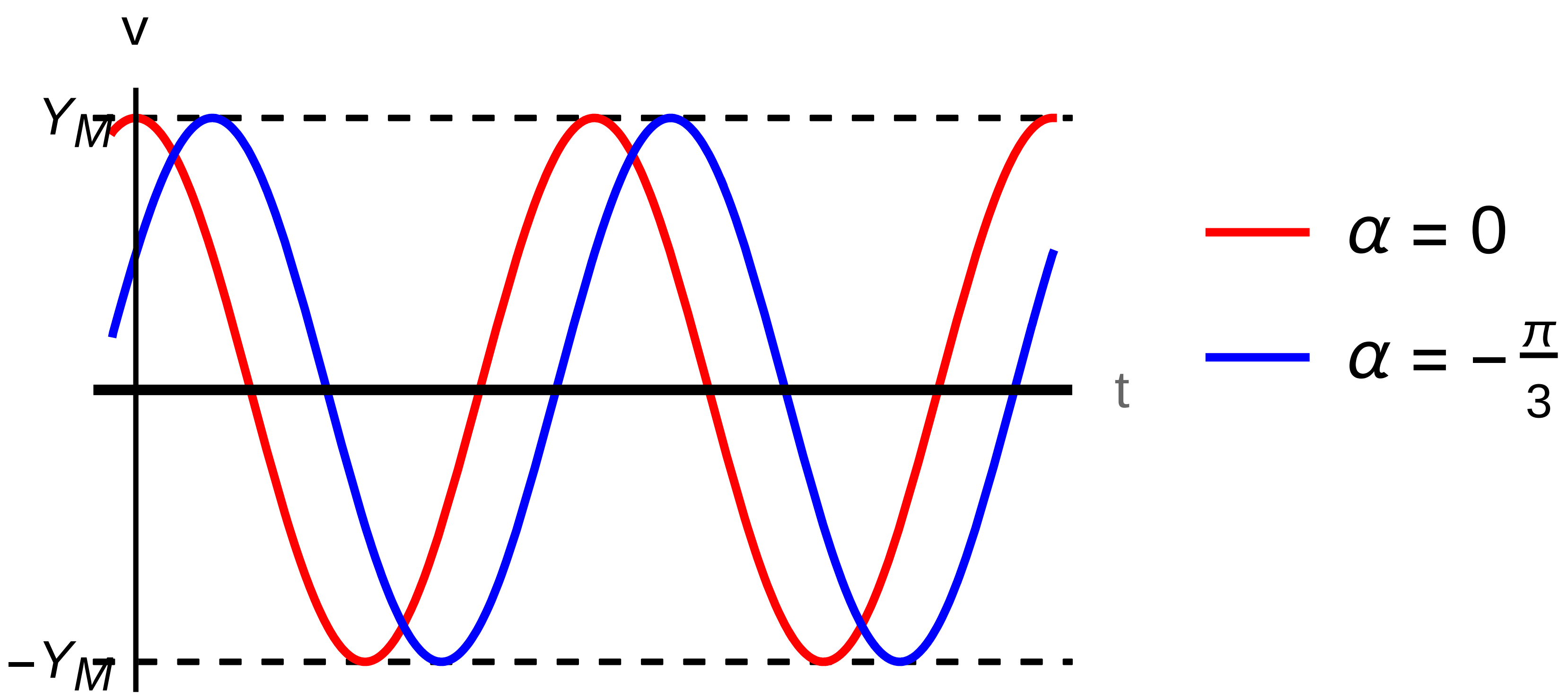
Circuitos CA

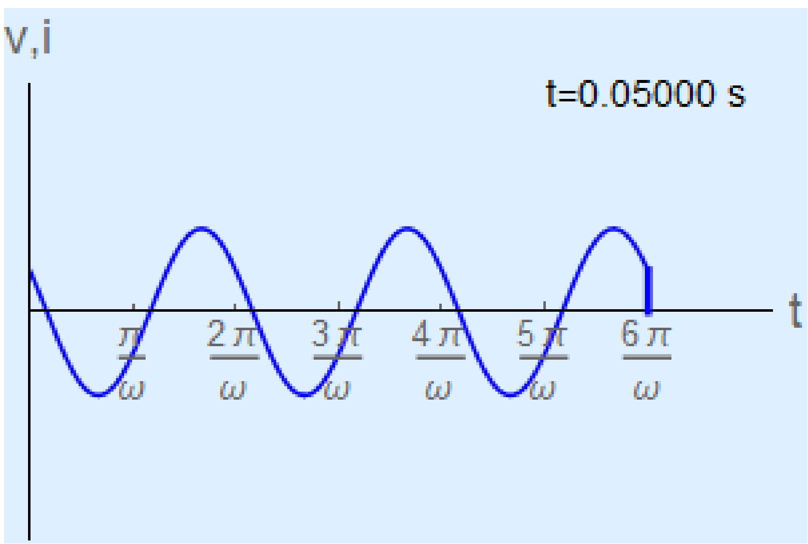
- Mais utilizados que circuitos CC
- conversão energia mecânica/elétrica (geração): geradores de usinas hidrelétricas e térmicas
- utilização: transformadores e motores de indução

