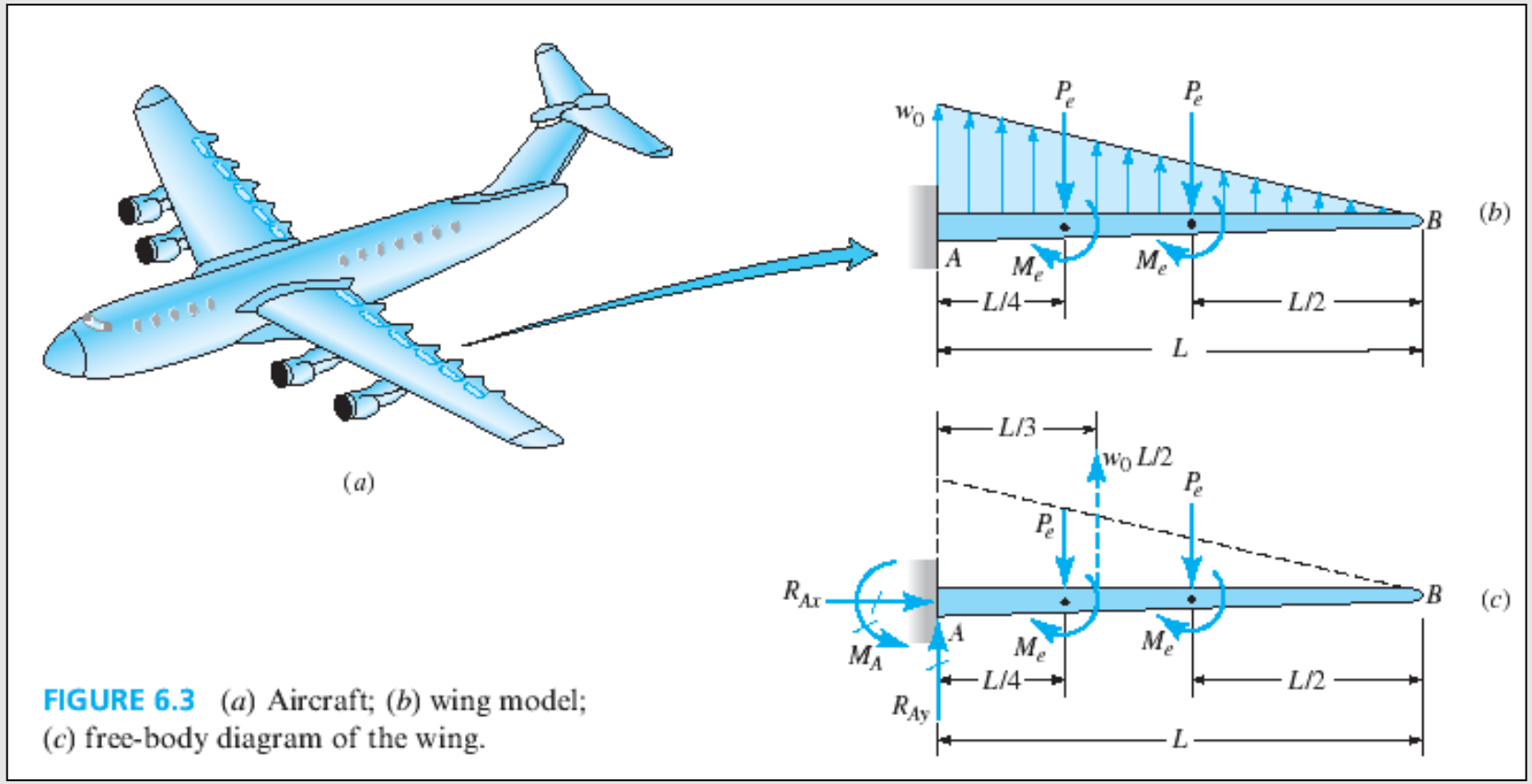


FIGURE 6.3 (*a*) Aircraft; (*b*) wing model; (*c*) free-body diagram of the wing.