Proper Orthogonal Decomposition - POD

PEF5737 - Non-linear dynamics and stability

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Outline

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Objective

To present an introduction to the Proper Orthogonal Decomposition (POD) method. Some examples are also discussed. This class is based in Feeny & Kappagantu (1998).

Outline

- POD is an empirical method for dynamic analysis and allows conclusions *a posteriori* about the investigated system;

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- Besides its use in dynamics of structures, POD is employed in flow analyses (turbulence), image processing among other applications. This class focuses on its use for vibration analyses.

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Investigated problem

- Consider the vibrations of a structure. Let M be the number of degrees of freedom (properly chosen) sampled at N time instants. The displacement on the m-th degree of freedom at t_n is $x_m(t_n)$;

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- Consider the vibrations of a structure. Let M be the number of degrees of freedom (properly chosen) sampled at N time instants. The displacement on the m-th degree of freedom at t_n is $x_m(t_n)$;
- We define the response matrix $\mathbf{X}$ as:

$$\mathbf{X} = \begin{pmatrix} x_1(t_0) & x_2(t_0) & x_3(t_0) & \dots & x_M(t_0) \\ x_1(t_1) & x_2(t_1) & x_3(t_1) & \dots & x_M(t_1) \\ x_1(t_2) & x_2(t_2) & x_3(t_2) & \dots & x_M(t_2) \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ x_1(t_N) & x_2(t_N) & x_3(t_N) & \dots & x_M(t_N) \end{pmatrix}$$

Correlation matrix

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- The correlation matrix $\mathbf{R} = \frac{1}{N}\mathbf{X}^t\mathbf{X}$.
- Provided $\mathbf{R}$ is symmetric and real, it can be put in the diagonal form. The eigenvalues of $\mathbf{R}$ are the proper orthogonal values (POVs) and the corresponding eigenvectors are the proper orthogonal modes (POMs).

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Geometric interpretation

Let $\mathbf{v}$ be a normalized POM. Consequently, $\mathbf{R}\mathbf{v} = \lambda\mathbf{v}$. Como $\mathbf{R} = \frac{1}{N}\mathbf{X}^t\mathbf{X}$, we obtain $(\mathbf{X}\mathbf{v})^t(\mathbf{X}\mathbf{v}) = \lambda N \leftrightarrow \frac{1}{N}(\mathbf{X}\mathbf{v})^t(\mathbf{X}\mathbf{v}) = \lambda$

Geometric interpretation

Each row of $\mathbf{X}$ can be interpreted as an snapshot (a "photograph"). Defining $\mathbf{p}_j$ as the snapshot at a particular instant t_j, $\mathbf{X}\mathbf{v}$ is given by:

$$\begin{bmatrix} \mathbf{v}^\mathsf{T}\mathbf{p_1} \\ \mathbf{v}^\mathsf{T}\mathbf{p_2} \\ \vdots \\ \mathbf{v}^\mathsf{T}\mathbf{p_N} \end{bmatrix}$$

Geometric interpretation

$\mathbf{Xv}$ can be interpreted as the projection of the experimental data onto $\mathbf{v}$. Consequently, λ plays the role of a mean squared distance from the origin. In mechanical systems, this distance is associated with energy.

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Formulação

Undamped system under free vibrations: $\mathbf{M}\ddot{\mathbf{x}} + \mathbf{K}\mathbf{x} = 0$. The response can be written as a function of the natural modes v_i by means of:

$$\mathbf{x}(t) = \sum_{i=1}^{M} A_i \sin(\omega_i t - \phi_i)\mathbf{v_i}$$

where A_i and ϕ_i depend on the initial condition.

- As showed in the class, an eigenvector of $\mathbf{R}$ (POM) converges to a modal vector. This is valid for low-damped systems.

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- POD can be used as an empiric scheme for determining the modal shape (a possible alternative do other methods such as, for example, the Circle Adjust Method)

A brief note

For a harmonically forced system, the POM do not tend to the modal vectors. However, close to the resonance, in which one mode dominates the response and the corresponding POV is much large than the others, the associated POM is a good approximation for the excited mode.

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Definition

- A non-linear mode can be interpreted as an invariant manifold in the state-space;

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- Fenny & Kappagantu (1998) deal with synchronous non-linear modes.

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- POMs are an optimal linear representation for the non-linear normal modes;
- The POD technique can be used obtaining reduced-order models for non-linear systems

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Final remarks

- For several applications, POMs can be assumed to match the natural modes of a linear system. This allows the definition of the modal shapes in a quick way from experiments;

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- Even for forced vibrations, the modal shapes can be obtained from POD;

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- POD can be easily programmed and is a powerful toll for quick or complex analyses;

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- For synchronous non-linear normal modes, POMs consist of the best linear fit (based on energy criterion);

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- POD can be easily programmed and is a powerful toll for quick or complex analyses;
- For synchronous non-linear normal modes, POMs consist of the best linear fit (based on energy criterion);
- Feeny & Kappagantu suggest, as further works, investigations on the POD use for non-synchronous non-linear modes.

Bibliography

Feeny, B. F. & Kapagantu, R. On the physical interpretation of proper orthogonal modes in vibrations, Journal of Sound and Vibration, 211(4), p.607-616, 1998.