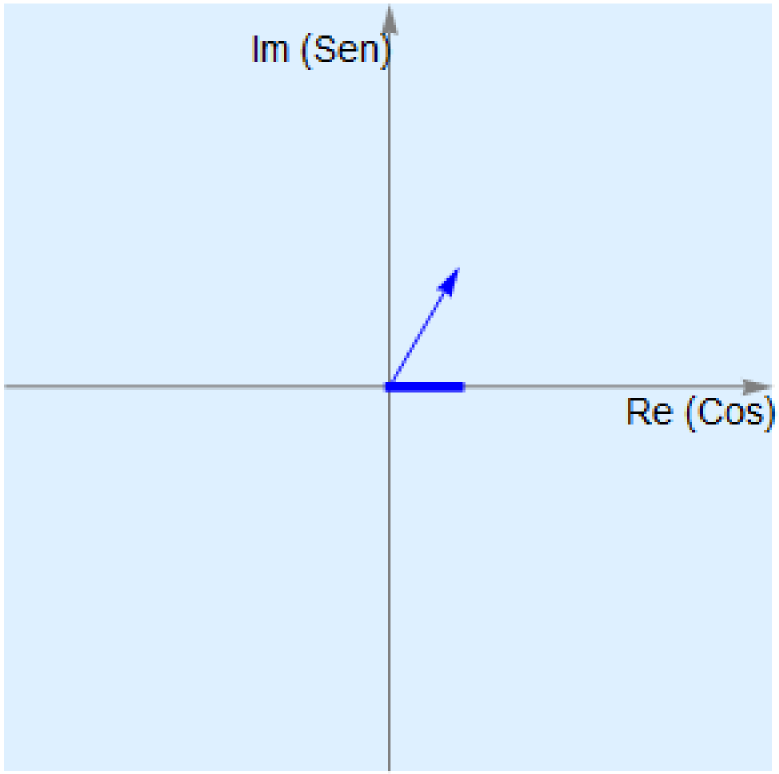
Grandeza cossenoidal

$v = Ym\cos(\omega t + \alpha)$



Representação de grandeza cossenoidal como vetor girante





Representação de grandezas cossenoidais

função no tempo $\Rightarrow$ $$v(t) = V_M \cos(\omega t + \theta)$$

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vetor girante $\Rightarrow$ $$\vec{V} = V_M\, e^{j\omega t} e^{j\theta}$$

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fasor $\Rightarrow$ $$\dot{V} = \frac{V_M}{\sqrt{2}} e^{j\theta} = \frac{V_M}{\sqrt{2}} \angle\theta$$

(pode ser representado por um número complexo)

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valor eficaz de uma grandeza cossenoidal $\Rightarrow$ $$\frac{V_M}{\sqrt{2}} = V_{ef}$$

Revisão números complexos

retangular

$$\bar{z} = a + jb = r\angle\theta$$

polar

$$a = r \cdot \cos\theta$$
$$b = r \cdot sen\theta$$

$\Longleftrightarrow$

$$r = \sqrt{a^2 + b^2}$$

$$\theta = \begin{cases} tg^{-1}\left(\dfrac{b}{a}\right), \text{se } a \leq 0 \;\; (1^{\text{o}} \text{ e } 4^{\text{o}} \text{ quadrantes}) \\ 180^0 + tg^{-1}\left(\dfrac{b}{a}\right), \text{se } a < 0 \;\; (2^{\text{o}} \text{ e } 3^{\text{o}} \text{ quadrantes}) \end{cases}$$

Conversão formas retangular $\leftrightarrow$ polar

Retangular	***Polar***
$3 + j4$	$5 \angle 53{,}13^{o}$
$3 - j4$	$5 \angle -53{,}13^{o}$
$-3 + j4$	$5 \angle 126{,}87^{o}$
$-3 - j4$	$5 \angle -126{,}87^{o}$

Operações com números complexos

Soma e subtração

- *forma retangular: somam-se ou subtraem-se as partes reais e imaginárias*

$$\bar{z}_1 = a_1 + jb_1$$

$$\bar{z}_2 = a_2 + jb_2$$

$$\bar{z}_3 = \bar{z}_1 + \bar{z}_2 = (a_1 + a_2) + j(b_1 + b_2)$$

$$\bar{z}_4 = \bar{z}_1 - \bar{z}_2 = (a_1 - a_2) + j(b_1 - b_2)$$

Operações com números complexos

Multiplicação

- *forma retangular: multiplicação termo a termo*
- *forma polar: coordenadas polares: multiplicam-se os módulos e somam-se os ângulos*

$$\bar{z}_1 = a_1 + jb_1 = r_1\angle\theta_1$$

$$\bar{z}_2 = a_2 + jb_2 = r_2\angle\theta_2$$

$$\bar{z}_5 = \bar{z}_1 \cdot \bar{z}_2 = (a_1 a_2 - b_1 b_2) + j(a_1 b_2 + a_2 b_1)$$

$$\bar{z}_5 = (r_1 r_2 \angle(\theta_1 + \theta_2))$$

Operações com números complexos

Divisão

forma retangular: artifício do conjugado do denominador

forma polar: dividem-se os módulos e subtraem-se as fases

$$\bar{z}_1 = a_1 + jb_1 = r_1\angle\theta_1$$

$$\bar{z}_2 = a_2 + jb_2 = r_2\angle\theta_2$$

$$\bar{z}_6 = \bar{z}_1 \div \bar{z}_2 = \frac{(a_1 + jb_1)(a_2 - jb_2)}{(a_2 + jb_2)(a_2 - jb_2)} = \frac{a_1 a_2 + b_1 b_2}{a_2^{\ 2} + b_2^{\ 2}} + j\,\frac{a_2 b_1 - a_1 b_2}{a_2^{\ 2} + b_2^{\ 2}}$$

$$\bar{z}_6 = (r_1 / r_2\angle(\theta_1 - \theta_2))$$

Operações com números complexos

Complexo conjugado

forma retangular: parte imaginária com sinal invertido

forma polar: ângulo com sinal invertido

$$\bar{z}_1 = a_1 + jb_1 = r_1\angle\theta_1$$
$$\bar{z}_1^{\,*} = a_1 - jb_1 = r_1\angle-\theta_1$$

Dicas:

$$\boxed{\frac{1}{\bar{z}_1} = \frac{1}{r_1}\angle-\theta_1}$$

$$\boxed{\frac{1}{j} = -j}$$

$$\boxed{\begin{aligned} -\bar{z}_1 &= r_1\angle(\theta_1 + 180^o) \\ &= r_1\angle(\theta_1 - 180^o) \end{aligned}}$$

ajustar ângulo para: $\boxed{-180^o < \theta < 180^o}$

Conceitos de circuitos CC são válidos...

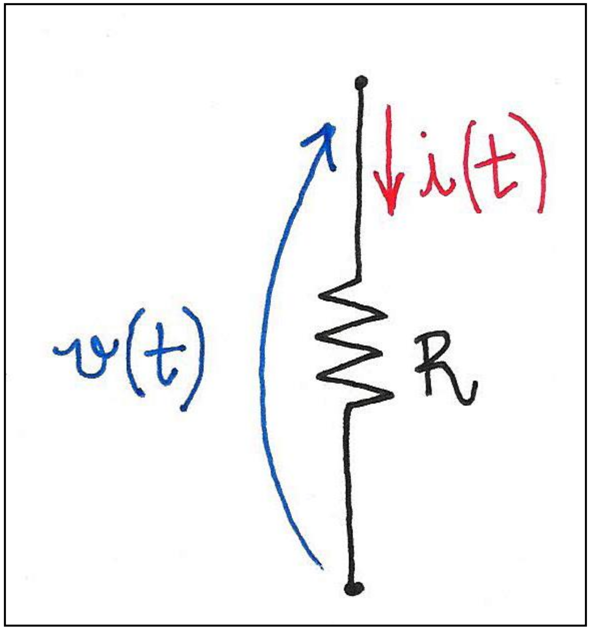
- Leis de Ohm e Kirchoff

$$v(t) = R\,i(t)$$

$$\text{PLK no nó k}: \sum i_k(t) = 0 \qquad \text{SLK na malha m}: \sum v_m(t) = 0$$

- Convenção de sentido da fonte / carga
- Associações série, paralelo, $\Delta \leftrightarrow Y$
- Técnicas de resolução
  - Conversão de fontes
  - Análise de malhas
  - Correntes de Maxwell
  - Superposição

Resistência em circuitos CA

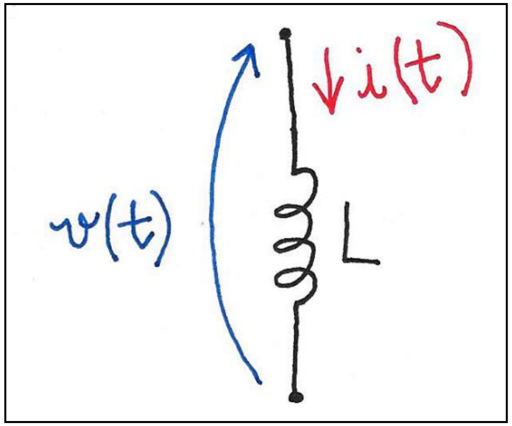


$$v(t) = V_m \cos(\omega t + \theta)$$

$$v(t) = R\, i(t)$$

$$\boxed{\dot{V} = R\, \dot{I}}$$

Indutância em circuitos CA



$$v(t) = V_M \cos(\omega t + \theta)$$

$$v(t) = L \frac{di(t)}{dt} \qquad i(t) = \int \frac{v(t)}{L} dt$$

$$i(t) = \int \frac{V_M}{L} \cos(\omega t + \theta)\, dt = \frac{V_M}{\omega L} \, sen(\omega t + \theta)$$

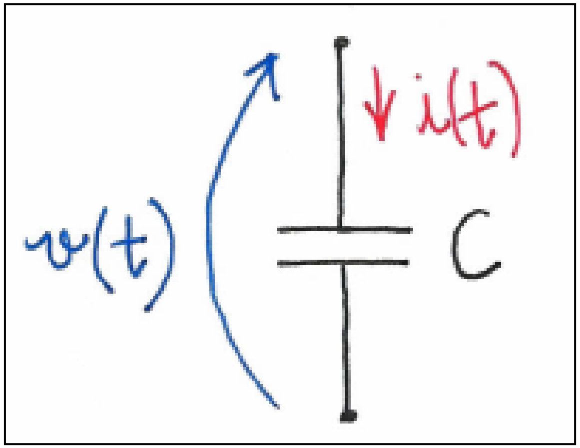
$$sen(x) = 0 \cdot \cos(x) + 1 \cdot sen(x)$$
$$= \cos(90^o) \cdot \cos(x) + sen(90^o) \cdot sen(x)$$
$$= \cos(x - 90^o)$$

$$i(t) = \frac{V_M}{\omega L} \cos(\omega t + \theta - 90^\circ)$$

$$\dot{I} = \frac{V_M}{\sqrt{2}\,\omega L} \angle (\theta - 90^\circ) = \frac{\left(\frac{V_M}{\sqrt{2}} \angle \theta\right)(1 \angle -90^\circ)}{\omega L} = \frac{\left(\frac{V_M}{\sqrt{2}} \angle \theta\right)}{(\omega L \angle 90^\circ)}$$

$$\boxed{\dot{I} = \frac{\dot{V}}{j\omega L}}$$

Capacitância em circuitos CA



$$v(t) = V_m \cos(\omega t + \theta)$$

$$q(t) = C\,v(t) \qquad i(t) = \frac{d}{dt} q(t) = C\frac{dv(t)}{dt}$$

$$i(t) = C\frac{d}{dt}\left(V_m \cos(\omega t + \theta)\right) = C\,V_m\,\omega\left(-\mathrm{sen}(\omega t + \theta)\right)$$

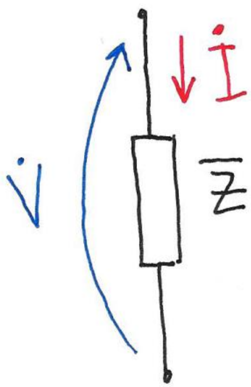
$$i(t) = +\omega C\,V_m \cos(\omega t + \theta + 90^\circ)$$

$$\begin{aligned} -sen(x) &= 0\cdot\cos(x) - 1\cdot sen(x) \\ &= \cos(90^o)\cdot\cos(x) - sen(90^o)\cdot sen(x) \\ &= \cos(x + 90^o) \end{aligned}$$

$$\dot{I} = \frac{\omega C V_m}{\sqrt{2}} \angle(\theta + 90^\circ) = \left(\frac{V_m}{\sqrt{2}}\angle\theta\right)\left(\omega C \angle 90^\circ\right) = \frac{\left(\frac{V_m}{\sqrt{2}}\angle\theta\right)}{\frac{1}{\omega C}\angle -90^\circ}$$

$$\boxed{\dot{I} = \frac{\dot{V}}{\frac{-j}{\omega C}}}$$

Impedâncias



$$\dot{I} = \frac{\dot{V}}{\bar{Z}}$$

$\bar{Z}$ é um número complexo, não um fasor!

R $\longrightarrow \bar{Z} = R \quad [\Omega]$

L $\longrightarrow \bar{Z} = j\omega L \quad [\Omega]$

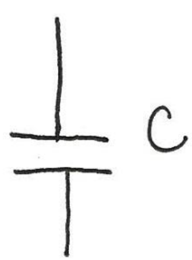
C $\longrightarrow \bar{Z} = \dfrac{-j}{\omega C} \quad [\Omega]$

Reatâncias

- Reatância indutiva

$$X_L = \omega L \quad \rightarrow \quad Z = jX_L$$

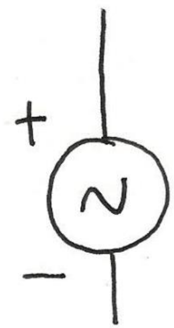
- Reatância capacitiva



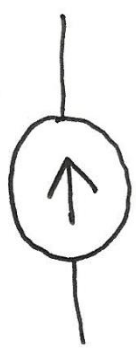
$$X_C = \frac{1}{\omega C} \quad \rightarrow \quad Z = -jX_C$$

Fontes CA

- Fonte de tensão CA ideal



- Fonte de corrente CA ideal



Associações de impedâncias

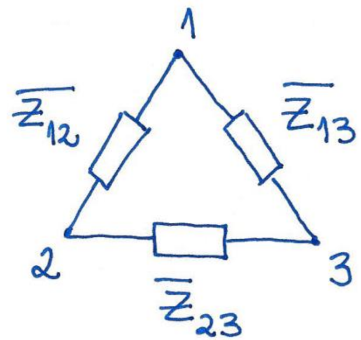
- Série

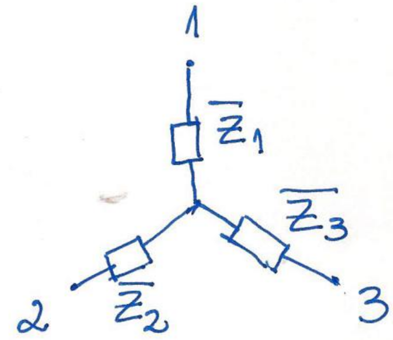
$$\overline{Z}_{eq} = \overline{Z}_1 + \overline{Z}_2 + \dots + \overline{Z}_m$$

- Paralelo

$$\overline{Z}_{eq} = \frac{1}{\frac{1}{\overline{Z}_1} + \frac{1}{\overline{Z}_2} + \dots + \frac{1}{\overline{Z}_n}}$$

Transformação $\Delta \leftrightarrow Y$





$$\bar{Z}_{12} = \frac{\bar{Z}_1 \bar{Z}_2 + \bar{Z}_1 \bar{Z}_3 + \bar{Z}_2 \bar{Z}_3}{\bar{Z}_3}$$

$$\bar{Z}_{13} = \frac{\bar{Z}_1 \bar{Z}_2 + \bar{Z}_1 \bar{Z}_3 + \bar{Z}_2 \bar{Z}_3}{\bar{Z}_2}$$

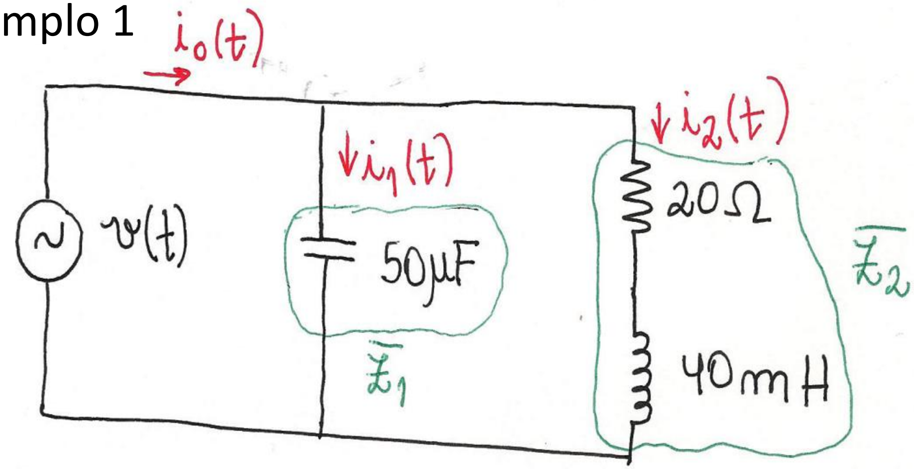
$$\bar{Z}_{23} = \frac{\bar{Z}_1 \bar{Z}_2 + \bar{Z}_1 \bar{Z}_3 + \bar{Z}_2 \bar{Z}_3}{\bar{Z}_3}$$

$$\bar{Z}_1 = \frac{\bar{Z}_{12} \, \bar{Z}_{13}}{\bar{Z}_{12} + \bar{Z}_{13} + \bar{Z}_{23}}$$

$$\bar{Z}_2 = \frac{\bar{Z}_{12} \, \bar{Z}_{23}}{\bar{Z}_{12} + \bar{Z}_{13} + \bar{Z}_{23}}$$

$$\bar{Z}_3 = \frac{\bar{Z}_{13} \, \bar{Z}_{23}}{\bar{Z}_{12} + \bar{Z}_{13} + \bar{Z}_{23}}$$

Exemplo 1



$$v(t) = 120 \operatorname{sen}(2\pi 60 t + 60°)\,V$$

$i_0(t), i_1(t), i_2(t) = ?$

$$\dot{V} = \frac{120}{\sqrt{2}} \angle (60° - 90°) = 84{,}853 \angle -30°\ V$$

$$\overline{Z}_1 = \frac{-j}{2\pi 60 \cdot 50 \cdot 10^{-6}} = -j\,53{,}052\,\Omega$$

$$\overline{Z}_2 = 20 + j2\pi 60 \cdot 40 \cdot 10^{-3} = 20 + j\,15{,}080\,\Omega$$

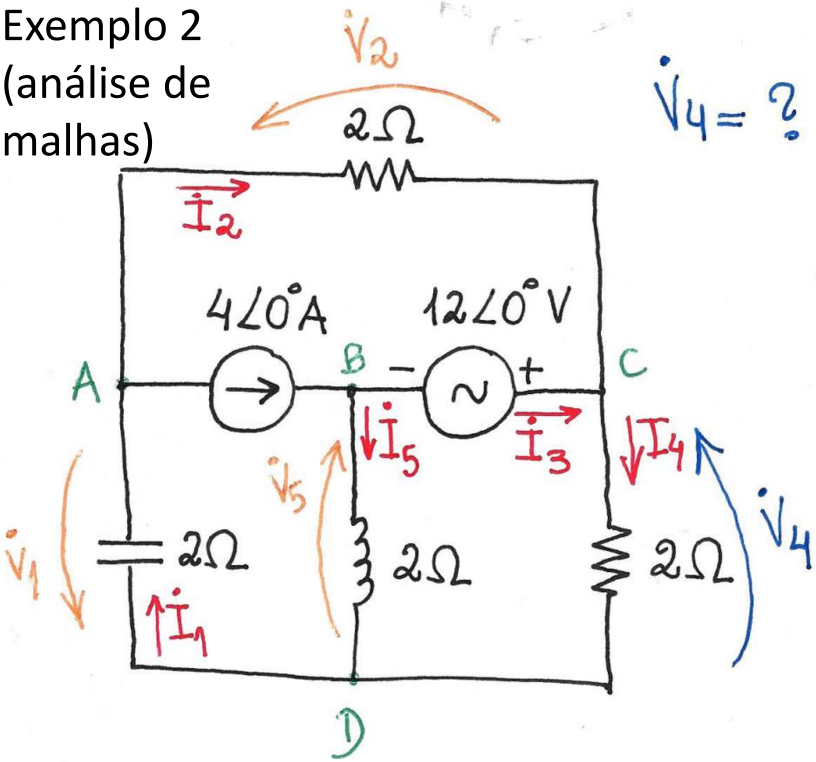
$$\dot{I}_1 = \frac{\dot{V}}{\overline{Z}_1} = \frac{84{,}853 \angle -30°}{-j53{,}052} = \frac{84{,}853 \angle -30°}{53{,}052 \angle -90} = 1{,}599 \angle 60° = 0{,}800 + j\,1{,}385\ A$$

$$\dot{I}_2 = \frac{\dot{V}}{\overline{Z}_2} = \frac{84{,}853 \angle -30°}{20 + j\,15{,}080} = \frac{84{,}853 \angle -30°}{25{,}048 \angle 37{,}02°} = 3{,}388 \angle -67{,}02° = 1{,}323 - j\,3{,}119\ A$$

$$\dot{I}_0 = \dot{I}_1 + \dot{I}_2$$
$$= 2{,}123 - j\,1{,}734 = 2{,}741 \angle -39{,}24°\ A$$

$$\boxed{\begin{aligned} i_0(t) &= 3{,}876 \cos(2\pi 60 t - 39{,}24°)\ A \\ i_1(t) &= 2{,}261 \cos(2\pi 60 t + 60°)\ A \\ i_2(t) &= 4{,}791 \cos(2\pi 60 t - 67{,}02°)\ A \end{aligned}}$$

Exemplo 2
(análise de malhas)



$\dot{V}_4 = ?$

PLK:

$$\dot{I}_1 - (4\angle 0^\circ) - \dot{I}_2 = 0$$
$$(4\angle 0^\circ) - \dot{I}_3 - \dot{I}_5 = 0$$
$$\dot{I}_2 + \dot{I}_3 - \dot{I}_4 = 0$$

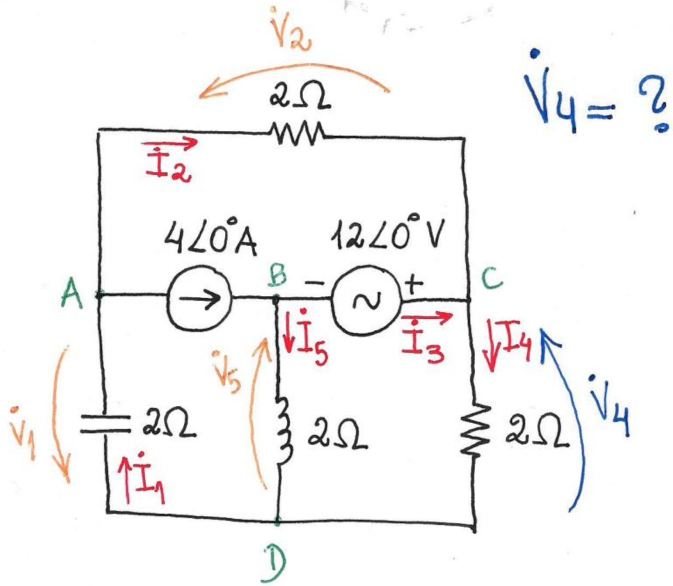
SLK:

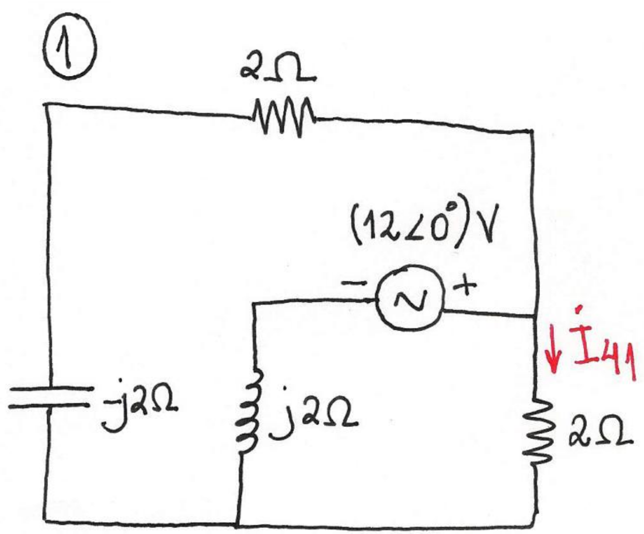
$$-2j\,\dot{I}_5 - (12\angle 0^\circ) + 2\,\dot{I}_4 = 0$$
$$-2j\,\dot{I}_1 + 2\,\dot{I}_2 + (12\angle 0^\circ) + 2j\,\dot{I}_5 = 0$$

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & -1 \\ 0 & 1 & 1 & -1 & 0 \\ 0 & 0 & 0 & 2 & -j2 \\ -j2 & 2 & 0 & 0 & j2 \end{bmatrix} \begin{bmatrix} \dot{I}_1 \\ \dot{I}_2 \\ \dot{I}_3 \\ \dot{I}_4 \\ \dot{I}_5 \end{bmatrix} = \begin{bmatrix} 4\angle 0^\circ \\ -(4\angle 0^\circ) \\ 0 \\ 12\angle 0^\circ \\ -(12\angle 0^\circ) \end{bmatrix} \Rightarrow \begin{bmatrix} \dot{I}_1 \\ \dot{I}_2 \\ \dot{I}_3 \\ \dot{I}_4 \\ \dot{I}_5 \end{bmatrix} = \begin{bmatrix} j2 \\ -4+j2 \\ 6-j4 \\ 2-j2 \\ -2+j4 \end{bmatrix} A = \begin{bmatrix} 2{,}000\angle 90^\circ \\ 4{,}472\angle 153{,}43^\circ \\ 7{,}211\angle -33{,}69^\circ \\ 2{,}828\angle -45^\circ \\ 4{,}472\angle 116{,}56^\circ \end{bmatrix} A$$

$$\dot{V}_4 = 2\,\dot{I}_4 = 2\,(2{,}828\angle -45^\circ) = (5{,}656\angle -45^\circ)\,V$$

Exemplo 2 (superposição/conversão de fontes)





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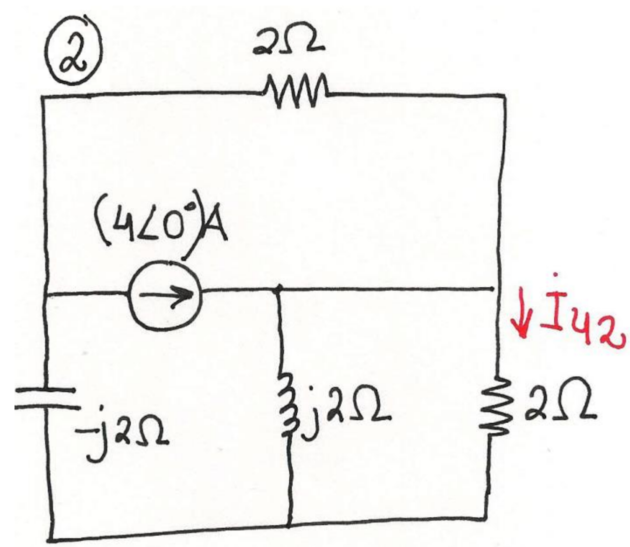

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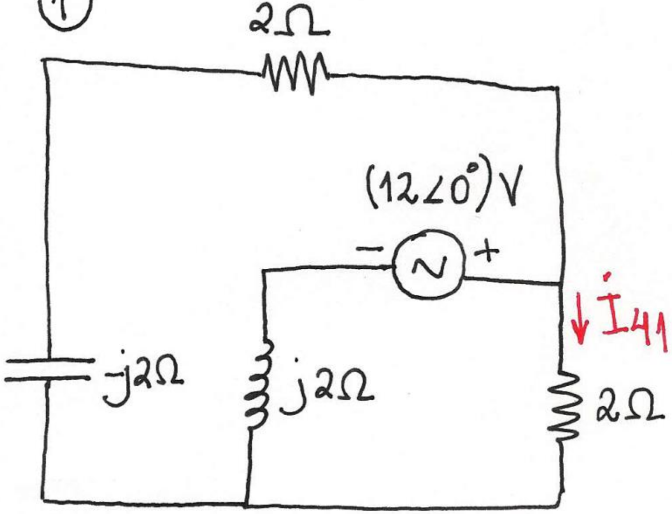
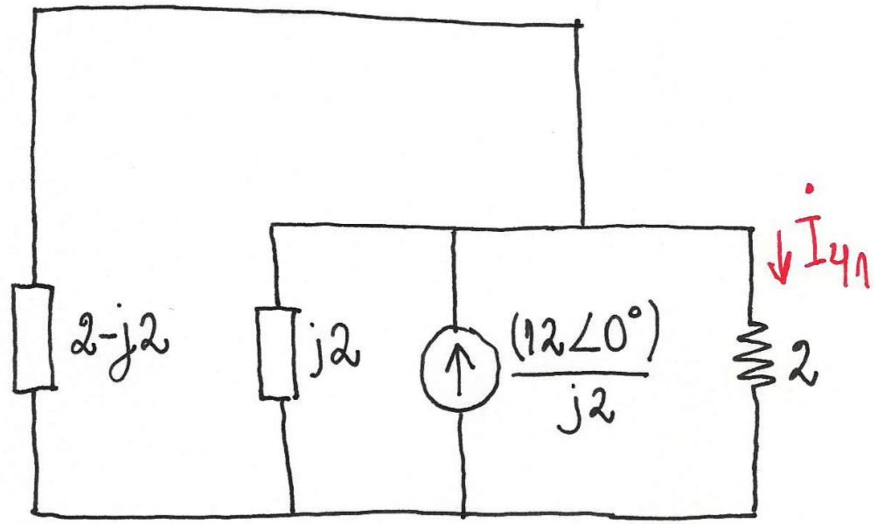
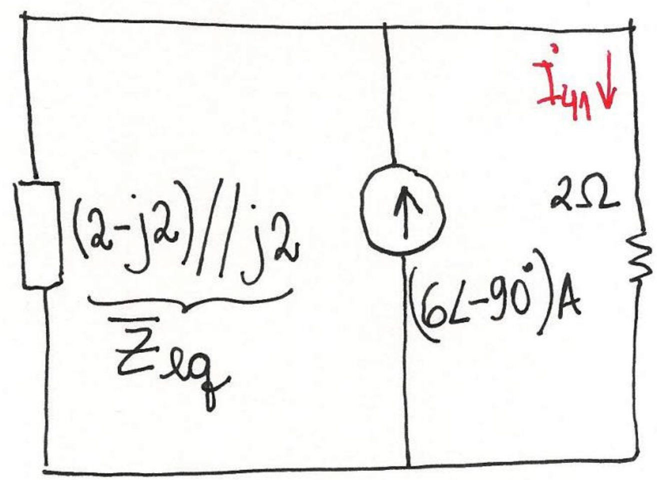
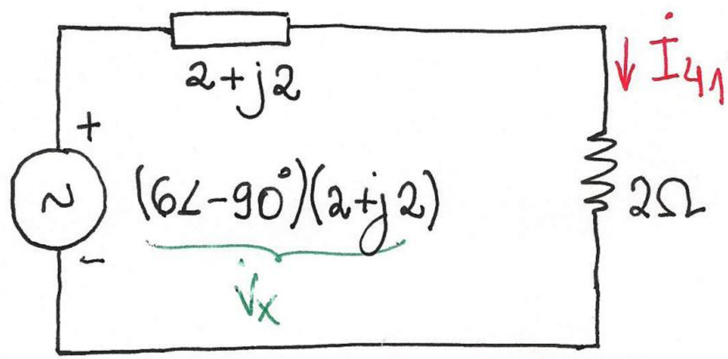








$$\dot{I}_{41} = \frac{\dot{V}_x}{(2+j2)+2} =$$

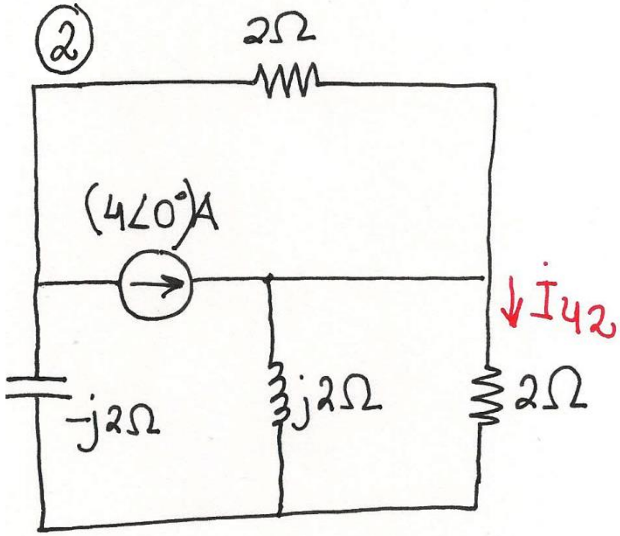
$$\dot{I}_{41} = \frac{16{,}968\angle -45^\circ}{4{,}472\angle 26{,}56^\circ}$$

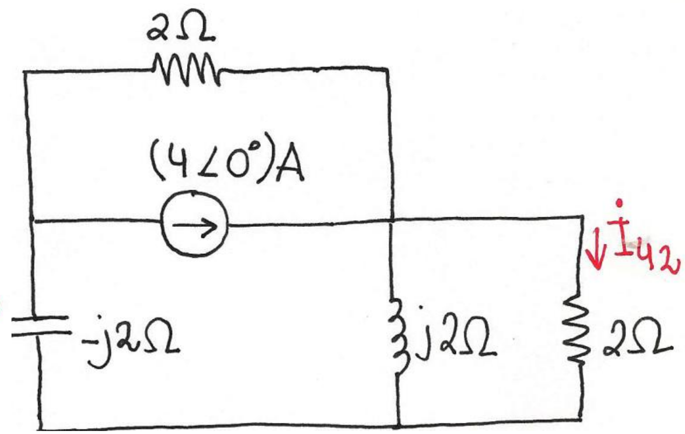
$$\dot{V}_x = (6\angle -90^\circ)(2{,}828\angle 45^\circ)\,V$$

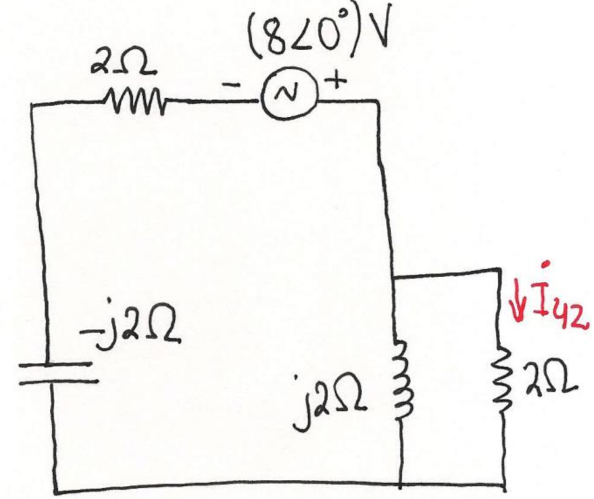
$$= (16{,}968\angle -45^\circ)\,V$$

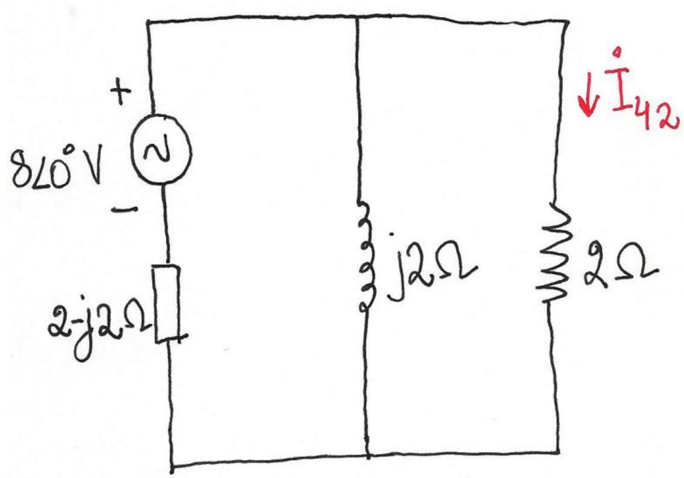
$$\bar{Z}_{eq} = \frac{j2(2-j2)}{2-j2+j2} = \frac{4+j4}{2} = 2+j2$$

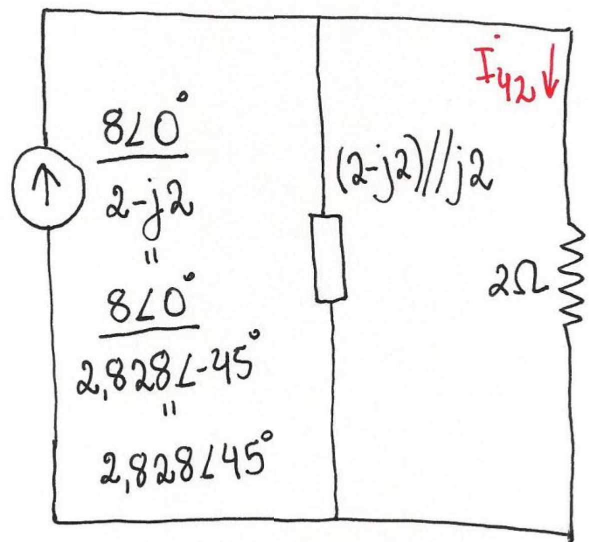
$$\boxed{\begin{aligned}\dot{I}_{41} &= (3{,}795\angle -71{,}56^\circ)A \\ &= (1{,}2 - j3{,}6)A\end{aligned}}$$

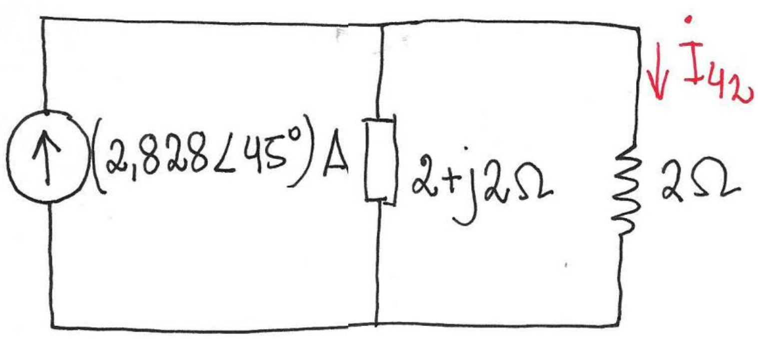


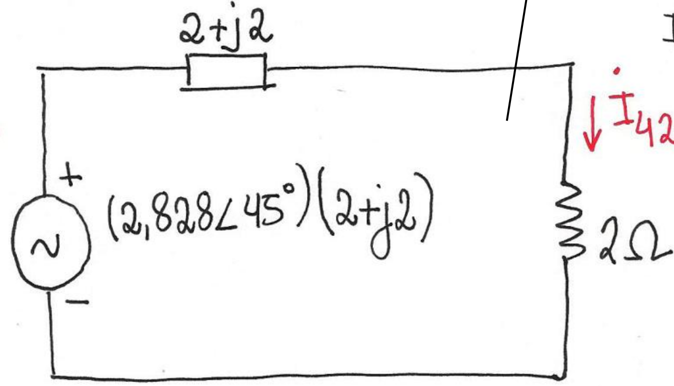












$$\dot{I}_{42} = \frac{(2{,}828\angle 45^\circ)(2+j2)}{(2+j2)+2} = \frac{(2{,}828\angle 45^\circ)(2{,}828\angle 45^\circ)}{4{,}472\angle 26{,}56^\circ} =$$

$$\boxed{\begin{aligned}\dot{I}_{42} &= (1{,}789\angle 63{,}43^\circ)\,A \\ &= (0{,}800 + j1{,}600)\,A\end{aligned}}$$

$$\dot{I}_4 = \dot{I}_{41} + \dot{I}_{42} = (2 - j2)\,A$$

$$\boxed{\begin{aligned}\dot{V}_4 &= 2\,\dot{I}_4 = (4 - j4)\,V \\ &= (5{,}657\angle -45^\circ)\,V\end{aligned}}$$